Gauss Elimination

Ques 1 2x+y+z=4,3x+5y+2z=15,2x+y+4z=8

```
In[189]:=  A = {{2, 1, 1}, {3, 5, 2}, {2, 1, 4}};
           A // MatrixForm
           b = {4, 15, 8};
           b // MatrixForm
           m1 = Length[A];
           m2 = Length[b];
           x = Table[0, {m1}];
           If[m1 ≠ m2, Print["The system cannot be solved"], Table[AppendTo[A[[i]], b[[i]]], {i, m1}];
           Print["[A|b]=", A // MatrixForm];
           For[i = 1, i ≤ m1 - 1, i++, s = Abs[A[[i, i]]];
           c = i;
           For[j = i + 1, j ≤ m1, j++, If[Abs[A[[j, i]]] > s, s = A[[j, i]];
           c = j;]];
           For[k = 1, k ≤ m1 + 1, k++, d[k] = A[[i, k]]; A[[i, k]] = A[[c, k]]; A[[c, k]] = d[k]];
           Print["Step=", i, A // MatrixForm];
           For[j = i + 1, j ≤ m1, j++, m = A[[j, i]] / A[[i, i]];
           For[k = 1, k ≤ m1 + 1, k++, A[[j, k]] = A[[j, k]] - (m * A[[i, k]])];];
           Print[A // MatrixForm];];
           For[i = 0, i ≤ m1 - 1, i++,
           x[[m1 - i]] = (A[[m1 - i, m1 + 1]] - Sum[A[[m1 - i, j]] * x[[j]], {j, m1 - i + 1, m1}]) / A[[m1 - i, m1 - i]];];
           Print["x = ", x // MatrixForm];]
```

Out[190]//MatrixForm=

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix}$$

Out[192]//MatrixForm=

$$\begin{pmatrix} 4 \\ 15 \\ 8 \end{pmatrix}$$

$$[A|b] = \begin{pmatrix} 2 & 1 & 1 & 4 \\ 3 & 5 & 2 & 15 \\ 2 & 1 & 4 & 8 \end{pmatrix}$$

$$\text{Step=1} \begin{pmatrix} 3 & 5 & 2 & 15 \\ 2 & 1 & 1 & 4 \\ 2 & 1 & 4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 5 & 2 & 15 \\ 0 & -\frac{7}{3} & -\frac{1}{3} & -6 \\ 0 & -\frac{7}{3} & \frac{8}{3} & -2 \end{pmatrix}$$

$$\text{Step=2} \begin{pmatrix} 3 & 5 & 2 & 15 \\ 0 & -\frac{7}{3} & -\frac{1}{3} & -6 \\ 0 & -\frac{7}{3} & \frac{8}{3} & -2 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 5 & 2 & 15 \\ 0 & -\frac{7}{3} & -\frac{1}{3} & -6 \\ 0 & 0 & 3 & 4 \end{pmatrix}$$

$$x = \begin{pmatrix} \frac{1}{7} \\ \frac{50}{21} \\ \frac{4}{3} \end{pmatrix}$$

Q2 2x1-x2+x3=5,2x1-3x2+x3=3,x1+3x2-4x3=4

```
In[205]:= A = {{2, -1, 1}, {2, -3, 1}, {1, 3, -4}};
A // MatrixForm
b = {5, 3, 4};
b // MatrixForm
m1 = Length[A];
m2 = Length[b];
x = Table[0, {m1}];
If[m1 ≠ m2, Print["The system cannot be solved"], Table[AppendTo[A〚i〛, b〚i〛], {i, m1}];
Print["[A|b]=", A // MatrixForm];
For[i = 1, i ≤ m1 - 1, i++, s = Abs[A〚i, i〛];
c = i;
For[j = i + 1, j ≤ m1, j++, If[Abs[A〚j, i〛] > s, s = A〚j, i〛;
c = j;]];
For[k = 1, k ≤ m1 + 1, k++, d[k] = A〚i, k〛; A〚i, k〛 = A〚c, k〛; A〚c, k〛 = d[k]];
Print["Step=", i, A // MatrixForm];
For[j = i + 1, j ≤ m1, j++, m = A〚j, i〛 / A〚i, i〛;
For[k = 1, k ≤ m1 + 1, k++, A〚j, k〛 = A〚j, k〛 - (m * A〚i, k〛)];];
Print[A // MatrixForm];];
For[i = 0, i ≤ m1 - 1, i++,
x〚m1 - i〛 = (A〚m1 - i, m1 + 1〛 - Sum[A〚m1 - i, j〛 * x〚j〛, {j, m1 - i + 1, m1}]) / A〚m1 - i, m1 - i〛;];
Print["x = ", x // MatrixForm];]
```

Out[206]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 1 \\ 2 & -3 & 1 \\ 1 & 3 & -4 \end{pmatrix}$$

Out[208]//MatrixForm=

$$\begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$$

$[A|b] = \begin{pmatrix} 2 & -1 & 1 & 5 \\ 2 & -3 & 1 & 3 \\ 1 & 3 & -4 & 4 \end{pmatrix}$

$\text{Step} = 1 \begin{pmatrix} 2 & -1 & 1 & 5 \\ 2 & -3 & 1 & 3 \\ 1 & 3 & -4 & 4 \end{pmatrix}$

$$\begin{pmatrix} 2 & -1 & 1 & 5 \\ 0 & -2 & 0 & -2 \\ 0 & \frac{7}{2} & -\frac{9}{2} & \frac{3}{2} \end{pmatrix}$$

$\text{Step} = 2 \begin{pmatrix} 2 & -1 & 1 & 5 \\ 0 & \frac{7}{2} & -\frac{9}{2} & \frac{3}{2} \\ 0 & -2 & 0 & -2 \end{pmatrix}$

$$\begin{pmatrix} 2 & -1 & 1 & 5 \\ 0 & \frac{7}{2} & -\frac{9}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{18}{7} & -\frac{8}{7} \end{pmatrix}$$

$x = \begin{pmatrix} \frac{25}{9} \\ 1 \\ \frac{4}{9} \end{pmatrix}$

Q3 3x1 - 6x2 + 2x3 = 14 , 4x1 - x2 + x3 = 2 , x1 - 3x2 + 7x3 = 22

```
In[213]:=  A = {{3, -6, 2}, {4, -1, 1}, {1, -3, 7}};
           A // MatrixForm
           b = {14, 2, 22};
           b // MatrixForm
           m1 = Length[A];
           m2 = Length[b];
           x = Table[0, {m1}];
           If[m1 ≠ m2, Print["The system cannot be solved"], Table[AppendTo[A〚i〛, b〚i〛], {i, m1}]];
           Print["[A|b]=", A // MatrixForm];
           For[i = 1, i ≤ m1 - 1, i++, s = Abs[A〚i, i〛];
           c = i;
           For[j = i + 1, j ≤ m1, j++, If[Abs[A〚j, i〛] > s, s = A〚j, i〛];
           c = j;]];
           For[k = 1, k ≤ m1 + 1, k++, d[k] = A〚i, k〛; A〚i, k〛 = A〚c, k〛; A〚c, k〛 = d[k]];
           Print["Step=", i, A // MatrixForm];
           For[j = i + 1, j ≤ m1, j++, m = A〚j, i〛 / A〚i, i〛;
           For[k = 1, k ≤ m1 + 1, k++, A〚j, k〛 = A〚j, k〛 - (m * A〚i, k〛)];];
           Print[A // MatrixForm];];
           For[i = 0, i ≤ m1 - 1, i++,
           x〚m1 - i〛 = (A〚m1 - i, m1 + 1〛 - Sum[A〚m1 - i, j〛 * x〚j〛, {j, m1 - i + 1, m1}]) / A〚m1 - i, m1 - i〛;];
           Print["x = ", x // MatrixForm];]
```

Out[214]//MatrixForm=

$$\begin{pmatrix} 3 & -6 & 2 \\ 4 & -1 & 1 \\ 1 & -3 & 7 \end{pmatrix}$$

Out[216]//MatrixForm=

$$\begin{pmatrix} 14 \\ 2 \\ 22 \end{pmatrix}$$

$$[A|b] = \begin{pmatrix} 3 & -6 & 2 & 14 \\ 4 & -1 & 1 & 2 \\ 1 & -3 & 7 & 22 \end{pmatrix}$$

$$\text{Step}=1 \begin{pmatrix} 4 & -1 & 1 & 2 \\ 3 & -6 & 2 & 14 \\ 1 & -3 & 7 & 22 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & 1 & 2 \\ 0 & -\frac{21}{4} & \frac{5}{4} & \frac{25}{2} \\ 0 & -\frac{11}{4} & \frac{27}{4} & \frac{43}{2} \end{pmatrix}$$

$$\text{Step}=2 \begin{pmatrix} 4 & -1 & 1 & 2 \\ 0 & -\frac{21}{4} & \frac{5}{4} & \frac{25}{2} \\ 0 & -\frac{11}{4} & \frac{27}{4} & \frac{43}{2} \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & 1 & 2 \\ 0 & -\frac{21}{4} & \frac{5}{4} & \frac{25}{2} \\ 0 & 0 & \frac{128}{21} & \frac{314}{21} \end{pmatrix}$$

$$x = \begin{pmatrix} -\frac{9}{16} \\ -\frac{115}{64} \\ \frac{157}{64} \end{pmatrix}$$