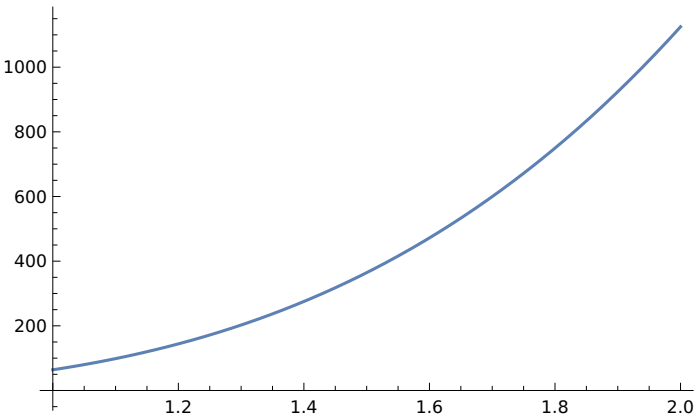
Newton Method

Q1: 27 x^4 + 162 x^3 -100 x^2 +62 x-7

In[145]:=
```
f[x_] := 27 x^4 + 162 x^3 - 180 x^2 + 62 x - 7
Subscript[x, 0] = 0;
ϵ = 5 * 10^-5;
Nmax = 10;
For[n = 1, n ≤ Nmax, n++,
Subscript[x, 1] = N[Subscript[x, 0] - (f[Subscript[x, 0]] / f'[Subscript[x, 0]])];
If[Abs[Subscript[x, 1] - Subscript[x, 0]] < ϵ,
    Return[Subscript[x, 1]], Subscript[x, 2] = Subscript[x, 0];
    Subscript[x, 0] = Subscript[x, 1]];
Print[n, "th iteration value is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 2]]]];
Print["The final approximate root is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 0]]]
Plot[f[x], {x, 1, 2}]
```

```
1th iteration value is 0.112903
Estimated error is : 0.112903
2th iteration value is 0.187147
Estimated error is : 0.0742436
3th iteration value is 0.236208
Estimated error is : 0.0490615
4th iteration value is 0.268729
Estimated error is : 0.0325205
5th iteration value is 0.290328
Estimated error is : 0.0215988
6th iteration value is 0.304691
Estimated error is : 0.0143635
7th iteration value is 0.314251
Estimated error is : 0.0095599
8th iteration value is 0.320617
Estimated error is : 0.00636631
9th iteration value is 0.324858
Estimated error is : 0.00424112
10th iteration value is 0.327685
Estimated error is : 0.00282605
The final approximate root is 0.327685
Estimated error is : 0.
```

Out[152]=



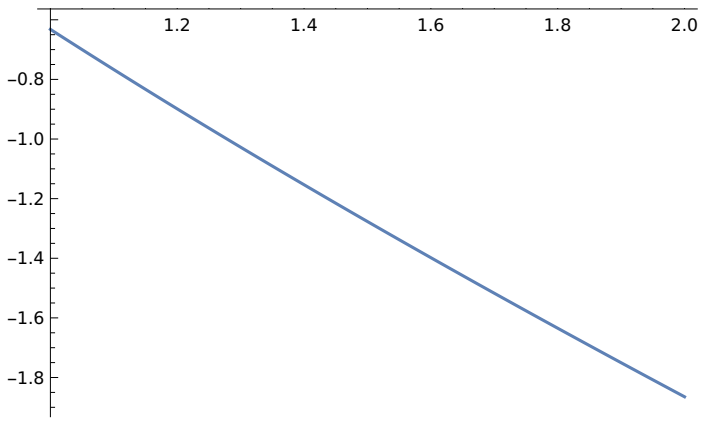
Q2: f(x)=e^-x - x

In[153]:=
```
f[x_] := Exp[-x] - x
Subscript[x, 0] = 0.5;
ϵ = 5 * 10^-4;
Nmax = 10;
For[n = 1, n ≤ Nmax, n++,
Subscript[x, 1] = N[Subscript[x, 0] - (f[Subscript[x, 0]] / f'[Subscript[x, 0]])];
If[Abs[Subscript[x, 1] - Subscript[x, 0]] < ϵ,
    Return[Subscript[x, 1]], Subscript[x, 2] = Subscript[x, 0];
    Subscript[x, 0] = Subscript[x, 1]];
Print[n, "th iteration value is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 2]]]];
Print["The final approximate root is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 0]]]
Plot[f[x], {x, 1, 2}]
```

```
1th iteration value is 0.566311

Estimated error is : 0.066311

2th iteration value is 0.567143

Estimated error is : 0.000832162
```

Out[157]=
```
0.567143

The final approximate root is 0.567143

Estimated error is : 1.25375 × 10^-7
```

Out[160]=



Q3 f(x) = x^3 - 3x + 1

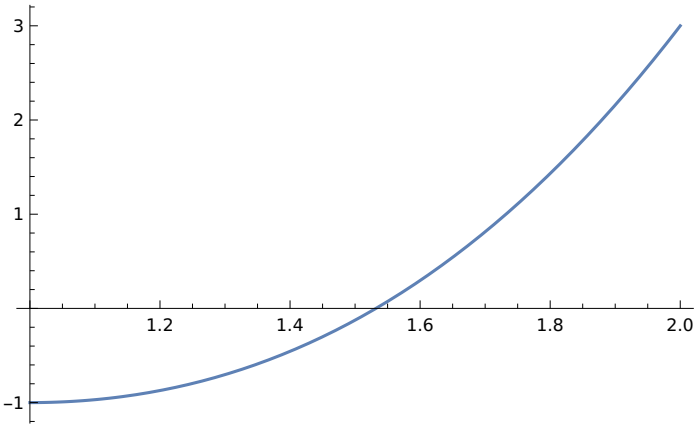
In[1]:=
```
f[x_] := x^3 - 3 x + 1
Subscript[x, 0] = 0;
ϵ = 5 * 10^-5;
Nmax = 10;
For[n = 1, n ≤ Nmax, n++,
Subscript[x, 1] = N[Subscript[x, 0] - (f[Subscript[x, 0]] / f'[Subscript[x, 0]])];
If[Abs[Subscript[x, 1] - Subscript[x, 0]] < ϵ,
    Return[Subscript[x, 1]], Subscript[x, 2] = Subscript[x, 0];
    Subscript[x, 0] = Subscript[x, 1]];
Print[n, "th iteration value is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 2]]]];
Print["The final approximate root is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 0]]]
Plot[f[x], {x, 1, 2}]
```

```
1th iteration value is 0.333333

Estimated error is : 0.333333

2th iteration value is 0.347222

Estimated error is : 0.0138889

3th iteration value is 0.347296

Estimated error is : 0.0000741309
```

Out[5]=
```
0.347296
```

```
The final approximate root is 0.347296

Estimated error is : 2.16999 × 10^-9
```

Out[8]=



Q4 f(x) = x^3 - 48

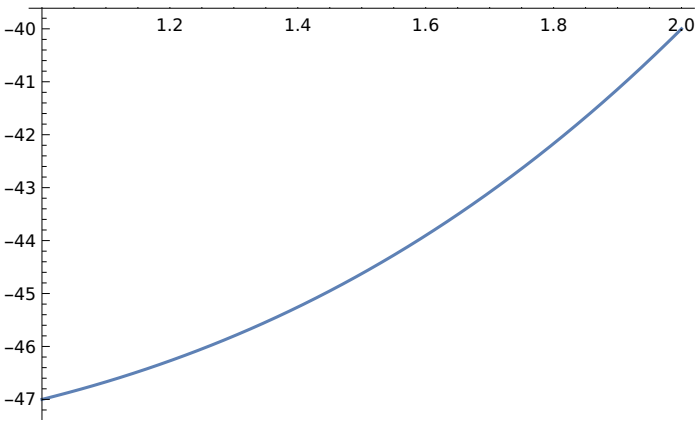
In[41]:=
```
f[x_] := x^3 - 48
Subscript[x, 0] = 1;
ϵ = 5 * 10^-5;
Nmax = 10;
For[n = 1, n ≤ Nmax, n++,
Subscript[x, 1] = N[Subscript[x, 0] - (f[Subscript[x, 0]] / f'[Subscript[x, 0]])];
If[Abs[Subscript[x, 1] - Subscript[x, 0]] < ϵ,
   Return[Subscript[x, 1]], Subscript[x, 2] = Subscript[x, 0];
   Subscript[x, 0] = Subscript[x, 1]];
Print[n, "th iteration value is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 2]]]];
Print["The final approximate root is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 0]]]
Plot[f[x], {x, 1, 2}]
```

```
1th iteration value is 16.6667
Estimated error is : 15.6667
2th iteration value is 11.1687
Estimated error is : 5.49796
3th iteration value is 7.57407
Estimated error is : 3.59464
4th iteration value is 5.32829
Estimated error is : 2.24578
5th iteration value is 4.11576
Estimated error is : 1.21253
6th iteration value is 3.68838
Estimated error is : 0.42738
7th iteration value is 3.63503
Estimated error is : 0.0533469
8th iteration value is 3.63424
Estimated error is : 0.000790566
```

Out[45]= 3.63424

```
The final approximate root is 3.63424
Estimated error is : 1.71999 × 10^-7
```

Out[48]=



Q5 f(x) = log[1+x]-cos x

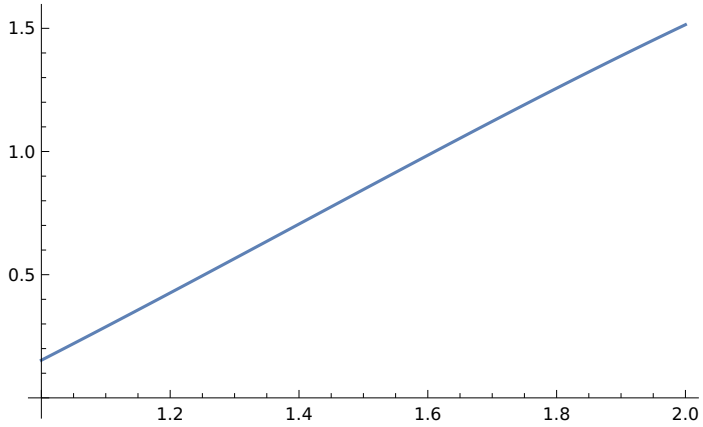
In[57]:=
```
f[x_] := Log[1 + x] - Cos[x]
Subscript[x, 0] = 0;
ϵ = 5 * 10^-5;
Nmax = 10;
For[n = 1, n ≤ Nmax, n++,
Subscript[x, 1] = N[Subscript[x, 0] - (f[Subscript[x, 0]] / f'[Subscript[x, 0]])];
If[Abs[Subscript[x, 1] - Subscript[x, 0]] < ϵ,
    Return[Subscript[x, 1]], Subscript[x, 2] = Subscript[x, 0];
    Subscript[x, 0] = Subscript[x, 1]];
Print[n, "th iteration value is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 2]]]];
Print["The final approximate root is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 0]]]
Plot[f[x], {x, 1, 2}]
```

```
1th iteration value is 1.

Estimated error is : 1.

2th iteration value is 0.886062

Estimated error is : 0.113938

3th iteration value is 0.884511

Estimated error is : 0.0015508
```

Out[61]= 0.884511

```
The final approximate root is 0.884511

Estimated error is : 3.24163 × 10^-7
```

Out[64]=



Q6 f(x) = (1/(1+x)) + sin x

In[73]:=

```
f[x_] := (1 / (1 + x)) + Sin[x]
Subscript[x, 0] = 0.5;
ϵ = 5 * 10 ^ -5;
Nmax = 10;
For[n = 1, n ≤ Nmax, n++,
Subscript[x, 1] = N[Subscript[x, 0] - (f[Subscript[x, 0]] / f '[Subscript[x, 0]])];
If[Abs[Subscript[x, 1] - Subscript[x, 0]] < ϵ,
    Return[Subscript[x, 1]], Subscript[x, 2] = Subscript[x, 0];
    Subscript[x, 0] = Subscript[x, 1]];
Print[n, "th iteration value is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 2]]]];
Print["The final approximate root is ", Subscript[x, 1]];
Print["Estimated error is : ", Abs[Subscript[x, 1] - Subscript[x, 0]]]
Plot[f[x], {x, 1, 2}]
```

```
1th iteration value is -2.14602

Estimated error is : 2.64602

2th iteration value is -3.4572

Estimated error is : 1.31118

3th iteration value is -3.54372

Estimated error is : 0.0865132

4th iteration value is -3.54535

Estimated error is : 0.00162893
```

Out[77]= -3.54535

The final approximate root is -3.54535

Estimated error is : 6.3412×10^{-7}

Out[80]=