BISECTION METHOD

Ques. 01 Find the root of function x^3+4x^2-10 using bisection method.

In[1]:=
```
z1 = FindRoot[x^3 + 4 x^2 - 10, {x, 1, 2}]
```

Out[1]=
```
{x → 1.36523}
```

In[2]:=
```
f[x_] := x^3 + 4 x^2 - 10
```

In[3]:=
```
a = 1;
```

In[4]:=
```
b = 2;
```

In[5]:=
```
ϵ = 5 * 10^-4;
```

In[6]:=
```
Nmax = 10;
```

In[7]:=
```
If[f[a] * f[b] > 0,
Print["These values do not satisfy the IVP so change the initial value"],
For[i = 1, i ≤ Nmax, i++, c = (a + b) / 2;
If[Abs[(b - a) / 2] < ϵ, Return[c],
Print["------------>  ", i, "th Iteration value is -> ", c];
Print["--> Estimated Error in ", i, "th Iteration is -> ", (b - a) / 2];
Print["--> Exact Error in ", i, "th Iteration is -> ", 1.36523 - c];

If[f[a] * f[c] < 0, b = c, a = c]]]];
Print["------------------------------>  The Approximate root is -> ", N[c]];
Plot[f[x], {x, 1, 2}]
```

```
------------>  1th Iteration value is -> 3/2

--> Estimated Error in 1th Iteration is -> 1/2

--> Exact Error in 1th Iteration is -> -0.13477

------------>  2th Iteration value is -> 5/4
```

--> Estimated Error in 2th Iteration is -> $\frac{1}{4}$

--> Exact Error in 2th Iteration is -> 0.11523

------------> 3th Iteration value is -> $\frac{11}{8}$

--> Estimated Error in 3th Iteration is -> $\frac{1}{8}$

--> Exact Error in 3th Iteration is -> -0.00977

------------> 4th Iteration value is -> $\frac{21}{16}$

--> Estimated Error in 4th Iteration is -> $\frac{1}{16}$

--> Exact Error in 4th Iteration is -> 0.05273

------------> 5th Iteration value is -> $\frac{43}{32}$

--> Estimated Error in 5th Iteration is -> $\frac{1}{32}$

--> Exact Error in 5th Iteration is -> 0.02148

------------> 6th Iteration value is -> $\frac{87}{64}$

--> Estimated Error in 6th Iteration is -> $\frac{1}{64}$

--> Exact Error in 6th Iteration is -> 0.005855

------------> 7th Iteration value is -> $\frac{175}{128}$

--> Estimated Error in 7th Iteration is -> $\frac{1}{128}$

--> Exact Error in 7th Iteration is -> -0.0019575

------------> 8th Iteration value is -> $\frac{349}{256}$

--> Estimated Error in 8th Iteration is -> $\frac{1}{256}$

--> Exact Error in 8th Iteration is -> 0.00194875

------------> 9th Iteration value is -> $\frac{699}{512}$

--> Estimated Error in 9th Iteration is -> $\frac{1}{512}$

--> Exact Error in 9th Iteration is -> -4.375×10^{-6}

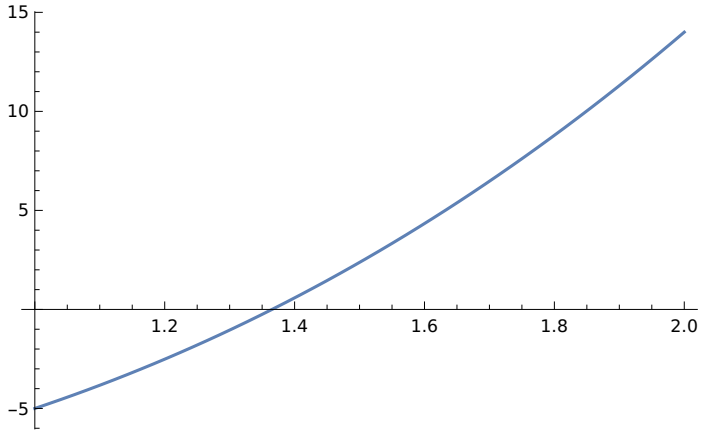
------------> 10th Iteration value is -> $\frac{1397}{1024}$

--> Estimated Error in 10th Iteration is -> $\frac{1}{1024}$

--> Exact Error in 10th Iteration is -> 0.000972187

----------> The Approximate root is -> 1.36426

Out[9]=



Ques. 02 Find the root of function Cos[x] using bisection method.

In[10]:=
```
z1 = FindRoot[Cos[x], {x, 1, 2}]
```

Out[10]= {x → 1.5708}

In[11]:=
```
f[x_] := Cos[x]
```

In[12]:=
```
a = 0;
```

In[13]:=
```
b = 2;
```

In[14]:=
```
ϵ = 5 * 10^-4;
```

In[15]:=
```
Nmax = 15;
```

In[16]:=
```
If[f[a] * f[b] > 0,
Print["These values do not satisfy the IVP so change the initial value"],
For[i = 1, i ≤ Nmax, i++, c = (a + b) / 2;
If[Abs[(b - a) / 2] < ϵ, Return[c],
Print["------------>  ", i, "th Iteration value is -> ", c];
Print["--> Estimated Error in ", i, "th Iteration is -> ", (b - a) / 2];
Print["--> Exact Error in ", i, "th Iteration is -> ", 1.5708 - c];

If[f[a] * f[c] < 0, b = c, a = c]]]];
Print["------------------------------>  The Approximate root is -> ", N[c]];
Plot[f[x], {x, 0, 2}]
```

```
------------>  1th Iteration value is -> 1

--> Estimated Error in 1th Iteration is -> 1

--> Exact Error in 1th Iteration is -> 0.5708

------------>  2th Iteration value is -> 3/2

--> Estimated Error in 2th Iteration is -> 1/2

--> Exact Error in 2th Iteration is -> 0.0708

------------>  3th Iteration value is -> 7/4

--> Estimated Error in 3th Iteration is -> 1/4

--> Exact Error in 3th Iteration is -> -0.1792

------------>  4th Iteration value is -> 13/8

--> Estimated Error in 4th Iteration is -> 1/8

--> Exact Error in 4th Iteration is -> -0.0542

------------>  5th Iteration value is -> 25/16

--> Estimated Error in 5th Iteration is -> 1/16

--> Exact Error in 5th Iteration is -> 0.0083

------------>  6th Iteration value is -> 51/32

--> Estimated Error in 6th Iteration is -> 1/32

--> Exact Error in 6th Iteration is -> -0.02295

------------>  7th Iteration value is -> 101/64
```

--> Estimated Error in 7th Iteration is -> $\frac{1}{64}$

--> Exact Error in 7th Iteration is -> -0.007325

------------> 8th Iteration value is -> $\frac{201}{128}$

--> Estimated Error in 8th Iteration is -> $\frac{1}{128}$

--> Exact Error in 8th Iteration is -> 0.0004875

------------> 9th Iteration value is -> $\frac{403}{256}$

--> Estimated Error in 9th Iteration is -> $\frac{1}{256}$

--> Exact Error in 9th Iteration is -> -0.00341875

------------> 10th Iteration value is -> $\frac{805}{512}$

--> Estimated Error in 10th Iteration is -> $\frac{1}{512}$

--> Exact Error in 10th Iteration is -> -0.00146563

------------> 11th Iteration value is -> $\frac{1609}{1024}$

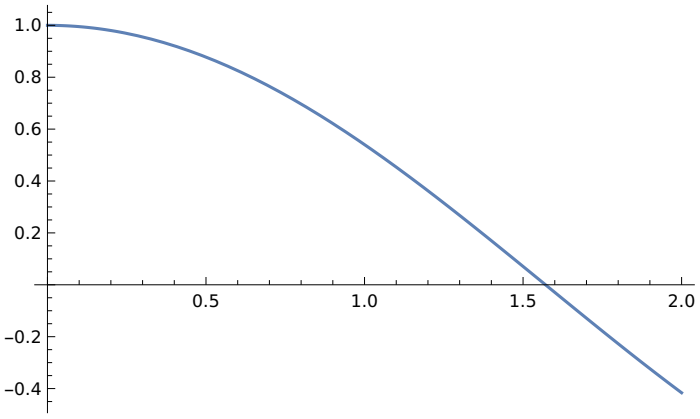
--> Estimated Error in 11th Iteration is -> $\frac{1}{1024}$

--> Exact Error in 11th Iteration is -> -0.000489063

Out[16]= $\frac{3217}{2048}$

------------------------------> The Approximate root is -> 1.5708

Out[18]=



Ques. 03 Find the root of function Exp[-x]-x using bisection method.

In[19]:=
```
z1 = FindRoot[Exp[-x] - x, {x, 0, 2}]
```

Out[19]=
```
{x → 0.567143}
```

In[20]:=
```
f[x_] := Exp[-x] - x
```

In[21]:=
```
a = 0;
```

In[22]:=
```
b = 0.6;
```

In[23]:=
```
ϵ = 5 * 10^-4;
```

In[24]:=
```
Nmax = 15;
```

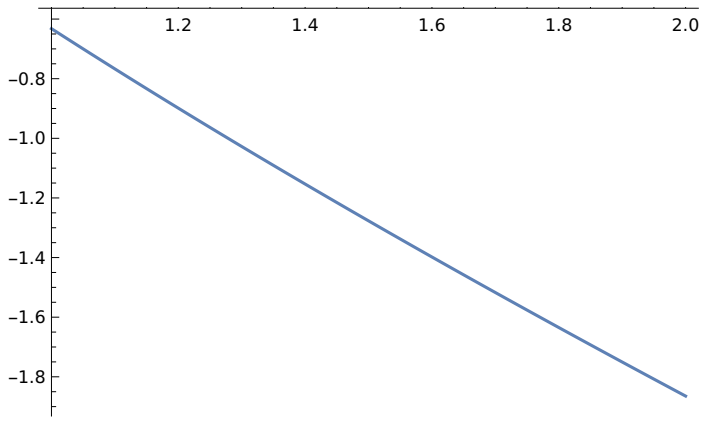
In[25]:=
```
If[f[a] * f[b] > 0,
Print["These values do not satisfy the IVP so change the initial value"],
For[i = 1, i ≤ Nmax, i++, c = (a + b) / 2;
If[Abs[(b - a) / 2] < ϵ, Return[c],
Print["------------>  ", i, "th Iteration value is -> ", c];
Print["--> Estimated Error in ", i, "th Iteration is -> ", (b - a) / 2];
Print["--> Exact Error in ", i, "th Iteration is -> ", 0.567143 - c];

If[f[a] * f[c] < 0, b = c, a = c]]]];
Print["------------------------------>  The Approximate root is -> ", N[c]];
Plot[f[x], {x, 1, 2}]
```

```
------------>  1th Iteration value is -> 0.3
--> Estimated Error in 1th Iteration is -> 0.3
--> Exact Error in 1th Iteration is -> 0.267143
------------>  2th Iteration value is -> 0.45
--> Estimated Error in 2th Iteration is -> 0.15
--> Exact Error in 2th Iteration is -> 0.117143
------------>  3th Iteration value is -> 0.525
--> Estimated Error in 3th Iteration is -> 0.075
--> Exact Error in 3th Iteration is -> 0.042143
------------>  4th Iteration value is -> 0.5625
--> Estimated Error in 4th Iteration is -> 0.0375
--> Exact Error in 4th Iteration is -> 0.004643
------------>  5th Iteration value is -> 0.58125
--> Estimated Error in 5th Iteration is -> 0.01875
--> Exact Error in 5th Iteration is -> -0.014107
------------>  6th Iteration value is -> 0.571875
--> Estimated Error in 6th Iteration is -> 0.009375
--> Exact Error in 6th Iteration is -> -0.004732
------------>  7th Iteration value is -> 0.567188
--> Estimated Error in 7th Iteration is -> 0.0046875
--> Exact Error in 7th Iteration is -> -0.0000445
------------>  8th Iteration value is -> 0.564844
--> Estimated Error in 8th Iteration is -> 0.00234375
--> Exact Error in 8th Iteration is -> 0.00229925
------------>  9th Iteration value is -> 0.566016
--> Estimated Error in 9th Iteration is -> 0.00117187
--> Exact Error in 9th Iteration is -> 0.00112738
------------>  10th Iteration value is -> 0.566602
--> Estimated Error in 10th Iteration is -> 0.000585938
--> Exact Error in 10th Iteration is -> 0.000541438
```

Out[25]= 0.566895

```
----------------------------->  The Approximate root is -> 0.566895
```

Out[27]=



Ques. 04 Find the root of function x^5-2x-1 using bisection method.

In[28]:=
```
z1 = FindRoot[x^5 - 2 x - 1, {x, -1, 1}]
```

Out[28]= $\{x \to -1.\}$

In[29]:=
```
f[x_] := x^5 - 2 x - 1
```

In[30]:=
```
a = -1;
```

In[31]:=
```
b = 0;
```

In[32]:=
```
ϵ = 5 * 10^-4;
```

In[33]:=
```
Nmax = 15;
```

In[34]:=
```
If[f[a] * f[b] > 0,
Print["These values do not satisfy the IVP so change the initial value"],
For[i = 1, i ≤ Nmax, i++, c = (a + b) / 2;
If[Abs[(b - a) / 2] < ϵ, Return[c],
Print["------------>  ", i, "th Iteration value is -> ", c];
Print["--> Estimated Error in ", i, "th Iteration is -> ", (b - a) / 2];
Print["--> Exact Error in ", i, "th Iteration is -> ", -1.0 - c];

If[f[a] * f[c] < 0, b = c, a = c]]]];
Print["------------------------------>  The Approximate root is -> ", N[c]];
Plot[f[x], {x, 1, 2}]
```

```
------------>  1th Iteration value is -> -1/2

--> Estimated Error in 1th Iteration is -> 1/2

--> Exact Error in 1th Iteration is -> -0.5
```

----------> 2th Iteration value is -> $-\frac{1}{4}$

--> Estimated Error in 2th Iteration is -> $\frac{1}{4}$

--> Exact Error in 2th Iteration is -> -0.75

----------> 3th Iteration value is -> $-\frac{1}{8}$

--> Estimated Error in 3th Iteration is -> $\frac{1}{8}$

--> Exact Error in 3th Iteration is -> -0.875

----------> 4th Iteration value is -> $-\frac{1}{16}$

--> Estimated Error in 4th Iteration is -> $\frac{1}{16}$

--> Exact Error in 4th Iteration is -> -0.9375

----------> 5th Iteration value is -> $-\frac{1}{32}$

--> Estimated Error in 5th Iteration is -> $\frac{1}{32}$

--> Exact Error in 5th Iteration is -> -0.96875

----------> 6th Iteration value is -> $-\frac{1}{64}$

--> Estimated Error in 6th Iteration is -> $\frac{1}{64}$

--> Exact Error in 6th Iteration is -> -0.984375

----------> 7th Iteration value is -> $-\frac{1}{128}$

--> Estimated Error in 7th Iteration is -> $\frac{1}{128}$

--> Exact Error in 7th Iteration is -> -0.992188

----------> 8th Iteration value is -> $-\frac{1}{256}$

--> Estimated Error in 8th Iteration is -> $\frac{1}{256}$

--> Exact Error in 8th Iteration is -> -0.996094

----------> 9th Iteration value is -> $-\frac{1}{512}$

--> Estimated Error in 9th Iteration is -> $\frac{1}{512}$

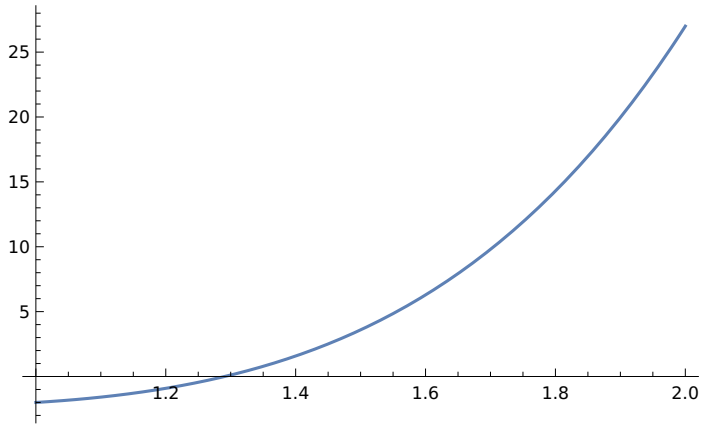
--> Exact Error in 9th Iteration is -> -0.998047

```
------------>  10th Iteration value is -> -1/1024

--> Estimated Error in 10th Iteration is -> 1/1024

--> Exact Error in 10th Iteration is -> -0.999023
```

Out[34]= $-\frac{1}{2048}$

```
------------------------------>  The Approximate root is -> -0.000488281
```

Out[36]=



Ques. 05 Find the root of function x^3-5x+1 using bisection method.

In[37]:=
```
z1 = FindRoot[x^3 - 5 x + 1, {x, 0, 1}]
```

Out[37]= {x → 0.20164}

In[38]:=
```
f[x_] := x^3 - 5 x + 1
```

In[39]:=
```
a = 0;
```

In[40]:=
```
b = 1;
```

In[41]:=
```
ϵ = 5 * 10^-4;
```

In[42]:=
```
Nmax = 15;
```

In[43]:=
```
If[f[a] * f[b] > 0,
Print["These values do not satisfy the IVP so change the initial value"],
For[i = 1, i ≤ Nmax, i++, c = (a + b) / 2;
If[Abs[(b - a) / 2] < ϵ, Return[c],
Print["------------>  ", i, "th Iteration value is -> ", c];
Print["--> Estimated Error in ", i, "th Iteration is -> ", (b - a) / 2];
Print["--> Exact Error in ", i, "th Iteration is -> ", 0.20164 - c];

If[f[a] * f[c] < 0, b = c, a = c]]];
Print["------------------------------>  The Approximate root is -> ", N[c]];
Plot[f[x], {x, 0, 1}]
```

```
------------>  1th Iteration value is -> 1/2

--> Estimated Error in 1th Iteration is -> 1/2

--> Exact Error in 1th Iteration is -> -0.29836

------------>  2th Iteration value is -> 1/4

--> Estimated Error in 2th Iteration is -> 1/4

--> Exact Error in 2th Iteration is -> -0.04836

------------>  3th Iteration value is -> 1/8

--> Estimated Error in 3th Iteration is -> 1/8

--> Exact Error in 3th Iteration is -> 0.07664

------------>  4th Iteration value is -> 3/16

--> Estimated Error in 4th Iteration is -> 1/16

--> Exact Error in 4th Iteration is -> 0.01414

------------>  5th Iteration value is -> 7/32

--> Estimated Error in 5th Iteration is -> 1/32

--> Exact Error in 5th Iteration is -> -0.01711

------------>  6th Iteration value is -> 13/64

--> Estimated Error in 6th Iteration is -> 1/64

--> Exact Error in 6th Iteration is -> -0.001485
```

----------> 7th Iteration value is -> $\frac{25}{128}$

--> Estimated Error in 7th Iteration is -> $\frac{1}{128}$

--> Exact Error in 7th Iteration is -> 0.0063275

----------> 8th Iteration value is -> $\frac{51}{256}$

--> Estimated Error in 8th Iteration is -> $\frac{1}{256}$

--> Exact Error in 8th Iteration is -> 0.00242125

----------> 9th Iteration value is -> $\frac{103}{512}$

--> Estimated Error in 9th Iteration is -> $\frac{1}{512}$

--> Exact Error in 9th Iteration is -> 0.000468125

----------> 10th Iteration value is -> $\frac{207}{1024}$

--> Estimated Error in 10th Iteration is -> $\frac{1}{1024}$

--> Exact Error in 10th Iteration is -> -0.000508437

Out[43]= $\frac{413}{2048}$

------------------------------> The Approximate root is -> 0.20166

Out[45]=