Corporate credit risk under misaligned transition expectations: a firm-level modeling approach

Elisa NDIAYE

December 29, 2025

Abstract

Climate transition stress tests typically rely on exogenously imposed transition scenarios, implicitly assuming that firms perfectly anticipate future policies, technologies and market conditions. In practice, however, corporate decision-making under transition uncertainty crucially depends on firms' own expectations, which may be misaligned with the realized transition pathway and significantly affect their financial resilience.

This paper develops a firm-level credit risk model to assess how misaligned transition expectations shape corporate default risk. Extending the framework of [Ndiaye et al., 2024a], we allow firms to form imperfect anticipations and to re-evaluate their strategies as the transition unfolds. Using a stochastic representation of sales, carbon intensity and production costs, combined with nested Monte Carlo simulations, we quantify how misanticipating carbon prices, abatement costs or demand-side adjustments affects probabilities of default.

Numerical illustrations on fictitious firms show that pessimistic anticipations may reduce default risk by inducing early adjustments, whereas overly optimistic expectations can sharply increase default probabilities, particularly for carbon-intensive firms. Overall, our results highlight that transition risk cannot be fully captured by carbon-price shocks alone and that expectation formation constitutes a key, yet overlooked, transmission channel of climate transition risk in corporate credit portfolios.

1 Introduction

The transition to a low-carbon economy raises new challenges for credit risk evaluation. A major source of risk stems from the uncertainty surrounding the transition itself, as multiple possible pathways can lead to different macroeconomic outcomes. These have been explored through various transition scenarios that aim to assess how policy changes, technological developments, and market adjustments can support the achievement of Net-Zero emissions as early as possible. Numerous such scenarios exist, including the NGFS scenarios [Richters et al., 2022] and the Shared Socioeconomic Pathways (SSPs, [Riahi et al., 2017]). These scenarios provide, albeit not exhaustively, trajectories for variables such as global GDP, inflation rates, carbon prices, carbon emissions, and energy consumption, at different geographical scales (from continental to country level in some cases). In Climate Stress-Tests, supervisors impose one or several transition scenarios, thereby removing the uncertainty about which pathway will unfold at the macro level. Yet this does not imply that firms themselves are free from such uncertainty: when designing their adaptation strategies, they must adopt a scenario as a working hypothesis, and some of these anticipations will inevitably turn out to be wrong. Moreover, stress-testing frameworks implicitly assume that firms correctly anticipate future transition pathways, often reducing transition risk to exogenous carbon-price shocks. This assumption overlooks the role of expectation formation, misalignment, and other transition risks drivers in shaping corporate decisions as well as credit risk outcomes. This paper explicitly challenges this assumption by modeling transition risk as an endogenous outcome of firms' imperfect anticipations, as well as encompassing all three transition risks drivers as defined in [Basel Committee on Banking Supervision, 2021].

A further issue is that Stress-Tests scenarios are macro-level, while Climate Stress-Tests require a downscaling to fit the granularity required to bypass the inexistence of historical data. This downscaling introduces an additional layer of uncertainty. Indeed, many ensembles of firm-level pathways may lead to the same aggregated results. Works by [Ndiaye et al., 2024b] and [Ndiaye et al., 2024a] tackle the question of the firm-level downscaling for transition scenarios in a credit risk Climate Stress-Tests framework and rely on a perfect foresight hypothesis. While in line with the usual Stress-Tests framework, this assumption may underestimate credit risk, as the credibility of announced climate policies is intertwined with the need for firms to form expectations when planning their adaptation strategy. Moreover, [Ndiaye et al., 2024b] has shown that distinct transition scenarios induce different business model adaptations at the firm level. What matters at the firm level is precisely these deviations: although the aggregate economy may be forced to follow an imposed transition pathway in the stress-test framework, individual companies may misanticipate it when formulating their business model. Evaluating credit risk one firm at a time thus requires accounting for such heterogeneous misalignments, all the more since no historical data exist to calibrate these dynamics in a Bayesian or copula-based fashion.

In practice, firms disclose their business model targets after designing them diligently. Organizations such as the *Science Based Targets initiative* (SBTi) propose tools for the design of corporates' business model adaptation based on transition scenarios. In 2023, over 4,000 companies and financial institutions had set reduction targets validated by the SBTi[1]. In practice, firms' targets are set with respect to different scenarios, depending on the firm considered. This highlights that companies necessarily build their transition strategies under an assumed scenario, and that these assumptions differ across firms. As a result, some firms are bound to misanticipate the actual transition pathway, reinforcing the uncertainty around how the transition will ultimately unfold and be managed at the firm level. Clearly, a firm's misanticipations can be another amplifier of credit risk and we propose to study their impact in this paper.

[1] The list of companies and the specific targets can be found on [Science Based Targets Initiative, 2024]

State of the art To the best of our knowledge, there is no existing framework that directly accounts for firms' anticipations within a conditional stress-testing setup where scenarios are imposed. The closest contributions address related but distinct questions. Early works by [Bell and Vuuren, 2022], use the KMV model [Crosbie and Bohn, 2003] with carefully calibrated parameters to assess the sensitivity of corporates' PDs to transition and physical risk shocks of varying frequencies and severities. Their findings highlight the sensitivity of PDs to the assumed transition pathway, but their framework does not explicitly account for firms' anticipations of such pathways, nor the consequences of being wrong. A first attempt was made in the Credit-Risk Climate Stress-Testing methodology developed by [Barclays PLC, 2021]. Precisely, they directly integrate firms' disclosed transition adaptation plans when available and deemed credible according to their criteria. This hands-on and expansive work permits testing of a single strategy - when available - that implicitly integrates real firms' anticipations along several scenarios under a deterministic credit risk assessment framework. They do not compare the impact of a potential re-evaluation of strategies when the climate policies do not match the anticipations, nor other potential strategies for firms that do not have disclosed strategies or credible ones. Furthermore, they do not question the fitness of the firm's anticipations.

Works by [Le Guenedal and Tankov, 2024] define a stochastic framework for the credit-risk evaluation of a firm under scenario uncertainty. They use an endogenous default barrier *'à la'* Leland and Toft [Leland and Toft, 1996] to price a transition risk-vulnerable corporate bond under a constant market shares assumption in a world where the carbon price trajectory is gradually revealed as time goes. They assume the carbon price is a time inhomogeneous compound Poisson process whose law is not entirely known by the market at the starting date. The only determinant of transition risk considered is the climate policy through the carbon price, which is a stochastic process whose intensity depends on the effective scenario. Moreover, there is no possibility for firms' adaptation to the transition. Note that they focus on the climate policy uncertainty rather than assessing the impact of a misanticipation regarding the climate scenario. In contrast, [Battiston et al., 2023] do not formulate probabilities on the occurrence of transition scenarios but analyze the impact of a re-evaluation of the market's anticipations regarding the climate policy on credit risk for bond pricing using a structural model. They allow for a linear adaptation of firms with respect to the anticipated scenario under a constant market shares assumption, with a possibility for a re-evaluation. The re-evaluation of the adaptation consists of following the plan that would have been used should the anticipations have been right. Similarly to [Le Guenedal and Tankov, 2024], the only transition risk driver considered is the climate policy. What's more is that they study the impact of collective misanticipations regarding the climate scenario, and not that of a single firm in a transition-aware market, which raises the question of the fulfillment of said scenario.

Our contribution This paper builds on the framework of [Ndiaye et al., 2024a] to evaluate the impact of a single firm's misanticipation of the transition scenario on credit risk. The aim is not to formulate probabilities on the transition scenario's likelihood of occurrence. In fact, the end goal of this study is to evaluate the impact of a marginal firm's transition-skepticism or transition-enthusiasm while a given transition scenario unfolds. We assume the firm's anticipations are solely formulated through the choice of business model. In contrast to [Battiston et al., 2023], who analyze the effects of a collective re-evaluation of climate policy anticipations at the market level, our focus is on the consequences of a single firm's misjudgment within an otherwise transition-aware market. While a scenario can still be attained when some firms deviate, it becomes difficult to reconcile its realization with the assumption that the entire market initially misanticipated it. Moreover, the model takes into account a broad set of transition drivers, and thus goes beyond the sole impact of a carbon price trajectory misanticipation. In that sense, we adopt a wider definition of scenarios than in [Le Guenedal and Tankov, 2024] and [Battiston et al., 2023], and we relax the constant market shares assumption to allow for both consumer sentiment and cost pass-through effects. We also examine the effect of a possible re-evaluation of anticipations during the scenario at various dates.

We show results under the forward-looking business model adaptation plan proposed in [Ndiaye et al., 2024b] and a central scenario. We find that, under these conditions, misanticipating the climate policy does not necessarily penalize the credit risk. In fact, being pessimistic reduces the default probability. Moreover, a transition-skeptic firm will always benefit from re-evaluating its business model in terms of credit risk. Taken together with [Ndiaye et al., 2024b] and [Ndiaye et al., 2024a], this paper contributes to a unified framework for climate-related credit risk modeling: from firm-level scenario downscaling, to PD estimation under imposed scenarios, and finally to the analysis of misanticipation effects at the firm level.

Organization of the paper The remainder of this paper is organized as follows. Section 2 introduces the problem setting and the representation of transition scenarios. Section 3 defines the probability of default under both correct and mistaken anticipations of the transition pathway. Section 4 extends the model by allowing for a re-evaluation of anticipations during the scenario. Section 5 provides a numerical illustration on the relative impact of misanticipation across different transition risk drivers, with and without re-evaluation.

2 The model setup

2.1 Definition of a scenario

The purpose of this paper is to evaluate the impact of a single firm's misanticipations regarding the transition on its probability of default (PD). To avoid negative effects on the macroeconomy, the climate policy is usually disclosed in advance. This way, firms may take precautions regarding future carbon price shocks or technology bans and design their business model diligently.

However, the company may not believe in the announcement and considers many plausible scenarios for the transition. Let $\mathcal{S} = \{\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_H\} \neq \varnothing$, $H \geq 2$ be the set of plausible transition scenarios. A transition scenario is a deterministic set of likely energy transition pathways for main macroeconomic variables ranging on the period $[0, T]$, where T denotes the end of the scenario. All scenarios have the same time horizon and are studied on the same set of N discrete dates $i = 0 : N$. Thus, the N-th date corresponds to T. The definition of a scenario is as follows:

Definition 2.1 (Scenario)**.** *A scenario* $\mathbf{S}_s \in \mathcal{S}, s = 1 : H$ *is the following deterministic 8-tuple:*

$$\left((\bar{S}_i^s)_{i=0:N}, (I^{\mathrm{ref,s}}s)_{i=0:N}, (\mathtt{cp}_i^s)_{i=0:N}, (I_i^{R,s})_{i=0:N}, \kappa^s, \alpha^s, (\lambda_0^s, g^s), \mu^s \right),$$

where

- $(\bar{S}_i^s)_{i=0:N} \in (\mathbb{R}^+)^N$ *are the cumulated sales revenue of the reference market between* $i-1$ *and* i*,*
- $(I^{\mathrm{ref,s}}s)_{i=0:N} \in (\mathbb{R}^+)^N$ *is the market reference for the intensity,*
- $(\mathtt{cp}_i^s)_{i=0:N} \in (\mathbb{R}^+)^N$ *is the carbon price,*
- $(I_i^{R,s})_{i=0:N} \in \mathbb{R}^N$ *is the inflation rate,*
- $\kappa^s \geq 0$ *is the market's sensitivity to the relative intensity,*
- $\alpha^s \geq 0$ *is the factor of autonomous decrease of abatement costs over time,*
- $(\lambda_0^s, g^s) \in \mathbb{R}^+ \times [1, \infty)$ *are respectively the mean occurrence of physical damages at time* 0*, and the growth factor of the physical risk damages counting process,*
- μ^s *is the probability law of the physical damages amplitude.*

Because there is more than one plausible transition scenario, with the true scenario being in $\mathcal{S}$, the firm will form anticipations on the scenario that will unfold among the plausible scenarios. The firm will transfer its anticipations to its business model: it tailors its carbon emissions mitigation plan in date 0 with respect to its anticipations on the transition. This plan may be constructed based on the scenario and the company's characteristics, or completely exogenous. Furthermore, it may either be a deterministic or constant function of some parameters or even stochastic. The firm's anticipations regarding the scenario are incorporated into its business model through its carbon emissions mitigation plan solely. In a nutshell, the firm anticipates at time 0 which transition scenario within $\mathcal{S}$ will unfold, and then uses this anticipation to craft its business model. We give more insights about the business model design in the next section.

2.2 Reminder of the baseline PD model (scenario-dependent)

We briefly recall the baseline firm-level framework of [Ndiaye et al., 2024b, Ndiaye et al., 2024a], restricted here to the ingredients needed in this paper. All quantities are *conditional on an imposed transition scenario* $\mathbf{S}_s \in \mathcal{S}$ (index $\mathbf{S}_s$ omitted only when clear). Scenario $\mathbf{S}_s$ provides the deterministic macro trajectories used below: carbon price $(\mathtt{cp}_i^s)_{i=0:N}$, reference market sales $(\bar{S}_i^s)_{i=0:N}$ (current prices), reference intensity $(I^{\mathrm{ref,s}}s)_{i=0:N}$, inflation $(I^{R,s})_{i=0:N}$, and market sensitivity to the relative intensity $\kappa^s \geq 0$.

Carbon mitigation strategy. Assume that the company will seek to adapt its business model to mitigate the effects of the climate policy on its financial cash flows. This adaptation consists only of reducing its relative emissions (i.e., intensity), using a forward-looking carbon mitigation plan, crafted in date zero. The firm's carbon mitigation strategy is the following sequence of abatement efforts:

$$\boldsymbol{\pi} := (\gamma_i)_{i=0:N-1}, \qquad \gamma_i \in [0, \gamma_{\max}], \quad \forall i = 0 : N-1,$$

which governs the business-model state $X_i^s := (I_i^s, S_i^s)$. The sequence of γ_i may be stochastic or deterministic, defined by functions or constant values, and may depend on the scenario through firm-level anticipations. In any case, this plan is decided at date 0, and applied throughout the scenario.

Intensity and sales dynamics. With $\delta := T/N$ a constant time-step, and idiosyncratic shocks $(\Delta\varepsilon_i^I, \Delta\varepsilon_i^S)$ (centered, bounded, and with respective log moment generating function ψ_I, ψ_S), we write the firm's intensity $(I_i)_{i\geq 0}$ and sales dynamics $(S_i)_{i\geq 0}$ as:

$$I_{i+1}^s = I_i^s \exp\big(- \gamma_i \delta + \sigma_I \Delta\varepsilon_i^I - \psi_i^I(\sigma_I)\big), \tag{2.1}$$

$$S_{i+1}^s = S_i^s \frac{\bar{S}_{i+1}^s}{\bar{S}_i^s} \exp\big(- \kappa^s (I_i^s - I^{\text{ref,s}} s)\delta + \sigma_S \Delta\varepsilon_i^S - \psi_i^S(\sigma_S)\big). \tag{2.2}$$

Since firm-level intensity must satisfy $I_i^s = E_i^s / S_i^s$ with E_i^s the emissions, we assume for consistency reasons that $\text{Cov}(\Delta\varepsilon_i^I, \Delta\varepsilon_i^S) < 0, \forall i = 0 : N$. Since the scenario stops in N, we assume $I_i = I_N$ and $S_i = S_N$ for all $i \geq N$.

Total operating costs. The carbon cost $\mathtt{C}^{\mathtt{C}}$ and total costs T^C follow

$$\begin{aligned} \mathtt{C}^{\mathtt{C}}(s, i, I, S) &= \mathtt{cp}_i^s I S, \\ T^C(s, i, I, S) &= \mathtt{cp}_i^s I S \ + \ k\,(S)^\nu, \quad k > 0,\ \nu \in \mathbb{R}_+, \end{aligned}$$

thereby capturing both explicit carbon taxation (via $\mathtt{cp}_i^s$) and other variable costs scaled by sales (allowing for economies/diseconomies of scale via ν).

Green investment cost. Abatement is costly and convex (decreasing over time with technology learning), '*à la DICE*' [Nordhaus, 2017]:

$$\text{IC}_i^s \ = \ S_i^s \cdot c \cdot \alpha^{\,i\delta} \, \frac{\big(1 - e^{-\gamma_i \delta}\big)^\beta}{\beta}, \qquad c > 0,\ \alpha \in [0.95, 1],\ \beta \geq 2.$$

This is the cost of replacing the production capital to produce S_i at a lesser intensity, given by the intensity reduction rate γ_i.

Non Green Investment function. Assume the selling price dynamics are $P_i^s = P_{i-1}^s\,(1+I^{R,s})$. The firm's production Y_i^s follows a reduced-form Cobb-Douglas production function $Y_i^s = a(K_i^s)^\theta$ $(a > 0,\ \theta > 0)$, with K_i^s the productive capital under scenario $\mathbf{S}_s$. By definition of the sales revenues as the product of production Y_i^s and selling price P_i^s, we get:

$$K_i^s = \left(\tfrac{S_i^s}{a\,P_i^s}\right)^{1/\theta},$$

Thus, from period $i-1$ to i, the required investment in productive capital (i.e. Non green) is:

$$\mathrm{NGI}_i^s = \left(K_i^s - (1-d\delta)K_{i-1}^s\right)^+,$$

with $d \in [0,1]$ the yearly capital depreciation rate.

3 Corporate probability of default with business model adaptation and anticipations regarding the transition scenario

3.1 Probability of default with an admissible intensity reduction strategy under perfect anticipations

At each decision date i, we wish to compute the probability of the company defaulting within the next period, given that it has not occurred before from an investors' standpoint. To do so, consider a filtered probability space $(\Omega, (\mathcal{F}_i)_{0\leq i\leq N}, \mathbb{P})$ where $(\mathcal{F}_i)_{0\leq i\leq N}$ represents the information flow known by the investors. We assume that the investors have a perfect anticipation of the scenario. Indeed, if the entire market is wrong, the transition scenario cannot be fulfilled. The considered firm is the only one that can be mistaken. Moreover, we assume an infinitely granular economy, i.e., a single's firm decisions have negligible impact on the macroeconomic variables' trajectories.

Assume we are in scenario $\mathbf{S}_s \in \mathcal{S}$. At date 0, the firm designs its carbon reduction strategy with respect to $\mathbf{S}_s$ and the value of the state variable $\mathbf{X}_0$. The strategy set at date 0 may be a random or a deterministic function of the scenario either directly or through the state variables. We will restrict our study to $\mathcal{F}$-adapted Markov strategies in the definition given by [Ndiaye et al., 2024b]. Since companies usually disclose their business model strategy in advance to reassure investors, this hypothesis is in line with real-life management practices. Let Π be the set of admissible strategies i.e., **Markov**, $\mathcal{F}$-adapted and such that each $\gamma_i \in [0, \gamma_{max}], \forall i = 0 : N-1$. Let $\boldsymbol{\pi}(s) := (\gamma_i^s)_{i=0:N-1} \in \Pi$ be this strategy. The argument s indicates that the strategy was designed in anticipation of scenario $\mathbf{S}_s$. Note that, in this Section, we assume the company's anticipations regarding the climate policy are perfect: the scenario the company has anticipated is the one that actually unfolds.

To compute the probability of default under any admissible strategy $\boldsymbol{\pi}(s)$ and scenario $\mathbf{S}_s$, we need to introduce new notation. Let $\mathbf{X}_i^{\boldsymbol{\pi}(s),s} = (I_i^{\boldsymbol{\pi}(s),s}, S_i^{\boldsymbol{\pi}(s),s})$ denote the value of the state variable at date i computed with strategy $\boldsymbol{\pi}(s)$ under scenario $\mathbf{S}_s$. We define the default under strategy $\boldsymbol{\pi}(s)$ in scenario $\mathbf{S}_s$ following the definition given in [Ndiaye et al., 2024a].

$$\{\tau^{\boldsymbol{\pi}(s),s} = i\} = \{i \geq 1, A_i^{\boldsymbol{\pi}(s),s} < D_i^{\boldsymbol{\pi}(s),s}, A_j^{\boldsymbol{\pi}(s),s} \geq D_j^{\boldsymbol{\pi}(s),s}, \forall j = 0, \cdots, i-1\},$$

where $A_i^{\boldsymbol{\pi}(s),s}$ and $D_i^{\boldsymbol{\pi}(s),s}, i = 0 : N$ are respectively the values for the company's assets and the debt computed under admissible strategy $\boldsymbol{\pi}(s)$ and scenario $\mathbf{S}_S$ at date i, which are computed as follows:

$$A_i^{\boldsymbol{\pi}(s),s} = \mathbb{E}\left[\sum_{j=i}^{N-1} \frac{S_j^{\boldsymbol{\pi}(s),s} - T^C(s,j,S_j^{\boldsymbol{\pi}(s),s}, I_j^{\boldsymbol{\pi}(s),s})}{(1+r\delta)^{j-i}} + \frac{S_N^{\boldsymbol{\pi}(s),s} - T^C(s,N,S_N^{\boldsymbol{\pi}(s),s}, I_N^{\boldsymbol{\pi}(s),s})}{r\delta(1+r\delta)^{N-i-1}} \mid \mathcal{F}_i\right] - \bar{Z}_i^s,$$

$$D_i^{\boldsymbol{\pi}(s),s} = (1-\zeta\delta) D_{i-1}^{\boldsymbol{\pi}(s),s} + I^G(s,i-1,S_{i-1}^{\boldsymbol{\pi}(s),s}, \gamma_{i-1}^s) + \left(\left(\frac{S_i^{\boldsymbol{\pi}(s),s}}{aP_i^s}\right)^{1/\theta} - (1-d\delta)\left(\frac{S_{i-1}^{\boldsymbol{\pi}(s),s}}{aP_{i-1}^s}\right)^{1/\theta}\right)^+,$$

Where $d \in [0,1]$ is the capital depreciation rate, r is the risk free rate, $\zeta \in [0,1]$ the debt amortization rate and $\bar{Z}_i^s$ the expected physical-damage shock to profits, modeled as an inhomogeneous compound Poisson process with scenario-dependent increasing intensity $\lambda_i = \lambda_0 g^i$, $\lambda_0 \geq 0$, $g \in [1, (1+r\delta)^{1/\delta})$ (see [Ndiaye et al., 2024a]).

Both $(A_i^{\boldsymbol{\pi}(s),s})_{i\geq 0}$ and $(D_i^{\boldsymbol{\pi}(s),s})_{i\geq 0}$ are $\mathcal{F}$-adapted processes. Note that, under $\mathbf{S}_s$, $Z_1 \sim \mu^s$ and:

$$\bar{Z}_i^s = \begin{cases} \lambda_0^s \delta \dfrac{(1+r\delta)^{i+1}}{r\delta} \mathbb{E}\left[Z_1\right], & \text{if} \quad g^s = 1, \\ \dfrac{\lambda_0^s \left[(g^s)^{i\delta} - (g^s)^{(i-1)\delta}\right]}{\delta \ln(g^s)} \times \dfrac{1+r\delta}{1+r\delta-(g^s)^{\delta}} \mathbb{E}\left[Z_1\right], & \text{if} \quad g^s > 1. \end{cases}$$

We can now define the 1-period probability of default at date 0 denoted $\mathtt{PD}_0^{\boldsymbol{\pi}(s),s}$ with strategy $\boldsymbol{\pi}(s)$ under scenario $\mathbf{S}_s \in \mathcal{S}$ as:

$$\begin{aligned}\mathtt{PD}_0^{\boldsymbol{\pi}(s),s} :=& \mathbb{P}(\tau^{\boldsymbol{\pi}(s),s} = 1 \mid \mathcal{F}_0) \\ =& \mathbb{P}(A_1^{\boldsymbol{\pi}(s),s} < D_1^{\boldsymbol{\pi}(s),s} \mid A_0^{\boldsymbol{\pi}(s),s} \geq D_0)\mathbf{1}\{A_0^{\boldsymbol{\pi}(s),s} \geq D_0\}.\end{aligned}$$

We now generalize this definition for any date i. The 1-period probability of default at date i, denoted by $\mathtt{PD}_i^{\boldsymbol{\pi}}$, is:

$$\begin{aligned}\mathtt{PD}_i^{\boldsymbol{\pi}(s),s} :=& \mathbb{P}(\tau^{\boldsymbol{\pi}(s),s} = i+1 \mid \mathcal{F}_i) \\ =& \mathbb{P}(A_{i+1}^{\boldsymbol{\pi}(s),s} < D_{i+1}^{\boldsymbol{\pi}(s),s} \mid A_j^{\boldsymbol{\pi}(s),s} \geq D_j^{\boldsymbol{\pi}(s),s}, \forall j = 0:i)\mathbf{1}\{A_j^{\boldsymbol{\pi}(s),s} \geq D_j^{\boldsymbol{\pi}(s),s}, \forall j = 0:i\}.\end{aligned}$$

3.2 Probability of default with an admissible intensity reduction strategy under wrong anticipations

Assume that the company has designed its carbon reduction strategy at date 0 while assuming that scenario $\mathbf{S}_z \in \mathcal{S}$ will unfold. Similarly to the preceding section, we denote this strategy by $\boldsymbol{\pi}(z) = (\gamma_i^z)_{i=0:N-1} \in \Pi$ and allow for this strategy to be a random or a deterministic function of the scenario either directly or via the state variable $\mathbf{X}$. In all cases, the strategy must be Markov. However, the company's anticipations regarding the future of the climate scenario turn out to be incorrect. Indeed, the scenario that does unfold is $\mathbf{S}_s \in \mathcal{S}$, $\mathbf{S}_s \neq \mathbf{S}_z$. The company may not undergo corrective measures, i.e. its strategy remains the same at later stages of the scenario, despite the impacts on its balance sheet. One important observation is that, since the strategy is defined as a function of the scenario parameters (either directly or indirectly), applying the same strategy in different scenarios may not lead to the same carbon mitigation levels. Moreover, the investors anticipate the climate policy perfectly, i.e. they use the right scenario in their evaluation of the firm's value.

Let $\mathbf{X}_i^{\boldsymbol{\pi}(z),s} = (I_i^{\boldsymbol{\pi}(z),s}, S_i^{\boldsymbol{\pi}(z),s})$ be the value of the state variable at date i computed under strategy $\boldsymbol{\pi}(z)$ under scenario $\mathbf{S}_s$. Precisely, the dynamics rewrite as follows:

$$I_{i+1}^{\boldsymbol{\pi}(z),s} = I_i^{\boldsymbol{\pi}(z),s} e^{-\gamma_i^z \delta} \times e^{\sigma_I \Delta\varepsilon_i^I - \psi_i^I(\sigma_I)}, \qquad i = 0 : N-1, \tag{3.1}$$

$$S_{i+1}^{\boldsymbol{\pi}(z),s} = S_i^{\boldsymbol{\pi}(z),s} \frac{\bar{S}_{i+1}^s}{\bar{S}_i^s} e^{-\kappa^s \left(I_i^{\boldsymbol{\pi}(z),s} - I_i^{\mathrm{ref},s}\right)\delta} \times e^{\sigma_S \Delta\varepsilon_i^S - \psi_i^S(\sigma_S)}, \qquad i = 0 : N-1. \tag{3.2}$$

The main difference between the dynamics of the intensity in (2.1) and those given in (3.1) is the carbon reduction strategy used. Regarding the sales revenue (3.2), the trajectories for reference market intensity and sales remain the same as in (2.2). The difference lies in the intensity trajectory used for the drift part. The carbon cost thus becomes:

$$\mathtt{C}^{\mathtt{C}}(s, i, I_i^{\boldsymbol{\pi}(z),s}, S_i^{\boldsymbol{\pi}(z),s}) = \mathtt{cp}_i^s I_i^{\boldsymbol{\pi}(z),s} S_i^{\boldsymbol{\pi}(z),s}, \quad i \geq 0.$$

It is worth noting that if $\kappa^s = 0$, the sales revenue is not impacted by the company's misanticipations regarding the climate policy. Given the dynamics described in the previous section, the firm's total assets and debt under admissible strategy $\boldsymbol{\pi}(z)$ in scenario $\mathbf{S}_s$ at date i are:

$$A_i^{\boldsymbol{\pi}(z),s} = \mathbb{E}\left[\sum_{j=i}^{N-1} \frac{S_j^{\boldsymbol{\pi}(z),s} - T^C(s, j, S_j^{\boldsymbol{\pi}(z),s}, I_j^{\boldsymbol{\pi}(z),s})}{(1+r\delta)^{j-i}} + \frac{S_N^{\boldsymbol{\pi}(z),s} - T^C(s, N, S_N^{\boldsymbol{\pi}(z),s}, I_N^{\boldsymbol{\pi}(z),s})}{r\delta(1+r\delta)^{N-i-1}} \mid \mathcal{F}_i\right] - \bar{Z}_i^s,$$

$$D_i^{\boldsymbol{\pi}(z),s} = (1-\zeta\delta) D_{i-1}^{\boldsymbol{\pi}(z),s} + I^G(s, i-1, S_{i-1}^{\boldsymbol{\pi}(z),s}, \gamma_{i-1}^z) + \left(\left(\frac{S_i^{\boldsymbol{\pi}(z),s}}{aP_i^s}\right)^{1/\theta} - (1-d\delta)\left(\frac{S_{i-1}^{\boldsymbol{\pi}(z),s}}{aP_{i-1}^s}\right)^{1/\theta}\right)^+.$$

We define the default under strategy $\boldsymbol{\pi}(z)$ in scenario $\mathbf{S}_s$ as the following stopping time:

$$\{\tau^{\boldsymbol{\pi}(z),s} = i\} = \{i \geq 1, A_i^{\boldsymbol{\pi}(z),s} < D_i^{\boldsymbol{\pi}(z),s}, A_j^{\boldsymbol{\pi}(z),s} \geq D_j^{\boldsymbol{\pi}(z),s}, \forall j = 0, \cdots, i-1\}, \tag{3.3}$$

The 1-period probability of default at date $i = 0 : N-1$ under scenario $\mathbf{S}_s$ with strategy $\boldsymbol{\pi}(z)$ denoted by $\mathtt{PD}_\mathtt{i}^{\boldsymbol{\pi}\mathbf{(z),s}}$ is:

$$\begin{aligned}\mathtt{PD}_\mathtt{i}^{\boldsymbol{\pi}\mathbf{(z),s}} :=& \mathbb{P}(\tau^{\boldsymbol{\pi}(z),s} = i+1 \mid \mathcal{F}_i) \\ =& \mathbb{P}(A_{i+1}^{\boldsymbol{\pi}(z),s} < D_{i+1}^{\boldsymbol{\pi}(z),s} \mid A_j^{\boldsymbol{\pi}(z),s} \geq D_j^{\boldsymbol{\pi}(z),s}, \forall j = 0 : i) \mathbf{1}\{A_j^{\boldsymbol{\pi}(z),s} \geq D_j^{\boldsymbol{\pi}(z),s}, \forall j = 0 : i\}.\end{aligned}$$

3.3 Numerical computation of the probability of default term structure with any admissible intensity reduction strategy under perfect and imperfect anticipations

Due to the complexity of the computation required, we propose to compute the 1-period default probability using Nested Monte Carlo Simulations for any admissible strategy computed with any type of anticipations. Let $\mathbf{S}_s$ and $\mathbf{S}_z$ be two scenarios in $\mathcal{S}$. Here, we may have $\mathbf{S}_s = \mathbf{S}_z$. For the sake of clarity, we will write as $\mathbf{X}_j^{\boldsymbol{\pi}(z),s}(i,\mathbf{x}) = (I_j^{\boldsymbol{\pi}(z),s}(i,\mathbf{x}), S_j^{\boldsymbol{\pi}(z),s}(i,\mathbf{x})), \forall i = 0:N, j = 0:N, i \leq j$ the value of the state variable at date j starting from the initial value $\mathbf{x} \in (\mathbb{R}^+)^2$ at date i and computed along scenario $\mathbf{S}_s$ with the admissible strategy $\boldsymbol{\pi}(z) = (\gamma_i^z)_{i=0:N-1} \in \Pi$ that has been designed at date 0 while anticipating that scenario $\mathbf{S}_z$ will unfold. We only consider Markov strategies, meaning that each γ_j^z is a stochastic kernel depending only on the last known value of the state variable $\mathbf{X}$: for all $i = 0:N-1$, each $\gamma_j^z : \mathbf{X} \mapsto \gamma_j^z(\mathbf{X})$, $\mathbf{X} \in \mathbb{R}^+$ is a measurable function computed on the last known value of $\mathbf{X}$. The methodology goes as follows:

1. Simulate $M_1 \gg 1$ trajectories $\{\left(\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)\right)_{i=0:N}\}_{m=1:M_1}$ starting from the initial value $\mathbf{X}_0 = (I_0, S_0)$ strategy $\boldsymbol{\pi}(z, X_0) = (\gamma_i^z)_{i=0:N-1}$ and scenario $\mathbf{S}_s$.
2. For $i = 0:N-1$:
   (a) Compute the expected sum of future discounted physical damages $\bar{Z}_i^s$.
   (b) For each simulated value $\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0), m = 1:M_1$:
      i. Consider an admissible strategy $\boldsymbol{\pi}_i\left(z, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}\right) = \left(\gamma_j^z(.)\right)_{j=i:N-1}$. Note that $\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}$ means $\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)$. This strategy starts in i and has been designed while anticipating scenario $\mathbf{S}_z$.
      ii. Resimulate M_2 trajectories starting from the initial value $\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)$ in i.
      $$\{\left(\mathbf{X}_j^{\boldsymbol{\pi}(z),s,m,l}\left(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}\right)\right)_{j=i:N}\}_{l=1:M_2}.$$
      iii. Compute the assets for trajectory m as the sum of future profits computed with the newly simulated trajectories under scenario $\mathbf{S}_s$. Since the investors have a perfect anticipation of the climate policy, the company's assets are computed as follows:

$$
\begin{aligned}
A_{i+1}^{\boldsymbol{\pi}(z),s,m,l} := \sum_{j=i+1}^{N-1} & \frac{S_j^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}) - T^C\left(s, j, S_j^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_j^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})\right)}{(1+r\delta)^{j-i-1}} \\
& + \frac{S_N^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}) - T^C\left(s, N,\ S_N^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_N^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})\right)}{r\delta(1+r\delta)^{N-i-2}} \\
& - \bar{Z}_i^s.
\end{aligned}
$$

iv. Compute the debt for trajectory m with the newly simulated trajectories and scenario $\mathbf{S}_s$:

$$D_{i+1}^{\boldsymbol{\pi}(z),s,m,l} = (1-\delta\zeta)D_i^{\boldsymbol{\pi}(z),s,m,l} + I^G\left(s, i, S_i^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), \gamma_i^z\right) + \left(\left(\frac{S_{i+1}^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})}{aP_{i+1}^s}\right)^{1/\theta} - (1-\delta d)\left(\frac{S_i^{\boldsymbol{\pi}(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})}{aP_i^s}\right)^{1/\theta}\right)^+.$$

(c) For each trajectory m, compute the default frequency as:

$$\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s,m} = \frac{1}{M_2}\sum_{l=1}^{M_2} \mathbf{1}\{A_{i+1}^{\boldsymbol{\pi}(z),s,m,l} < D_{i+1}^{\boldsymbol{\pi}(z),s,m,l}\}.$$

We thus get:

$$\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s} = \frac{1}{M_1}\sum_{m=1}^{M_1} \widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s,m}.$$

(d) Since the trajectories for which $\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s,m} = 1$ are defaulted a.s., they can no longer be considered in the computation of the default frequency due to the default condition (3.3). Hence, we need to remove them and reduce the number of trajectories M_1 accordingly: $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1} \mathbf{1}\{\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s,m} = 1\}$.

We detail this methodology in Algorithm 1:

Algorithm 1: Computation of the Probability of default under perfect or wrong anticipations with Nested Monte Carlo Simulations

Input: *Number of Trajectories:* $M_1 >> 1$, *Number of Nested Simulations:* $M_2 >> 1$, *True Scenario:* $\mathbf{S}_s$, *Strategy:* $\boldsymbol{\pi}(z, \mathbf{X}_0) = (\gamma_i^z)_{i=0:N-1}$.

Output: *PD term structure:* $\left(\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s}\right)_{i=0:N-1}$

Compute M_1 Monte Carlo paths for $\{(\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0))_{i=1:N}\}_{m=1:M_1} = \{\left(I_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0), S_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)\right)_{i=0:N}\}_{m=1:M_1}$ with strategy $\boldsymbol{\pi}$.

for $i = 0 : N-1$ **do**

  Compute $\bar{Z}_i^s$.

  for $m = 1 : M_1$ **do**

    for $l = 1 : M_2$ **do**

      Set strategy $\boldsymbol{\pi}_i(z, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)) := \left(\gamma_j^z(.)\right)_{j=i:N-1}$.

      Simulate a trajectory $\left(S_j^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_j^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})\right)_{j=i:N}$ starting with value $\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}$ in i.

      Compute the assets and debt:

$$A_{i+1}^{\boldsymbol{\pi}(z),s,m,l} := \sum_{j=i+1}^{N-1} \frac{S_j^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}) - T^C(s, j, S_j^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_j^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}))}{(1+r\delta)^{j-i-1}}$$

$$+ \frac{S_N^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}) - T^C(s, N, S_N^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_N^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}))}{r\delta(1+r\delta)^{N-i-2}} - \bar{Z}_i^s,$$

$$D_{i+1}^{\boldsymbol{\pi}(z),s,m,l} = (1-\delta\zeta) D_i^{\boldsymbol{\pi}(z),s,m,l} + I^G\left(s, i, S_i^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), \gamma_i^z(\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0))\right)$$

$$+ \left(\left(\frac{S_{i+1}^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})}{aP_{i+1}^s}\right)^{1/\theta} - (1-\delta d)\left(\frac{S_i^{\boldsymbol{\pi}_i(z),s,m,l}(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})}{aP_i^s}\right)^{1/\theta}\right)^+.$$

  Compute the default frequency as:

$$\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s} = \frac{1}{M_1}\sum_{m=1}^{M_1}\frac{1}{M_2}\sum_{l=1}^{M_2}\mathbf{1}\{A_{i+1}^{\boldsymbol{\pi}(z),s,m,l} < D_{i+1}^{\boldsymbol{\pi}(z),s,m,l}\}$$

  Remove all trajectories such that $\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s,m} = 1$.

  Update $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1}\mathbf{1}\{\widehat{\mathtt{PD}}_i^{\boldsymbol{\pi}(z),s,m} = 1\}$.

4 Impact of the re-evaluation of the anticipations on the transition scenario

4.1 Statement of the problem

In the previous section, we have considered both cases of perfect and imperfect anticipations regarding the future of the energy transition. In the latter, we have not allowed for the company to correct its carbon mitigation plan. This is the object of this section. Assume the company has designed its carbon reduction strategy at date 0 with respect to the state of its business model $\mathbf{X}_0$ and while anticipating that scenario $\mathbf{S}_z \in \mathcal{S}$ would unfold. We denote this strategy by $\pi(z) := (\gamma_i^z)_{i=0:N-1}$ with $\pi(z) \in \Pi$. However, the scenario that unfolds is $\mathbf{S}_s \in \mathcal{S}$, with $\mathbf{S}_s \neq \mathbf{S}_z$. The company becomes aware of its mistaken anticipation at some date $t \in \{1, \ldots, N-1\}$ and corrects its strategy for the latter part of the scenario. We assume that the investors know that the company is mistaken, but do not know when it will correct its strategy before it does happen. We assume the stopping time t assigns mass one to some integer in $\{1, \ldots, N-1\}$.

The trajectories are divided into 3 parts:

1. First, for any $i \in [0, t)$: The company does not know they are mistaken. Thus, they apply the original strategy $\boldsymbol{\pi}(z) = (\gamma_j^z)_{j=0:N-1}$.
2. Then, when $i = t$: The company realizes its mistake and designs a new admissible strategy $\boldsymbol{\pi}_t(s, \mathbf{X}_t^{\boldsymbol{\pi}(z),s}) = (\gamma_j^s)_{j=t:N-1}$ given its current business model's state $\mathbf{X}_t^{\boldsymbol{\pi}(z),s}$ and while anticipating that scenario $\mathbf{S}_s$ is unfolding. The starting point of this strategy is $\mathbf{X}_t^{\boldsymbol{\pi}(z),s}(0, \mathbf{X}_0)$ which is stochastic and $\mathcal{F}$-measurable, meaning the value of the state variable at date j computed with strategy $\boldsymbol{\pi}_{0:t}(z) = (\gamma_i^z)_{i=0:t-1}$ along scenario $\mathbf{S}_s$. This value is stochastic. Hence, the corrected strategy may not be the same depending on the state of the world at date t. Note that here, the goal is not to *"fill the gap"* between what the company has done and what it should have done, should its anticipations have been right. On the contrary, the company will factor in its current characteristics in its new business model and do "*what is best*" given these.
3. For $i \in [t, N]$: The company applies the newly designed strategy $\boldsymbol{\pi}_t(s) = (\gamma_j^s)_{j=t:N-1}$ and the change of strategy is announced to the investors that will now integrate it in their valuation.

We write as $\mathbf{X}_i^{t,z\to s} = (I_i^{t,z\to s}, S_i^{t,z\to s})$ the effective trajectory of the state variable at date $i \geq 0$ computed along scenario $\mathbf{S}_s$ with strategy $\boldsymbol{\pi}_{0:i}(z)$ until $i = t$, and strategy $\boldsymbol{\pi}_t(s) := (\gamma_j^s)_{j=t:N-1}$ after t. We denote the effectively applied strategy by $\boldsymbol{\pi}^{t,z\to s} = (\gamma_0^z, \ldots, \gamma_{t-1}^z, \gamma_t^s, \ldots, \gamma_{N-1}^s)$. This strategy is not fully known at date 0 and is gradually learned as the scenario unfolds. In particular, for $\mathbf{X}_0^{t,z\to s} = (I_0^{t,z\to s}, S_0^{t,z\to s}) = (I_0, S_0) \in \mathbb{R}^+ \times \mathbb{R}^+$, the dynamics are:

$$I_{i+1}^{t,z\to s} = \begin{cases} I_i^{t,z\to s} e^{-\delta(\gamma_i^z \mathbf{1}\{i<t\} + \gamma_i^s \mathbf{1}\{t\leq i<N\})} \times e^{\sigma_I \Delta\varepsilon_i^I - \psi_i^I(\sigma_I)}, & i = 0 : N-1, \\ I_N^{t,z\to s}, & i \geq N. \end{cases}$$

$$S_{i+1}^{t,z\to s} = \begin{cases} S_i^{t,z\to s} \times \frac{\bar{S}_{i+1}^s}{\bar{S}_i^s} e^{-\kappa^s \left(I_i^{t,z\to s} - I_i^{\mathrm{ref},s}\right)\delta} \times e^{\sigma_S \Delta\varepsilon_i^S - \psi_i^S(\sigma_S)}, & i = 0 : N-1 \\ S_N^{t,z\to s}, & i \geq N. \end{cases}$$

4.2 Probability of default

Under the assumptions described before, the investors do not know when the firm will correct its choice of business model in advance. Thus, they cannot factor in this into the firm's valuation before t. Hence, the firm's value for any i in $[0,t)$ is computed under the assumption that the company applies $\pi(z)$ until the end of the scenario:

$$A_i^{t,z\to s} = \mathbb{E}\left[\sum_{j=i}^{N-1} \frac{S_j^{\boldsymbol{\pi}(z),s} - T^C(s,j,S_j^{\boldsymbol{\pi}(z),s}, I_j^{\boldsymbol{\pi}(z),s})}{(1+r\delta)^{j-i}} + \frac{S_N^{\boldsymbol{\pi}(z),s} - T^C(s,N,S_N^{\boldsymbol{\pi}(z),s}, I_N^{\boldsymbol{\pi}(z),s})}{r\delta(1+r\delta)^{N-i-1}} \mid \mathcal{F}_i\right] - \bar{Z}_i^s, \quad i < t.$$

Then, in t, the firm designs its new strategy while anticipating the right scenario $\mathbf{S}_s$ and with respect to the current state of its business model $\mathbf{X}_t^{\boldsymbol{\pi}(z),s}$. The change of strategy is disclosed to the investors that can now factor in this in the firm's valuation. Note that the new strategy may directly depend on $\mathbf{X}_t^{\boldsymbol{\pi}(z),s}$, i.e. the state of the world at date t. The firm's assets become:

$$A_i^{t,z\to s} = \mathbb{E}\left[\sum_{j=i}^{N-1} \frac{S_j^{t,z\to s} - T^C(s,j,S_j^{t,z\to s}, I_j^{t,z\to s})}{(1+r\delta)^{j-i}} + \frac{S_N^{t,z\to s} - T^C(s,N,S_N^{t,z\to s}, I_N^{t,z\to s})}{r\delta(1+r\delta)^{N-i-1}} \mid \mathcal{F}_i\right] - \bar{Z}_i^s, \quad i \geq t.$$

Regarding the firm's debt, the dynamics are as follows:

$$D_i^{t,z\to s} = \begin{cases} (1-\zeta\delta)D_{i-1}^{t,z\to s} + I^G(s,i-1,S_{i-1}^{t,z\to s},\gamma_{i-1}^{z}) + \left(\left(\frac{S_i^{t,z\to s}}{aP_i^s}\right)^{1/\theta} - (1-d\delta)\left(\frac{S_{i-1}^{t,z\to s}}{aP_{i-1}^s}\right)^{1/\theta}\right)^+, & i < t, \\ (1-\zeta\delta)D_{i-1}^{t,z\to s} + I^G(s,i-1,S_{i-1}^{t,z\to s},\gamma_{i-1}^{s}) + \left(\left(\frac{S_i^{t,z\to s}}{aP_i^s}\right)^{1/\theta} - (1-d\delta)\left(\frac{S_{i-1}^{t,z\to s}}{aP_{i-1}^s}\right)^{1/\theta}\right)^+, & i = t : N. \end{cases}$$

We define the default under strategy $\boldsymbol{\pi}(z)$ in scenario $\mathbf{S}_s$ as the following stopping time:

$$\{\tau^{t,z\to s} = i\} = \{A_i^{t,z\to s} < D_i^{t,z\to s}, A_j^{t,z\to s} \geq D_j^{t,z\to s}, \forall j = 0,\cdots,i-1\}, \qquad (4.1)$$

The 1-period probability of default at date $i = 0 : N-1$ under scenario $\mathbf{S}_s$ with re-evaluation of the strategy at date $t > 0$ is:

$$\begin{aligned} \mathtt{PD}_{\mathtt{i}}^{\mathtt{t,z\to s}} :=& \mathbb{P}(\tau^{t,z\to s} = i+1 \mid \mathcal{F}_i) \\ =& \mathbb{P}(A_{i+1}^{t,z\to s} < D_{i+1}^{t,z\to s} \mid A_j^{t,z\to s} \geq D_j^{t,z\to s}, \forall j = 0:i)\mathbf{1}\{A_j^{t,z\to s} \geq D_j^{t,z\to s}, \forall j = 0:i\}. \end{aligned}$$

4.3 Numerical computation of the probability of default under imperfect anticipations with re-evaluation of the strategy

We propose another numerical methodology to compute the 1-period default probability based on Nested Monte Carlo Simulations. Let $\mathbf{S}_s$ and $\mathbf{S}_z$ be two scenarios in $\mathcal{S}$ such that $\mathbf{S}_s \neq \mathbf{S}_z$ and $\boldsymbol{\pi}^{t,z\to s} = (\gamma_0^z, \dots, \gamma_{t-1}^z, \gamma_t^s, \dots, \gamma_{N-1}^s) \in \Pi$ an admissible strategy. We write as $\mathbf{X}_j^{t,z\to s}(i,\mathbf{x}) = (I_j^{t,z\to s}(i,\mathbf{x}), S_j^{t,z\to s}(i,\mathbf{x})), \forall i = 0:N, j = 0:N, i \leq j$ the value of the state variable at date j starting with value $\mathbf{x} \in (\mathbb{R}^+)^2$ at date i and computed along scenario $\mathbf{S}_s$ under admissible strategy $\boldsymbol{\pi}^{t,z\to s} \in \Pi$.

1. Simulate $M_1 \gg 1$ trajectories $\{\left(\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0)\right)_{i=0:N}\}_{m=1:M_1} = \{\left(\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)\right)_{i=0:N}\}_{m=1:M_1}$ with starting value $\mathbf{X}_0 = (I_0, S_0)$ and strategy $\boldsymbol{\pi}(z)$ along scenario $\mathbf{S}_s$. At this stage, the re-evaluation of the strategy is not yet taken into account. We assume that the entire trajectory is initially computed under the misaligned strategy $\boldsymbol{\pi}(z)$ designed at date 0.
2. For $i = 0 : t-1$:
   (a) Compute the expected sum of future discounted physical damages in scenario $\mathbf{S}_s$:
   $$\bar{Z}_i^s = \begin{cases} \lambda_0^s \delta \dfrac{(1+r\delta)^{i+1}}{r\delta} \mathbb{E}\left[Z_1\right], & \text{if} \quad g^s = 1, \\ \dfrac{\lambda_0^s \left[(g^s)^{i\delta} - (g^s)^{(i-1)\delta} \right]}{\delta \ln(g^s)} \times \dfrac{1+r\delta}{1+r\delta-(g^s)^{\delta}} \mathbb{E}\left[Z_1\right], & \text{if} \quad (g^s) > 1. \end{cases} \tag{4.2}$$
   (b) For each simulated value $\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0), i < t, m = 1 : M_1$:
      i. Consider an admissible strategy $\boldsymbol{\pi}_i(z) = \left(\gamma_j^z(.)\right)_{j=i:N-1}$. This strategy starts in i and has been designed while anticipating scenario $\mathbf{S}_z$ at date 0. It does not encompass the re-evaluation of the firm's anticipations.
      ii. Resimulate M_2 trajectories starting with value $\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0) = \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)$ in i:
      $$\{\left(\mathbf{X}_j^{\boldsymbol{\pi}(z),s,m,l}\left(i, \mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}\right)\right)_{j=i:N}\}_{l=1:M_2}.$$
      iii. Compute the assets for trajectory m as the sum of future profits computed with the newly simulated trajectories on scenario $\mathbf{S}_s$:

$$\begin{aligned} A_{i+1}^{t,z\to s,m,l} := & \sum_{j=i+1}^{N-1} \frac{S_j^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}) - T^C\left(s,j,S_j^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_j^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})\right)}{(1+r\delta)^{j-i-1}} \\ & + \frac{S_N^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}) - T^C\left(s,N,\ S_N^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_N^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})\right)}{r\delta(1+r\delta)^{N-i-2}} \\ & - \bar{Z}_i^s. \end{aligned}$$

iv. Compute the debt for trajectory m with the newly simulated trajectories and scenario $\mathbf{S}_s$:

$$D_{i+1}^{t,z\to s,m,l} = (1-\delta\zeta)D_i^{\boldsymbol{\pi}(z),s,m,l} + I^G\left(s,i,S_i^{\boldsymbol{\pi}(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}),\gamma_i^z\right) + \left(\left(\frac{S_{i+1}^{\boldsymbol{\pi}(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})}{aP_{i+1}^s}\right)^{1/\theta} - (1-\delta d)\left(\frac{S_i^{\boldsymbol{\pi}(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})}{aP_i^s}\right)^{1/\theta}\right)^+.$$

v. For each trajectory m, compute the default frequency as:

$$\widehat{\mathtt{PD}}_i^{t,z\to s,m} = \frac{1}{M_2}\sum_{l=1}^{M_2}\mathbf{1}\{A_{i+1}^{t,z\to s,m,l} < D_{i+1}^{t,z\to s,m,l}\}.$$

We thus get:

$$\widehat{\mathtt{PD}}_i^{t,z\to s} = \frac{1}{M_1}\sum_{m=1}^{M_1}\widehat{\mathtt{PD}}^{t,z\to s,m}.$$

vi. Because the trajectories such that $\widehat{\mathtt{PD}}_i^{t,z\to s,m} = 1$ are defaulted a.s., they can no longer be considered in the computation of the default frequency due to the default condition (4.1). Hence, we need to remove them and reduce the number of trajectories M_1 accordingly: $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1}\mathbf{1}\{\widehat{\mathtt{PD}}_i^{t,z\to s,m} = 1\}$.

3. When $i = t$:

(a) Compute for each trajectory $\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0), m = 1,\dots,M_1$ a new strategy $\boldsymbol{\pi}_t^m(s,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) = \left(\gamma_j^s(\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})\right)_{j=t:N-1}$ with respect to the current state of the business model $\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}$ and along scenario $\mathbf{S}_s$. We stress that, for each trajectory m, a corresponding strategy $\boldsymbol{\pi}_t^m(s,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})$ is computed, which may depend on the state $\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}$ at date t. However, if the strategy is independent from the starting point, the complexity of the methodology reduces consequently. For readability purposes, we will write this strategy as $\boldsymbol{\pi}^{t,m}(s) = (\gamma_j^{t,s,m})_{j=t:N-1}$.

(b) Recompute each trajectory $m = 1,\dots,M_1$ with the new strategy from t to N:

$$\{\left(\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0)\right)_{i=t:N}\}_{m=1:M_1}$$

Where for any $i\in[t,N]$ and $m = 1:M_1$:

$$\begin{aligned}\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0) &= \mathbf{X}_i^{\boldsymbol{\pi}^{t,m}(s),s}\left(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)\right)\\ &= \left(I_i^{\boldsymbol{\pi}^{t,m}(s),s}(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}), S_i^{\boldsymbol{\pi}^{t,m}(s),s}(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})\right).\end{aligned}$$

In particular, $I_i^{\boldsymbol{\pi}^{t,m}(s),s}(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})$ and $S_i^{\boldsymbol{\pi}^{t,m}(s),s}(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})$, $i = t:N-1$, $m = 1:M_1$ are the respective value for the intensity and the sales at date $i \geq t$, with starting value $\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}$ at date t and strategy $\boldsymbol{\pi}^{t,m}(s)$ under trajectory m. Their dynamics write as:

$$I_{i+1}^{\boldsymbol{\pi}^{t,m}(s),s}(t, \mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) = I_i^{\boldsymbol{\pi}^{t,m}(s),s}(t, \mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})e^{-\delta\gamma_i^{t,s,m}} \times e^{\sigma_I \Delta\varepsilon_i^I - \psi_i^I(\sigma_I)},$$
$$S_{i+1}^{\boldsymbol{\pi}^{t,m}(s),s}(t, \mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) = S_i^{\boldsymbol{\pi}^{t,m}(s),s}(t, \mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})\frac{\bar{S}_{i+1}^s}{\bar{S}_i^s}e^{-\kappa^s\left(I_i^{\boldsymbol{\pi}^{t,m}(s),s}(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) - I_i^{\mathrm{ref},s}\right)\delta - \sigma_S\Delta\varepsilon_i^S - \psi_i^S(\sigma_S)}.$$

Thanks to the exponential feature of the state variable, one gets:

$$I_{i+1}^{\boldsymbol{\pi}^{t,m}(s),s}(t, \mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) = I_t^{\boldsymbol{\pi}(z),s,m}(0, X_0)e^{-\delta\sum_{j=t}^{i}(\gamma_j^{t,s,m} - \gamma_j^z)},$$
$$S_{i+1}^{\boldsymbol{\pi}^{t,m}(s),s}(t, \mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) = S_t^{\boldsymbol{\pi}(z),s,m}(0, X_0)e^{-\kappa^s\sum_{j=t}^{i}\left(I_j^{\boldsymbol{\pi}^{t,m}(s),s}(t,\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}) - I_j^{\boldsymbol{\pi}(z),s,m}(0,X_0)\right)\delta}.$$

We can thus adjust the latter part of the trajectories analytically without resimulating additional randomness. The beginning, i.e. $\{\left(\mathbf{X}_i^{t,z\to s,m}(0, \mathbf{X}_0)\right)_{i=0:t-1}\}_{m=1:M_1}$ is left unchanged.

4. For $i = t : N-1$: We now consider trajectories starting in t with value $\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}(0, \mathbf{X}_0)$.
   (a) Compute $\bar{Z}_i$ as in (4.2).
   (b) Consider an admissible strategy $\boldsymbol{\pi}_i^{t,m}(s) = \left(\gamma_j^{t,s,m}(.)\right)_{j=i:N-1}$. This strategy starts in $i \geq t$ and has been designed while anticipating the right scenario, i.e. $\mathbf{S}_s$ and with respect to the value of the state variable in t $\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m}(0, \mathbf{X}_0)$. The strategy may differ depending on the trajectory considered.
   (c) For each trajectory $m = 1 : M_1$:
      i. Resimulate M_2 trajectories starting with value $\mathbf{X}_i^{t,z\to s,m}(0, \mathbf{X}_0)$ in i and strategy $\boldsymbol{\pi}^{t,m}(s)$:

$$\{\left(\mathbf{X}_j^{\boldsymbol{\pi}^{t,m}(s),s,m,l}\left(i, \mathbf{X}_i^{t,z\to s,m}\right)\right)_{j=i:N}\}_{l=1:M_2}.$$

      ii. Compute the assets for trajectory m as the sum of future profits computed with the newly simulated trajectories under strategy $\boldsymbol{\pi}^{t,m}(s)$ on scenario $\mathbf{S}_s$:

$$A_{i+1}^{t,z\to s,m,l} := \sum_{j=i+1}^{N-1}\frac{S_j^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m}) - T^C\left(s, j, S_j^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m}), I_j^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m})\right)}{(1+r\delta)^{j-i-1}}$$
$$+ \frac{S_N^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m}) - T^C\left(s, N,\ S_N^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m}), I_N^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m})\right)}{r\delta(1+r\delta)^{N-i-2}}$$
$$- \bar{Z}_i^s.$$

iii. Compute the debt for trajectory m with the newly simulated trajectories and scenario $\mathbf{S}_s$:

$$D_{i+1}^{t,z\to s,m,l} = (1-\delta\zeta)D_i^{t,z\to s,m,l} + I^G\left(s, i, S_i^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m}), \gamma_i^{t,s,m}\right)$$
$$+ \left(\left(\frac{S_{i+1}^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m})}{aP_{i+1}^s}\right)^{1/\theta} - (1-\delta d)\left(\frac{S_i^{\boldsymbol{\pi}_i^{t,m}(s),s,l}(i, \mathbf{X}_i^{t,z\to s,m})}{aP_i^s}\right)^{1/\theta}\right)^+.$$

(d) For each trajectory m, compute the default frequency as:

$$\widehat{\mathtt{PD}}_i^{t,z\to s,m} = \frac{1}{M_2}\sum_{l=1}^{M_2} \mathbf{1}\{A_{i+1}^{t,z\to s,m,l} < D_{i+1}^{t,z\to s,m,l}\}.$$

We thus get:

$$\widehat{\mathtt{PD}}_i^{t,z\to s} = \frac{1}{M_1}\sum_{m=1}^{M_1} \widehat{\mathtt{PD}}_i^{t,z\to s,m}.$$

(e) Because the trajectories such that $\widehat{\mathtt{PD}}_i^{t,z\to s,m} = 1$ are defaulted a.s., they can no longer be considered in the computation of the default frequency due to the default condition (3.3). Hence, we need to remove them and reduce the number of trajectories M_1 accordingly: $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1} \mathbf{1}\{\widehat{\mathtt{PD}}_i^{t,z\to s,m} = 1\}$.

We detail this methodology in Algorithm 2:

Algorithm 2: Computation of the Probability of default with re-evaluation of the strategy using Nested Monte Carlo Simulations

Input: *Number of Trajectories:* $M_1 >> 1$,
Number of Nested Simulations: $M_2 >> 1$,
True scenario : $\mathbf{S}_s$,
Anticipated scenario : $\mathbf{S}_z$,
Re-evaluation date : t,
Strategy: $\boldsymbol{\pi}(z, \mathbf{X}_0) = (\gamma_i^z)_{i=0:N-1}$.

Output: *PD term structure:* $\left(\widehat{\mathtt{PD}}_i^{t,z\to s}\right)_{i=0:N-1}$.

1 Compute M_1 Monte Carlo paths for $\{(\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0))_{i=1:N}\}_{m=1:M_1} = \{(\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0))_{i=1:N}\}_{m=1:M_1} = \{\left(I_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0), S_i^{\boldsymbol{\pi}(z),s,m}(0,\mathbf{X}_0)\right)_{i=0:N}\}_{m=1:M_1}$ under strategy $\boldsymbol{\pi}$.

2 **for** $i = 0 : t-1$ **do**

3   Compute $\bar{Z}_i^s$.

4   **for** $m = 1 : M_1$ **do**

5   **for** $l = 1 : M_2$ **do**

6   Set strategy $\boldsymbol{\pi}_i(z) := \left(\gamma_j^z(.)\right)_{j=i:N-1}$.

7   Simulate a trajectory $\left(S_j^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}), I_j^{\boldsymbol{\pi}_i(z),s,m,l}(i,\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m})\right)_{j=i:N}$ starting with value $\mathbf{X}_i^{\boldsymbol{\pi}(z),s,m}$ in i.

8   Compute the assets $A_{i+1}^{t,z\to s,m,l} = A_{i+1}^{\boldsymbol{\pi}(z),s,m,l}$ and $D_{i+1}^{t,z\to s,m,l} = D_{i+1}^{\boldsymbol{\pi}(z),s,m,l}$

9   Compute the default frequency as:

$$\widehat{\mathtt{PD}}_i^{t,z\to s} = \frac{1}{M_1}\sum_{m=1}^{M_1}\widehat{\mathtt{PD}}_i^{t,z\to s,m} = \frac{1}{M_1}\sum_{m=1}^{M_1}\frac{1}{M_2}\sum_{l=1}^{M_2}\mathbf{1}\{A_{i+1}^{t,z\to s,m,l} < D_{i+1}^{t,z\to s,m,l}\}$$

10   Remove all trajectories such that $\widehat{\mathtt{PD}}_i^{t,z\to s,m} = 1$.

11   Update $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1}\mathbf{1}\{\widehat{\mathtt{PD}}_i^{t,z\to s,m} = 1\}$.

12 Compute $\boldsymbol{\pi}^{t,m}(s) = \left(\gamma_j^s(\mathbf{X}_t^{\boldsymbol{\pi}(z),s,m})\right)_{j=t:N-1}$ for each $m = 1 : M_1$.

13 Update $\{(\mathbf{X}_i^{t,z\to s,m}(0,\mathbf{X}_0))_{i=t:N}\}_{m=1:M_1}$.

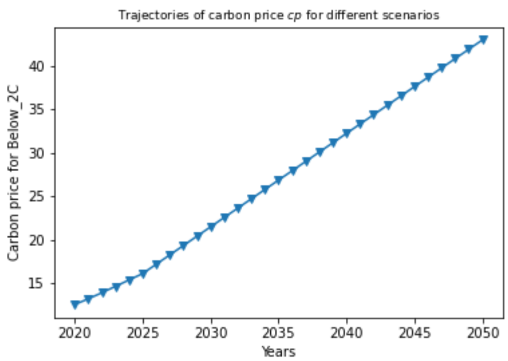
for $i = t : N-1$ **do**

14   Repeat steps 3 to 11 with the updated trajectories and the new strategy $\boldsymbol{\pi}^{t,m}(s)$.

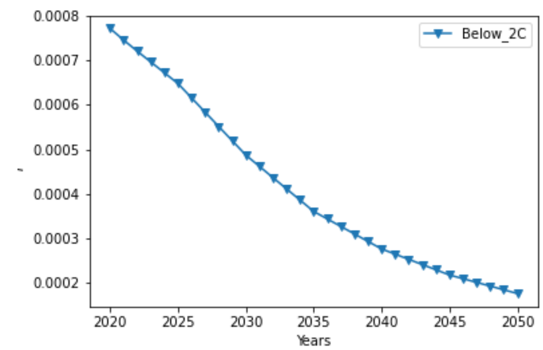
5 Application to fictitious companies

5.1 Description of the scenarios

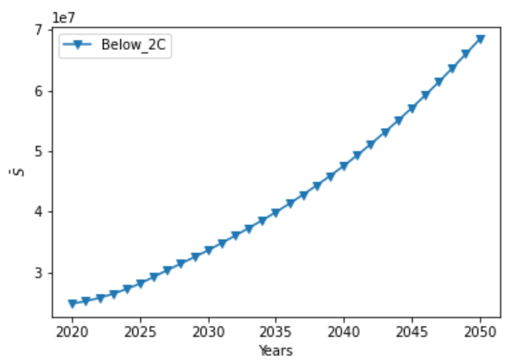
The application of the model is based on the NGFS phase II *Below 2 Degrees* (B2C) energy transition scenario [Richters et al., 2022]. In this scenario, the climate policy stringency increases gradually, yielding a 67% chance of limiting warming to below 2°C. We focus on NACE Rev2 sector D35 :*"Electricity, gas, steam and air conditioning supply"*. An overview of the relevant trajectories is shown in Figure 1.



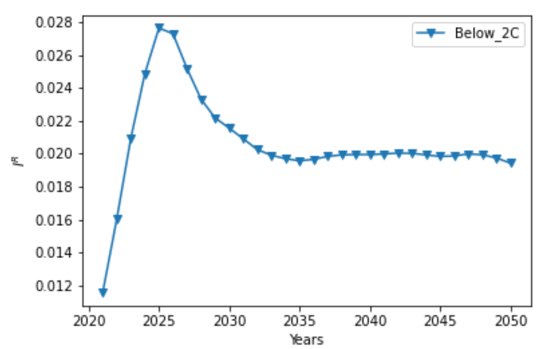
(a) Carbon price $\mathtt{cp}$.



(b) Reference intensity.



(c) Sales revenue from the reference market $\bar{S}$.



(d) Inflation rate.

Figure 1: Trajectories of carbon price ($\mathtt{cp}$), reference intensity (I^{ref}), sector sales revenue ($\bar{S}$) and inflation rate ($I^R s$) extracted from the NGFS phase II *'Below 2 Degrees'* scenario for sector D35 for France. The carbon price trajectory for scenario DT should be read on the right-hand-side y-axis.Starting point: $\mathtt{cp}_0 = 12.56\mathtt{USD}/\mathtt{tCO}_2$, $I_0^{ref} = 7.71E{-}4\mathtt{tCO}_2/\mathtt{USD}$, $\bar{S}_0 = \mathtt{USD}2,48 \times 10^7$.

5.2 Choice of parameters

We have considered a 6 dates problem ($N = 6, \delta = 5$), and have run tests on three fictitious companies, that only differ in their initial intensity level: $I_0^1 = 1.16E{-}3$ (brown), $I_0^2 = 7.71E{-}4$ (yellow), $I_0^3 = 3.86E{-}4$ (green). Note that $I_0^{ref} = I_0^2$ in all scenarios. Table 1 summarizes all other parameters used, unless stated otherwise. Due to the stability of companies' revenues and intensity, we have chosen low volatility levels (2%). The disturbance process is $(\Delta\varepsilon_i^I, \Delta\varepsilon_i^S) = (\sqrt{\delta}\min(\max(Z^1, -3), 3); \sqrt{\delta}\min(\max(Z^2, -3), 3))$ for all i, where (Z^1, Z^2) is a centered and standardized bivariate Gaussian variable with $\text{cov}(Z^1, Z^2) = -0.3$.

The parameters chosen for the green investment function are the same as those proposed by default in [Nordhaus, 2017]. We have set γ_{max} such that the maximum yearly decrease rate of intensity is no greater than 40%. Note that this is quite a high upper bound. The parameters of the production and cost functions are all set to 1, we thus consider the profit as a linear function of S.

We do not specify a distribution for the physical shock Z since it is not required for the PD computation (see Algorithm 1). We set its expectation to 100. We assume physical risks shocks have an average frequency of once every five years, and keep this frequency constant throughout the analysis ($g = 1$). The initial outstanding debt $D_0 = 1.5E5$, and is the same across all firms and strategies. Finally, we set $\zeta = 12\%$ and $d = 5.5\%$. This amounts to an average residual maturity of around 8.3 years for the outstanding debt, and an average life of approximately 18 years for the productive capital. We ran $M_1 = 10\text{E}4$ simulations with $M_2 = 200$.

σ_I	S_0	σ_S	κ	α	β	c	r	γ_{max}	k	ν	a	θ	$\mathbb{E}[Z_1]$	λ_0	g	D_0	ζ	d
2%	$8.4E5$	2%	30	0.95	2.8	1.26	6%	$-\log(0.6)$	1	1	1	1	100	1/5	1	$1.5E5$	12%	5.5%

Table 1: Values of model parameters for the toy example unless stated otherwise.

5.3 Description of the strategy

For the numerical application, we have considered the "minimum transition costs" strategy ("*min costs*"). This strategy is interesting because it is sensitive to all the components of an energy transition scenario. Indeed, it is defined so as to minimize the firm's future carbon cost in the cheapest way over the time horizon $[0, T]$. At the end of the time horizon, it is assumed that the company may no longer reduce its carbon cost and has to endure some uncontrollable discounted carbon cost until the end of times. This is illustrated with some non-negative function of our controlled system denoted by $\mathcal{C}_N$. Assume a constant carbon cost $\texttt{cp}_N I_N S_N$ for any $j \geq N$, to be paid indefinitely by the company after the end of the scenario. This quantity is then discounted, which leads to:

$$\mathcal{C}_N(\mathbf{X}_N) := \texttt{cp}_N \mathbf{X}_N^{(1)} \mathbf{X}_N^{(2)} \frac{1 + r\delta}{r\delta}.$$

With initial state space $\mathbf{X}_0 = \mathbf{x} = (I, S)$ (for intensity and sales level), and given a random intensity reduction strategy $\boldsymbol{\pi} = (\gamma_i)_{i=0:N-1}$, the total expected cost of the strategy $\boldsymbol{\pi}$ is:

$$J_{\boldsymbol{\pi}}(0, \mathbf{x}) := \mathbb{E}\left[\sum_{i=0}^{N-1} \frac{\mathtt{cp}_i \mathbf{X}_i^{(1)} \mathbf{X}_i^{(2)} + I^G(i, \mathbf{X}_i^{(2)}, \gamma_i)}{(1 + r_i\delta)^i} + \frac{\mathcal{C}_N(\mathbf{X}_N)}{(1 + r\delta)^N} \mid \mathbf{X}_0 = \mathbf{x}\right].$$

The company selects $\boldsymbol{\pi}^{\text{min costs}} = (\gamma_i)_{i=0:N-1}$ such that:

$$\boldsymbol{\pi}^{\text{min costs}} := \arg \inf_{\boldsymbol{\pi} \in \Pi} J_{\boldsymbol{\pi}}(0, \mathbf{x}),$$

The existence of a Markov non randomized solution for this discrete time stochastic problem has been proven in [Ndiaye et al., 2024b] based on the dynamic programming principle. Moreover, they have proposed an algorithm for its numerical resolution called *Backwards Sampling*. For the following, we will denote this strategy by π. Other strategies such as following the same decrease rate of the sector's average intensity or choosing γ such that the green investment equals the carbon cost at each date i could also be considered. However, the former is only depends to $I^{\text{ref,s}}$. Since we intend to focus on each transition risk transmission channel independently in our analysis, this strategy lacks pertinence in this setting. Regarding the latter, it is not designed in a forward looking manner, i.e. without any anticipations regarding the scenario. The inadequacy of this strategy for our analysis is also clear.

5.4 PD under wrong anticipations with no strategy re-evaluation

To isolate the impact of the misanticipation of each of the transition risk drivers on the credit risk, we focus on reduced scenarios $\mathbf{S}_s$ and $\mathbf{S}_z$, $z, s \in \{1, \ldots, H\}, z \neq s$ which only differ in one component. We study the impact of each transition risk driver on credit risk individually, namely policy (through the carbon price $\mathtt{cp}$), technology (through the autonomous factor of abatement cost decrease α), and the consumer sentiment (through the market sensitivity to the relative intensity κ).

5.4.1 Main Findings.

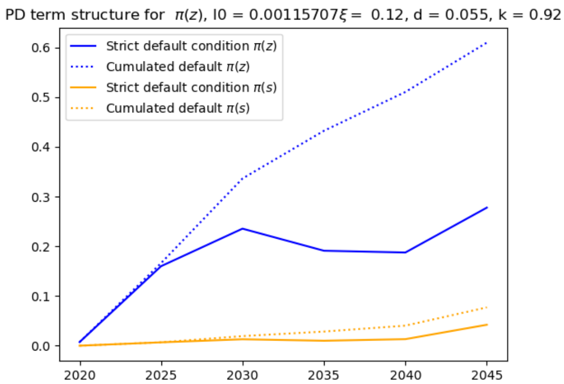
- The anticipations regarding the consumer sentiment for firms with a much lower initial intensity than that of their sector have negligible impact on its default probability.
- Anticipating a more favorable scenario than the actual one regarding the policy and consumer sentiment driver, leads to a greater credit risk than a correct anticipation. The gap widens with the initial intensity.
- Anticipating a more stringent scenario than the actual one regarding the policy and consumer sentiment driver, leads to a lower credit risk than correct anticipations. The gain increases with the initial intensity.
- Regarding the technology driver of transition risks, ambitious carbon reduction strategies are rewarded with lower credit risk, despite more investment. Conversely, shy strategies are penalized by augmented credit risk.
- In turn, this suggests that it is *"better safe than sorry"* regarding the transition.

5.5 Misanticipation of the carbon price trajectory $\mathtt{cp}^s$

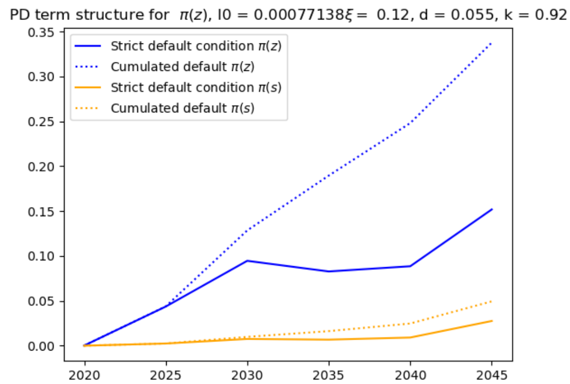
We evaluate the impact of the misanticipation of the carbon price trajectory by considering two possibilities: the trajectory from the B2C scenario and a flat carbon tax $\mathtt{cp}_i = \mathtt{cp}_0$ $\forall i = 0 : N$.

Assuming a flat carbon price. In this case, the 3 firms (brown, yellow, and green) anticipate a flat carbon price in date 0 when designing their strategies: $\mathtt{cp}_i^z = \mathtt{cp}_0$ for all $i = 0 : N$. However, the actual carbon price trajectory is not flat and follows exactly the one described in scenario B2C. For the brown company (Fig. 2a), the misanticipation of the carbon price trajectory largely increases the credit risk. Table 2 shows that despite conducting a carbon reduction strategy, anticipating a gentler climate policy than the actual one can multiply the PDs by up to 20 for a brown company. Indeed, the misanticipation of future carbon costs leads the company to conduct a shy carbon reduction strategy. In turn, the carbon costs are driven up and sales growth is slowed down, thus slashing the firm's valuation, in spite of lower debt.

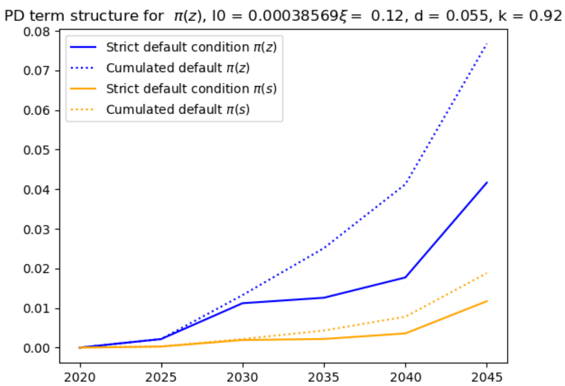
Similar results are seen for the yellow (Fig. 2b) and green (Fig. 2c) companies. In particular, the impact of the misanticipation decreases with the company's initial intensity level (Tables 3 and 4). Thus, anticipating a gentler carbon price trajectory than the actual scenario augments the firm's credit risk, regardless of its initial relative emissions.



(a) Brown Company



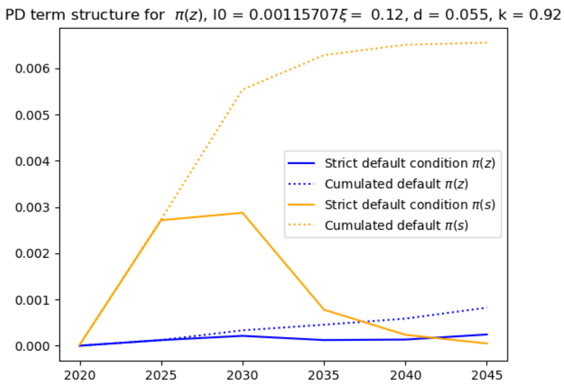
(b) Yellow Company



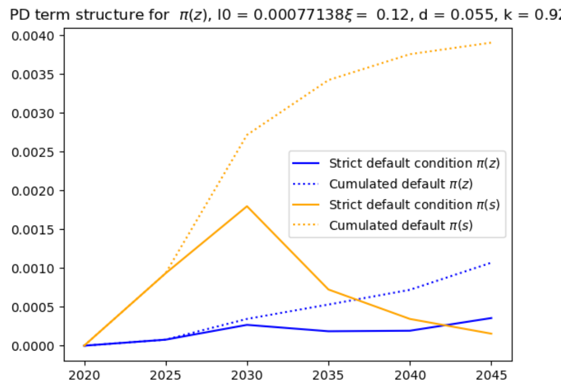
(c) Green Company

Figure 2: PD term structure for the three firms under a **flat carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating a flat carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating the carbon price.

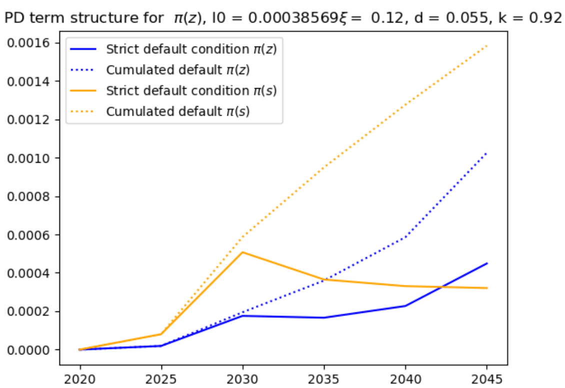
Assuming an increasing carbon price In this case, the 3 firms (brown, yellow, and green) anticipate the carbon price from the scenario B2C in date 0. However, the actual carbon price trajectory is flat: $\mathtt{cp}_i^s = 0$ for all $i = 0 : N$. For the three firms (Fig. 3), misanticipating an increasing carbon tax leads to a lower credit risk with respect to correct anticipations. In this case, the firms actually anticipate a more stringent climate policy than what actually happens. In turn, they increase their cuts in relative emissions. Despite more carbon mitigation, thus more debt, the companies' financial health is actually reinforced thanks to boosted sales and lower carbon costs, thus better total assets. In particular, the gains in terms of PD increase with the company's initial intensity level. In other words, the "*browner*" a company is, the more it benefits by doing more than the scenario requires.



(a) Brown Company



(b) Yellow Company



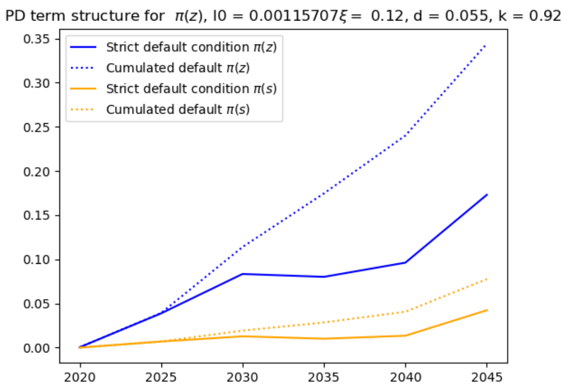
(c) Green Company

Figure 3: PD term structure for the three firms under an **increasing carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating an increasing carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating the carbon price.

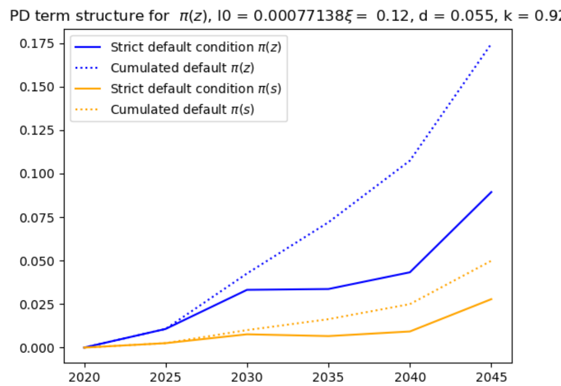
5.6 Misanticipation of the autonomous factor of abatement cost decrease α

We evaluate the impact of the misanticipation of the autonomous factor of abatement cost decrease by considering two possibilities: constant unitary abatement costs $\alpha = 1$ and decreasing unitary abatement costs $\alpha = 0.95$.

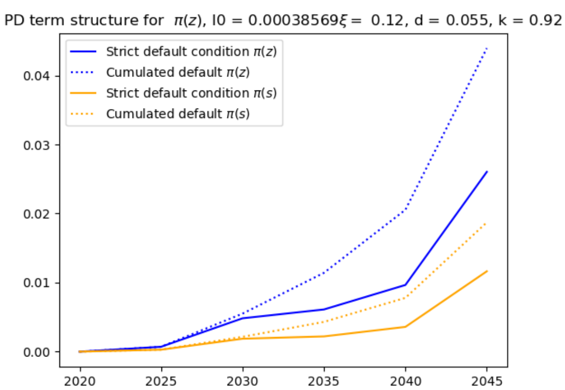
Assuming constant abatement costs. In this case, the three companies assume that the marginal technology costs will remain constant throughout the scenario $\alpha^z = 1$, whereas the costs actually decrease $\alpha^s = 0.95$. Thus, they assume a more stringent scenario in terms of green technology costs than the actual one. This assumption leads to less actual carbon reduction than with $\boldsymbol{\pi}(s)$ to manage the green investment costs. Despite managed debt, the firm's assets are negatively impacted on two fronts: on the one hand, the carbon cost is inflated; on the other hand, the sales are slowed down due to higher intensity. This leads to degraded credit risk (Figure 4), which increases with time. Indeed, the strategy $\pi(s)$ takes advantage of the future decline of green technology to undertake more carbon cuts by the end of the scenario. Furthermore, this degradation increases with the starting level intensity.



(a) Brown Company



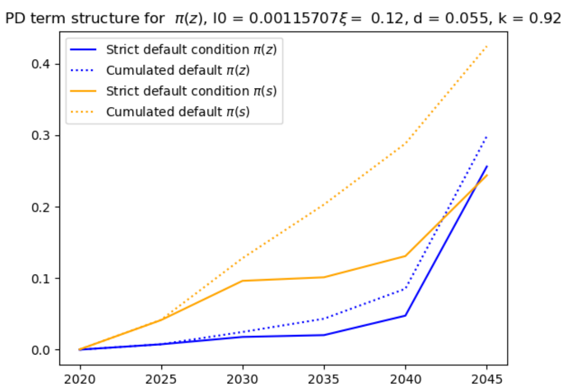
(b) Yellow Company



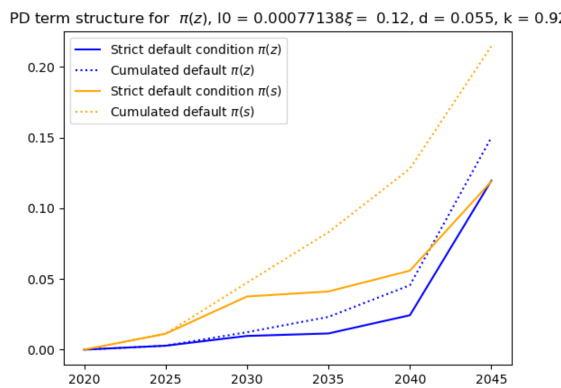
(c) Green Company

Figure 4: PD term structure for the three firms under a **constant unitary abatement costs misanticipation**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating $\alpha^z = 1$. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating $\alpha^s = 0.95$.

Assuming decreasing unitary abatement costs. Here, the companies assume decreasing unitary abatement costs ($\alpha^z = 0.95$), whereas the scenario has constant unitary abatement costs. In other words, they anticipate a more favorable scenario than the one that does unfold in terms of technology costs ($\alpha^s = 1$). This misassumption leads to a different timing of the carbon reduction strategy. With decreasing costs, the firm is more inclined to invest towards the end of the scenario. This leads to greater carbon reduction perspectives than with $\boldsymbol{\pi}(s)$. In turn, the firm's value with strategy $\boldsymbol{\pi}(z)$ is greater than with $\boldsymbol{\pi}(s)$ from the beginning of the scenario, driving the PD down. Nonetheless, because the gap in the green investment costs' anticipation and the actual costs widens, this strategy requires more debt than $\boldsymbol{\pi}(s)$, which makes the PD shoot up in the final years of the scenario. However, both in strict and cumulated terms, the firm is less likely to default with $\boldsymbol{\pi}(z)$ thanks to more carbon reduction (Fig. 5).

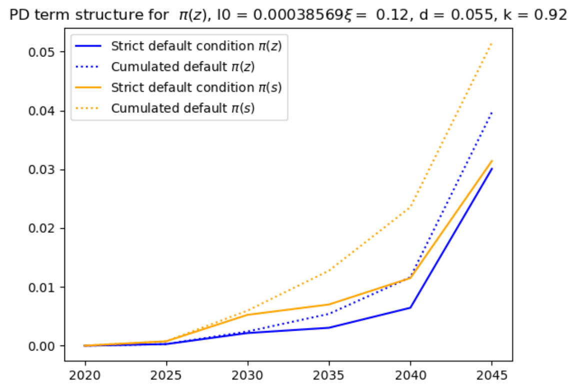


(a) Brown Company



(b) Yellow Company

Figure 5: PD term structure for the three firms under a **misanticipation of decreasing unitary abatement costs**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating $\alpha^z = 0.95$. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating $\alpha^s = 1$.



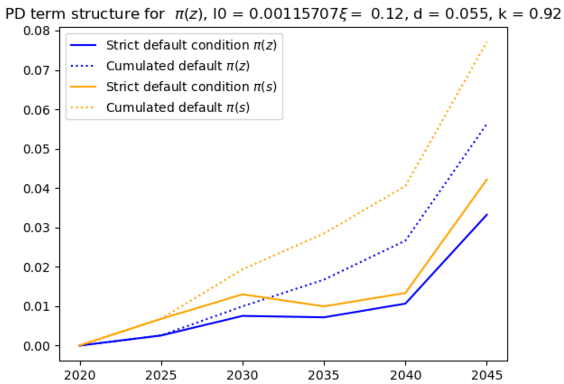
(c) Green Company

Figure 5: PD term structure for the three firms under a **misanticipation of decreasing unitary abatement costs**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating $\alpha^z = 0.95$. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating $\alpha^s = 1$.

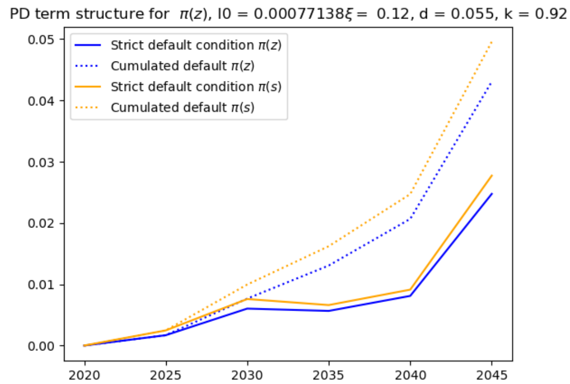
5.7 Misanticipation of the market sensitivity to the relative intensity κ

We evaluate the impact of the misanticipation of the market sensitivity to the relative intensity by considering two possibilities: constant market shares $\kappa = 0$ and adaptive market shares $\kappa = 30$.

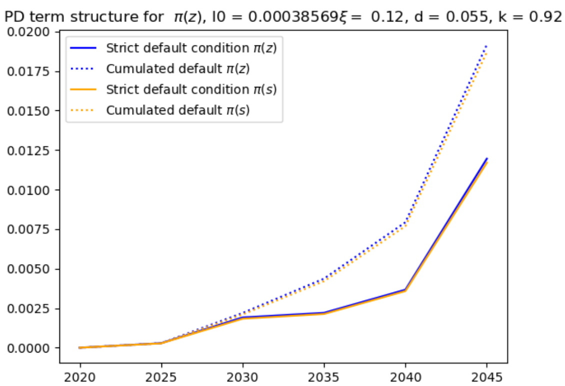
Assuming constant market shares. The firms assume constant market shares ($\kappa^z = 0$) whereas the market shares adapt with respect to the firms' relative intensity to the sectoral average ($\kappa^s = 30$). Because the consumer sentiment driver of transition risk may boost sales due to lower intensity, thus improving the firm's credit risk through the assets, anticipating constant market shares actually amounts to anticipating a more stringent scenario. Similarly to the other two drivers, misanticipating constant market shares reduces the firm's credit risk for brown and yellow companies (Figs 6a and 6b). The reduction in the PD grows with the starting level intensity. However, for green companies, the error in anticipation has little impact on the default probability, suggesting that the perspective of a boost in sales does not affect the firms' minimum transition costs carbon reduction strategy in this case.



(a) Brown Company



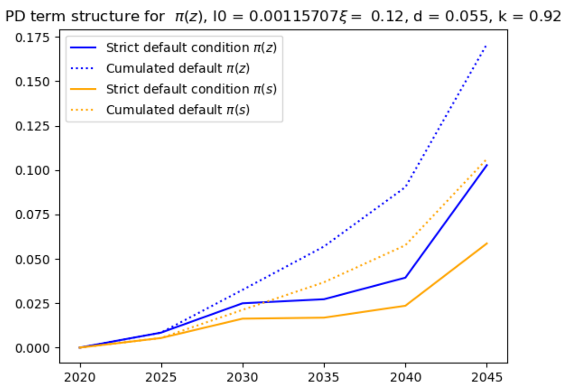
(b) Yellow Company



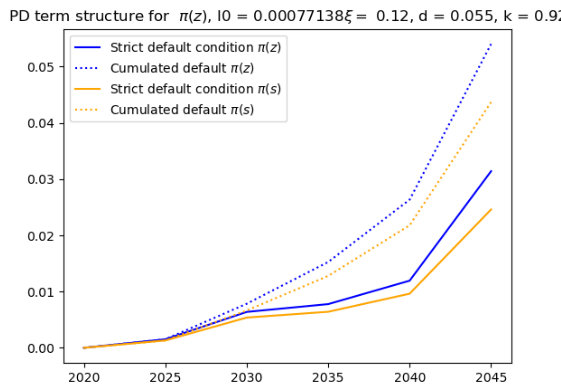
(c) Green Company

Figure 6: PD term structure for the three firms under a **constant market shares misanticipation**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating $\kappa^z = 0$. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating $\kappa^s = 30$.

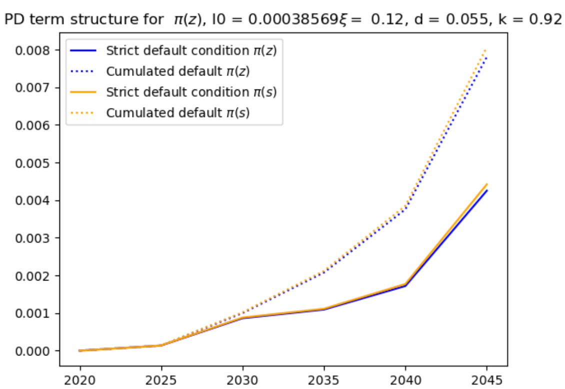
Assuming adaptive market shares. The firms anticipate adaptive market shares $\kappa^z = 30$. However, the market is not sensitive to the firms' relative emissions compared to the rest of its sector ($\kappa^s = 0$). Because $\kappa > 0$ offers a boost in a company's sales when its intensity decreases, assuming $\kappa^z = 30$ actually amounts to anticipating a more favorable scenario. Here, this misanticipation leads the firm to underestimate the impact of the transition, leading to an increased probability of default in all years for both the yellow (Fig. 7b) and the brown (Fig. 7a) companies. Similarly to the case $\kappa^z = 0$, the green company is barely affected by the error in the firm's anticipations (Fig. 7c).



(a) Brown Company



(b) Yellow Company



(c) Green Company

Figure 7: PD term structure for the three firms under a **adaptive market shares misanticipation**. $\boldsymbol{\pi}(z)$ refers to the Min Costs strategy while misanticipating $\kappa^z = 30$. $\boldsymbol{\pi}(s)$ refers to the Min Costs strategy while correctly anticipating $\kappa^s = 0$.

5.8 PD under wrong anticipations with strategy re-evaluation

5.8.1 Methodology for the computation of the re-evaluated strategy.

We study the impact of a wrong anticipation on the climate policy with subsequent re-evaluation of the strategy for a company. Given the results of the past section, we choose to focus solely on the yellow firm, and a misassumption regarding the future carbon price trajectory. Precisely, we suppose that the company anticipates a flat carbon tax ($\mathtt{cp}_i^z = \mathtt{cp}_0$ for all $i = 0 : N$) while designing its carbon reduction strategy. However, the carbon price is increasing and follows directly the trajectory described in the NGFS Phase II *Below 2 Degrees* scenario (B2C). We have thus considered a case where the firm anticipates a more favorable transition scenario than the one that actually unfolds. We consider the impact of re-evaluating the strategy at different dates $t \in 1, \dots, N-1$ on the firm's probability of default. For computational reasons, we consider the minimum transition costs in a deterministic setting (i.e. $\sigma_S = \sigma_I = 0$) for the re-evaluation in date $t \in \{1, \dots, N-1\}$. The objective function is written as:

$$
\begin{aligned}
J_{\boldsymbol{\pi}_t}(t, \mathbf{x}) &:= \mathbb{E}\left[\sum_{i=t}^{N-1} \frac{\mathtt{cp}_i \mathbf{X}_i^{(1)} \mathbf{X}_i^{(2)} + I^G(i, \mathbf{X}_i^{(2)}, \gamma_i)}{(1+r\delta)^i} + \frac{\mathcal{C}_N(\mathbf{X}_N)}{(1+r\delta)^N} \mid \mathbf{X}_t = \mathbf{x}\right] \\
&= \mathbb{E}\left[\sum_{i=t}^{N-1} \frac{\mathcal{C}_i(\mathbf{X}_i, \gamma_i)}{(1+r\delta)^i} + \frac{\mathcal{C}_N(\mathbf{X}_N)}{(1+r\delta)^N} \mid \mathbf{X}_t = \mathbf{x}\right]
\end{aligned}
$$

where r is the risk-free rate and $\mathcal{C}_i(I, S, \gamma)$ is the one-stage cost function defined by:

$$
\mathcal{C}_i(\mathbf{X}, \gamma) = \mathtt{cp}_i \mathbf{X}^{(1)} \mathbf{X}^{(2)} + I^G(i, \mathbf{X}^{(2)}, \gamma) \qquad \text{if} \quad i \in [t, N).
$$

The company selects $\boldsymbol{\pi}_t = (\gamma_i)_{i=t:N-1}$ such that:

$$
J^*(0, \mathbf{x}) := \inf_{\boldsymbol{\pi}_t \in \Pi_t} J_{\boldsymbol{\pi}}(0, \mathbf{x}),
$$

where Π_t is the set of admissible strategies $\boldsymbol{\pi}_t = (\gamma_i)_{i=t:N-1}$ i.e. $\mathcal{F}$-measurable and such that $\gamma_i \in [0, \gamma_{max}]$ for all $i = t : N-1$. Since the value of the sales revenues does not depend on the control process at date i in this particular setting, we consider the following dynamic programming equation.

$$
\gamma_i^*(\mathbf{x}) = \arg\min_{\gamma \in [0, \gamma_{max}]} \left(\mathcal{C}_i(\mathbf{x}, \gamma) + \frac{1}{1+r\delta} J^*(i+1, \mathbf{X}_{i+1}(\mathbf{x}))\right), \quad \forall i = N-1, \dots, t, \quad \mathbf{x} \in (\mathbb{R}^+)^2,
$$

$$
J^*(N, \mathbf{x}) = \mathcal{C}_N(\mathbf{x}).
$$

We have adapted the Backwards Sampling algorithm from [Ndiaye et al., 2024b, Algorithm 1] to fit this deterministic setting.

1. Starting with a given deterministic strategy $\boldsymbol{\pi}_t^0$, we work our way from $N-1$ towards $t \in 1, \dots, N-1$, and aim to solve the dynamic programming equation at each date i. We design $\boldsymbol{\pi}_t^0$ such that the intensity reduction rate between any two dates i and $i+1$ is the same as the sector's average intensity.

$$
\boldsymbol{\pi}_t^0 := \left(\gamma_i^0\right)_{i=0:N}, \quad \text{s.t.} \quad e^{-\gamma_i^0 \delta} = \max\left(\frac{I_{i+1}^{ref}}{I_i^{ref}}; e^{-\gamma_{max}\delta}\right).
$$

2. We compute the single trajectory for the state variable under this strategy:

$$\left(\mathbf{X}_i^{\boldsymbol{\pi}_t^0}(t, \mathbf{X}_i^{t,z\to s,m})\right)_{i=t:N} = \left(I_i^{\boldsymbol{\pi}_t^0}(t, \mathbf{X}_i^{t,z\to s,m}), S_i^{\boldsymbol{\pi}_t^0}(t, \mathbf{X}_i^{t,z\to s,m})\right)_{i=t:N}, \quad m \in 1, \ldots, M_1.$$

3. For $i = N-1 : t$, we seek the optimal process as a function of I by computing the trajectories over a grid of 100 possible values for $\gamma \in [0, \gamma_{max}]$ and by selecting the value yielding the lowest cost. We repeat this step for 100 values for I, $\{I_i^{u,m}\}_{u=1:100}$ taken uniformly in $(0, I_i^{\boldsymbol{\pi}_t^0}(t, \mathbf{X}_i^{t,z\to s,m})]$ since the intensity can only decrease asymptotically towards 0. For each I_i^u, $u = 1 : 100$ we get one $\gamma_i^{u,m}$ that confers the lowest value for the dynamic programming algorithm on the grid considered.
4. We then learn the optimal process function by regressing the optimal values for $\{\gamma_i^{u,m}\}_{u=1:100}$ on $\{I_i^{u,m}\}_{u=1:100}$ and get $\boldsymbol{\pi}_t^{m,1} = (\hat{\gamma}_i^{m,1}(.))_{i=t:N-1}$
5. We repeat this process by using the previously computed strategy $\boldsymbol{\pi}_t^{m,k-1}$, $k = 2\ldots$ as the new starting strategy, and until stability of the results. In practice, only 3 repetitions suffice.
6. We repeat this step for the M_1 Monte Carlo points $\{\mathbf{X}_i^{t,z\to s,m}(0, \mathbf{X}_0)\}_{m=1:M_1}$

5.8.2 Results with a re-evaluation of the anticipations.

Figure 8 shows the different PD term structures obtained for the yellow firm in scenario B2C with an increasing carbon price while the yellow firm anticipates a flat carbon price trajectory. We study the impact of different dates for the re-evaluation of the strategy, and add the PD term structures for the two extreme cases: when there is a correct anticipation in $i = 0$, and when there is a misanticipation but no re-evaluation. The results show that if the firm corrects its mistake as soon as $t = 1$ (2025), its PD term structure is barely impacted by the error. However, the longer the company waits before re-evaluating its strategy, and the higher its PD becomes. Nonetheless, a re-evaluation is always beneficial for the firm. Indeed, regardless of the year it re-evaluates its strategy (given that it has not defaulted before), its PD converges with that of the perfect anticipation case except for the final year. Most of the impact of the misanticipation is borne at the end of the scenario (2045), because the strategy designed with the wrong anticipations has not conferred enough carbon mitigation in the first part of the scenario. In turn, the difference in intensity in 2050 leads to a tremendous difference in the firm's value because the lack of further perspectives for intensity reduction.

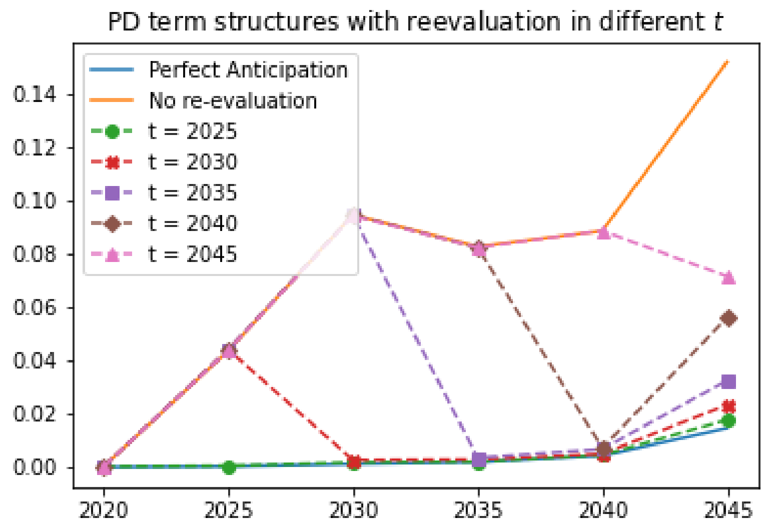


Figure 8: PD term structures for the **yellow** company in scenario B2C with a **flat carbon price misanticipation** at starting point ($\mathtt{cp}_i^z = \mathtt{cp}_0$) and a re-evaluation of the carbon reduction strategy at different dates $t = 2020, \ldots, 2045$.

6 Conclusion

We have designed a model for the evaluation of the probability of default of a firm that encompasses the firm's anticipations regarding the future of the state of the economy. The aim is to evaluate the impact of the firm's misassumption on the scenario when deciding on its business model in the context of the energy transition, while the market correctly anticipates the future state of the economy. We have distinguished three cases: (1) perfect anticipations, (2) wrong anticipations with no re-evaluation of the business model, (3) wrong anticipations with re-evaluation of the business model. Our approach is based on a financial cash-flow logic designed to bring forth the transition risk through its various drivers as described in [Basel Committee on Banking Supervision, 2021]: policy, technology, and sentiment. Indeed, it evaluates the effect of the firm's business model on the probability of default using transition-contingent financials under an adaptive market shares assumption. The firm's total assets have been computed as a stochastic process based on a discounted cash-flow approach that considers potential physical risk damages. The debt is also a stochastic process modeled by directly estimating the firm's total investments with respect to its business model and the scenario variables. The probability of default is then evaluated using a Black-Cox default condition [Black and Cox, 1976] and Nested Monte Carlo simulations for which we have proposed two algorithms. Then, we have computed the different probability of default term structures under various assumptions regarding the future of the economy and one scenario-sensitive business model strategy (see [Ndiaye et al., 2024b]). For this step, the NGFS Phase II *Below 2 Degrees* scenario has been used as the central scenario to some extent. Moreover, we have only considered results under the strategy proposed in [Ndiaye et al., 2024b] forward-looking.

Under these conditions, we have found that for firms with much lower initial intensity levels than their sector, the anticipations on the consumer sentiment have minimal impact on default probability, however, for average firms and above, anticipating a more sensitive market to the consumer sentiment leads to degraded credit risk. Indeed, the firm's business model would rely too much on the potential gains in sales thanks to the reallocation of the market shares inherent to small carbon cuts. Conversely, anticipating constant market shares encourages the company to do more which is rewarded with a lower credit risk than if the company had perfectly anticipated the future state of the economy. Similarly, anticipating a more stringent climate policy than reality leads to a lower probability of default, while the opposite augments the credit risk. Regarding the technology driver of transition risks, anticipating cheaper technology reduces the credit risks thanks to more investment, whereas the opposite leads to inflated default probabilities. In turn, under these conditions, having pessimistic views regarding the transition may be rewarded. Lastly, anticipating a more favorable climate policy enhances the firm's credit risk. However, in this case, the company benefits from correcting its business model with the right anticipations, regardless of the delay. Our results suggest that transition risk cannot be adequately captured by carbon-price-based stress tests alone, and that expectation misalignment constitutes a first-order driver of corporate default risk under climate transition. These results are interesting but should be confronted under other scenarios and different strategies.

Appendices

C.1 Assumptions on the carbon price

C.1.1 Flat carbon price assumption.

BROWN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Flat Carbon Price	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,74%	0,74%
2025	0,68%	0,68%	16,00%	16,61%
2030	1,30%	1,93%	23,54%	33,62%
2035	1,00%	2,85%	19,10%	43,23%
2040	1,32%	4,04%	18,75%	51,04%
2045	4,21%	7,71%	27,77%	60,96%

Table 2: PD term structure for the **brown** under a **flat carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating a flat carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating the carbon price.

YELLOW	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Flat Carbon Price	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,02%	0,02%
2025	0,24%	0,24%	4,36%	4,37%
2030	0,75%	0,98%	9,45%	12,84%
2035	0,67%	1,62%	8,27%	18,95%
2040	0,91%	2,47%	8,85%	24,80%
2045	2,75%	4,94%	15,18%	33,76%

Table 3: PD term structure for the **yellow** under a **flat carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating a flat carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating the carbon price.

GREEN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Flat Carbon Price	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,03%	0,03%	0,22%	0,22%
2030	0,19%	0,22%	1,12%	1,32%
2035	0,22%	0,43%	1,26%	2,51%
2040	0,36%	0,78%	1,77%	4,12%
2045	1,17%	1,88%	4,17%	7,67%

Table 4: PD term structure for the **green** under a **flat carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating a flat carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating the carbon price.

C.1.2 Increasing Carbon Price assumption.

BROWN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Increasing Carbon Price	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,27%	0,27%	0,01%	0,01%
2030	0,29%	0,55%	0,02%	0,03%
2035	0,08%	0,63%	0,01%	0,05%
2040	0,02%	0,65%	0,01%	0,06%
2045	0,00%	0,66%	0,02%	0,08%

Table 5: PD term structure for the **brown** under an **increasing carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating an increasing carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating the carbon price.

YELLOW	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Increasing Carbon Price	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,09%	0,09%	0,01%	0,01%
2030	0,18%	0,27%	0,03%	0,03%
2035	0,07%	0,34%	0,02%	0,05%
2040	0,03%	0,38%	0,02%	0,07%
2045	0,02%	0,39%	0,04%	0,11%

Table 6: PD term structure for the **yellow** under an **increasing carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating an increasing carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating the carbon price.

GREEN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Increasing Carbon Price	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,01%	0,01%	0,00%	0,00%
2030	0,05%	0,06%	0,02%	0,02%
2035	0,04%	0,09%	0,02%	0,04%
2040	0,03%	0,13%	0,02%	0,06%
2045	0,03%	0,16%	0,04%	0,10%

Table 7: PD term structure for the **green** under an **increasing carbon price misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating an increasing carbon price. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating the carbon price.

C.2 Assumptions on α

C.2.1 Constant abatement costs assumption.

BROWN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Constant Abatement Costs	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,04%	0,04%	0,00%	0,00%
2025	3,87%	3,91%	0,68%	0,69%
2030	8,37%	11,42%	1,30%	1,94%
2035	8,02%	17,47%	1,01%	2,86%
2040	9,56%	23,96%	1,33%	4,06%
2045	17,25%	34,33%	4,21%	7,73%

Table 8: PD term structure for the **brown** under a **constant abatement misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating constant abatement misassumption. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating decreasing abatement costs.

YELLOW	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Constant Abatement Costs	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	1,05%	1,06%	0,25%	0,25%
2030	3,31%	4,24%	0,76%	1,00%
2035	3,38%	7,19%	0,67%	1,63%
2040	4,34%	10,75%	0,93%	2,50%
2045	8,90%	17,42%	2,77%	4,97%

Table 9: PD term structure for the **yellow** under a **constant abatement misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating constant abatement misassumption. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating decreasing abatement costs.

GREEN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Constant Abatement Costs	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,07%	0,07%	0,03%	0,03%
2030	0,49%	0,56%	0,19%	0,21%
2035	0,61%	1,15%	0,22%	0,43%
2040	0,96%	2,06%	0,35%	0,77%
2045	2,62%	4,41%	1,17%	1,87%

Table 10: PD term structure for the **green** under a **constant abatement misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating constant abatement misassumption. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating decreasing abatement costs.

C.2.2 Decreasing Abatement Costs Assumption.

BROWN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Decreasing Abatement Costs	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,04%	0,04%	0,00%	0,00%
2025	4,15%	4,19%	0,76%	0,77%
2030	9,62%	12,79%	1,76%	2,46%
2035	10,10%	20,28%	2,03%	4,31%
2040	13,07%	28,81%	4,77%	8,51%
2045	24,33%	42,41%	25,58%	29,83%

Table 11: PD term structure for the **brown** under a **decreasing abatement misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating decreasing abatement misassumption. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating constant abatement costs.

YELLOW	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Decreasing Abatement Costs	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	1,13%	1,13%	0,27%	0,27%
2030	3,75%	4,73%	0,96%	1,22%
2035	4,12%	8,31%	1,15%	2,31%
2040	5,63%	12,85%	2,44%	4,55%
2045	11,91%	21,48%	11,96%	15,00%

Table 12: PD term structure for the **yellow** under a **decreasing abatement misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating decreasing abatement misassumption. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating constant abatement costs.

GREEN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Decreasing Abatement Costs	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,08%	0,08%	0,03%	0,03%
2030	0,52%	0,60%	0,22%	0,24%
2035	0,69%	1,27%	0,30%	0,54%
2040	1,14%	2,34%	0,64%	1,16%
2045	3,14%	5,15%	3,00%	3,97%

Table 13: PD term structure for the **green** under a **decreasing abatement misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating decreasing abatement misassumption. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating constant abatement costs.

C.3 Assumptions on κ

C.3.1 Constant Market Shares Assumption.

BROWN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Constant Market Shares	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,68%	0,68%	0,26%	0,26%
2030	1,30%	1,94%	0,75%	1,00%
2035	1,00%	2,85%	0,72%	1,67%
2040	1,34%	4,05%	1,07%	2,67%
2045	4,21%	7,71%	3,32%	5,63%

Table 14: PD term structure for the **brown** under a **constant market shares misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating constant market shares. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating adaptive market shares.

YELLOW	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Constant Market Shares	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,25%	0,25%	0,17%	0,17%
2030	0,76%	1,00%	0,60%	0,77%
2035	0,66%	1,62%	0,57%	1,31%
2040	0,91%	2,47%	0,81%	2,07%
2045	2,77%	4,96%	2,48%	4,31%

Table 15: PD term structure for the **yellow** under a **constant market shares misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating constant market shares. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating adaptive market shares.

GREEN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Constant market shares	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,03%	0,03%	0,03%	0,03%
2030	0,18%	0,21%	0,19%	0,22%
2035	0,21%	0,42%	0,22%	0,44%
2040	0,36%	0,77%	0,37%	0,79%
2045	1,17%	1,87%	1,20%	1,92%

Table 16: PD term structure for the **green** under a **constant market shares misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating constant market shares. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating adaptive market shares.

C.3.2 Adaptive Market Shares Assumption.

BROWN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Adaptive Market Shares	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,54%	0,54%	0,84%	0,85%
2030	1,63%	2,13%	2,50%	3,25%
2035	1,69%	3,68%	2,72%	5,69%
2040	2,36%	5,76%	3,94%	9,04%
2045	5,86%	10,62%	10,27%	17,08%

Table 17: PD term structure for the **brown** under an **adaptive market shares misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating adaptive market shares. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating constant market shares.

YELLOW	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Adaptive Market Shares	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,13%	0,13%	0,15%	0,15%
2030	0,54%	0,66%	0,64%	0,78%
2035	0,64%	1,28%	0,78%	1,53%
2040	0,96%	2,18%	1,19%	2,63%
2045	2,46%	4,37%	3,14%	5,40%

Table 18: PD term structure for the **yellow** under an **adaptive market shares misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating adaptive market shares. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating constant market shares.

References

[Barclays PLC, 2021] Barclays PLC (2021). Corporate transition risk forecast model. https://home.barclays/content/dam/home-barclays/documents/citizenship/ESG/2021/Corporate-Transition-Forecast-Model-2021.pdf.

[Basel Committee on Banking Supervision, 2021] Basel Committee on Banking Supervision (2021). Climate-related risk drivers and their transmission channels. https://www.bis.org/bcbs/publ/d517.pdf.

[Battiston et al., 2023] Battiston, S., Mandel, A., Monasterolo, I., and Roncoroni, A. (2023). Climate credit risk and corporate valuation. *Available at SSRN 4124002*.

GREEN	$\boldsymbol{\pi}(s)$		$\boldsymbol{\pi}(z)$	
Adaptive Market Shares	**Strict PD**	**Cumulated PD**	**Strict PD**	**Cumulated PD**
2020	0,00%	0,00%	0,00%	0,00%
2025	0,01%	0,01%	0,01%	0,01%
2030	0,09%	0,10%	0,09%	0,10%
2035	0,11%	0,21%	0,11%	0,21%
2040	0,18%	0,39%	0,17%	0,38%
2045	0,44%	0,81%	0,43%	0,78%

Table 19: PD term structure for the **green** under an **adaptive market shares misassumption**. $\boldsymbol{\pi}(z)$ refers to the Min Costs trajectory while misanticipating adaptive market shares. $\boldsymbol{\pi}(s)$ refers to the Min Costs trajectory while correctly anticipating constant market shares.

[Bell and Vuuren, 2022] Bell, F. and Vuuren, G. V. V. (2022). The impact of climate risk on corporate credit risk. *Cogent Economics & Finance*, 10.

[Black and Cox, 1976] Black, F. and Cox, J. C. (1976). Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, 31(2):351–367.

[Crosbie and Bohn, 2003] Crosbie, P. and Bohn, J. (2003). *Modeling Default Risk*, pages 471–506.

[Le Guenedal and Tankov, 2024] Le Guenedal, T. and Tankov, P. (2024). Corporate debt value under transition scenario uncertainty. *Mathematical Finance*.

[Leland and Toft, 1996] Leland, H. E. and Toft, K. B. (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *The journal of finance*, 51(3):987–1019.

[Ndiaye et al., 2024a] Ndiaye, E., Bezat, A., Gobet, E., Guivarch, C., and Jiao, Y. (2024a). Corporate probability of default under an energy transition scenario with business model adaptation to the transition. working paper or preprint.

[Ndiaye et al., 2024b] Ndiaye, E., Bezat, A., Gobet, E., Guivarch, C., and Jiao, Y. (2024b). Optimal business model adaptation plan for a company under a transition scenario. working paper or preprint.

[Nordhaus, 2017] Nordhaus, W. D. (2017). Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7):1518–1523.

[Riahi et al., 2017] Riahi, K., van Vuuren, D. P., Kriegler, E., Edmonds, J., O'Neill, B. C., Fujimori, S., Bauer, N., Calvin, K., Dellink, R., Fricko, O., Lutz, W., Popp, A., Cuaresma, J. C., KC, S., Leimbach, M., Jiang, L., Kram, T., Rao, S., Emmerling, J., Ebi, K., Hasegawa, T., Havlik, P., Humpenöder, F., Da Silva, L. A., Smith, S., Stehfest, E., Bosetti, V., Eom, J., Gernaat, D., Masui, T., Rogelj, J., Strefler, J., Drouet, L., Krey, V., Luderer, G., Harmsen, M., Takahashi, K., Baumstark, L., Doelman, J. C., Kainuma, M., Klimont, Z., Marangoni, G., Lotze-Campen, H., Obersteiner, M., Tabeau, A., and Tavoni, M. (2017). The shared socioeconomic pathways and their energy, land use, and greenhouse gas emissions implications: An overview. *Global Environmental Change*, 42:153–168.

[Richters et al., 2022] Richters, O., Bertram, C., Kriegler, E., Anz, J., Beck, T., Bresch, D. N., Charles, M., Clarke, L., and Cui, R. (2022). NGFS climate scenario database: Technical documentation v2. 2. https://data.ece.iiasa.ac.at/ngfs-phase-2/#/downloads.

[Science Based Targets Initiative, 2024] Science Based Targets Initiative (2024). Companies taking action - science based targets. https://sciencebasedtargets.org/companies-taking-action#dashboard.