Supplement materials of “Analysis of a queue-length-dependent vacation queue with bulk service, N-policy, set-up time and cost optimization”

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1. Practical application of the concerned model (corresponding to Section 2, main paper)

Here we point out the application of our concerned model in two major real-life circumstances.

(i) Group screening of blood samples: A paramount importance has to be given during blood transfusion as the presence of various pathogens such as Hepatitis B (HBV), Hepatitis C (HCV), Human Immunodeficiency Virus (HIV), Syphilis etc. in blood units might be a question of life and death. This eventually leads to the requirement of a meticulous inspection of a lot of donated blood (generally known as blood samples) which arrive at the central blood bank store in bulk/groups. The operational and management system of blood bank may consist of two crucial factors: testing procedure and perishability. Each blood unit has an expiration date; after that, it is perished. Hence we need to minimize the testing period which can be possible by adopting a group screening policy and batch-size-dependent service mechanism. In the first phase, blood samples are pooled in groups of certain sizes and are tested by the Enzyme Linked Immuno-Sorbent Assay (ELISA). This signifies the use of our concerned batch-service queue. However, ELISA test fails for just recently been infected person which may result in a false negative. To overcome this, in the second phase, testing of individual blood samples (those groups found clean at the first phase) take place using Polymerase Chain Reaction (PCR) to detect the presence of pathogens with a much higher probability of a positive result for an infected person.

If the minimum threshold value of blood units is not accumulated, the server may leave for a certain amount of time depending on the number of blood samples already waiting for testing. During that period, maintenance of equipment, collection of required chemicals etc. can be completed without wasting time. This exactly signifies a queue-length-dependent vacation period which leads to a cost-efficient operating and management system. Before starting the next batch screening process using ELISA, the complete set-up of the testing facility is required if the number of accumulated samples equals the pre-decided threshold value N which are another two significant characteristic features of our concerned model. During the first phase of blood screening using ELISA, N-policy can be incorporated to avoid frequent switches over between vacation

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and busy state which reduces the associated testing cost.

(iii) Ferry crossing: Ferry systems are crucial in numerous areas, providing a key transportation way across bodies of water. Due to high demand and the necessity of efficiency, effective management of these systems are of paramount importance. One such method to optimize ferry operations is the use of a bulk service queue with vacation period, set-up time and N-policy. This approach not only streamlines the boarding process but also ensures efficient resource management and enhances customer satisfaction.

Consider a ferry service dock where only one boat is available, operated by a single driver. The boat starts the crossing from the terminal only when a minimum number (lower threshold value 'a') of passengers are available, and on a particular trip the ferry can accommodate passengers up to a predetermined maximum capacity 'b'. Under this service policy, more number of passengers can be transported in a shorter period of time. After completing a trip, if there are fewer number of passengers (less than 'a') present, the boat driver takes a vacation of random duration. During the vacation period, it may happen that enough number of passengers arrive and the boat has to be ready to depart. However, it could not depart due to the absence of the boat driver as he/she has left for vacation. It may increase the impatience among passengers who are seated inside the boat as well as those waiting in the queue, resulting in a loss of revenue for the boat owner. In order to minimize this loss, the boat owner imposes a penalty on the boat driver for his/her late arrival from vacation. The imposition of penalty on the boat driver may result in reduced earning. To minimize the loss of earnings of the driver as well as the loss of revenue of the owner, instead of taking a vacation of constant mean length by the driver, he/she may decide to take a vacation of variable mean length depending on the queue length at vacation initiation epoch (i.e., shorter vacation when the queue is long, and longer vacation when the queue is short). This eventually reduces the revenue loss of the boat owner and a penalty is imposed on the boat driver. In general, during the vacation period, the machine that drives the boat is turned off and after returning from vacation period the boat driver takes some amount of time to start the machine which may be considered as a set-up time. After returning from a vacation, if the service provider (the ferry) starts the service process with a sufficiently large number, say, 'N', then, after completing one trip, the boat driver may not need to take another vacation and can continue the service process which increases the revenue for both the boat driver and the owner. This scenario can be adequately modeled in the framework of our concerned model. For more applications of these types of queues in transportation systems and manufacturing industries see Gupta et al. [1], Yang and Wu [2].

2. Numerical tables of example (corresponding to Section 8, main paper)

Table 1: Probability distribution at service completion epoch for single vacation

n	$p^{+}_{n,7}$	$p^{+}_{n,8}$	$p^{+}_{n,9}$	$p^{+}_{n,10}$	$p^{+}_{n,11}$	$p^{+}_{n,12}$	$p^{+}_{n,13}$	$p^{+}_{n,14}$	$p^{+}_{n,15}$	$p^{+}_{n,16}$	p^{+}_{n}
0	0.008344	0.008340	0.007969	0.007920	0.004656	0.004319	0.003956	0.000701	0.000536	0.000404	0.047145
1	0.001318	0.001371	0.001355	0.001385	0.001397	0.001336	0.001259	0.000658	0.000536	0.000815	0.011430
2	0.001530	0.001600	0.001588	0.001629	0.001711	0.001647	0.001559	0.000966	0.000804	0.001430	0.014464
3	0.001775	0.001866	0.001860	0.001917	0.002088	0.002022	0.001924	0.001371	0.001162	0.002318	0.018303
4	0.002061	0.002178	0.002181	0.002257	0.002540	0.002473	0.002363	0.001894	0.001631	0.004002	0.023580
5	0.001075	0.001171	0.001203	0.001272	0.001683	0.001677	0.001637	0.001906	0.001703	0.005689	0.019016
6	0.001039	0.001141	0.001180	0.001254	0.001700	0.001707	0.001677	0.002136	0.001945	0.007712	0.021491
7	0.000964	0.001068	0.001112	0.001191	0.001653	0.001674	0.001658	0.002292	0.002127	0.010057	0.023796
8	0.000836	0.000937	0.000987	0.001065	0.001516	0.001553	0.001552	0.002327	0.002206	0.012683	0.025662
9	0.000642	0.000733	0.000783	0.000857	0.001260	0.001311	0.001329	0.002181	0.002123	0.015091	0.026310
10	0.000573	0.000661	0.000712	0.000785	0.001160	0.001218	0.001245	0.002086	0.002070	0.017415	0.027925
15	0.000269	0.000328	0.000372	0.000428	0.000607	0.000675	0.000726	0.001068	0.001175	0.023631	0.029279
30	0.000033	0.000048	0.000062	0.000080	0.000059	0.000078	0.000098	0.000012	0.000018	0.005547	0.006036
50	0.000003	0.000005	0.000007	0.000011	0.000002	0.000003	0.000005	0.000000	0.000000	0.0000164	0.000200
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000001
≥ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

n	$q^{[0]+}_{n}$	$q^{[1]+}_{n}$	$q^{[2]+}_{n}$	$q^{[3]+}_{n}$	$q^{[4]+}_{n}$	$q^{[5]+}_{n}$	$q^{[6]+}_{n}$	q^{+}_{n}	r^{+}_{n}
0	0.011123	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.002706	0.004766	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.003219	0.000805	0.007682	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
3	0.003821	0.000913	0.001001	0.011136	0.000000	0.000000	0.000000	0.000000	0.000000
4	0.004528	0.001035	0.001106	0.001184	0.015657	0.000000	0.000000	0.000000	0.000000
5	0.002651	0.001172	0.001222	0.001287	0.001407	0.013422	0.000000	0.000000	0.000000
6	0.002597	0.000522	0.001349	0.001397	0.001511	0.001045	0.015870	0.000000	0.000000
7	0.002440	0.000479	0.000488	0.001517	0.001621	0.001112	0.001091	0.000000	0.000000
8	0.002149	0.000417	0.000432	0.000463	0.001740	0.001183	0.001152	0.000000	0.000000
9	0.001684	0.000332	0.000358	0.000398	0.000459	0.001258	0.001218	0.000000	0.000000
10	0.001506	0.000218	0.000266	0.000320	0.000388	0.000293	0.001286	0.000000	0.000000
15	0.000687	0.000062	0.000041	0.000039	0.000046	0.000037	0.000041	0.000000	0.000000
30	0.000059	0.000001	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
50	0.000002	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
≥ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.0000000	0.000000

Table 2: Probability distribution at arbitrary epoch for single vacation

n	ν_n	$p_{n,7}$	$p_{n,8}$	$p_{n,9}$	$p_{n,10}$	$p_{n,11}$	$p_{n,12}$	$p_{n,13}$	$p_{n,14}$	$p_{n,15}$	$p_{n,16}$
0	0.002834	0.003937	0.004413	0.004673	0.005097	0.006228	0.006483	0.006614	0.007421	0.007324	0.007113
1	0.002612	0.000648	0.000754	0.000823	0.000921	0.001201	0.001281	0.001333	0.001688	0.001694	0.008457
2	0.004344	0.000757	0.000884	0.000970	0.001090	0.001421	0.001521	0.001590	0.002031	0.002050	0.010024
3	0.006746	0.000883	0.001037	0.001143	0.001289	0.001680	0.001806	0.001894	0.002436	0.002471	0.011859
4	0.010124	0.001031	0.001217	0.001346	0.001524	0.001986	0.002142	0.002255	0.002911	0.002969	0.021763
5	0.011349	0.000556	0.000675	0.000764	0.000882	0.001144	0.001260	0.001351	0.001781	0.001862	0.023532
6	0.014330	0.000542	0.000663	0.000755	0.000887	0.001125	0.001247	0.001345	0.001746	0.001842	0.025195
7	0.012867	0.000507	0.000626	0.000719	0.000839	0.001063	0.001187	0.001289	0.001635	0.001744	0.026670
8	0.014088	0.000446	0.000556	0.000645	0.000759	0.000943	0.001063	0.0001165	0.001425	0.001542	0.027847
9	0.014612	0.000349	0.000443	0.000521	0.000621	0.000747	0.000856	0.000949	0.001091	0.001207	0.027728
10	0.015064	0.000315	0.000404	0.000478	0.000574	0.000673	0.000778	0.000870	0.000943	0.001056	0.027239
15	0.015439	0.000156	0.000212	0.000264	0.000330	0.000320	0.000390	0.000457	0.000296	0.000361	0.020095
30	0.000000	0.000021	0.000035	0.000049	0.000069	0.000027	0.000040	0.000054	0.000002	0.000002	0.002282
50	0.000000	0.000002	0.000003	0.000006	0.000009	0.000001	0.000002	0.000002	0.000000	0.000000	0.000072
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
$\geq$ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.0000000	0.000000	0.000000	0.000000

n	$q_n^{[0]}$	$q_n^{[1]}$	$q_n^{[2]}$	$q_n^{[3]}$	$q_n^{[4]}$	$q_n^{[5]}$	$q_n^{[6]}$	r_n	q_n^{queue}
0	0.009179	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.071316
1	0.001605	0.001698	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.024715
2	0.001876	0.000219	0.001727	0.000000	0.000000	0.000000	0.000000	0.000000	0.030504
3	0.002191	0.000247	0.000177	0.001826	0.000000	0.000000	0.000000	0.000000	0.037685
4	0.002559	0.000277	0.000194	0.000155	0.002019	0.000000	0.000000	0.000000	0.054474
5	0.001382	0.000311	0.000213	0.000167	0.000146	0.001425	0.000000	0.000000	0.048800
6	0.001341	0.000131	0.000234	0.000181	0.000156	0.000090	0.001432	0.000000	0.053232
7	0.001247	0.000119	0.000080	0.000196	0.000167	0.000095	0.000080	0.000000	0.051130
8	0.001085	0.000103	0.000071	0.000057	0.000179	0.000101	0.000084	0.000000	0.052159
9	0.000834	0.000082	0.000059	0.000049	0.000045	0.000107	0.000089	0.000000	0.050389
10	0.000743	0.000053	0.000043	0.000039	0.000038	0.000024	0.000094	0.000000	0.049428
15	0.000332	0.000011	0.000007	0.000005	0.000005	0.000003	0.000003	0.000000	0.038689
30	0.000028	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.003324
50	0.000001	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000103
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.00000	0.000000	0.000000	0.000000
$\geq$ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

L_q=10.990536, L=20.854676, L_s=9.864139, P_{dor}=0.247607, P_{idle}=0.331103

Q_{vac}=0.043893, W_q=2.198107, W=4.170935, P_{set}=0.039603

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Table 3: Probability distribution at service completion epoch for multiple vacation

n	$p^+_{n,7}$	$p^+_{n,8}$	$p^+_{n,9}$	$p^+_{n,10}$	$p^+_{n,11}$	$p^+_{n,12}$	$p^+_{n,13}$	$p^+_{n,14}$	$p^+_{n,15}$	$p^+_{n,16}$	p^+_n	$q^{[0]+}_n$	$q^{[1]+}_n$	$q^{[2]+}_n$	$q^{[3]+}_n$	$q^{[4]+}_n$
0	0.001402	0.001401	0.001338	0.001330	0.000783	0.000726	0.000667	0.000119	0.000091	0.000069	0.007926	0.002447	0.000000	0.000000	0.000000	0.000000
1	0.000222	0.000230	0.000227	0.000233	0.000235	0.000225	0.000212	0.000111	0.000091	0.000139	0.001925	0.000595	0.001803	0.000000	0.000000	0.000000
2	0.000257	0.000269	0.000267	0.000274	0.000288	0.000277	0.000263	0.000163	0.000136	0.000243	0.002437	0.000708	0.000304	0.003907	0.000000	0.000000
3	0.000298	0.000313	0.000312	0.000322	0.000351	0.000340	0.000324	0.000232	0.000196	0.000395	0.003085	0.000841	0.000345	0.000509	0.007427	0.000000
4	0.000346	0.000366	0.000366	0.000379	0.000427	0.000416	0.000398	0.000321	0.000276	0.000674	0.003969	0.000996	0.000391	0.000563	0.000790	0.013258
5	0.000181	0.000197	0.000202	0.000214	0.000283	0.000282	0.000275	0.000322	0.000288	0.000956	0.003200	0.000583	0.000443	0.000622	0.000858	0.001192
6	0.000175	0.000192	0.000198	0.000211	0.000286	0.000287	0.000283	0.000360	0.000329	0.001294	0.003615	0.000571	0.000197	0.000686	0.000932	0.001279
7	0.000162	0.000179	0.000187	0.000200	0.000278	0.000282	0.000279	0.000387	0.000360	0.001685	0.003999	0.000537	0.000181	0.000248	0.001012	0.001373
8	0.000141	0.000157	0.000166	0.000179	0.000255	0.000261	0.000262	0.000393	0.000374	0.002122	0.004310	0.000473	0.000158	0.000219	0.000309	0.001473
9	0.000108	0.000123	0.000132	0.000144	0.000211	0.000220	0.000224	0.000369	0.000360	0.002527	0.004418	0.000370	0.000126	0.000182	0.000266	0.000389
10	0.000096	0.000111	0.000120	0.000132	0.000195	0.000205	0.000210	0.000352	0.000351	0.002919	0.004691	0.000331	0.000083	0.000135	0.000214	0.000328
15	0.000045	0.000055	0.000062	0.000071	0.000102	0.000113	0.000122	0.000180	0.000199	0.004006	0.004959	0.000151	0.000023	0.000021	0.000026	0.000040
30	0.000006	0.000008	0.000010	0.000013	0.000010	0.000013	0.000017	0.000002	0.000003	0.000984	0.001066	0.000013	0.000001	0.000000	0.000000	0.000000
50	0.000000	0.000001	0.000001	0.000002	0.000000	0.000001	0.000001	0.000000	0.000000	0.000029	0.000035	0.000000	0.000000	0.000000	0.000000	0.000000
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
≥ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

n	$q^{[5]+}_n$	$q^{[6]+}_n$	$q^{[7]+}_n$	$q^{[8]+}_n$	$q^{[9]+}_n$	$q^{[10]+}_n$	$q^{[11]+}_n$	$q^{[12]+}_n$	$q^{[13]+}_n$	$q^{[14]+}_n$	$q^{[15]+}_n$	$q^{[16]+}_n$	$q^{[17]+}_n$	$q^{[18]+}_n$	$q^{[19]+}_n$	q^+_n	r^+_n
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.002447	0.000000
1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.002398	0.000000
2	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.004919	0.000000
3	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.009122	0.000000
4	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.015998	0.000000
5	0.016552	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.020250	0.000000
6	0.001289	0.024196	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.029151	0.000000
7	0.001372	0.001663	0.020736	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.027122	0.000000
8	0.001459	0.001757	0.001275	0.026153	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.033276	0.000000
9	0.001552	0.001856	0.001340	0.001456	0.030868	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.038405	0.000000
10	0.000361	0.001961	0.001408	0.001523	0.001569	0.035770	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.043683	0.000000
15	0.000045	0.000063	0.000107	0.000172	0.000226	0.000274	0.001889	0.001913	0.001914	0.001909	0.058288	0.000000	0.000000	0.000000	0.000000	0.067061	0.000000
30	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000002	0.000004	0.000021	0.000528
50	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000004
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
≥ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.0000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 4: Probability distribution at arbitrary epoch for multiple vacation

n	$p_{n,7}$	$p_{n,8}$	$p_{n,9}$	$p_{n,10}$	$p_{n,11}$	$p_{n,12}$	$p_{n,13}$	$p_{n,14}$	$p_{n,15}$	$p_{n,16}$	$q_n^{[0]}$	$q_n^{[1]}$	$q_n^{[2]}$	$q_n^{[3]}$	$q_n^{[4]}$
0	0.003881	0.004347	0.004601	0.005021	0.006140	0.006399	0.006540	0.007353	0.007273	0.007083	0.011843	0.000000	0.000000	0.000000	0.000000
1	0.000639	0.000743	0.000811	0.000908	0.001184	0.001263	0.001317	0.001672	0.001683	0.008441	0.002071	0.003765	0.000000	0.000000	0.000000
2	0.000746	0.000871	0.000955	0.001073	0.001401	0.001501	0.001572	0.002012	0.002035	0.010024	0.002420	0.000487	0.005154	0.000000	0.000000
3	0.000870	0.001022	0.001125	0.001269	0.001657	0.001782	0.001873	0.002413	0.002454	0.011877	0.002828	0.000547	0.000528	0.007143	0.000000
4	0.001016	0.001199	0.001326	0.001502	0.001957	0.002115	0.002230	0.002885	0.002949	0.020924	0.003302	0.000615	0.000580	0.000605	0.010024
5	0.000547	0.000665	0.000752	0.000868	0.001127	0.001244	0.001336	0.001765	0.001849	0.022813	0.001784	0.000691	0.000637	0.000655	0.000724
6	0.000534	0.000653	0.000744	0.000863	0.001109	0.001231	0.001330	0.001730	0.001830	0.024605	0.001730	0.000290	0.000699	0.000708	0.000776
7	0.000500	0.000617	0.000708	0.000826	0.001047	0.001172	0.001275	0.001620	0.001732	0.026222	0.001608	0.000265	0.000240	0.000766	0.000830
8	0.000440	0.000548	0.000635	0.000748	0.000929	0.001050	0.001152	0.001412	0.001532	0.027554	0.001399	0.000229	0.000211	0.000222	0.000888
9	0.000344	0.000437	0.000513	0.000612	0.000736	0.000844	0.000938	0.001081	0.001198	0.027557	0.001077	0.000181	0.000175	0.000191	0.000224
10	0.000311	0.000397	0.000471	0.000566	0.000664	0.000767	0.000860	0.000934	0.001049	0.027184	0.000958	0.000118	0.000129	0.000153	0.000189
15	0.000154	0.000209	0.000260	0.000325	0.000315	0.000385	0.000452	0.000293	0.000359	0.020404	0.000429	0.000033	0.000019	0.000019	0.000022
30	0.000021	0.000034	0.000048	0.000068	0.000027	0.000039	0.000053	0.000001	0.000002	0.002394	0.000037	0.000001	0.000000	0.000000	0.000000
50	0.000002	0.000003	0.000006	0.000010	0.000001	0.000001	0.000002	0.000000	0.000000	0.000074	0.000001	0.000000	0.000000	0.000000	0.000000
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
$\geq$ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

n	$q_n^{[5]}$	$q_n^{[6]}$	$q_n^{[7]}$	$q_n^{[8]}$	$q_n^{[9]}$	$q_n^{[10]}$	$q_n^{[11]}$	$q_n^{[12]}$	$q_n^{[13]}$	$q_n^{[14]}$	$q_n^{[15]}$	$q_n^{[16]}$	$q_n^{[17]}$	$q_n^{[18]}$	$q_n^{[19]}$	r_n	p_n^{queue}
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.070481
1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.024497
2	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.030251
3	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.037388
4	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.053229
5	0.010307	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.047766
6	0.000651	0.012805	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.052288
7	0.000690	0.000716	0.009542	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.050376
8	0.000732	0.000755	0.000480	0.010644	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.051560
9	0.000776	0.000796	0.000503	0.000486	0.011261	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.049930
10	0.000173	0.000839	0.000527	0.000507	0.000470	0.011824	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.049090
15	0.000021	0.000025	0.000038	0.000055	0.000065	0.000071	0.000469	0.000438	0.000407	0.000380	0.013109	0.000000	0.000000	0.000000	0.000000	0.000000	0.038756
30	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000001	0.000789	0.003516
50	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000006	0.000106
100	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
$\geq$ 150	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

L_q=11.130534, L=21.004972, L_s=9.874438, P_{idle}=0.331170,
Q_{vac}=0.292086, W_q=2.226107, W=4.200994, P_{set}=0.039083

3. Tables of system cost optimization (corresponding to Subsection 8.2, main paper)

Table 5: The effect of minimum threshold value (a) on TC^*

	Single vacation					Multiple vacation				
a	$E[V]$	$E[I]$	$E[B]$	$E[L_c]$	TC^*	$E[V]$	$E[I]$	$E[B]$	$E[L_c]$	TC^*
1	1.176471	1.889471	1.970145	3.859616	11.971815	1.243791	1.529506	1.591924	3.121429	12.709743
2	1.132929	1.860042	1.934440	3.794482	11.926355	1.198656	1.484371	1.541021	3.025392	**12.604084**
3	1.065270	1.813679	1.880679	3.694378	11.893852	1.127957	1.413672	1.463398	2.877069	12.740866
4	0.896115	1.691479	1.742354	3.433833	**11.884734**	0.957706	1.243421	1.279161	2.522582	12.943154
5	0.743394	1.572490	1.610226	3.182716	11.948348	0.801590	1.087304	1.112374	2.199678	13.274444
6	0.627757	1.473749	1.502484	2.976234	12.063505	0.682041	0.967755	0.986002	1.953758	13.663073
7	0.539339	1.390272	1.412828	2.803100	12.211276	0.589740	0.875455	0.889283	1.764738	14.077042
8	0.457885	1.305064	1.322577	2.627642	12.411333	0.503540	0.789254	0.794640	1.588894	14.584224
9	0.382442	1.217442	1.230905	2.448347	12.673918	0.422484	0.708199	0.715934	1.424133	15.205077
10	0.317099	1.132953	1.143484	2.276437	12.989622	0.351186	0.636899	0.642785	1.279685	15.909970
11	0.260881	1.052196	1.060750	2.112945	13.350732	0.288845	0.574559	0.579222	1.153781	16.686964
12	0.212225	0.974962	0.982317	1.957279	13.754191	0.234026	0.519741	0.523661	1.043402	17.532566

Table 6: The effect of service rate on TC^*

	Single vacation					Multiple vacation				
α	$E[V]$	$E[I]$	$E[B]$	$E[L_c]$	TC^*	$E[V]$	$E[I]$	$E[B]$	$E[L_c]$	TC^*
23.5	0.883604	1.682656	2.452525	4.135182	12.449569	1.190463	1.476177	2.170386	3.646563	13.241464
25.5	0.890224	1.687325	2.036897	3.724222	12.098813	1.195003	1.480718	1.802249	3.282967	12.905690
27.5	0.896115	1.691480	1.742354	3.433833	11.884734	1.198656	1.484371	1.541021	3.025392	12.704084
29.5	0.901394	1.695203	1.522647	3.217850	11.748499	1.201667	1.487381	1.346003	2.833384	12.578495
31.5	0.906156	1.698561	1.352427	3.050988	11.659481	1.204196	1.489910	1.194837	2.684748	12.498736
33.5	0.910474	1.701607	1.216640	2.918246	11.600472	1.206353	1.492068	1.074220	2.566287	12.447896
35.5	0.914410	1.704383	1.105776	2.810159	11.561224	1.208217	1.493932	0.975735	2.469667	12.415943
37.5	0.918013	1.706924	1.013538	2.720462	11.535358	1.209845	1.495560	0.893800	2.389360	12.396649
39.5	0.921325	1.709260	0.935585	2.644844	11.518749	1.211280	1.496994	0.824566	2.321560	12.386008
41.5	0.924381	1.711415	0.868829	2.580244	11.508654	1.212554	1.498269	0.765290	2.263558	12.381368
43.5	0.927209	1.713410	0.811014	2.524423	11.503202	1.213694	1.499409	0.713967	2.213376	**12.380926**
45.5	0.929836	1.715262	0.760452	2.475713	**11.501090**	1.214721	1.500435	0.669097	2.169532	12.383439
47.5	0.932282	1.716986	0.715855	2.432841	11.501393	1.215650	1.501364	0.629535	2.130899	12.388029
49.5	0.934565	1.718596	0.676225	2.394821	11.503448	1.216495	1.502209	0.594391	2.096600	12.394071
51.5	0.936701	1.720102	0.640773	2.360874	11.506771	1.217267	1.502982	0.562964	2.065945	12.401114

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