Homework 4 Question 1: Moving averages

There are many ways to model the relationship between an input sequence $\{u_1, u_2, \ldots\}$ and an output sequence $\{y_1, y_2, \ldots\}$. In class, we saw the moving average (MA) model, where each output is approximated by a linear combination of the k most recent inputs:

$$MA : y_t \approx b_1 u_t + b_2 u_{t-1} + \cdots + b_k u_{t-k+1}$$

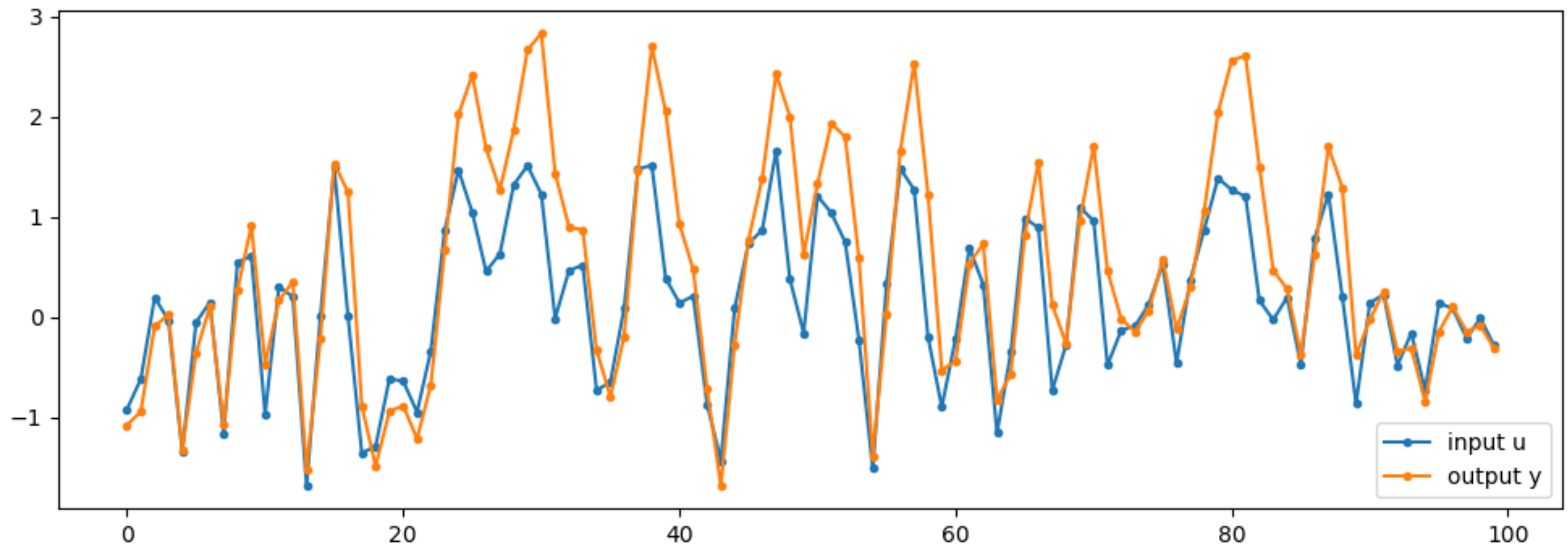
We then used least-squares to find the coefficients $b_1, \ldots, b_k$. What if we didn't have access to the inputs at all, and we were asked to predict future y values based only on the previous y values? One way to do this is by using an autoregressive (AR) model, where each output is approximated by a linear combination of the ℓ most recent outputs (excluding the present one):

$$AR : y_t \approx a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_\ell y_{t-\ell}$$

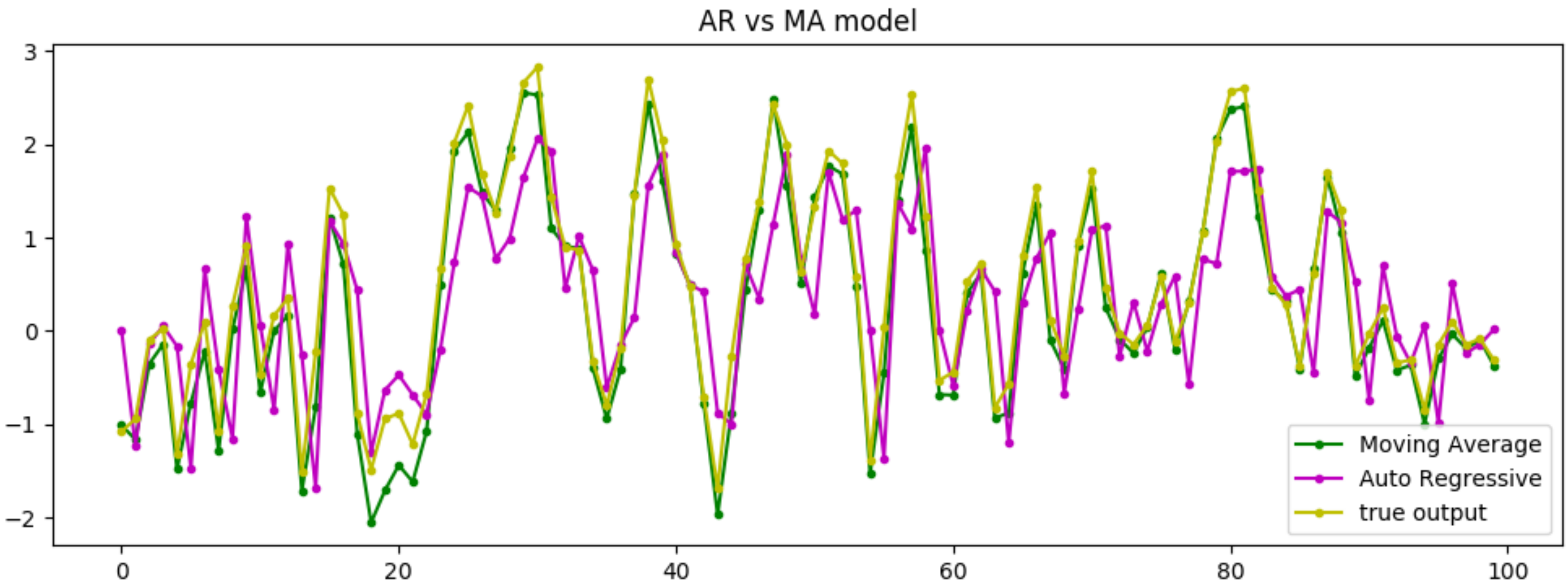
Of course, if the inputs contain pertinent information, we shouldn't expect the AR method to outperform the MA method!

a) Using the same dataset from class uy_data.csv, plot the true y, and on the same axes, also plot the estimated $\hat{y}$ using the MA model and the estimated $\hat{y}$ using the AR model. Use k = 5 for both models. To quantify the difference between estimates, also compute $\|y - \hat{y}\|$ for both cases.

In [1]:
```
raw = readcsv("uy_data.csv");
u = raw[:,1];
y = raw[:,2];
T = length(u)

# plot the u and y data
using PyPlot
figure(figsize=(12,4))
plot([u y],".-");
legend(["input u", "output y"], loc="lower right");
```



In [4]:
```
width = 5
AR = zeros(T,width)
for i = 1:width
    AR[i+1:end,i] = y[1:end-i]
end
woptAR = AR\y
yestAR = AR*woptAR

MA = zeros(T,width)
for i = 1:width
    MA[i:end,i] = u[1:end-i+1]
end
woptMA = MA\y
yestMA = MA*woptMA

figure(figsize=(12,4))
plot(yestMA,"g.-",yestAR,"m.-", y, "y.-")
legend(["Moving Average", "Auto Regressive","true output"], loc="lower right");
title("AR vs MA model");
println()
println("Norm for MA :",norm(yestMA-y))
println("Norm for AR :",norm(yestAR-y))
```



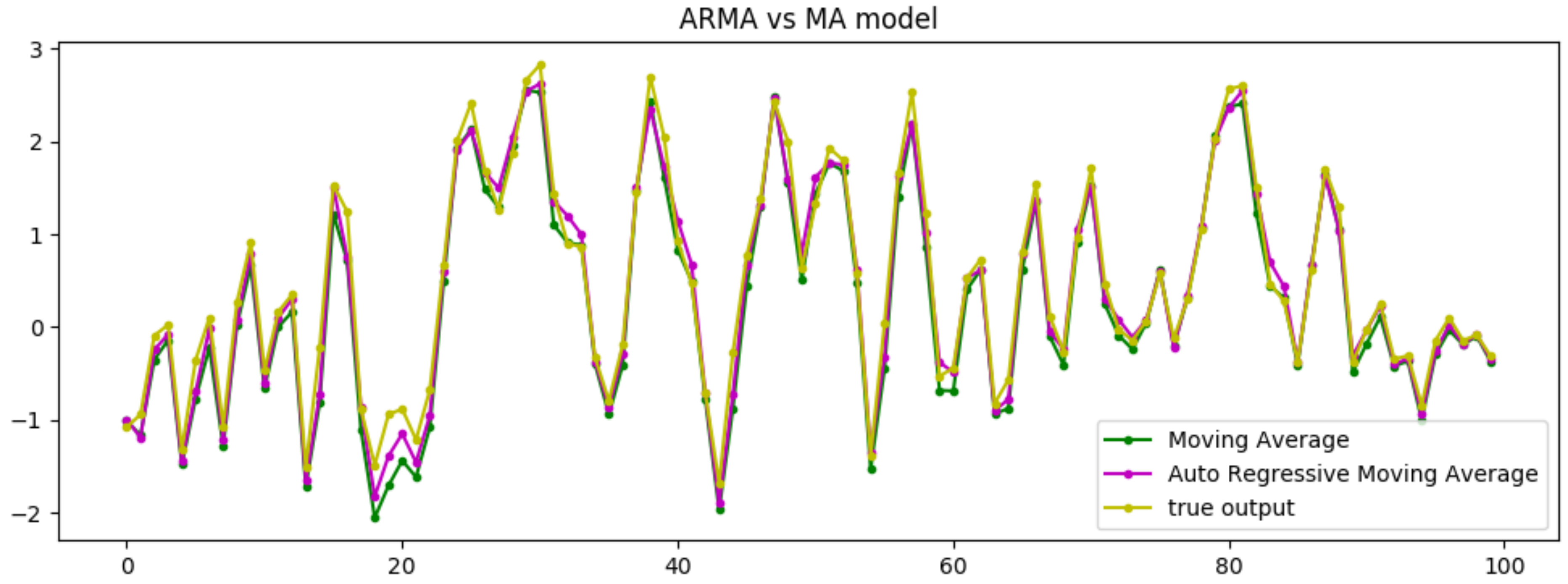

```
Norm for MA :2.460854388269911
Norm for AR :7.436691765656794
```

b) Yet another possible modeling choice is to combine both AR and MA. Unsurprisingly, this is called the autoregressive moving average (ARMA) model:

$$ARMA : y_t \approx a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_\ell y_{t-\ell} + b_1 u_t + b_2 u_{t-1} + \cdots + b_k u_{t-k+1}$$

Solve the problem once more, this time using an ARMA model with $k = \ell = 1$. Plot y and $\hat{}y$ as before, and also compute the error $\|y - y\hat{}\|$.

In [3]:
```
k = 1
l = 1
width = k + l
ARMA = zeros(T,width)
for i = 1:l
    ARMA[i+1:end,i] = y[1:end-i]
end
for i = l+1:width
    ARMA[i-l:end,i] = u[1:end-(i-l)+1]
end
woptARMA = ARMA\y
yestARMA = ARMA*woptARMA

figure(figsize=(12,4))
plot(yestMA,"g.-",yestARMA,"m.-", y, "y.-")
legend(["Moving Average", "Auto Regressive Moving Average","true output"], loc="lower right");
title("ARMA vs MA model");
println()
println("Norm for ARMA :",norm(yestARMA-y))
```




```
Norm for ARMA :1.8565828148734607
```