

Experiment 3 Report

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The calculated value of α for the Roll Number : 191EE126 is

$$\alpha = 1 + \text{mod}(126, 3) = 1$$

Problem 1

(Computing DFT)

(Solution)

Given the signal, $x(t) = \sin(2\pi 15\alpha t)$, for a duration of 2 seconds with a resolution of 120 samples/sec,

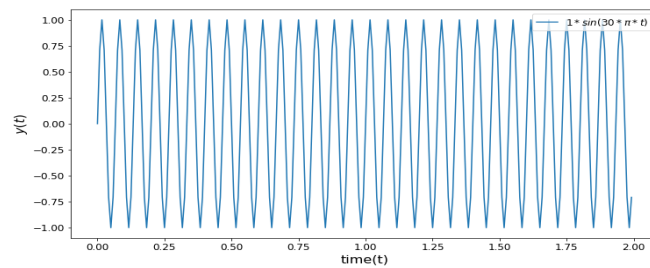


Figure 1: Generated Signal

1. The Discrete fourier transform is calculated and the magnitude spectrum is plotted for the first 120 samples of the signal.

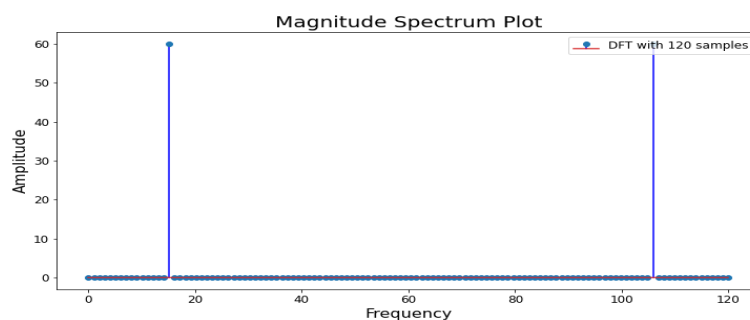


Figure 2: DFT of first 120 Samples

Here we can see that there is no spectral leakage. There is a clear peak at 15Hz and the spectrum is not spread.

2. The Discrete Fourier transform is calculated and the magnitude spectrum is plotted again, but for the first 130 samples of the signal.

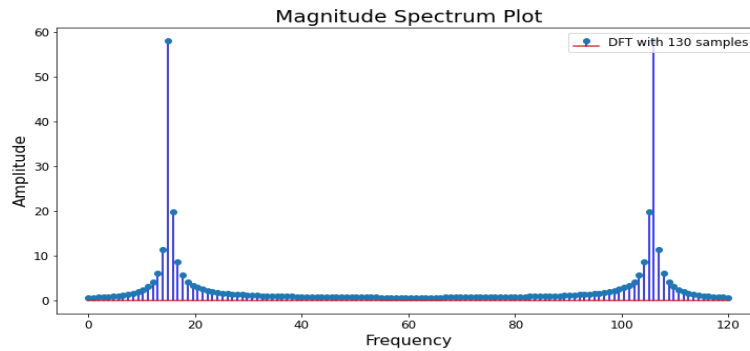


Figure 3: DFT of first 130 Samples

Clearly there is an spectral leakage here. The peak is at 15Hz but there is an spreading of the spectrum near it.

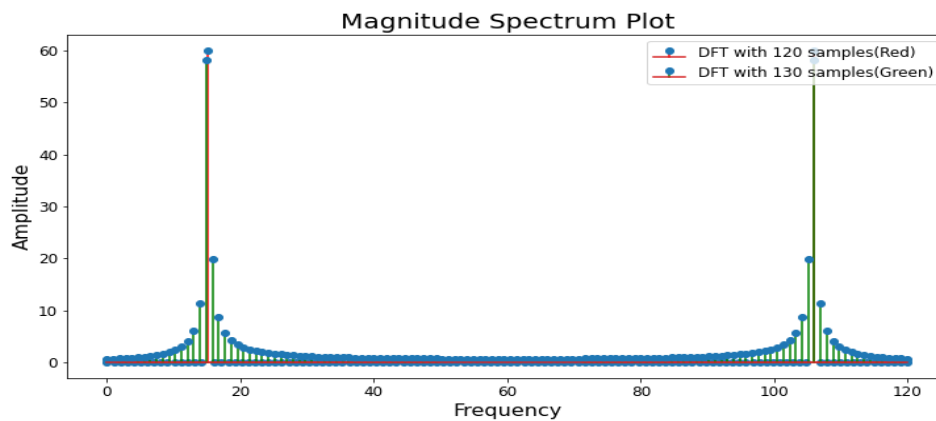


Figure 4: DFT of first 120 and 130 Samples

For multiples of 8, for example first 240 samples, the normalized magnitude spectrum of discrete Fourier transform is the same as the first 120 samples. There is no spectral leakage.

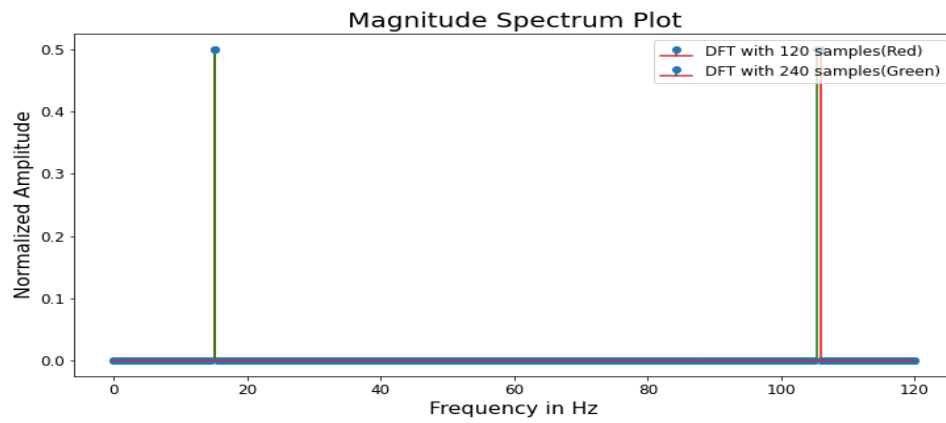


Figure 5: DFT of first 120 and 240 Samples

Problem 2**(Resolution of DFT)****(Solution)**

Given the signal, $x_a(t) = 0.1\sin(A\pi t) + \cos(B\pi t)$, for a duration of 10 seconds with a resolution of 200 samples/sec, the Discrete Fourier transform is plotted for with the following different samples :

1. 215 samples

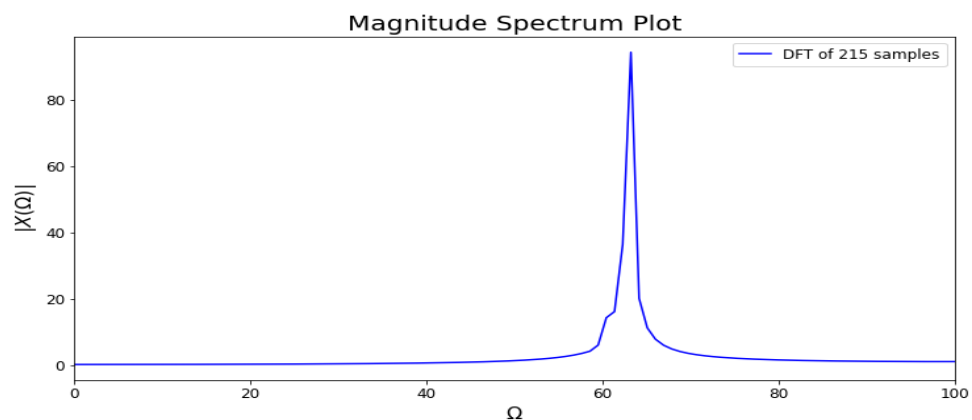


Figure 6: DFT of 215 Samples

2. 415 samples

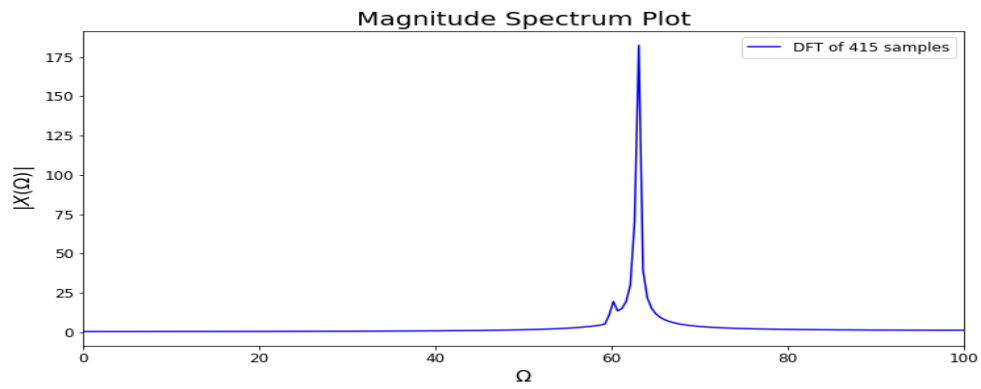


Figure 7: DFT of 415 Samples

3. 1115 samples

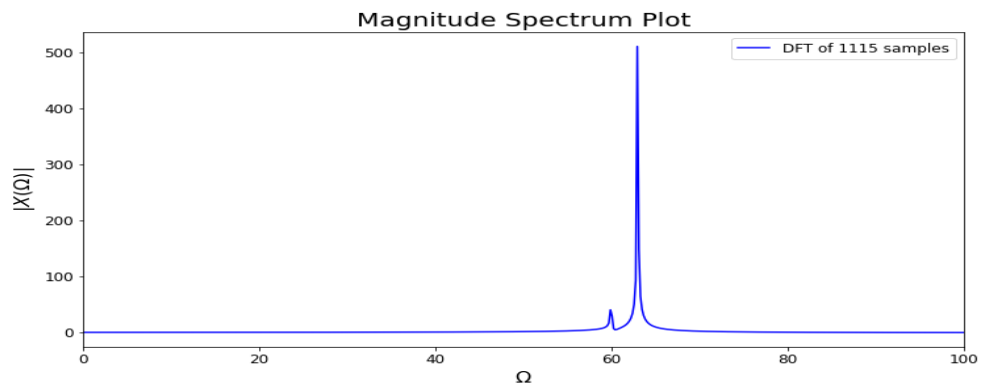


Figure 8: DFT of 1115 Samples

4. 1515 samples

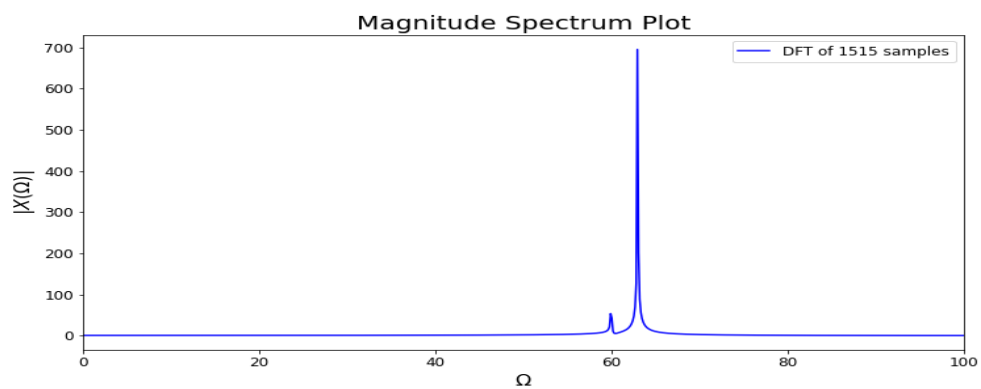


Figure 9: DFT of 1515 Samples

5. 1915 samples

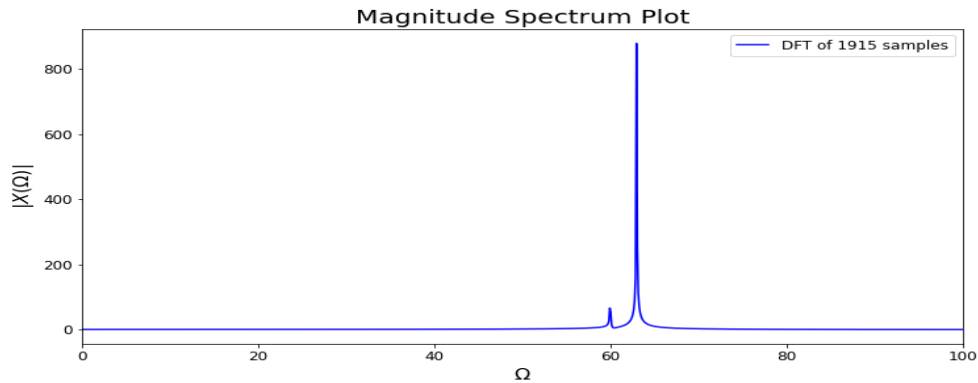


Figure 10: DFT of 1915 Samples

As the number of samples increases from 215 to 1915, the peak becomes more and more sharp. When only 215 samples are the width of the peaks are large. There is spectral leakage. But as the resolution increases or the number of samples, the peaks become for sharp. Hence the peaks are finely distinguishable.

Problem 3**(Resolution of DFT with Windowing)****(Solution)**

The same signal as in problem 3 is considered. But the signal is windowed using the hamming window. The hamming window is the extension of the Hann window :

$$h(n) = \alpha + (1.0 - \alpha) \cos \left(\left(\frac{2\pi}{N} \right) n \right)$$

The same samples are taken and magnitude spectra of the DFT is plotted for these samples.

1. 215 samples

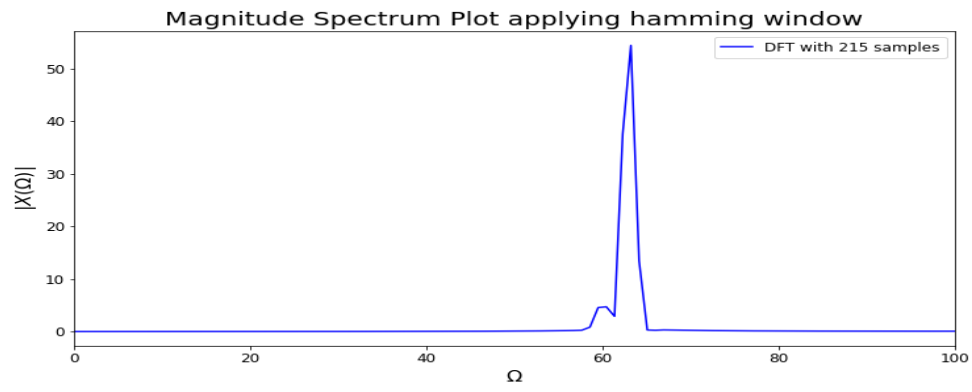


Figure 11: DFT of 215 Samples with Hamming window

2. 415 samples

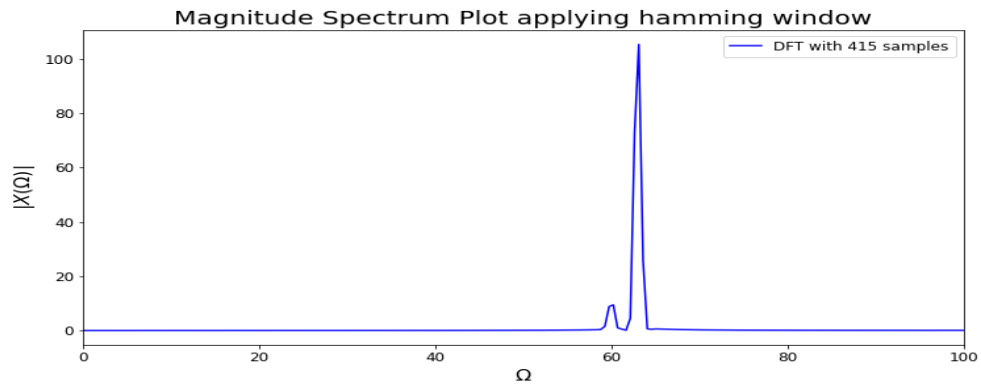


Figure 12: DFT of 415 Samples with Hamming window

3. 1115 samples

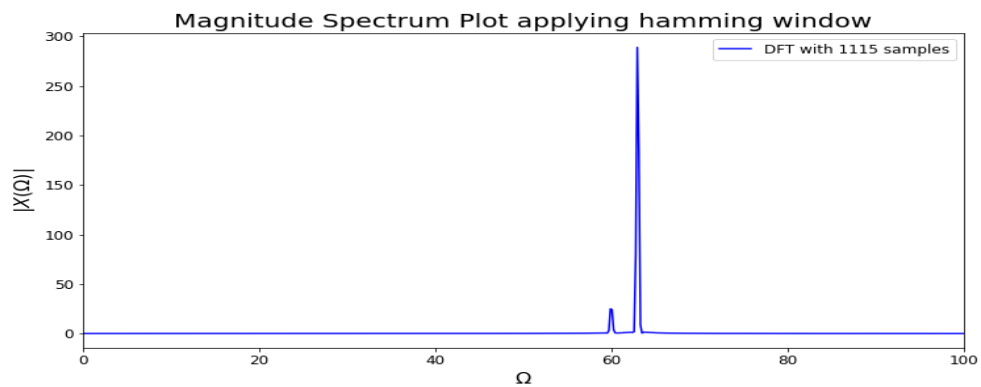


Figure 13: DFT of 1115 Samples with Hamming window

4. 1515 samples

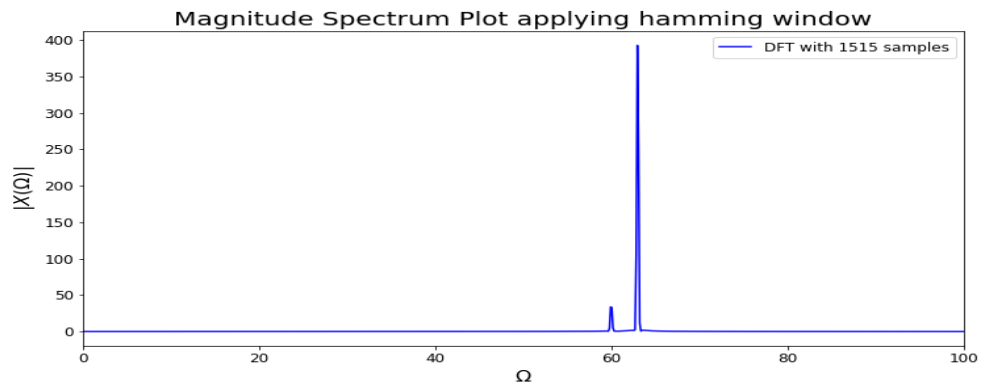


Figure 14: DFT of 1515 Samples with Hamming window

5. 1915 samples

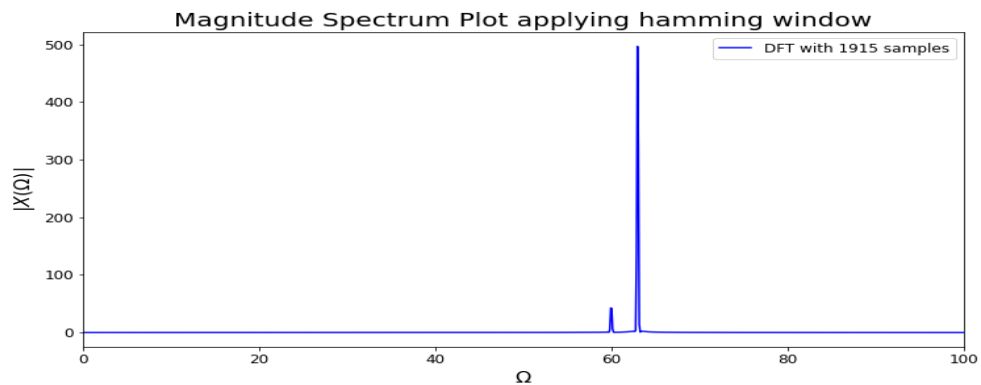


Figure 15: DFT of 1915 Samples with Hamming window

Windowing increases the resolution of the DFT. The width of the peaks decreases and hence the spectral leakage decreases. Windowing also results in smoothing of the plot and decreases the amplitude in the frequency domain.

Problem 4**(Frequency Estimation using Windowing)****(Solution)**

The data is extracted from *Exp4Data1.txt*. Each data point is separated by a ',' delimiter. It contains 500 samples of dual tone. It is padded with 100 zeros to improve the interpolation of the curve. New length of the signal or the number of samples is 600. DFT is calculated for the signal using the following windowing operations :

1. Hamming Window

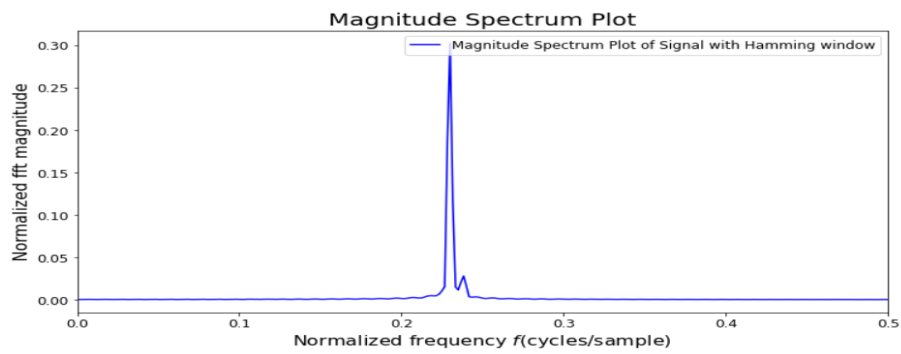


Figure 16: DFT of Signal with Hamming window

The Magnitude spectrum is plotted using normalized parameters. The frequency is normalized with respect to F_s . The two peaks are found for the hamming window at :

$$F_1 = 0.228F_s$$

$$F_2 = 0.236F_s$$

2. Rectangular Window

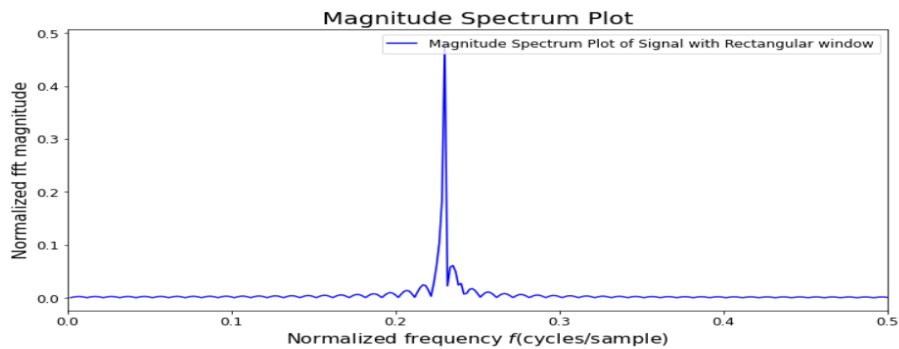


Figure 17: DFT of Signal with Rectangular window

The magnitude spectrum has many small peaks but two prominent peaks, hence a threshold value has to be chosen to find the peaks. Hence, the peaks are at :

$$F_1 = 0.228F_s$$

$$F_2 = 0.233F_s$$

The rectangular window has a smaller width (in frequency domain), due to which the peaks are cannot be properly distinguished.

Code Repository

The code, input and output of all the problems is in the following repository :

<https://github.com/KaranTejas/DSP-Lab/tree/main/Experiment4>.