EE386 Digital Signal Processing Lab **Jul-Dec 2021**

Experiment 7 Report

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The calculated value of α for the Roll Number : 191EE126 is

$$\alpha = 1 + mod(126, 3) = 1$$

Problem 1

(Window Functions)

(Solution)

A window function is a mathematical function that is zero-valued outside of some chosen interval, normally symmetric around the middle of the interval, usually near a maximum in the middle, and usually tapering away from the middle.

1. Rectangular Window:

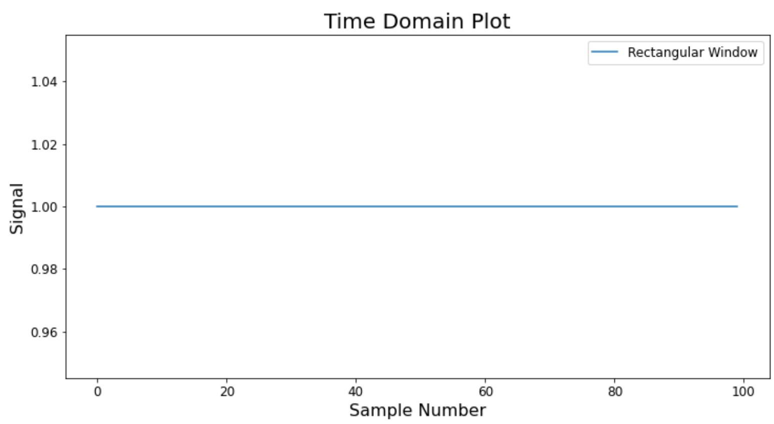

Figure 1: Rectangular Window

The (zero-centered) rectangular window may be defined by :

$$W = \begin{cases} 1, & -\frac{M-1}{2} < n < \frac{M-1}{2} \\ 0, & elsewhere \end{cases}$$

where, M is the window length. If we consider the length as 2τ, the Fourier Transform of the rectangular window is $W(\Omega) = 2\tau sinc(\Omega\tau)$. The rectangular window is the simplest window, equivalent to replacing all but N values of a data sequence by zeros. It has the narrowest main lobe, which allows it to have high frequency resolution but the side lobes are quite large. This causes spectral leakage.

2. Hamming Window and Hanning Window

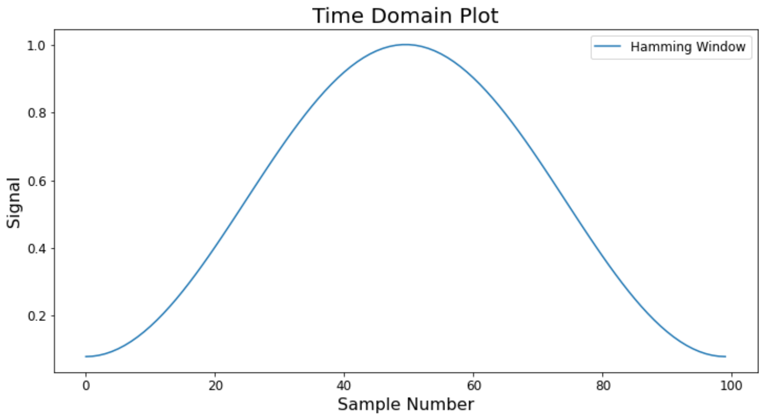

Figure 2: Hamming Window

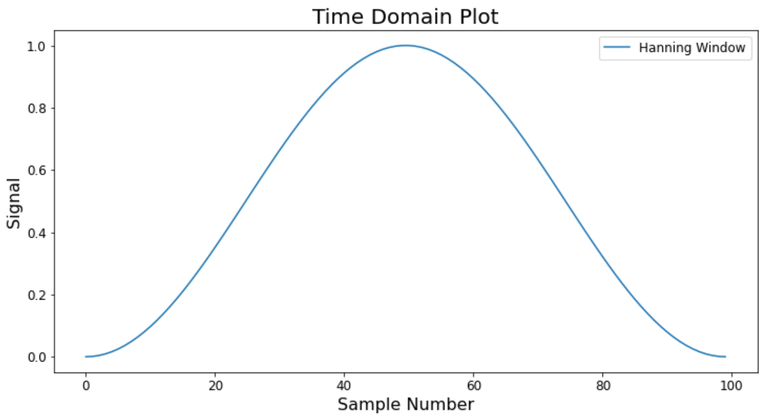

Figure 3: Hanning Window

The hamming window and hanning window may be defined as :

$$W_{hanning}(t) = \begin{cases} 0.5 + 0.5cos(\frac{\pi t}{\tau}), & -\tau < n < \tau \\ 0, & elsewhere \end{cases}$$

$$W_{hamming}(t) = \begin{cases} 0.54 + 0.46cos(\frac{\pi t}{\tau}), & -\tau < n < \tau \\ 0, & elsewhere \end{cases}$$

where 2τ is the window length. Both Hamming and Hanning window functions have a sinusoidal shape, which result in a wide peak but low side lobes. Due to this, there is very low spectral leakage. However, the Hanning window touches zero at both ends eliminating all discontinuity. The Hamming window doesn't quite touch zero and thus still has a small discontinuity in the signal. Due to this difference, the Hamming window does a better job in cancelling the nearest side lobe. But it does a poorer job of canceling any other side lobes. Both the window functions are useful for noise measurements because of better frequency resolution.

Window	Main lobe width	Minimum stopband attenuation	Transition Region
Rectangular	2	13dB	Narrow
Hanning	4	31dB	Intermediate
Hamming	4	43dB	Intermediate

The magnitude spectrum of the Hamming Window for different window lengths using 1024 point DFT and normalized magnitude is plotted.

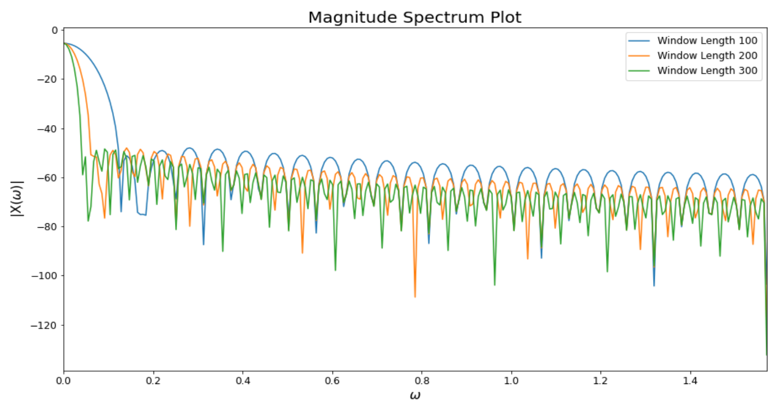

Figure 4: Rectangular Window

Problem 2

(FIR Filter Design)

(Solution)

Low pass FIR filter is designed using Rectangular and Hamming window using *firwin()* function of scipy library with a cutoff frequency of $\omega_c = \frac{\pi}{\alpha+1} = \frac{\pi}{2}$ and window length 21. The *firwin()* return the coefficients of a finite impulse response filter. The impulse resonses of both the filters are plotted.

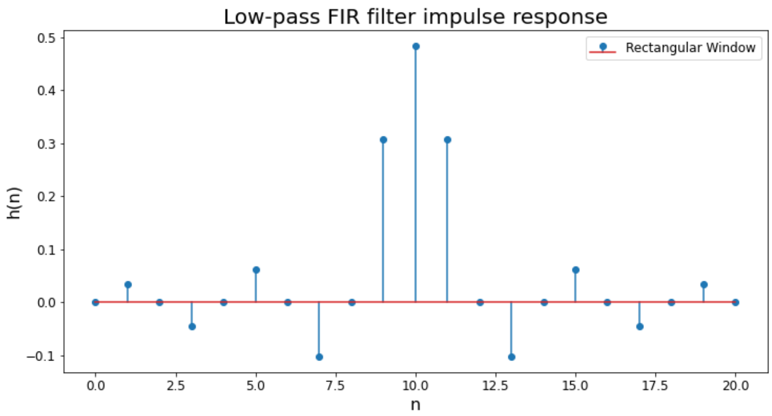

Figure 5: Rectangular Window

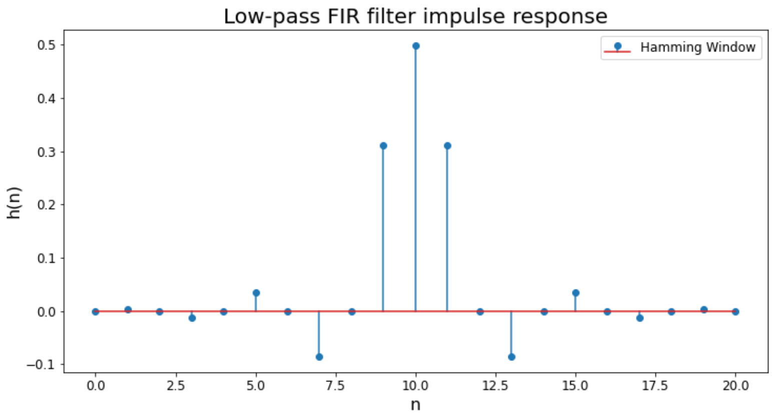

Figure 6: Hamming Window

(Bode Plot)

(Solution)

The Bode Plot of the both the filters is plotted using *freqz()* of the scipy library which takes the transfer function as the input and returns the frequency response. The magnitude and phase is plotted against frequency. The scales are logarithmic using *semilogx()*.

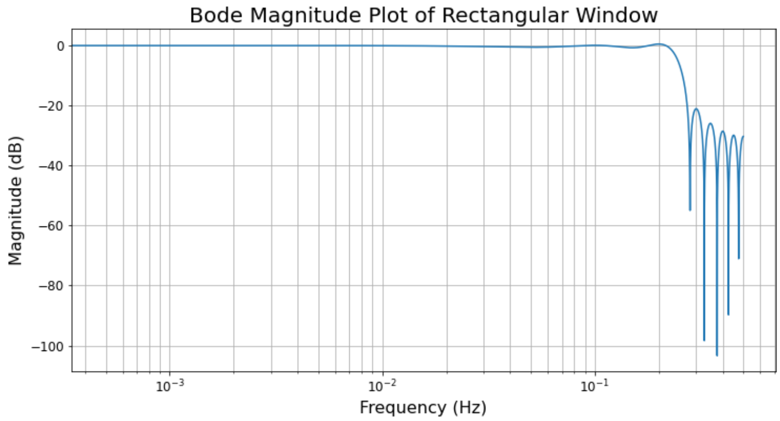

Figure 7: Rectangular Window

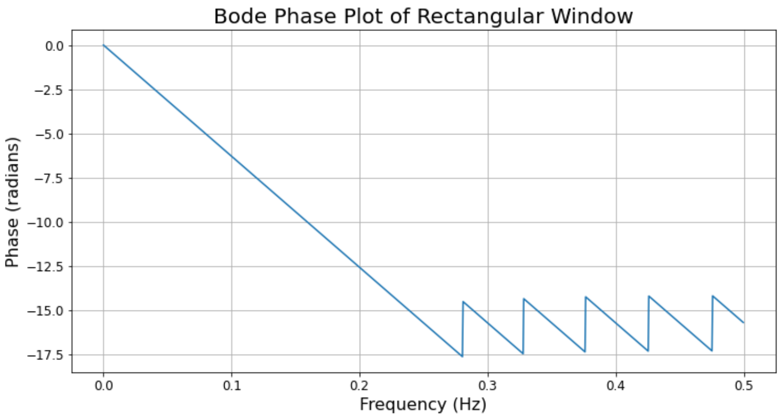

Figure 8: Rectangular Window

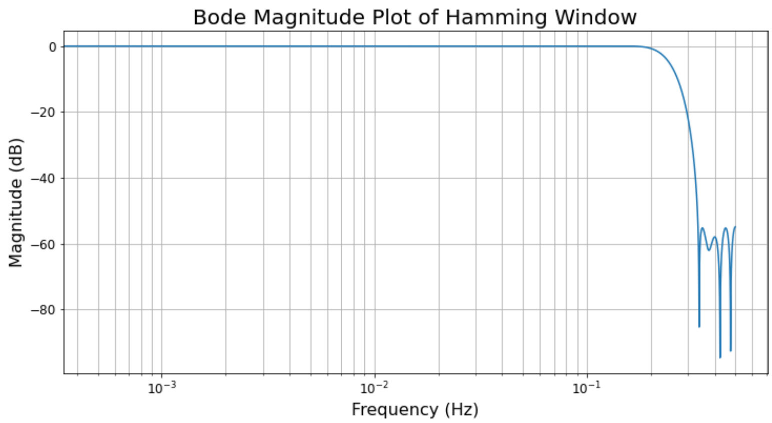

Figure 9: Hamming Window

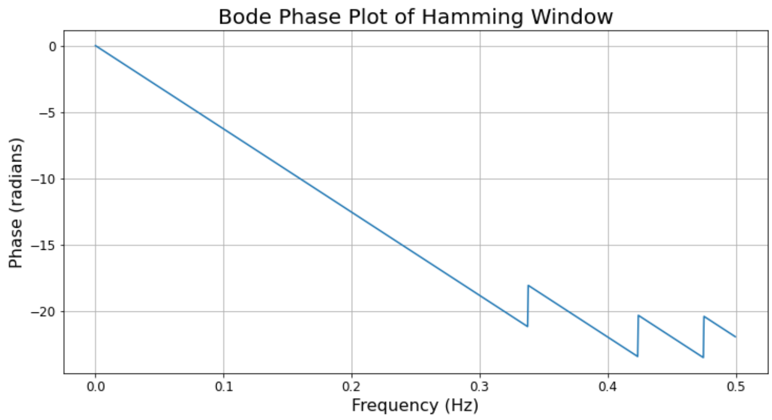

Figure 10: Hamming Window

Problem 3

(Filtering using FIR Filters)

(Solution)

The audio signal is read from the *intru1.wav* file and stored. The spectrogram of the signal is plotted using *specgram()* function in matplotlib library using hamming window.

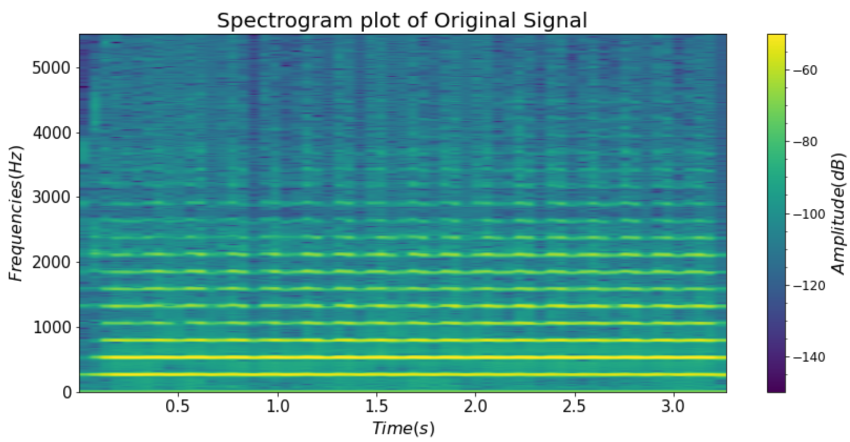

Figure 11: Spectrogram of the Audio Signal

The fundamental frequency of the signal is found to be 256.6924 Hz. The specification of filter is :

- $\Omega'_l = 400\pi \frac{rad}{sec}$
- $\Omega'_u = 600\pi \frac{rad}{sec}$
- $\Omega'_{s1} = 200\pi \frac{rad}{sec}$

- $\Omega'_{s2} = 800\pi \frac{rad}{sec}$

The filter is designed using the *firwin()* function of scipy library similar to problem 2. The audio signal is convolved with the given signal using *lfilter()* function and the spectrogram is plotted.

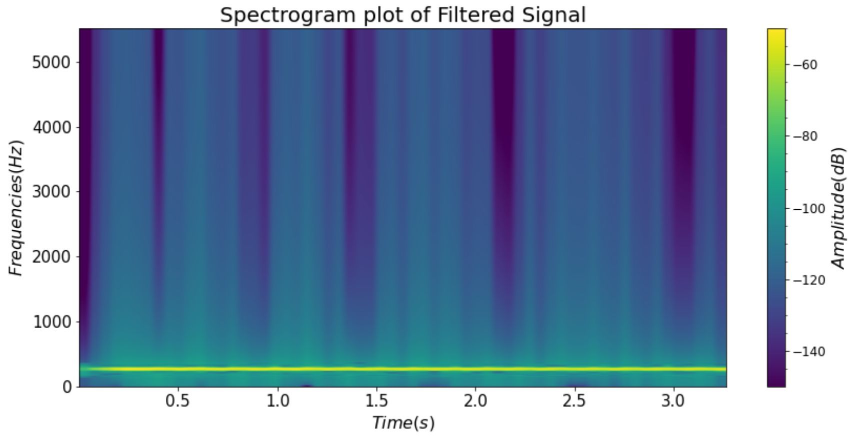

Figure 12: Spectrogram of the Filtered Signal

The fundamental frequency and the only peak of the filtered signal is 256.6924 Hz which is equal to that of the original signal. The fltered signal is stored in *'instrument_filtered.wav'*.

Problem 4

(Time-domain windowing and Window-Based FIR filter design)

(Solution)

In spectral analysis, we expect the signal to have a few finite stationary frequency components and try the find these peaks in the frequency components. In windowing, we multiply L-length window function, with the signal in the time domain to select its first L samples. The multiplication in the time domain is interpreted as convolution in the frequency domain, i.e., DTFT of window signal is convolved with that of the given signal, $X(\omega) * W(\omega)$. This results in shifted and scaled copies of $W(\omega)$ in the spectrum. Because of the structure of the magnitude spectrum of $W(\omega)$, the position of the mainlobes of the shifted $W(\omega)$ tell the frequency components. Similarly in FIR filter design, a shifted ideal filter impulse response is multiplied with a causal window in the time domain, which results in convolution in the frequency domain.

The performance of a filter with a window can be associated with the window's main lobes width and the height of the side lobes. For a given window, the height of the side lobes are fixed but the main lobe width can be decreased by increasing the window length. The frequency resolution of the filter increases as the main lobe width decreases. The spectral leakage of the filter increases as the side lobes height increases. This is because the smaller peaks get created in the neighbourhood of the main peaks during convolution due to the side lobes. The height of the side lobes is dependent on the type of the window, it cannot be changed with the window length. Hence a window with smaller side lobes should be chosen to reduce spectral leakage.

Code Repository

The code, input and output of all the problems is in the following repository :
`https://github.com/KaranTejas/DSP-Lab/tree/main/Experiment7`.