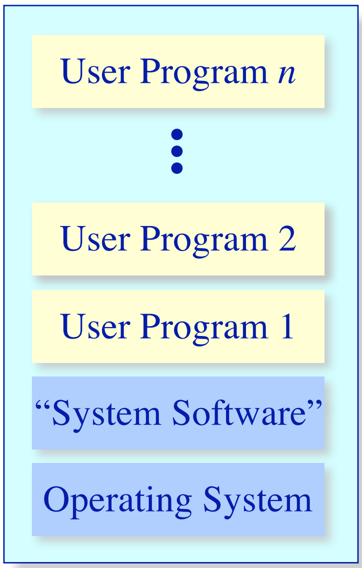
Page Replacement Algorithms

Virtual Memory Management

Fundamental issues : A Recap

- Key concept: Demand paging
  - Load pages into memory only when a page fault occurs
- Issues:
  - Placement strategies
    - Place pages anywhere – no placement policy required
  - Replacement strategies
    - What to do when there exist more jobs than can fit in memory
  - Load control strategies
    - Determining how many jobs can be in memory at one time



Memory

Page Replacement Algorithms
Concept

- Typically Σ_i *VAS_i* >> *Physical Memory*
- With demand paging, physical memory fills quickly
- When a process faults & memory is full, some page must be swapped out
  - Handling a page fault now requires ***2*** disk accesses not 1!

Which page should be replaced?

Local replacement — Replace a page of the faulting process
Global replacement — Possibly replace the page of another process

Page Replacement Algorithms
Evaluation methodology

- Record a *trace* of the pages accessed by a process
  - Example: (Virtual) address trace...
    (3,0), (1,9), (4,1), (2,1), (5,3), (2,0), (1,9), (2,4), (3,1), (4,8)
  - generates page trace
    3, 1, 4, 2, 5, 2, 1, 2, 3, 4 (represented as *c*, *a*, *d*, *b*, *e*, *b*, *a*, *b*, *c*, *d*)

Simulate the behavior of a page replacement algorithm on the trace and record the number of page faults generated

fewer faults ⇨ *better performance*

Optimal Page Replacement
Clairvoyant replacement

- Replace the page that won't be needed for the longest time in the future

Time	0	1	2	3	4	5	6	7	8	9	10
Requests		*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames 0	*a*										
Page Frames 1	*b*										
Page Frames 2	*c*										
Page Frames 3	*d*										
Faults											
Time page needed next											

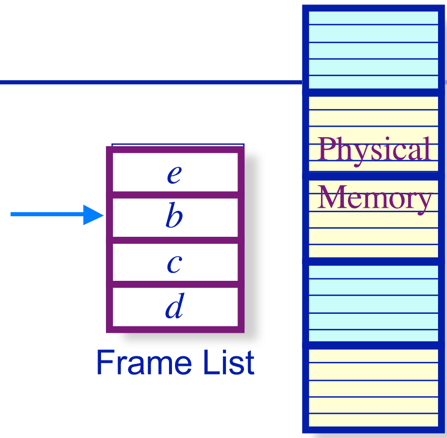
Optimal Page Replacement
Clairvoyant replacement

- Replace the page that won't be needed for the longest time in the future

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	***d***
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*
	3	*d*	*d*	*d*	*d*	*d*	***e***	*e*	*e*	*e*	*e*	*e*
Faults							•					•
Time page needed next							$a = 7$ $b = 6$ $c = 9$ $d = 10$					$a = 15$ $b = 11$ $c = 13$ $e = 14$

Local Page Replacement
FIFO replacement

- Simple to implement
  - A single pointer suffices

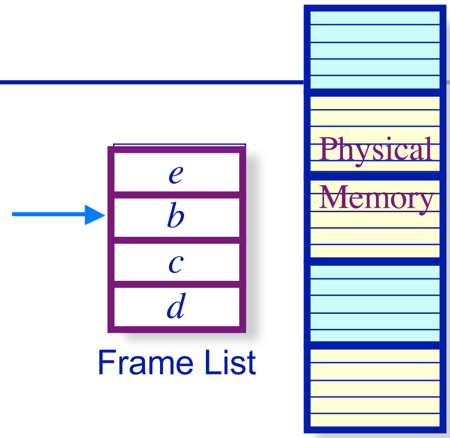


- Performance with 4 page frames:

		Time	0	1	2	3	4	5	6	7	8	9	10
Requests				*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0		*a*										
	1		*b*										
	2		*c*										
	3		*d*										
Faults													

Local Page Replacement
FIFO replacement

- Simple to implement
  - *A single pointer suffices*



- Performance with 4 page frames:

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	***e***	*e*	*e*	*e*	*e*	***d***
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	***a***	*a*	*a*	*a*
	2	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	***b***	*b*	*b*
	3	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults							•		•	•	•	•

Least Recently Used Page Replacement

Use the recent past as a predictor of the near future

- Replace the page that hasn't been referenced for the longest time

				1	2	3	4	5	6	7	8	9	10
Time			0	1	2	3	4	5	6	7	8	9	10
Requests				*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0		*a*										
	1		*b*										
	2		*c*										
	3		*d*										
Faults													
Time page last used													

Least Recently Used Page Replacement
Use the recent past as a predictor of the near future

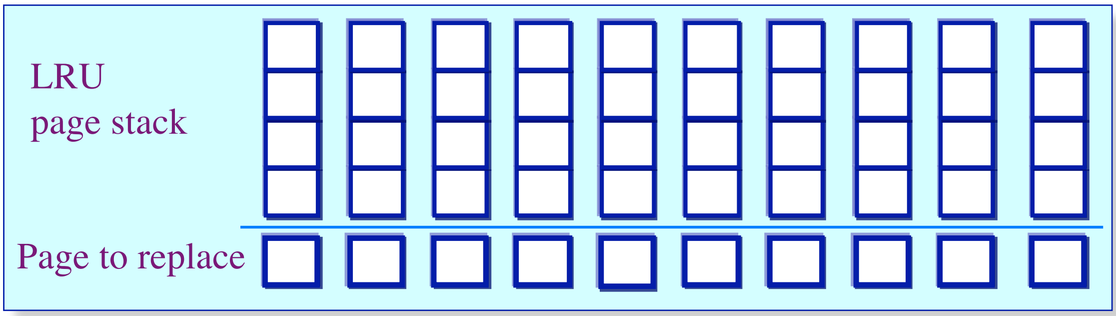
- Replace the page that hasn't been referenced for the longest time

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	*c*	*c*	*c*	*c*	*c*	***e***	*e*	*e*	*e*	*e*	***d***
	3	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults							•				•	•
Time page last used						*a* = 2 *b* = 4 *c* = 1 *d* = 3				*a* = 7 *b* = 8 *e* = 5 *d* = 3	*a* = 7 *b* = 8 *e* = 5 *c* = 9	

Least Recently Used Page Replacement
Implementation

- Maintain a "stack" of recently used pages

		Time	0	1	2	3	4	5	6	7	8	9	10
		Requests		*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0		*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
	1		*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2		*c*	*c*	*c*	*c*	*c*	***e***	*e*	*e*	*e*	*e*	***d***
	3		*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults								•				•	•



Least Recently Used Page Replacement
Implementation

- Maintain a "stack" of recently used pages

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
	1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
	2	*c*	*c*	*c*	*c*	*c*	***e***	*e*	*e*	*e*	*e*	***d***
	3	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*	***c***	*c*
Faults							•				•	•

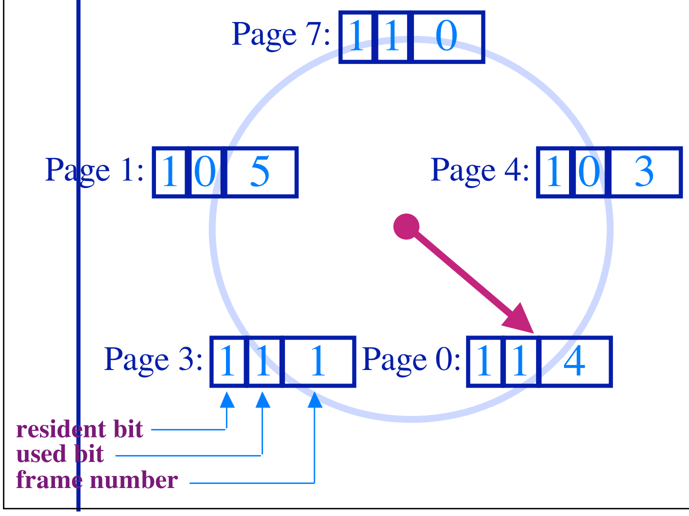
	1	2	3	4	5	6	7	8	9	10
LRU page stack	*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
		c	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*
			c	*a*	*d*	*d*	*e*	*e*	*a*	*b*
				c	*a*	*a*	*d*	*d*	*e*	*a*
Page to replace					***c***				***d***	***e***

Least Recently Used Page Replacement
Alternate Implementation --- Aging Register

- Maintain an n-bit aging register $R = R_{n-1}R_{n-2}...R_0$ for each page frame
  - On a page reference, set R_{n-1} to 1
  - Every T units of time, shift the aging vector right by one bit
- Key idea:
  - Aging vector can be interpreted as a positive binary number
  - Value of R decreases periodically unless the page is referenced
- Page replacement algorithm:
  - On a page fault, replace the page with the smallest value of R

Approximate LRU Page Replacement
The Clock algorithm

- Maintain a circular list of pages resident in memory
  - Use a *clock* (or *used/referenced*) bit to track how often a page is accessed
  - The bit is set whenever a page is referenced
- Clock hand sweeps over pages looking for one with *used* bit = 0
  - Replace pages that haven't been referenced for one complete revolution of the clock




```
func Clock_Replacement
begin
   while (victim page not found) do
      if (used bit for current page = 0) then
         replace current page
      else
         reset used bit
      end if
      advance clock pointer
   end while
end Clock_Replacement
```

Clock Page Replacement

Example

		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*a*	*d*	*b*	*e*	*b*	*a*	*b*	*c*	*d*
Page Frames	0	*a*	*a*	*a*	*a*	*a*						
	1	*b*	*b*	*b*	*b*	*b*						
	2	*c*	*c*	*c*	*c*	*c*						
	3	*d*	*d*	*d*	*d*	*d*						
Faults												

Page table entries for resident pages:

1	*a*
1	*b*
1	*c*
1	*d*

Clock Page Replacement
Example

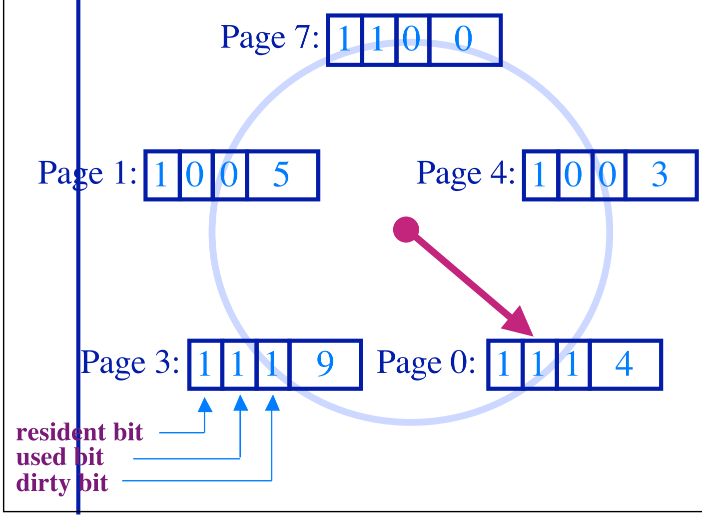
Time	0	1	2	3	4	5	6	7	8	9	10
Requests		c	a	d	b	e	b	a	b	c	d
Page Frames 0	a	a	a	a	a	e	e	e	e	e	d
Page Frames 1	b	b	b	b	b	b	b	b	b	b	b
Page Frames 2	c	c	c	c	c	c	c	a	a	a	a
Page Frames 3	d	d	d	d	d	d	d	d	d	c	c
Faults						•		•		•	•

Page table entries for resident pages:

Time 0	Time 5	Time 6	Time 7	Time 8	Time 9	Time 10
1 a	1 e	1 e	1 e	1 e	1 e	1 d
1 b	0 b	1 b	0 b	1 b	1 b	0 b
1 c	0 c	0 c	1 a	1 a	1 a	0 a
1 d	0 d	0 d	0 d	0 d	1 c	0 c

Optimizing Approximate LRU Replacement
The Second Chance algorithm

- There is a significant cost to replacing "dirty" pages
- Modify the Clock algorithm to allow dirty pages to always survive one sweep of the clock hand
  - Use both the *dirty bit* and the *used bit* to drive replacement



Second Chance Algorithm

Before clock sweep: *used*	Before clock sweep: *dirty*	After clock sweep: *used*	After clock sweep: *dirty*
0	0	*replace page*	
0	1	0	0
1	0	0	0
1	1	0	1

The Second Chance Algorithm

Example

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			c	a^w	d	b^w	e	b	a^w	b	c	d
Page Frames	0	a	a	a	a	a						
	1	b	b	b	b	b						
	2	c	c	c	c	c						
	3	d	d	d	d	d						
Faults												

Page table entries for resident pages:

10	a
10	b
10	c
10	d

The Second Chance Algorithm

Example

Time	0	1	2	3	4	5	6	7	8	9	10
Requests		c	a^w	d	b^w	e	b	a^w	b	c	d
Page Frames 0	a	a	a	a	a	a	a	a	a	a	a
1	b	b	b	b	b	b	b	b	b	b	d
2	c	c	c	c	c	e	e	e	e	e	e
3	d	d	d	d	d	d	d	d	d	c	c
Faults						•				•	•

Page table entries for resident pages:	10 a	11 a	00 a^*	00 a^*	11 a	11 a	00 a^*
	10 b	11 b	00 b^*	10 b^*	10 b^*	10 b^*	10 d
	10 c	10 c	10 e	10 e	10 e	10 e	00 e
	10 d	10 d	00 d	00 d	00 d	10 c	00 c

The Problem With Local Page Replacement

How much memory do we allocate to a process?

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Requests		*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*

Page Frames														
	0	*a*												
	1	*b*												
	2	*c*												
Faults														

Page Frames														
	0	*a*												
	1	*b*												
	2	*c*												
	3	–												
Faults														

The Problem With Local Page Replacement

How much memory do we allocate to a process?

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Requests		*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*	*a*	*b*	*c*	*d*

Page Frames													
0	*a*	*a*	*a*	*a*	***d***	*d*	*d*	***c***	*c*	*c*	***b***	*b*	*b*
1	*b*	*b*	*b*	*b*	*b*	***a***	*a*	*a*	***d***	*d*	*d*	***c***	*c*
2	*c*	*c*	*c*	*c*	*c*	*c*	***b***	*b*	*b*	***a***	*a*	*a*	***d***
Faults					•	•	•	•	•	•	•	•	•

Page Frames													
0	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*
1	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*	*b*
2	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*
3	–				***d***	*d*	*d*	*d*	*d*	*d*	*d*	*d*	*d*
Faults					•								

Page Replacement Algorithms

Performance

- Local page replacement
  - LRU — Ages pages based on when they were last used
  - FIFO — Ages pages based on when they're brought into memory
- Towards global page replacement ... with variable number of page frames allocated to processes

The principle of locality

- 90% of the execution of a program is sequential
- Most iterative constructs consist of a relatively small number of instructions
- When processing large data structures, the dominant cost is sequential processing on individual structure elements
- Temporal vs. physical locality

Optimal Page Replacement

For processes with a variable number of frames

- *VMIN* — Replace a page that is not referenced in the *next* τ accesses
- Example: τ = 4

		0	1	2	3	4	5	6	7	8	9	10
Time												
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	• $t = 0$										
	Page *b*	-										
	Page *c*	-										
	Page *d*	• $t = -1$										
	Page *e*	-										
Faults												

Optimal Page Replacement

For processes with a variable number of frames

- *VMIN* — Replace a page that is not referenced in the *next* τ accesses
- Example: τ = 4

		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	• $t = 0$	-	-	-	-	-	-	-	-	*F*	-
	Page *b*	-	-	-	-	*F*	-	-	-	-	-	-
	Page *c*	-	*F*	•	•	•	•	•	•	-	-	-
	Page *d*	• $t = -1$	•	•	•	-	-	-	-	-	-	*F*
	Page *e*	-	-	-	-	-	-	*F*	•	•	-	-
Faults			•			•		•			•	•

Explicitly Using Locality
The working set model of page replacement

- Assume recently referenced pages are likely to be referenced again soon…
- … and *only* keep those pages recently referenced in memory (called *the working set*)
  - Thus pages may be removed even when no page fault occurs
  - The number of frames allocated to a process will vary over time
- A process is allowed to execute only if its working set fits into memory
  - The working set model performs implicit load control

Working Set Page Replacement
Implementation

- Keep track of the last τ references
  - The pages referenced during the last τ memory accesses are the working set
  - τ is called the *window size*
- Example: Working set computation, τ = 4 references:

Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	• $t = 0$										
	Page *b*	-										
	Page *c*	-										
	Page *d*	• $t = -1$										
	Page *e*	• $t = -2$										
Faults												

Working Set Page Replacement
Implementation

- Keep track of the last τ references
  - The pages referenced during the last τ memory accesses are the working set
  - τ is called the *window size*
- Example: Working set computation, τ = 4 references:

Time	0	1	2	3	4	5	6	7	8	9	10
Requests		*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory: Page *a*	• $t = 0$	•	•	•	-	-	-	-	-	*F*	•
Page *b*	-	-	-	-	*F*	•	•	•	-	-	-
Page *c*	-	*F*	•	•	•	•	•	•	•	•	•
Page *d*	• $t = -1$	•	•	•	•	•	•	-	-	-	*F*
Page *e*	• $t = -2$	•	-	-	-	-	*F*	•	•	•	•
Faults		•			•		•			•	•

Page-Fault-Frequency Page Replacement
An alternate working set computation

- Explicitly attempt to minimize page faults
  - When page fault frequency is high — *increase working set*
  - When page fault frequency is low — *decrease working set*

Algorithm:

Keep track of the rate at which faults occur

When a fault occurs, compute the time since the last page fault

Record the time, t_{last}, of the last page fault

If the time between page faults is "large" then reduce the working set

If $t_{current} - t_{last} > \tau$, then remove from memory all pages not referenced in $[t_{last}, t_{current}]$

If the time between page faults is "small" then increase working set

If $t_{current} - t_{last} \leq \tau$, then add faulting page to the working set

Page-Fault-Frequency Page Replacement

Example, window size = 2

- If $t_{current} - t_{last} > 2$, remove pages not referenced in $[t_{last}, t_{current}]$ from the working set
- If $t_{current} - t_{last} \le 2$, just add faulting page to the working set

		0	1	2	3	4	5	6	7	8	9	10
Time		0	1	2	3	4	5	6	7	8	9	10
Requests			*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	•										
	Page *b*	-										
	Page *c*	-										
	Page *d*	•										
	Page *e*	•										
Faults												
$t_{cur} - t_{last}$												

Page-Fault-Frequency Page Replacement

Example, window size = 2

- If $t_{current} - t_{last} > 2$, remove pages not referenced in $[t_{last}, t_{current}]$ from the working set
- If $t_{current} - t_{last} \leq 2$, just add faulting page to the working set

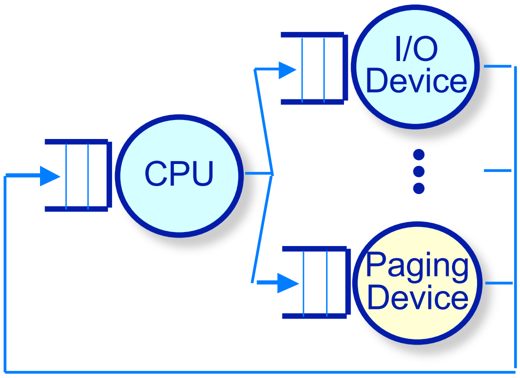
	Time	0	1	2	3	4	5	6	7	8	9	10
	Requests		*c*	*c*	*d*	*b*	*c*	*e*	*c*	*e*	*a*	*d*
Pages in Memory	Page *a*	•	•	•	•	-	-	-	-	-	*F*	•
	Page *b*	-	-	-	-	*F*	•	•	•	•	-	-
	Page *c*	-	*F*	•	•	•	•	•	•	•	•	•
	Page *d*	•	•	•	•	•	•	•	•	•	-	*F*
	Page *e*	•	•	•	•	-	-	*F*	•	•	•	•
	Faults		•			•		•			•	•
	$t_{cur} - t_{last}$		1			3		2			3	1

Load Control
Fundamental tradeoff

- High multiprogramming level
  - $$MPL_{max} = \frac{\text{number of page frames}}{\text{minimum number of frames required for a process to execute}}$$
- Low paging overhead
  - MPL_{min} = 1 process
- Issues
  - What criterion should be used to determine when to increase or decrease the *MPL*?
  - Which task should be swapped out if the *MPL* must be reduced?

Load Control

How not to do it: Base load control on CPU utilization

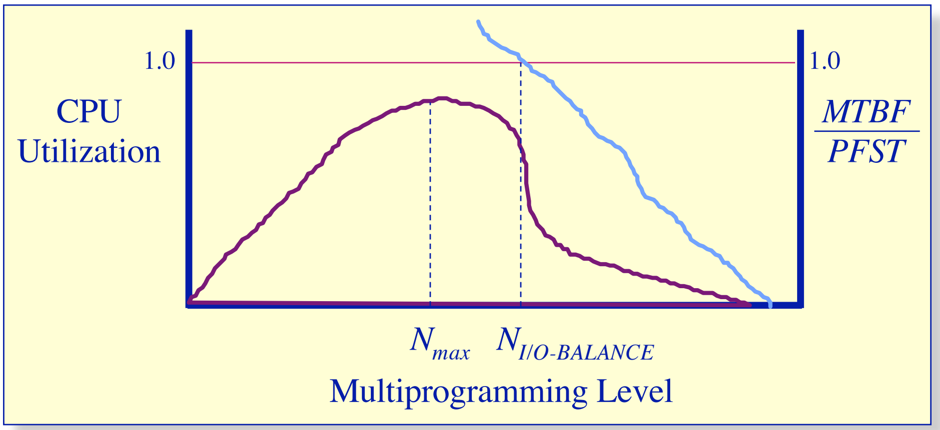


- Assume memory is nearly full
- A chain of page faults occur
  - A queue of processes forms at the paging device
- CPU utilization falls
- Operating system increases *MPL*
  - New processes fault, taking memory away from existing processes
- CPU utilization goes to 0, the OS increases the *MPL* further...

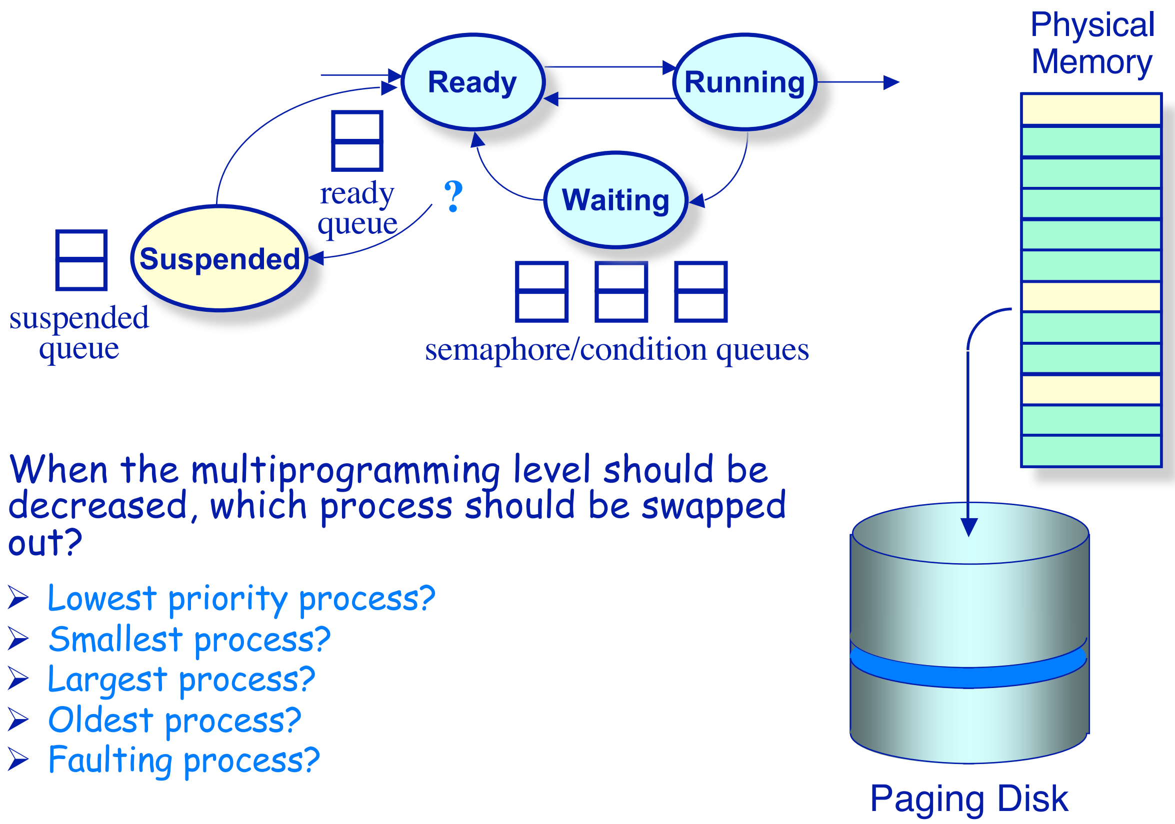
System is *thrashing* — spending all of its time paging

Load Control
Thrashing

- Thrashing can be ameliorated by *local* page replacement
- Better criteria for load control: Adjust MPL so that:
  - *mean time between page faults (MTBF) = page fault service time (PFST)*
  - $\Sigma\ WS_i$ *= size of memory*



Load Control
Thrashing



- When the multiprogramming level should be decreased, which process should be swapped out?
  - Lowest priority process?
  - Smallest process?
  - Largest process?
  - Oldest process?
  - Faulting process?