

# GUJARAT TECHNOLOGICAL UNIVERSITY

## B. E. FIRST YEAR

Subject Name: **Vector Calculus and Linear Algebra**

Subject Code: **110015**

### **Course Objectives of Vector Calculus and Linear Algebra:**

This course is designed to develop the intuitive understanding and computational skills necessary for the concepts of calculus of functions of several variables by tying together vector calculus and vector algebra. To be successful in this course, student should have adequate knowledge of elementary calculus and set theory.

The course objective is to introduce the student to the concepts of vector-valued functions, vector analysis and vector algebra. Students will learn in detail about Gradient, Curl, Divergence, Line integration, Surface integration. It is leading to learn the Green's theorem, Stoke's theorem and Divergence Theorem. They will also learn all the basics of vector spaces like subspaces, linear span, linear independence of vectors, basis of vector space, solution space of simultaneous linear system of equations, determinants and its properties of higher order, linear transformation of real vector spaces, matrix of linear transformations, change of basis, inner product space and gram-schmidt process, eigen values and eigen vectors and application to quadric forms.

After the successful completion of the course, students will be able to

- Determine the line integration, surface integration
- Understand the vector space and its properties
- Establish the linear transformations and inner product
- Recognize applications to quadratic forms

The course will help students to apply these concepts to the problems related to models in work, circulation and flux Problems, hydrodynamics and fluid dynamics, electrical circuits, networking, linear programming, graph theory, computer graphics, construction of curves and surfaces through specified points etc.

The course is designed in such a way that it can be covered comprehensively in a semester course.

| Sr. No. | Contents  | Total Hrs |
|---------|---|-----------|
| 1       | Gradients and Directional derivatives   | 01        |
| 2       | Parametrization of curves, Arc length and surface area of parametrized curves and surfaces.   | 02        |
| 3       | Line integrals, Work, circulation, flux, path independence, conservative field, surface integrals   | 03        |
| 4       | Divergence and curl, Green's theorem, Stoke's theorem and Divergence theorem.   | 05        |
| 5       | Vectors in $R^n$ , properties of $R^n$ , dot product, norm and distance properties in $R^n$ , Pythagorean theorem in $R^n$ , Vector Spaces, Vector Subspaces, Linear independence and dependence, Linear span, Basis of subspaces, Extension to basis.  | 07        |
| 6       | Methods to solve system of linear equations: Gauss elimination, Gauss Jordan method, Elementary row operation, Row reduced echelon form, Inverse of matrices, Row space, Column space, Null space, Rank.  | 07        |
| 7       | Determinants and its properties, Cramer's rule.   | 02        |
| 8       | Linear transformation in $R^n$ and basic properties, Linear transformation as Rotation, reflection, expansion, contraction, shear, projection, Matrix of linear transformations, Change of basis and Similarity, Rank Nullity theorem.  | 05        |
| 9       | Inner product space and properties, Angle and orthogonality and normality of basis, Gram Schmidt's process, Least squares approximations (linear system)  | 05        |
| 10      | Eigen values and eigen vectors, Cayley-Hamilton theorem, Eigen values of orthogonal, symmetric, skew symmetric, hermitian, skew hermitian, unitary and normal matrices, Algebraic and Geometric multiplicity, Diagonalization by similarity transformation, Spectral theorem for real symmetric matrices, Application to quadratic forms. | 05        |

### Text Books:

1. E. Kreyszig, Advanced Engineering Mathematics (8<sup>th</sup> Edition), Wiley-India (1999) Sections: 8.9-8.11, 9.1-9.9
2. H. Anton, Elementary Linear Algebra with applications (9<sup>th</sup> Edition), Wiley-India (2008). Chapters 1 to 8 and Articles 9.5-9.7.

### Reference Books:

1. Hughes – Hallett et al., Calculus – Single and Multivariable (3<sup>rd</sup> Edition), John-Wiley and Sons (2003).
2. G. Strang, Linear Algebra and its applications (4<sup>th</sup> Edition), Thomson, (2006).
3. S. Kumaresan, Linear Algebra – A Geometric Approach, Prentice Hall India (2006).
4. Maurice D. Weir, Joel Hass, Frank R. Giordano, Thomas' Calculus, Person Education (11<sup>th</sup> edition)