Lab 9 Solution

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Problem 1

Minimize

$$f(x, y) = -xy$$

subject to

$$x + y^2 \leq 2 \text{ and } x, y \geq 0$$

```
# Load the nloptr library
library(nloptr)

# Objective function
func <- function(x) {
  c(-x[1] * x[2])
}
```

The above code defines the objective function that needs to be minimized.

```
# Nonlinear inequality constraints:

eval_g_ineq <- function(x) {
  c(x[1] +  x[2]^2 - 2, -x[1], -x[2])
}
```

The above code defines the inequality constraints subject to which the objective function is to be minimized.

```
# Run optimization using NLOPT_GN_ISRES
res <- nloptr(
  x0 = c(0.5, 0.5),                          # Feasible start point
  eval_f = func,
  eval_g_ineq = eval_g_ineq,
```

```
  lb = c(0, 0),                              # Lower bounds for x and y
  ub = c(10, 10),                            # Large but finite upper bounds
  opts = list(
    "algorithm" = "NLOPT_GN_ISRES",
    "xtol_rel" = 1e-8, "maxeval" = 1e4
  )
)
```

The variable **res** stores the required output obtained after minimization of the given function.

```
res
```

```
Call:
nloptr(x0 = c(0.5, 0.5), eval_f = func, lb = c(0, 0), ub = c(10,
    10), eval_g_ineq = eval_g_ineq, opts = list(algorithm = "NLOPT_GN_ISRES",
    xtol_rel = 1e-08, maxeval = 10000))


Minimization using NLopt version 2.8.0

NLopt solver status: 5 ( NLOPT_MAXEVAL_REACHED: Optimization stopped because
maxeval (above) was reached. )

Number of Iterations....: 10000
Termination conditions:  xtol_rel: 1e-08    maxeval: 10000
Number of inequality constraints:  3
Number of equality constraints:    0
Current value of objective function:  -1.08529541072842
Current value of controls: 1.326188 0.8183571
```

From the above, we can see that the minimum value of the function for relative tolerance 10^{-8} and maximum number of function evaluations as 10^4 is quite close to the actual minimum value of the function i.e. -1.088662. The function attains its minimum value at $(\frac{4}{3}, \sqrt{\frac{2}{3}})$.

Problem 2

Minimize

$$f(x, y) = 2x + y$$

subject to

$$\sqrt{x^2 + y^2} \leq 2,\ x \geq 0 \text{ and } y \geq 0.5x - 1$$

```r
# Objective function
func <- function(x) {
  2 * x[1] + x[2]
}
```

The above code defines the objective function that needs to be minimized.

```r
# Inequality Constraints:

eval_g_ineq <- function(x) {
  c(sqrt(x[1]^2 + x[2]^2) - 2,
    -x[1],
    (0.5 * x[1] - 1) - x[2]          # y   0.5x -1
  )
}
```

The above code defines the inequality constraints subject to which the objective function is to be minimized.

```r
res <- nloptr(
  x0 = c(1, 1),  # start point satisfying constraints approximately
  eval_f = func,
  eval_g_ineq = eval_g_ineq,
  lb = c(0, -3),
  ub = c(10, 10),
  opts = list(
    "algorithm" = "NLOPT_GN_ISRES",
    "xtol_rel" = 1e-8, "maxeval" = 10000
  )
)
```

The variable **res** stores the required output obtained after minimization of the given function.

```r
res
```

```
Call:
nloptr(x0 = c(1, 1), eval_f = func, lb = c(0, -3), ub = c(10,
    10), eval_g_ineq = eval_g_ineq, opts = list(algorithm = "NLOPT_GN_ISRES",
    xtol_rel = 1e-08, maxeval = 10000))
```

```
Minimization using NLopt version 2.8.0

NLopt solver status: 5 ( NLOPT_MAXEVAL_REACHED: Optimization stopped because
maxeval (above) was reached. )

Number of Iterations....: 10000
Termination conditions:  xtol_rel: 1e-08    maxeval: 10000
Number of inequality constraints:  3
Number of equality constraints:    0
Current value of objective function:  -0.999962271400933
Current value of controls: 1.364148e-05 -0.9999896
```

From the above, we can see that the minimum value of the function for relative tolerance 10^{-8} and maximum number of function evaluations as 10^4 is quite close to the actual minimum value of the function i.e. -1. The function attains its minimum value at $(0, -1)$.

Problem 3

Minimize

$$f(x_1, \dots, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

subject to

$$x_1 + x_2 + x_3 + x_4 = 1 \text{ and } x_4 \geq A.$$

Consider three cases:

1. $A < 1/4$
2. $A = 1/4$
3. $A > 1/4$

```r
# Objective Function
func <- function(x){
  sum(x^2)
}
```

The above code defines the objective function that needs to be minimized.

Equality Constraints:

```
# Equality Constraints:

eval_g_eq <- function(x){
  sum(x) - 1
}
```

The above code defines the equality constraints subject to which the objective function is to be minimized.

```
solve_p3 <- function(A) {
  eval_g_ineq <- function(x) {
    A - x[4]
  }

  res <- nloptr(
    x0 = c(rep(A / 3, 3), 1 - A),  # reasonable start
    eval_f = func,
    eval_g_ineq = eval_g_ineq,
    lb = rep(-2, 4),
    ub = rep(10, 4),
    eval_g_eq = eval_g_eq,
    opts = list(
      "algorithm" = "NLOPT_GN_ISRES",
      "xtol_rel" = 1e-8, "maxeval" = 1e8
    )
  )
  return(res)
}
```

The above code minimizes the objective function subject to different values of the variable A.

```
# Case (i) A < 1/4
res_p3_case1 <- solve_p3(A = 0.2)
print(res_p3_case1)
```

```
Call:
nloptr(x0 = c(rep(A/3, 3), 1 - A), eval_f = func, lb = rep(-2,
    4), ub = rep(10, 4), eval_g_ineq = eval_g_ineq, eval_g_eq = eval_g_eq,
    opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,
```

```
        maxeval = 1e+08))


Minimization using NLopt version 2.8.0

NLopt solver status: 4 ( NLOPT_XTOL_REACHED: Optimization stopped because
xtol_rel or xtol_abs (above) was reached. )

Number of Iterations....: 88193
Termination conditions:  xtol_rel: 1e-08    maxeval: 1e+08
Number of inequality constraints:  1
Number of equality constraints:    1
Optimal value of objective function:  0.25000023517728
Optimal value of controls: 0.250289 0.2498277 0.2496966 0.2501867
```

```r
# Case (ii) A = 1/4
res_p3_case2 <- solve_p3(A = 0.25)
print(res_p3_case2)
```

```


Call:
nloptr(x0 = c(rep(A/3, 3), 1 - A), eval_f = func, lb = rep(-2,
    4), ub = rep(10, 4), eval_g_ineq = eval_g_ineq, eval_g_eq = eval_g_eq,
    opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,
        maxeval = 1e+08))


Minimization using NLopt version 2.8.0

NLopt solver status: 4 ( NLOPT_XTOL_REACHED: Optimization stopped because
xtol_rel or xtol_abs (above) was reached. )

Number of Iterations....: 90939
Termination conditions:  xtol_rel: 1e-08    maxeval: 1e+08
Number of inequality constraints:  1
Number of equality constraints:    1
Optimal value of objective function:  0.250000344384973
Optimal value of controls: 0.2496812 0.2504248 0.2497717 0.2501223
```

```r
# Case (iii) A > 1/4
res_p3_case3 <- solve_p3(A = 0.3)
print(res_p3_case3)
```

```
Call:
nloptr(x0 = c(rep(A/3, 3), 1 - A), eval_f = func, lb = rep(-2,
    4), ub = rep(10, 4), eval_g_ineq = eval_g_ineq, eval_g_eq = eval_g_eq,
    opts = list(algorithm = "NLOPT_GN_ISRES", xtol_rel = 1e-08,
        maxeval = 1e+08))


Minimization using NLopt version 2.8.0

NLopt solver status: 4 ( NLOPT_XTOL_REACHED: Optimization stopped because
xtol_rel or xtol_abs (above) was reached. )

Number of Iterations....: 80214
Termination conditions:  xtol_rel: 1e-08    maxeval: 1e+08
Number of inequality constraints:  1
Number of equality constraints:    1
Optimal value of objective function:  0.253333415319365
Optimal value of controls: 0.2333537 0.2335305 0.2331158 0.3
```

From the above results we can see that the objective function has a minimum value of $\frac{1}{4}$ irrespective of the value of the variable A, subject to the given constraints.