Econometric Modelling for Value-at-Risk

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Contents

Objective of the project

- History and Importance of Value-at-Risk
- VaR models like Historical Simulation, Variance Covariance Approach (Delta-Normal Approach), Monte Carlo Simulaton, Extreme Value Theory Approach (Semi-Parametric Approach).
- Methods of Backtesting
- Interpretation of Backtesting Results

- The first attempt to quantify risk was made by Francis Edgeworth in 1888.
- VAR was indirectly first mentioned by Dickson.H.Leavers in 1945.
- Harry Markowitz (1990) and Arthur D Roy (1952) both were looking for a strategy to maximize profit at a particular level of risk.
- Under the Chairmanship of Dennis Weatherstone of JP Morgan Bank, VaR was created by using Markowitz Portfolio Theory.
- Till Guldimann can be viewed as the creator of the term ”Value at Risk” while the head of Global Research at JP Morgan.

- Risk Measurement: It quantifies the market risk
- Portfolio diversification: Using VaR, investors can determine how much risk each item in a portfolio contributes.
- Decision Making: Financial managers use VaR to set risk limits, allocate capital, and develop hedging strategies.
- Stress Testing: Involves simulating extreme market scenarios to evaluate the resilience of portfolios and financial systems during crises.

Methods for Calculation of VaR

Historical Simulation

- A risk management tool used to calculate the possible losses of an investment or financial portfolio based on previous market data is known as historical simulation.

- This method applies current weights to a time series of historical asset returns

$$R_{p,k} = \sum_{i=1}^{N} w_{i,t} R_{i,k} \text{ for } k = 1, \ldots, t$$

- The method can use full valuation,which are obtained from applying historical changes in prices to the current level of prices

$$S^*_{i,k} = S_{i,0} + \Delta S_{i,k} \quad i = 1, ..., N$$

- Then new portfolio V_p^*, k then is computed perhaps incorporating Non linear relationships

$$V_k^* = V(s_i^*, k)$$

- The set of risk factors can incorporate implied volatility measures. This create hypothetical return corresponding to Simulation

$$R_{p,k} = \frac{V_k^* - V_0}{V_0}$$

- VAR then is obtained from the entire distribution of hypothetical returns, where each historical scenario is assigned the same weight of $(\frac{1}{t})$.

Methods for Calculation of VaR

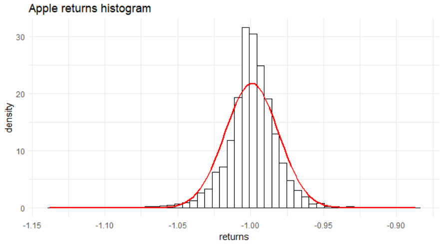
Variance Covariance Approach (Delta-Normal Approach)

- Variance Covariance approach works on the assumption that data is normally distributed.

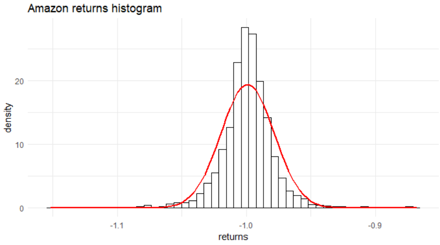
- The VaR calculation formula is as follows:
  Portfolio Value * Portfolio Standard Deviation * Z-Score of Confidence

- Where the relationship between a value and a set of value's mean is described by the statistical measurement known as the Z-Score.

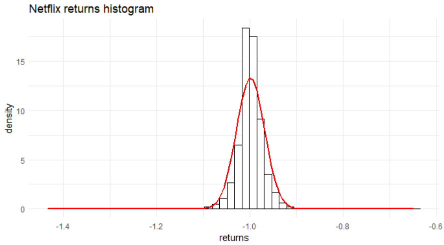
- The covariance matrix of the portfolio's assets can be used to calculate the standard deviation for the portfolio.



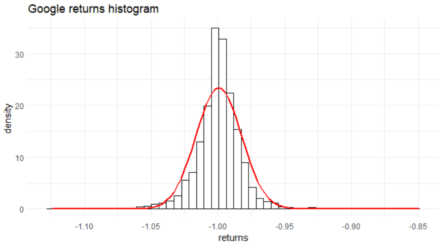
(a) Apple Returns Histogram



(b) Amazon Returns Histogram



(a) Netflix Returns Histogram



(b) Google Returns Histogram

Methods for Calculation of VaR

Monte Carlo Simulation

- The Monte Carlo Simulation approach is founded on the practise of obtaining numerical results through repeated random sampling.

- Formula for Calculation of VaR

$$VaR = Q(P - CL)$$

- Q- It is the quantile function, which provides the inverse of the Cumulative Distribution Function.
  P- It represents the simulated returns, which are generated from the normal distribution with the mean and standard deviation of the actual returns.
  CL- It is the desired Confidence Level.

- The study of extreme events or outliers that occur in the tail area of a distribution is the focus of the statistical field known as extreme value theory (EVT).

- The semi-parametric approach often involves the following steps- Threshold Selection, Tail Fitting, Risk Measure.

- The cumulative distribution function (CDF) has been used to characterize the GPD
$F(y;\xi,\beta) = 1 - \left(1 + \xi \frac{y}{\beta}\right)^{-\frac{1}{\xi}}$

- **Quantile Function (QF)** or the inverse of the CDF of the GPD, which is used to calculate VaR:
$Q(p;\xi,\beta) = \beta \left[(1-p)^{-\xi} - 1\right]$

Introduction of Backtesting

- VaR Backtesting compares losses predicted by a VaR model with the actual losses.
- Backtesting is required to evaluate model's effectiveness.
- "VaR is only as good as its Backtest. When someone shows me VaR number, I don't ask how it is computed, I ask to see the backtest" (K. Dowd 2005).

- Paul Kupiec proposed the Kupiec test, technically known as the Proportion of Failures (POF) test.

- The test statistic for the Kupiec test is calculated as follows:

$$POF = 2\left[\ln((1-p)^{N-n} \cdot p^{n})\right] - 2\left[\ln((1-x)^{N-n} \cdot x^{n})\right]$$

  Where:
  p is the expected proportion of exceptions (1 minus the confidence level).
  N is the total number of observations.
  n is the actual number of exceptions.
  x is the observed proportion of exceptions (n/N).

- The results of this test on different models are further projected.

Kupiec Backtesting Results for Historical Simulation, Variance Covariance Approach and Monte Carlo Simulation

Table: Kupiec Test Results for Historical Simulation, Variance Covariance Approach and Monte Carlo Simulation at different confidence interval for all the selected stocks and for last 500 observations

Stock	**Confidence Level**	**Kupiec Test Result**
Apple	95%	Rejected
Apple	97.5%	Rejected
Apple	99%	Rejected
Amazon	95%	Rejected
Amazon	97.5%	Rejected
Amazon	99%	Rejected
Netflix	95%	Rejected
Netflix	97.5%	Rejected
Netflix	99%	Rejected
Google	95%	Rejected
Google	97.5%	Rejected
Google	99%	Rejected

Kupiec Backtesting Results for Extreme Value Theory Approach (Semi-Parametric Approach)

Table: Kupiec Test Results for Extreme Value Theory Approach at different confidence interval for all the selected stocks and for last 500 observations

Stock	Confidence Level	Result	Result for 500
Apple	95%	Accepted	Accepted
Apple	97.5%	Rejected	Rejected
Apple	99%	Rejected	Rejected
Amazon	95%	Rejected	Rejected
Amazon	97.5%	Rejected	Accepted
Amazon	99%	Rejected	Rejected
Netflix	95%	Accepted	Accepted
Netflix	97.5%	Accepted	Accepted
Netflix	99%	Accepted	Accepted
Google	95%	Accepted	Accepted
Google	97.5%	Accepted	Rejected
Google	99%	Accepted	Accepted

Methods of Backtesting

Traffic Light, Unconditional Coverage, Frequency, and Failure Rate (TUFF) Backtesting

- **Traffic Light Test (TLT)**: This test measures the proportion of times the actual return is less than the predicted value at risk (VaR). A lower value indicates better performance.

- **Unconditional Coverage Test (UCT)**: This test measures the accuracy of the model's VaR predictions. This value should ideally be close to confidence level used.

- **Frequency Test (FT)**: This test measures the proportion of 'hits', instances when the actual return is less than the predicted VaR, in the total dataset.

- **Failure Rate (FR)**: This test calculates the proportion of 'hits' in the total number of 'hits' expected. This value should ideally be close to 1.

TUFF Backtest Results for Historical Simulation

Table: TUFF Backtest Results for Selected Stocks

Stock	TLT	UCT	FT	FR
Apple	0.9731	1	0.9731	1
Amazon	0.9716	1	0.9716	1
Netflix	0.9653	1	0.9653	1
Google	0.9728	1	0.9728	1

Table: TUFF Backtest Results for 500 Observations

Stock	TLT	UCT	FT	FR
Apple	0.968	1	0.968	1
Amazon	0.966	1	0.966	1
Netflix	0.948	1	0.948	1
Google	0.954	1	0.954	1

TUFF Backtest Results for Variance Covariance Approach

Table: TUFF Backtest Results for Selected Stocks (Data normally distributed)

Stock	TLT	UCT	FT	FR
Apple	0.975	1	0.975	1
Amazon	0.974	1	0.974	1
Netflix	0.977	1	0.977	1
Google	0.975	1	0.975	1

Table: TUFF Backtest Results for Selected stocks (Data not normally distributed)

Stock	TLT	UCT	FT	FR
Apple	0.975	1	0.975	1
Amazon	0.974	1	0.974	1
Netflix	0.977	1	0.977	1
Google	0.975	1	0.975	1

TUFF Backtest Results for Variance Covariance Approach

Table: TUFF Backtest Results for 500 Observations (Data normally distributed)

Stock	**TLT**	**UCT**	**FT**	**FR**
Apple	0.975	1	0.975	1
Amazon	0.974	1	0.974	1
Netflix	0.977	1	0.977	1
Google	0.975	1	0.975	1

Table: TUFF Backtest Results for 500 Observations (Data not normally distributed)

Stock	**TLT**	**UCT**	**FT**	**FR**
Apple	0.975	1	0.975	1
Amazon	0.974	1	0.974	1
Netflix	0.977	1	0.977	1
Google	0.975	1	0.975	1

TUFF Backtest Results for Monte Carlo Simulation

Table: TUFF Backtest Results for Selected stocks

Stock	TLT	UCT	FT	FR
Apple	0.9684418	1	0.9684418	1
Amazon	0.9727811	1	0.9727811	1
Netflix	0.9704142	1	0.9704142	1
Google	0.9759369	1	0.9759369	1

Table: TUFF Backtest Results for 500 Observations

Stock/Portfolio	TLT	UCT	FT	FR
Apple	0.974359	1	0.974359	1
Amazon	0.97357	1	0.97357	1
Netflix	0.9755424	1	0.9755424	1
Google	0.9767258	1	0.9767258	1

TUFF Backtest Results for Extreme Value Theory Approach (Semi-Parametric Approach)

Table: TUFF Backtest Results for Selected stocks

Stock	Confidence Level	P-Value
Apple	95%	1
Amazon	95%	N/A
Netflix	95%	1
Google	95%	1
Apple	97.5%	N/A
Amazon	97.5%	N/A
Netflix	97.5%	1
Google	97.5%	1
Apple	99%	N/A
Amazon	99%	N/A
Netflix	99%	1
Google	99%	1

TUFF Backtest Results for Extreme Value Theory Approach (Semi-Parametric Approach)

Table: TUFF Backtest Results for 500 Observations

Stock	Confidence Level	P-Value
Apple	95%	1
Amazon	95%	N/A
Netflix	95%	1
Google	95%	1
Apple	97.5%	N/A
Amazon	97.5%	N/A
Netflix	97.5%	1
Google	97.5%	1
Apple	99%	N/A
Amazon	99%	N/A
Netflix	99%	0.9999997
Google	99%	0.8413447

Conclusion

- we ought to be able to identify most bad VaR models, but the worrying issue is whether we can find any good ones.

- The backtesting result raise concern about the model's ability to estimate risk in satisfactory precision because of volatility.

- The empirical research demonstrates that VaR figures should never be taken as 100 percent. regardless of how advanced the systems are,correct.

Future Work

- **Machine learning and AI integration**: Neural networks can identify complex non-linear relationships in the data.
- **Big Data Integration**: Incorporating data sources like social media, news can allow for more precise VaR calculations.
- **Real Time VaR Calculations**: Allowing institutions to respond to risk as they emerge with advent computational tools.
- **Hybrid Models**: Combining the strengths of different VaR calculation methodologies.

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Inquiries

Dear Supervisors,
I kindly invite any comments, insights, or questions you may have.
Thank you for your guidance and feedback.