ENPM 673

HOMEWORK 1

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PROBLEM 1

Assume that you have a camera with a resolution of 5MP where the camera sensor is square shaped with a width of 14mm. It is also given that the focal length of the camera is 15mm.

1. Compute the Field of View of the camera in the horizontal and vertical direction.
Answer :
Resolution - 5MP
sensor dimension - 14mm X 14mm
focal length (f) - 15mm
Field of View (θ)−?

$\theta = 2Tan^{-1}\left(\frac{d}{2f}\right)$

$\theta = 2Tan^{-1}\left(\frac{14}{2*15}\right)$

$\theta = 50.03$

2. Assuming you are detecting a square shaped object with width 5cm, placed at a distance of 20 meters from the camera, compute the minimum number of pixels that the object will occupy in the image.
Answer :
Height of object (H) - 5cm
Distance - 2000cm
Resolution - 5MP
Sensor dimension - 14mm X 14mm
Focal length (f) - 15mm
Number of pixels occupied - ?

$\theta = Tan^{-1}\left(\frac{H}{D}\right)$

$\theta = Tan^{-1}\left(\frac{5}{2000}\right)$

$\theta = 0.1432$

By using the property of similar triangles we can find the height of the object on the sensor.

$\theta = Tan^{-1}\left(\frac{h}{f}\right)$

OR, $h = Tan(\theta) * f$

$h = Tan(0.1432) * 15$

$h = 0.0375$

Since the object is a square the image on the sensor is also a square. Hence the area occupied on the sensor is $0.0375x0.0375 = 0.00140625mm^2$

Area of the sensor = $14x14 = 196mm^2$

Area of one pixel = $\frac{Areaofthesensor}{totalnumberofpixels}$

Area of one pixel = $\frac{196}{5000000}$

Area of one pixel = 0.0000392

Therefore minimum number of pixels = $\frac{areaoccupiedbytheimageonthesensor}{Areaofonepixel}$

Therefore minimum number of pixels = $\frac{0.00140625}{0.0000392}$

Therefore minimum number of pixels = 35.87

Rounding to 36 pixels

PROBLEM 2

Two files of 2D data points are provided in the form of CSV files (Dataset 1 and Dataset 2). The data represents measurements of a projectile with different noise levels and is shown in figure 1. Assuming that the projectile follows the equation of a parabola,

1. Find the best method to fit a curve to the given data for each case. You have to plot the data and your best fit curve for each case. Submit your code along with the instructions to run it.

2. Briefly explain all the steps of your solution and discuss why your choice of outlier rejection technique is best for that case.

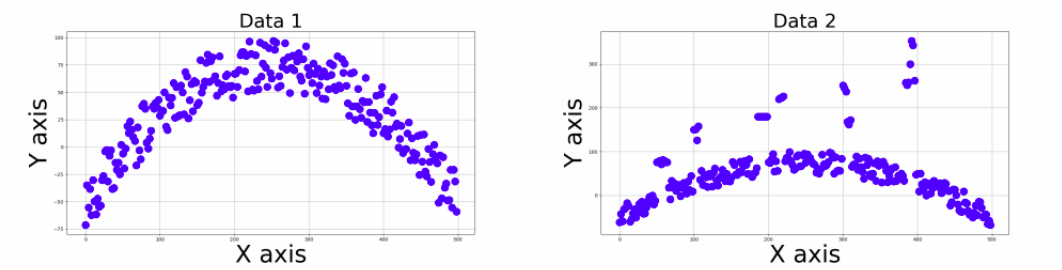

Figure 1: from left to right (a) Dataset 1 and (b) Dataset 2

Answer 1 :

Python program to fit a curve can be found in ENPM673 problem 2.py
Instructions to run the program can be found in README.md
The best method to find solution for data set 1 is by using Least square method and solution for data set 2 is by using RANSAC method.

Answer for data set 1 :

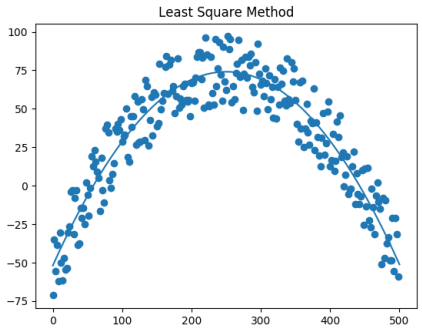

Figure 2: Least square method

Answer for data set 2 :

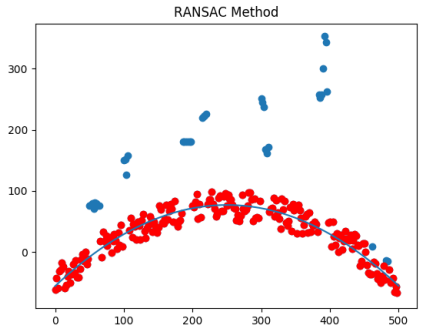

Figure 3: RANSAC Method

Answer 2 :

For data set 1 we have chosen Least square method as there are no outliers. It gives you the best possible solution for dataset 1.
The general parabola equation is $y = ax^2 + bx + c$
a, b, c are are unknowns and we have found the unknowns using the equation $X = (A^T A)^{-1} * A^T B$
using a, b, c we plotted the best solution for the data set 1.

For data set 2 we have chosen RANSAC as there are outliers. It gives you the best possible solution for dataset 2 by neglecting the outliers and considering the inliers.
For RANSAC method we found the best possible parabola using unknown and considering three random points. Considering the threshold probability as 0.8 and threshold distance as 70. We made a loop to randomly select three random points from the data set 2, after that we made a parabolic curve using least square method and we continuously calculated the vertical distance of all the point from the parabola by iteration. We iterated the loop

till we had all the points less that the threshold distance. Then we appended these points as inliers. To check the best solution, we checked the probability of the whole sample to get more that threshold probability, once we attain the probability more than 0.8, we have the best solution to the data set 2 as RANSAC method.
from the figure we can see the red colour dots as inliers and the parabola attained is the best curve for the data set.
The general parabola equation is $y = ax^2 + bx + c$
a, b, c are are unknowns and we have found the unknowns using the equation $X = (A^T A)^{-1} * A^T B$
using a, b, c we plotted the best solution for the data set 1.

PROBLEM 3

The concept of homography in Computer Vision is used to understand, explain and study visual perspective, and, specifically, the difference in appearance of two plane objects viewed from different points of view. This concept will be taught in more detail in the coming lectures. For now, you just need to know that given 4 corresponding points on the two different planes, the homography between them is computed using the following system of equations Ax = 0, where:

$$A = \begin{bmatrix} x1 & y1 & 1 & 0 & 0 & 0 & x1xp1 & y1xp1 & xp1 \\ 0 & 0 & 0 & x1 & y1 & 1 & x1yp1 & y1yp1 & yp1 \\ x2 & y2 & 1 & 0 & 0 & 0 & x2xp2 & y2xp2 & xp2 \\ 0 & 0 & 0 & x2 & y2 & 1 & x2yp2 & y2yp2 & yp2 \\ x3 & y3 & 1 & 0 & 0 & 0 & x3xp3 & y3xp3 & xp3 \\ 0 & 0 & 0 & x3 & y3 & 1 & x3yp3 & y3yp3 & yp3 \\ x4 & y4 & 1 & 0 & 0 & 0 & x4xp4 & y4xp4 & xp4 \\ 0 & 0 & 0 & x4 & y4 & 1 & x4yp4 & y4yp4 & yp4 \end{bmatrix}$$

$$X = \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix}$$

For the given point correspondences,

	x	y	xp	yp
1	5	5	100	100
2	150	5	200	80
3	150	150	220	80
4	5	150	100	200

find the homography matrix

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}$$

1. Show mathematically how you will compute the SVD for the matrix A.
Answer :

SVD of $A = U\Sigma V^T$
Substituting the values into matrix A

$$A = \begin{bmatrix} 5 & -5 & -1 & 0 & 0 & 0 & 500 & 500 & 100 \\ 0 & 0 & 0 & 5 & 5 & 1 & 500 & 500 & 100 \\ -150 & 5 & 1 & 0 & 0 & 0 & 30000 & 1000 & 200 \\ 0 & 0 & 0 & 150 & 5 & 1 & 12000 & 400 & 80 \\ 150 & -150 & 1 & 0 & 0 & 0 & 33000 & 12000 & 220 \\ 0 & 0 & 0 & -150 & 150 & 1 & 12000 & 12000 & 80 \\ -5 & -150 & 1 & 0 & 0 & 0 & 500 & 15000 & 100 \\ 0 & 0 & 0 & -5 & -150 & 1 & 1000 & 30000 & 200 \end{bmatrix}$$

Taking A^t

$$A^t = \begin{bmatrix} -5 & 0 & -150 & 0 & -150 & 0 & -5 & 0 \\ -5 & 0 & -5 & 0 & -150 & 0 & -150 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -5 & 0 & -150 & 0 & -150 & 0 & -5 \\ 0 & -5 & 0 & -5 & 0 & -150 & 0 & -150 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ 500 & 500 & 30000 & 12000 & 33000 & 12000 & 500 & 1000 \\ 500 & 500 & 1000 & 400 & 12000 & 12000 & 15000 & 30000 \\ 100 & 100 & 200 & 80 & 80 & 80 & 100 & 200 \end{bmatrix}$$

Multiplying A with A^t to obtain AA^t

$$AA^t = \begin{bmatrix} 510051 & 510000 & 15520776 & 6208000 & 22509501 & 12008000 & 7760776 & 15520000 \\ 510000 & 510051 & 15520000 & 6208776 & 22508000 & 12009501 & 7760000 & 15520776 \\ 15520776 & 15520000 & 901062526 & 360416000 & 1002039251 & 372016000 & 30021501 & 60040000 \\ 6208000 & 6208776 & 360416000 & 144188926 & 400806400 & 148829651 & 12008000 & 24017501 \\ 22509501 & 22508000 & 1002039251 & 400806400 & 1233051401 & 540006400 & 196531251 & 393016000 \\ 12008000 & 12009501 & 372016000 & 148829651 & 540006400 & 288051401 & 186008000 & 372039251 \\ 7760776 & 7760000 & 30021501 & 12008000 & 196531251 & 186008000 & 225282526 & 450520000 \\ 15520000 & 15520776 & 60040000 & 24017501 & 393016000 & 372039251 & 450520000 & 901062526 \end{bmatrix}$$

By using the below equation we can calculate the value of λ which is the eigen values

$|AA^t - \lambda I| = 0$

λ_1 = 2.59663813e+09
λ_2 = 1.09695312e+09
λ_3 = 6.41021467e+04
λ_4 = 4.05655526e+04
λ_5 = 2.12053510e+04
λ_6 = 2.23413513e+03
λ_7 = 6.48615897e-01
λ_8 = 4.52174595e+01

By substituting the values of $\lambda in |\mathrm{AA}^t - \lambda I| = 0$ we get the eigen vectors

$$U = \begin{bmatrix} 0.01323849 & 0.00646185 & 0.10425695 & 0.41551129 & -0.25053827 & -0.24426678 & 0.80887413 & -0.19887385 \\ 0.01323822 & 0.00646269 & 0.13991737 & 0.40171863 & -0.23959338 & -0.24622078 & -0.53211918 & -0.64625674 \\ 0.53169496 & -0.39015654 & 0.09872668 & 0.55087994 & 0.17563415 & -0.03941962 & -0.14965124 & 0.44392108 \\ 0.21267852 & -0.1560573 & 0.48478221 & -0.14024586 & -0.51167439 & 0.64022783 & 0.02819686 & -0.05686699 \\ 0.68331921 & -0.13702738 & -0.48005121 & -0.3147369 & 0.03029149 & 0.01642001 & 0.12720284 & -0.40917248 \\ 0.31747076 & 0.15487638 & 0.5829539 & -0.42776589 & 0.01033054 & -0.58543948 & -0.011553 & 0.09709349 \\ 0.14422946 & 0.39511051 & -0.37081729 & 0.05147831 & -0.68759 & -0.166967 & -0.14921056 & 0.40001036 \\ 0.28844902 & 0.79023096 & 0.13039554 & 0.24112635 & 0.33669459 & 0.31205209 & 0.02834518 & -0.07512711 \end{bmatrix}$$

$\Sigma = diag(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8)$

$\sigma = \sqrt{\lambda}$

$$\Sigma = \begin{bmatrix} 5.09572186e{+}04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.31202827e{+}04 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.53184017e{+}02 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.01408919e{+}02 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.45620572e{+}02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.72666386e{+}01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8.05366933e{-}01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.72439288e{+}00 \end{bmatrix}$$

To find V matrix we multiply A^T matrix with A matrix and find out the eigen vectors. The transpose of the eigen vectors give us the desired V matrix.

$$A^T A = \begin{bmatrix} 45050 & 24025 & 310 & 0 & 0 & 0 & -9455000 & -2027500 & -43000 \\ 24025 & 45050 & 310 & 0 & 0 & 0 & -5177500 & -4057500 & -28500 \\ 310 & 310 & 4 & 0 & 0 & 0 & -64000 & -28500 & -480 \\ 0 & 0 & 0 & 45050 & 24025 & 310 & -3607500 & -2012500 & -25500 \\ 0 & 0 & 0 & 24025 & 45050 & 310 & -2012500 & -6304500 & -42900 \\ 0 & 0 & 0 & 310 & 310 & 4 & -25500 & -42900 & -460 \\ -9455000 & -5177500 & -64000 & -3607500 & -2012500 & -25500 & 2278750000 & 612800000 & 10910000 \\ -2027500 & -4057500 & -28500 & -2012500 & -6304500 & -42900 & 612800000 & 1414660000 & 9752000 \\ -43000 & -28500 & -480 & -25500 & -42900 & -460 & 10910000 & 9752000 & 129200 \end{bmatrix}$$

By using the below equation we can calculate the value of λ which is the eigen values (eigenvalues of AA^T is the same as the eigenvaluse of $A^T A$)

$$|A^T A - \lambda I| = 0$$

λ_1 = 2.59663813e+09
λ_2 = 1.09695312e+09
λ_3 = 6.41021467e+04
λ_4 = 4.05655526e+04
λ_5 = 2.12053510e+04
λ_6 = 2.23413513e+03
λ_7 = 6.48615897e-01
λ_8 = 4.52174595e+01

By substituting the values of λ in $|A^T A - \lambda I| = 0$ we get the eigen vectors

$$V = \begin{bmatrix} 3.59202e{-}03 & 2.32696e{-}03 & -2.31181e{-}01 & -1.87461e{-}01 & -1.79907e{-}01 & -1.16490e{-}01 & 9.24689e{-}01 & -8.56031e{-}02 & 5.55037e{-}02 \\ 2.48947e{-}03 & -1.11092e{-}03 & -5.00092e{-}01 & 1.72071e{-}01 & 6.79638e{-}01 & -5.07767e{-}01 & -2.21702e{-}02 & -6.26301e{-}03 & -4.96061e{-}03 \\ 2.69340e{-}05 & 3.79258e{-}06 & -2.55894e{-}03 & -3.49107e{-}03 & 5.02815e{-}03 & -9.18688e{-}03 & 3.50790e{-}02 & 7.91211e{-}01 & 6.10431e{-}01 \\ 1.59017e{-}03 & -1.14924e{-}04 & 6.37923e{-}01 & 4.07070e{-}01 & 5.13087e{-}01 & 1.80833e{-}01 & 3.60933e{-}01 & -2.76772e{-}02 & 1.86298e{-}02 \\ 1.80578e{-}03 & -4.25775e{-}03 & 4.34963e{-}01 & 1.32509e{-}01 & -3.31666e{-}01 & -8.25911e{-}01 & -3.28149e{-}02 & -9.74034e{-}04] & -3.96849e{-}03 \\ 1.63242e{-}05 & -2.40188e{-}05 & 5.28488e{-}03 & -3.71548e{-}04 & 2.77599e{-}03 & 2.55189e{-}03 & -1.01296e{-}01 & -6.04854e{-}01 & 7.89840e{-}01 \\ -8.87724e{-}01 & -4.60337e{-}01 & -7.12424e{-}05 & -2.88016e{-}03 & 2.41216e{-}03 & -1.20650e{-}03 & 4.27146e{-}03 & -3.90376e{-}04 & 2.47755e{-}04 \\ -4.60315e{-}01 & 8.87723e{-}01 & 3.29433e{-}03 & -3.33579e{-}03 & 2.40715e{-}03 & -5.32965e{-}03 & -4.99436e{-}04 & -9.41566e{-}06 & -4.96061e{-}05 \\ -5.45902e{-}03 & 3.31403e{-}03 & -3.16666e{-}01 & 8.67152e{-}01 & -3.63899e{-}01 & 1.16832e{-}01 & 3.99896e{-}02 & -1.19663e{-}03 & 7.64072e{-}03 \end{bmatrix}$$

Finally, taking transpose of v gives us the desired V^T matrix

$$V^T = \begin{bmatrix} 3.59202e{-}03 & 2.48947e{-}03 & 2.69340e{-}05 & 1.59017e{-}03 & 1.80578e{-}03 & 1.63242e{-}05 & -8.87724e{-}01 & -4.60315e{-}01 & -5.45902e{-}03 \\ 2.32696e{-}03 & -1.11092e{-}03 & 3.79258e{-}06 & -1.14924e{-}04 & -4.25775e{-}03 & -2.40188e{-}05 & -4.60337e{-}01 & 8.87723e{-}01 & 3.31403e{-}03 \\ -2.31181e{-}01 & -5.00092e{-}01 & -2.55894e{-}03 & 6.37923e{-}01 & 4.34963e{-}01 & 5.28488e{-}03 & -7.12424e{-}05 & 3.29433e{-}03 & -3.16666e{-}01 \\ -1.87461e{-}01 & 1.72071e{-}01 & -3.49107e{-}03 & 4.07070e{-}01 & 1.32509e{-}01 & -3.71548e{-}04 & -2.88016e{-}03 & -3.33579e{-}03 & 8.67152e{-}01 \\ -1.79907e{-}01 & 6.79638e{-}01 & 5.02815e{-}03 & 5.13087e{-}01 & -3.31666e{-}01 & 2.77599e{-}03 & 2.41216e{-}03 & 2.40715e{-}03 & -3.63895e{-}01 \\ -1.16490e{-}01 & -5.07767e{-}01 & -9.18687e{-}03 & 1.80833e{-}01 & -8.25911e{-}01 & 2.55189e{-}03 & -1.20650e{-}03 & -5.32965e{-}03 & 1.16832e{-}01 \\ 9.24689e{-}01 & -2.21702e{-}02 & 3.50790e{-}02 & 3.60933e{-}01 & -3.28149e{-}02 & -1.01296e{-}01 & 4.27146e{-}03 & -4.99436e{-}04 & 3.99896e{-}02 \\ -8.56031e{-}02 & -6.26301e{-}03 & 7.91211e{-}01 & -2.76772e{-}02 & -9.74034e{-}04 & -6.04854e{-}01 & -3.90376e{-}04 & -9.41566e{-}06 & -1.19663e{-}03 \\ 5.55037e{-}02 & -4.96061e{-}03 & 6.10431e{-}01 & 1.86298e{-}02 & -3.96849e{-}03 & 7.89840e{-}01 & 2.47755e{-}04 & -4.96061e{-}05 & 7.64072e{-}03 \end{bmatrix}$$

The SVD of A matrix has been obtained mathematically

For a homography matrix Ax=o

when we multiply matrix A with matrix x we get,

$$Ax = \begin{bmatrix} 5H_{11} & -5H_{12} & -1H_{13} & 0H_{21} & 0H_{22} & 0H_{23} & 500H_{31} & 500H_{32} & 100H_{33} \\ 0H_{11} & 0H_{12} & 0H_{13} & 5H_{21} & 5H_{22} & 1H_{23} & 500H_{31} & 500H_{32} & 100H_{33} \\ -150H_{11} & 5H_{12} & 1H_{13} & 0H_{21} & 0H_{22} & 0H_{23} & 30000H_{31} & 1000H_{32} & 200H_{33} \\ 0H_{11} & 0H_{12} & 0H_{13} & 150H_{21} & 5H_{22} & 1H_{23} & 12000H_{31} & 400H_{32} & 80H_{33} \\ 150H_{11} & -150H_{12} & 1H_{13} & 0H_{21} & 0H_{22} & 0H_{23} & 33000H_{31} & 12000H_{32} & 220H_{33} \\ 0H_{11} & 0H_{12} & 0H_{13} & -150H_{21} & 150H_{22} & 1H_{23} & 12000H_{31} & 12000H_{32} & 80H_{33} \\ -5H_{11} & -150H_{12} & 1H_{13} & 0H_{21} & 0H_{22} & 0H_{23} & 500H_{31} & 15000H_{32} & 100H_{33} \\ 0H_{11} & 0H_{12} & 0H_{13} & -5H_{21} & -150H_{22} & 1H_{23} & 1000H_{31} & 30000H_{32} & 200H_{33} \end{bmatrix}$$

When we solve for the values of H we get :

$H_{11} = 5.55037621e-02$

$H_{12} = -4.96061429e-03$
$H_{13} = 6.10431147e-01$
$H_{21} = 1.86298626e-02$
$H_{22} = -3.96849143e-03$
$H_{23} = 7.89840031e-01$
$H_{31} = 2.47755125e-04$
$H_{32} = -4.96061429e-05$
$H_{33} = 7.64072395e-03$

Therefore,

$$x = \begin{bmatrix} 5.55037621e-02 - 4.96061429e-036.10431147e-01 \\ 1.86298626e-02 - 3.96849143e-037.89840031e-01 \\ 2.47755125e-04 - 4.96061429e-057.64072395e-03 \end{bmatrix}$$

2. Write python code to compute the SVD.

Answer 2 :

Python code to compute SVD can be found in Problem3.py file