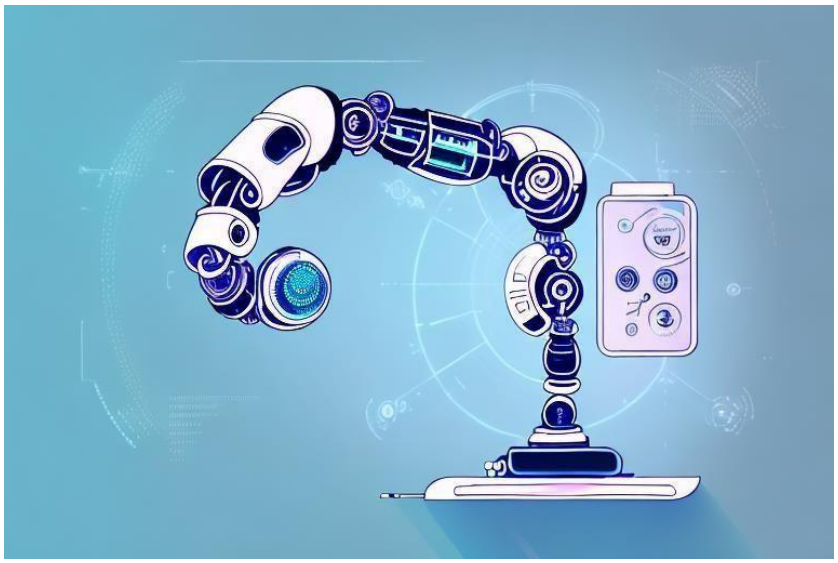
MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)



Digital Control Systems

PRACTICAL
20ES-BATCH

Submitted To: Engr. Mehran M. Memon

Name: KARAN

Roll No.: 20ES062

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

CERTIFICATE

This is to certify that Mr/Ms. ____KARAN_____, bearing Roll Number ___20ES062_____, studying in Fourth year, 7th semester in the year 2024 in Department of Electronic Engineering, MUET, Jamshoro, has completed practical course based on the course contents of subject Digital Control Systems-(ES-413) and given a satisfactory account of it in this lab-book containing a record the laboratory work.

(Subject Teacher)
Engr. Mehran M. Memon

Dated: _____

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

INDEX

Sr. #	LIST OF EXPERIMENTS	CLO	Taxonomy Level	Obt. Marks
1	Introduction To MATLAB/Simulink Software	4,5	P3, A4	
2	Plotting Of Discrete Time Testing Signals	4,5	P3, A4	
3	To Analyze Z-Transforms and Inverse Z-Transforms	4,5	P3, A4	
4	Discrete Time Root Locus	4,5	P3, A4	
5	Mathematical Modelling of Electrical System and Dc Circuit Analysis Using MATLAB	4,5	P3, A4	
6	The Step Response of a Discrete Time System and Effect of Sampling Time on System Response	4,5	P3, A4	
7	The Sample Response Analysis and The Computer Simulation of a Sampled-Data System	4,5	P3, A4	
8	Discrete-Time State Space Representation of a System and Modelling of Magnetically Suspended Ball System	4,5	P3, A4	
9	The Controllability and Observability of a System in Discrete Time Domain	4,5	P3, A4	
10	Frequency Response and Construction of Bode Plot	4,5	P3, A4	
11	Performance Of Pid and Digital Pid Controller	4,5	P3, A4	
12	Tuning Of Digital Pid Controller	4,5	P3, A4	
13	Operation Of Compensator and Design of Lead Compensator	4,5	P3, A4	
14	Performance And Design of Digital Filter	4,5	P3, A4	
15	Low Pass, High Pass & Band Pass Digital Filters	4,5	P3, A4	
16	Open Ended Lab	4,5	P3, A4	
			Total Marks Obtained (Out of 15):	

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)
LAB # 01: INTRODUCTION TO MATLAB/SIMULINK SOFTWARE

OBJECTIVE(S):

Name: KARAN Roll # 20ES062

Signature of Lab Tutor: _____ Date: _____

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	*The purpose of this lab is to introduce you to software tool namely MATLAB/SIMULINK*	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

EQUIPMENTS REQUIRED:

- Personal Computer
- MATLAB R2013a or above

THEORY:

Millions of engineers and scientists worldwide use MATLAB® to analyze and design the systemsand products transforming our world. The matrix-based MATLAB language is the world's most natural way to express computational mathematics. Built-in graphics make it easy to visualize andgain insights from data. The desktop environment invites experimentation, exploration, anddiscovery. These MATLAB tools and capabilities are all rigorously tested and designed to work together.MATLAB helps you take your ideas beyond the desktop. You can run your analyses on larger datasets and scale up to clusters and clouds. MATLAB code can be integrated with other languages, enabling you to deploy algorithms and applications within web, enterprise, and production systems.MATLAB Control System Toolbox™

Control System Toolbox of MATLAB provides algorithms and apps for systematically analyzing, designing, and tuning linear control systems. You can specify your system as a transfer function, state-space, zero-pole-gain, or frequency-response model. Apps and functions, such as step response plot and Bode plot, let you analyze and visualize system behavior in the time and frequencydomain

You can tune compensator parameters using interactive techniques such as Bode loop shaping andthe root locus method. The toolbox automatically tunes both SISO and MIMO compensators, including PID controllers. Compensators can include multiple tunable blocks spanning several feedback loops. You can tune gain-scheduled controllers and specify multiple tuning objectives, such as reference tracking, disturbance rejection, and stability margins. You can validate your design by verifying rise time, overshoot, settling time, gain and phase margins, and other requirements.

Getting Started with Control System Toolbox

- Dynamic System Models: Represent simple and complex dynamic systems, discretize models, and reduce model order.

  With Control System Toolbox™ software, you represent dynamic systems as model objects. Model objects are specialized data containers that encapsulate model data and other attributes in a structured way. Model objects enable you to manipulate linear systemsas single entities rather than keeping track of multiple data vectors, matrices, or cell arrays.Once you have a linear model, you can perform time-domain or frequency-domain analysisof it, design a controller for it, and perform other design or analysis tasks. It includes following key features:
  - Linear System Representation: Models of linear time-invariant systems
  - Model Interconnection: Series, parallel, and feedback connections; block diagram building
  - Model Transformation: Model type conversion, continuous-discrete conversion, order reduction.
  - Model Reduction: Model order reduction, low-order approximation, pole-zero cancellation

- Linear Analysis: Time- and frequency-domain responses, stability margins, parameter sensitivity.

  Control System Toolbox™ software lets you analyze the dynamics of linear systems. You can visualize system behavior in time domain and frequency domain. You can extract system characteristics such as rise time, overshoot, and settling time. You can also analyze system stability. Use these tools to analyze the behavior of plant models or validate the performance of a tuned control system. Key features include:
  - Linear System Analyzer: Use this app to examine the time-domain and frequency-domain behavior of SISO and MIMO systems.
  - Time and Frequency Domain Analysis: System responses such as Bode plots and step responses; system characteristics such as response time and overshoot; simulation
  - Stability Analysis: Gain and phase margins, pole and zero locations
  - Sensitivity Analysis: Robustness of control systems
  - Passivity and Sector Bounds: Analyze systems for passivity and arbitrary conic-sector bounds
  - Plot Customization: Customize analysis plot appearance and units
- Control System Design and Tuning: Tune PID controllers and other control architectures automatically or interactively, design Kalman filters.

  Control System Toolbox™ control design tools let you design and tune single-loop and multi loop control systems. Use these techniques and tools to:

  i. Automatically tune common control components such as PID controllers, lead-lag networks, LQG Controllers, and Kalman filters.
  ii. Graphically tune SISO compensators using classical tools such as root locus, Bode diagrams, and Nichols charts.
  iii. Automatically tune SISO or MIMO control systems to meet high-level design goals such as reference tracking, disturbance rejection, and stability margins, regardless of control system architecture.

  The key features include:

  - PID Controller Tuning: Automatic and interactive tuning of PID gains
  - Classical Control Design: Design, tuning, and analysis of single-input, single-output (SISO) feedback systems
  - State-Space Control Design and Estimation: Linear-Quadratic-Gaussian control, pole placement, Kalman estimators.
  - Multiloop, Multiobjective Tuning: Automated tuning of control systems to meet design requirements
  - Gain Scheduling: Tuning of gain-scheduled controllers for nonlinear plants

- Matrix Computations: Controllability and observability, Lyapunov and Riccati equations

  Control System Toolbox™ employs various functions using which you can solve controllability and observability matrices, continuous-time and discrete-time Lyapunov equations, continuous-time and discrete-time Riccati equations.

Introduction to Simulink

Simulink is a software package for modeling, simulating, and analyzing dynamic systems. It supports linear and nonlinear systems, modeled in continuous time, sampled time, or a hybrid of the two. One can easily build, analyze and visualize models. Models are represented graphically in Simulink as block diagrams.

One of the primary advantages of employing Simulink (and simulation in general) for the analysis of dynamic systems is that it allows us to quickly analyze the response of complicated systems that may be prohibitively difficult to analyze analytically. Simulink is able to numerically approximate the solutions to mathematical models that we are unable to, or don't wish to, solve "by hand."

Simulink® is a block diagram environment for multi domain simulation and Model-Based Design. It supports system-level design, simulation, automatic code generation, and continuous test and verification of embedded systems. Simulink provides a graphical editor, customizable block libraries, and solvers for modeling and simulating dynamic systems. It is integrated with

MATLAB®, enabling you to incorporate MATLAB algorithms into models and export simulationresults to MATLAB for further analysis.

Simulink handles data in three categories:

- Signals — Block inputs and outputs, computed during simulation
- States — Internal values, representing the dynamics of the block, computed during simulation
- Parameters — Values that affect the behavior of a block, controlled by the user

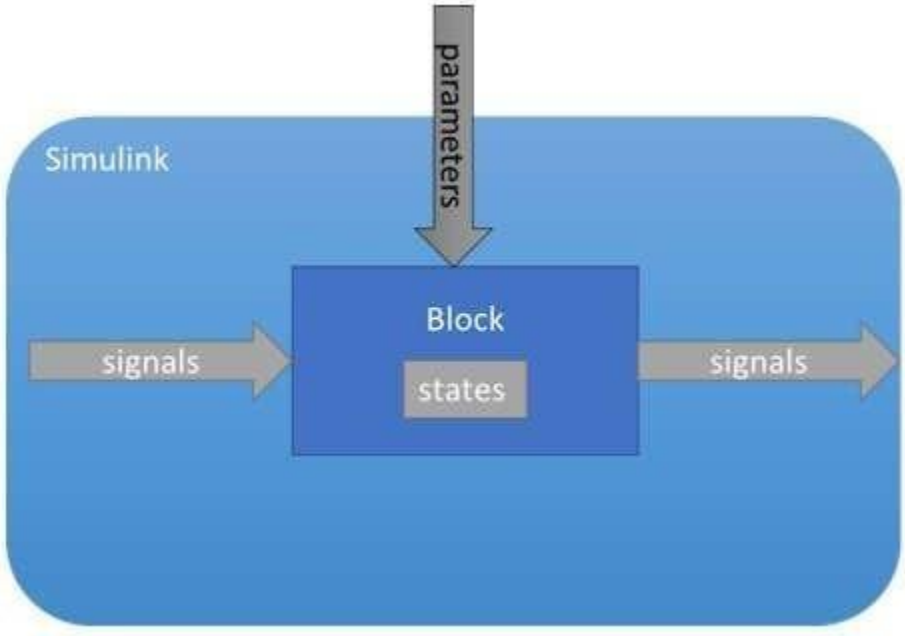


Figure 1.1. Simulink Manipulation

At each time step, Simulink computes new values for signals and states. By contrast, you specify parameters when you build the model and can occasionally change them while simulation is running.

1.1 Creating discrete time signal in MATLAB

1.1.1 Example1: The MATLAB function hn= dimpulse(P,Q,n) returns the discrete impulse response consisting of n samples, applied to the discrete transfer function H(Z)= P(Z)/Q(Z)
Let

$$H(Z) = \frac{2.24z^2 + 2.5z + 2.25}{z^2 + 0.5z + 0.68}$$

Create the script file disc_imp that returns the discrete response in the form of a table and a plot in the form of sequence of length n=10

MATLAB Solution

```
% Script file: disc _ imp
clc; clf;
n =10;
P = [2.24 2.5 2.25];
Q = [1 .5 .68];
hn = dimpulse(P,Q,n);
nn = 0:1:9;
results = [ nn' hn];
disp('************************************************************');
disp('The impulse response sequence h(n) for the first 10 samples is:');
disp('^^^^^^^^^^^^^^^^');
disp('   n       h(n)');
disp('^^^^^^^^^^^^^^^^');
disp(results);
disp('************************************************************');
yzero = zeros(1,10);
stem(nn,hn); hold on; plot(nn,yzero)
title('Discrete impulse response h(n) vs n')
xlabel('time index n');
ylabel('Amplitude [h(n)]');
```

The script file *disc_ imp* is executed and the results are shown as follows:

```
**********************************************************************
The impulse response sequence h(n) for the first 10 samples is:
^^^^^^^^^^^^^^^^^^^^^^
     n          h(n)
^^^^^^^^^^^^^^^^^^^^^^
     0          2.2400
     1.0000     1.3800
     2.0000     0.0368
     3.0000    -0.9568
     4.0000     0.4534
     5.0000     0.4239
     6.0000    -0.5203
     7.0000    -0.0281
     8.0000     0.3679
     9.0000    -0.1648
**********************************************************************
```

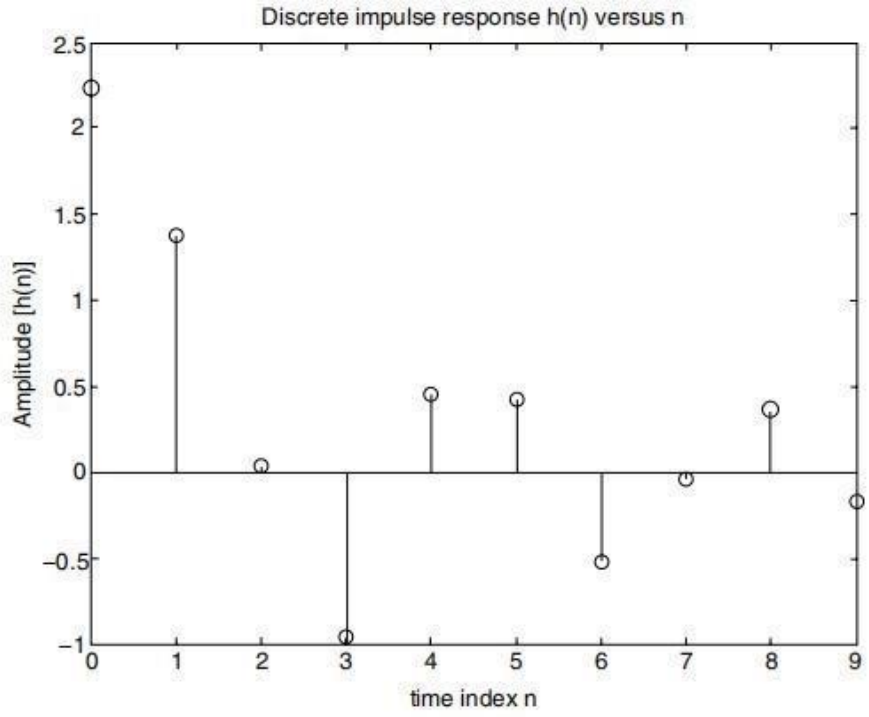


Figure 1.2- Discrete impulse response h(n)

1.1.2 Example2: The MATLAB function un= dstep(P,Q,n) returns the discrete step response with length of n of the system defined by the transfer function H(Z)= P(Z)/Q(Z).
Let

$$H(Z) = \frac{2.24z^2 + 2.5z + 2.25}{z^2 + 0.5z + 0.68}$$

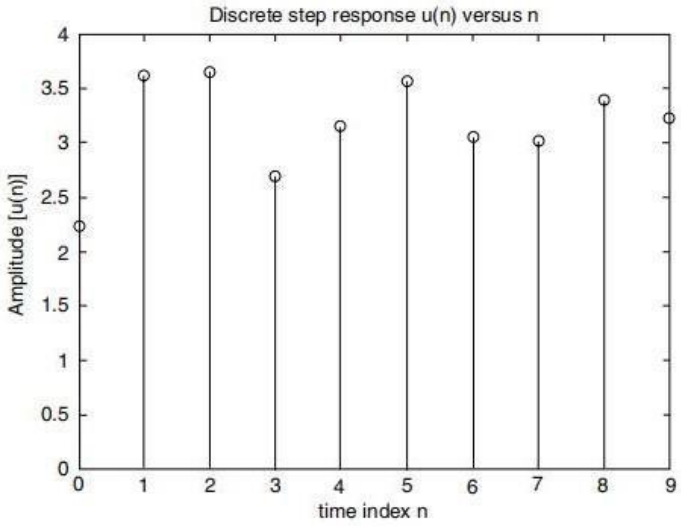
Create the script file disc_step that returns the discrete response in the form of a table and a plotfor a sequence of length n=10

MATLAB Solution

```
% Script file: disc _ step
clc; clf;
n = 10;
P = [2.24 2.5 2.25];
Q = [1 .5 .68];
un = dstep(P,Q,n);
nn = 0:1:9;
results = [nn' un];
disp('*****************************************************************');
disp('The discrete step sequence u(n) for the first 10 samples is:');
disp('^^^^^^^^^^^^^^^');
disp('   n      u(n)');
disp('^^^^^^^^^^^^^^^');
disp(results);
disp('*****************************************************************');
yzero = zeros(1,10);
stem (nn,un); hold on; plot (nn,yzero)
title ('Discrete step response u(n) vs n')
xlabel ('time index n');
ylabel ('Amplitude[u(n)]');
```

```
*********************************************************************
The discrete step sequence u(n) for the first 10 samples is:
^^^^^^^^^^^^^^^^^^^^
    n          u(n)
^^^^^^^^^^^^^^^^^^^^
    0          2.2400
    1.0000     3.6200
    2.0000     3.6568
    3.0000     2.7000
    4.0000     3.1534
    5.0000     3.5773
    6.0000     3.0570
    7.0000     3.0289
    8.0000     3.3968
    9.0000     3.2320
*********************************************************************
```

Figure 1.3 Discrete step response



1.1.1 Example3: Create the script file signal_noise that returns the plot of the following signals.

A. Gaussian white noise

B. The signal - Signal =cos(2πn/64)

C. The contaminated signal plus noise using n=128 elements. Let the white Gaussian signal power be P=2.7 watt.

MATLAB Solution

```
% Script file: signal _ noise
n = 1:128; P =2.7;
figure(1)
subplot(2,1,1)
white _ noise = sqrt(P)*randn(1,128);
stem(n,white _ noise); hold on;plot(n,white _ noise);
xlabel('discrete time n');
ylabel('Amplitude'); title('Gaussian noise');
subplot(2,1,2)
signal = cos(2*pi*n/64);
stem(n,signal);hold on;plot(n,signal);
title('cosine wave with period=64')
xlabel('discrete time n');
ylabel('Amplitude');

figure(2)
signal _ noise = white _ noise+signal;
stem(n,signal _ noise);hold on;plot(n,signal _ noise);
title('cosine wave plus white Gaussian noise')
xlabel('discrete time n');
ylabel('Amplitude');
```

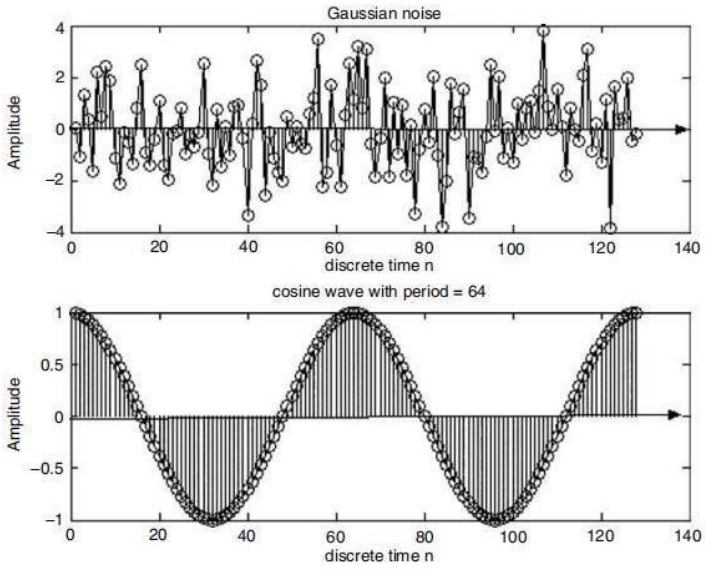


Figure 1.4- Plots of part A and B

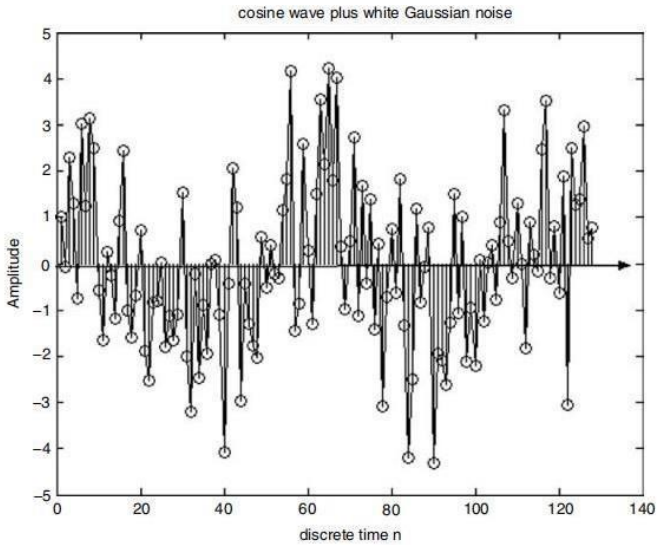


Figure 1.5- Plot of part C

1.1.2 Example4: Let the discrete sequence

$$f(n) = 4n0.8^n u(n)$$

be contaminated by a random noisy signal with a magnitude less than 1.
Create the script file *averages* that returns the following plots:

1. f(n) versus n
2. noise signal[n(t)] versus n
3. [f(n) + n(t)] versus n
4. [f(n) + n(t)] moving average using two terms versus n
5. [f(n) + n(t)] moving average using three terms versus n
6. [f(n) + n(t)] moving average using four terms versus n

Estimate the best moving average approximation for [f(n) + n(t)] versus n, by executing the script file at least three times and observing and recording the results

Where the L point moving average is defined by

$$\{\text{moving aver. } L\} = \frac{1}{L}\sum_{k=0}^{L-1} f(n-k)$$

for L = 2, 3, and 4, over the range $0 \leq n \leq 20$.

MATLAB Solution

```
% Script file: averages
n = 0:1:20;

figure(1)
subplot(3,1,1)
signal = 4.*n.*(.8.^n);stem(n,signal);hold on;
plot(n,signal,n,signal,'o');
title('Signal vs. n')
ylabel('Amplitude');xlabel('time index n');
subplot(3,1,2)
noise = 2.*rand(1,21)-1.0;y = zeros(1,21);

plot(n,y);title('Noise vs. n');hold on;
stem(n,noise);hold on;plot(n,y,n,noise);
ylabel('Amplitude');xlabel('time index n');
axis([0 20 -1.3 1.3]);
subplot(3,1,3);
signoi =signal+noise;
stem(n,signoi);title('[Signal + Noise] vs. n');hold on;
plot(n,signoi);ylabel('Amplitude[ signal+noise]');
xlabel(' time index n')

figure(2)
subplot(3,1,1)
N = [.5 .5];
D = 1;
movave2 = filter(N,D,signoi);
plot(n,signal,n,signoi,'s--',n,movave2,'o-.');
legend('signal','signal+noise','moving ave/2term')
title('Various moving averages');ylabel('magnitude')
subplot(3,1,2)
N=[.33 .33 .33];
D=1;
movave3=filter(N,D,signoi);
plot(n,signal,n,signoi,'s--',n,movave3,'o-.');ylabel('magnitude')
legend('signal','signal+noise','moving ave/3term')
subplot(3,1,3)
N = [.25 .25 .25 .25];
D = 1;
```

```
movave4 = filter(N,D,signoi);
plot(n,signal,n,signoi,'s--',n,movave4,'o-.');
legend('signal','signal+noise','moving ave/4term')
ylabel('magnitude');xlabel(' time index n')

figure(3)
plot(n,signal,n,signoi,'ks--',n,movave2,'ko-.');
legend('signal','signal+noise','moving ave/2term');
title('Best approximation using moving averages');
ylabel('magnitude');xlabel(' time index n')

figure(4)
err2 = signal-movave2;
stem(n,err2);hold on;plot(n,y,n,err2);
title('error=[mov.ave/2terms ] vs t');
ylabel('error');xlabel(' time index n')
error1= sum(abs(signal-signoi)/21);
error2 = sum(abs(signal-movave2)/21);
error3 = sum(abs(signal-movave3)/21);
error4 = sum(abs(signal-movave4)/21);
disp('************************************************')
disp('***************ANALYSIS OF ERROR***************')
disp('************************************************')
disp('  no ave   2 term ave   3 term ave   4 term ave ');
disp([error1 error2 error3 error4])
disp('************************************************')
disp('************************************************')
```

Back in the command window the script file *averages* is executed three times, and the results are shown as follows and in the plots of Figures

```
averages
*********************************************************
*******************ANALYSIS OF ERROR*******************
*********************************************************
   no ave    2 term ave     3 term ave     4 term ave
   0.5389    0.5276         0.7011         0.9793
*********************************************************
*********************************************************

>> averages
*********************************************************
*******************ANALYSIS OF ERROR*******************
************************************************
   no ave 2 term ave  3 term ave  4 term ave 0.5272
   0.4794 0.6574  0.9033
*********************************************************
*********************************************************

>> averages
*******************ANALYSIS OF ERROR*******************
************************************************
   no ave 2 term ave  3 term ave  4 term ave 0.4144
   0.3750 0.5572  0.8223
*********************************************************
*********************************************************
```

Observe that the results obtained by executing the script file *averages* three times are not the same, but in each case the best results are achieved when the moving average uses two terms.

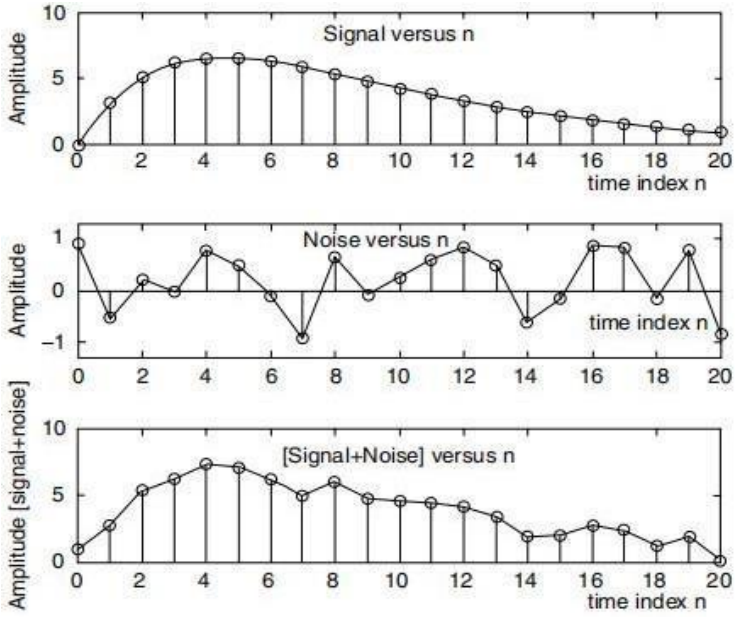


Figure 1.6- Plots of f(n), noise signal[n(t)], and [f(n) + n(t)]

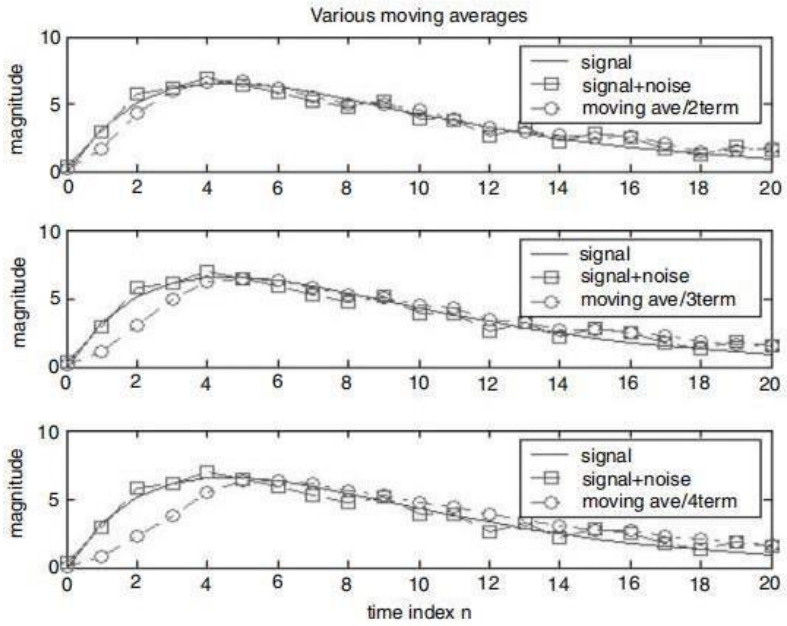


Figure 1.7- Plots of various moving averages

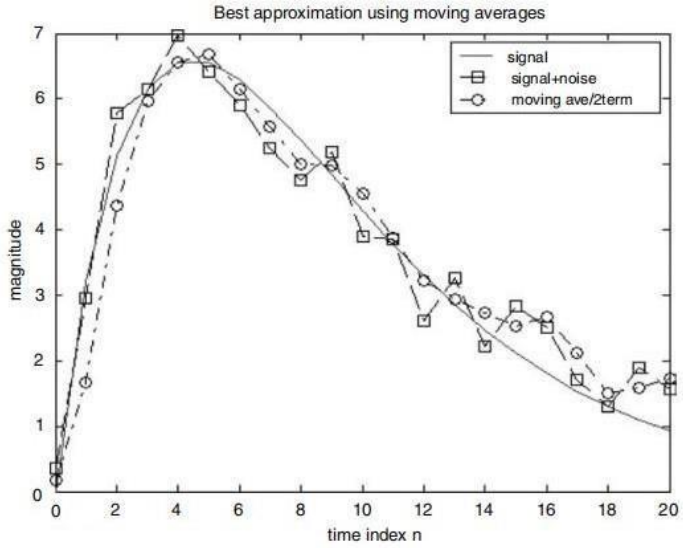


Figure 1.8- Plots of f(n), [f(n) + n(t)], and best moving average

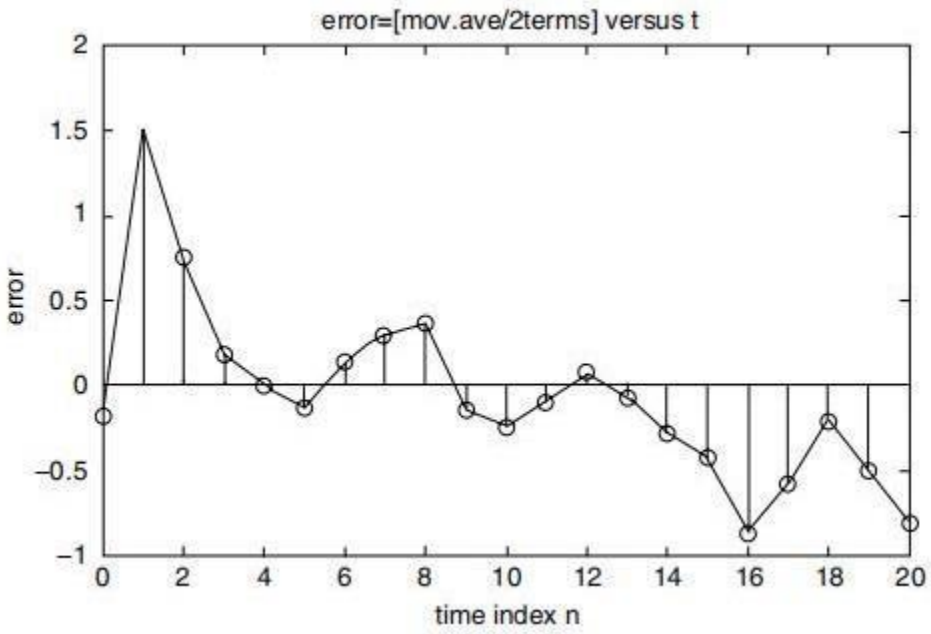


Figure 1.9- Error plot of moving average using two terms

1.2 Review Exercise?

a) Enlist the main features of MATLAB Control System Toolbox.

Transfer-function, state-space, zero-pole-gain, and frequency-response, Step response, Nyquist plot, and other time-domain and frequency-domain tools for analyzing stability and performance. PID Controller Tuning, gain-scheduled, and arbitrary SISO and MIMO control systems.

b) For what purpose Simulink is used?

Simulink provides a graphical editor, customizable block libraries, and solvers for modeling and simulating dynamic systems. It is integrated with MATLAB, enabling you to incorporate MATLAB algorithms into models and export simulation results to MATLAB for further analysis.

c) What is real time application of digital control system?

Real-time applications of digital control systems are industrial automation, power systems, aerospace and defense, robotics, communication, automotive systems, renewable energy systems, Environmental control, and traffic control system.

d) What are examples of digital devices?

Examples of such devices are

- ✓ Personal Computers (Desktops and Laptops),
- ✓ Smartphones, and Tablets,
- ✓ Calculators,
- ✓ Digital Weighing Machine,
- ✓ Accounting machines,
- ✓ Digital clock,
- ✓ ATM (Automated Teller Machine).

e) Attach the Plot of Example 1 to Example 4 and write a MATLAB program for the following system

Example-1:

```
n = 10;
P = [2.24 2.5 2.25];
Q = [1 .5 .68];
hn = dimpulse(P,Q,n);
nn = 0:1:9;
results = [nn' hn];
disp('************************************************************');
disp ('The discrete Impulse sequence u(n) for the first 10 samples is:');
disp('^^^^^^^^^^^^^^^^^');
disp('   n    h(n)');
disp('^^^^^^^^^^^^^^^^^');
disp('disp(results)');
disp('************************************************************');
yzero = zeros(1,10);
stem (nn,hn); hold on;
plot (nn,yzero)
title ('Discrete Impulse response h(n) vs n')
xlabel ('time index n');
ylabel ('Amplitude[h(n)]');
```

I	[0.8000;0.400...
n	10
nn	[0,1,2,3,4,5,6,...
P	[2.2400,2.500...
Q	[1,0.5000,0.6...
R	[-1,1,1;10,5,0;...
result	[0.8000,0.400...
results	*10x2 double*
sys	*1x1 tf*
sysc	*1x1 tf*
syscz	*1x1 tf*
syscz2	*1x1 tf*
syscz3	*1x1 tf*
sysz	*1x1 tf*
sysz1	*1x1 tf*
sysz2	*1x1 tf*
tout	*63x1 double*
V	[0;10;10]
yzero	[0,0,0,0,0,0,0,...

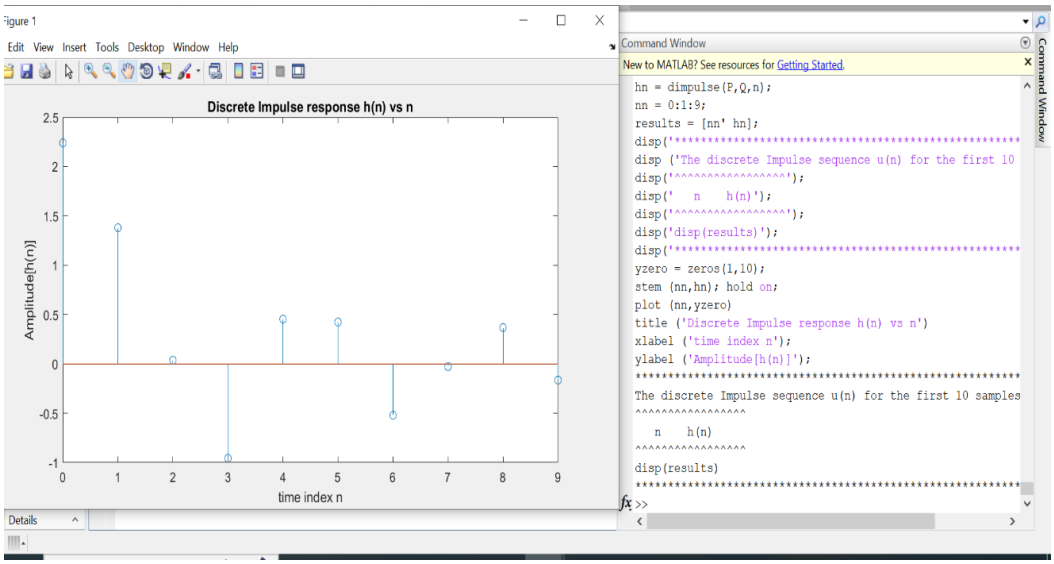


Figure 1
Edit View Insert Tools Desktop Window Help
Discrete Impulse response h(n) vs n
Amplitude[h(n)]
time index n
Details
Command Window
New to MATLAB? See resources for Getting Started.
hn = dimpulse(P,Q,n);
nn = 0:1:9;
results = [nn' hn];
disp ('The discrete Impulse sequence u(n) for the first 10
disp(' n   h(n)');
disp('disp(results)');
yzero = zeros(1,10);
stem (nn,hn); hold on;
plot (nn,yzero)
title ('Discrete Impulse response h(n) vs n')
xlabel ('time index n');
ylabel ('Amplitude[h(n)]');
The discrete Impulse sequence u(n) for the first 10 samples
n   h(n)
disp(results)

Example-2:


% Script file: disc_impulse
n = 10;
P = [2.24 2.5 2.25];
Q = [1 .5 .68];
un = dstep(P,Q,n);
nn = 0:1:9;
results = [nn' un];
disp ("The discrete step sequence u(n) for the first 10 samples is:");
disp(" n   u(n)");
disp('disp(results)');
yzero = zeros(1,10);
stem (nn,un); hold on; plot (nn,yzero)
title ("Discrete step response u(n) vs n")
xlabel ("time index n");
ylabel ("Amplitude[u(n)]");

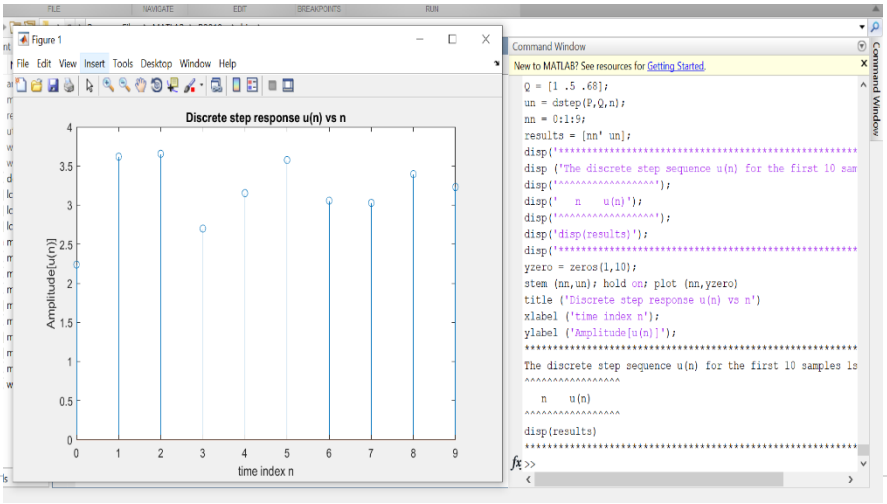
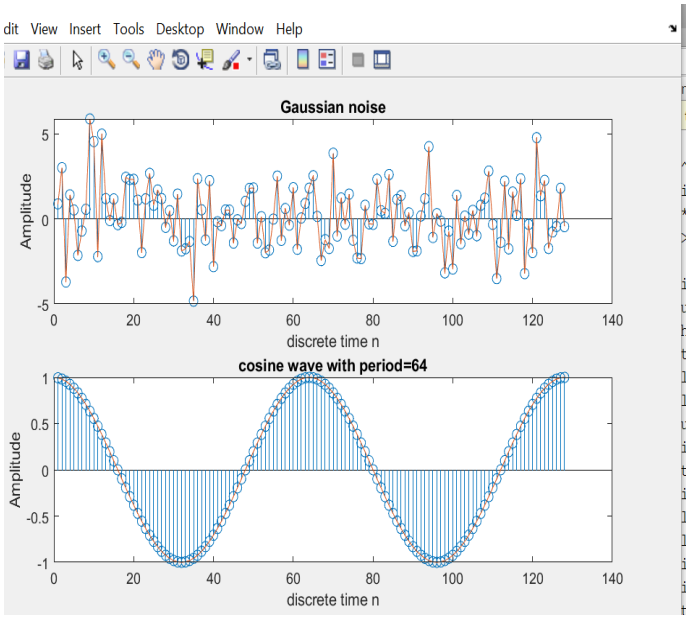


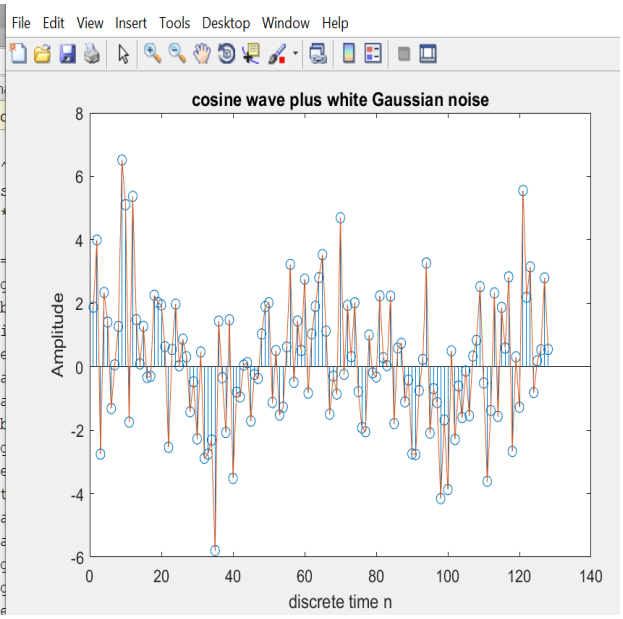
Figure 1
File Edit View Insert Tools Desktop Window Help
Discrete step response u(n) vs n
Amplitude[u(n)]
time index n
Command Window
New to MATLAB? See resources for Getting Started.
Q = [1 .5 .68];
un = dstep(P,Q,n);
nn = 0:1:9;
results = [nn' un];
disp ('The discrete step sequence u(n) for the first 10 sam
disp(' n   u(n)');
disp('disp(results)');
yzero = zeros(1,10);
stem (nn,un); hold on; plot (nn,yzero)
title ('Discrete step response u(n) vs n')
xlabel ('time index n');
ylabel ('Amplitude[u(n)]');
The discrete step sequence u(n) for the first 10 samples is
n   u(n)
disp(results)

Example-3:

```
% Script file: signal_noise
n = 1:128; P =2.7;
figure(1)
subplot (2,1,1)
white_noise = sqrt(P)*randn(1,128);
stem (n, white_noise); hold on; plot (n,white_noise);
xlabel('discrete time n');
ylabel('Amplitude'); title('Gaussian noise');
subplot (2,1,2)
signal = cos(2*pi*n/64);
stem (n,signal); hold on; plot (n,signal);
title( 'cosine wave with period=64');
xlabel('discrete time n');
ylabel ('Amplitude');
figure(2)
signal_noise = white_noise+signal;
stem(n,signal_noise);hold on;plot (n,signal_noise);
title('cosine wave plus white Gaussian noise')
xlabel ('discrete time n');
ylabel ('Amplitude');
```

```
^^^^^^^^^^^^^^^^
disp(results)
************************************************************
>> % Script file: signal_noise
n = 1:128; P =2.7;
figure(1)
subplot (2,1,1)
white_noise = sqrt(P)*randn(1,128);
stem (n, white_noise); hold on; plot (n,white_noise);
xlabel('discrete time n');
ylabel('Amplitude'); title('Gaussian noise');
subplot (2,1,2)
signal = cos(2*pi*n/64);
stem (n,signal); hold on; plot (n,signal);
title( 'cosine wave with period=64');
xlabel('discrete time n');
ylabel ('Amplitude');
figure(2)
signal_noise = white_noise+signal;
stem(n,signal_noise);hold on;plot (n,signal_noise);
title('cosine wave plus white Gaussian noise')
xlabel ('discrete time n');
ylabel ('Amplitude');
fx >>
```





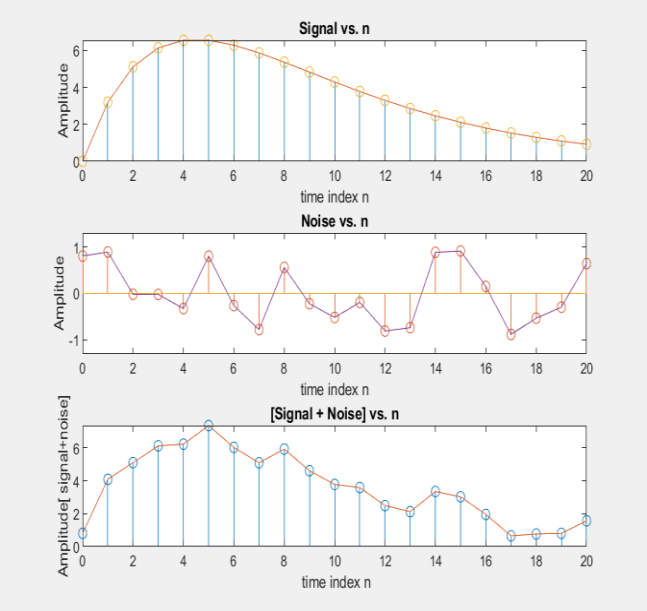
Example-4:

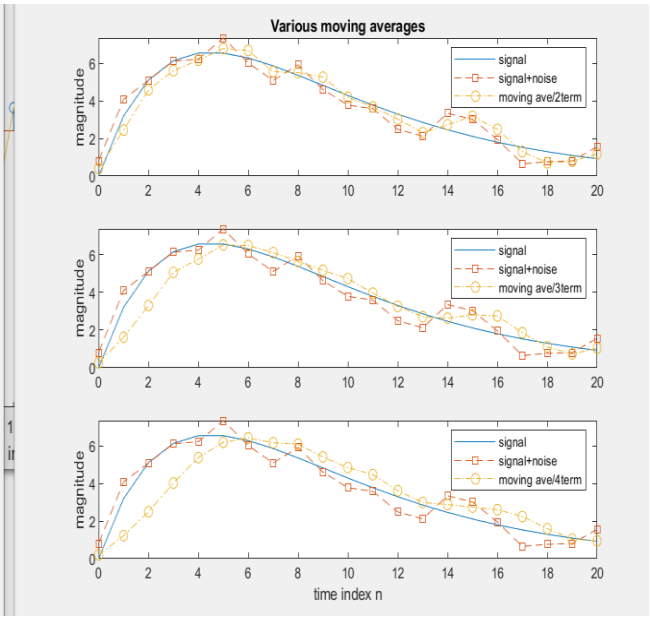
```
% Script file: averages
n = 0:1:20;
figure (1)
subplot (3,1,1)
signal = 4.*n.*(.8.^n);stem(n, signal);hold on;
plot (n,signal,n,signal, 'o');
title('Signal vs. n')
ylabel ('Amplitude');xlabel('time index n');
subplot (3,1,2)
noise = 2.*rand(1,21)-1.0; y = zeros(1,21);
plot(n,y);title( 'Noise vs. n');hold on;
stem (n, noise);hold on;plot (n,y,n,noise);
ylabel ('Amplitude');xlabel('time index n');
axis ([0 20 -1.3 1.3]);
subplot (3,1, 3);
signoi =signal+noise;
stem(n,signoi);title('[Signal + Noise] vs. n');hold on;
plot (n, signoi);ylabel('Amplitude[ signal+noise]');
xlabel(' time index n')
figure(2)
subplot (3,1,1)
N = [.5 .5];
D = 1;
movave2 = filter(N,D,signoi);
```

```
movave2 = filter(N,D,signoi);
plot (n, signal,n,signoi, 's--',n, movave2, 'o-.');
legend('signal', 'signal+noise', 'moving ave/2term')
title('Various moving averages');ylabel('magnitude')
subplot (3,1,2)
N= [.33 .33 .33];
D=1;
movave3=filter (N,D,signoi);
plot (n, signal,n,signoi, 's--',n, movave3, 'o-.');
ylabel ('magnitude')
legend('signal','signal+noise', 'moving ave/3term')
subplot (3,1,3)
N = [.25 .25 .25 .25];
D = 1;
movave4 = filter (N,D,signoi);
plot (n, signal,n,signoi, 's--',n, movave4, 'o-.');
legend('signal', 'signal+noise', 'moving ave/4term')
ylabel ('magnitude');xlabel( 'time index n')
figure(3)
plot (n, signal,n,signoi, 'ks--', n, movave2, 'ko-.');
legend('signal', 'signal+noise', 'moving ave/2term');
title('Best approximation using moving averages');
ylabel ('magnitude');xlabel( 'time index n')
figure(4)
err2 = signal-movave2;
```

```
plot (n, signal,n,signoi, 'ks--', n, movave2, 'ko-.');
legend('signal', 'signal+noise', 'moving ave/2term');
title('Best approximation using moving averages');
ylabel ('magnitude');xlabel( 'time index n')
figure(4)
err2 = signal-movave2;
stem (n,err2);hold on;plot (n,y,n,err2);
title('error=[mov.ave/2terms ] vs t');
ylabel('error');xlabel(' time index n')
errorl= sum(abs(signal-signoi)/21);
error2 = sum(abs(signal-movave2)/21);
error3 = sum(abs (signal-movave3)/21);
error4 = sum(abs(signal-movave4)/21);
disp('***********************************')
disp('*****ANALYSIS OF ERROR****' )
disp('***********************************')
disp(' no ave 2 term ave 3 term ave4 term ave ');
disp([errorl error2 error3 error4])
disp('***********************************')
disp('***********************************')
```

err2
error2
error3
error4
errorl
hn
l
movave2
movave3
movave4
n
N
nn
noise
P
Q
R
result
results
signal
signal_nois
signoi
sys
sysc
syscz


Signal vs. n
Amplitude
time index n
Noise vs. n
Amplitude
time index n
[Signal + Noise] vs. n
Amplitude[signal+noise]
time index n


Various moving averages
magnitude
signal
signal+noise
moving ave/2term
moving ave/3term
moving ave/4term
time index n

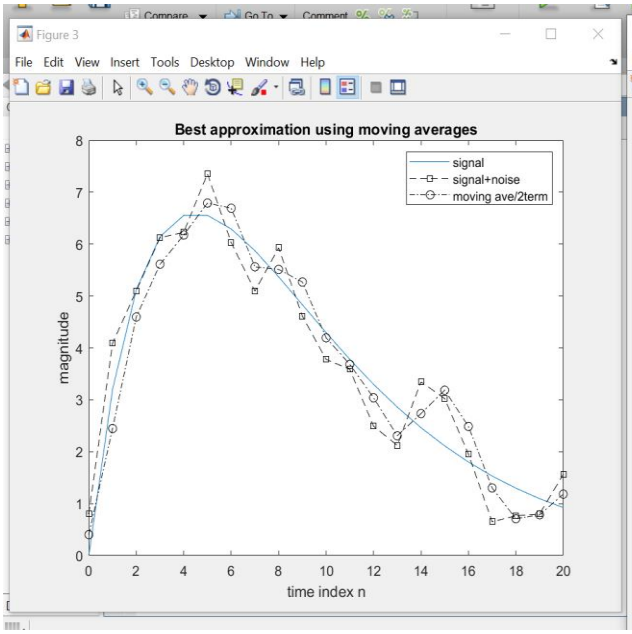


Figure 3
File Edit View Insert Tools Desktop Window Help
Best approximation using moving averages
signal
signal+noise
moving ave/2term
magnitude
time index n

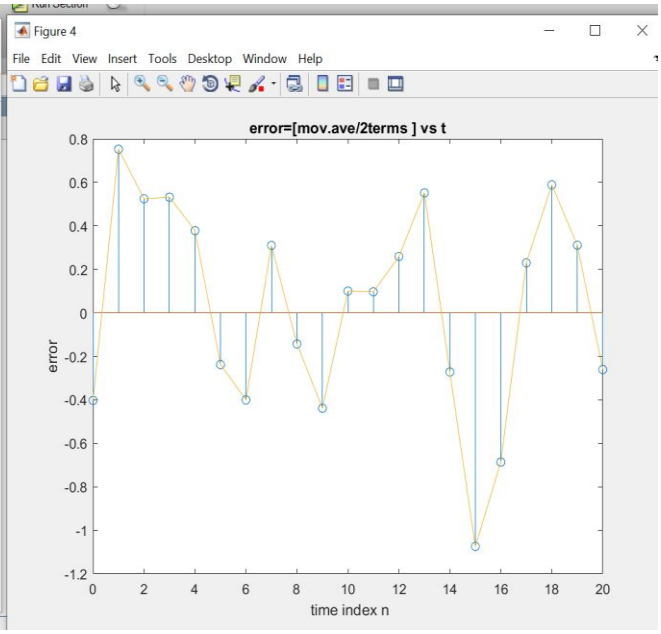
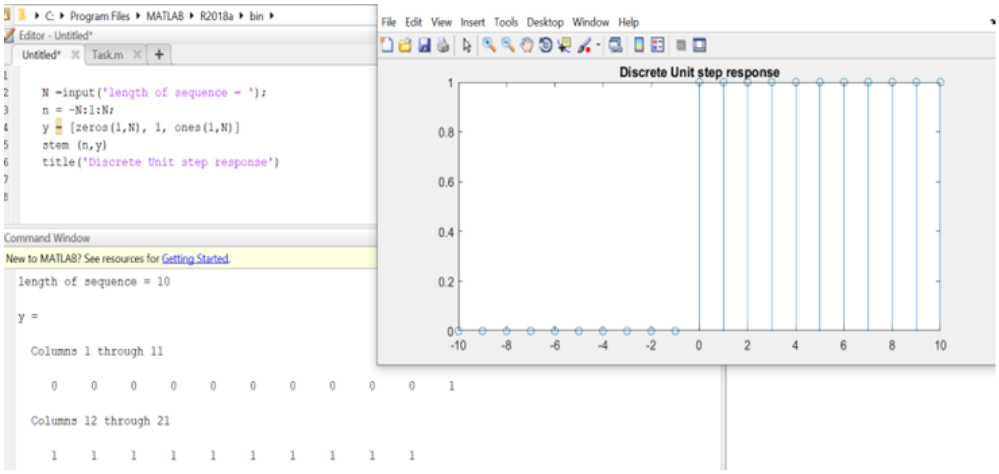


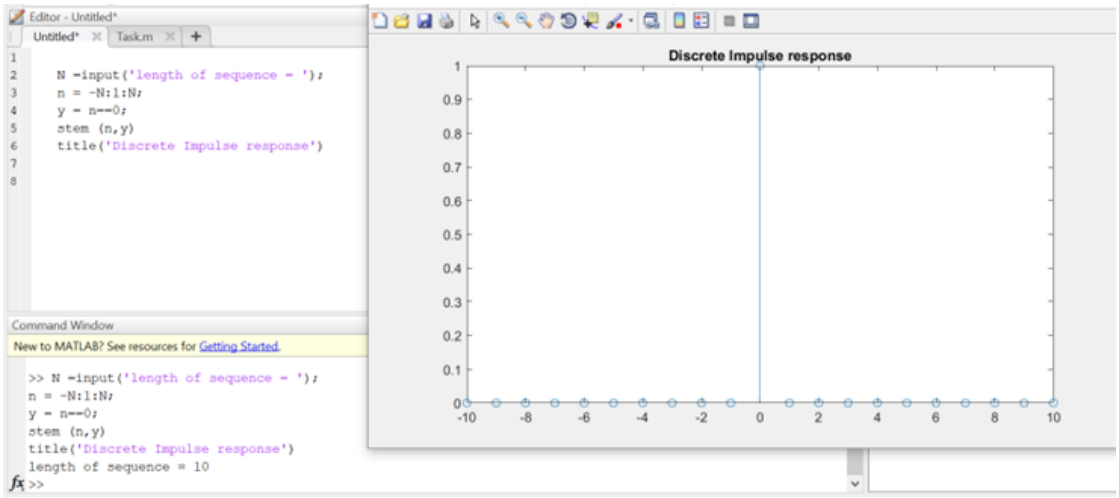
Figure 4
File Edit View Insert Tools Desktop Window Help
error=[mov.ave/2terms] vs t
error
time index n

1. Discrete step and impulse response plots by creating an impulse and step sequence as input

Discrete Step response:



Discrete Impulse response:



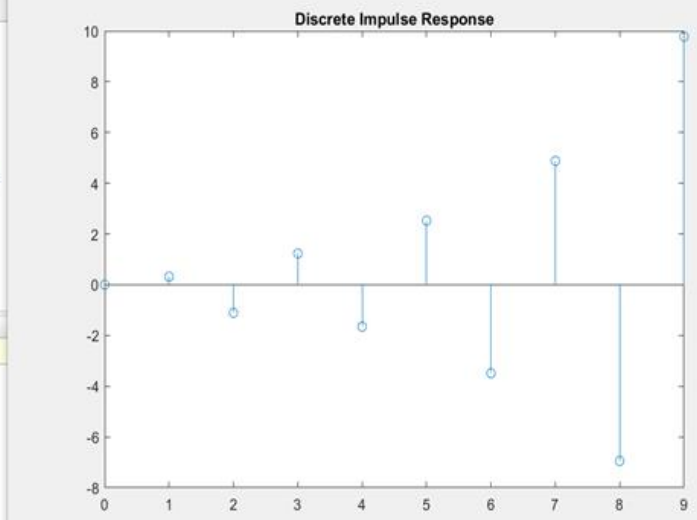
2 Repeat part a by using the discrete MATLAB functions dimpulse and dstep, andcompare the result obtained with the result of part a1

$$H(z) = \frac{0.32 + 0.43z^{-1} - 0.85z^{-2} + 0.2z^{-3}}{1 + 0.802z^{-1} - 0.42z^{-2} + 0.62z^{-3}}$$

Discrete impulse response:

```
n = 10;
P = [0 .32 -0.85 0.21];
Q = [1 .802 -0.42 0.62];
hn = dimpulse(P,Q,n);
nn = 0:1:9;
stem(nn,hn)
title('Discrete Impulse Response');
```

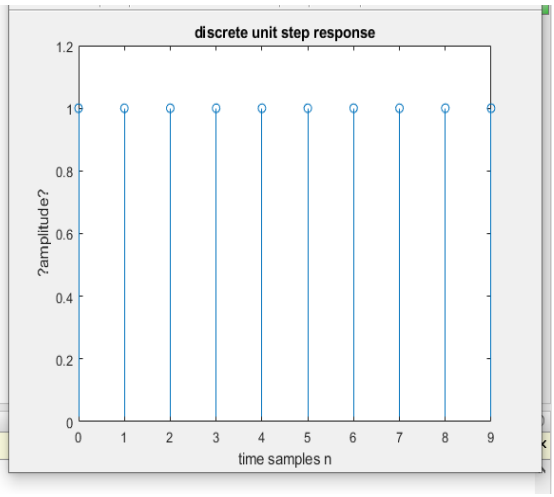
```
>> n = 10;
P = [0 .32 -0.85 0.21];
Q = [1 .802 -0.42 0.62];
hn = dimpulse(P,Q,n);
nn = 0:1:9;
stem(nn,hn)
title('Discrete Impulse Response');
>>
```



Discrete step response:

```
n=10;
E=[0.32 0.43 -0.85 0.21];
P=[1 0.802 -0.42 0.62];
hn=dstep (P, Q, n) ;
nn=0:9;
stem (nn, hn) ;
title ('discrete unit step response')
xlabel ('time samples n')
ylabel ('amplitude')
```

```
>> dstep_response
```



REFERENCE(S)

1. https://www.mathworks.com
2. Matlab: A Practical Introduction to Programming and Problem Solving by Stormy Attaway.

FINAL CHECK LIST

1. Before you leave, properly save your work and close the software tool.
2. Submit your completed laboratory work, before the next laboratory session.

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 02: PLOTTING OF FISCRETE TIME TESTING SIGNALS

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	To illustrate how Discrete Time simulation is performed in Simulink	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: Computer with MATLAB software

Approximate Time Required: Two to three hours

Introduction: Control systems implemented on a digital computer are called digital control systems . Digital control systems differ from their analog counterparts in two important ways.
Digital systems operate in discrete time, not continuous time. That is, control computations do not occur continuously (as they do using op-amps for analog systems),but occur at discrete instants in time. These instants are usually regular periodic times, separated by the sampling period T [sec]. The sampling operation introduces artifacts into the signal which are known as signal aliases. It also introduces delay into the closed-loop system which tends to destabilize the control.

Digital systems do not use in finite-precision mathematics. To store the value of a signal such as a control signal in computer memory, the value must be quantized. This means that the value is rounded to the nearest number which can be stored. The coefficients of the transfer function of the controller must also be quantized. These are two separate issues: signal quantization and coefficient quantization.
In this lab period you will implement and simulate a discrete-time system in Simulink.

Notation for Sampling

Consider a continuous-time signal x(t) If we look at the signal at discrete points in time that are separated by a constant sampling period T , then we have the set of values x(Kt) where k is an integer. To simplify notation, we say that x[k] = x(kT) where x[k] is a discrete-time signal (denoted using square brackets) and x(t) is a continuous-time signal (denoted by parentheses). When you read $x[k]$ you should understand that some sampling period T is implied, which must be somehow specified else where.

Linear Constant Coefficient Difference Equations

The continuous-time systems you have seen are defined by Linear Constant Coefficient .Ordinary Differential Equations (LCCODEs). Linear, time invariant, lumped discrete-time systems may be defined by Linear Constant Coefficient Difference Equations (LCCDEs). A controller would be implemented with a LCCDE such as:

$$\sum_{i=0}^{n} a_i u[k-i] = \sum_{i=0}^{m} b_i e[k-i],$$

with input e[k] and output u[k] and a_i and b_i which determine the transfer function of the controller.An example discrete-time system is a discrete-time integrator. If the sampling period is T , then the output of an integrator may be approximated by(rectangular rule)

$$y[k] = T \sum_{i=0}^{k} x[i],$$

where y[k] is the output of the integral and x[k] is the input signal.
Note that

$$y[k-1] = T \sum_{i=0}^{k-1} x[i] \text{ so we can write}^2$$

$$y[k] = y[k-1] + Tx[k].$$

Discrete-Time Simulation

When we consider implementing the above equation, we nd that we need two quantities: x[k] and y[k -1]. The input to the system is x[k], and y[k-1] is a delayed version of the system's output. Therefore, we can implement the above equation with a feedback loop which has a delay in it.

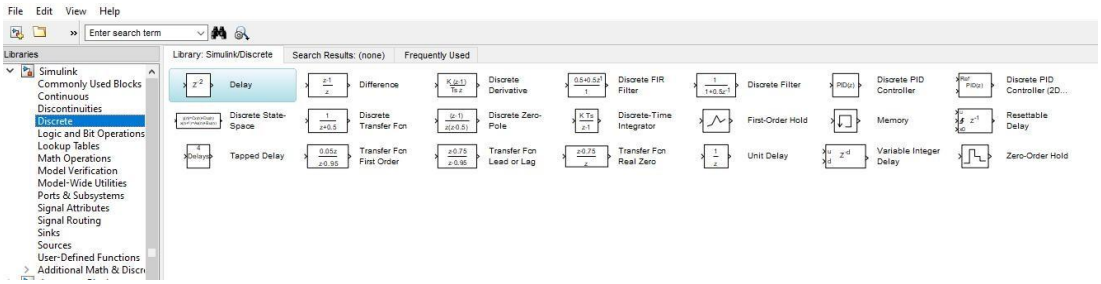


Figure 2.1- Simulink's discrete-time library.

Simulink has a number of blocks which will aid you in making discrete-time simulations. These are part of the Discrete-Time Simulink library, and are shown in Fig. 2.1. The two blocks of interest to us now are the Zero-Order Hold and Unit Delay blocks. Over the course of the semester, you will learn to use many of the other blocks as well. The reason for the label 1/z on the unit-delay will also become apparent.

To continue with the integrator example, consider the block diagram in Fig. 2.2. A source (in this case a sine wave) generates an input signal x(t). This signal must be sampled to makex[k]. The Simulink block which does this function is the Zero-Order Hold block. The x[k] signal is scaled by T to produce T x[k]. The output of the integrator is y[k] which is computedas x[k] + y[k-1]. The final remaining signal y[k-1] is computed from y[k] by passing it through a Unit Delay block.

The zero-order hold, amplifier and unit delay blocks require the sampling period T . You will find that it is best to enter these as symbolic T and define the value of T in the MATLAB workspace. This allows easy and consistent change of sampling rate.

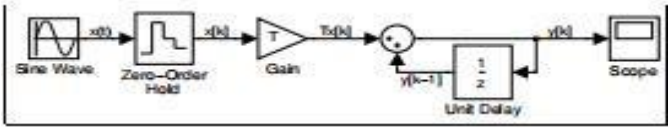


Figure 2.2- Discrete-time integrator implemented in Simulink

.

Laboratory Experiment

A leaky integrator is defined by the difference equation

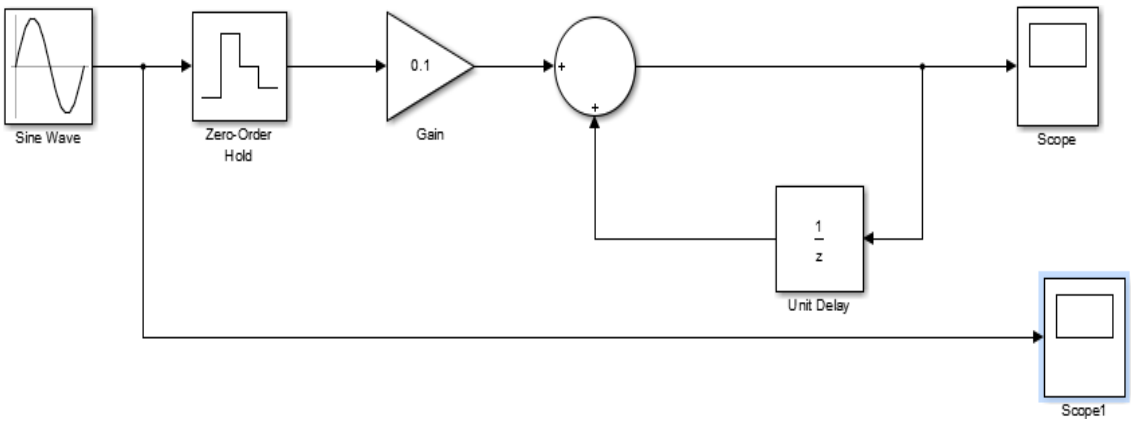
$$y[k] = ay[k-1] + Tx[k],$$

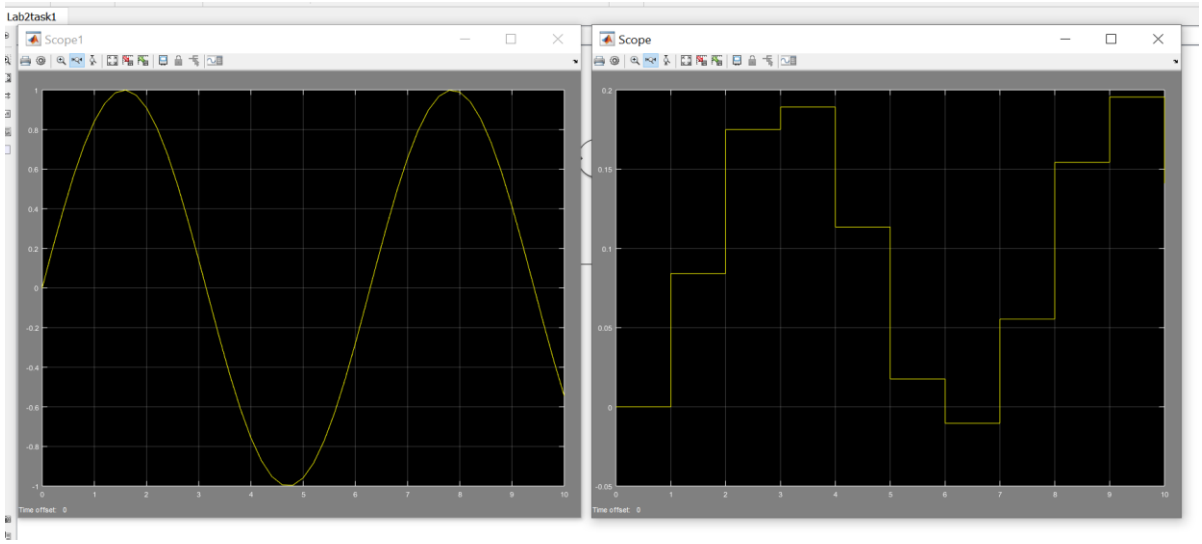
where usually 0 <a < 1. The system is stable, however, for 1 < a < 1.

1. Implement a leaky integrator in Simulink. Plot the step response of the system for a number of values of 0 <=a <= 1. Also plot the step response of the system for a number of values -1 <= a < 0. Use T= 0.1 sec.
2. Plot the response of the system with the input being a sine wave and a=0.9. Use a sinewave of frequency 1 rad=sec. Use sampling periods T = 0.1; 0.5; 1 sec.

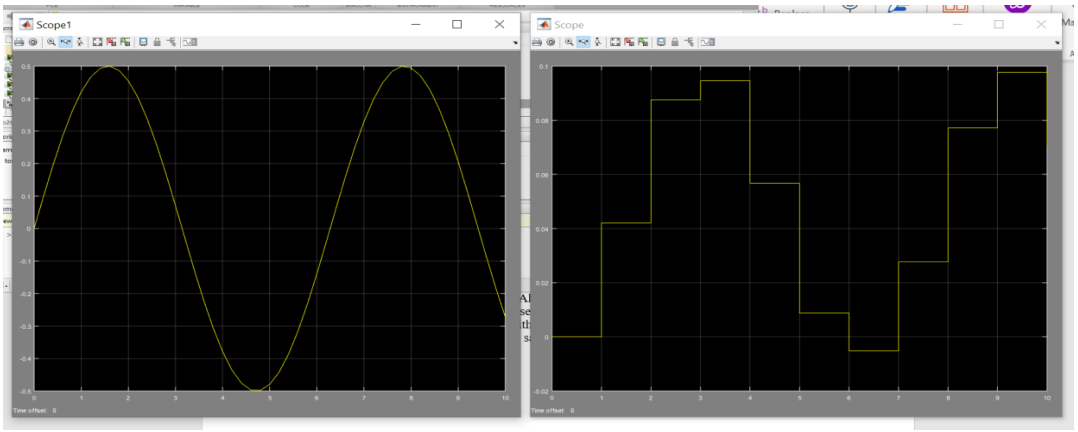
Review Exercise

a) Include your Simulink diagram and plots in your report. Comment on the shapes of the output curves. What prominent features do you see? How do you explain them? Your leaky integrator also has another name suggest what kind of discrete-time filter it is implementing (Hint: It is either a low-pass, band-pass or high-pass filter).

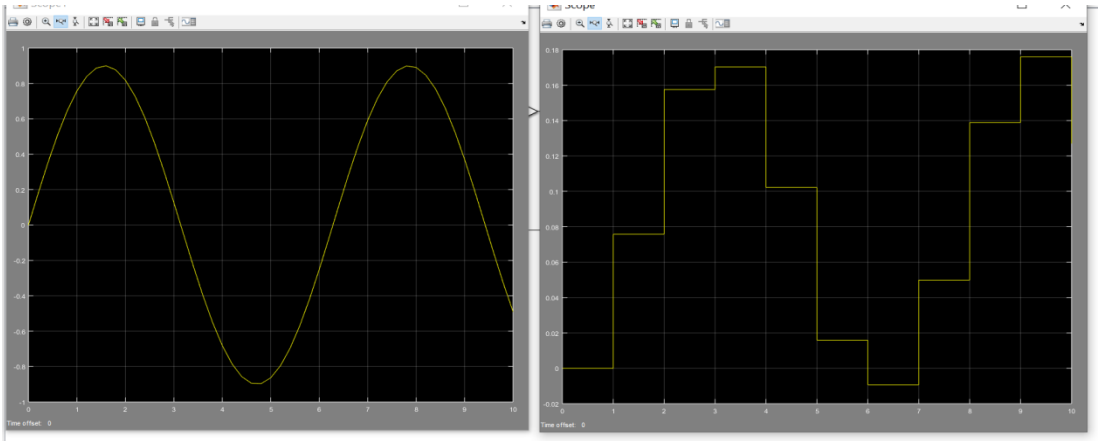




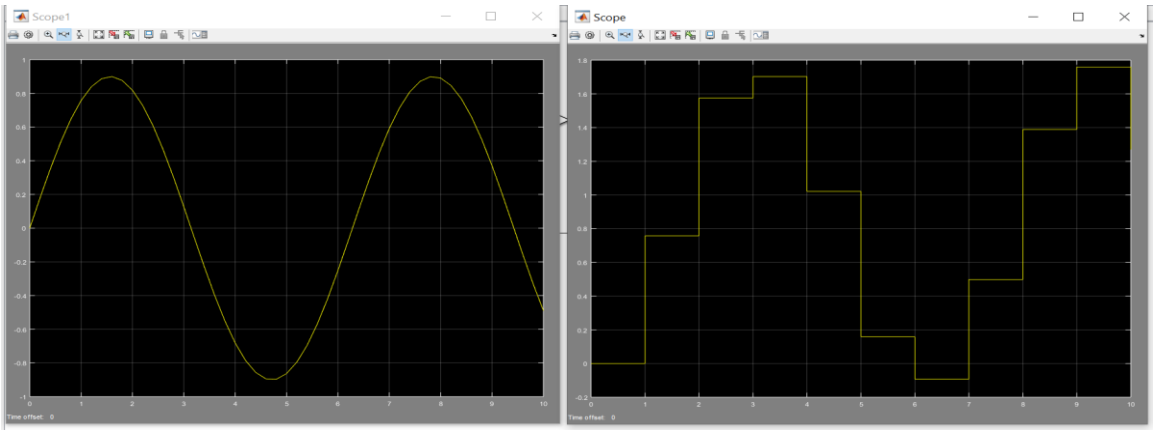
(When a=1, t=0.1)



(a=0.5, t=0.1)



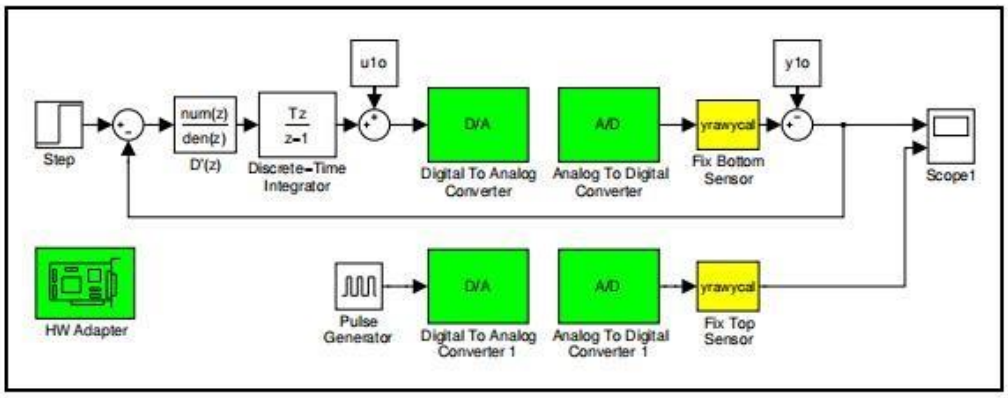
(a=0.9, t=0.1)



(a=0.9, t=1)

Comments:
Increasing the gain of a leaky integrator amplifies input signal response and accelerates system dynamics, potentially leading to instability and saturation if not carefully controlled.

b) Simulate the Single Input Multi Output (SIMO) system in simulink



MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 03: Z-Transform & Inverse Z-Transform

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	To analyze Z-Transform and inverse Z-Transform practically using MATLAB	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: Computer with MATLAB software

Approximate Time Required: Two to three hours

Z-transform

It converts a discrete time-domain signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation. The z-transform, like many other integral transforms, can be defined as either a one-sided or two-sided transform. The bilateral or two-sided Z-transform of a discrete-time signal x[n] is the function X(z)। alternatively, in cases where $x[n]$ is defined only for $n \geq 0$, the single-sided or unilateral Z- transform this definition is used when the signal is causal. As analog filters are designed using the Laplace transform, recursive digital filters are developed with a parallel technique called the z-transform. The overall strategy of these two transforms is the same: probe the impulse response with sinusoids and exponentials to find the system's poles and zeros. The Laplace transforms deals with differential equations, the s-domain, and the s-plane. Correspondingly, the z-transform deals with difference equations, the z-domain, and the z-plane. However, the two techniques are not a mirror image of each other; the s-plane is arranged in a rectangular coordinate system, while the z-plane uses a polar format. Recursive digital filters are often designed by starting with one of the classic analog filters, such as the Butterworth, Chebyshev, or elliptic. A series of mathematical conversions are then used to obtain the desired digital filter. The Z transform of a discrete time system X[n] is defined as Power Series.

ztrans

z-transform.

Syntax

F = ztrans(f)

Description

F = ztrans(f) is the z-transform of the scalar symbol f with default independent variable n. The default return is a f unction of z.

Example 1: Find the z-transform of:

$$x(n) = \frac{1}{4^n} u(n)$$

Solution

```
syms z n

ztrans((1/4)^n)
```

Result

```
ans =
z/(z - 1/4)
```

Z-Transfer Function

The z-transfer function, also known as the Pulse Transfer Function H(z) is defined as the ratio of the z-transform of the output to the z-transform of the input when all initial conditions are zero.

Mathematically,

$$\mathbf{H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_N z^{-N}}} \qquad (1)$$

The roots of the denominator of H(z) are called "poles" and those of the numerator are called "zeros". The above equation has M zeros and N poles

A system is said to be stable if all of its poles lie inside of a unit circle on the z-plane. The following example illustrate the use of MATLAB to compute zero and poles of a transfer function and to plot them onto the z-plane.

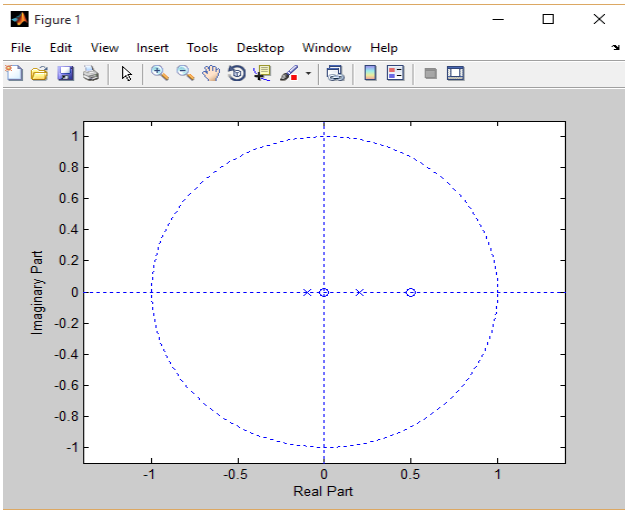
Example 2: Plot poles and zeros of the following pulse transfer function onto the z-plane and check its stability.

$$\mathbf{H(z) = \frac{2 - z^{-1}}{1 - 0.1z^{-1} - 0.02z^{-2}}}$$

Solution

```
num = [2    -1];                    % numerator of H(z)
den   = [1   -0.1    -0.02];        % denominator of H(z)
zplane(num,den)
```

Resul



"x" indicate the poles and "0" indicate the zeros.

Figure:3.1 Since all the poles lie inside the unit circle, the system is stable.

Computing Inverse Z-Transform using MATLAB function "iztrans":

Syntax

f = iztrans(F)

Description

f = iztrans(F) is the inverse z-transform of the scalar symbolic object F with default independent variable z. The default return is a function of n.

Example 3: Find the inverse z-transform:

$$x(z) = \frac{2z}{2z-1}$$

Solution:

```
syms z n
iztrans((2*z)/(2*z-1))
```

Result:

```
ans =
(1/2)^n
```

As we know, there are two methods to find inverse z-transform: Partial fraction method and Power series method MATLAB has built-in functions to find the inverse z-transform by using both of the above methods.

(a) Partial Fraction Method:

Example 4: Using MATLAB determine the partial fraction expansion of

$$\mathbf{H(z) = \frac{18z^3}{18z^3 + 3z^2 - 4z - 1}}$$

Solution:

Use the following MATLAB program:

```
num=[18  0   0   0];
den = [18   3   -4   -1];
[r, p, k] = residuez(num,den);    % students are advised to get on-help about  "residuez" to
                                  % findout more about it.
```

```
disp('Residues'); disp(r)
disp('Poles'); disp(p)
disp('Direct terms'); disp(k)
```

(b) Power Series Method

To find inverse z-transform by power series, one may use the built-in function "impz".

You may also use MATLAB function "filter" for the same purpose.

Example:5 Using MATLAB determine the power series expansion of $\frac{1}{1-1.5z^{-1}+0.5z^{-2}}$

Solution:

Use the following MATLAB program

```
L = 11;                 % Length of output vector
num = [1   0   0];
den  = [1  -1.5   0.5];
[y, t] = impz(num,den,L);
disp('Coefficients of the power series expansion');
disp(y)
```

Example 6: Repeat example 5 with the built-in function "filter"

Solution:

```
N = 11;
num = [1    0    0];
den   = [1    -1.5    0.5];
x =[1  zeros(1,N-1)];
y = filter(num,den, x);
disp('Coefficients of the power series expansion');
disp(y)
```

Frequency Response:

The Freqz function computes and display the frequency response of given z-Transform of the function

Freqz(b,a,Fs)

b= Coeff. Of Numerator a= Coeff. Of Denominator Fs= Sampling Frequency

Example7: Create the script file that analyzes the discrete system transfer function given by

$$H(e^{jW}) = \frac{G(e^{jW})}{F(e^{jW})} = \frac{5 + 0.9e^{-jW} + e^{-2jW}}{1 - 0.7e^{-jW} + 0.3e^{-j2W}}$$

and returns the following:

1. The pole/zero plot of H(e^{jW}) (evaluates the system stability)
2. The plot of abs{H(e^{jW})} versus W
3. The plot of angle {H(e^{jW})} versus W
4. List all the poles, zeros, and stand alone term of the transfer function H($z=e^{jW}$) and compare the results obtained with the results of part a
5. Determine the ROC of H($z = e^{jW}$)
6. The magnitude of the system poles (length from origin)
7. The partial fraction expansion of H(z)
8. h(n) by using the symbolic MATLAB inverse ZT of H(z)

MATLAB Solution

```
%Script file = analys
num = [5 0.9 1];
den = [1 -0.7 0.3];
figure(1)
zplane(num,den); title('Pole and zero plot of H(z)')
figure(2)
W = -pi:pi/99:pi;
[H,W] = freqz(num,den,W);
 magH = abs(H);phaseH = angle(H);
 subplot (2,1,1);
 plot (W,magH);title ('abs[H(exp(-jW))] vs W');ylabel('Magnitude');
 subplot (2,1,2);
 plot (W,phaseH);title('angle[H(exp(-jW))] vs W');
 ylabel ('Phase in rad.');xlabel('frequency W in rad');
 [z,p,k] = tf2zp(num,den);
 m = abs(p);
 disp('****************************************');
 disp('************ R E S U L T S *************');
 disp('****************************************')
 disp('The zeros of H(z) are:');disp(z);
 disp('The poles of H(z) are:');disp(p);
 disp('The gain  of H(z) is:');disp(k);
 disp('The magnitude of the poles are:');
 disp(m);disp('The ROC is outside the circle with radius:');
 disp(max(m))
 % partial fraction expansion
 [r,pp,kk] = residuez (num,den);
 disp('Partial fraction coefficients of H(z)')
 disp('the residues are:')
 disp(r);disp('the stand alone term is :');disp(kk);
 syms z Fz;
 disp('The partial fraction expansion is:')
 Fz = r(1)/(z-pp(1))+r(2)/(z-pp(2))+ kk
 hn = iztrans(Fz);disp('The impulse response h(n) is:');hn
 disp('^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^')
```

The script file *analys* is executed, and the results are as follows:

```
>> analys
*****************************************
************ R E S U L T S **************
*****************************************
The zeros of H(z) are:
                          -0.0900 + 0.4381i
                          -0.0900 - 0.4381i
The poles of H(z) are:
                          0.3500 + 0.4213i
                          0.3500 - 0.4213i
The gain of H(z) is:
                         5
The magnitude of the poles are:
                                    0.5477
                                    0.5477
The ROC is outside the circle with radius:
                                                0.5477
Partial fraction coefficients of H(z)
the residues are:
                      0.8333 - 4.5295i
                      0.8333 + 4.5295i

the stand alone term is:
 kk =
       3.3333
```

```
The partial fraction expansion is:
Fz =
(5/6-1/426*i*3723311^(1/2))/(z-7/20-1/20*i*71^(1/2))+
(5/6+1/426*i*3723311^(1/2))/(z-7/20+1/20*i*71^(1/2))+10/3

The impulse response h(n) is:
hn =
    127/9*charfcn[0](n)-97/18*(-6/(i*71^(1/2)-7))^n-979/1278*i*(-6/(i*71^
(1/2)-7))^n*71^(1/2)97/18*6^n*(1/(7+i*71^(1/2)))^n+979/1278*i*6^n*(1/(7+i*71^
(1/2)))^n*71^(1/2)
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
```

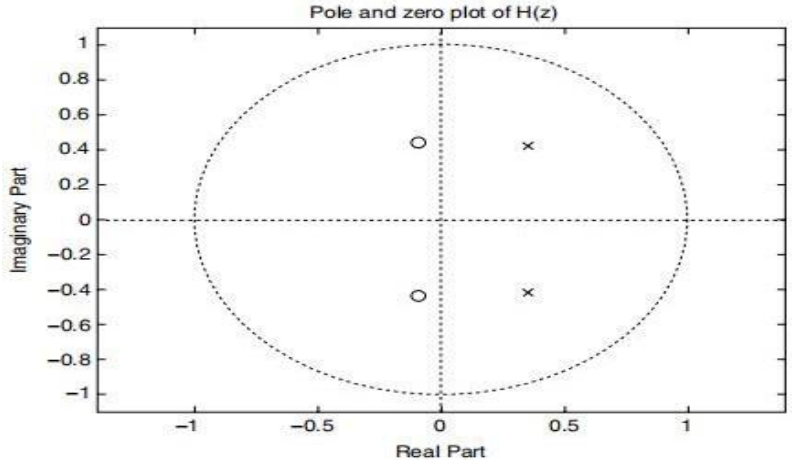


Figure 3.2- Pole/Zero plot of $H(e^{jW})$

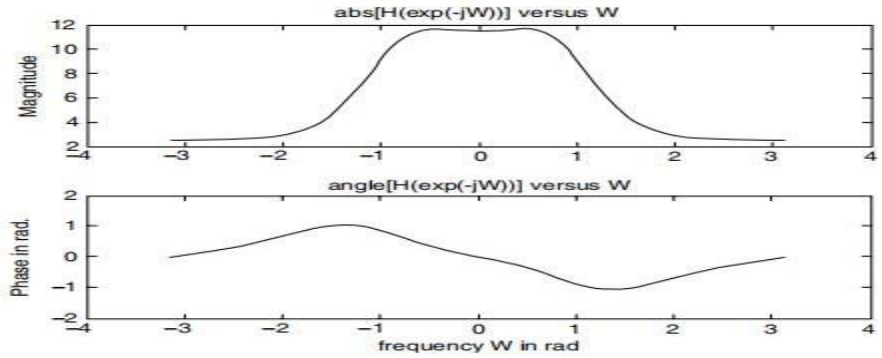


Figure 3.3- Plot of abs $H(e^{jW})$ and angle $H\{(e^{jW})\}$

Example8: Create the script file pole_zeroes that returns the rational transfer function H(z), the numerator and denominator coefficients of H(z), and the first 11 coefficients of their system impulse response h(n) given the following system's zeros, poles, and gain

$z_1 = 0.21$ $p_1 = -0.45$
$z_2 = 3.14$ $p_2 = 0.67$
$z_3 = -0.3 + j5$ $p_3 = 0.81 + j0.72$
$z_4 = -0.3 - j5$ $p_4 = 0.81 - j0.72$
$k = 2.3$ $p_5 = -0.33$

MATLAB Solution

```
% Script file: pole _ zeroes
z1 = 0.21 ; p1 =-0.45;
z2 = 3.14 ; p2 = 0.67;
z3 =-0.3 + 5j; p3 = 0.81+0.72j;
z4 = -0.3-5j; p4 = 0.81-0.72j;
k =2.3; p5 = 0.33;
zr = [z1 z2 z3 z4];
pl = [p1 p2 p3 p4 p5];
z = zr'; p=pl';
[num,den] = zp2tf(z,p,k);
n =0:10;
disp ('*************************************************')
disp ('**************** R E S U L T S ****************')
disp ('*************************************************')
tf(num,den,1)
disp('**********************')
disp ('Coefficients of the numerator polynomial of H(z)'); disp(num');
disp('**********************')
disp ('Coefficients of the denominator polynomial of H(z)');
disp(den');L=11; [y,t] = impz(num,den,L);
disp('**********************')
disp ('The first 11 coefficients of the impulse response h(n) are:');
disp('**********************')
disp('  coef. num.       h(n)')
disp('**********************')
[n' y]
disp ('*************************************************')
```

The script file *pole_zeroes* is executed, and the results are as follows:

```
>> pole _ zeroes
**************************************************************
*****************R E S U L T S ***************************
**************************************************************
Transfer function:
2.3 z^4 - 6.325 z^3 + 54.6 z^2 - 192.4 z + 38.05
------------------------------------------------------------
z^5 - 2.17 z^4 + 1.837 z^3 - 0.1757 z^2 - 0.43 z + 0.1169
Sampling time: 1
****************************
Coefficients of the numerator polynomial of H(z)

         0
    2.3000
   -6.3250
   54.6006
 -192.4085
   38.0520
****************************
Coefficients of the denominator polynomial of H(z)
    1.0000
   -2.1700
    1.8366
   -0.1757
   -0.4300
    0.1169
****************************
The first 11 coefficients of the impulse response h(n) are:
****************************
  coef. num.      h(n)
****************************
ans =
      0              0
      1.0000         2.3000
      2.0000        -1.3340
      3.0000        47.4817
      4.0000       -86.5192
      5.0000      -236.1448
      6.0000      -346.0348
      7.0000      -311.8158
      8.0000      -125.3485
      9.0000       148.4517
     10.0000       376.3726
****************************
```

Note that the system transfer function is given by

$$H(z) = \frac{2.3z^{-1} - 6.325z^{-2} + 54.6006z^{-3} - 192.4085z^{-4} + 38.0520z^{-5}}{1 - 2.17z^{-1} + 1.8366z^{-2} - 0.1757z^{-3} - 0.43z^{-4} + 0.1169z^{-5}}$$

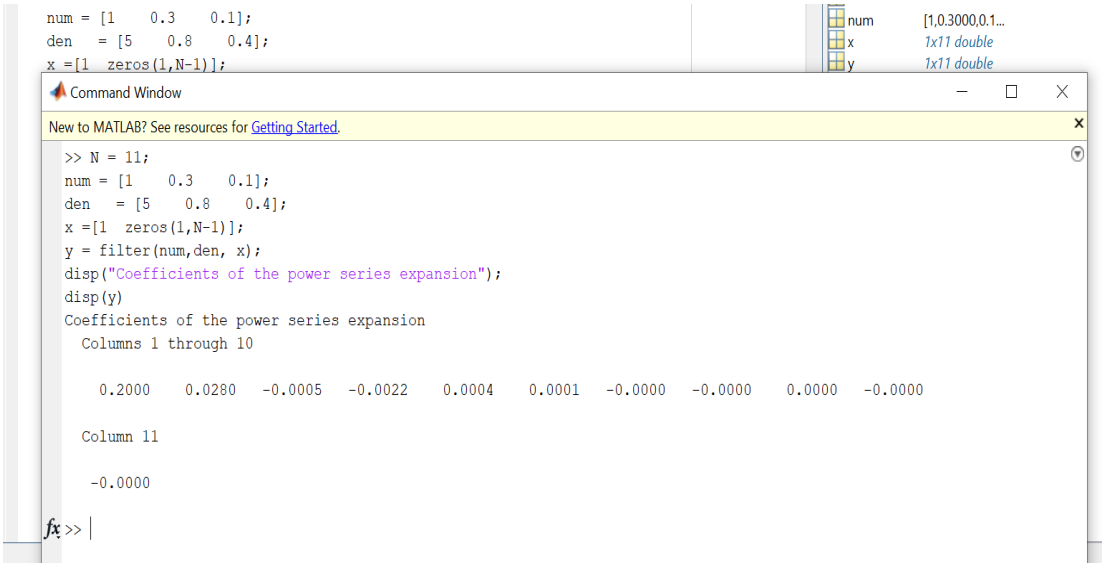
Note also that the corresponding system impulse response $h(n) = Z^{-1}[H(z)]$ is given by
$h(n) = 0 + 2.3\delta(n-1) - 1.3403\delta(n-2) + 47.4817\delta(n-3) - 86.5192\delta(n-4) - 236.1448\delta(n-5) - 346.0348\delta(n-6) - 311.8158\delta(n-7) - 155.3485\delta(n-8) + 148.4517\delta(n-9) + 376.3726\delta(n-10)$

Review exercise

a) What is ROC? Repeat example 6 by following transfer function

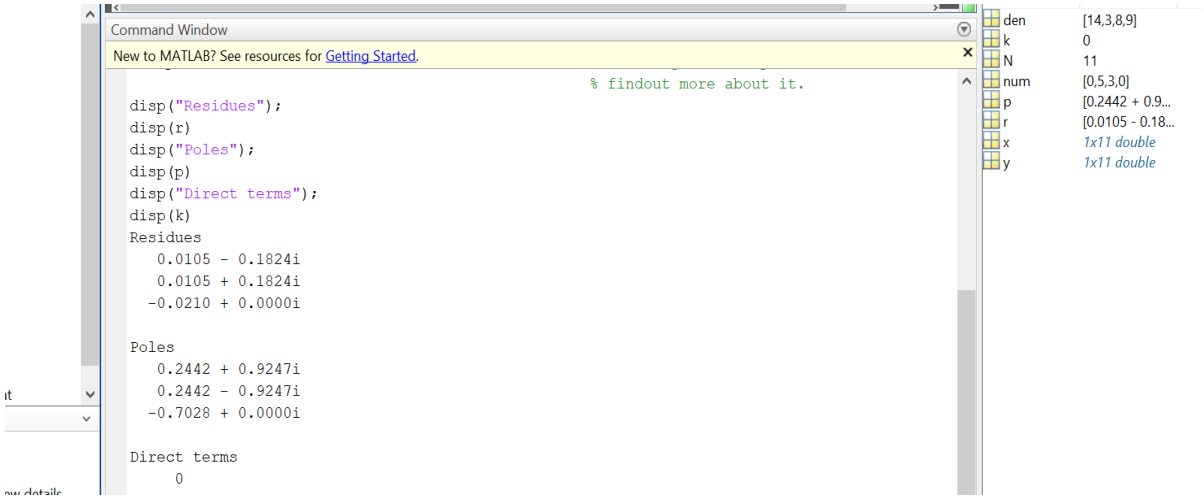
$$H(Z)= \frac{1+0.3Z^{-1}+0.1Z^{-2}}{5+0.8Z^{-1}+0.4Z^{-2}}$$

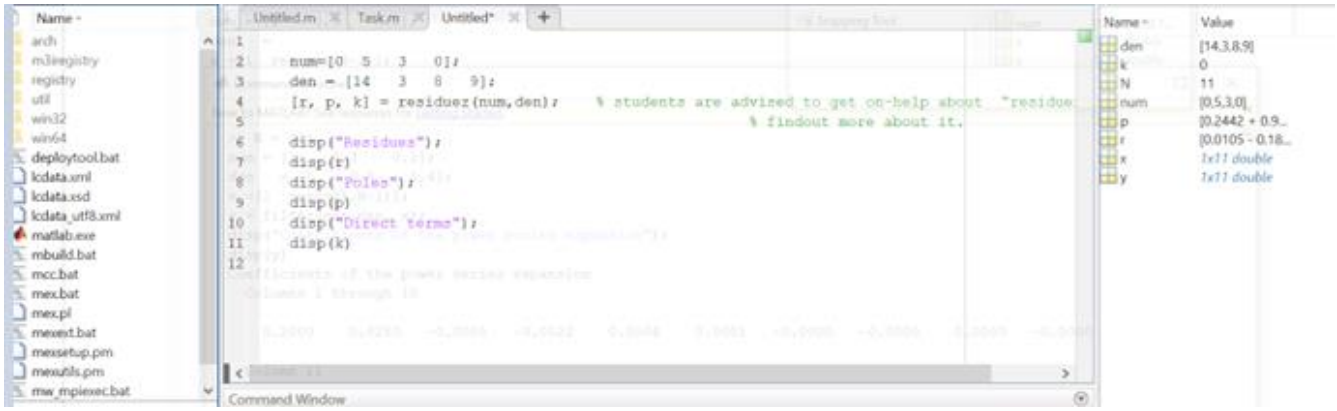
The region of convergence, known as the ROC, is important to understand because it defines the region where the z-transform exists. The ROC for a given x[n], is defined as the range of z for which the z-transform converges.



b) Determine the partial fraction expansion of the z-transform G(z) given by:

$$\frac{5z^2+3z}{14z^3+3z^2+8z^1+9z^o}$$





a) What is difference between z-transform and Fourier transform?

The Fourier Transform is employed for continuous-time signals in the frequency domain, representing them as a sum of sinusoidal components, while the Z-Transform is designed for discrete-time signals, operating in the complex z-plane and providing information about system behavior in terms of poles and zeros, making it particularly useful in digital signal processing applications.

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 04: DISCRETE TIME ROOT LOCUS

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	• Know about the Root locus technique. • Analyze the system performance in discrete-time domain using Root locus technique with the help of MATLAB.	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: PC with MATLAB software

Approximate Time Required: Two to three hours

Theory:

The relative stability and the transient performance of a closed-loop control system are directly related to the location of the closed-loop roots of the characteristic equation in the z-plane. It is frequently necessary to adjust one or more system parameters in order to obtain suitable root locations. Therefore, it is worthwhile to determine how the roots of the characteristic equation of a given system migrate about the z-plane as the parameters are varied; that is, it is useful to determine the Locus of roots in the z-plane as a parameter is varied. The root locus technique is a graphical method for sketching the locus of roots in the z-plane as a system parameter is varied. The root locus method provides graphical information, and therefore an approximate sketch can be used to obtain qualitative information concerning the stability and performance of the system.

Discrete Root Locus

The root-locus is the locus of points where roots of the characteristic equation can be found as a single parameter is varied from zero to infinity. The characteristic equation of our unity-feedback system with simple proportional gain, K, is:

$$1 + KG'(z)H_{zoh}(z) = 0$$

where G'(z) is the digital controller and $H_{zoh}(z)$ is the plant transfer function in z-domain (obtained by implementing a zero-order hold).

Commands used for Root Locus

- *rlocus(SYS)*: computes and plots the root locus of the single-input, single-output LTI model SYS.
- *rlocfind:* Finds root locus gain K for a given set of roots.
- *[K,POLES] = rlocfind(SYS)* is used for interactive gain selection from the root locus plot of the SISO system SYS generated by rlocus. rlocfind puts up a crosshair cursor in the graphics window which is used to select a pole location on an existing root locus. The root locus gain associated with this point is returned in K and all the system poles for this gain are returned in POLES.

Example 1: Given, G(z) = K[0.3678(z+0.7189) / (z-1)(z+0.24)] & H(z) =1. Using root locus, determine the range of K for which system becomes stable, with the help of MATLAB.

Solution

Since, the characteristic equation is given by 1+G(z)H(z)=0, this yields

$$1+ K[0.3678(z+0.7189) / (z-1)(z+0.24)]*(1)=0$$

Comparing it with $1+KG'(z)H_{zoh}(z)=0$, we have

$$G'(z) = [0.3678(z+0.7189) / (z-1)(z+0.24)]$$

Now, type the following MATLAB code:

```
close all

n =[0.3678 0.2644];

d= [1 -0.76 -0.24];
s=tf(n,d,0.1); rlocus(s) [k,poles]=rlocfind(s)
%verification figure

num= k*[0.3678 0.2644];

den=[1 0.368*k-0.76 0.2644*k-0.24];

sys = tf(num,den,0.1)

pzmap(sys)

p = pole(sys)
```

selected_point = -0.4810 + 0.8727i

k = 4.6782

poles =

-0.4803 + 0.8753i

-0.4803 - 0.8753i

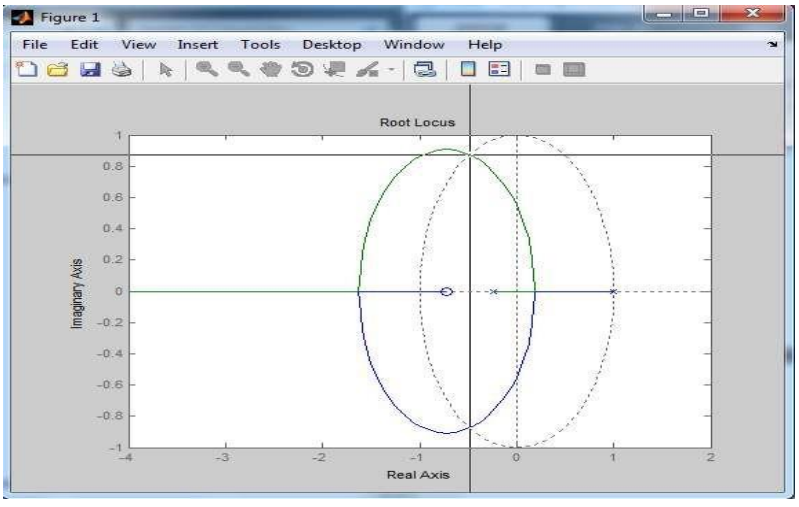


Figure-4.1 Root loci of given system

Verification

N

```
sys =

     1.721 z + 1.237

  ---------------

  z^2 + 0.9616 z + 0.9969

Sample time: 0.1 seconds Discrete-time

transfer function. p =

  -0.4808 + 0.8751i

  -0.4808 - 0.8751i
```

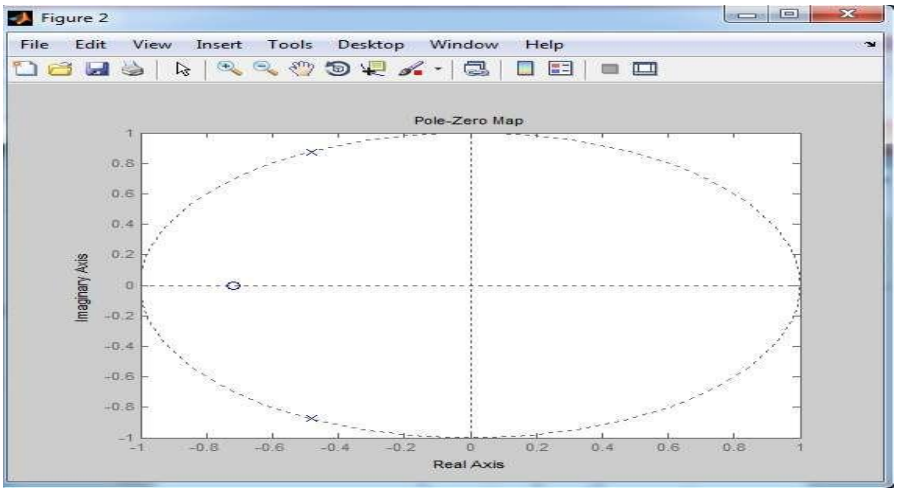


Figure-4.2 Location of poles and zeros in pzmap

Thus, the given system is stable for 0 < K < 4.6782

Example 2: Plot the Root loci of following system

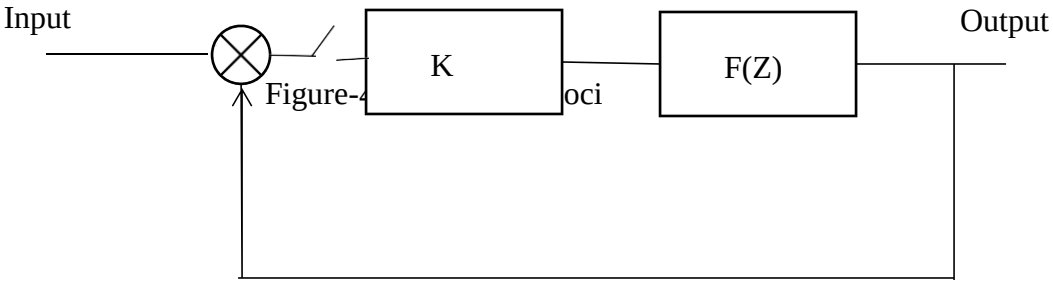


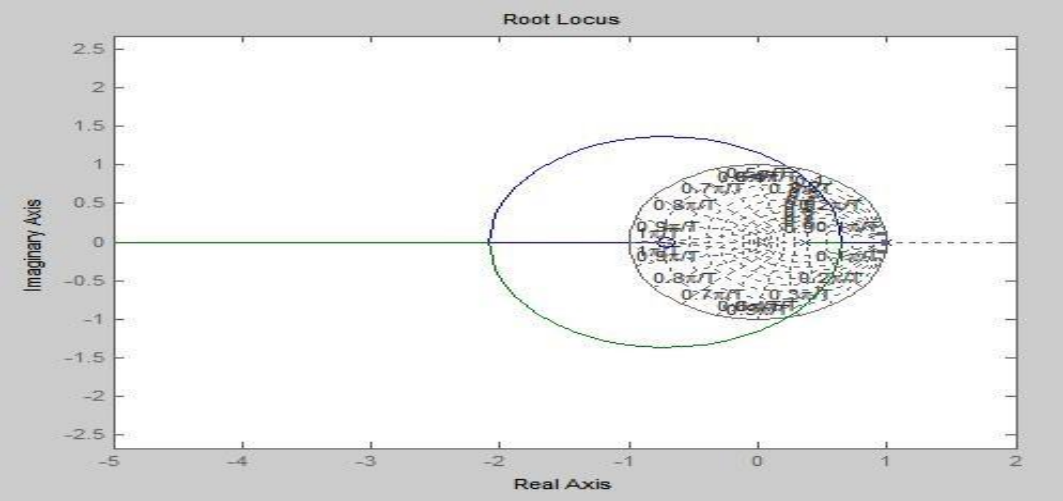
Figure-4.3 Block diagram of system

$$F(Z) = \frac{0.368(Z+0.717)}{(Z-1)(Z-0.368)}$$

MATLAB Code:

```
num=[0.368 0.368*0.717];
den=[1 -1.368 0.368];
sysd=tf(num,den,-1);
rlocus(sysd);
zgrid;
axis('equal');
```

Figure-4.4 z-plan root loc



Stability and Transient Response:

For continuous systems, we know that certain behaviors result from different pole locations in the s- plane. For instance, a system is unstable when any pole is located to the right of the imaginary axis. For discrete systems, we can analyze the system behaviors from different pole

$$z = e^{sT}$$

locations in the z-plane. The characteristics in the z-plane can be related to those in the s-plane by the following expression.

- T = sampling time (sec/sample)
- s = location in the s-plane
- z = location in the z-plane

The figure below shows the mapping of lines of constant damping ratio (ζ) and natural frequency (ω_n) from the s-plane to the z-plane using the expression shown above

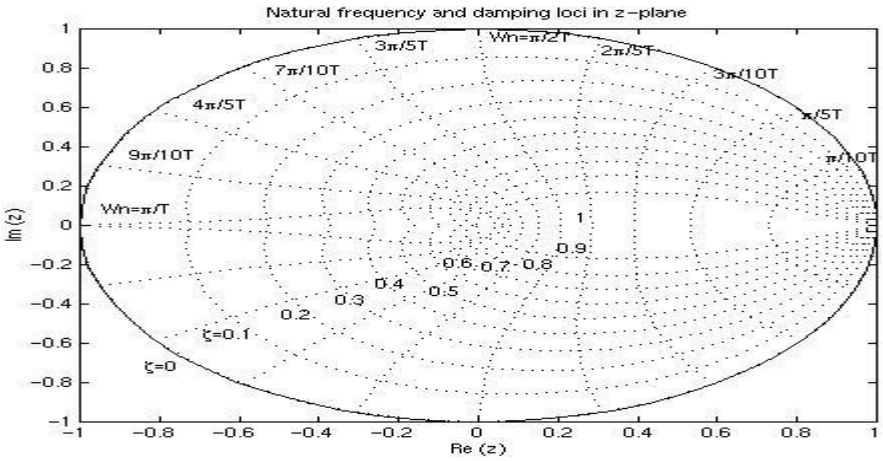


Figure-4.5 z-plan root loci

You may have noticed that in the z-plane the stability boundary is no longer the imaginary axis, but rather is the circle of radius one centered at the origin, |z| = 1, referred to as the **unit circle**. A discrete system is stable when all poles are located inside the unit circle and unstable when any pole is located outside the circle.

For analyzing the transient response from pole locations in the z-plane, the following three equations used in continuous system designs are still applicable.

where,

- ζ= damping ratio
- ω_n= natural frequency (rad/sec)
- Ts= 1% settling time
- Tr= 10-90% rise time
- Mp = maximum overshoot

$$\zeta\omega_n \geq \frac{4.6}{Ts}$$

$$\omega_n \geq \frac{1.8}{Tr}$$

$$\zeta = \frac{-\ln(Mp)}{\sqrt{\pi^2 + \ln^2(Mp)}}$$

Example 3: Suppose we have the following discrete transfer function

$$\frac{Y(z)}{F(z)} = \frac{1}{z^2 - 0.3z + 0.5}$$

Create a new m-file and enter the following commands. Running this m-file in the command window gives you the following plot with the lines of constant damping ratio and natural frequency.

MATLAB Code:

```
numDz = 1;
denDz = [1 -0.3 0.5];
sys = tf(numDz,denDz,-1); % the -1 indicates that the sample time is
undetermined
pzmap(sys)
axis([-1 1 -1 1])
zgrid
```

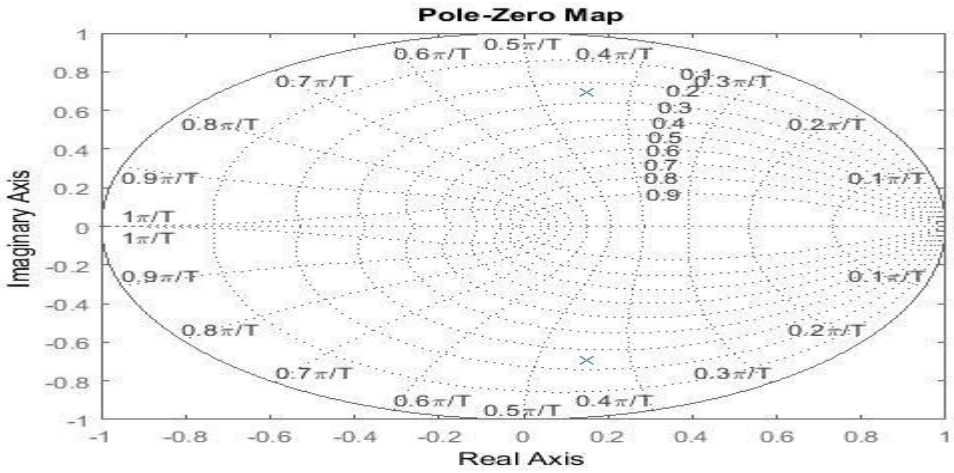


Figure-4.6 z-plan root loci

From this plot, we see the poles are located approximately at a natural frequency of 9π/20T (rad/sample) and a damping ratio of 0.25. Assuming that we have a sampling time of 1/20 sec (which leads to = 28.2 rad/sec) we can determine that this system should have a rise time of 0.06 sec, a settling time of 0.65 sec, and a maximum percent overshoot of 45% (0.45 times more than the steady-state value). Let's obtain the step response and see if these are correct. Add the following commands to the above m-file and rerun it in the command window. You should obtain the following step response.

MATLAB Code:

```
sys = tf(numDz, denDz,1/20);
```

step(sys,2.5);

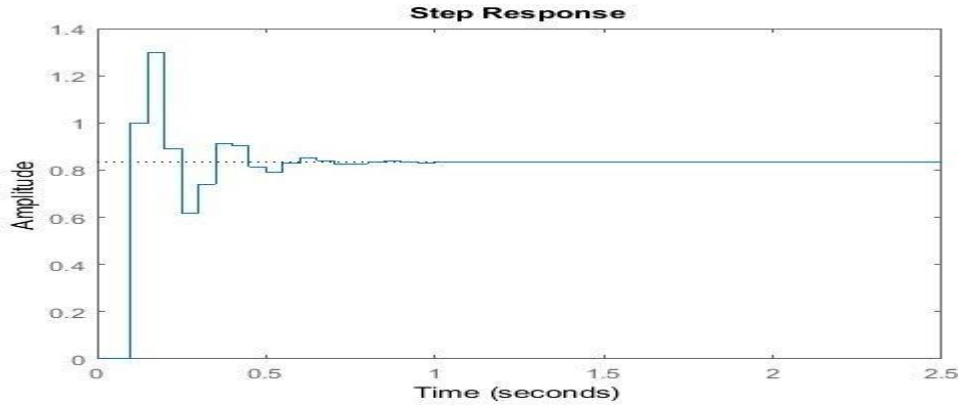


Figure-4.7 Step response of discrete root loci system

As we can see from the plot, the rise time, settling time and overshoot came out to be what we expected. This demonstrates how we can use the locations of poles and the above three equations to analyze the transient response of a discrete system

LAB TASK(S):

1. **Given, G(z) = K[(z+1) / (z^2-1.2z+0.2)] & H(z) =1. Using root locus, determine the range of K for which system becomes stable, with the help of MATLAB.**

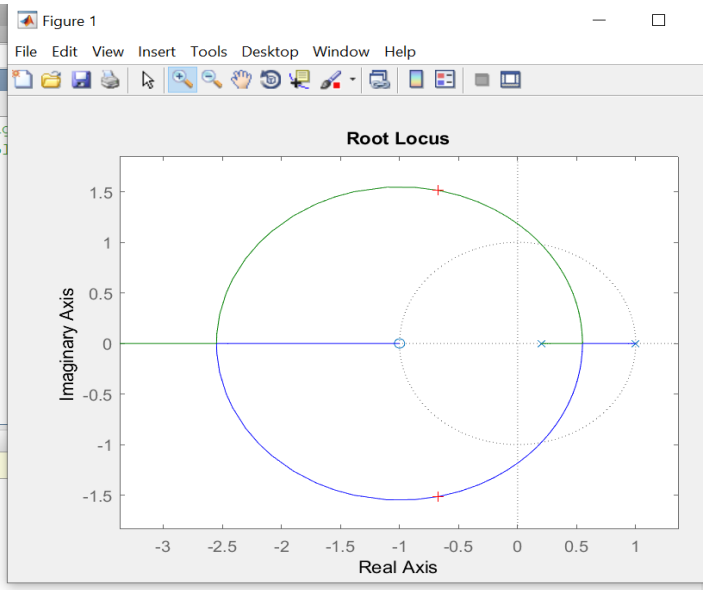
   Note: Attach code & results with your handout.

MATLAB Code:

```
% Given, G(z) = K[ (z+1) / (z2-1.2z+0.2)] & H(z) =1. Using root locus,
% determine the range of K for which  system becomes stable, with the help of MATLAB.
num= [1 1];
den= [1 -1.2 0.2];
openlooptf=tf(num, den, 0.1); % 0.1 is the sampling time
rlocus(openlooptf)
[k,poles]=rlocfind(openlooptf)
% Verification
num= k*[1 1];
den=[1 k-1.2 k+0.2];
sys = tf(num,den,0.1)
figure;
pzmap(sys)
p = pole(sys)
```

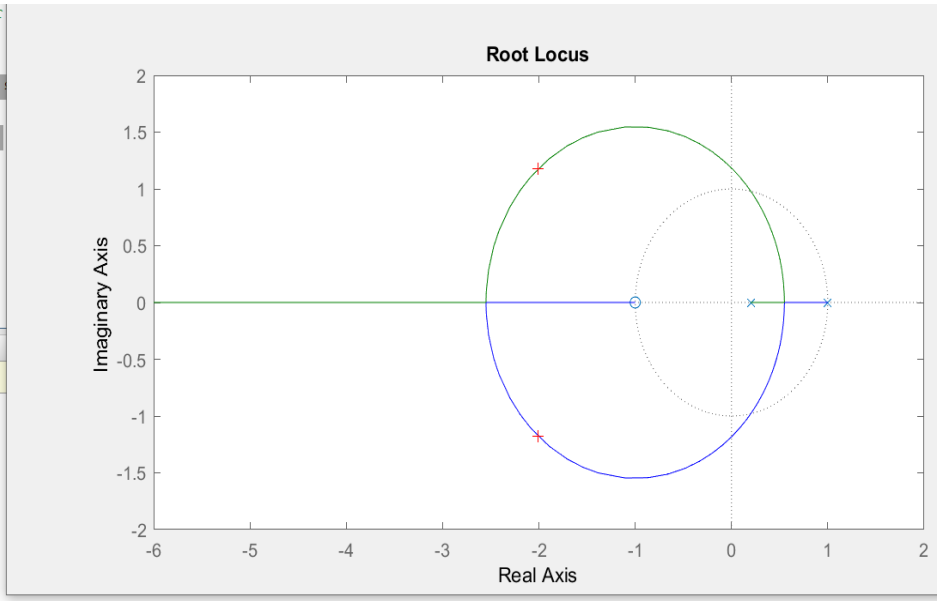
```
% Given, G(z) = K[ (z+1) / (z2-1.2z+0.2)] & H(z) =1. Using
% determine the range of K for which  system becomes stabl
num= [1 1];
den= [1 -1.2 0.2];
openlooptf=tf(num, den, 0.1); % 0.1 is the sampling time
rlocus(openlooptf)
[k,poles]=rlocfind(openlooptf)
% Verification
num= k*[1 1];
den=[1 k-1.2 k+0.2];
sys = tf(num, den, 0.1) ;
figure
pzmap(sys)
p = pole(sys)
```

```
openlooptf =

        z + 1
 -----------------
 z^2 - 1.2 z + 0.2

Sample time: 0.1 seconds
Discrete-time transfer function.
```



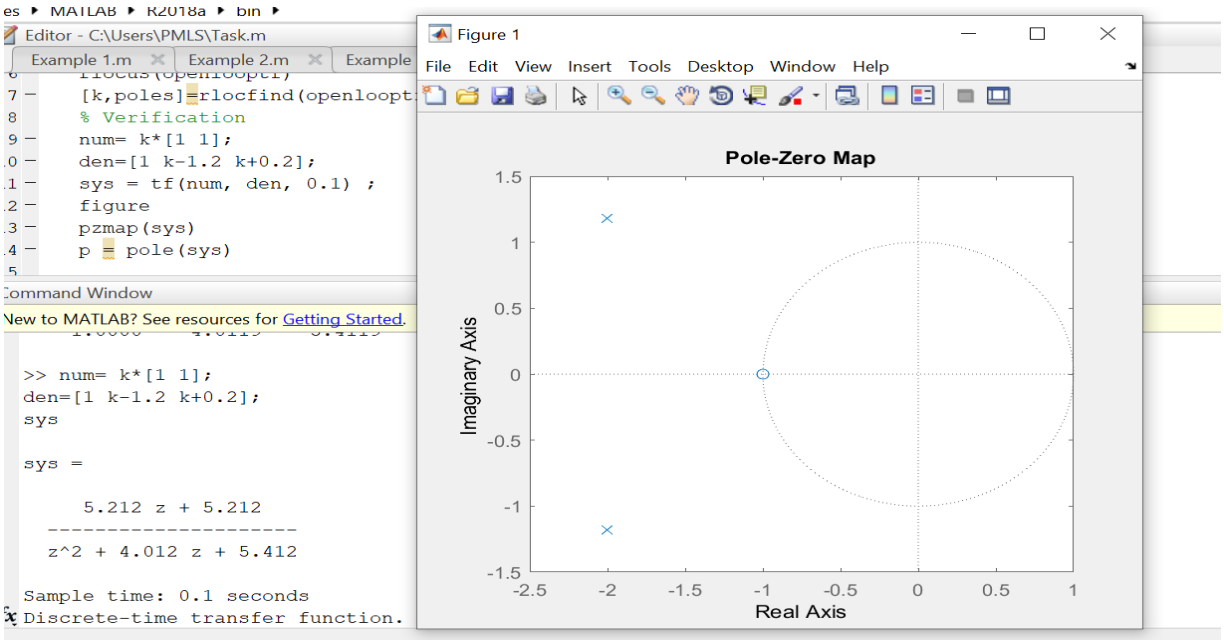

```
% determine the range of K for
num= [1 1];
den= [1 -1.2 0.2];
openlooptf=tf(num, den, 0.1);
rlocus(openlooptf)
[k,poles]=rlocfind(openlooptf)
% Verification
num= k*[1 1];
den=[1 k-1.2 k+0.2];
sys = tf(num, den, 0.1) ;
figure
pzmap(sys)
p = pole(sys)
```

```
mmand Window
w to MATLAB? See resources for Getting Started.
k =

    5.2119


poles =

  -2.0060 + 1.1782i
  -2.0060 - 1.1782i
```



selected_point = -2.2275 + 1.5579i
k = 5.2119
poles =
-2.0060 + 1.1782i
-2.0060 - 1.1782i

Verification:



num= k*[1 1];
den=[1 k-1.2 k+0.2];
sys =

$$\frac{5.212\,z + 5.212}{z^2 + 4.012\,z + 5.412}$$

Sample time: 0.1 seconds
Discrete-time transfer function.
p =
-2.0060 + 1.1782i
-2.0060 - 1.1782i

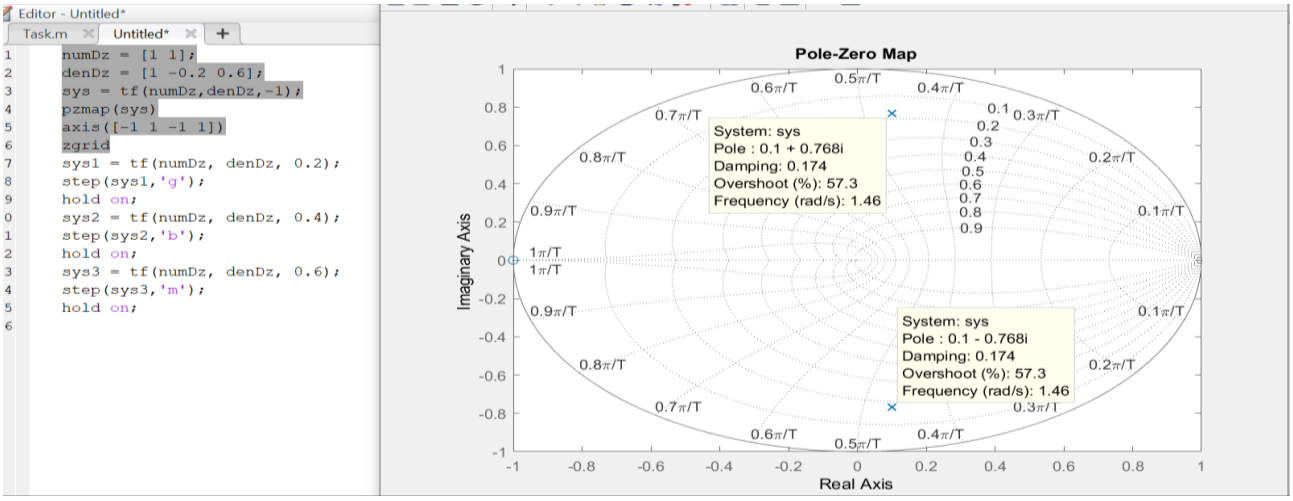
From Code Execution k = 5.2119
Thus the given system is stable between $0<K< 5.2119$

2. **Given, G(z) = K[(z+1) / (z^2-0.2z+0.6)] & H(z) =1. Plot pz-map, also step response with different time samples, with the help of MATLAB.**

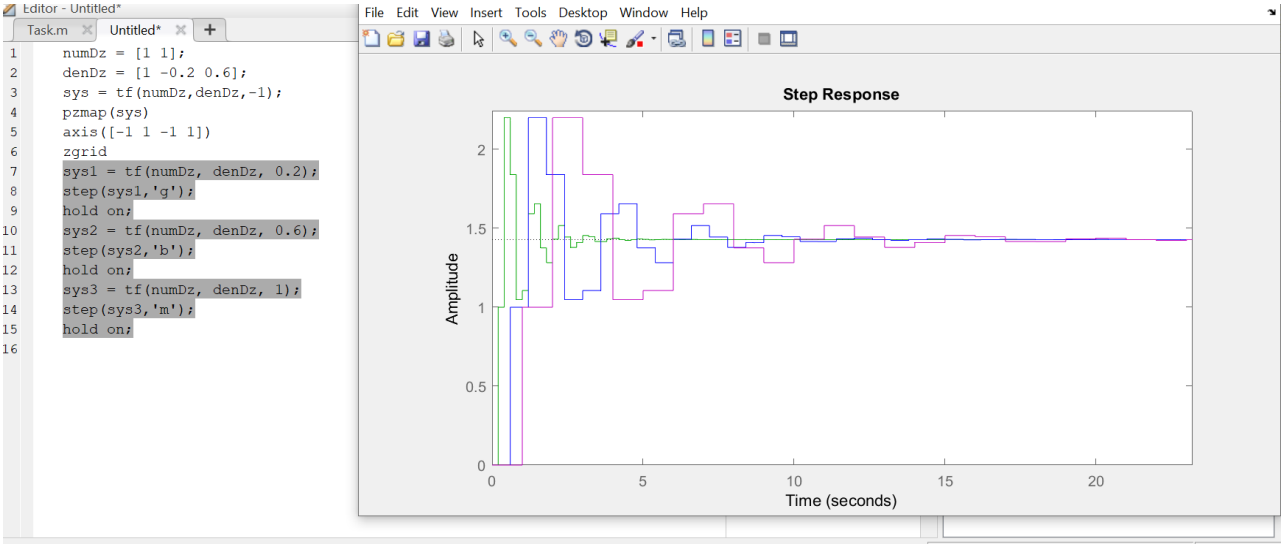
Matlab code:

```
numDz = [1 1];
denDz = [1 -0.2 0.6];
sys = tf(numDz,denDz,-1);
pzmap(sys)
axis([-1 1 -1 1])
zgrid
sys1 = tf(numDz, denDz, 0.2);
step(sys1,'g');
hold on;
sys2 = tf(numDz, denDz, 0.4);
step(sys2,'b');
hold on;
sys3 = tf(numDz, denDz, 0.6);
step(sys3,'m');
hold on;
```

denDz	[1,-0.2000,0.6...
numDz	[1,1]
sys	1x1 tf
sys1	1x1 tf
sys2	1x1 tf
sys3	1x1 tf
sys4	1x1 tf



From the pz-map, it is shown that damping ratio is 0.17, and a maximum percent overshoot of 57% (0.57 times more than the steady-state value).



From plot of step responses at different time samples, it is observed that by decreasing sampling time, system comes at stability quickly.

REFERENCE(S)

1. https://www.mathworks.com
2. Matlab: A Practical Introduction to Programming and Problem Solving by Stormy Attaway.

FINAL CHECK LIST

1. Before you leave, properly save your work and close the software tool.
2. Submit your completed laboratory work, before the next laboratory session.

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 05: MATHMATICALLY MODELING OF ELECTRICAL SYSTEM AND DC CIRCUIT ANALYSIS USING MATLAB

Name: karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	Mathematical modeling of electrical system and DC circuit analysis using MATLAB	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: PC with MATLAB software

Approximate Time Required: Two to three hours

Introduction

There are various types of physical systems, namely we have:

1. Mechanical systems
2. Electrical systems
3. Electronic systems
4. Thermal systems
5. Hydraulic systems
6. Chemical systems

First we need to understand why do we need to model these systems in the first place? Mathematical modeling of a control system is the process of drawing the block diagrams for these types of systems in order to determine their performance and transfer functions.

In an electrical type of system we have three variables

1. Voltage which is represented by 'V'.
2. Current which is represented by 'I'.
3. Charge which is represented by 'Q'.

And also we have three parameters which are active and passive components:

1. Resistance which is represented by 'R'.
2. Capacitance which is represented by 'C'.
3. Inductance which is represented by 'L'.

System response

The natural overdamped RLC series circuit response occurs for the condition $\alpha^2 > w_o^2$.

Then the roots s_1 and s_2 are real, unequal, and negative numbers

The natural critical-damped RLC series circuit response occurs for the condition $\alpha^2 = w^2_o$. Then the roots s_1 and s_2 are real $(-\alpha)$ and repeated, resulting in a solution of the form $i(t) = A_1e^{-\alpha t} + A_2te^{-\alpha t}$, where the constants A_1 and A_2 can be evaluated from the system's initial conditions.

The natural underdamped RLC series circuit response occurs for the condition $\alpha^2 < w^2_o$. Then the roots s_1 and s_2 are a complex conjugate pair, resulting in a solution of the form

The transient analysis of a circuit that contains more than one loop can be described by a set of differential equations. The set of differential equations may consist of either loop or node equations where the unknowns may be either the loop cur-rents or the node voltages.

Example1:

Using two loop equations and one node equation, and the matrix operations $I = inv(R) * V$ (where R is the [equivalent] impedance matrix of the network and $V$the voltage vector), solve for the three branch currents—I_1, I_2, and I_3—shown in the circuit diagram of figure1

The node and the two loop equations are shown as follows:

Node A; $-I_1 + I_2 + I_3 = 0$

Loop# 1; $10 * I_1 + 5 * I_2 = 10$

Loop# 2; $-5 * I_2 + (10 + 20) * I_3 = 0$

The preceding equations in matrix form is given by

$$\begin{bmatrix} -1 & 1 & 1 \\ 10 & 5 & 0 \\ 0 & -5 & 30 \end{bmatrix} * \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

Then the 3 × 3 matrix becomes the resistance matrix *R*, and the voltage *V* isgiven by the column vector $[0\ 10\ 0]^T$ as illustrated by the following matrix equation:

$$[R] * [I]\ [V]$$

then

$$I = \text{inv}\,(R) * V$$

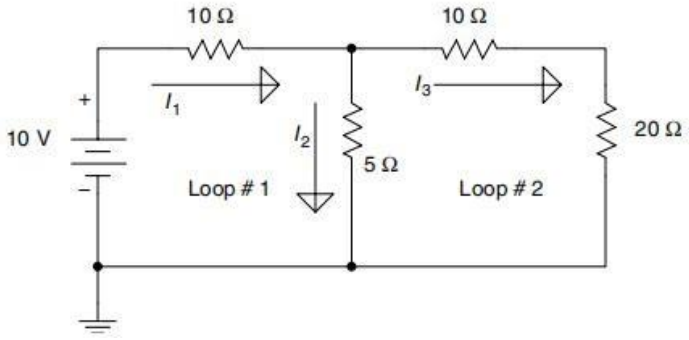


Figure5.1:Network Example

MATLAB Solution

```
>> R = [-1 1 1;10 5 0;0 -5 30];
>> V = [0;10;0];
>> I = inv(R)*V;  % Solves for the loop currents
>> Result = [I(1) I(2) I(3)];
>> % Results are printed
>> disp('**********************************');
>> disp('**********R E S U L T S*********');
>> disp('**********************************');
>> disp('   The currents I1, I2, I3 are:');
>> disp(Result)
>> disp('    amp.    amp.    Amp.');
>> disp('**********************************');
```

```
**********************************
**********R E S U L T S*********
**********************************
 The currents I1, I2, I3 are:
       0.7000    0.6000    0.1000
         amp.      amp.      amp.
**********************************
```

Example2: For the circuit diagram shown in Figure 2.60

7. Write the three system mesh (loop) equations 2.Arrange the result of part 1 into a matrix equation
3. Using MATLAB, solve for the three loop currents (I_1, I_2, and I_3)

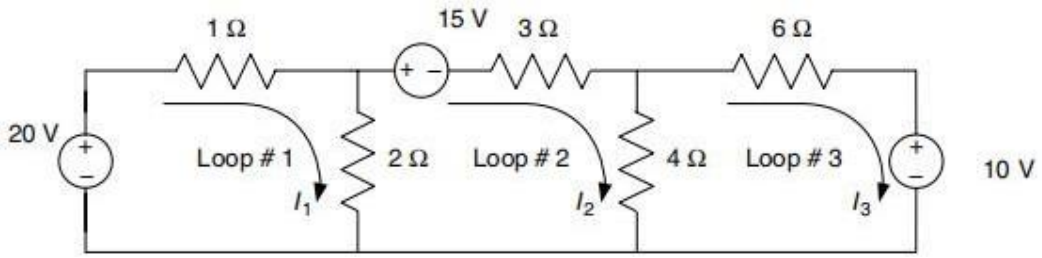


Figure5.2: Network Example

ANALYTICAL Solution

The loop equations are

$$\text{Loop 1:} \quad 20 = 3I_1 - 2I_2 + 0I_3$$
$$\text{Loop 2:} \quad -15 = -2I_1 + 9I_2 - 4I_3$$
$$\text{Loop 3:} \quad -10 = 0I_1 - 4I_2 + 10I_3$$

The resulting matrix equation is

$$\underbrace{\begin{bmatrix} 3 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 10 \end{bmatrix}}_{R} * \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} 20 \\ -15 \\ -10 \end{bmatrix}}_{V}$$

MATLAB Solution

```
>> R = [3 -2 0; -2 9 -4; 0 -4 10];
>> V = [20; -15; -10];
>> I = inv(R)*V;
>> Result = [I(1) I(2) I(3)];
>> disp('************************************************')
>> disp('The loop currents I1, I2 and I3(in amp) are:');
>> disp(Result');
>> disp('************************************************')
```

```
************************************************
The loop currents I1, I2 and I3 (in amp) are:
    6.0440
   -0.9341
   -1.3736
************************************************
```

Example3: The switch shown in the circuit diagram in Figure:5.3 hasbeen in position a for a long time. At t = 0, the switch is moved to position b where it remains for 2 s and then moves back to position a, where itremains indefinitely.

Obtain analytical expressions for $v_C(t)$ and $i_C(t)$ for all t 2.Use MATLAB to obtain plots over the range 0 s ≤ t ≤ 10 s of

A. The voltage $v_C(t)$ versus t
B. The current $i_C(t)$ versus t

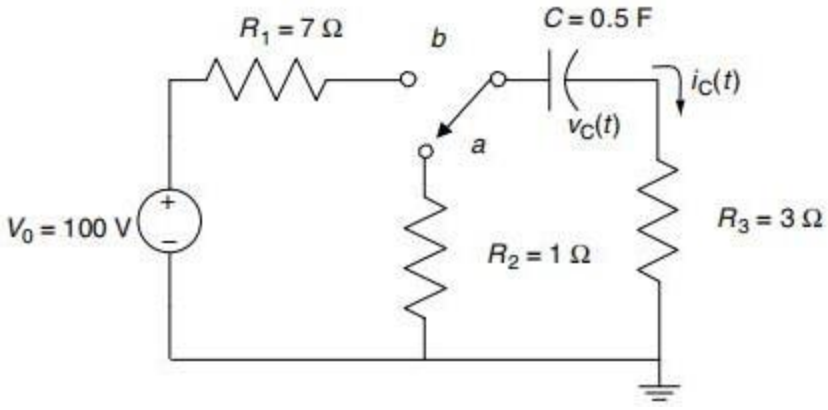


Figure5.3-Network example

ANALYTICAL Solution

For $t < 0$, the switch is in position *a*. Therefore,

$$v_c(t) = 0$$

and

$$i_c(t) = 0$$

For $0 < t < (t_1 = 2)$ seconds, the switch is in position *b*. Then,

$$v_C(t) = V_0(1 - e^{-t/\tau_1})$$

and

$$i_C(t) = \frac{V_0 - v_C(t)}{R_1 + R_3}$$

where $\tau_1 = (R_1 + R_3) * C = 10\ \Omega * 0.5$ F = 5 s.
At time $t = t_1 = 2$ s, the voltage across the capacitor is

$$v_C(t_1) = V_{C\max} = V_0(1 - e^{-2/\tau_1})$$

For $t > (t_1 = 2\text{ s})$, the switch is back in position *a*. Then

$$v_C(t) = V_{C\max} * e^{-(t-t_1)/\tau_2}$$

and

$$i_C(t) = \frac{-v_C(t)}{R_3 + R_2}$$

where $\tau_2 = (R_2 + R_3) * C = 4\ \Omega * 0.5\ \text{F} = 2\text{ s}.$

MATLAB Solution

```
>> V = 100; R1 =7; R2 =1; R3=3; C =0.5;     % circuit elements
>> tau1 = (R1+R3)*C                          % time constant #1
   tau1 =
        5
>> tau2 = (R2+R3)*C                          % time constant #2
   tau2 =
        2
>> for k =1:40
       t(k) = k/20;   % 0 < t < 2
       v(k) = V*(1-exp(-t(k)/tau1));
       i(k) = (V-v(k))/(R1+R2);
   end

   >> Vmax = v(40)
      Vmax =
           32.9680
   >> for k=41:200
        t(k)=k/20;   % 2 < t < 10
        v(k)=Vmax*exp(-(t(k)-t(40))/tau2);
        t(40)=40/20=2sec=t1
        i(k)= -v(k)/(R2+R3);
      end
   >>                                % plot the Voltage VC(t)
   >> subplot(2,1,1)
   >> plot(t,v)
   >> axis([0 10 0 40]);
   >> title ('Transients in the circuit of Fig 2.62')
   >> xlabel('time in sec'), ylabel('Voltage vc(t)')
   >> grid on
   >>                                % plot the current IC (t)
   >> subplot(2,1,2)
   >> plot(t,i)
   >> axis([0 10 -13 15]);
   >> xlabel('time in sec'),ylabel('Current ic(t)')
   >> grid on
```

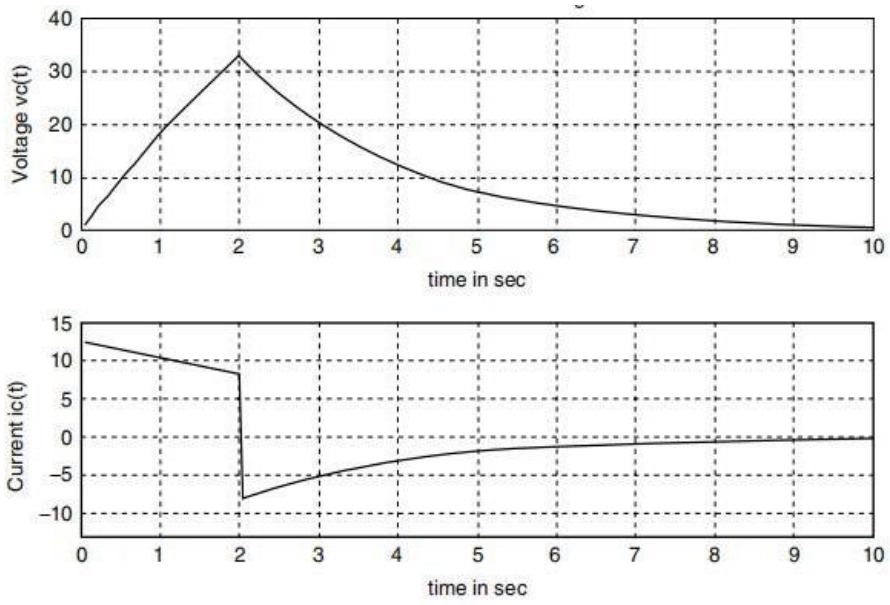


Figure5.4: Transient plots

Example4: Steady-state conditions exist in the network shown in Figure 5.5 at t = 0 when the V_1 = 120 V source is connected to the RCL parallel circuit. At t = 0, the switch moves downward, and the source V_1 = 120 V and resistor R = 10 Ω are disconnected from the parallel (RLC) structure.

Analyze the transient response (t > 0) of the source-free parallel RLCcircuit, for each of the following values of R, R = 3, 9, and 72 Ω.

1. Determine the analytical response $v_C(t)$ for each value of *R*, for $t \geq 0$2.Create the script file transient_RLC_parallel that returns the MATLAB solutions of part 1 and its corresponding voltage plots
4. Compare the MATLAB solutions of part 2 with the analytical solutionsof part 1

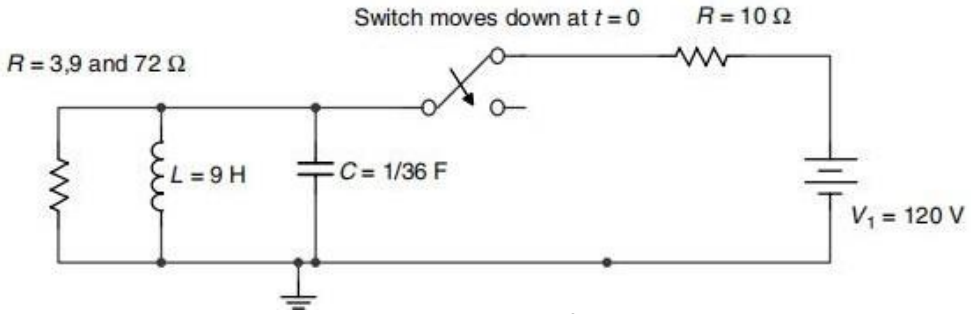


Figure 5.5: Network of example

ANALYTICAL Solution

From the circuit diagram of Figure 5.5 for $t \leq 0$, the initial conditionsare $v_C(0) = 0$ and $i_L(0) = 120/10 = 12$ A, and

$$C\frac{dv_C(t)}{dt}\Big|_{t=0} = i_C(t=0) = i_C(0) = i_L(0) + i_R(0)$$

then

$$\frac{dv_C(t)}{dt}\Big|_{t=0} = 36i_C(0) = 36(12+0) = 432 \text{ V/s}$$

Recall that the node equation is

$$\frac{d^2v_C(t)}{dt^2} + \frac{1}{RC}\frac{dv_C(t)}{dt} + \frac{1}{CL}v_C(t) = 0 \quad \text{for } t \geq 0$$

For $R = 3\ \Omega$, the resonant frequency is

$$w_0 = \frac{1}{\sqrt{LC}} = 2 \text{ rad/s}$$

The neper frequency is

$$\alpha = \frac{1}{2RC} = 6 \text{ Hz}$$

The complex frequencies are s_1 and s_2, given by

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$$

then $0 > \sqrt{\alpha^2 - w_0^2}$ and is clearly the overdamped case.
Then the solution $v_C(t)$ is of the form $v_C(t) = A_1e^{-s_1t} + A_2e^{-s_2t}$
For $R = 9\ \Omega$, the resonant frequency is

$$w_0 = \frac{1}{\sqrt{LC}} = 2 \text{ rad/s}$$

The neper frequency is

$$\alpha = \frac{1}{2RC} = 2 \text{ Hz}$$

Then s_1 and s_2 are negative and repeated frequencies, which are given by

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -\alpha$$

Since $0 = \sqrt{\alpha^2 - w_0^2}$, the case is critical damped.
Then the solution $v_C(t)$ is of the form $v_C(t) = A_1 e^{-at} + A_2 t e^{-at}$
For $R = 72\ \Omega$, the resonant frequency is

$$w_0 = \frac{1}{\sqrt{LC}} = 2 \text{ rad/s}$$

The neper frequency is

$$\alpha = \frac{1}{2RC} = \frac{1}{4} \text{ Hz}$$

The complex frequencies s_1 and s_2 are given by $s_{1,2} = -\alpha \pm j\sqrt{w_0^2 - \alpha^2}$ (complex conjugate), and is clearly the underdamped case. Then the solution $v_C(t)$ is of the form

$$v(t) = e^{-\alpha t}[A_1 \cos(w_d t) + A_2 \sin(w_d t)]$$

where $w_d = \sqrt{w_0^2 - \alpha^2}$.

Let us use MATLAB to obtain and plot the solutions just presented, and compare them with the analytical results.

MATLAB Solution

```
% Script file : transient _ RLC _ parallel
% source free RLC parallel circuit analysis
disp('****************************************')
disp('*******R E S U L T S******************')
disp('Source free parallel RLC circuit')
disp('****************************************')
disp('Solution of diff. equation for R=3 ohms')
overdamped = dsolve('(1/36)*D2y+(1/3)*Dy+(1/9)*y=0','y(0) = 0','Dy(0)= 32','t')
figure(1)
ezplot(overdamped)
xlabel('time (sec)')
ylabel('voltage(volts)')
title('v(t) vs t (overdamped case; R=3 ohms)')
axis([0 3 0 40]);
disp('****************************************')
figure(2)
disp('Solution of diff. equation for R=9 ohms')
criticaldamped = dsolve('(1/36)*D2y+(1/9)*Dy+(1/9)*y=0','y(0)=0','Dy(0)=432','t')
simple _ crit=simple(criticaldamped);
ezplot(criticaldamped)
xlabel('time (sec)')
ylabel('voltage(volts)')
title('v(t) vs t (criticaldamped case; R=9 ohms)')
axis([0 4 0 90]);
disp('****************************************')
disp('Solution of diff. equation for R=72 ohms')
underdamped = dsolve('(1/36)*D2y+(1/72)*Dy+(1/9)*y=0','y(0)=0','Dy(0)=432','t')
figure(3)
ezplot(underdamped)
xlabel('time (sec)'); ylabel('voltage(volts)')
title('v(t) vs t (underdamped case; R=72 ohms)')
axis([0 6 -150 200]);
disp('****************************************')
```

Results are:

```
>> transient _ RLC _ parallel
*******************************
******* *******R E S U L T S******************
Source freeparallel RLC circuit
*******************************
*******
Solution of diff. equation for R=3ohms overdamped =
  27*2^(1/2)*exp(2*(-3+2*2^(1/2))*t)-27*2^(1/2)*exp(-2*(
3+2*2^(1/2))*t) ***************************************Solution of diff.
equation for R=9
ohms criticaldamped =432*exp(-2*t)*t*******************************
******* Solution of diff. equationfor R=72 ohms
underdamped =
              576/7*7^(1/2)*exp(-1/4*t)*sin(3/4*7^(1/2)*t)
*************************************
```

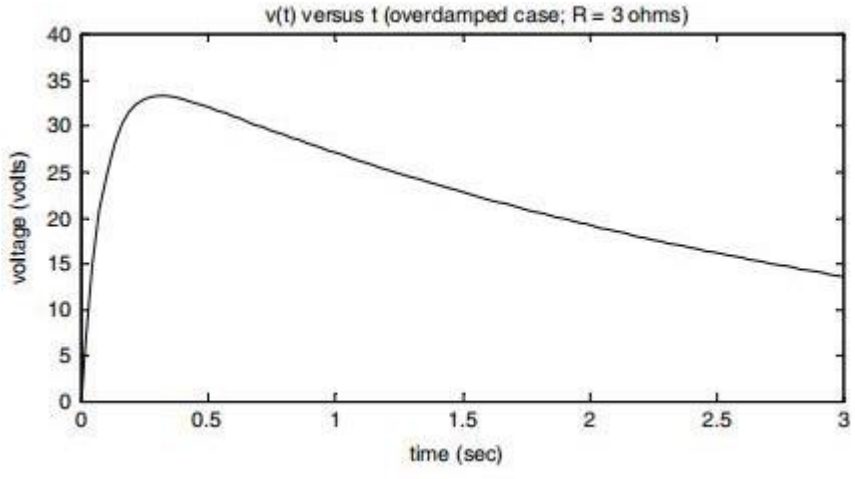


Figure5.6: Transient over damped plot ($t > 0$) of the source-free parallelRLC

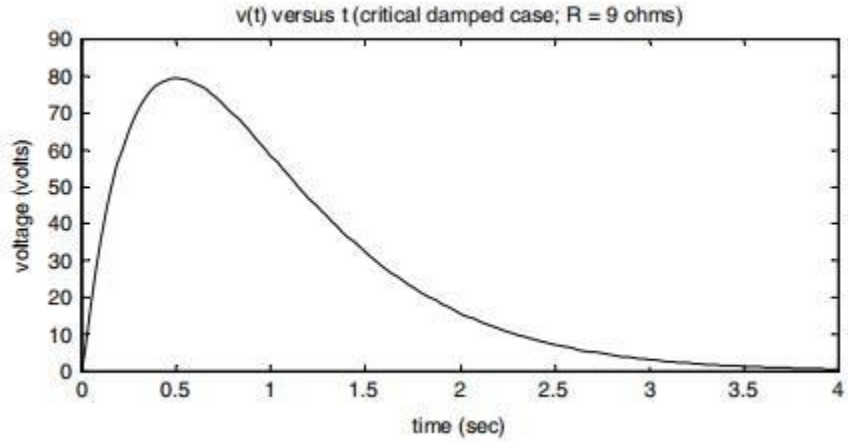


Figure 5.7: Transient critical damped plot ($t > 0$) of the source-free parallelRLC

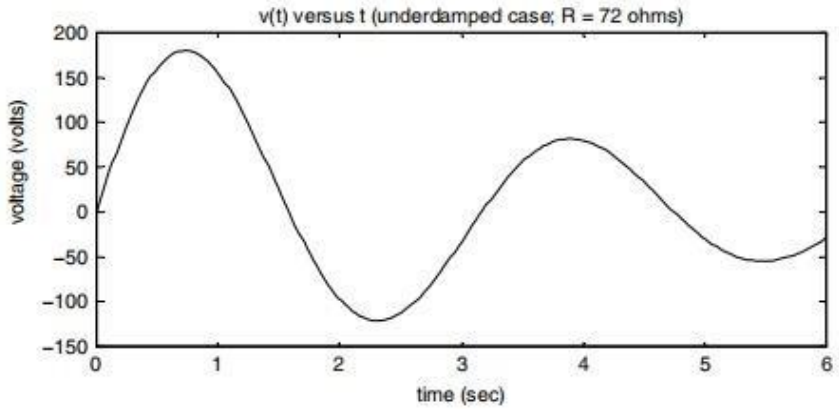


Figure 5.8: Transient under damped plot ($t > 0$) of the source-free parallelRLC

Observe that by changing the value of R (R = 3, 9, and 72 Ω), the three solutions for the second-order differential equation—over, critical, and under damped—are obtained. Observe that the analytical solutions completely agree with the MATLAB solutions

Example5: Steady-state conditions exist in the circuit of Figure 5.9 for$t \leq 0$, while the voltage source $V_1 = 6$ V is connected to the network. At $t = 0^+$, the switch moves downward disconnecting the source while connecting R_1 to the rest of the circuit.

1. Obtain analytically the loop differential equation set and the initialconditions
2. Using the MATLAB symbolic solver, create the script file transient_2loops that returns the transient currents for each loop, andtheir respective plots, for $t \geq 0$
3. Also obtain simplify and *pretty* expressions for each of thetransient loop currentsof part 2.

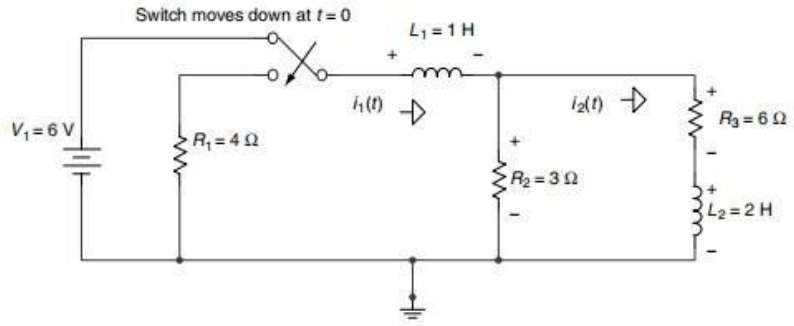


Figure5.9: Network of example

ANALYTICAL Solution

The conditions at $t = 0$ are $i_1(0) = 3$ A and $i_2(0) = 2$ A. Applying KVL to the two-mesh network of Figure 5.9 for $t \geq 0$, results in the following set of simultaneous differential equations:

$$(R_1 + R_2)i_1(t) + L_1\frac{di_1(t)}{di} - R_2 i_2(t) = 0$$

$$-R_2 i_1(t) + (R_3 + R_2)i_2(t) + L_2\frac{di_2(t)}{di} = 0$$

Replacing the elements by their values yields the following set of differential equations:

$$7i_1(t) + \frac{di_1(t)}{di} - 3i_2(t) = 0$$

or

$$\frac{di_1(t)}{di} = -7i_1(t) + 3i_2(t)$$

and

$$-3i_1(t) + 9i_2(t) + 2\frac{di_2(t)}{di} = 0$$

or

$$2\frac{di_2(t)}{dt} = 3i_1(t) - 9i_2(t)$$

MATLAB Solution

```
% Script file : transient _ 2loops
% transient solutions for two loop network
disp('****************************************')
disp('*******R E S U L T S******************')
disp('****************************************')
disp('Solution of the two diff. loop equations for t>0 ')
[y1 y2] = dsolve('Dy1=-7*y1+3*y2,2*Dy2=3*y1- 9*y2','y1(0)=3','y2(0)=2','t');
figure(1)
ezplot (y1,[0 3])
axis([0 1.5 0 3.5])
xlabel ('time (sec)')
ylabel ('current i1(t) in amps')
title ('i1(t) vs t ')
figure(2)
ezplot(y2,[0 3])
axis([0 1.5 0 2.3])
xlabel('time (sec)')
ylabel('current i2(t) in amps')
title('i2(t) vs t ')
disp('****************************************')
disp('The currents i1(t) and i2(t), in amps are :')
[y1 y2]'
disp('****************************************')
disp('The simplify solution for i1(t) is:')
simplify(y1)
disp('****************************************')
disp('The simplify solution for i2(t) is :')
simplify(y2)
disp('****************************************')
disp('The pretty solution for i1(t) is :')
pretty(y1)
disp('****************************************')
disp('The pretty solution for i2(t) is :')
pretty(y2)
disp('****************************************')
```

```
figure(3)
ezplot(y1)
hold on
ezplot(y2)
xlabel('time (sec)')
axis([0 1.0 0 3.5])
ylabel('currents [i1(t)&i2(t)] in amps')
title('i1(t) & i2(t) vs t ')
```

The script file *transient_2loops* is executed, and the results and plots (Figures 2.106 through 2.108) are as follows:

```
>> transient _ 2loops

**************************************************
*******R E S U L T S*****************************
**************************************************
Solution of the two diff. loop equations for t>0
**************************************************
The currents i1(t) and i2(t), in amps are :
ans =
[ conj(-9/194*97^(1/2)*exp(-1/4*(23+97^(1/2))*t)+9/194*97^(1/2)*exp(1/4*(-
23+97^(1/2))*t)+3/2*exp(1/4*(-23+97^(1/2))*t)+3/2*exp(-1/4*(23+97^(1/2))*t))]
[  conj(exp(1/4*(-23+97^(1/2))*t)-14/97*97^(1/2)*exp(-1/4*(23+97^
(1/2))*t)+14/97*97^(1/2)*exp(1/4*(-23+97^(1/2))*t)+exp(-1/4*(23+97^(1/2))*t))]
**************************************************************
The simplify solution for i1(t) is:
ans =
-9/194*97^(1/2)*exp(-1/4*(23+97^(1/2))*t)+9/194*97^(1/2)*exp(1/4*(-23+97^
(1/2))*t)+3/2*exp(1/4*(-23+97^(1/2))*t)+3/2*exp(-1/4*(23+97^(1/2))*t)
**************************************************************************
The simplify solution for i2(t) is :
ans =
exp(1/4*(-23+97^(1/2))*t)-14/97*97^(1/2)*exp(-1/4*(23+97^(1/2))*t)+14/97*97^
(1/2)*exp(1/4*(-23+97^(1/2))*t)+exp(-1/4*(23+97^(1/2))*t)
**************************************************************************
    The pretty solution for i1(t) is :
              1/2                    1/2
    - 9/194 97     exp(- 1/4 (23 + 97    ) t)
              1/2                 1/2                              1/2
    + 9/194 97   exp(1/4 (-23 + 97    ) t) + 3/2 exp(1/4 (-23 + 97    ) t)
                          1/2
    + 3/2 exp(- 1/4 (23 + 97    ) t)
**************************************************************************
    The pretty solution for i2(t) is :
                   1/2              1/2                    1/2
    exp(1/4 (-23 + 97    ) t) -  14/97     exp(- 1/4 (23 + 97    ) t)
              1/2                   1/2                              1/2
    + 14/19 97     exp(1/4 (-23 + 97    ) t) + exp(- 1/4 (23 + 97    ) t)
**************************************************************************
```

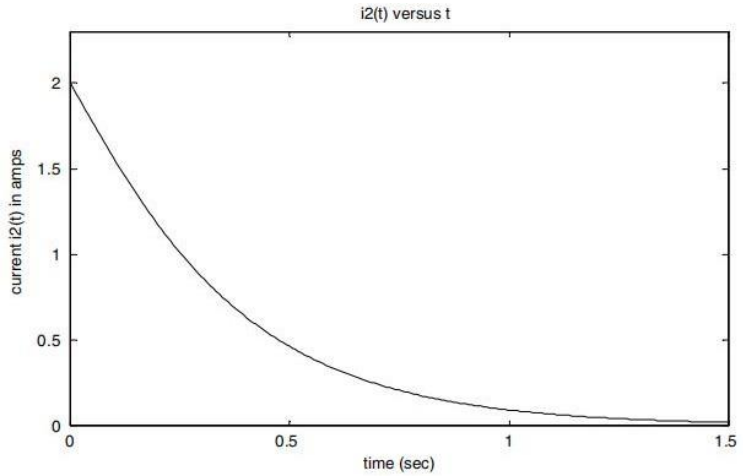


Figure5.10:Plot of I1(t)

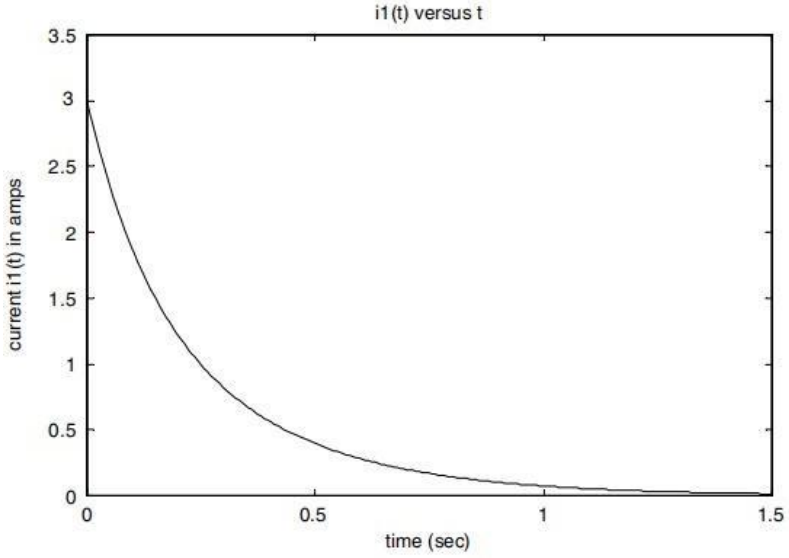


Figure 5.11: Plot of I2(t)

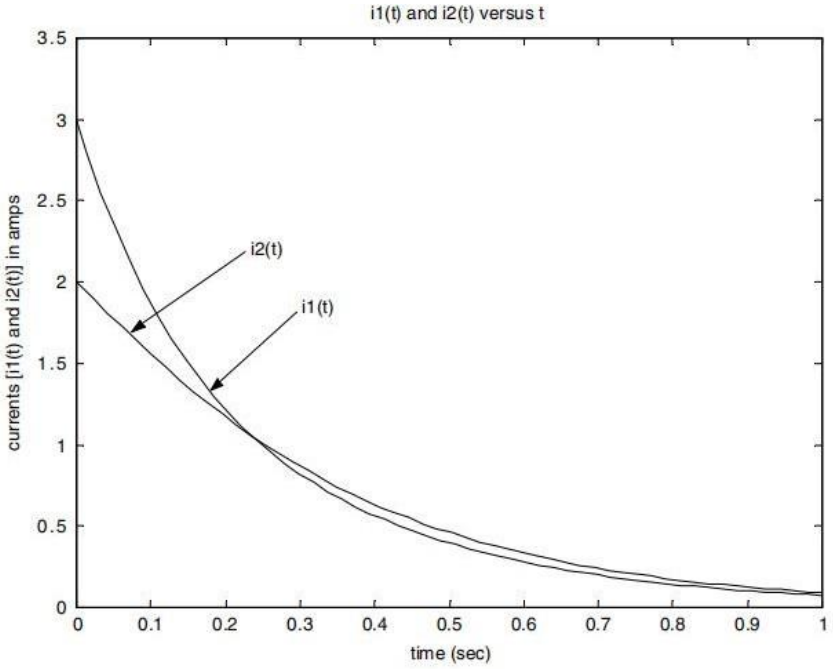


Figure 5.12: Plot of i1(t) and i2(t)

Review Exercise?

a) What is Mathematical Modelling of any type of system?

Mathematical modeling of a control system is the process of drawing the block diagrams for these types of systems in order to determine their performance and transfer functions .

b) What is transient and steady state response of system?

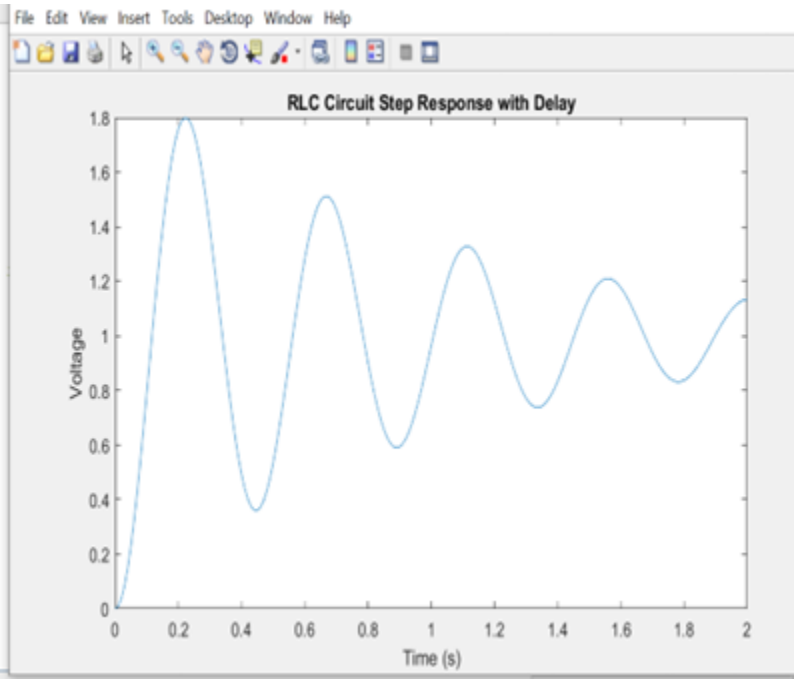
The transient response is the circuit's temporary response that will die out with time. The steady state response is the behavior of the circuit a long time after an external excitation is applied .

c) Why do you get delay in system output response?

Delays can be attributed to acquiring information to make a decision, creating a control decision and/or executing the decisions. For example in the control system of an aircraft delays can be cause by actuators and sensors .

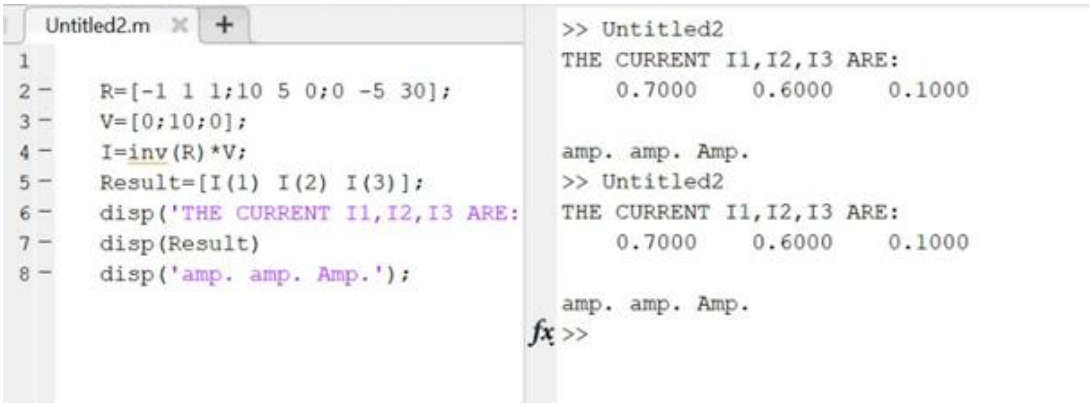
d) Take any RLC circuit by your choice and write the program for that circuit in MATLAB. Include delay in system response and explain the behavior of system?

```
R = 1;
L = 0.5;
C = 0.01;
num = [1];
den = [L*C, R*C, 1];
sys = tf(num, den);
delay = 0.2;
sys_with_delay = pade(sys, delay); % pade command
t = 0:0.01:2;
u = ones(size(t)); % Step input
y = lsim(sys_with_delay, u, t);
figure;
plot(t, y);
title('RLC Circuit Step Response with Delay');
xlabel('Time (s)');
ylabel('Voltage');
```

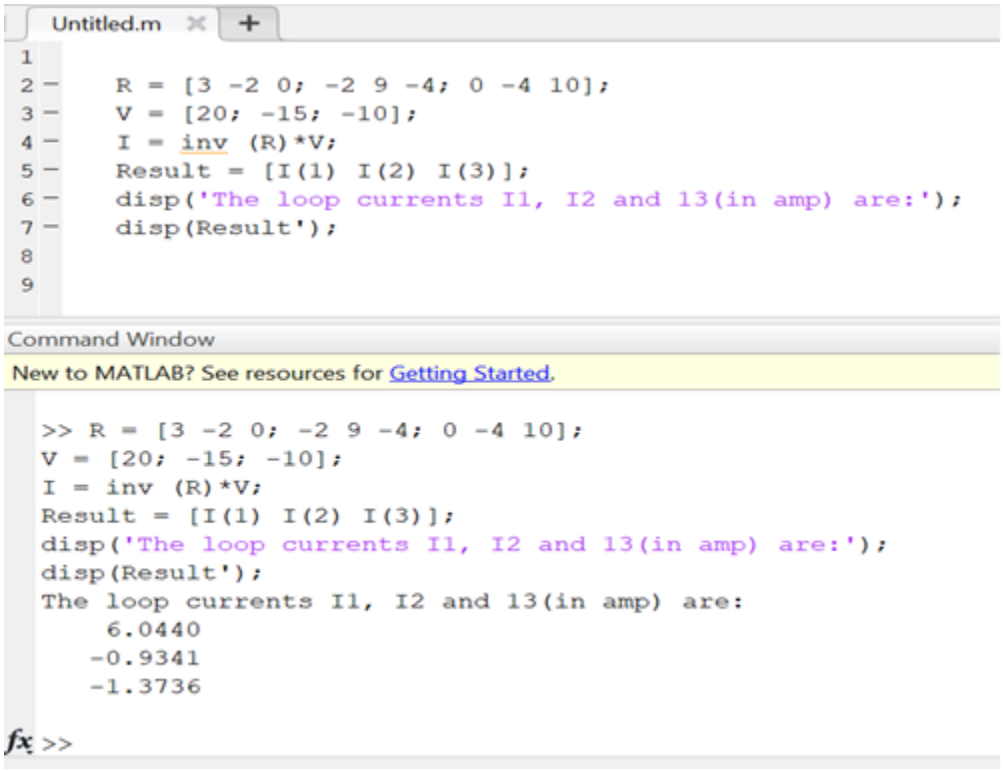


e) Attach the screenshots of example1 to example5 (including codes and Outputplots/results)

Example: 1

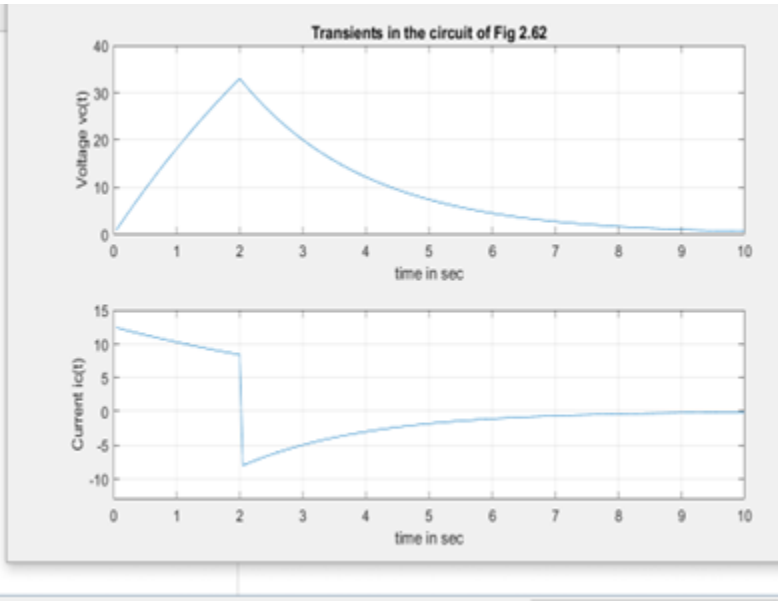


Example: 2



Example:3

```
V = 100; R1 = 7; R2 = 1; R3 = 3; C = 0.5;
taul = (R1 + R3) * C;
tau2 = (R2 + R3) * C;
for k = 1:40
    t(k) = k/20; % 0 < t < 2
    v(k) = V * (1 - exp(-t(k)/taul));
    i(k) = (V - v(k))/(R1 + R2);
end
Vmax = 32.9680;
for k = 41:200
    t(k) = k/20; % 2 < t < 10
    v(k) = Vmax * exp(-(t(k) - t(40))/tau2);
    i(k) = -v(k)/(R2 + R3);
end
subplot(2, 1, 1)
plot(t, v)
axis([0 10 0 40]);
title('Transients in the circuit of Fig 2.62')
xlabel('time in sec'), ylabel('Voltage vc(t)')
grid on
subplot(2, 1, 2)
plot(t, i)
axis([0 10 -13 15]);
xlabel('time in sec'), ylabel('Current ic(t)')
```



Example: 4

```
disp('****RESULTS****')
% Overdamped case (R = 3 ohms)
disp('Source free parallel RLC circuit')
disp('Solution of diff. equation for R=3 ohms')
overdamped = dsolve('(1/36)*D2y+(1/3)*Dy+(1/9)*y=0',' y(0)=0','Dy(0)=32','t');
figure(1)
ezplot(overdamped)
xlabel('time (sec)');
ylabel('voltage (volts)');
title('v(t) vs t (overdamped case; R=3 ohms)')
axis([0 3 0 40]);
% Critically damped case (R = 9 ohms)
disp('Solution of diff. equation for R=9 ohms')
criticaldamped = dsolve('(1/36)*D2y+(1/9)*Dy+(1/9)*y=0','y(0)=0','Dy(0)=432','t');
simple_crit = simplify(criticaldamped);
figure(2)
ezplot(criticaldamped)
xlabel('time (sec)');
ylabel('voltage (volts)');
title('v(t) vs t (criticaldamped case; R=9 ohms)')
axis([0 4 0 90]);
% Underdamped case (R = 72 ohms)
disp('Solution of diff. equation for R=72 ohms')
underdamped = dsolve('(1/36)*D2y+(1/72)*Dy+(1/9)*y=0', 'y(0)=0', 'Dy(0)=432','t');
figure(3)
ezplot(underdamped)
xlabel('time (sec)');
ylabel('voltage(volts)')
title('v(t) vs t (underdamped case; R=72 ohms)')
axis([0 6 -150 200]);
```

Command Window

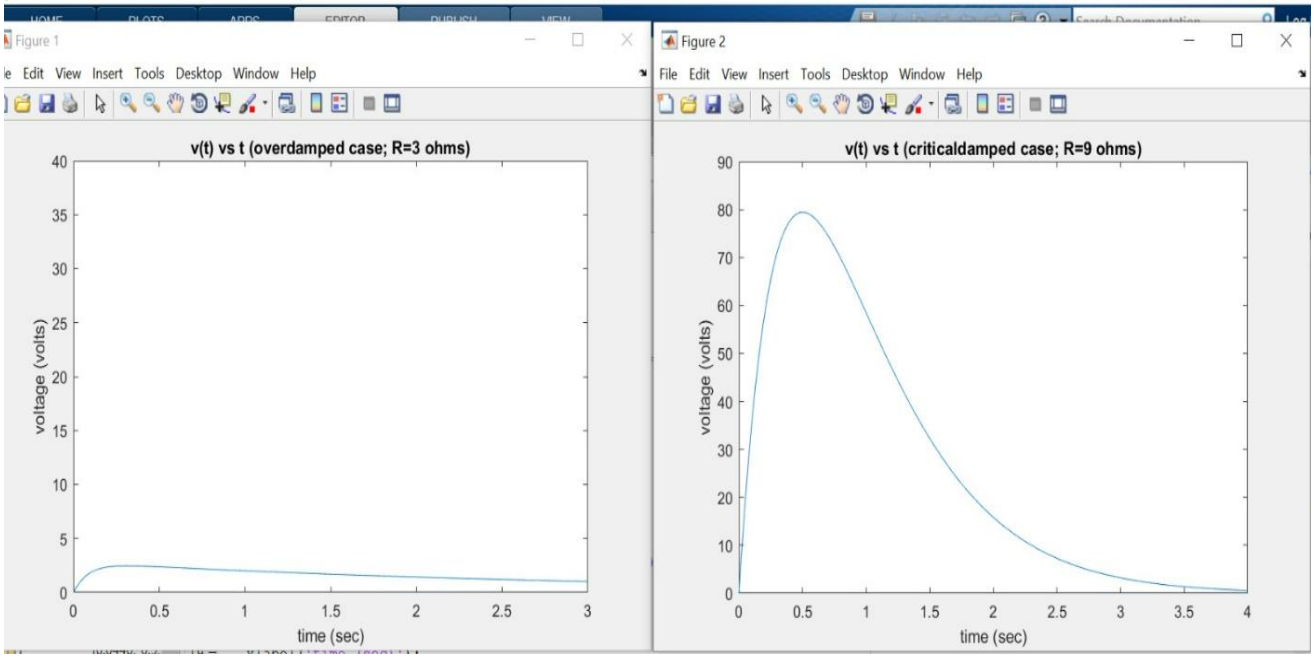


Figure 1
File Edit View Insert Tools Desktop Window Help
v(t) vs t (overdamped case; R=3 ohms)
voltage (volts)
time (sec)
Figure 2
File Edit View Insert Tools Desktop Window Help
v(t) vs t (criticaldamped case; R=9 ohms)
voltage (volts)
time (sec)

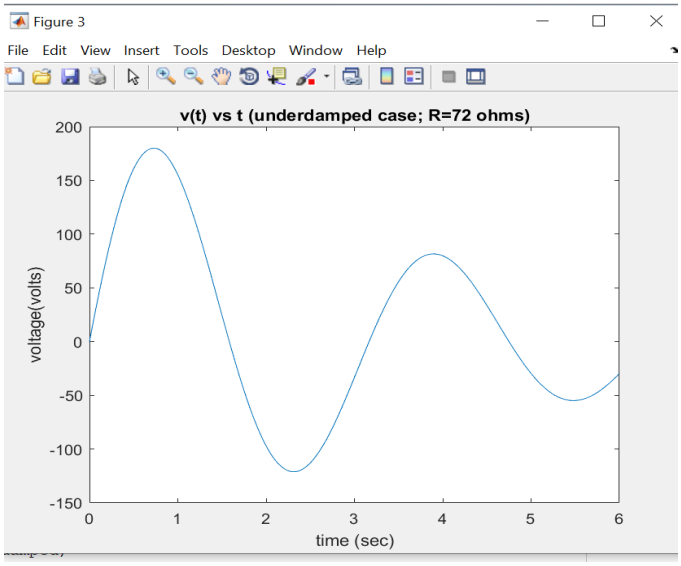
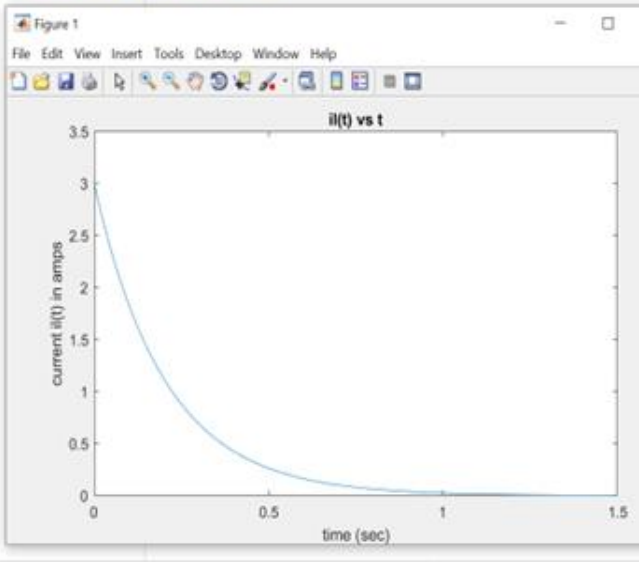
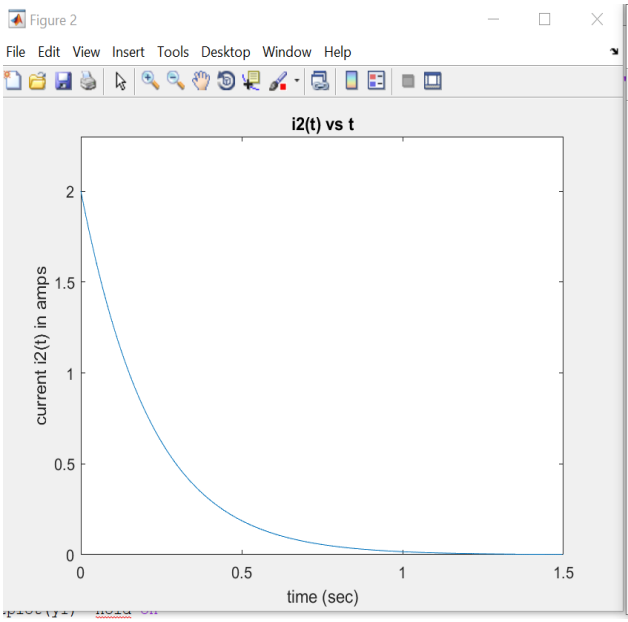


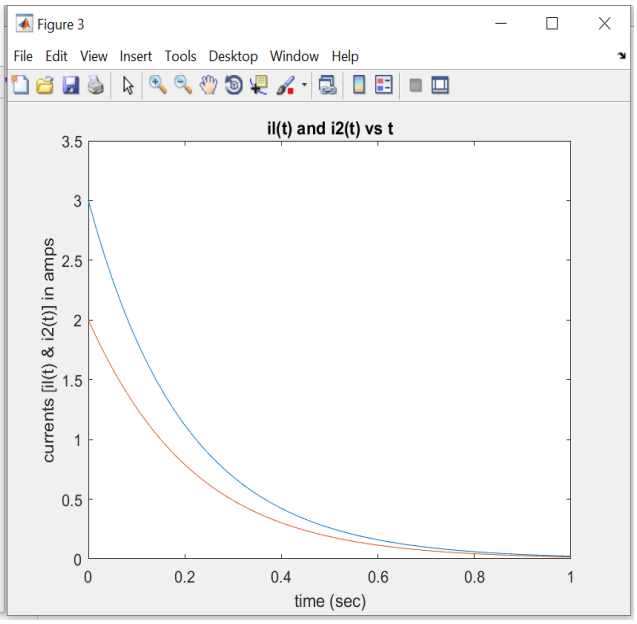
Figure 3
File Edit View Insert Tools Desktop Window Help
v(t) vs t (underdamped case; R=72 ohms)
voltage(volts)
time (sec)

Example: 5

```
disp('**RESULTS**')
[y1, y2] = dsolve('Dy1 = -7*y1 + 3*y2', 'Dy2 = 3*y1 - 9*y2', 'y1(0) = 3', 'y2(0) = 2', 't');
figure(1)
ezplot(y1, [0 3])
axis([0 1.5 0 3.5])
xlabel('time (sec)')
ylabel('current i1(t) in amps')
title('i1(t) vs t')
figure(2)
ezplot(y2, [0 3])
axis([0 1.5 0 2.3])
xlabel('time (sec)')
ylabel('current i2(t) in amps')
title('i2(t) vs t')
disp('The currents i1(t) and i2(t) in amps are:')
[y1 y2]
disp('The simplified solution for i1(t) is:')
simplify(y1)
disp('The simplified solution for i2(t) is:')
simplify(y2)
disp('The pretty solution for i1(t) is:')
pretty(y1)
disp('The pretty solution for i2(t) is:')
pretty(y2)
figure(3)
ezplot(y1)  hold on
ezplot(y2)
xlabel('time (sec)')
axis([0 1.0 0 3.5])
```







MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 06: THE STEP RESPONSE OF DISCRETE TIME SYSTEM AND EFFECTOF SAMPLING TIME ON SYSTEM RESPONSE

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	To analyze the step response of a discrete time system and effect of samplingtime on system response	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

LAB RUBRIC(S):

Equipment/software Required: Computer with MATLAB

Approximate Time Required: Two to three hours

Introduction

Absolute stability is a basic requirement of all control systems. Apart from that, good relative stability and steady state accuracy are also required in any control system, whether continuous time or discrete time. Transient response corresponds to the system closed loop poles and steady state response corresponds to the excitation poles or poles of the input function. In many practical control systems, the desired performance characteristics arespecified in terms of time domain quantities. Unit step input is most commonly used in analysis of a system since it is easy to generate and represent a sufficiently drastic changethus providing useful information on both transient and steady state responses. The transient response of a system depends on the initial conditions. It is a common practice to considerthe system initially at rest. Consider the digital control system shown in Figure-5.1

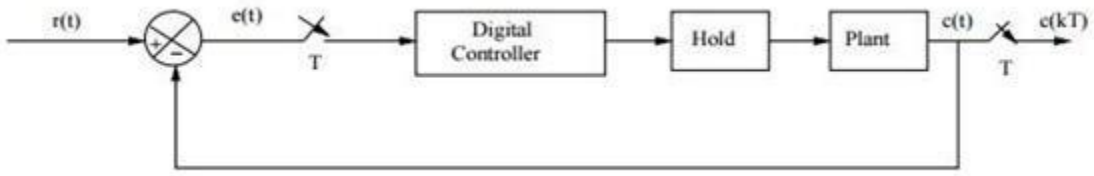


Figure-6.1: Block diagram of a digital control system

Similar to the continuous time case, transient response of a digital control system can also be characterized by the following.

1. Rise time (tr): Time required for the unit step response to rise from 0% to 100% of its final value in case of under damped system or 10% to 90% of its final value in case of overdamped system.
2. Delay time (td): Time required for the the unit step response to reach 50% of its final value.
3. Peak time (tp): Time at which maximum peak occurs.
4. Peak overshoots (Mp): The difference between the maximum peak and the steady state value of the unit step response.
5. Settling time (ts): Time required for the unit step response to reach and stay within 2% or 5% of its steady state value.

However since the output response is discrete the calculated performance measures may be slightly different from the actual values as shown in Figure-5.2. The output has a maximumvalue cmax whereas the maximum value of the discrete output is c ∗ max which is always less than or equal to cmax. If the sampling period is small enough compared to the oscillations of the response then this difference will be small otherwise c ∗ max may be completely erroneous.

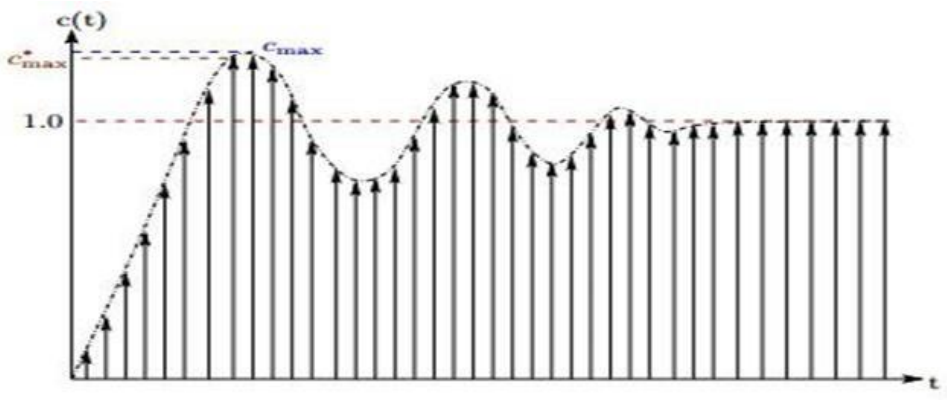


Figure-6.2: Unit step response of discrete time system

6.1 Example1: Consider a unity feedback control system having forward path transfer function

$$G(s)=\frac{1}{s^2+s}$$

Determine (i) step response in continuous and discrete domain.

(ii) effect of sampling time on system response

Matlab Code:

```
n=[1]
d=[1 1 0]
sys=tf(n,d)
sysz=c2d(sys,1,'zoh')
sysc=feedback(sys,1)
syscz=feedback(sysz,1)
step(syscz,'b')
hold on
sysz1=c2d(sys,.5,'zoh')
syscz2=feedback(sysz1,1)
step(syscz2,'g')
hold on
sysz2=c2d(sys,.10,'zoh')
syscz3=feedback(sysz2,1)
step(syscz3,'m')
hold on
```

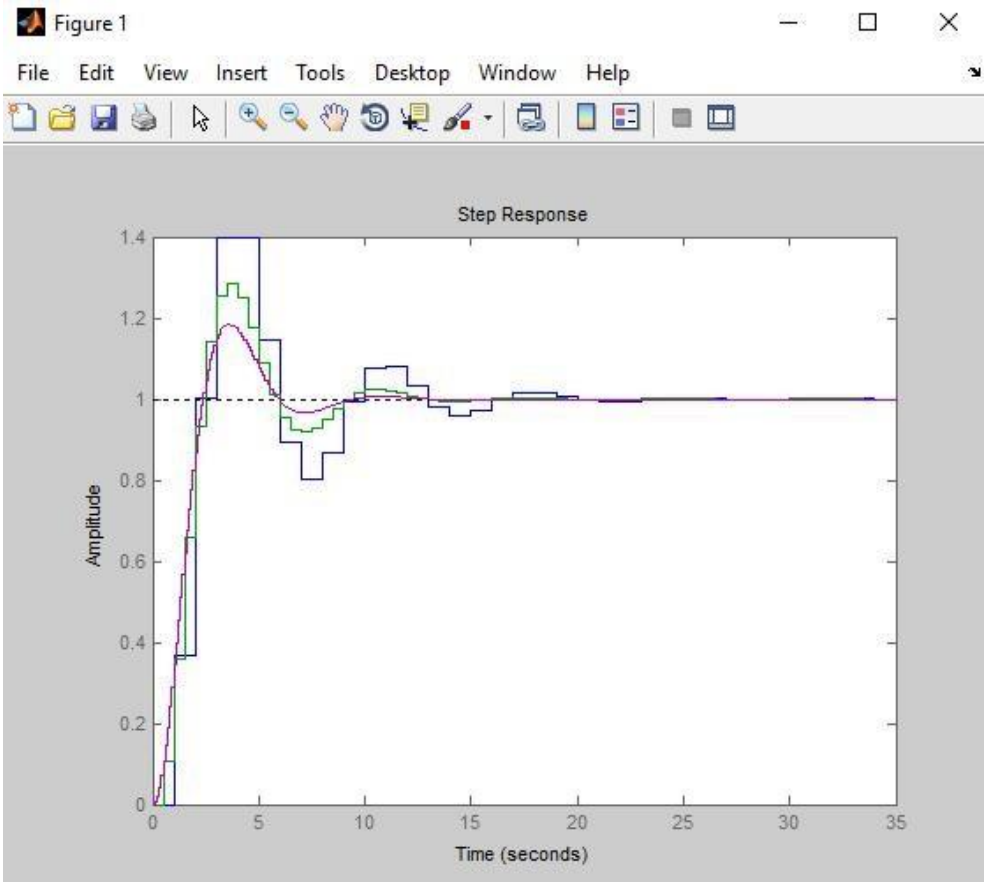


Figure-6.3: Sampling response of example1 at different sampling time

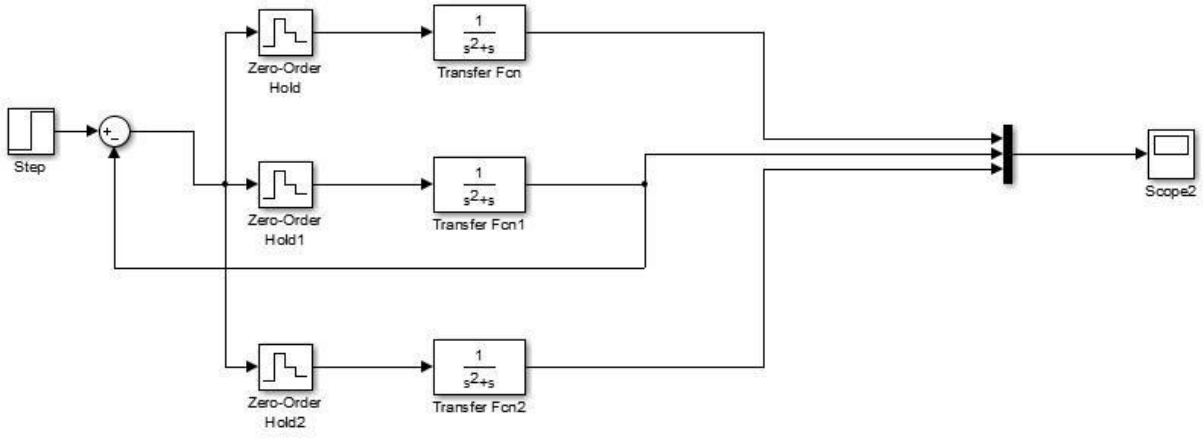


Figure-6.4: Simulink model of step response of system at different sampling time

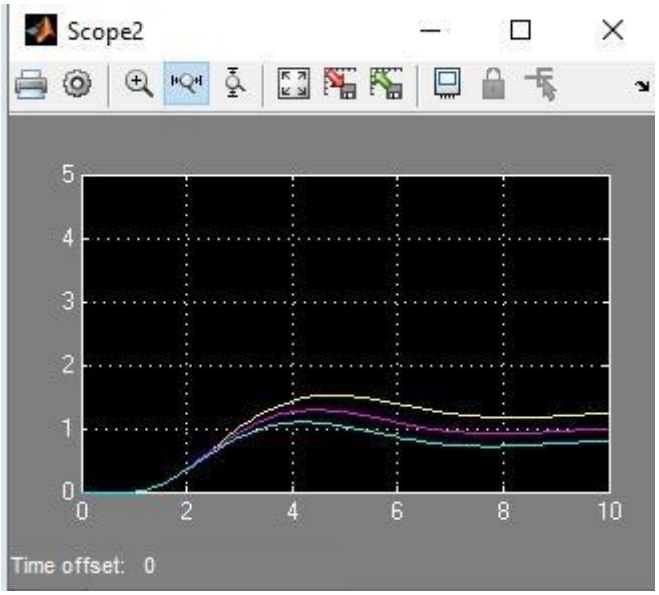


Figure-6.5: Output response of simulink model design

6.2 Example2: Consider a unity feedback control system having forward path transfer function

$$G(s)=\frac{2}{s^2+3s+1}$$

Determine (i) step response in continuous and discrete domain.

(ii) effect of sampling time on system response

Matlab Code:

```
n=[2 ];
d=[1 3 1];
sys=tf(n,d);
sysz=c2d(sys,1,'zoh');
sysc=feedback(sys,1);
syscz=feedback(sysz,1);
step(syscz,'b');
hold on;
sysz1=c2d(sys,.5,'zoh');
syscz2=feedback(sysz1,1);
step(syscz2,'g');
hold on;
sysz2=c2d(sys,.10,'zoh');
syscz3=feedback(sysz2,1);
step(syscz3,'m');
hold on;
```

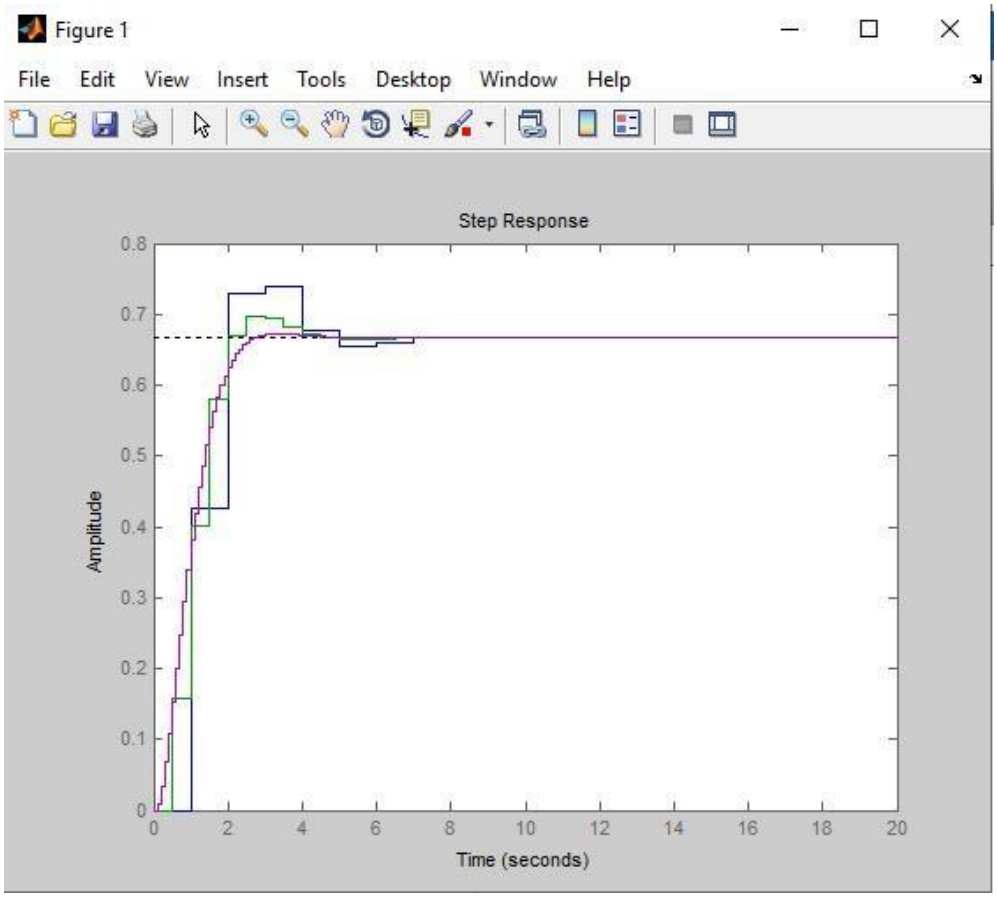


Figure-6.6: Sampling response of example2 at different sampling time

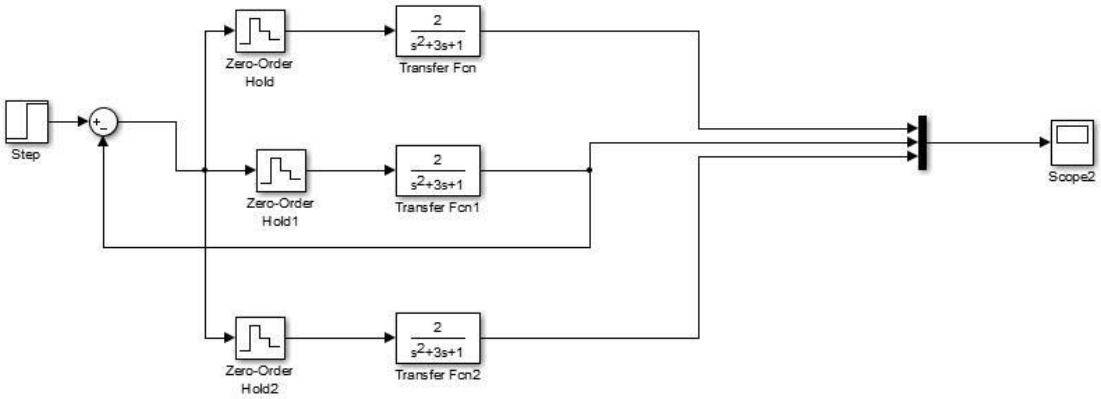


Figure-6.7: Simulink model of step response of system at different sampling time

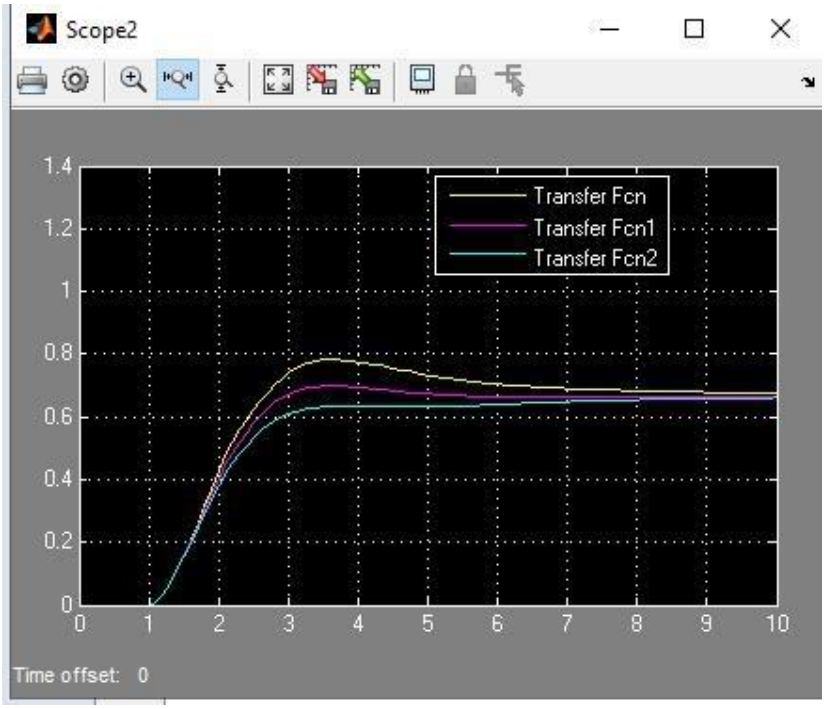


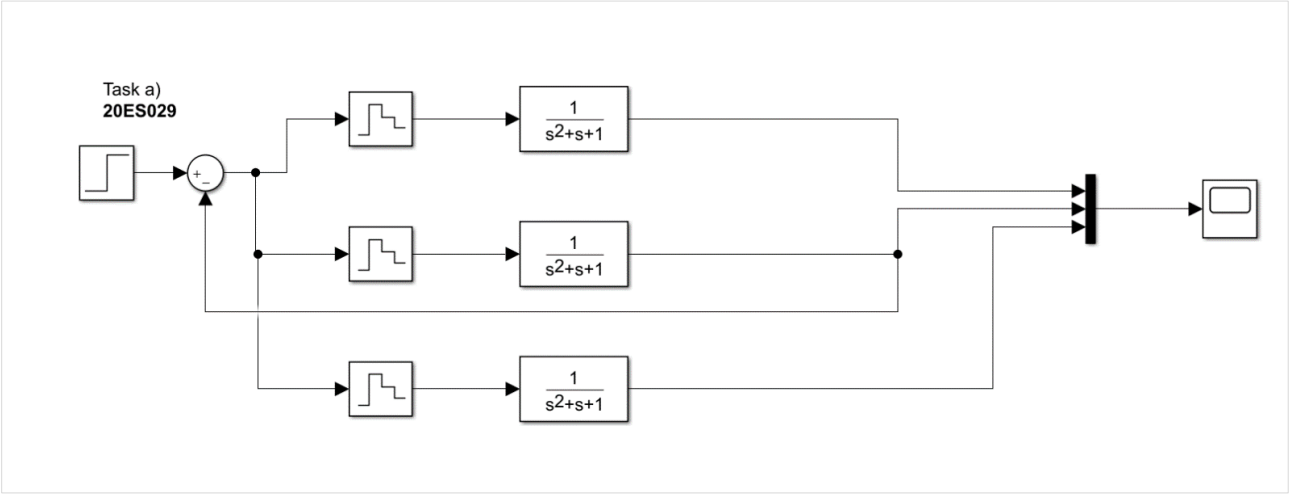
Figure-6.8: Output response of simulink model design

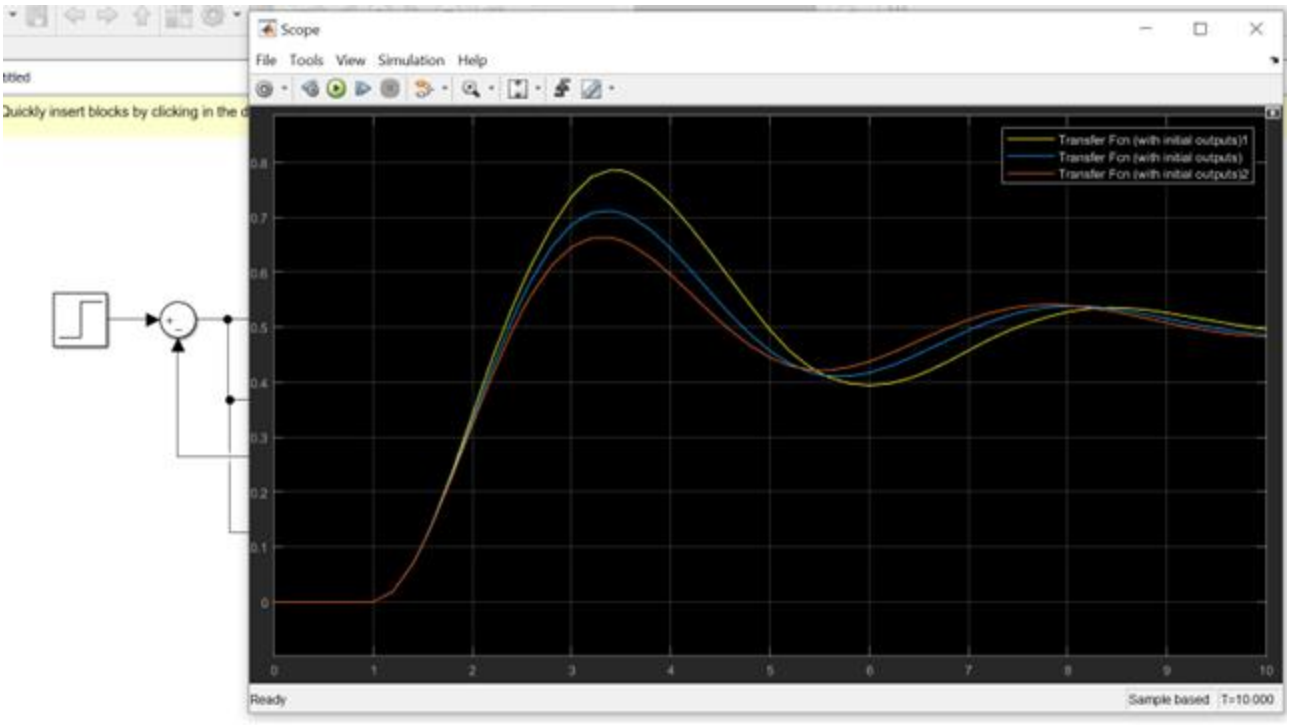
6.3 Review Exercise

a) Build Simulink model to obtain step response of a unity feedback system whose closed loop transfer function is given by:

$$Tts) = \frac{1}{s^2 + s + 1}$$

Also show effect of sampling time on time response specification parameters.





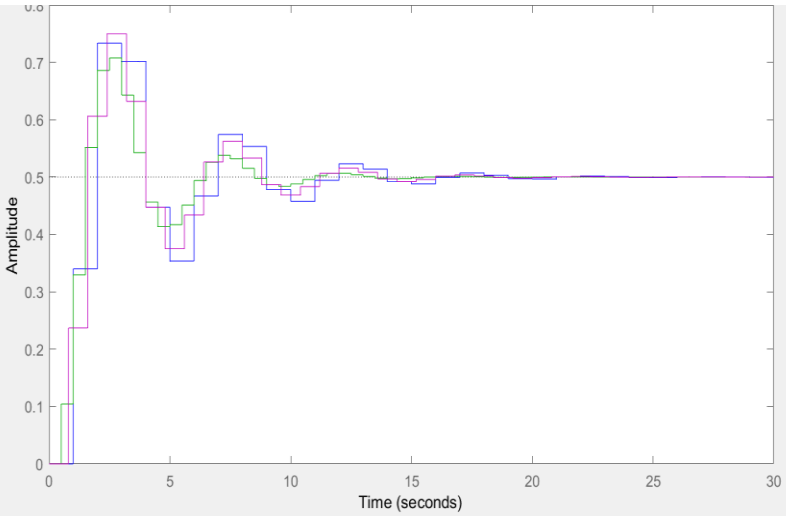
b) Write down MATLAB code to obtain step response of a unity feedback system having forward path transfer function

$$G(s) = \frac{1}{s^2 + s + 1}$$

```
n=[1]
d=[1 1 1]
sys=tf(n,d)
sysz=c2d(sys,1,'zoh')
sysc=feedback(sys,1)
syscz=feedback(sysz,1)
step(syscz,'b')
hold on
sysz1=c2d(sys,.5,'zoh')
syscz2=feedback(sysz1,1)
step(syscz2,'g')
hold on
sysz2=c2d(sys,.8,'zoh')
syscz3=feedback(sysz2,1)
step(syscz3,'m')
hold on
```

```
n=[1]
d=[1 1 1]
sys=tf(n,d)
sysz=c2d(sys,1,'zoh'
sysc=feedback(sys,1)
syscz=feedback(sysz,
step(syscz,'b')
hold on
sysz1=c2d(sys,.5,'zo
syscz2=feedback(sysz
step(syscz2,'g')
hold on
sysz2=c2d(sys,.8,'zo
syscz3=feedback(sysz
step(syscz3,'m')
hold on
```

```
ommand Window
ew to MATLAB? See resources for Getti
Sample time: 0.8 seconds
Discrete-time transfer fu
>>
```



c) Briefly illustrate the effect of sampling time on time response specification parameters with the aid of simulation results?

Shorter sampling times in a sample and hold circuit mean it can quickly grab information about the input signal. This helps in capturing the signal accurately and making the output settle faster. However, if the sampling time is too short, the circuit may become more sensitive to small timing errors, called jitter, which can affect accuracy. Also, longer sampling times can lead to the stored voltage (held value) dropping more quickly. In fast situations, short sampling times are needed for quick and accurate responses .

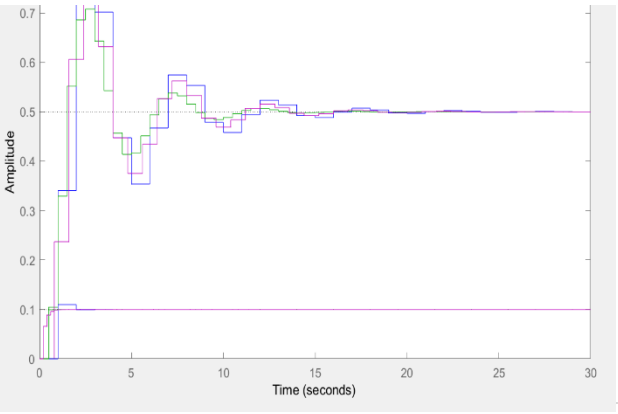
d) How practical sample and hold circuit works?

A practical sample and hold circuit samples an analog signal during a short period, holds that value, and repeats the process at regular intervals, providing a way to digitize continuous signals. The sample and hold circuit is often used in analog-to-digital converters (ADCs). It allows the conversion of continuous analog signals into discrete digital values by capturing and holding the analog voltage at specific points in time .

e) Take different systems transfer function. Perform simulation of all systems and explain the behavior of all the systems. Include your simulations and resulting plots.

```
n=[1]
d=[0 2 9]
sys=tf(n,d)
sysz=c2d(sys,1,'zoh')  %Samp tim =1
sysc=feedback(sys,1)
syscz=feedback(sysz,1)
step(syscz,'b')
hold on
sysz1=c2d(sys,.5,'zoh') %Samp tim =0.5
syscz2=feedback(sysz1,1)
step(syscz2,'g')
hold on
sysz2=c2d(sys,.2,'zoh') %Samp tim =0.2
syscz3=feedback(sysz2,1)
step(syscz3,'m')
hold on
```

```
Command Window
New to MATLAB? See resources for Getting Started.
Discrete-time transfer function.
```



MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 07: The sample response analysis and the computer simulation of a sampled-data system

Name: KARAN	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	To perform the sample response analysis and the computer simulation of a sampled-data system using MATLAB	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment/software Required: Computer with MATLAB software

Approximate Time Required: Two to three hours

Introduction

A typical sampled-data system is shown in Figure-7.1, where *S* and *H* represent the Sampler (A/D converter) and the Holder (D/A converter) respectively.

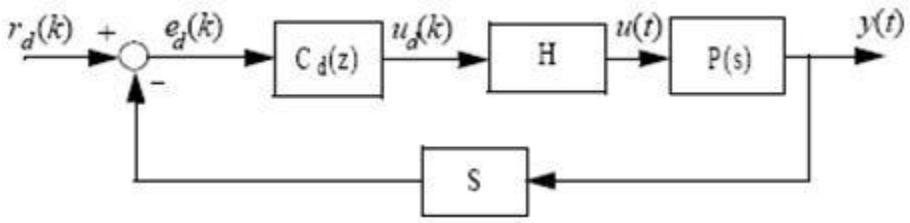


Figure-7.1 Sampled-Data System

Unlike a pure analog system (that has an analog controller and the continuous-time plant), or a pure discrete-time system (with a digital controller and the discretized model of the plant), the sampled-data system has both the digital controller and the continuous-time plant. The signals passing through the entire system is a mixture of sampled data and continuous time signals. Most control system simulation software packages such as MATLAB only have functions for continuous-time and discrete-time simulations, e.g., lsim, step, and dstep, dlsim, etc., but none for sampled data simulation. This lab is to write a general MATLAB program (function) to simulate the step response of a sampled-data (digital) control system.

Experiment Procedure

The analog plant $P(s)$ that we considered in this lab is a flexible beam system:

$$P(s) = \frac{1.6188s^2 - 0.1575s - 43.9425}{s^4 + 0.1736s^3 + 27.9001s^2 + 0.0186s}$$

Assume that you have designed an advanced digital controller for this complex system (at sampling period T = 0.5 sec), i.e. as in Figure-7.1:

$$C_d(z) = \frac{-0.1084z^5 - 0.01202z^4 + 0.1708z^3 + 0.08469z^2 - 0.09198z - 0.04313}{z^6 - 0.6528z^5 - 0.8377z^4 + 0.4495z^3 + 0.4709z^2 - 0.5820z + 0.1521}$$

Before we can implement this digital controller on-line and perform real-time control on the real flexible beam system, first of all, we need to test the control system performance in a computer environment. This can be achieved by sampled-data (digital) simulation.
The following steps describe the procedure for simulating the step response of the sampled data system using existing MATLAB functions:

1. Define the models of the digital controller and the analog plant, e.g.
   sysC d = tf(numC d, denC d, T) %Create a discrete-time transfer function with sampling period T 髣$T = 0.5$ 鼠
   sysP = tf(numP,denP)

2. Discretize the continuous-time plant P髣s鼠 to obtain the ZOH equivalent $P_z d$髣z鼠髣P_d髣z鼠 $= SP$髣s鼠H 鼠 via the MATLAB function c2dThe

discretized system is shown in Figure-7.2:

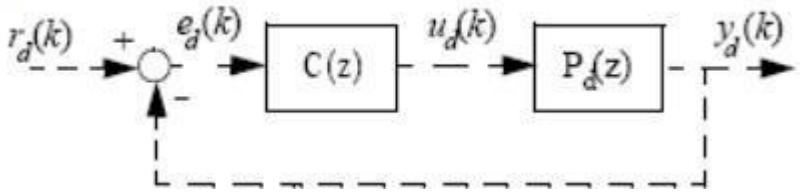


Figure-7.2: The Discretized System

Here r_d is the sampled input sequence, which is a discrete-time unit step. Figure-7.2 is the exact model of Figure-7.1 at sampling instants.

3. Obtain the control signal sequence u_d. First of all, compute the transfer function from r_d to u_d in Figure-7.2,

$$G_1(z) = \frac{U_d(z)}{R_d(z)} = \frac{C_d(z)}{1 + C_d(z)P_d(z)}$$

   G1 = feedback(sysC_d, sysP_d)

   and then compute the discrete-time control sequence u_d via dstep, e.g.

   [numG1,denG1] = tfdata(G1, ‘v’)

   U_d = dstep(numG1,denG1,N)

   Note: N is the number of samples to be simulated

4. Simulate zero-order-hold, i.e., compute the continuous-time control input u after the ZOH in Figure-8.1 ($u = Hu_d$). Suppose we would like to compute n inter-sample points for every sampling period (of length T), the time increment (step size) in u is then T/n. Thus the vector u is obtained by holding each value of (u_d for n time, i.e.

$$u = [u_d(1)u_d(1)...u_d(1)u_d(2)...u_d(2).....u_d(N)...u_d(N)]$$

5. With input u to $P(s)$ obtained from above, the output $y(t)$ of $P(s)$ in Figure-7.1 can finally be computed by continuous-time simulation, e.g. lsim.

(The obtained $y(t)$ is in fact the computer approximation of the actual continuous-time output in the real system. It should be a good approximation because 10 points of the inter-sample response are calculated during each sampling interval T.)
Finally, the above steps can be integrated into a general function named sdstep (or choose your own preferred name) in MATLAB:

[y,u,t] = sdstep(numC_d,denC_d,numP,denP,T,N,n)

For this function, the user can define the model of the digital controller in *numC_d* and *denC_d*, and the model of the continuous-time plant in *numP* and *denP*. *T* and *N* are sampling period and number of samples to be simulated, respectively; and the integer n is the ratio of sampling period T to the time increment $T1$ used when calculating the sample response. The returned variables of the above function are the output from continuous-time plant $P(s)$, $y(t)$ (see Figure-7.1, and the corresponding input $u(t)$ and the time $vector\ t$.

Practical Analysis of sampling and reconstruction

Example1: Let $f(t) = te^{-1000t}\,u(t)$
Create the script file sample_data that returns the following plots:

1. f(t) versus t, over the range $0 \le t \le 5$ ms
2. f(nT) versus nt, for T = 0.2 ms, over the range $1 \le n \le 26$
3. Reconstruct f(t) from f(nT) using
4. Summations of sinc functions (emulating a low-pass filter)
5. Stair function
6. Spine function
7. error(t) versus t, for the reconstruction process when a low-pass filter is used, where

$$error(t) = \left| f(t) - \sum_{n=1}^{N=26} fn(nT_s)\,\mathrm{sinc}[F_s(t - nT_S)] \right|, \quad \text{for } F_S = \frac{1}{T_S}$$

MATLAB Solution

```
% Script file: sample _ data
t = 0:0.0005:0.005;          %  26 time samples
fa = t.*exp(-1000.*t);

figure(1)
subplot(2,1,1)
plot(t,fa);title('f(t) vs t')
ylabel('Amplitude');xlabel('time t (msec)');
subplot(2,1,2)
Ts = 0.0002; n = 0:1:25; Fs=1/Ts;
nTs = n*Ts;
fd = nTs.*exp(-1000*nTs);
stem(nTs,fd);hold on;
plot(nTs,fd);
title('f(t) sample with Ts=.2msec.');
ylabel('Amplitude'); xlabel('time index n (msec)')
```

```
figure(2)                    % reconstructions
f25 = 0;
for k = 1:1:26;
    fr25 = fd(k)*sinc(Fs*(t-k*Ts));
    f25 = f25+fr25;
end
subplot(3,1,1)
plot(t,f25,'ko-',t,fa,'ks-.'); legend('sinc-reconst','f(t)');
title('Reconstruction of f(t) ')
ylabel('Amplitude')
subplot(3,1,2)
stem(nTs,fd);hold on;
stairs(nTs,fd); ylabel('Amplitude');legend('stairs')
subplot(3,1,3)
y = spline(nTs,fd,t);
plot(nTs(1:2.5:26.5),y); legend('spline');
ylabel('Amplitude')
xlabel('time (sec)')

figure(3)
error=abs(fa-f25);
plot(t,error);ylabel('magnitude')
title('error(t) = abs [f(t) - Reconstruction (Sums(sinc))]')
xlabel('time(sec)')
```

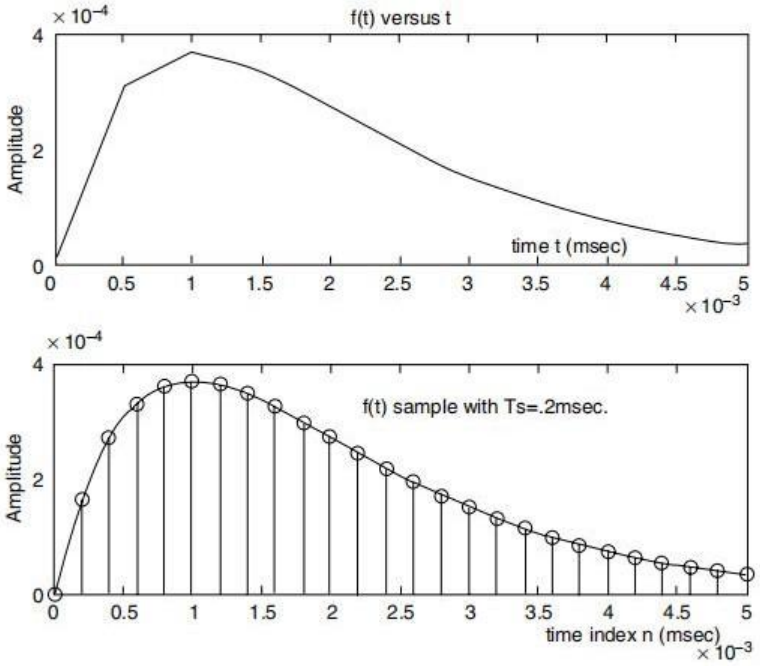


Figure-7.3: Plot of F(t) anf F(nT)

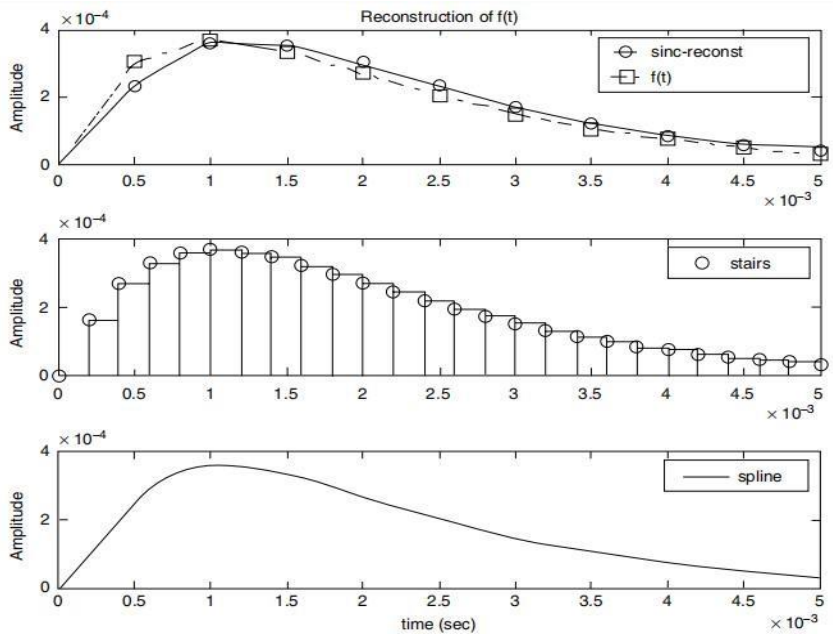


Figure-7.4: Reconstruction plot for F(t)

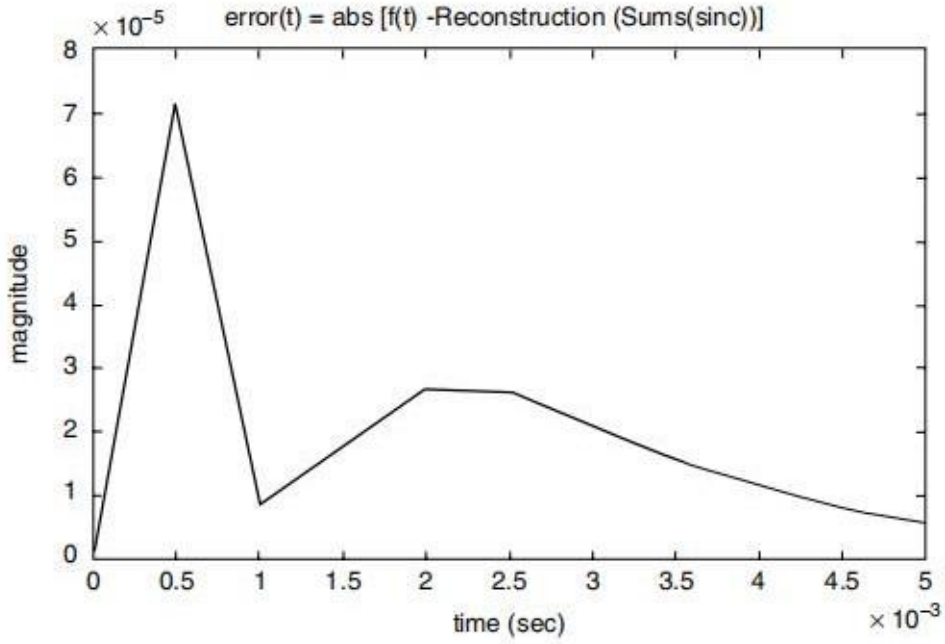


Figure-7.5: Reconstruction plot error for F(t)

Example2: Create the script file up_down_samples that returns the approximationplots of the fol-lowing discrete time function:

$$f(n) = \cos(2\ 0.05n) + 2\sin(2\ 0.03n)$$

for a length of N = 100 samples, for the following cases:

1.Down-sample or decimate the sequence f(n) with the integer factors of M = 2, 4, and 10.
2.Up-sample or interpolate the sequence f(n) with the integer factors of L = 2, 4, and 10.
3.Resample f(n) by the ratio of the two integers given by L/M, for the following cases: L = 3, M = 2, and L = 2, M = 3.

MATLAB Solution

```
% Script file: up_down_samples
n = 0:1:99;
f = cos(2*pi*.05*n)+2*sin(2*pi*.03*n);
yzero = zeros(1,500);


figure(1);                          % down-sample with M=2, 4, 10
subplot(2,2,1)
stem(n,f);hold on; plot(n,f,n,yzero(1:100))
title('f(n) vs n'); ylabel('Amplitude');
xlabel('time index n');
subplot(2,2,2)
gm2 = decimate(f,2,'fir');
nm2 = 0:100/2-1;
stem(nm2,gm2(1:50)); hold on; plot(nm2,yzero(1:50));
title('f(n) is decimated with M=2'); ylabel('Amplitude');
xlabel('time index n');
subplot(2,2,3)
gm4 = decimate(f,4,'fir');
nm4 = 0:100/4-1;
stem(nm4,gm4(1:25));hold on;
plot(nm4,gm4(1:25),nm4,yzero(1:25));
ylabel('Amplitude'); xlabel('time index n');
title('f(n) is decimated with M=4')
subplot(2,2,4)
gm10 = decimate(f,10,'fir');
nm10=0:100/10-1;
stem(nm10,gm10(1:10));hold on;
plot(nm10,gm10(1:10),nm10,yzero(1:10));
```

```
ylabel('Amplitude');xlabel('time index n')
title('f(n) is decimated with M=10')

figure(2);                          % up-sample with L = 2, 4, 10
subplot(2,2,1)
stem(n,f); hold on; plot(n,f,n,yzero(1:100))
title('f(n) vs n');ylabel('Amplitude');
xlabel('time index n');
subplot(2,2,2)
gL2 = interp(f,2);
nL2 = 0:100*2-1;
stem(nL2,gL2(1:200));hold on; plot(nL2,yzero(1:200));
title('f(n) is upsampled with L=2'); ylabel('Amplitude');
xlabel('time index n');
subplot(2,2,3)
gL4 =  interp(f,4);
nL4 = 0:100*4-1;
stem(nL4,gL4(1:400)); hold on;
plot (nL4,yzero(1:400));
ylabel('Amplitude');xlabel('time index n');
title('f(n) is upsampled with L=4')
subplot(2,2,4)
gL10= interp(f,10);
nL10 = 0:100*10-1;
stem(nL10,gL10(1:1000));hold on;
ylabel('Amplitude'); xlabel('time index n')
title('f(n) is upsampled with L=10')

figure(3)
-
                % re-sample by ratio of L=3/M=2 & L=2/M=3
subplot(3,1,1)
stem(n,f);hold on; plot(n,f,n,yzero(1:100))
title('f(n) vs n');
ylabel('Amplitude'); xlabel('time index n');
subplot(3,1,2)
gr32= resample(f,3,2);
nr32 = 0:100*3/2-1;yzeros = zeros(1,length(nr32));
stem(nr32,gr32(1:100*3/2));hold on;plot(nr32,yzeros);
ylabel('Amplitude'); xlabel('time index n');
subplot(3,1,3)
gr23 = resample(f,2,3);
nr23 = 0:100*2/3-1;
yzero = zeros(1,length(nr23));
stem(nr23,gr23(1:100*2/3));
hold on;plot(nr23,yzero);
axis([0 65 -5 5]);
ylabel('Amplitude'); xlabel('time index n')
```

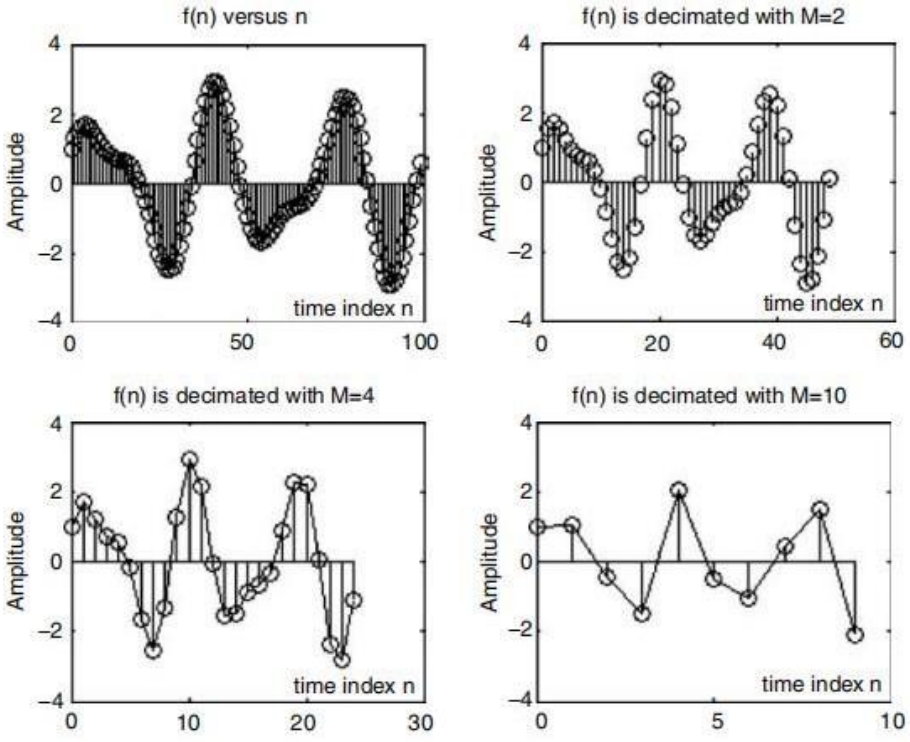


Figure-7.6: Decimation plot of F(t)

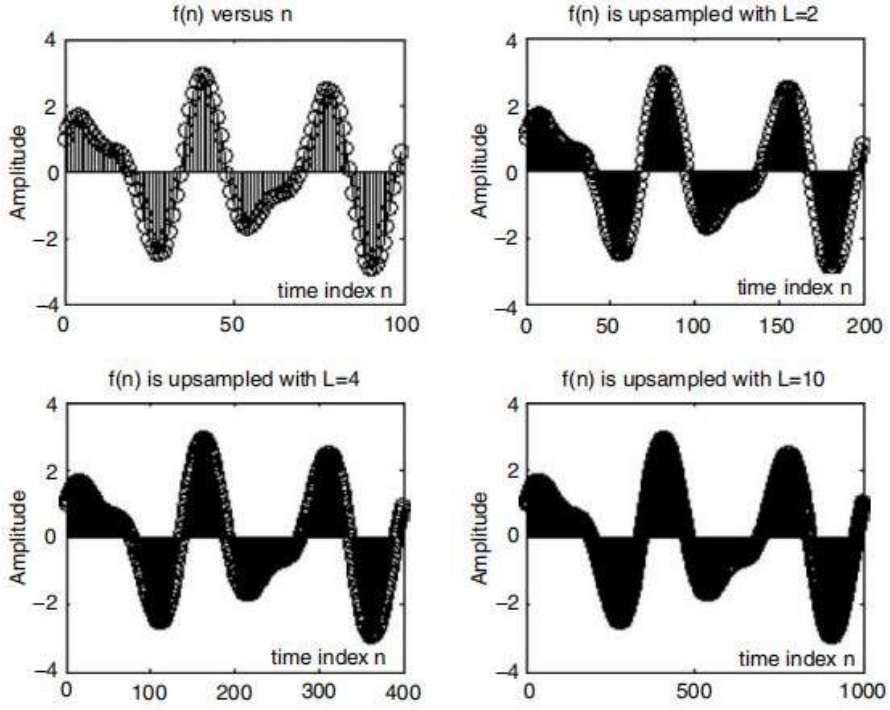


Figure-7.8: Interpolation plots of F(t)

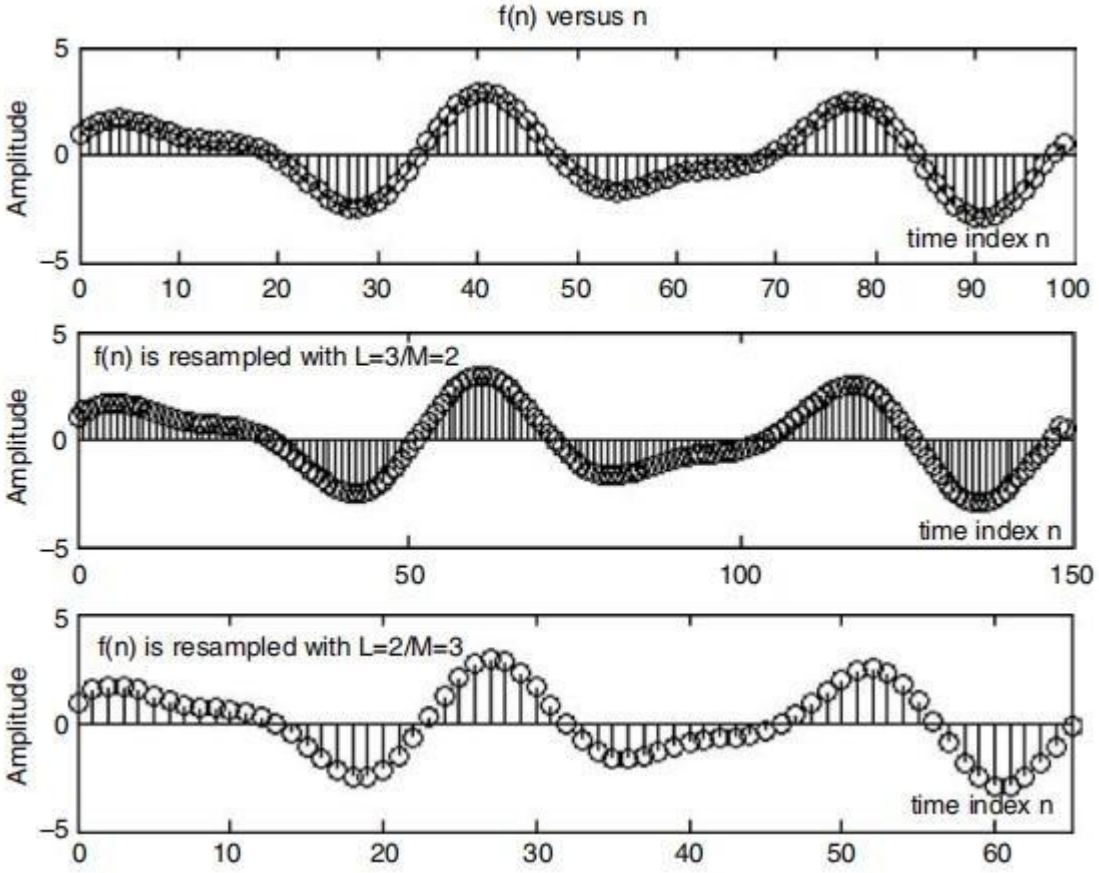
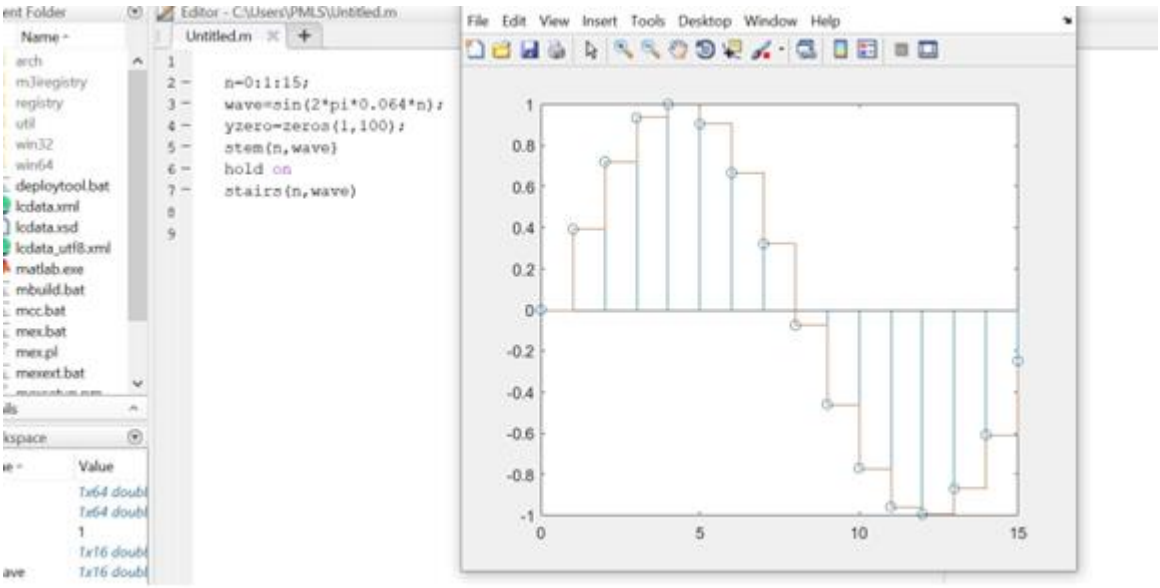


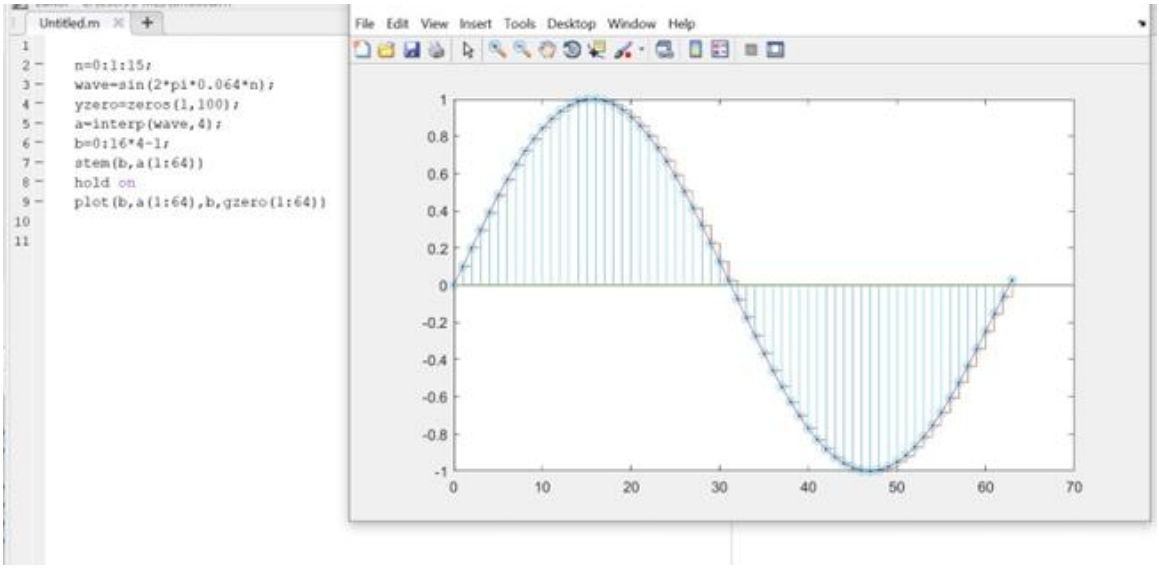
Figure-7.9: Plots of *f(t)* and resampled *f(t)* with notes given by *L*/*M*

Review Exercise?

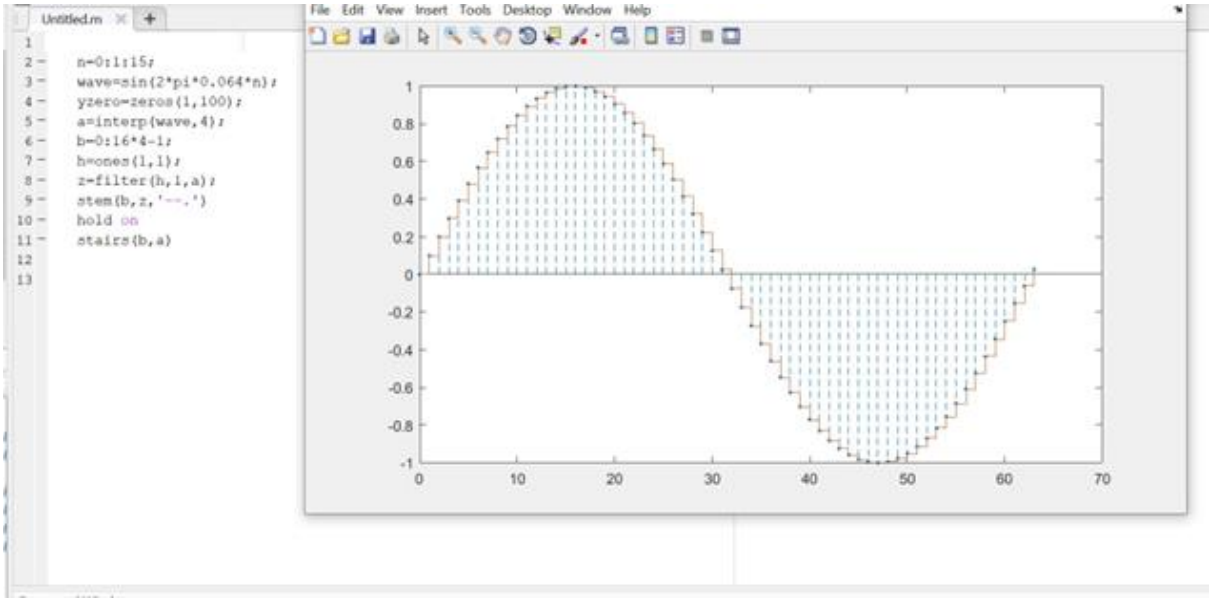
a) Construct a sinusoidal signal. Specify a sample rate such that 16 samples corresponds to exactly one signal period. Draw a stem plot of the signal. Overlay a stair step graph for sample and hold visualization.?



b) Up sample the signal by a factor of four. Plot the result along with the original signal. Up sample increases the sample rate of the signal by adding zeroes between the existing samples?



c) Filter with moving average FIR filter to fil in the zeros with sample and hold values?

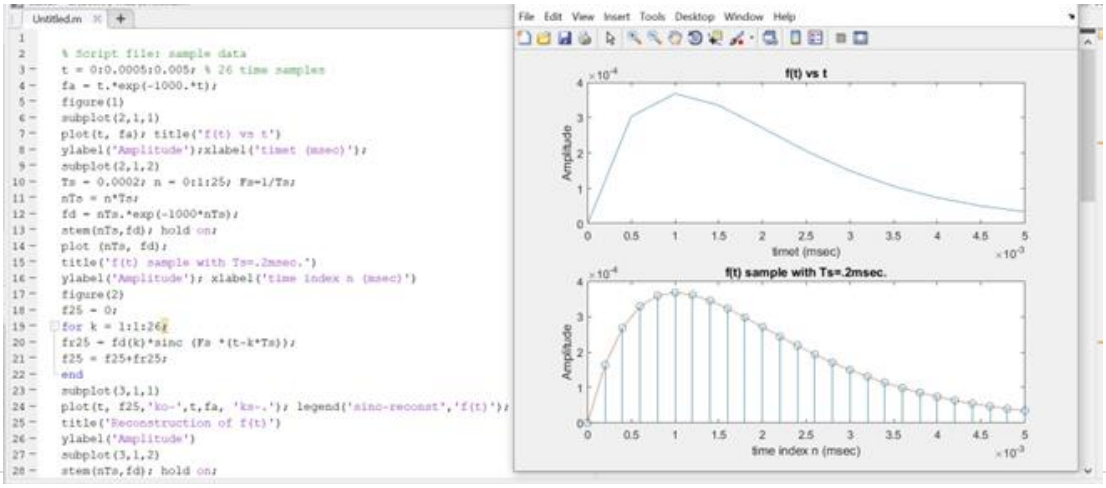


d) What is the effect of Zero-order-hold?

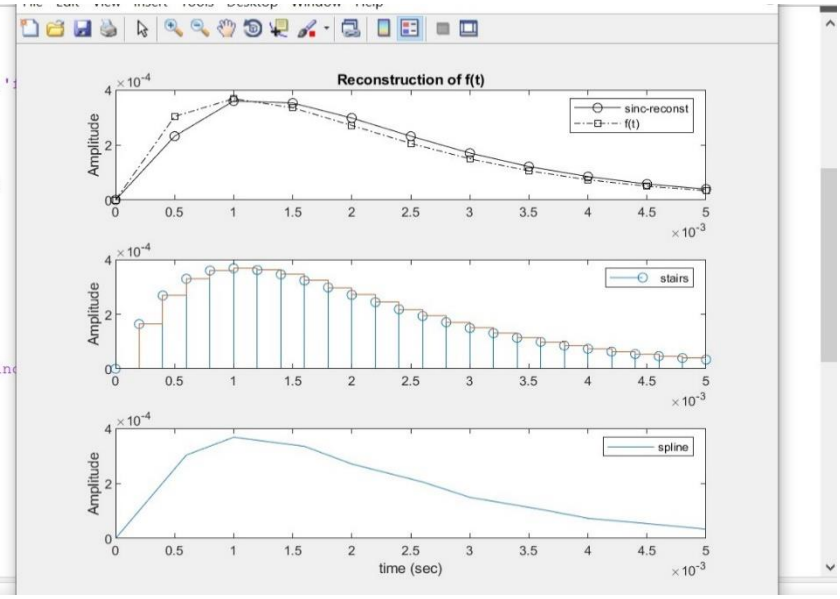
The zero-order hold is a method which is widely used to reconstruct the signals in real time. In the zero-order hold reconstruction method, the continuous signal is reconstructed from its samples by holding the given sample for an interval until the next sample is received.Therefore, the zero-order hold generates the step approximations.

e) Attach the screenshots of MATLAB code and plots of example1 and example2 in your report

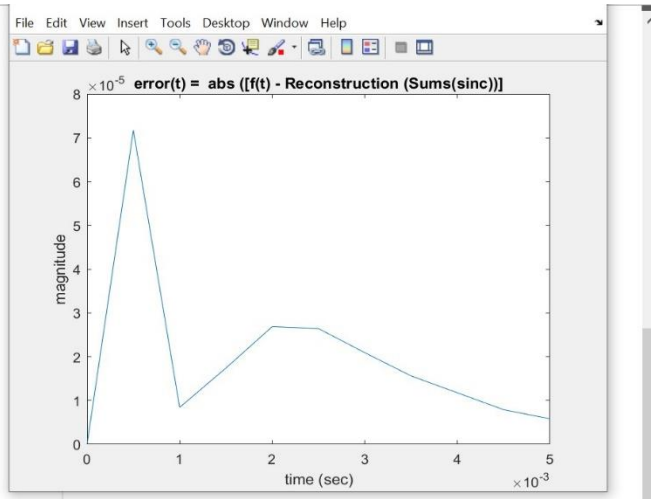
Example 1



```
f25 = f25+fr25;
end
subplot(3,1,1)
plot(t, f25,'ko-',t,fa, 'ks-.'); legend('sinc-reconst','
title('Reconstruction of f(t)')
ylabel('Amplitude')
subplot(3,1,2)
stem(nTs,fd); hold on;
stairs (nTs, fd); ylabel('Amplitude'); legend('stairs')
subplot(3,1,3)
y = spline (nTs,fd,t);
plot(nTs(1:2.5:26.5),y); legend('spline');
ylabel('Amplitude')
xlabel('time (sec)')
figure(3)
error=abs (fa-f25);
plot(t, error); ylabel('magnitude')
title('error(t) =  abs ([f(t) - Reconstruction (Sums(sin
xlabel('time (sec)')
t = 0:0.0005:0.005; % 26 time samples
fa = t.*exp(-1000.*t);
figure(1)
subplot(2,1,1)
plot(t, fa); title('f(t) vs t')
ylabel('Amplitude');xlabel('timet (msec)');
subplot(2,1,2)
Ts = 0.0002; n = 0:1:25; Fs=1/Ts;
nTs = n*Ts;
```

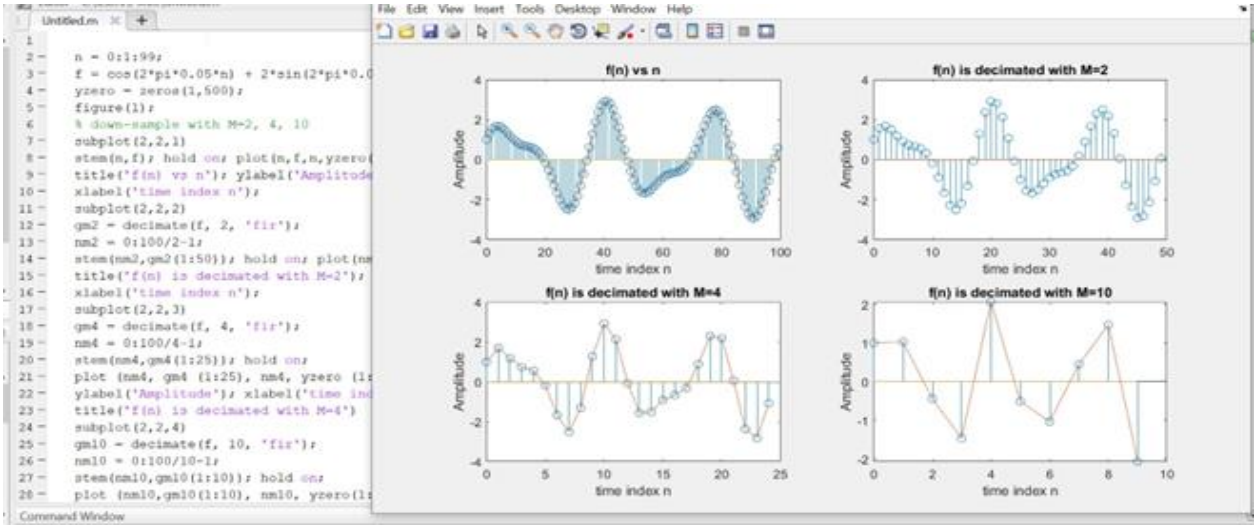


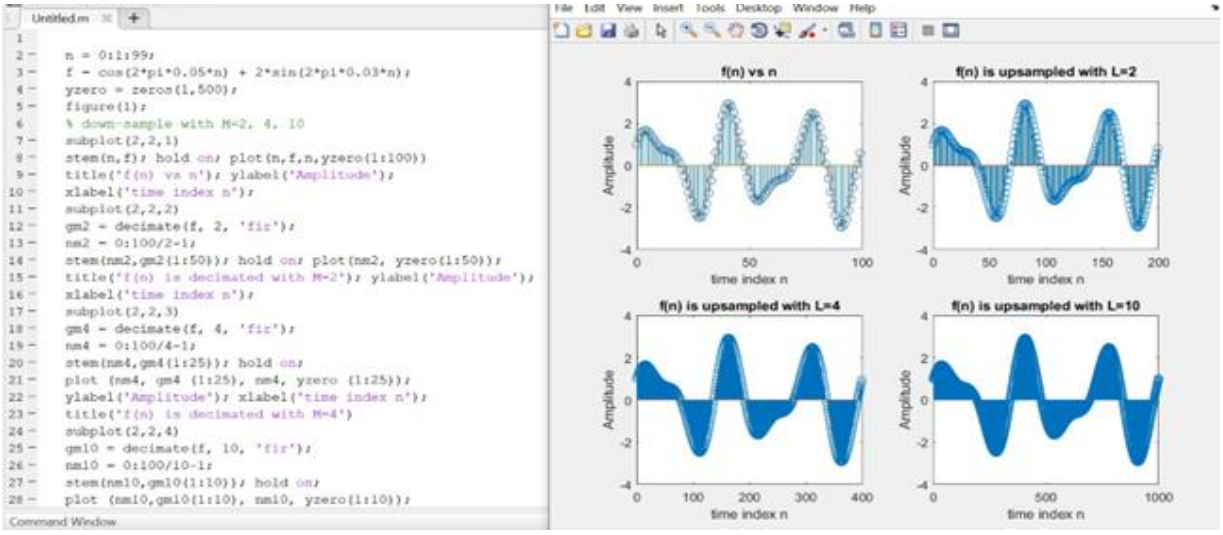

```
49 -    fd = nTs.*exp(-1000*nTs);
50 -    stem(nTs,fd); hold on;
51 -    plot (nTs, fd);
52 -    title('f(t) sample with Ts=.2msec.')
53 -    ylabel('Amplitude'); xlabel('time index n (msec)')
54 -    figure(2)
55 -    f25 = 0;
56 -    for k = 1:1:26;
57 -    fr25 = fd(k)*sinc (Fs *(t-k*Ts));
58 -    f25 = f25+fr25;
59 -    end
60 -    subplot(3,1,1)
61 -    plot(t, f25,'ko-',t,fa, 'ks-.'); legend('sinc-reconst','f(t)');
62 -    title('Reconstruction of f(t)')
63 -    ylabel('Amplitude')
64 -    subplot(3,1,2)
65 -    stem(nTs,fd); hold on;
66 -    stairs (nTs, fd); ylabel('Amplitude'); legend('stairs')
67 -    subplot(3,1,3)
68 -    y = spline (nTs,fd,t);
69 -    plot(nTs(1:2.5:26.5),y); legend('spline');
70 -    ylabel('Amplitude')
71 -    xlabel('time (sec)')
72 -    figure(3)
73 -    error=abs (fa-f25);
74 -    plot(t, error); ylabel('magnitude')
75 -    title('error(t) =  abs ([f(t) - Reconstruction (Sums(sinc))]')
76 -    xlabel('time (sec)')
```



Command Window
New to MATLAB? See resources for Getting Started.

Example 2:





f) Write conclusion/observation of this laboratory?

An analog signal can only be effectively utilized after conversion to digital form if it is sampled accurately. Otherwise, the resulting digital representation may not faithfully capture the original signal, rendering it unusable or unreliable for further processing or analysis.

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 08: THE DISCRETE TIME STATE SPACE REPRESENTATION OF A SYSTEM AND MODELING OF MAGNETICAL SUSPENDED BALL SYSTEM

Name: Karan Roll # 20ES062

Signature of Lab Tutor: _____ Date: _____

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	To illustrate the Discrete-Time State Space representation of a System and modeling of magnetically suspended ball system using MATLAB	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: Computer with MATLAB

Approximate Time Required: Two to three hours.

Introduction: The Transfer function approach is applicable only to linear time-invariant systems having a single-input and single output. Thus, the state-space methods for the analysis and synthesis of control systems are best suited for dealing with linear/non linear time variant, time invariant and multiple-input-multiple-output systems along with linear time-invariant and single-input-single- output systems.

Concept of State-Space Method: State-space method is based on the description of system in terms of n first-order difference equations (DTS) or differential equations (CTS) which can be represented in vector-matrixnotation to simplify the mathematical representation of system of equations.

State & State Variables: The *state* of a system is the smallest set of variables called *state variables,* such that the knowledge of these variables at t=0, together with the knowledge of input for $t \geq 0$, completely determines the behaviour of the system for any time $t \geq 0$. In state-space analysis, we are concerned with three types of variables; input variables, output variables and state variables.

For time-varying (linear or nonlinear) discrete-time systems, the state equation may be written as

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), k]$$

and the output equation as

$$\mathbf{y}(k) = \mathbf{g}[\mathbf{x}(k), \mathbf{u}(k), k]$$

For linear time-varying discrete-time systems, the state equation and output equation may be simplified to

$$\mathbf{x}(k+1) = \mathbf{G}(k)\mathbf{x}(k) + \mathbf{H}(k)\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)\mathbf{u}(k)$$

where

$\mathbf{x}(k) = n$-vector	(state vector)
$\mathbf{y}(k) = m$-vector	(output vector)
$\mathbf{u}(k) = r$-vector	(input vector)
$\mathbf{G}(k) = n \times n$ matrix	(state matrix)
$\mathbf{H}(k) = n \times r$ matrix	(input matrix)
$\mathbf{C}(k) = m \times n$ matrix	(output matrix)
$\mathbf{D}(k) = m \times r$ matrix	(direct transmission matrix)

The appearance of the variable k in the arguments of matrices $\mathbf{G}(k)$, $\mathbf{H}(k)$, $\mathbf{C}(k)$, and $\mathbf{D}(k)$ implies that these matrices are time varying. If the variable k does not appear explicitly in the matrices, they are assumed to be time invariant, or constant. That is, if the system is time invariant, then the last two equations can be simplified to

$$\mathbf{x}(k+1) = \mathbf{G}\mathbf{x}(k) + \mathbf{H}\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k)$$

Note: The notation *x(k)* simply means the vector *x(t)* at t=kT, where k=0,1,2,… & T is the sampling period.

In matrix form,

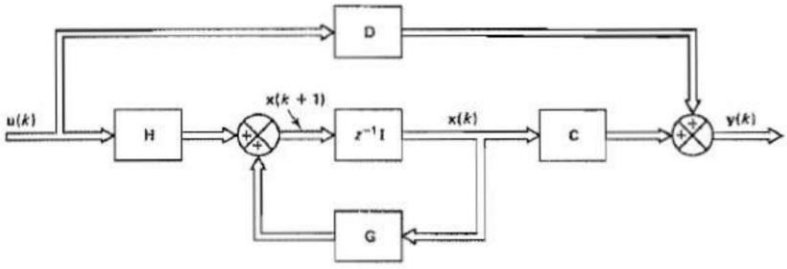
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{n-1}(k) \\ x_n(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [b_n - a_n b_0 \vdots b_{n-1} - a_{n-1} b_0 \vdots \cdots \vdots b_1 - a_1 b_0] \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} + b_0 u(k)$$

Figure-8.1: Block Diagram Representation of Discrete-Time State Equations

State Equations of Sampled Data Systems

Let us assume that the following continuous time system is subject to sampling process with aninterval of T



$x(t) = Ax(t) + Bu(t)$:*state equation* , $y\ t\ = Cx\ t\ + Du\ t$:*Output equation*

We know that the solution to above state equation is:

$(t) = \varphi(t\ t\ to\ )x\ (t\ ) + \int_{t\theta}^{t} \varphi\ t(t\ T\ B)u\ hT$ 10)

Since the inputs are constants in between two sampling instants, one can write

$U(T) = u(kT)\quad for\ kT \le t \le (k+h)T$ 11)

We consider $to = kT$

$x(\ ) = \varphi\ (t\ t\ kT\ x)\ k(T\ ) + \int_{KT}^{t} \varphi\ t(t\ T\ B)u\ (\ \ )$ 13)

$kT\ dT$

$\int_{Kt}^{t\ t\ T} (Bdu\ )\quad \theta\ t\ t\ KT\quad (\ \ \ \ )$

$\theta(T\ ) = \int k(+h\ T)\theta\ k(+h\ \ )(\ \ \ \ )$ $T\ t\ t\ du$

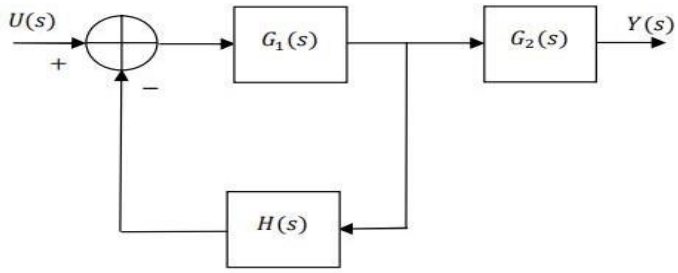
When a discrete system is composed of all digital signals, the state and output equations can bedescribed by

$x(K+\mathrm{h}) = Ax(k) + Bu(k)$

$y(k) = Cx(k) + Duhk$

$(\Rightarrow \frac{Yhz}{Uhz} = Chz(I - A)^{-1}B + D$ 14)

Example 1: Example. Using MATLAB, obtain the transfer function for the control system in the figure below.



where

$$\mathbf{G}_1 = \begin{pmatrix}1 & 3\\ 2 & 5\end{pmatrix}x + \begin{pmatrix}0\\1\end{pmatrix}u; \qquad \mathbf{C}_1 = (1 \quad 2)$$

$$\mathbf{G}_2 = \begin{pmatrix}-4 & 0\\ 0 & 3\end{pmatrix}x + \begin{pmatrix}1\\0\end{pmatrix}u; \qquad \mathbf{C}_2 = (3 \quad 0)$$

$$\mathbf{H} = \begin{pmatrix}3 & 2\\ 1 & 0\end{pmatrix}x + \begin{pmatrix}1\\1\end{pmatrix}u; \qquad \mathbf{C}_3 = (1 \quad 1)$$

MATLAB Solution:

```
a1 = [1  3; 2  5];
b1 = [0;  1];
c1 = [1  2];
d1 = 0;
a2 = [-4  0; 0  3];
b2 = [1;  0];
c2= [3  0];
d2 = 0;
a3 = [3  2; 1  0];
b3 = [1;  1];c3= [1  1]
d3 = 0;
```

The following MATLAB code will convert the system from state-space to transfer function:

```
[numg1, deng1] = ss2tf(a1,b1,c1,d1);
[numg2, deng2] = ss2tf(a2,b2,c2,d2);
[numh, denh] = ss2tf(a3,b3,c3,d3);
[numf, denf] = feedback(numg1,deng1,numh,denh);
[nums, dens] = series(numf,denf,numh,denh);
printsys(nums,dens)

num/den =
```

```
          4 s^4 - 10 s^3 - 14 s^2 - 4 s
-----------------------------------------------------
s^6 - 12 s^5 + 44 s^4 - 22 s^3 - 87 s^2 - 40 s - 4
```

Example 2: Discretize the following continuous-time transfer function

$$H(s) = e^{-0.3s}\frac{s-1}{s^2+4s+5}.$$

MATLAB Solution:

```
H = tf([1 -1],[1 4 5],'InputDelay', 0.3);
Hd = c2d(H,0.1,'foh');
step(H,'-',Hd,'--')
```

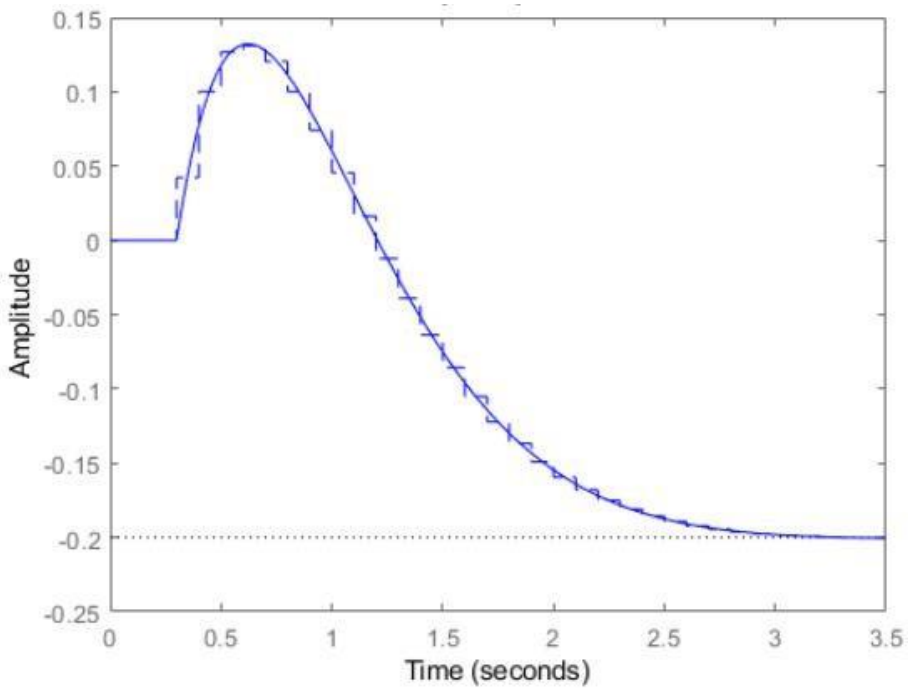


Figure-8.2: output response of transfer function

Example 3: Discretize the following delayed transfer function using zero-order hold on the input, and a 10-Hz sampling rate.

$$H(s) = e^{-0.25s}\frac{10}{s^2 + 3s + 10}.$$

MATLAB Solution:

```
h = tf(10,[1 3 10],'IODelay',0.25);
hd = c2d(h,0.1)
step(h,'--',hd,'-')
```

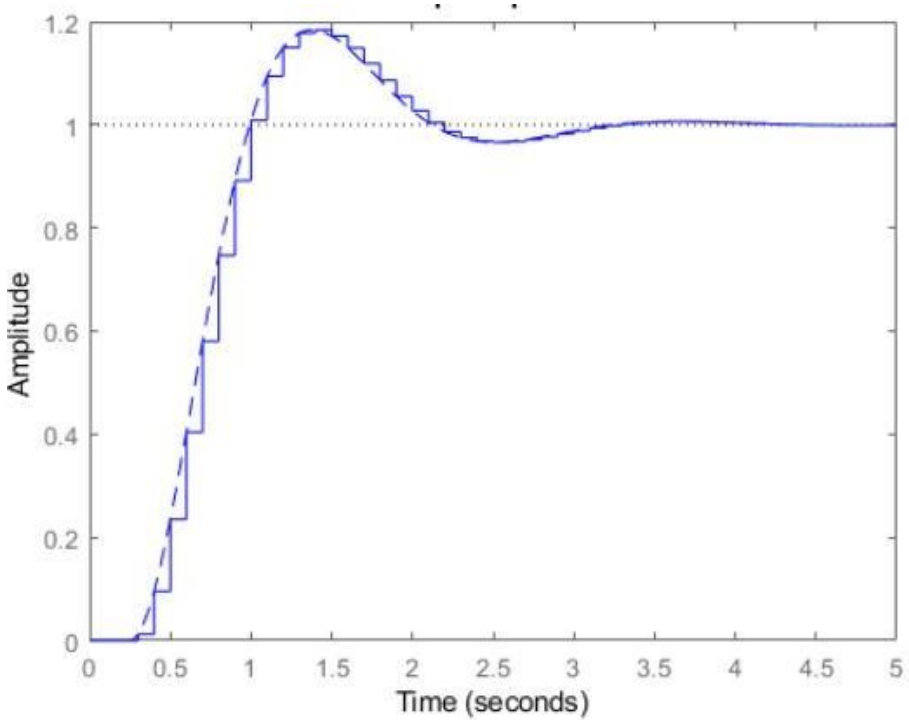


Figure-8.3: output response of transfer function

Example 4: Given a continuous-time state-space model with the state-space matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

Convert it into discrete-time state-space model, using MATLAB.

Take sample time of 1 second.

MATLAB Solution:

```
a=[0 1; -1 -2];
b=[0;1];
c=[1 1];
d=0;
sys_c=ss(a,b,c,d) % creates/stores continuous-time state-space model
ts=1;
sys_d=c2d(sys_c,ts,'zoh') % gives discrete-time state-space model
```

Result

```
sys_c =
a =
      x1 x2
  x1   0   1
  x2 -1 -2

 b =
      u1
x1                                                    0
  x2   1
 c =
      x1 x2
  y1   1   1

 d =
      u1
```

```
  y1   0
```

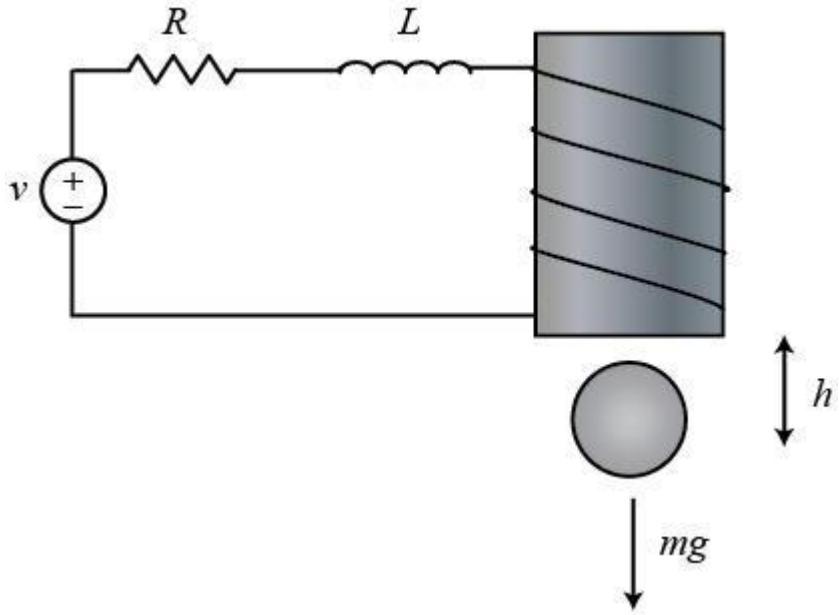
Continuous-time state-space model.

```
sys_d =
  a =
         x1       x2
   x1   0.7358   0.3679
   x2  -0.3679 1.92e-17
  b =
         u1
   x1 0.2642
   x2 0.3679
  c =
       x1 x2
   y1   1   1
d =
       u1
   y1   0
```

Sample time: 1 seconds

Discrete-time state-space model.

8.1 Example 5: To introduce the state-space control design method, we will use the magnetically suspended ball as an example. The current through the coils induces a magnetic force which can balance the force of gravity and cause the ball (which is made of a magnetic material) to be



suspended in mid-air.

Figure-8.4: Magnetically suspended ball

We linearize the equations about the point h= 0.01 m (where the nominal current is about 7 Amps) and obtain the linear state-space equations:

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + Bu$$

$$y = C\mathbf{x} + Du$$

$$x = \begin{bmatrix} \Delta h \\ \Delta \dot{h} \\ \Delta i \end{bmatrix}$$

Here the values of A = [0 1 0
980 0 -2.8
0 0 -100]
B= [0
0
100]
C= [1 0 0]

MATLAB Solution:

```
A = [ 0 1 0 ; 980  0 -2.8 ; 0 0 -100 ];
B = [ 0; 0 ;100 ];
```

```
C = [ 1 0 0 ];
```

8.1.1 Stability Analysis

One of the first things we want to do is analyze whether the open-loop system (without any control) is stable. As discussed in the Introduction: System Analysis section, the eigenvalues of the system matrix, , (equal to the poles of the transfer function) determine stability. The eigenvalues of the matrix are the values of that are solutions of $\det(sI - A) = 0$.

Poles= eig(A)

```
poles =

  31.3050

 -31.3050

-100.0000
```

From inspection, it can be seen that one of the poles is in the right-half plane (i.e. has positive real part), which means that the open-loop system is unstable.

To observe what happens to this unstable system when there is a non-zero initial condition, add the following lines to your m-file and run it again:

```
t = 0:0.01:2;
u = zeros(size(t));
x0 = [0.01 0 0];
sys = ss(A,B,C,0);
[y,t,x] = lsim(sys,u,t,x0);
plot(t,y)
title('Open-Loop Response to Non-Zero Initial Condition')
xlabel('Time (sec)')
ylabel('Ball                                                    Position (m)')
```

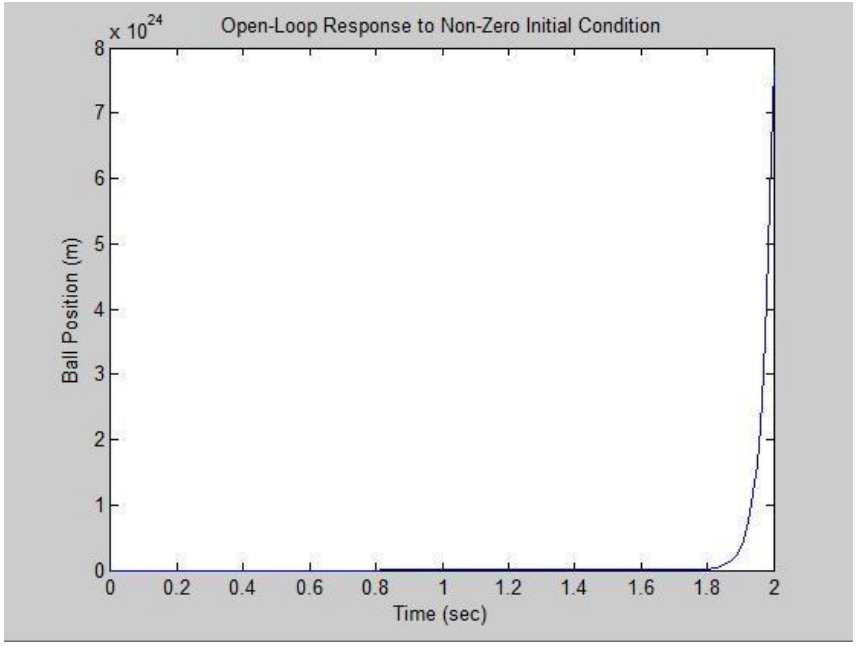


Figure-8.5: output response

It looks like the distance between the ball and the electromagnet will go to infinity, but probably the ball hits the table or the floor first (and also probably goes out of the range where our linearization is valid).

8.2 Review Exercise?

a) What is state space representation of Discrete-Time system?

The dynamics of a linear time (shift) invariant discrete-time system may be expressed in terms state (plant) equation and output(observation or measurement) equation as follows: x(k+1) = Ax(k)+Bu(k) and y(k) = Cx(k)+Du(k) where x(k) an n dimensional slate rector a time t=kT an r-dimensional control (input) vector and y(k) an m-dimensional output vector respectively are represented as x(k)=[x1(l), x2(l), x3(l),..... xn(l)] T,u(k)=[u1(k), u2(k), u3(k), u4(k),..... ur(k)]T,y(k)=[y1(k), y2(k), y3(k), y4(k),..... ym(k)]T .

b) What is a state model and what are state variables?

State model: The state of a dynamic system is the smallest set of variables and the knowledge of these variables at t=10 together with inputs fot t>=10 completely determines the behaviour of the system at t>=10. A compact and concise representation of the past history of the system can be termed as the state of the system.

State Variable: The smallest set of variables that determine the state of the system are kown as state variable.

c) What are the advantages of state space techniques?

This technique can be used for linear and non-linear time-variant and time invariant systems.

- It is easier to apply where Laplace transforms cannot be applied.
- The nth order differential equation can be expressed as 'n' equation of first order.
- It is time domain method.

d) A continuous-time state-space model of Mass-Spring-Damper is given as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

Where m = 1; b = 10; k = 20.

Using MATLAB, obtain its discrete-time state-space model. Take sample time of 1/100seconds.

Solution:

```
4 -  m = 1;
5 -  b = 10;
6 -  k = 20;
7 -  a=[0 1; -k/m -b/m];
8 -  b=[0;1/m];
9 -  c=[1 0];
10 -  d=[0];
11 -  sys_c=ss(a,b,c,d) % creates/stores continuous-time state-space model
12 -  ts=1/100;
13 -  sys_d=c2d(sys_c,ts,'zoh') % gives discrete-time state-space model
14
15
```

Result:

```
FILE | NAVIGATE | EDIT | BREAKPOINTS
C: ▸ Program Files ▸ MATLAB ▸ R2018a ▸ bin ▸
Command Window
New to MATLAB? See resources for Getting Started.
ts=1/100;
sys_d=c2d(sys_c,ts,'zoh') % gives discrete-time state-space model

sys_c =

  A =
        x1    x2
   x1    0     1
   x2  -20   -10

  B =
       u1
   x1   0
   x2   1

  C =
       x1  x2
   y1   1   0

  D =
       u1
   y1   0

Continuous-time state-space model.
fx
```

C: ▸ Program Files ▸ MATLAB ▸ R2018a ▸ bin ▸

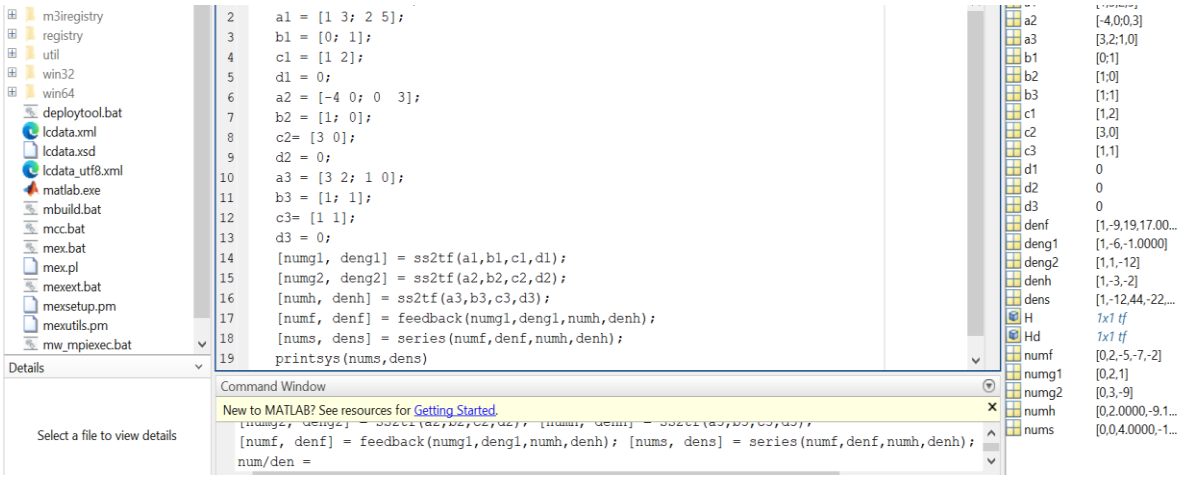
Command Window

New to MATLAB? See resources for Getting Started.

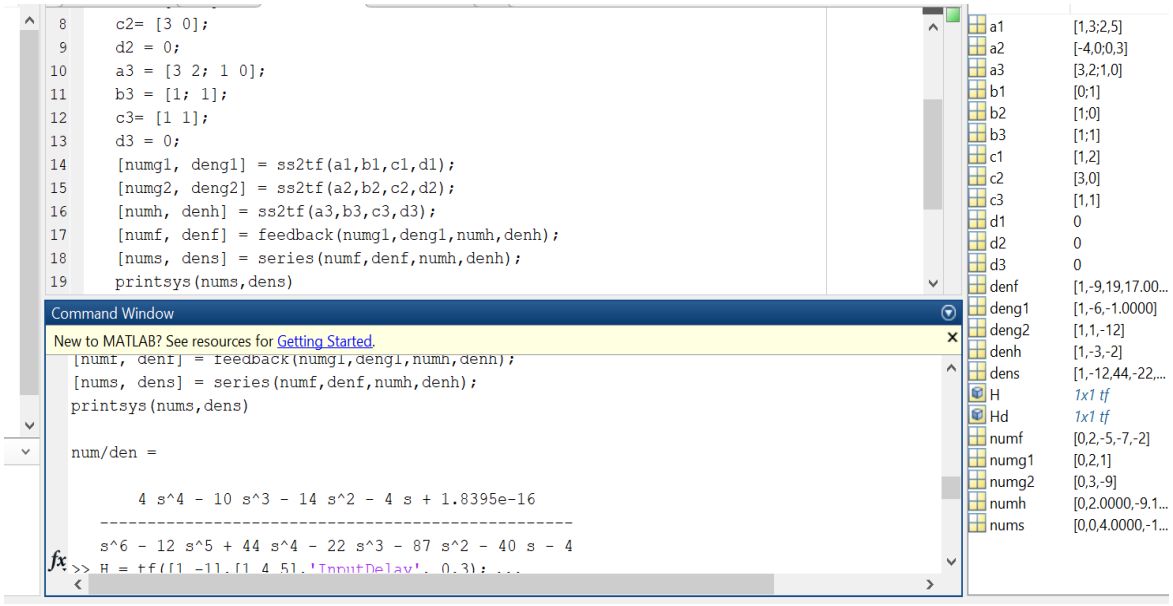
```
sys_d =

  A =
             x1         x2
   x1      0.999   0.009513
   x2    -0.1903     0.9039

  B =
              u1
   x1   4.837e-05
   x2    0.009513

  C =
       x1  x2
   y1   1   0

  D =
       u1
   y1   0

Sample time: 0.01 seconds
Discrete-time state-space model.
```

e) Attach the plots and results of example1 to example5. Also explain example 5 briefly

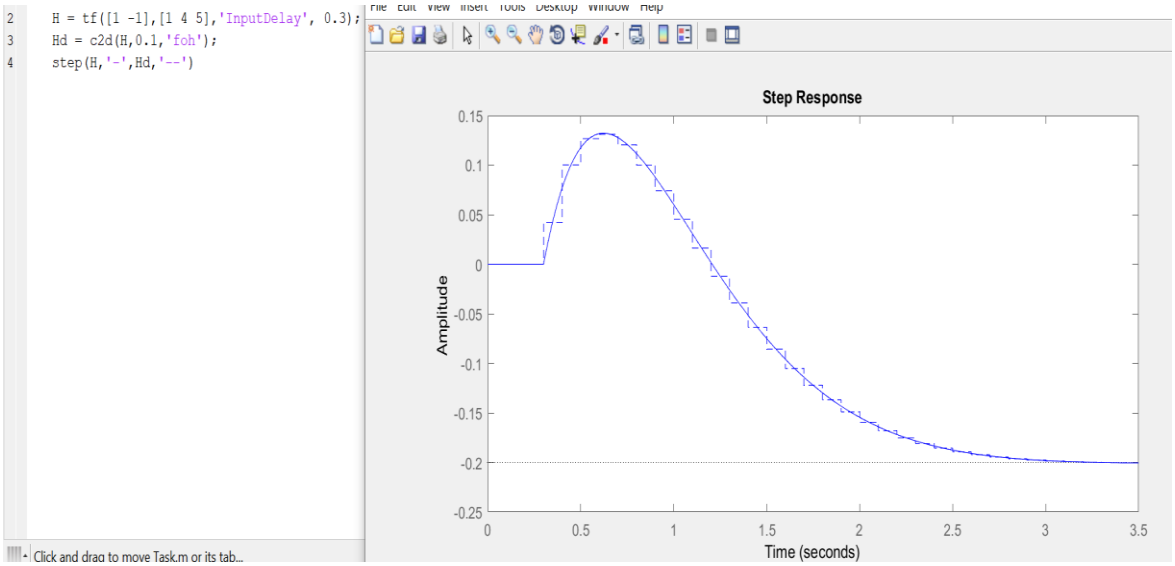
Example 1



Example 2

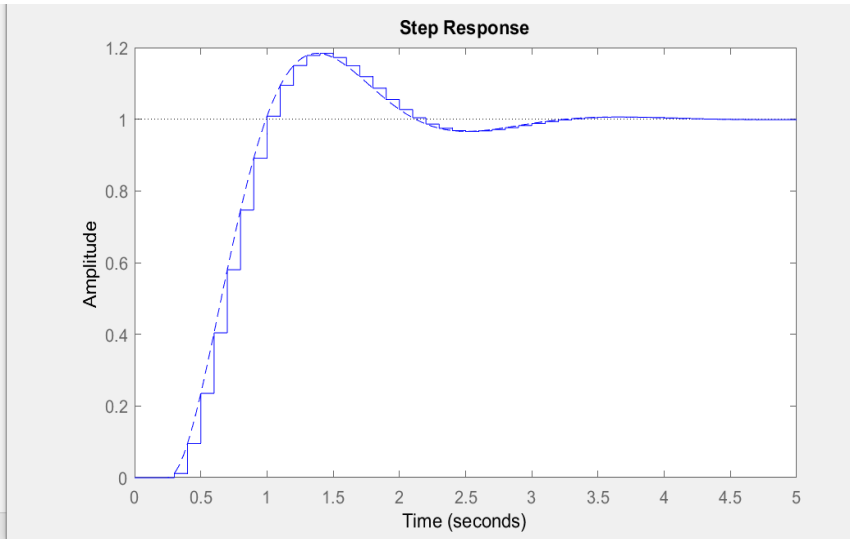


Example 3



Example 4

```
h = tf(10,[1 3 10],'IODelay',0.25);
hd = c2d(h,0.1)
step(h,'--',hd,'-')
```




```
a=[0 1; -1 -2];
b=[0;1];
c=[1 1];
d=0;
sys_c=ss(a,b,c,d) % creates/stores continuous-time state-space model
ts=1;
sys_d=c2d(sys_c,ts,'zoh') % gives discrete-time state-space model
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> a=[0 1; -1 -2];
b=[0;1];
c=[1 1];
d=0;
sys_c=ss(a,b,c,d) % creates/stores continuous-time state-space model
ts=1;
sys_d=c2d(sys_c,ts,'zoh') % gives discrete-time state-space model

sys_c =

  A =
       x1  x2
```

Command Window

New to MATLAB? See resources for Getting Started.

```
sys_c =

  A =
       x1  x2
   x1   0   1
   x2  -1  -2

  B =
       u1
   x1   0
   x2   1

  C =
       x1  x2
   y1   1   1

  D =
       u1
   y1   0

Continuous-time state-space model.
```

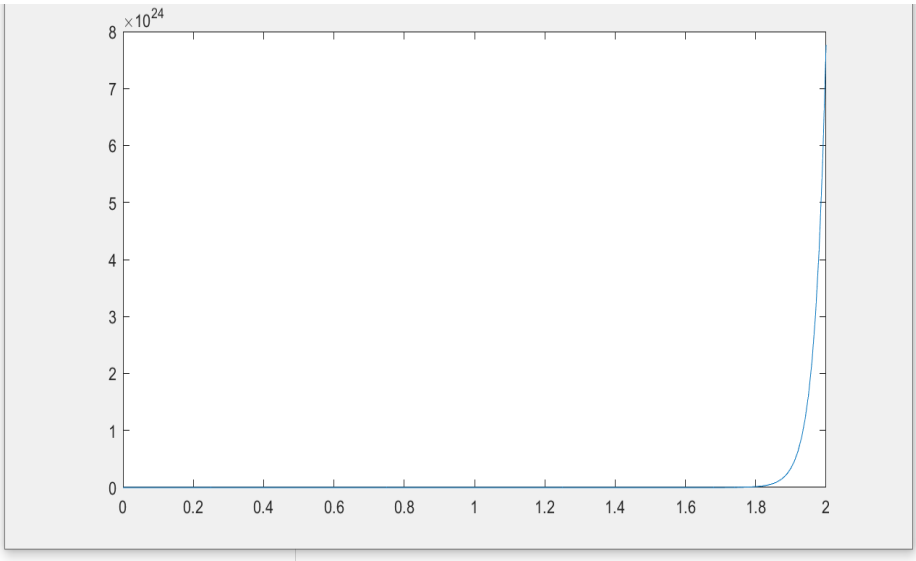
Command Window

New to MATLAB? See resources for Getting Started.

```
sys_d =

  A =
             x1        x2
   x1    0.7358    0.3679
   x2   -0.3679         0

  B =
            u1
   x1   0.2642
   x2   0.3679

  C =
       x1  x2
   y1   1   1

  D =
       u1
   y1   0

Sample time: 1 seconds
Discrete-time state-space model.
fx ...
```

Example 5

```
A = [ 0 1 0 ; 980 0 -2.8 ; 0 0 -100 ];
B = [ 0; 0 ;100 ];
C = [ 1 0 0 ];
Poles= eig(A)
poles = 31.3050
-31.3050
-100.0000
t = 0:0.01:2;
u = zeros(size(t)); x0 = [0.01 0 0];
sys = ss(A,B,C,0);
[y,t,x] = lsim(sys,u,t,x0); plot(t,y)
```



It is shown that the distance between the ball and the electromagnet goes to infinity, but probably the ballhits the table or the floor first (and also probably goes out of the range where our linearization is valid).

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 09 DETERMINING THE CONTROLLABILITY AND OBSERVABILITY OF A SYSTEM IN DISCRETE TIME DOMAIN

Name: karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	Determining the Controllability and Observability of a system in Discrete-Time domain.	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: PC with MATLAB software

Approximate Time Required: Two to three hours

Introduction

Controllability and observability are two important properties of state models which are to be studied prior to designing a controller.Controllability deals with the possibility of forcing the system to a particular state by application of a control input. If a state is uncontrollable then no input will be able to control that state.On the other hand whether or not the initial states can be observed from the output is determined using observability property. Thus if a state is not observable then the controller will not be able to determine its behavior from the system output and hence not be able to use that state to stabilize the system.

Controllability is an important property of a control system, and the Controllability property plays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control. Controllability and observability are dual aspects of the same problem.

Controllability:

A system is completely controllable if there exists an unconstrained control u(t) that can transfer any initial state x(to) to any other desired location x(t) in a finite time, to ≤ t ≤ T.

Observability:

A system is completely observable if and only if there exists a finite time T such that the initial state x(0) can be determined from the observation history y(t) given the control u(t), 0≤ t ≤ T.

Controllability

To determine Controllability of a system, consider a dynamical system given as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + B\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= C\boldsymbol{x}(k) + D\boldsymbol{u}(k) \end{aligned} \tag{9.1}$$

The state equation (9.1) or the pair (A,B) is controllable if and only if the *n* x *nm* Controllability matrix:

$$S = \begin{bmatrix} B & AB & A^2B & \ldots\ldots & A^{n-1}B \end{bmatrix}$$

has rank n, i.e., full row rank.

In MATLAB we can obtain the Controllability matrix S by using built-in function ctrb.

In order to check whether the given system is controllable or not, we compare the rank of Controllability matrix S with the length of state matrix A. The system is said to be controllable if

rank(S) = length(A)

Observability

The state equation (9.1) or the pair (A,C) is observable if and only if the *np* x *n* observability matrix:

$$V=\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

has rank *n*, i.e., full column rank.

In MATLAB we can obtain the observability matrix V by using built-in function obsv.

In order to check whether the given system is observable or not, we compare the rank of observability matrix V with the length of state matrix A. The system is said to be observable if

rank(V) = length(A)

Determining System Controllability & Observability in discrete-time domain using MATLAB

Example 1: Given the state-space matrices of a discrete-time state-space model with a sample time of 0.25 seconds:

$$A=\begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \quad B=\begin{bmatrix} 0 \\ 3 \end{bmatrix} \quad C=\begin{bmatrix} 0 & 1 \end{bmatrix} \quad D=\begin{bmatrix} 0 \end{bmatrix}$$

Determine the system Controllability & observability in discrete-time domain, using MATLAB.

MATLAB Code

```
A = [0 1;-5 -2];
B = [0;3];
C=[0 1];
D=0;
Ts = 0.25;
sys = ss(A,B,C,D,Ts)%creates/stores discrete-time state-space
                    %model
LengthofA= length(sys.A)

s=ctrb(sys)%gives controllability matrix

RankofS= rank(s)

v=obsv(sys)%gives observability matrix

RankofV= rank(v)
```

Result

```
sys  =

a =

      x1 x2
  x1   0   1
  x2  -5  -2


 b =

      u1
  x1   0
  x2   3

 c =

      x1 x2
  y1   0   1


 d =

      u1
  y1   0

Sample time: 0.25 seconds

Discrete-time state-space model.

LengthofA = 2

s =

     0     3
     3    -6
```

RankofS = 2

v =

0 1

-5 -2

RankofV = 2

Hence, the given system is controllable as well as observable in discrete-time domain.

Example 2: Given the state-space matrices of a continuous-time state-space model:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad D = [0]$$

Determine the system Controllability & observability in discrete-time domain, using MATLAB.

Take sample time of 1 second.

MATLAB Code

```
a=[0 1; -1 -2];
b=[0;1];
c=[1 1];
d=0;
sys_c=ss(a,b,c,d) % creates/stores continuous-time state-space model
ts=1;
sys_d=c2d(sys_c,ts,'zoh') % gives discrete-time state-space
model
 lengthofA=length(sys_d.a)
s=ctrb(sys_d) %gives controllability
matrix
rankofS=rank(s)
v=obsv(sys_d) %gives observability
matrix
 rankofV=rank(v)
```

Result

```
sys_c =

a =
      x1 x2
  x1   0   1
  x2  -1  -2

 b =
      u1
x1     0
  x2   1

 c =
      x1 x2
  y1   1   1

 d =
      u1
  y1   0

Continuous-time state-space model.

sys_d =

 a =
            x1        x2
  x1    0.7358    0.3679
  x2   -0.3679  1.92e-17

 b =
         u1
  x1 0.2642
  x2 0.3679

 c =
      x1 x2
  y1   1   1
```

d =

u1

y1 0

Sample time: 1 seconds

Discrete-time state-space model.

lengthofA = 2

s =

0.2642 0.3298

0.3679 -0.0972

rankofS = 2

v =

1.0000 1.0000

0.3679 0.3679

rankofV = 1

Hence, the given system is controllable but not observable in discrete-time domain.

Review Exercise?

a. What is Controllability?

A system is completely controllable if there exists an unconstrained control u(t) that can transfer any initial state x(to) to any other desired location x(t) in a finite time, to ≤ t ≤ T. Controllability is an important property of a control system, and the Controllability propertyplays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control. Controllability and observability are dual aspects of the same problem.

b. What is observability?

A system is completely observable if and only if there exists a finite time T such that the initialstate x(0) can be determined from the observation history y(t) given the control u(t), 0≤ t ≤ T.

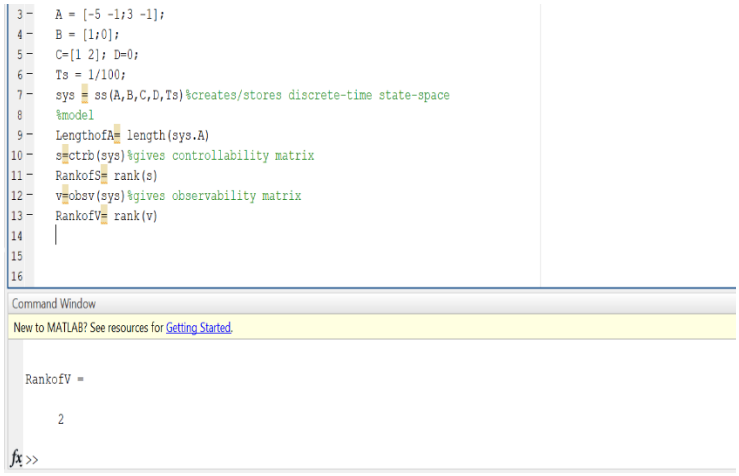
c. Given the state-space matrices of a continuous-time state-space model:

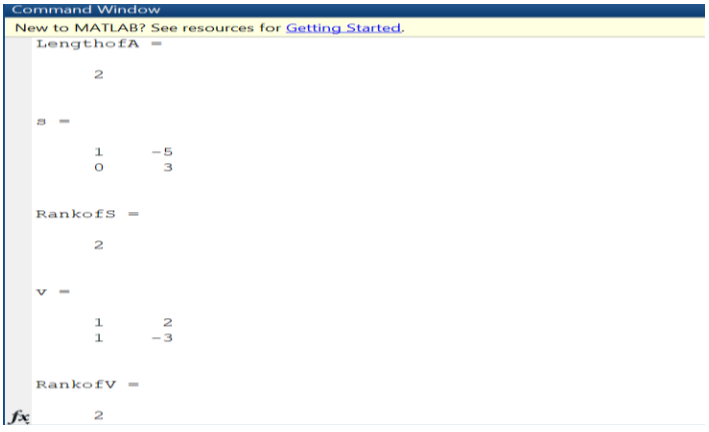
$$A = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

Determine the system Controllability & observability in discrete-time domain, using MATLAB. Take sample time of 1/100 seconds.

Program:

```
A = [-5 -1;3 -1];
B = [1;0];
C=[1 2]; D=0;
Ts = 1/100;
sys = ss(A,B,C,D,Ts)%creates/stores discrete-time state-space
%model
LengthofA= length(sys.A)
s=ctrb(sys)%gives controllability matrix
RankofS= rank(s)
v=obsv(sys)%gives observability matrix
RankofV= rank(v)
```





MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 10: FREQUENCY RESPONSE AND CONSTRUCTION OF BODE PLOT

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	*To analyse the frequency response and construction of bode plot using MATLAB*	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: Computer with MATLAB

Approximate Time Required: Two to three hours.

Introduction

Bode plot is a graph of the frequency response of a system. It is usually a combination of a Bode magnitude plot, expressing the magnitude of the frequency response and a Bode phase plot, expressing the phase shift.
We are interested in the frequency response of an LTI system. The transfer function can be written like this:

$$H(s) = K\frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)(s-p_3)\cdots}$$

such that when we plug in jω for s, we get

$$H(j\omega) = K\frac{(j\omega-z_1)(j\omega-z_2)\cdots}{(j\omega-p_1)(j\omega-p_2)(j\omega-p_3)\cdots}$$

That's a product (or quotient) of a bunch of complex numbers. Using polar form, we can say that the angle of the product (quotient) is the sum of the angles of each term (except for division we subtract, so it's the sum of the angles for the top terms, minus the sum of the angles for the terms in the denominator). Similarly, the magnitude is the product of the magnitude of all the terms. Summing terms is easy to do graphically; products are harder. However, on a log scale (e.g., dB), the product turns into a sum. Thus, if we plot the behavior of each term, we can then simply add the plots to find the total behavior.

For the poles, we could either plot the behavior of (s - p) and subtract it, or plot the behavior of 1/(s - p) and add that behavior. We'll plot the behavior of 1/(s - p), such that we only need to add terms.
The general plan for how to sketch a Bode plot by hand is, then, to first gain an understanding of what individual poles and individual zeros do, and then add the responses together. It is easiest to understand complex poles and zeros by looking at the response of a complex conjugate pair, rather than trying to look at the complex poles or zeros individually

Effect of Constant Terms:

Constant terms such as K contribute a straight horizontal line of magnitude 20 log10(K)

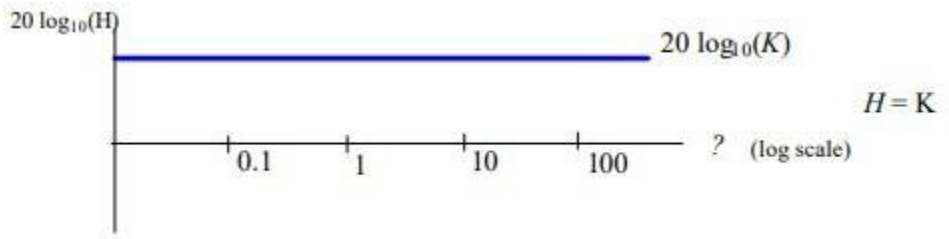


Figure-10.1: Effect of Constant Term

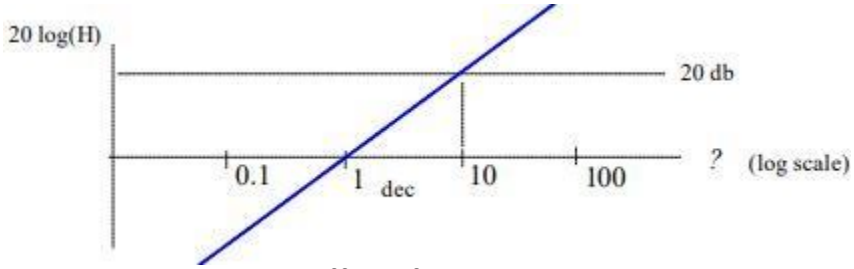


Figure-10.2: effect of pole and zero at origin

Effect of Individual Zeros and Poles Not at the Origin Zeros and Poles not at the origin are indicated by the $(\mathrm{T} + \square/zi\grave{A}$ and $(\mathrm{T} + \square/pi\grave{A}$. The values zi and pi in each of these expression is called a critical frequency (or break frequency). Below their critical frequency these terms do not contribute to the log magnitude of the overall plot. Above the critical frequency, they represent a ramp function of $20\ db$ per decade. Zeros give a positive slope. Poles produce a negative slope.

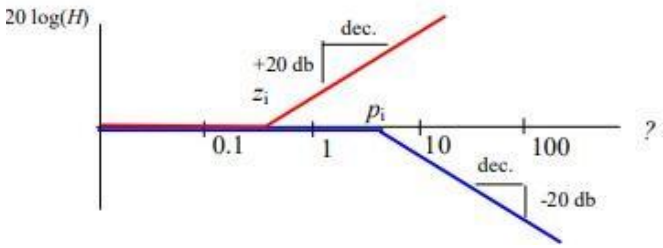


Figure-10.3 Effect of Pole and Zero at origin

To complete the log magnitude vs. frequency plot of a Bode diagram, we superposition all the lines of the different terms on the same plot.

10.1 Technique to get started:

1) Draw the line of each individual term on the graph
2) Follow the combined pole-zero at the origin line back to the left side of the graph.
3) Add the constant offset, 20 log10(K), to the value where the pole/zero at the origin line intersects the left side of the graph.
4) Apply the effect of the poles/zeros not at the origin. working from left (low values) to right (higher values) of the poles/zeros

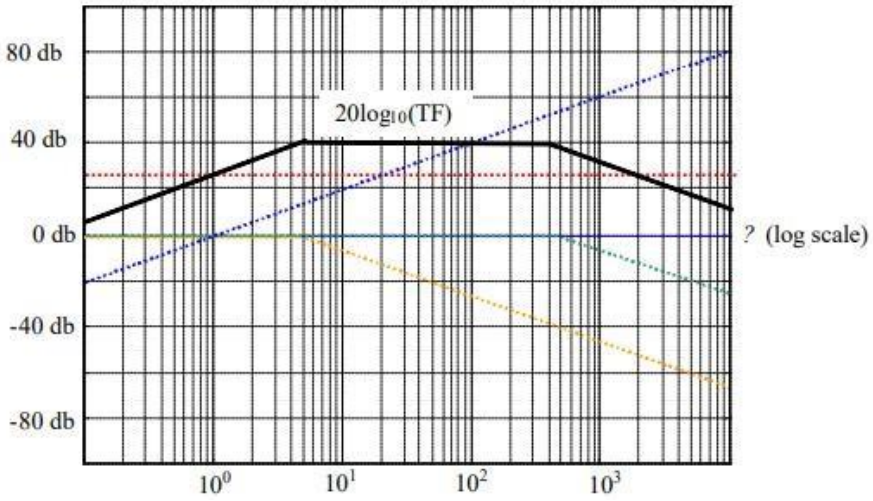


Figure-10.4: bode log plot

Example: Find the bode plot of following system using MATLAB

$$G = \frac{2}{S+1}$$

Matlab code

```
% Numerator
num = [2];
% Denominator
den = [1 1];
% Transfer Function
G = tf(num, den)
```

```
% Plot Frequency Response
bode(G), grid
```

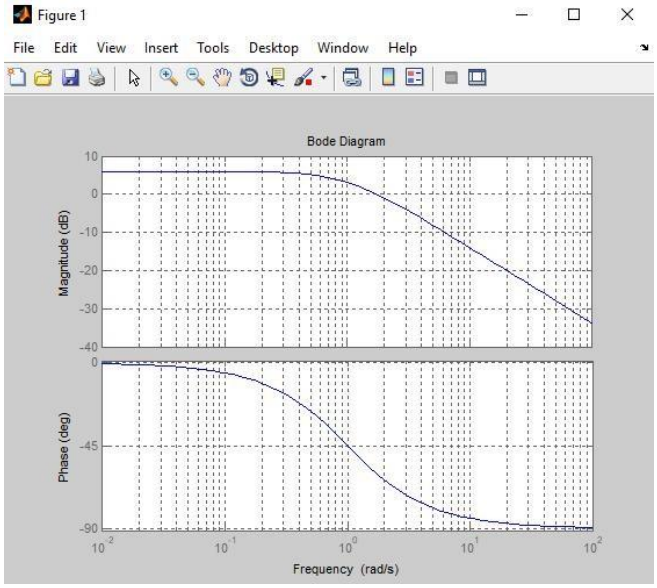


Figure-10.5: Output waveform

Bode Plot of dynamic system

Create a Bode plot of the following continuous-time SISO dynamic system.

MATLAB Code:

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);
bode(H)
```

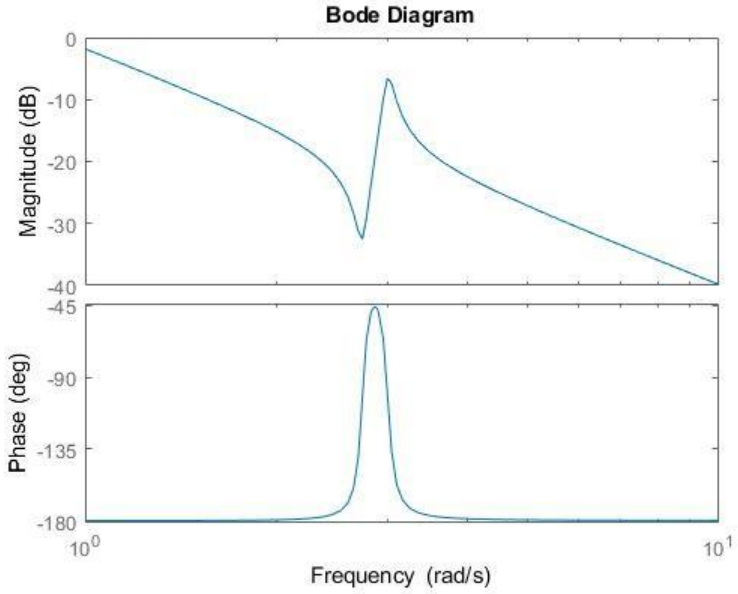


Figure-10.6: Bode Plot of above system

Bode Plot at Specified Frequencies

Create a Bode plot over a specified frequency range. Use this approach when you want tofocus on the dynamics in a particular range of frequencies

MATLAB Code:

```
H = tf([-0.1,-2.4,-181,-1950],[1,3.3,990,2600]);
bode(H,{1,100})
grid on
```

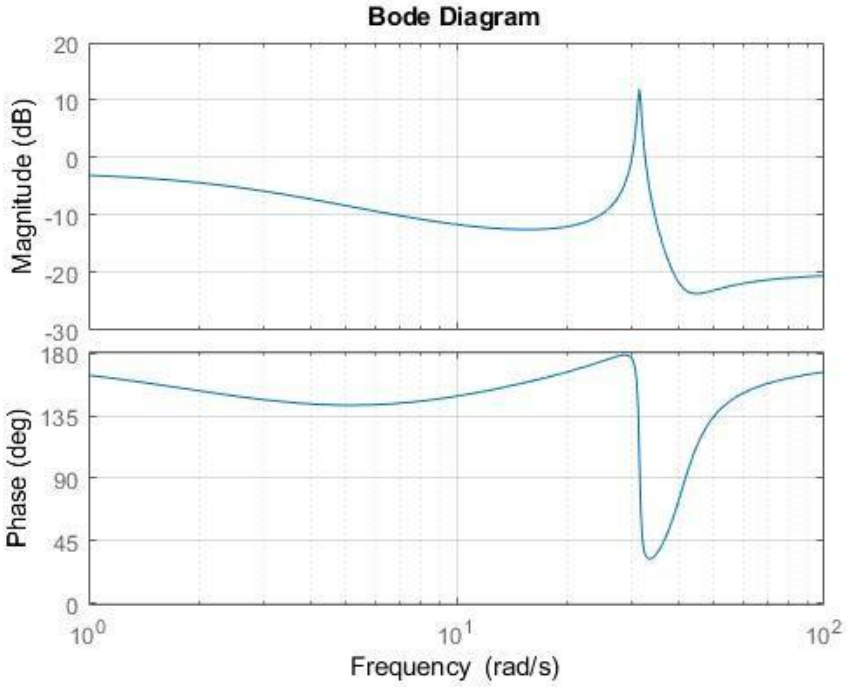


Figure-10.7: Output plot

The cell array {1,100} specifies the minimum and maximum frequency values in the Bode plot. When you provide frequency bounds in this way, the function selects intermediate points for frequency response data.

Alternatively, specify a vector of frequency points to use for evaluating and plotting the frequency response.

Create the bode plot at specified frequency points

MATLAB Code:

```
w = [1 5 10 15 20 23 31 40 44 50 85 100];
bode(H,w,'.-')
grid on
```

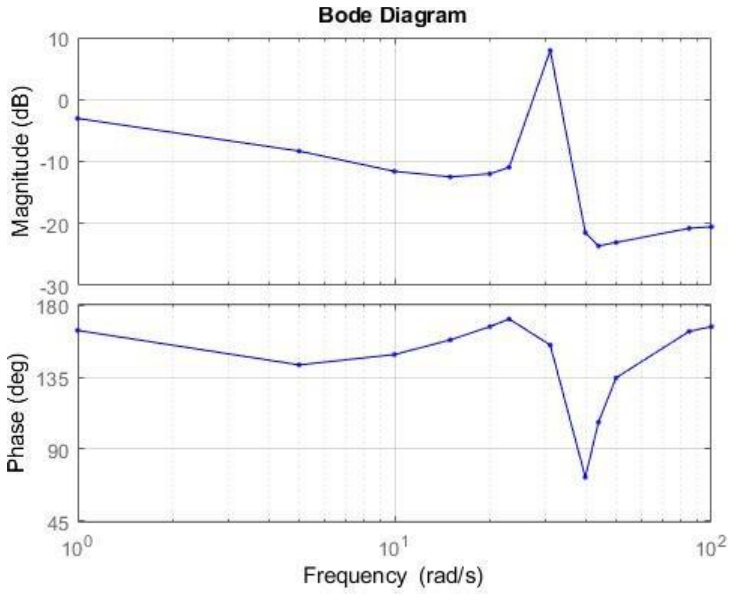


Figure-10.8: Bode Plots the frequency response at the specified frequencies

Compare Bode Plots of Several Dynamic Systems

Compare the frequency response of a continuous-time system to an equivalent discretized system on the same Bode plot.

Create continuous-time and discrete-time dynamic systems.

MATLAB Code:

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);
Hd = c2d(H,0.5,'zoh');
bode(H,Hd)
```

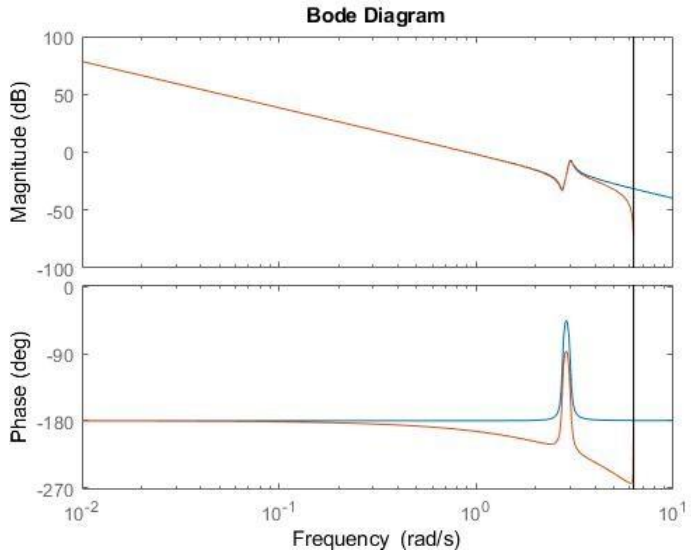


Figure-10.9: Output plot

The Bode plot of a discrete-time system includes a vertical line marking the Nyquist frequency of the system.

Bode Plot with Specified Line Attributes

Specify the line style, color, or marker for each system in a Bode plot using the LineSpec input argument.

MATLAB Code:

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);
Hd = c2d(H,0.5,'zoh');
bode(H,'r',Hd,'b--')
```

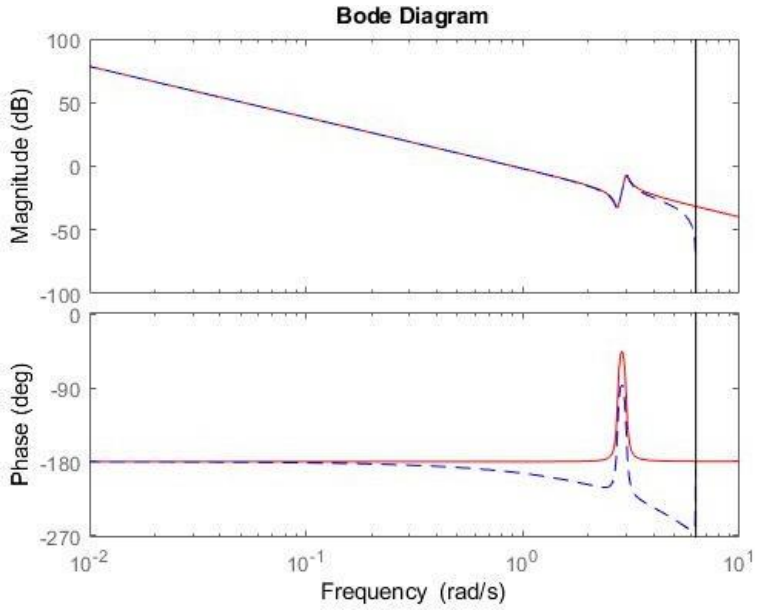


Figure-10.10: Output plot

The first LineSpec, 'r', specifies a solid red line for the response of H. The second LineSpec, 'b--', specifies a dashed blue line for the response of Hd.

Getting Magnitude and Phase Data for a System

The Bode plot of a system, G, shows the magnitude, G(jw)(in dB), and phase<G(jw) (degrees) over a range of frequencies. Often it is necessary to access this frequency response data directly, which is accomplished easily using the following commands:

MATLAB Code:

```
win = logspace(-2,2,5)
[mag,phase,wout] = bode(G,win);
size(wout)
size(mag)
size(phase)
```

Results:

```
win = 0.0100 0.1000 1.0000 10.0000 100.0000
ans = 5    1
ans = 1    1    5
ans = 1    1    5
```

where win is the input frequency vector in rad/s, mag is the magnitude in absolute units (i.e. not in dB), phase is the phase in degrees, and wout is the output frequencies at which the response was evaluated, useful for instance if you let MATLAB automatically choose the frequency vector by specifying only the range, win = {wmin, wmax}.

You'll notice that though wout is a standard (n x 1) column vector, the magnitude and phase are actually (1 x 1 x n). This is because the bode command accepts transfer function matrices in which case the (i,j,k) element of magnitude and phase corresponds to the jth input and ith output. For the SISO systems we most often encounter, i = j =1, these singleton dimensions are an inconvenience for matrix operations, etc. Fortunately MATLAB offers the function, squeeze, to fix just this problem:

```
mag = squeeze(mag)

phase = squeeze(phase)
```

Results:

```
mag = 1.0000  0.9950 0.7071 0.0995 0.0100

phase = -0.5729 -5.7106 -45.0000 -84.2894 -89.4271
```

Review exercise?

a) What is bode plot?

Bode plot is a graph of the frequency response of a system. It is usually a combination of a Bode magnitude plot, expressing the magnitude of the frequency response and a Bode phase plot, expressing the phase shift.

b) Why Bode plot is used?

Bode plot is like a map that shows how a system responds to different frequencies. It helps engineers understand and control things like stability, amplification, and filtering in systems like electronic circuits. By looking at a Bode plot, they can make sure a system works well at various frequencies and avoid problems like distortion or instability. It's a useful tool for designing and tweaking systems to do what they're supposed to do.

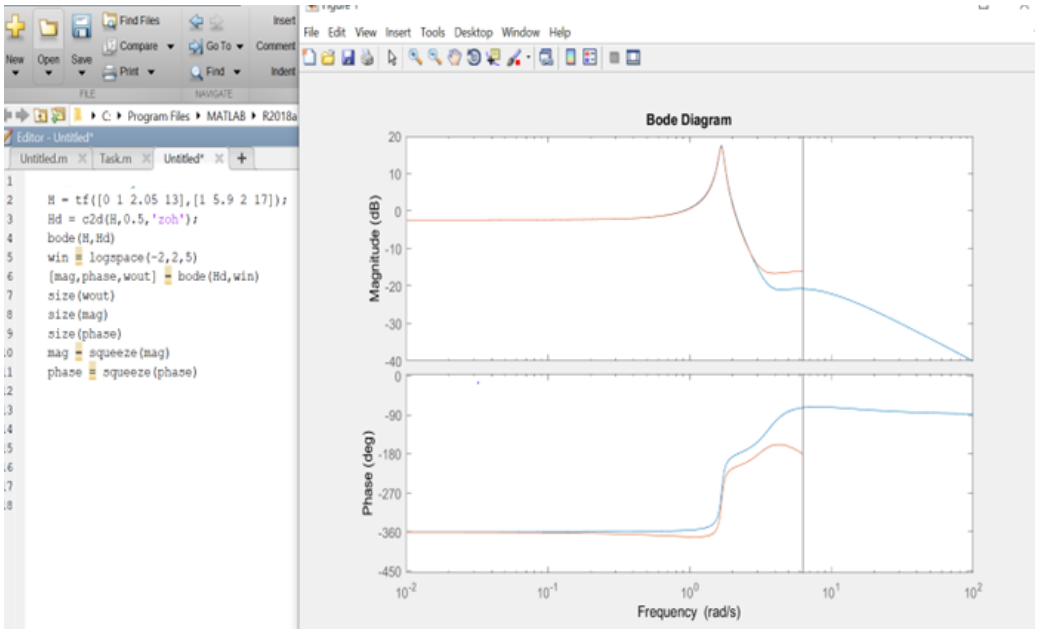
c) Define phase crossover frequency and gain cross over frequency in Bode plot?

The phase crossover frequency tells us when the phase of the system's response becomes zero degrees, and the gain crossover frequency indicates when the system's gain is unity. These frequencies are key parameters in Bode plots, aiding engineers in analyzing and designing systems, especially in the context of control theory and stability analysis.

d) Write the MATLAB Code to create the bode plot of continuous time and discrete timesystem?

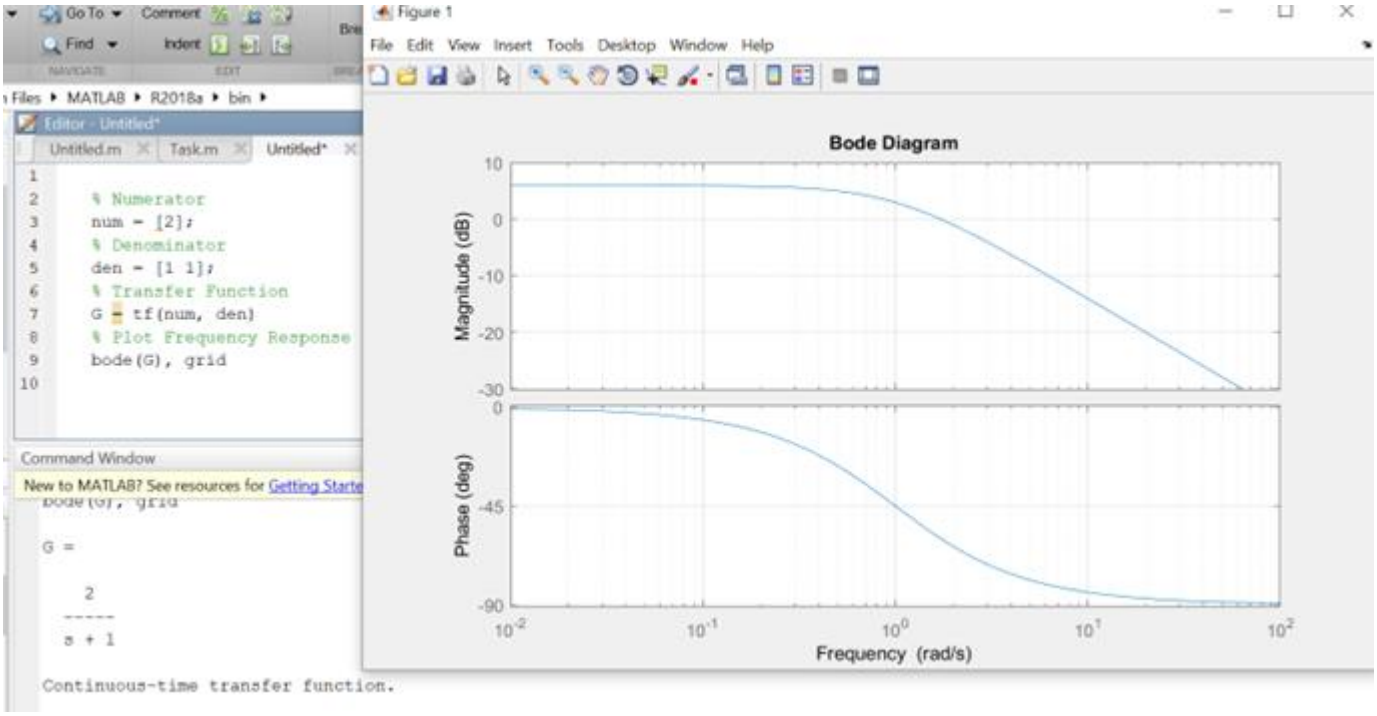
$$G(S) = \frac{S^2 + 2.05S + 13}{S^3 + 5.9S^2 + 2S + 17}$$

```
H = tf([0 1 2.05 13],[1 5.9 2 17]);
Hd = c2d(H,0.5,'zoh');
bode(H,Hd)
win = logspace(-2,2,5)
[mag,phase,wout] = bode(Hd,win)
size(wout)
size(mag)
size(phase)
mag = squeeze(mag)
phase = squeeze(phase)
```

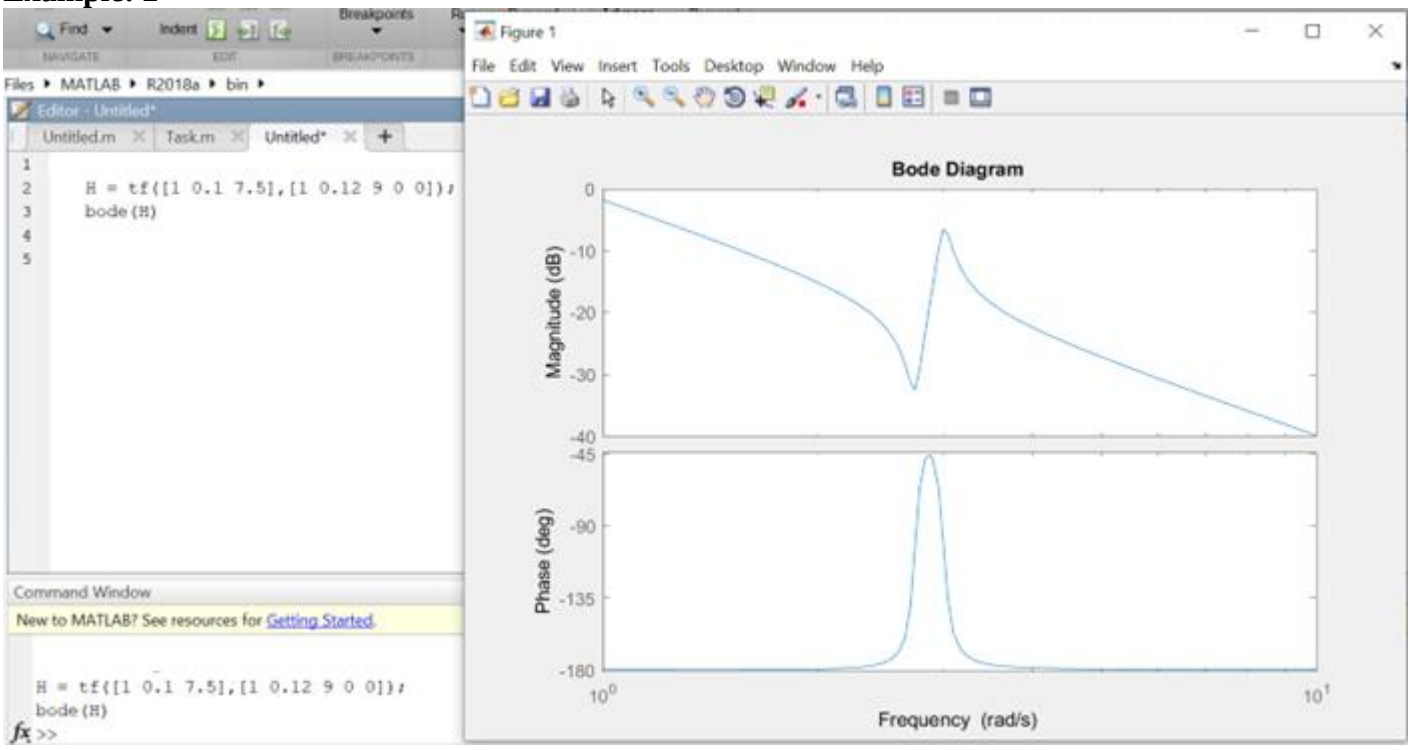


e) Attach the plots and results of all examples done in this laboratory

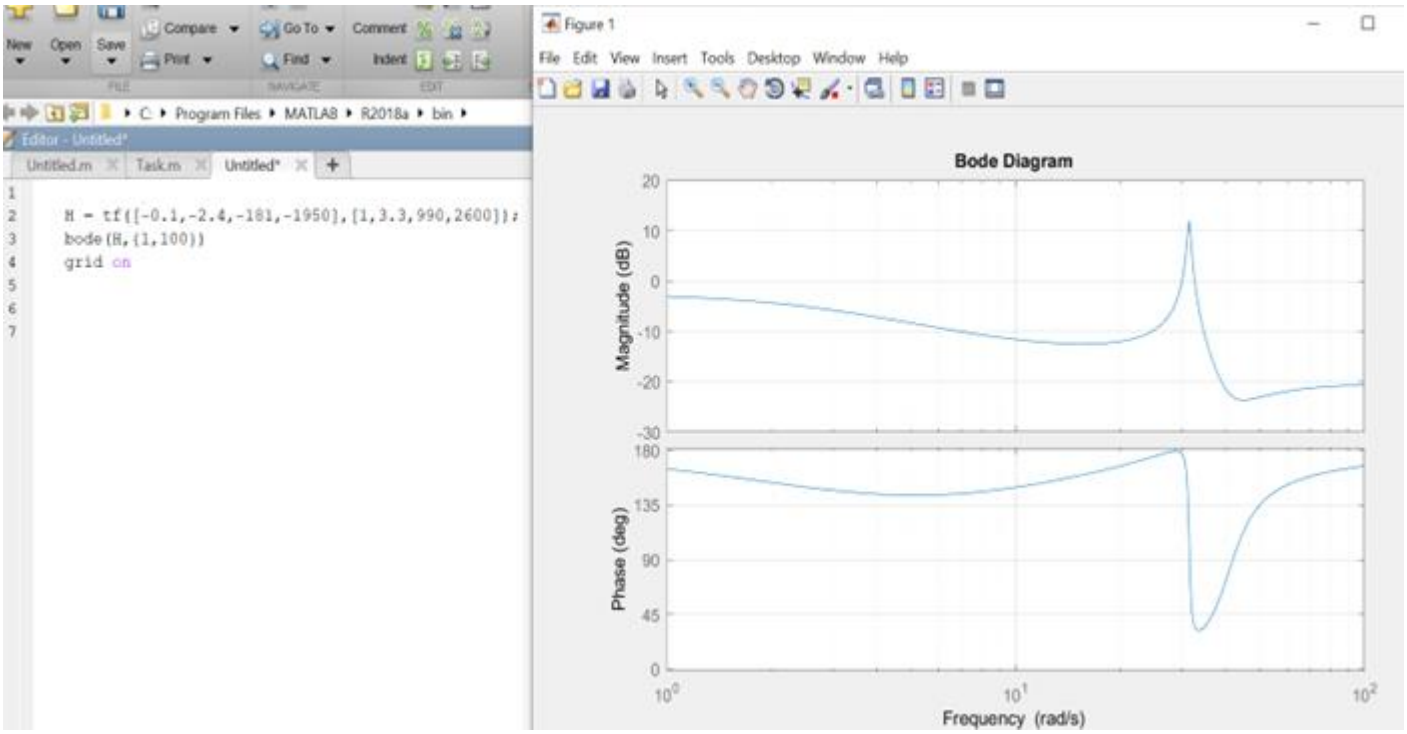
Example:1



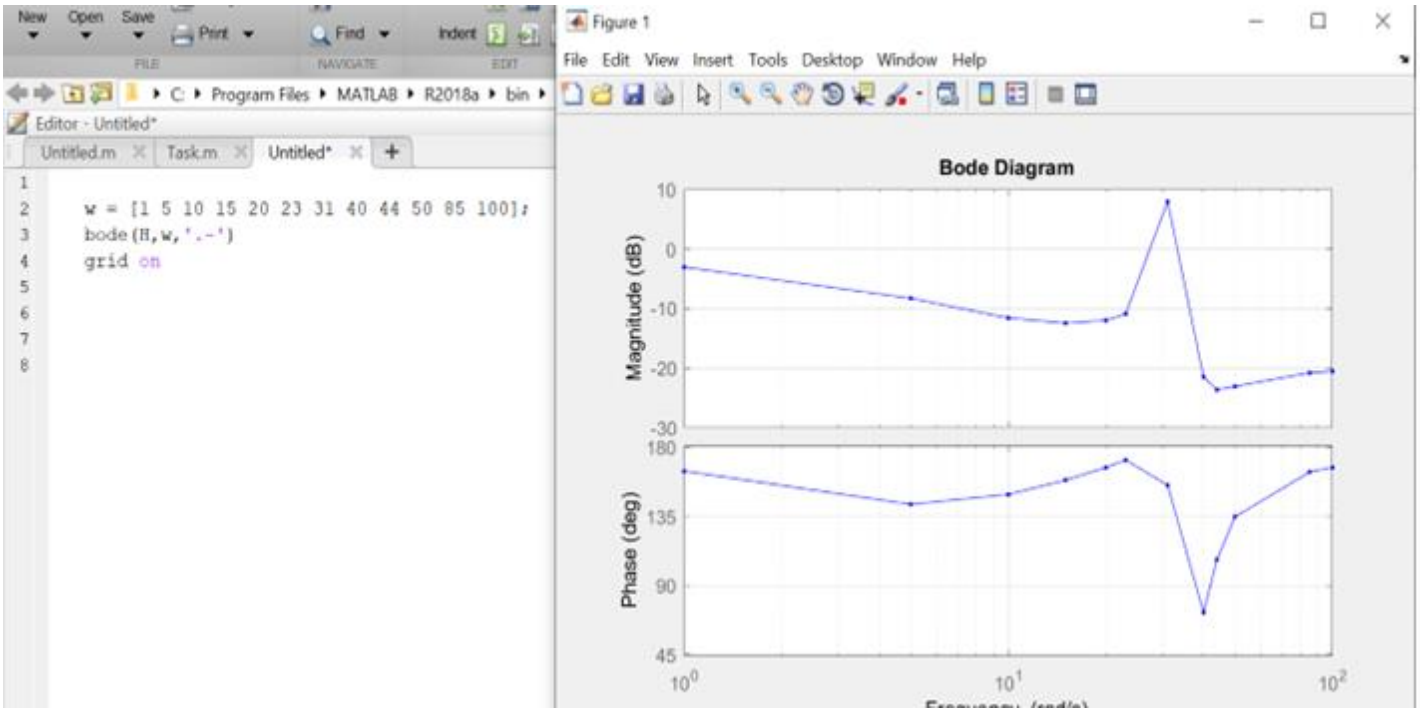
Example: 2



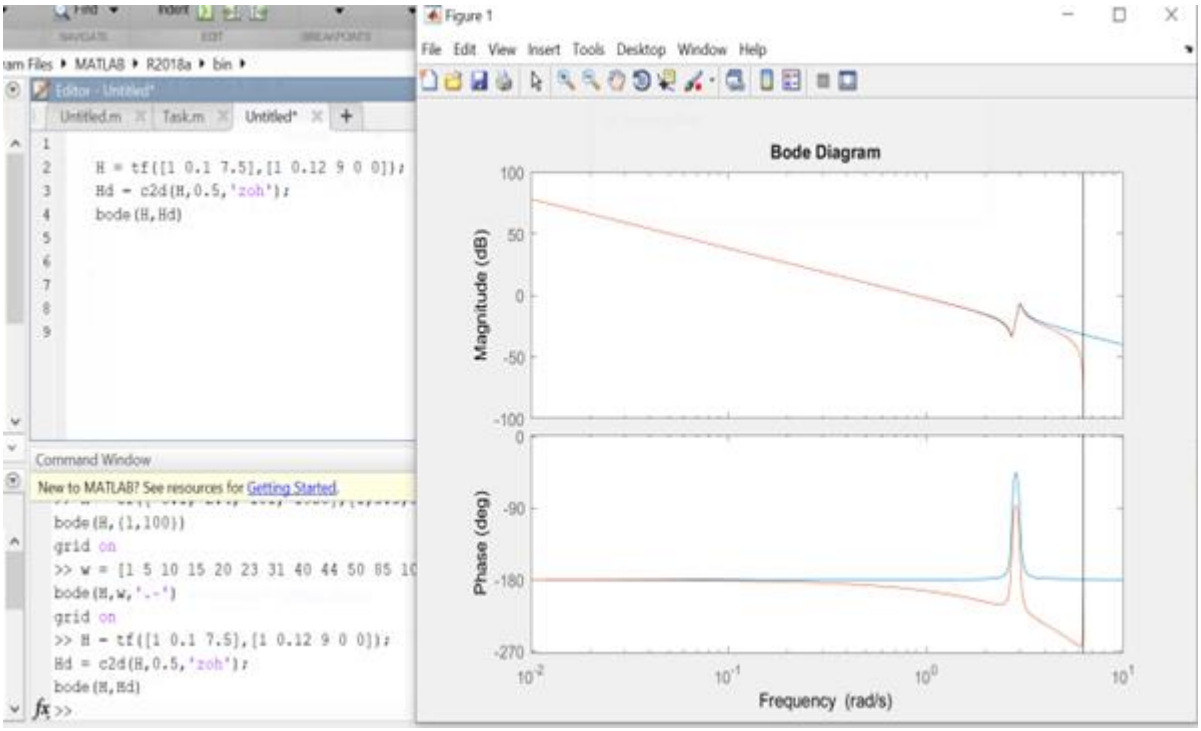
Example: 3



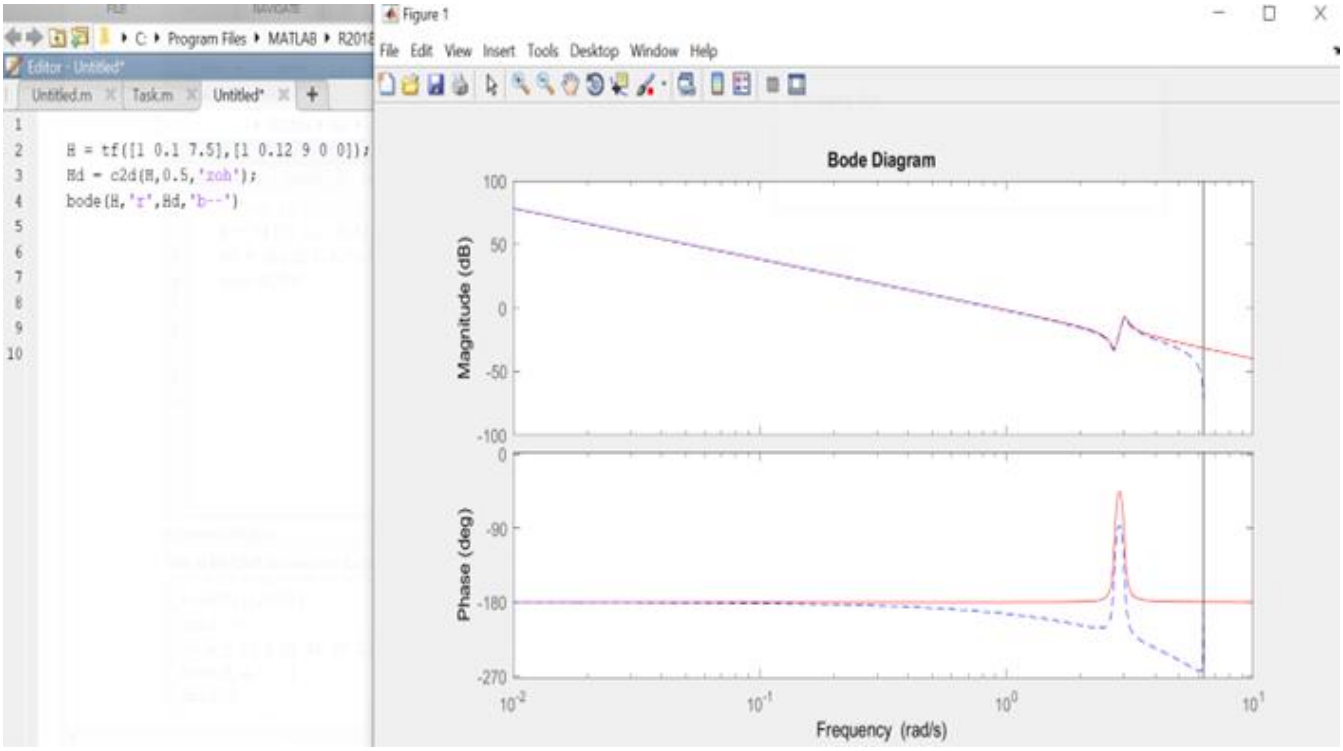
Example: 4



Example: 5



Example: 6



Example: 7

```
    win = logspace(-2,2,5)
    [mag,phase,wout] = bode(G,win);
    size(wout)
    size(mag)
    size(phase)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
win = logspace(-2,2,5)
[mag,phase,wout] = bode(G,win);
size(wout)
size(mag)
size(phase)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
[mag,phase,wout] = bode(G,win);
size(wout)
size(mag)
size(phase)

win =

    0.0100    0.1000    1.0000   10.0000  100.0000


ans =

     5     1


ans =

     1     1     5


ans =

     1     1     5

>>
```

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 11: PERFORMANCE OF PID AND DIGITAL PID CONTROLLER

Name: Karan Roll # 20ES062

Signature of Lab Tutor: _____ Date: _____

OBJECTIVE(S):

#	Topic	# Of Lectures	CLO	Taxonomy level
1	*To illustrate the performance of PID and digital PID controller using MATLAB/Simulink*	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

Equipment Required: Computer with MATLAB

Approximate Time Required: Two to three hours

Introduction

Digital PID Controller

A digital PID Controller works on the same principle as an analog PID. For a pulse transfer function a digital PID Controller is given as follows:

$G_D(z) = Kp+Ki*z/(z-1) + Kd*(z-1)/z$

Advantages of Digital PID Controllers

There are many useful options in digital controllers that are unavailable in analog controllers, such as variable tuning; selectable PID structure [P and D on process variable (PV) or error] and form; gap control; error squared; tuning algorithms; accurate tuning settings; and others. However, for loops that require a very fast response, the analog controller is faster than most digital controllers.

The characteristics of P, I, and D controllers

The proportional controller (K_P) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error.

An integral controller (K_I) will have the effect of eliminating the steady state error, but it may make the transient response worse.

A derivative control (K_D) will have the effect of increasing the stability of the system, reducing the overshoot and improving the transient response.

Effect of each controller K_P, K_I and K_D on the closed-loop system are summarized below:

CL Response	Rise Time	Overshoot	Settling Time	S-S Error
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increases	Eliminate
K_D	Small Change	Decreases	Decreases	Small Change

Table -11.1 Characteristics of Kp,Ki & Kd

MATLAB Code to design Digital PID Controller for plant given by

Example 1: G(s) = 1/s(s+1) & H(s) =1

```
close all
s=tf('s');
csys=1/(s*(s+1));
csys_closed=feedback(csys,1)
step(csys_closed)%cont: time step response
hold on
dsys=c2d(csys,1,'zoh');
dsys_closed=feedback(dsys,1)
step(dsys_closed,'y')%discrete time step response without pid
stepinfo(dsys_closed)%discrete time transient response ch.
%without pid
hold on
z=tf('z',1);
kp=1;ki=0.2;kd=0.2;
dpid=((kp+ki+kd)*z^2-(kp+2*kd)*z+kd)/(z^2-z) %pulse t/f of
%digital pid
dsys_closed_pid=feedback(dpid*dsys,1)
step(dsys_closed_pid,'g')%discrete time step response with pid
stepinfo(dsys_closed_pid)%discrete time transient response ch.
%with pid
```

Result

```
csys_closed =
1
_____
s^2 + s + 1
Continuous-time transfer function.
dsys_closed =
0.3679 z + 0.2642
_____________
z^2 - z + 0.6321
Sample time: 1 seconds
```

Discrete-time transfer function.
ans =
RiseTime: 1
SettlingTime: 16
SettlingMin: 0.8015
SettlingMax: 1.3996
Overshoot: 39.9576
Undershoot: 0
Peak: 1.3996
PeakTime: 3
dpid =
1.4 z^2 - 1.4 z + 0.2
---
z^2 - z
Sample time: 1 seconds
Discrete-time transfer function.
dsys_closed_pid =
0.515 z^3 - 0.1451 z^2 - 0.2964 z + 0.05285
---
z^4 - 1.853 z^3 + 1.591 z^2 - 0.6642 z + 0.05285
Sample time: 1 seconds
Discrete-time transfer function.
ans =
RiseTime: 1
SettlingTime: 18
SettlingMin: 0.7669
SettlingMax: 1.7079
Overshoot: 70.7898
Undershoot: 0
Peak: 1.7079
PeakTime: 3

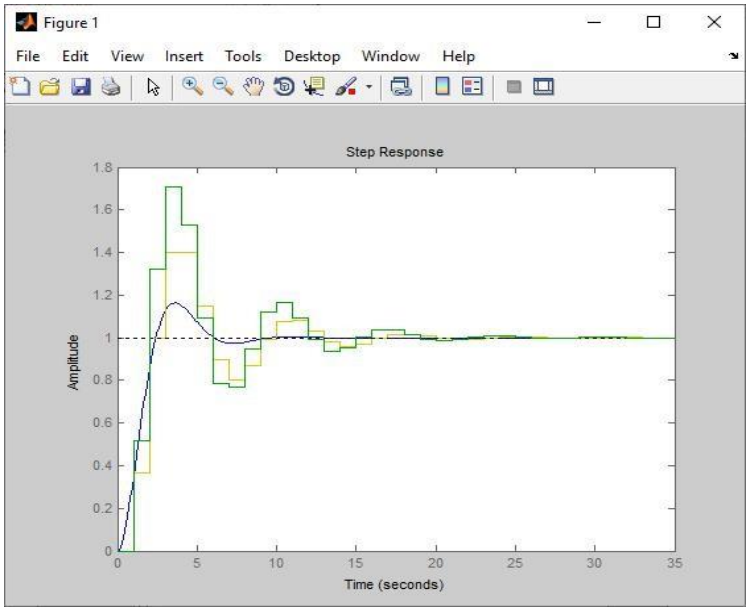


Figure-11.1: Output waveform

There are two ways to design a digital controller or a discrete-time control system. In first way design a continuous-time controller and then discretize it using ZOH or bilinear transformation or any discretization technique to obtain an equivalent digital controller as shown in Figure-11.2. Alternatively, one could discretize the plant first to obtain a sampled-data system or discrete-time system and then apply digital control system design techniques to design a digital controller as shown in Figure-11.3.

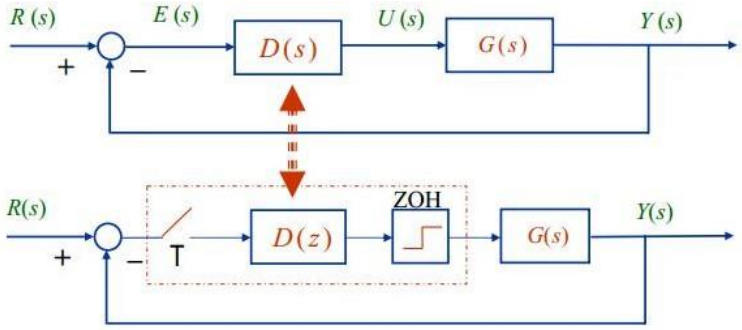


Figure-11.2: Digital controller using continuous-time controller

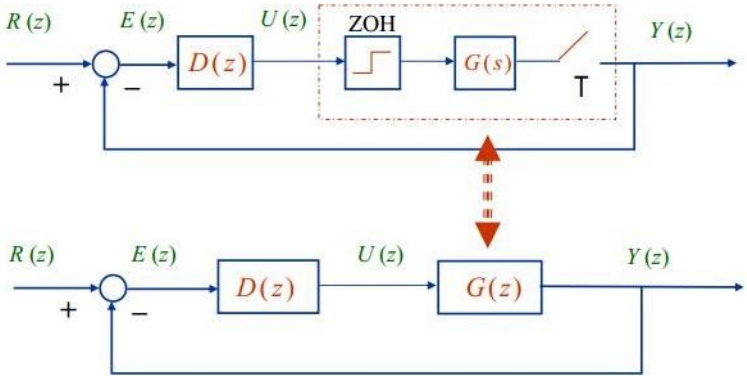


Figure-11.3 Digital controller using sampled-data system

The three term Proportional-Integral-Derivative (PID) control is still widely used in industries because of its simplicity. No need for a plant model and no design to be performed. The user just installs a controller and adjusts three gains to get the best achievable performance. PID regulation is the most mature and the most widely used technology of continuous system. The substance of its regulation is based on the deviation of the input value, a function of theproportional, integral and differential operator and the result of calculation for the output to control. In practical applications, depending on the circumstances, the structure of the PIDcontrol can be flexibly changed. Most PID controllers nowadays are digital.
For the continuous-time PID, start with the so-called parallel form

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt}e(t) \quad (1)$$

with its Laplace transform

$$G(s) = K_p + K_i \frac{1}{s} + K_D s \quad (2)$$

$$G(s) = K_p + K_i \frac{1}{s} + K_D s = K_p \left[1 + \frac{1}{T_i s} + T_D s\right] \quad (3)$$

Where K : proportional gain, K_i : integral gain, K_D : derivative gain, T_i : integral time constant and T_d : derivative time constant.

In MATLAB, the script code of parallel form may be represented by:

```
s = tf('s');  %% PID Parallel form
Kp=10;
Td=0.1;
Ti=0.1;
G = Kp  (1 + 1/(Ti*s) + Td*s)
```

The control parameters are:

- The proportional term: providing an overall control action proportional to the error signal through the constant gain factor.
- The integral term: the action is to reduce steady-state errors through low-frequency compensation by an integrator.
- The derivative term: improves transient response through high-frequency compensation by a differentiator.

The very same system may be designed at SIMULINK Toolbox, represented in Figure-8.3

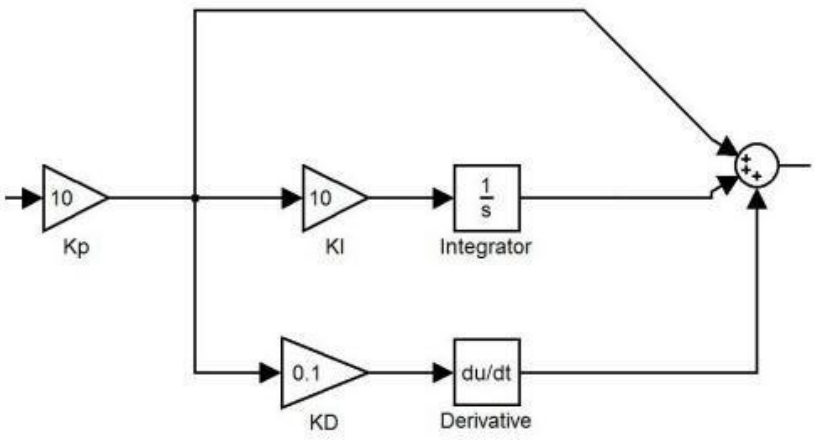


Figure-11.4 Simulink PID Control

11.1 Discrete-time PID Algorithm

For digital implementation, z-transform of equation-1 becomes:

$$U(z)=\left[K_p+\frac{K_i}{1-z^{-1}}+K_d(1-z^{-1})\right]E(z)$$

11.1.1 Example: 2

Consider a unity feedback system with forward path transfer function:

$$G_{(s)}=\frac{1}{s^2+10s+20}$$

Convert this system in z-domain with $T = 0\text{t}1$ sec. Show the effect of addition of a PD controller on the system performance.Eliminate overshoots in the controller

```
num=1;
den=[1 10 20];
g1=tf (num,den)
t1=feedback(g1,1)
d1=c2d(t1,.1,'zoh')
step(d1,'g')
hold on
num1=10;
den1=[1 10 20];
g2=tf  (num1,den1)
t2=feedback(g2,1)
d2=c2d(t2,.1, 'zoh')
step(d2,'m')
hold on
Kp=500;
Kd=10;
numc=[Kd Kp];
numo=conv(numc,num)
deno=den
g3=tf(numo,deno)
t3=feedback(g3,1)
d3= c2d(t3,.1,'zoh')
step(d3,'b')
hold on
Kp=500;
Kd=5;
numc=[Kd Kp];
numo=conv(numc,num)
deno=den
g3=tf(numo,deno)
t3=feedback(g3,1)
d3= c2d(t3,.1,'zoh')
step(d3,'y')
hold on
Kp=500;
Kd=.01;
numc=[Kd Kp];
numo=conv(numc,num)
deno=den
g3=tf(numo,deno)
t3=feedback(g3,1)
d3= c2d(t3,.1,'zoh')
step(d3,'r')
hold on
```

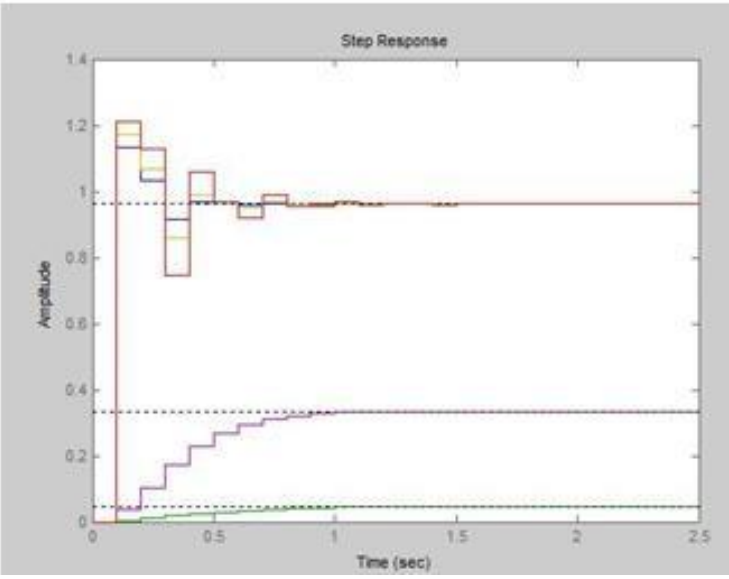


Figure-11.5 output results

11.2 Review Exercise

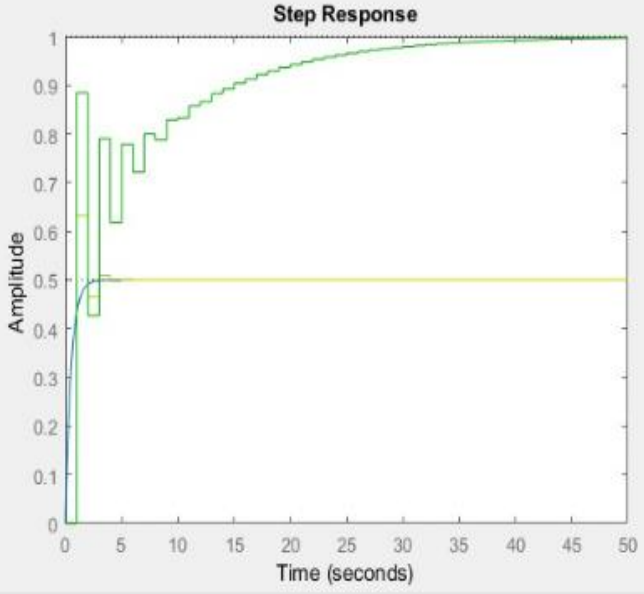
a) What is digital PID controller?

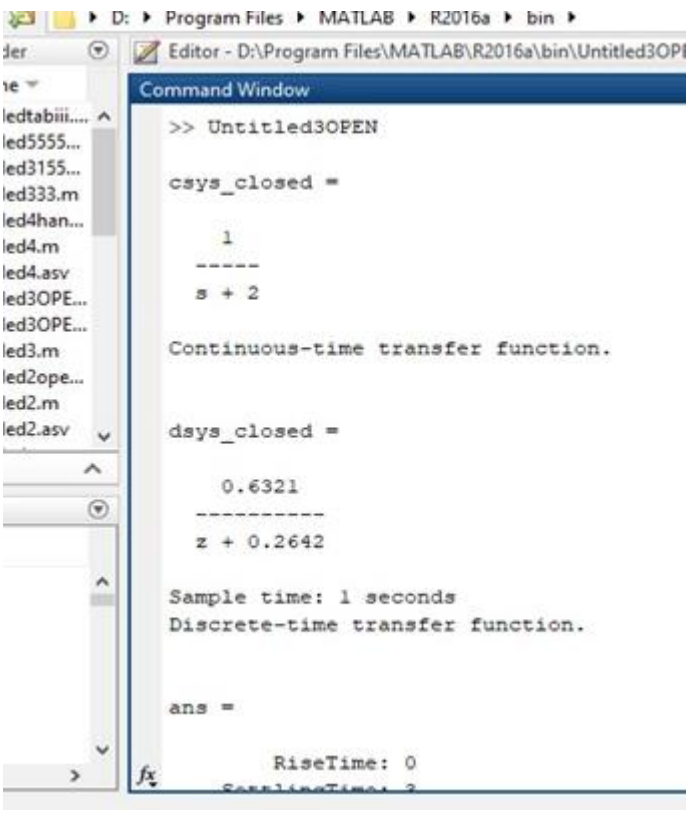
A Digital Proportional-Integral-Derivative (PID) controller is a type of feedback control system commonly used in industrial processes and various control applications. PID controllers are designed to regulate a system's output by continuously adjusting the input based on the difference between the desired setpoint and the actual process variable. A digital PID Controller works on the same principle as an analog PID. For a pulse transferfunction a digital PID Controller is given as follows:

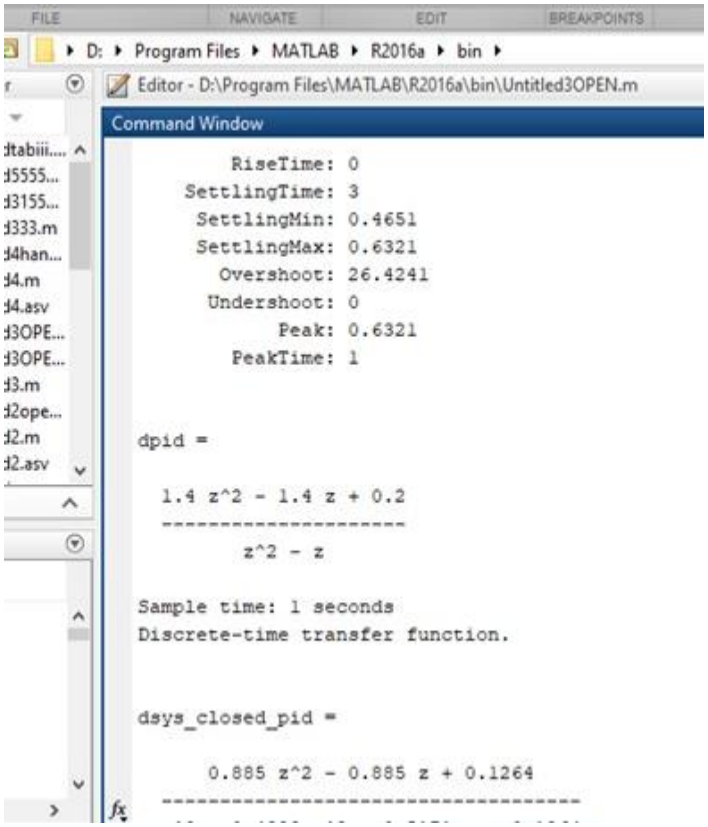
GD(z) = Kp+Ki*z/(z-1) + Kd*(z-1)/z

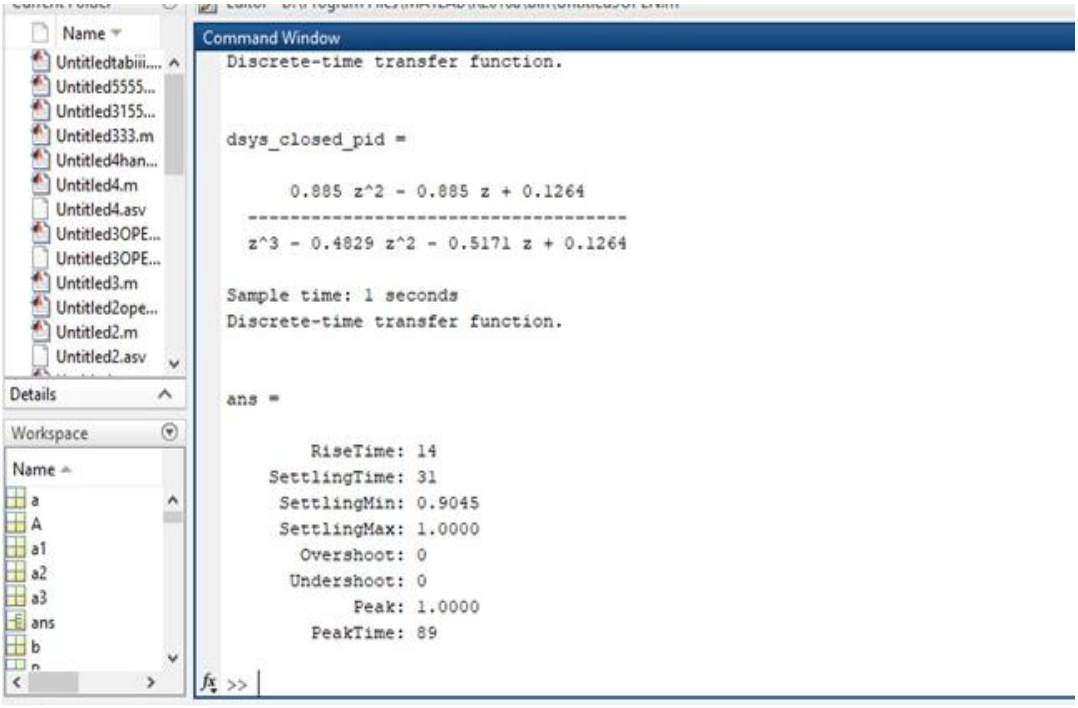
b) Write a MATLAB code to design Digital PID Controller for plant G(s) = 1/(s+1) & H(s) =1. Choosing an appropriate sampling time, tune Kp, Ki & Kd to get an optimized step response.

```
close all
s=tf('s');
csys=1/(1*(s+1));
csys_closed=feedback(csys, 1)
step(csys_closed)   %cont: time step response
hold on
dsys=c2d(csys,1,'zoh');
dsys_closed=feedback(dsys, 1)
step(dsys_closed, 'y') %discrete time step response without pid
stepinfo(dsys_closed) %discrete time transient response ch.
%without pid
hold on
z=tf('z', 1);
kp=1;ki=0.2;kd=0.2;
dpid=((kp+ki+kd)*z^2-(kp+2*kd)*z+kd)/(z^2-z) %pulse t/f of
%digital pid
dsys_closed_pid=feedback(dpid*dsys, 1)
step (dsys_closed_pid,'g')%discrete time step response with pid
stepinfo(dsys_closed_pid)%discrete time transient response ch.
%with pid
```




D: ▸ Program Files ▸ MATLAB ▸ R2016a ▸ bin ▸
Editor - D:\Program Files\MATLAB\R2016a\bin\Untitled3OPI
Command Window
>> Untitled3OPEN
csys_closed =
1
-----
s + 2
Continuous-time transfer function.
dsys_closed =
0.6321
----
z + 0.2642
Sample time: 1 seconds
Discrete-time transfer function.
ans =
RiseTime: 0

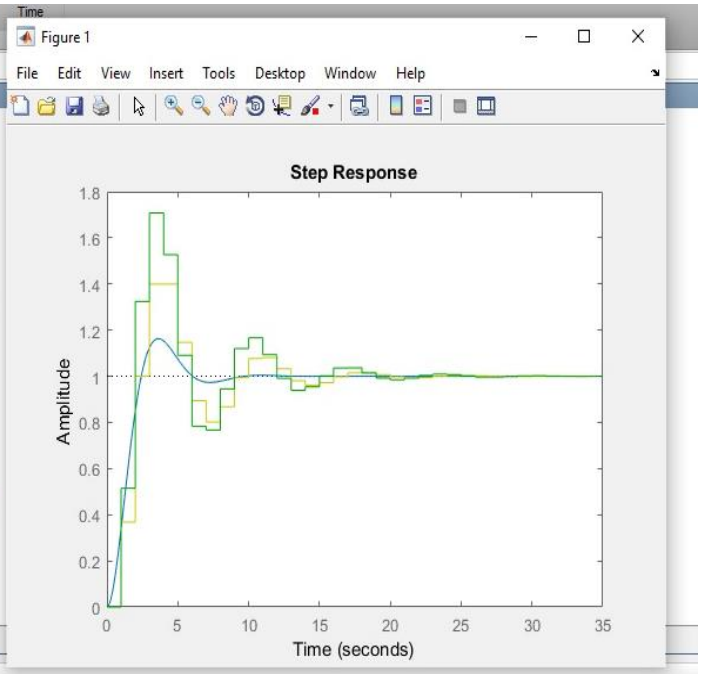

FILE
NAVIGATE
EDIT
BREAKPOINTS
D: ▸ Program Files ▸ MATLAB ▸ R2016a ▸ bin ▸
Editor - D:\Program Files\MATLAB\R2016a\bin\Untitled3OPEN.m
Command Window
RiseTime: 0
SettlingTime: 3
SettlingMin: 0.4651
SettlingMax: 0.6321
Overshoot: 26.4241
Undershoot: 0
Peak: 0.6321
PeakTime: 1
dpid =
1.4 z^2 - 1.4 z + 0.2
----
z^2 - z
Sample time: 1 seconds
Discrete-time transfer function.
dsys_closed_pid =
0.885 z^2 - 0.885 z + 0.1264


Name
Untitledtabiii....
Untitled5555...
Untitled3155...
Untitled333.m
Untitled4han...
Untitled4.m
Untitled4.asv
Untitled3OPE...
Untitled3OPE...
Untitled3.m
Untitled2ope...
Untitled2.m
Untitled2.asv
Details
Workspace
Name
a
A
a1
a2
a3
ans
b
Command Window
Discrete-time transfer function.
dsys_closed_pid =
0.885 z^2 - 0.885 z + 0.1264
z^3 - 0.4829 z^2 - 0.5171 z + 0.1264
Sample time: 1 seconds
Discrete-time transfer function.
ans =
RiseTime: 14
SettlingTime: 31
SettlingMin: 0.9045
SettlingMax: 1.0000
Overshoot: 0
Undershoot: 0
Peak: 1.0000
PeakTime: 89
>>

c) Attach the code and results of example1 and example2.

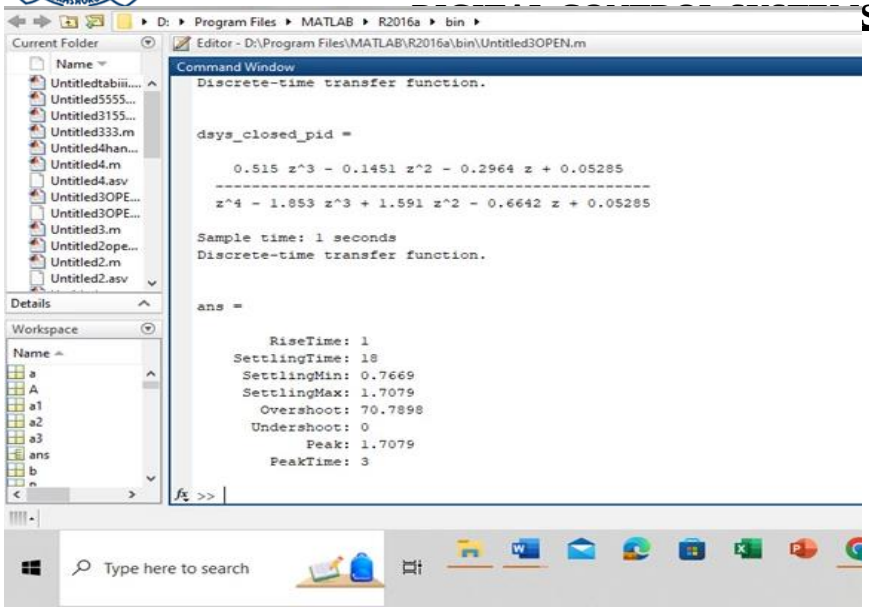
Example:1

```
%MATLAB CODE 20ES029
close all
s=tf('s'); csys=1/(s*(s+1));
csys_closed=feedback(csys,1)
step(csys_closed)%cont: time step response
hold on
dsys=c2d(csys,1,'zoh');
dsys_closed=feedback(dsys,1)
step(dsys_closed,'y')%discrete time step response without pid
stepinfo(dsys_closed)%discrete time transient response ch.
%without pid
hold on z=tf('z',1);
kp=1;ki=0.2;kd=0.2;
dpid=((kp+ki+kd)*z^2-(kp+2*kd)*z+kd)/(z^2-z) %pulse t/f of
%digital pid
dsys_closed_pid=feedback(dpid*dsys,1)
step(dsys_closed_pid,'g')%discrete time step response with pid
stepinfo(dsys_closed_pid)%discrete time transient response ch.
%with pid
```




```
>> Untitled3OPEN

csys_closed =

          1
  -----------
  s^2 + s + 1

Continuous-time transfer function.


dsys_closed =

  0.3679 z + 0.2642
  -----------------
  z^2 - z + 0.6321

Sample time: 1 seconds
Discrete-time transfer function.


ans =

        RiseTime: 1
```

```
ans =

        RiseTime: 1
    SettlingTime: 16
     SettlingMin: 0.8015
     SettlingMax: 1.3996
       Overshoot: 39.9576
      Undershoot: 0
            Peak: 1.3996
        PeakTime: 3


dpid =

  1.4 z^2 - 1.4 z + 0.2
  ---------------------
         z^2 - z

Sample time: 1 seconds
Discrete-time transfer function.


dsys_closed_pid =

    0.515 z^3 - 0.1451 z^2 - 0.2964 z + 0.05285
```

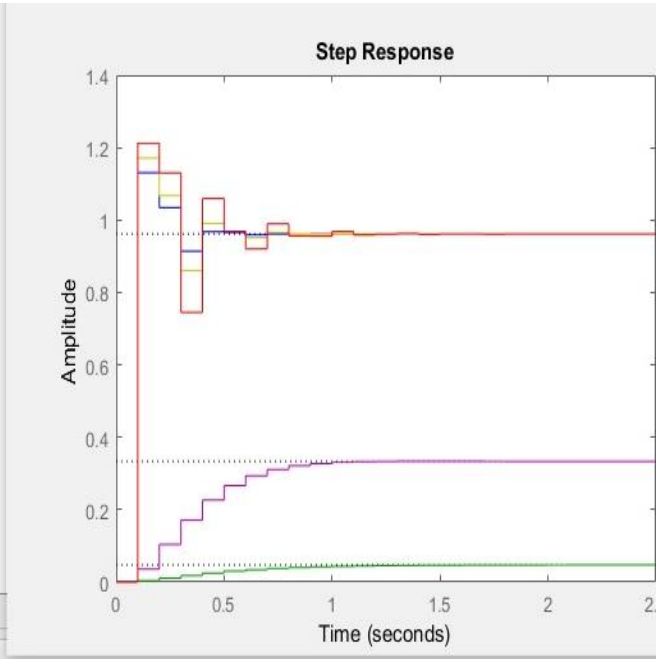


Example:2

```
num=1;
den=[1 10 20];g1=tf (num,den)
t1=feedback(g1,1) d1=c2d(t1,.1,'zoh')step(d1,'g')
hold on
num1=10;den1=[1 10 20];g2=tf (num1,den1) t2=feedback(g2,1)
d2=c2d(t2,.1, 'zoh')step(d2,'m')
hold on
Kp=500; Kd=10;numc=[Kd Kp];
numo=conv(numc,num) deno=den
g3=tf(numo,deno) t3=feedback(g3,1)
d3= c2d(t3,.1,'zoh')step(d3,'b')
hold on
Kp=500; Kd=5;numc=[Kd Kp]; numo=conv(numc,num) deno=den
g3=tf(numo,deno) t3=feedback(g3,1)
d3= c2d(t3,.1,'zoh')step(d3,'y')
hold on
Kp=500; Kd=.01;numc=[Kd Kp]; numo=conv(numc,num)
deno=den g3=tf(numo,deno) t3=feedback(g3,1)
d3= c2d(t3,.1,'zoh')
step(d3,'r')
hold on
```

Untitled333.m | Untitled3.m | Untitled3OPEN.m

Command Window



MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 12: TUNING OF DIGITAL PID CONTROLLER USING SIMULINK

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	Tuning of digital PID controller using simulink	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: PC with MATLAB

Approximate Time Required: Two to three hours

Introduction

Tuning of PID Controller

PID tuning is the process of finding the values of proportional, integral, and derivativegains of a PID controller to achieve desired performance and meet design requirements. PID controller tuning appears easy, but finding the set of gains that ensures the best performance of your control system is a complex task.

Traditionally, PID controllers are tuned either manually or using rule-based methods. Manual tuning methods are iterative and time-consuming, and if used on hardware, they can cause damage. Rule-based methods also have serious limitations: they do not support certain types of plant models, such as unstable plants, high-order plants, or plants with little or no time delay.

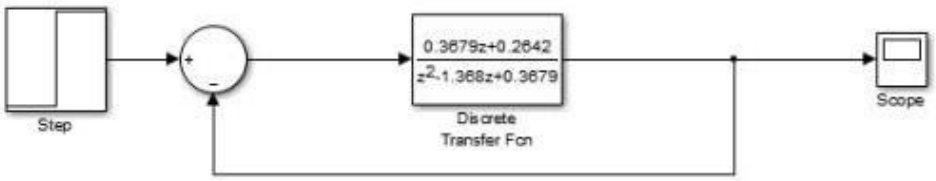
PID tuning using Simulink

The Simulink Control Design™ tools let you tune single-loop control systems

Example: Consider a plant given by

$G(z) = (0.3679z+0.2642)/(z^2-1.368z+0.3679)$ & $H(z) = 1$

First, the closed-loop step response of the plant can be observed using the followingSimulink model:



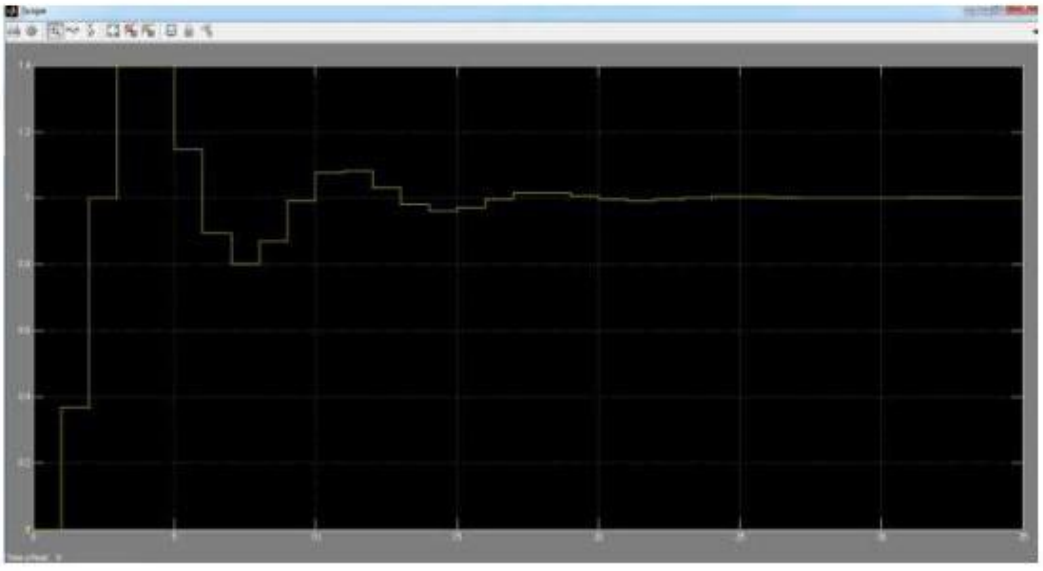


Figure-12.1 Closed Loop Response of plant G(z)

Then, keeping in view the pulse transfer function of a Digital PID Controller, i.e.
$G_D(z) = Kp+Ki*z/(z-1) + Kd*(z-1)/z$

a Simulink model for digital PID controller can be designed using Simulink blocks such as Discrete-Transfer Fcn Block(s), Gain Block(s), a source block such as Step Block and a sink block such as Scope Block.
Simulink Design of a digital PID controller for the plant G(z) specified before, is shown as follows.
Here Kp=1, Ki= 0.2 and Kd= 0.2.

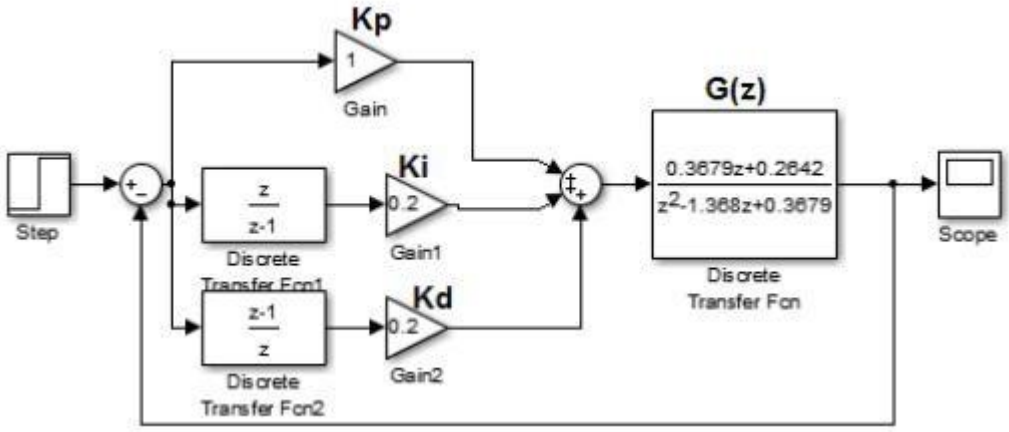


Figure 12.2- Simulink diagram

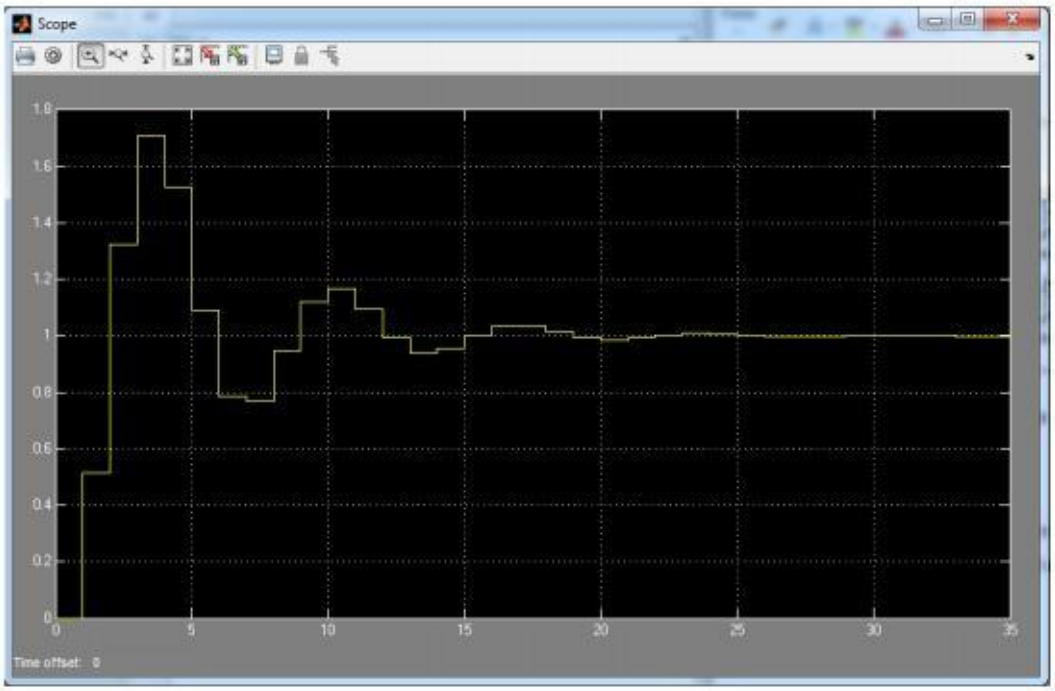


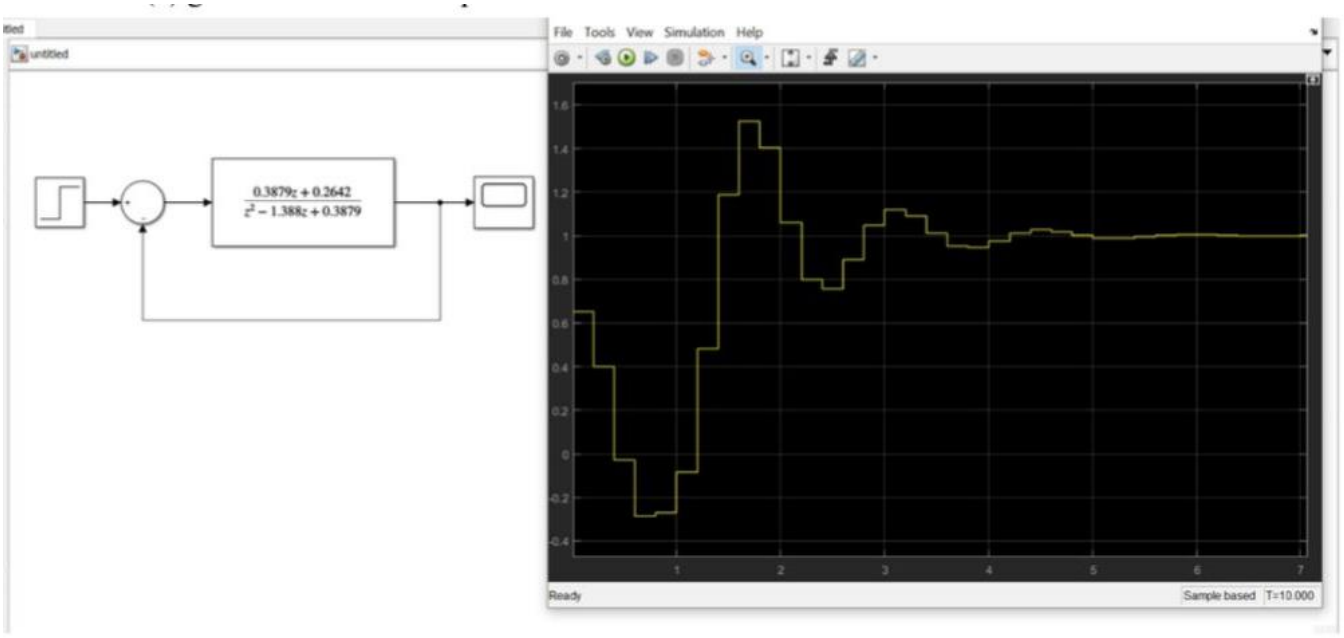
Figure -12.3 Simulink Design of digital PID controller for the plant G(z)

These values of Kp, Ki and Kd are tuned on hit-&-trial basis in order to obtain desired system response.
Likewise, you can design digital PI- Controller, PD-Controller in Simulink

Review Exercise?

a) Using Simulink, perform PID tuning to get an optimized step response of the plant G(z) given in the lab example

.



MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 13: OPERATION OF COMPENSATOR & DESIGN OF LEAD COMPENSATOR

Name: karan ,	Roll # 20ES062 ,
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	To illustrate the operation of compensator and design of Lead compensator	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

LAB RUBRIC(S)

Equipment Required: PC with MATLAB

Approximate Time Required: Two to three hours

Introduction

A feedback control system that provides in optimum performance without any necessary adjustment is rare, in building a control system the proper modification of plant dynamics may be a simple way to meet the performance specifications, this however may not be possible in many practical situations because the plant may be fixed and not modifiable then we must adjust parameters other than those in the fixed plant.

It is then required to reconsider the structure of the system and redesign the system, the design problems therefore become those of improving system performance by insertion of compensator

Compensator

Compensator is an additional component or circuit that is inserted into a control system to equalize or compensate for a deficient performance Compensation schemes commonly used for feedback control systems are

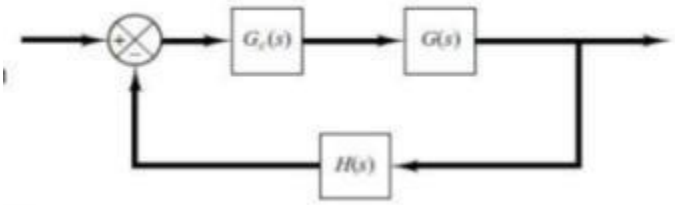


Figure-13.1: Series compensator

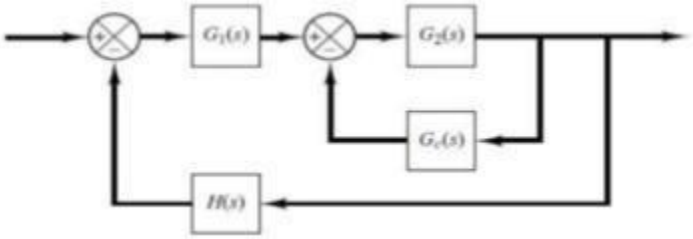


Figure-13.2: Parallel Compensator

Phase Lead or Lag Compensator

A discrete time phase lead /lag compensator is designed by using the bilinear transformation that maps the inside of the unit circle in Z-Plan to the entire the LHP of w-plane. The phase lead lag compensator is designed in the w-domain utilizing the techniques available for continuous time compensator design, then map it back to the z-domain by the bilinear transformation. The first order w-domain compensator is given by analogy to s-domain as

$$D(W) = a\,0\frac{1+\frac{W}{Ww0}}{1+\frac{W}{Ww0}} \qquad 1)$$

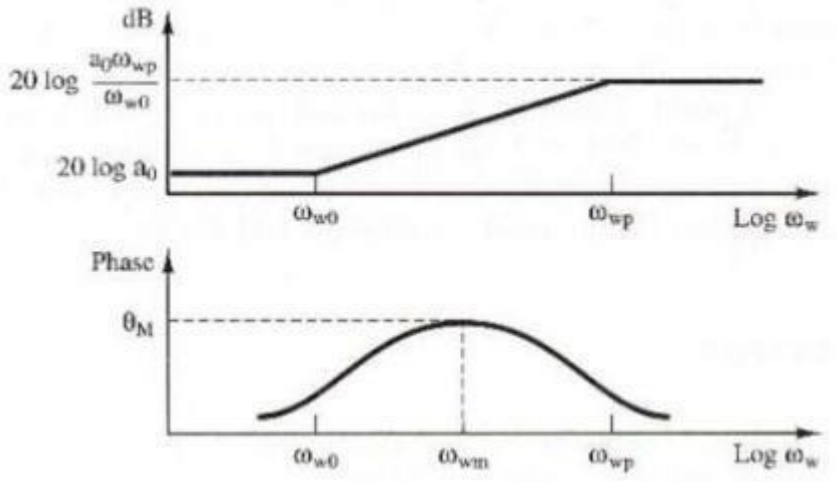


Figure-13.3: a Pole at $w = \ -wnp$ and a zero is at $w = \ -Wwo$

Phase lead compensator: $Wwp > \ Wwo$

Substituting $W = \frac{2z-1}{Tz+1}$ for w in the compensator transfer function

$$D(z) \ = \frac{1+\frac{\frac{2z-1}{Tz+1}}{\omega o}}{1+\frac{\frac{2z-1}{Tz+1}}{\omega wp}} \qquad 2)$$

$$= \ ao\frac{\omega wo\,\omega T(Z+1)+2\omega wp(z-1)}{\omega wo\,\omega T(Z+1)+2\omega wo(z-1)} \qquad 3)$$

Digital Compensator Design

1 Frequency Response Method

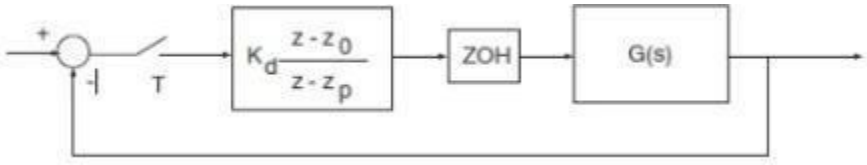


Figure-13.4: Digital compensator design block diagram

Design a phase Lead or Phase lag compensator

$$D(z) = \frac{kd(z-zo)}{z-zp}$$

Such that the (open loop system satisfies a given phase margin of $\varphi m \approx (100\varepsilon)$ so that the close loop system has damping ratio of ε since the frequency response method based on phase margin

and gain margin established in the Laplace domain (s-domain) is used here the frequency response of $D(z)$ combined with a plant into $w-domain$ by using the bilinear transformation because stable region of the z-domain plan that is the inside of a unit circle is mapped into the entire $LHP\ (Left\ half\ plan)$ of the w- plane. The parameters kd, zc and zp of the first order phase lead.lag compensator are determined in terms of $a0$, Wwo and Wwp the $w-domian$

Design a phase lag compensator that achieve a phase margin of
$\varphi m = 55°\ for\ ao = 1$

$$< G(J\omega\omega 1) = -180° + 55° + 5° = -120°$$

By the bode diagram $\omega\omega 1 \approx 0.36$. at this frequency
$G(J\omega\omega 1) = 2.57 = 8.2\,DB$

therefore
$\omega\omega o = 0.1\omega\omega 1 = 0.036$

$$\omega\omega\rho = \frac{0.1\omega\omega 1}{ao[G(J\omega 1)]} = 0.036$$

Using these values $of\ ao, wwo\ and\ wwp$

$$D(z) = 0.3891\ \frac{z - 0.9982}{z - 0.9993}$$

The frequency response after compensator is shown in the frequency axis of $\omega\omega$

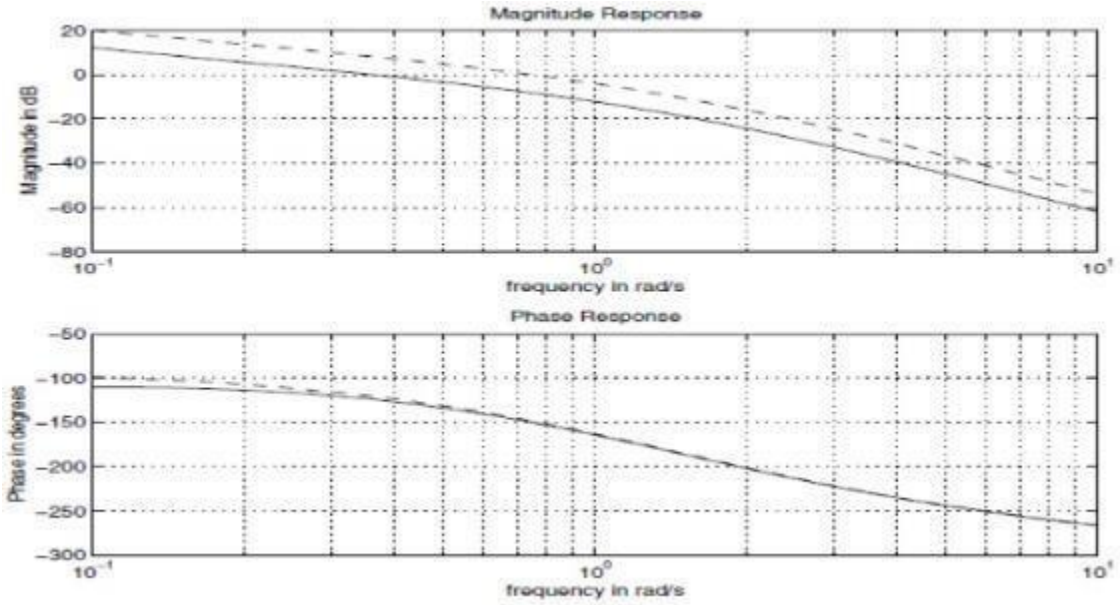
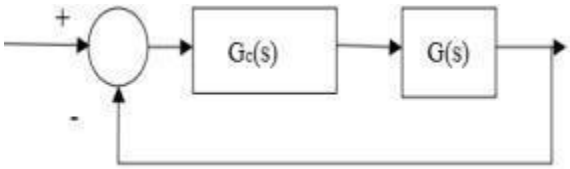


Figure-13.5: compensator output response

Matlab Code For Phase Lead Compensation




```
wm = 4.5;
 alpha = 0.3; T =
1/(wm*sqrt(alpha)); k=
10; gnum = [k];
gden = [1 1 0]; uncompensated = tf(gnum,gden)
cnum = [T 1];
cden = [T*alpha 1];
compensator = tf(cnum,cden) numo = conv(cnum,gnum); deno
= conv(cden,gden);
compensated = tf(numo,deno) bode(uncompensated,'r',compensated ,'g')
uncomtr=feedback(uncompensated,1)
comtr=feedback(compensated,1) step(uncomtr,'y')
hold on step(comtr,'b')
```

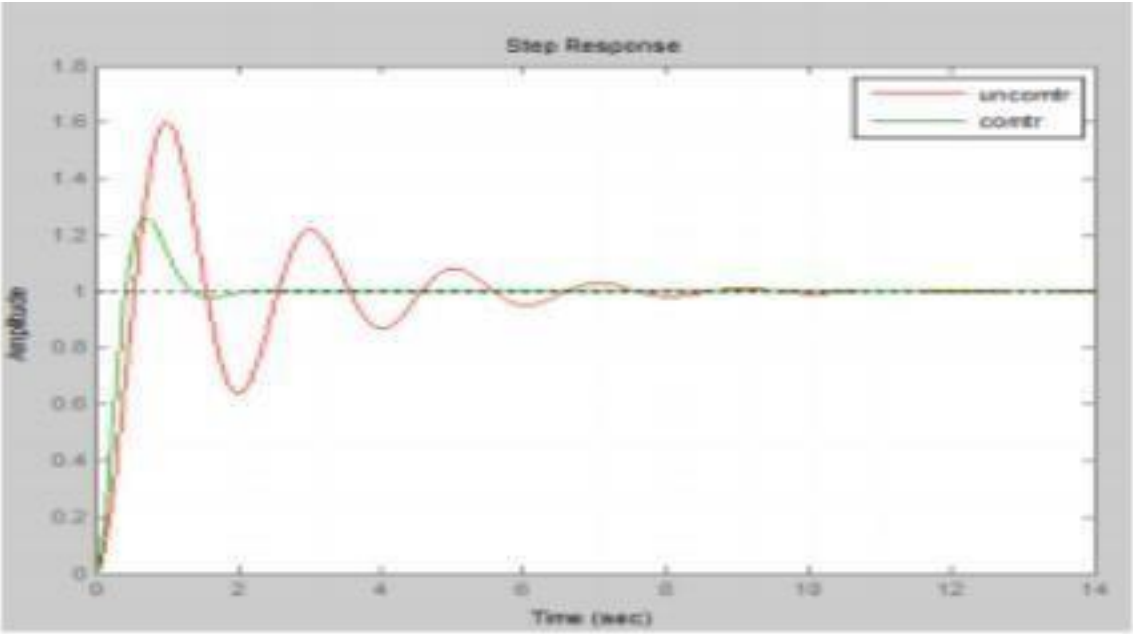


Figure-13.6: Output response of Phase Lead compensator

Design of lead Compensator :

Example: Design a lead compensator for the following system. The system achieve the following requirement. Mp≤20% , tr≤0.3 sec

$$G(S) = \frac{1}{s(s+1)}$$

MATLAB Code:

```
G=tf(1, [1 1 0]) z=
8.0501 k= 4.472
C=tf(k*[1 z],[1])
T=feedback(C*G,1
)
step(T) Sisotool(G)
```

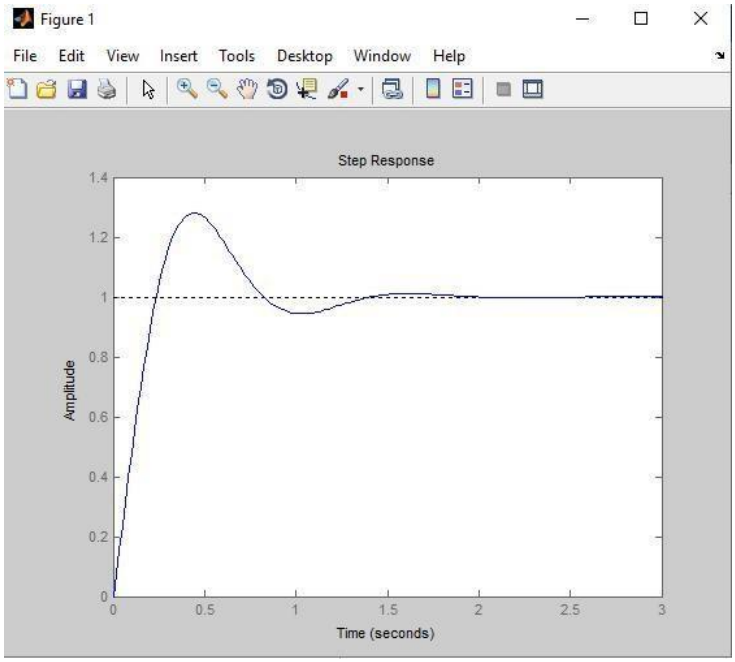


Figure-13.7: Output response of Phase Lead

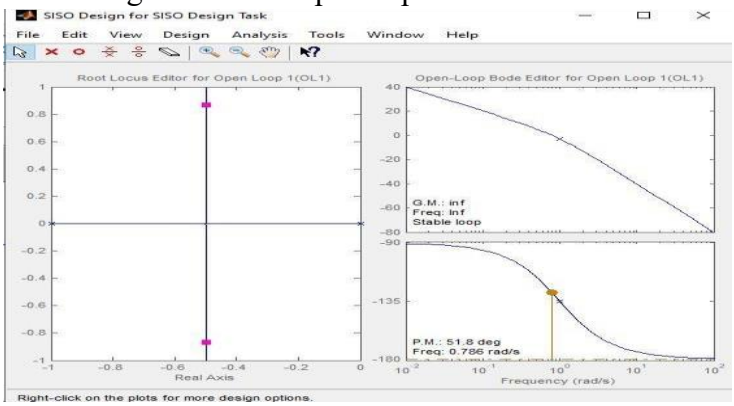


Figure-13.8: Output response using SISO tool

Example2:

```
Gv = tf(0.02,[4 1]); G1 = tf(70,[50 1]); H = tf(1,[12 1]);
Gc = tf(1); % dummy static gain

Gol = Gc*Gv*G1; % Openloop: I'm assuming Gp = G1
Gcl = feedback(Gol,H); % closed loop
step(Gcl) % look at the response stepinfo(Gcl) %
```

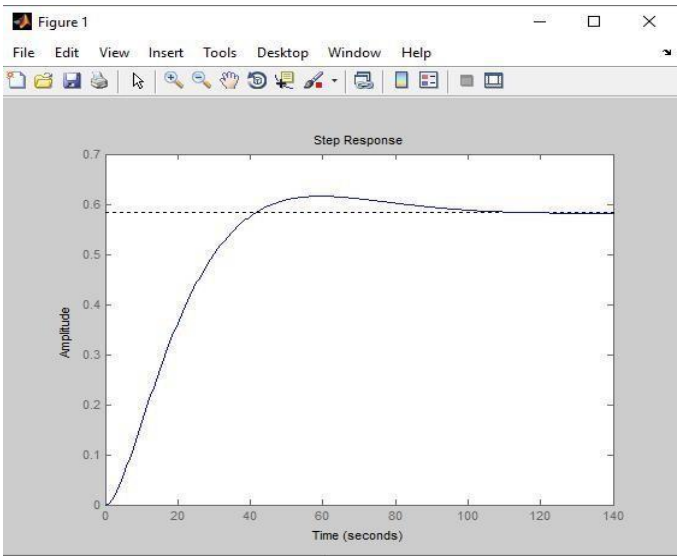


Figure-13.9: Output response of compensator

ans =

RiseTime: 27.5867
SettlingTime: 88.7066
SettlingMin: 0.5274
SettlingMax: 0.6159 Overshoot:
5.5879
Undershoot: 0
Peak: 0.6159
PeakTime: 59.4622

Example3:

```
% ---------- Unit ramp response ----------

% ***** Unit-ramp responses of compensated system and
% uncompensated system *****

% ***** Unit-ramp response will be obtained as the unit-step
% response of C(s)/[sR(s)] *****

% ***** Enter the numerators and denominators of C1(s)/[sR(s)]
% and C2(s)/[sR(s)], where C1(s) and C2(s) are Laplace
% transforms of the outputs of the compensated and un-
% compensated systems, respectively. *****

numc = [0   0   0   0   1.0235   0.0512];
denc = [1   3.005   2.015   1.0335   0.0512   0];
num = [0   0   0   0   1.06];
den = [1   3   2   1.06   0];

% ***** Specify the time range (such as t = 0:0.1:50) and enter
% step command and plot command. *****

t = 0:0.1:50;
[c1,x1,t] = step(numc,denc,t);
[c2,x2,t] = step(num,den,t);
plot(t,c1,'-',t,c2,'.',t,t,'- -')
grid
text(2.2,27,'Compensated system');
text(26,21.3,'Uncompensated system')
title('Unit-Ramp Responses of Compensated and Uncompensated Systems')
xlabel('t Sec');
ylabel('Outputs c1 and c2')
```

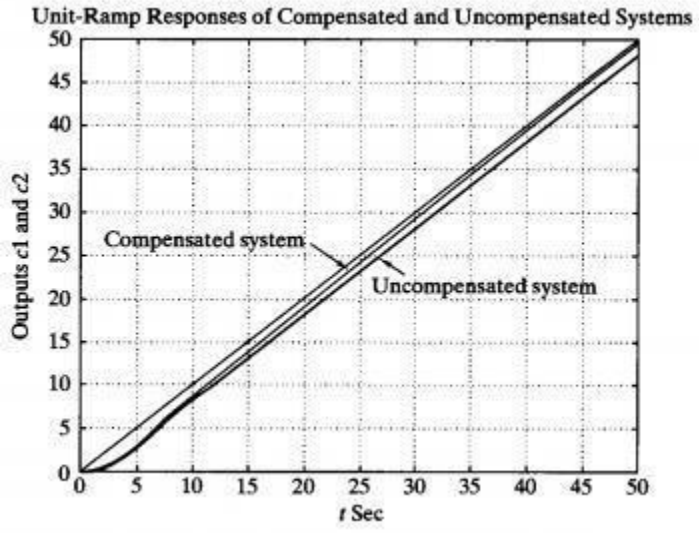


Figure-13.10: Compensated and uncompensated output response of a system

Example4:

```
% ---------- Unit-step response ----------

% ***** Unit-step responses of compensated system and
% uncompensated system *****

% ***** Enter the numerators and denominators of the
% compensated and uncompensated systems *****

numc = [0   0   0   1.0235   0.0512];
denc = [1   3.005   2.015   1.0335   0.0512];
num = [0   0   0   1.06];
den = [1   3   2   1.06];

% ***** Specify the time range (such as t = 0:0.1:40) and enter
% step command and plot command. *****

t = 0:0.1:40;
[c1,x1,t] = step(numc,denc,t);
[c2,x2,t] = step(num,den,t);
plot(t,c1,'-',t,c2,'.')
grid
text(13,1.12,'Compensated system')
text(13.6,0.88,'Uncompensated system')
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs c1 and c2')
```

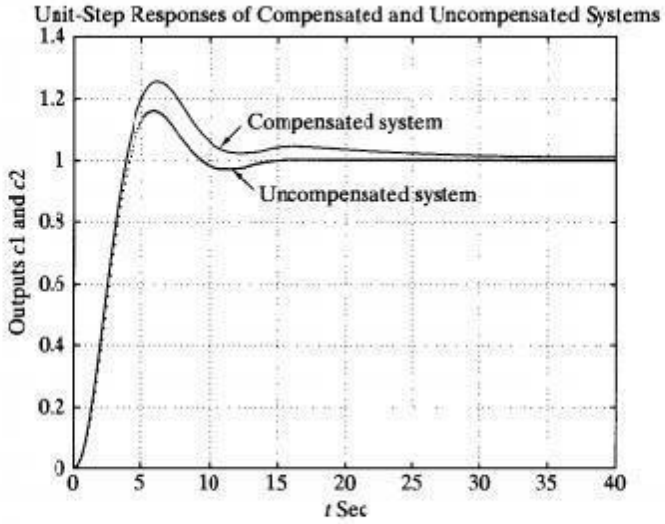


Figure-13.11: Compensated and uncompensated output response of a system

lag-compensated system exhibits a larger maximum overshoot and slower response than the original uncompensated system. Notice that a pair of the pole at $s = -0.0549$ and zero at $s = -0.05$ generates a long tail of small amplitude in the transient response. If a larger maximum overshoot and a slower response are not desired, we need to use a lag–lead compensator as presented in Section 7–5.

Review Questions

a) What is lead compensator in control system?

A lead compensator is a control system element that introduces phase advance, improving stability and transient response by boosting the phase of the system's open-loop transfer function.

b) What is the difference between controller and compensator?

Both controllers and compensators are used to influence the behavior of control systems, controllers directly adjust the system's input based on feedback, while compensators modify the system's dynamics to achieve desired performance characteristics.

c) What is the effects of the phase lead compensator on gain cross over frequency and the bandwidth?

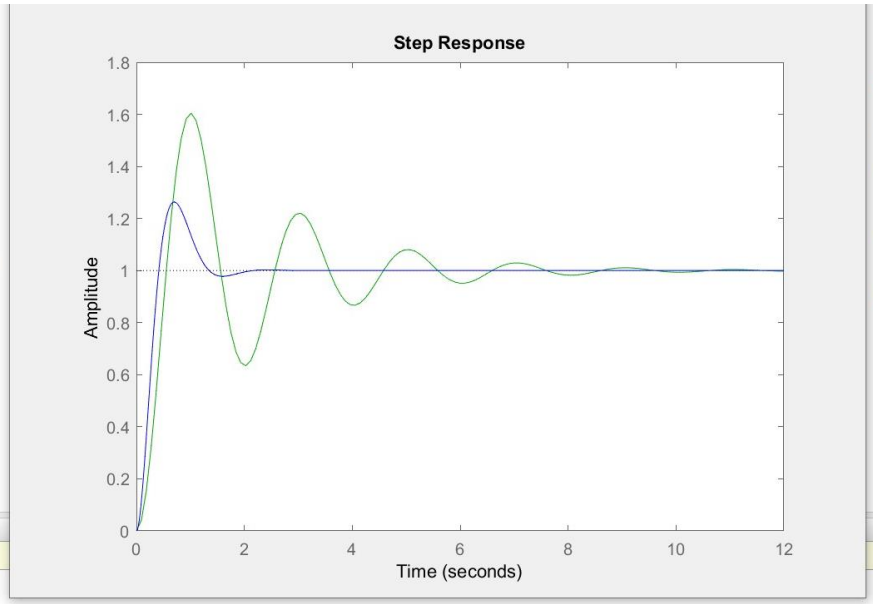
The phase lead compensator increases the gain crossover frequency and broadens the bandwidth of the control system, leading to improved stability, faster response, and enhanced control performance.

d) Attach screenshots of all tasks done in this laboratory.

EXAMPLE 1:

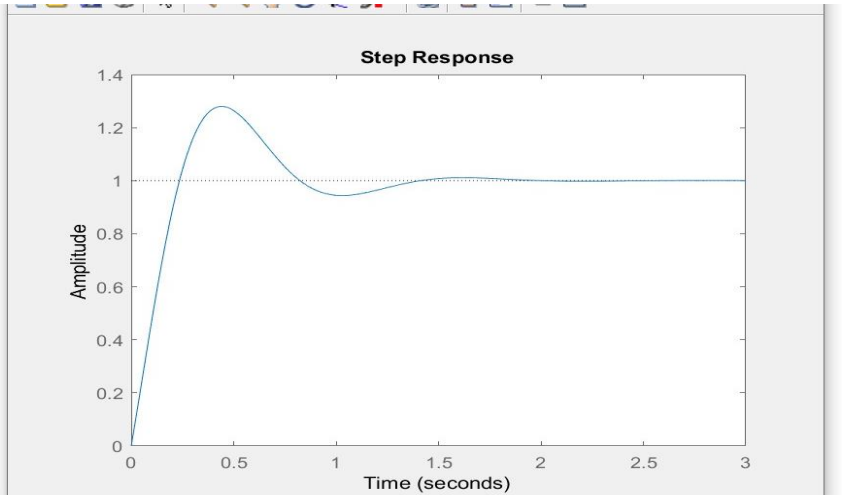
```
wm = 4.5;
alpha = 0.3;
T = 1/(wm*sqrt(alpha)); k= 10;
gnum = [k];
gden = [1 1 0];
uncompensated = tf(gnum,gden)
cnum = [T 1]; cden = [T*alpha 1];
compensator = tf(cnum,cden)
numo = conv(cnum,gnum);
deno = conv(cden,gden);
compensated = tf(numo,deno)
bode(uncompensated,'r',compensated ,'g')
uncomtr=feedback(uncompensated,1)
comtr=feedback(compensated,1)
step(uncomtr,'g')
hold on
step(comtr,'b')
```

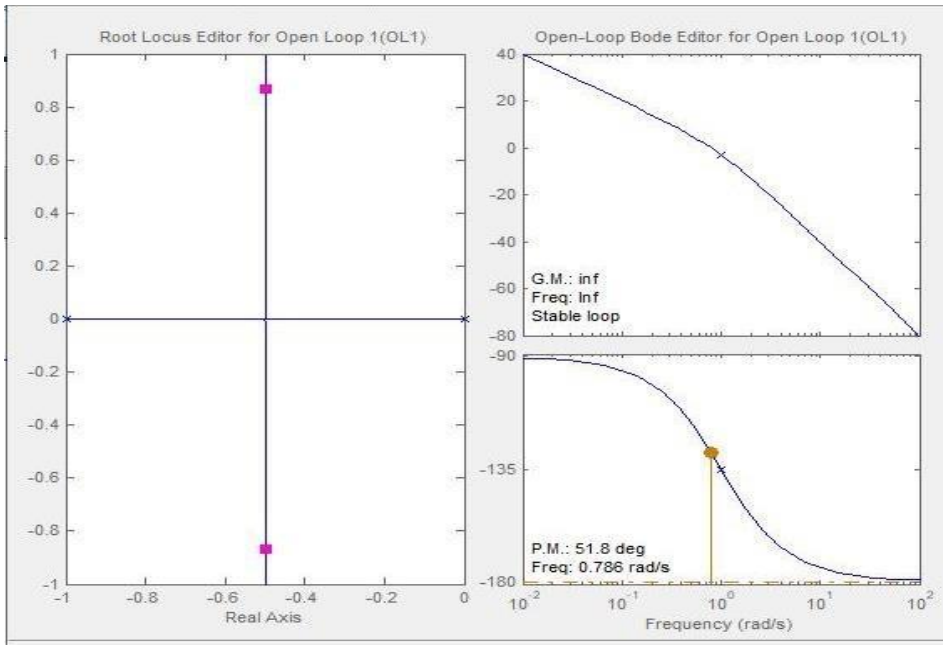
```
Command Window
New to MATLAB? See resources for Getting Started.
Continuous-time transfer function.
```



EXAMPLE 2:

```
G=tf(1, [1 1 0])
z= 8.0501
k= 4.472
C=tf(k*[1 z],[1])
T=feedback(C*G,1)
step(T)
Sisotool(G)
```





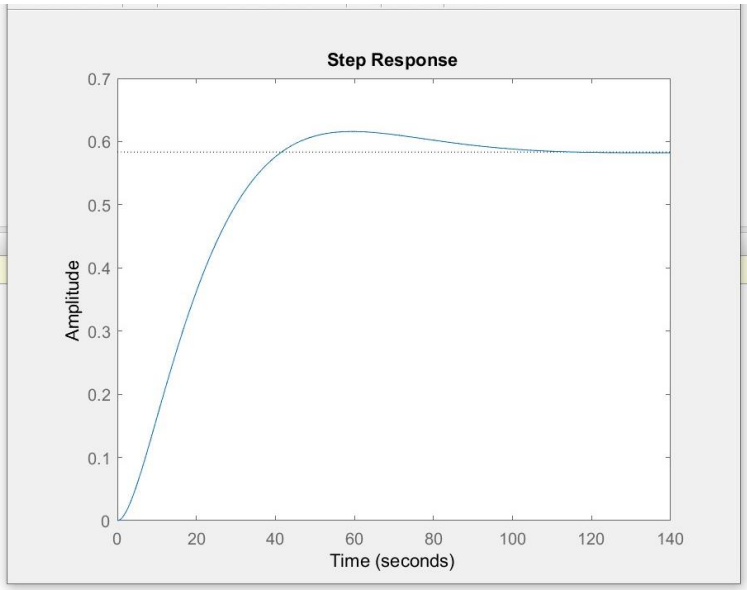
EXAMPLE 3:

```
Gv = tf(0.02,[4 1]);
G1 = tf(70,[50 1]);
H = tf(1,[12 1]);
Gc = tf(1); % dummy static gain
Gol = Gc*Gv*G1; % Openloop: I'm assuming Gp = G1
Gcl = feedback(Gol,H); % closed loop
step(Gcl) % look at the response
stepinfo(Gcl) % extract key data for your design
```

Command Window

New to MATLAB? See resources for Getting Started.

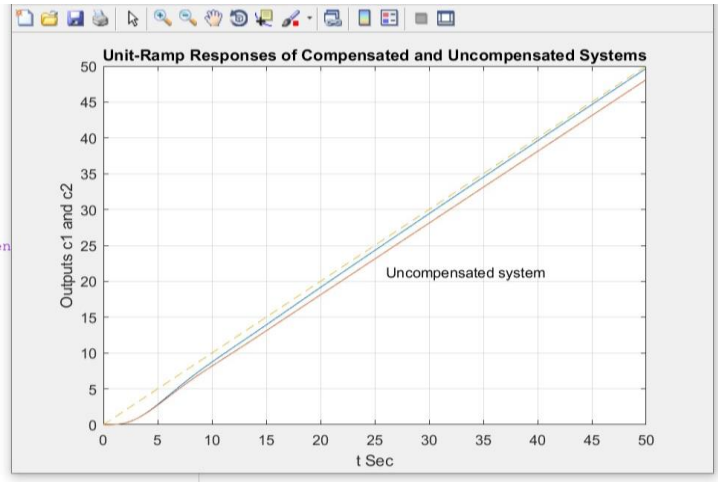
```
stepinfo(Gcl) % extract key data for your design

ans =

  struct with fields:

        RiseTime: 27.5867
    SettlingTime: 88.7066
     SettlingMin: 0.5274
     SettlingMax: 0.6159
       Overshoot: 5.5879
      Undershoot: 0
            Peak: 0.6159
        PeakTime: 59.4622

fx >>
```



EXAMPLE 4:

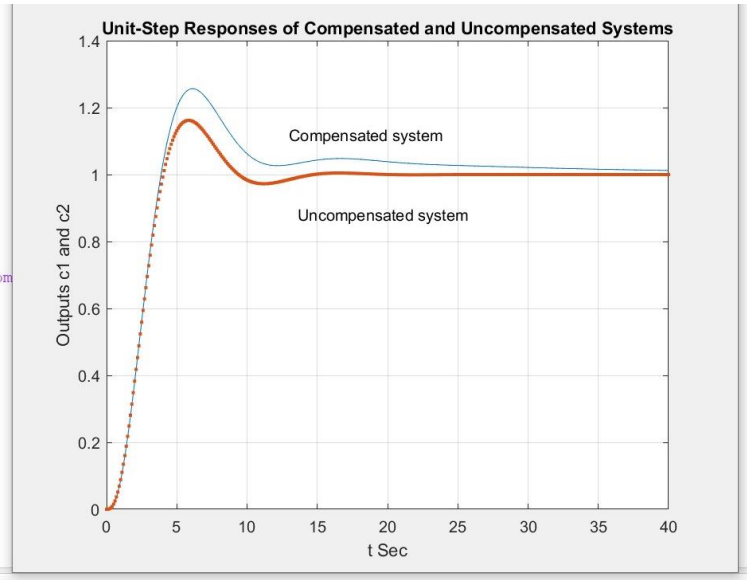
```
numc = [0 0 0 0 1.0235 0.0512];
denc =[1 3.005 2.015 1.0335 0.0512 0];
num = [0 0 0 0 1.06];
den = [1 3 2 1.06 0];
t = 0:0.1:50;
[c1,x1,t] = step(numc, denc,t);
[c2,x2,t] = step(num, den,t);
plot(t,c1,'-',t,c2,t,t,'--')
grid
text(2,2,27, 'Compensated system');
text(26,21.3,'Uncompensated system')
xlabel('t Sec');
title('Unit-Ramp Responses of Compensated and Uncompen
ylabel('Outputs c1 and c2')
```

Command Window



EXAMPLE 5:

```
numc=[0 0 0 1.0235 0.0512];
denc= [1 3.005 2.015 1.0335 0.0512];
num = [0 0 0 1.06];
den= [1 3 2 1.06];
t = 0:0.1:40;
[c1,x1,t] = step(numc,denc,t);
[c2,x2,t] = step(num,den,t);
plot(t,c1,'-',t,c2,'.')
grid
text(13,1.12,'Compensated system')
text(13.6,0.88,'Uncompensated system')
xlabel('t Sec')
title('Unit-Step Responses of Compensated and Uncom
ylabel('Outputs c1 and c2')
```



MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 14: PERFORMANCE AND DESIGN OF DIGITAL FILTER

Name: Karan	Roll # 20ES062
Signature of Lab Tutor:	Date:

OBJECTIVE(S):

#	*Topic*	# Of Lectures	CLO	Taxonomy level
1	*To illustrate the performance of digital filter and design of digital filter using MATLAB*	3	4,5	P3, A4

OUTCOME(S):

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRIC(S):

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

Equipment Required: PC with MATLAB

Approximate Time Required: Two to three hours

Introduction

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialized DSP. The analog input signal must first be sampled and digitized using an ADC (analog to digital converter). The resulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calculations on them. These calculations typically involve multiplying the input values by constants and adding the products together. If necessary, the results of these calculations, which now represent sampled values of the filtered signal, are output through a DAC (digital to analog converter) to convert the signal back to analog form. In a digital filter, the signal is represented by a sequence of numbers, rather than a voltage or current.

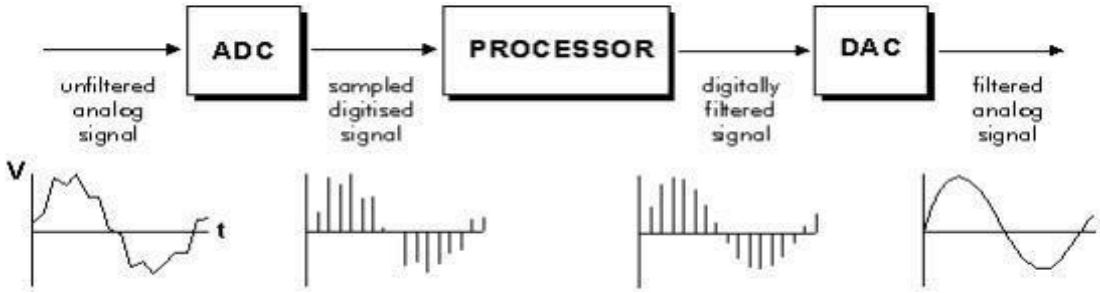


Figure-14.1: Basic setup of Digital filter

A digital filter is programmable, i.e. its operation is determined by a program stored in the processor's memory. This means the digital filter can easily be changed without affecting the circuitry (hardware). An analog filter can only be changed by redesigning the filter circuit.

Digital filters are easily designed, tested and implemented on a general-purpose computer or workstation.

The characteristics of analog filter circuits (particularly those containing active components) are subject to drift and are dependent on temperature. Digital filters do not suffer from these problems, and so are extremely stable with respect both to time and temperature.

Unlike their analog counterparts, digital filters can handle low frequency signals accurately. As the speed of DSP technology continues to increase, digital filters are being applied to high frequency signals in the RF (radio frequency) domain, which in the past was the exclusive preserve of analog technology.

Digital filters are very much more versatile in their ability to process signals in a variety of ways; this includes the ability of some types of digital filter to adapt to changes in the characteristics of the signal.

Fast DSP processors can handle complex combinations of filters in parallel or cascade (series), making the hardware requirements relatively simple and compact in comparison with the equivalent analog circuitry

Operation of digital filters

Basic theory of the operation of digital filters. This is essential to an understanding of how digital filters are designed and used,
Suppose the "raw" signal which is to be digitally filtered is in the form of a voltage waveform described by the function

$$= x\hat{}t) \qquad 1)$$

where t is time. This signal is sampled at time intervals h (the sampling interval). The sampled value at time

$$= \text{it} \quad \text{is}$$

$$\text{Xt} = \hat{}\,\text{it}) \qquad 2)$$

Thus the digital values transferred from the ADC to the processor can be represented by the sequence

$$, x\ , x\ , x\ , \ldots 0\ \text{t}\ 2\ 3$$

corresponding to the values of the signal waveform at

$$t\ = 0, \text{t}, 2\text{t}, 3\text{t}, \ldots\ a\ n\ d\ \ t\ = 0$$

is the instant at which sampling begins.
At time

$$t\ = n\ \text{t}$$

(where n is some positive integer), the values available to the processor, stored in memory, are x

$$, x\ , x\ , \ldots x\ \ 0\ \text{t}\ 2\ 3$$

Note that the sampled values

$$x\ n\ \ \text{h t}, x\ n\ \ \text{h}\ 2 \qquad 3)$$

etc. are not available, as they haven't happened yet! The digital output from the processor to the DAC consists of the sequence of values

$$, y\ , y\ , y\ , \ldots y\ \ 0\ \text{t}\ 2\ 3\ n$$

In general, the value of yn is calculated from the values

$$x\ 0, x\ \text{t}, x\ 2, x\ 3, \ldots , x\ n\ .$$

The way in which the y's are calculated from the x's determines the filtering action of the digital filter. In the next section, we will look at some examples of simple digital filters.

14.6 Unity gain filter: y = x n
Each output value yn is exactly the same as the corresponding input value

$$y\ \text{t} = x\ \text{t}$$
$$y\ 2 = x\ 2$$
$$y\ 3 = x\ 3$$

14.6.1 Simple gain filter: $y\ n\ = K\ x\ n$

where K = constant.

This simply applies a gain factor K to each input value. $K > t$ makes the filter an amplifier, while $0 < K < t$ makes it an attenuator. $K < 0$ corresponds to an inverting amplifier.

14.6.2 Digital filter coefficients

All of the digital filter examples given above can be written in the following general forms: Zero order:

$$y\,n = a\,0x\;n$$

First order: $y\,n = a\,o\,\mathrm{X}n\;\mathrm{h}\,a\,\mathrm{tX}\,n - \mathrm{t}$ 4)

Second order: $y\,n = a\,o\,\mathrm{X}n\;\mathrm{h}\,a\,\mathrm{tX}\,n - \mathrm{t}+ a\,2\mathrm{X}\,n - 2$ 5)

14.7 Example1: Design the following function using matlab

$$H(z) = \frac{b(1)}{a(1) + a(2)z^{-1}} = \frac{1}{1 - 0.2z^{-1}}$$

MATLAB Code:

```
rng default %initialize random number generator
x = rand(2,15); b = 1; a = [1 -0.2]; y =
filter(b,a,x,[],2); t = 0:length(x)-1; %index
vector plot(t,x(1,:)) hold on plot(t,y(1,:))
legend('Input Data','Filtered Data')
title('First Row')
```

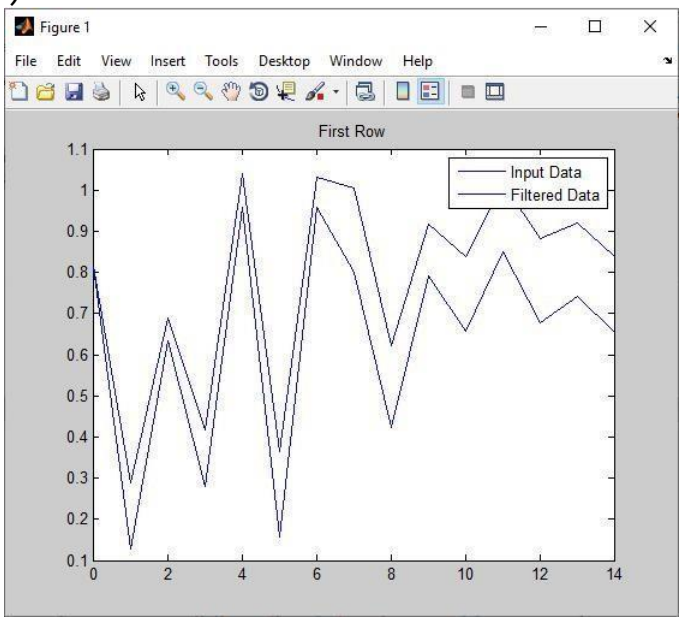


Figure-14.2: Figure first row

Example2: Plot the second row of input data against the filtered data**.**

MATLAB Code:

```
figure
plot(t,x(2,:)) hold
on plot(t,y(2,:))
legend('Input Data','Filtered Data') title('Second
Row')
```

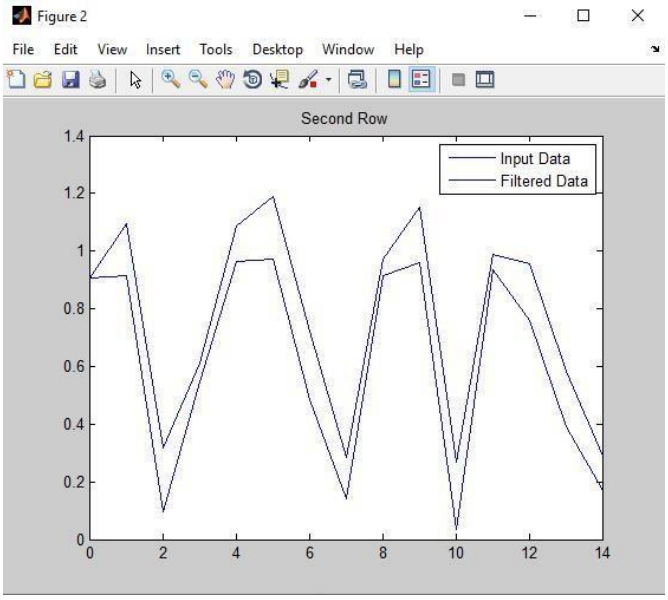


Figure-14.3: Second row

Example3: Create the script file direc_butt_disig that solves this by directly obtaining the magnitude and phase plots of the discrete transfer function H(z) (LPF third order, with T = 0.3 and a cutoff frequency of $w_c = 3$ rad/s).

MATLAB Code:

```
% Script file : direc _ butt _ disig
n =3;
T=.03;wc=3;w =-5:.1:5;
wz = w*T;
[Pz,Qz] = butter(n,wc*T/pi);
[magz,phasez]=freqz(Pz,Qz,wz/pi);
subplot(1,2,1)
plot(wz,magz);
title('Mag. of digital LPF');axis([-5 5 0.4 1.1]);
xlabel('w in rad')
ylabel('magnitude')
subplot(1,2,2)
plot(wz,phasez*180/pi);
title('Phase of digital LPF vs w')
xlabel('w in rad')
ylabel('phase in degrees')
```

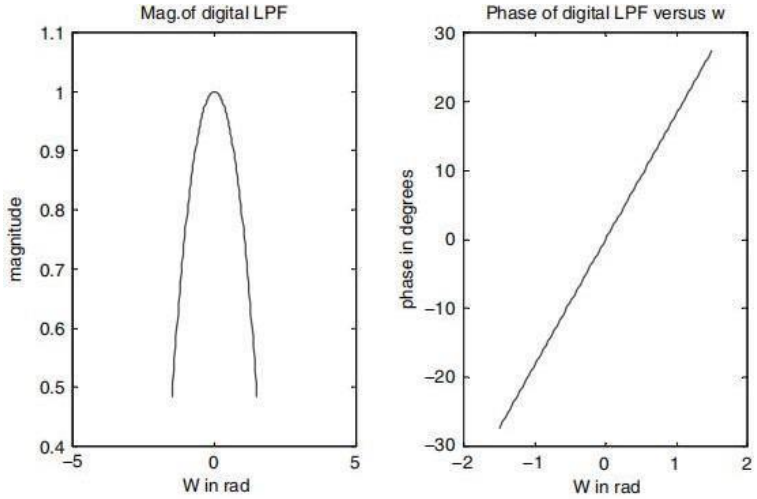


Figure14.4: Magnitude and phase plots of H(z)

Example4: Create the script file cheby_HPF that returns the coefficients of the digital transfer function and the magnitude and phase plots of a third-order Chebyshev type-1 and -2 digital HPFs with the following specs:

1. $w_c = 0.5$ rad/s
2. T=0.3
3. $R_p = 2$ dB
4. $R_s = 3$ dB

MATLAB Code:

```
% Script file: cheby _ HPF
N =3;                                   % order
Rp = 2; Rs = 3;                         % ripples
T = .3;                                 % sampling period
Wc = .5;                                % cut off frequency
Wz = -2*pi:.01*pi:3*pi;
wzc = Wc*T/pi;                          % normalized dig. frequency
[Pz1, Qz1] = cheby1(N, Rp, wzc, 'high');
disp('*****************************************')
disp('*************R E S U L T S *****************')
disp('*****************************************')
disp('For the transfer function H(z) of the Cheby/type1 HPF')
disp('the numerators coefficients are:')
disp(Pz1)
disp('The denominator coefficients are:')
disp(Qz1)
[Pz2, Qz2] = cheby2(N, Rs, wzc, 'high');
disp(' For the transfer function H(z) of the Cheby/type2 HPF')
disp('The numerators coefficients are:')
```

```
disp(Pz2)
disp('The denominator coefficients are:'); disp(Qz2)
disp('*******************************************')
[mag1, phase1] = freqz (Pz1,Qz1,Wz);
[mag2, phase2] = freqz (Pz2,Qz2,Wz);
subplot (2,2,1)
plot (Wz, mag1)
title ('Mag of Cheby. type/1, HPF');
axis([0 5 0 1.1]);
xlabel (' W in rad')
ylabel ('Magnitude')
subplot (2,2,2)
plot (Wz, phase1* 180/pi)
title ('Phase of Cheby. type/1,HPF ')
ylabel ('Angle in degrees')
xlabel (' W in rad')
subplot (2,2,3)
plot (Wz, mag2)
title ('Mag.of Cheby. type/2, HPF'); axis([0 1 0 1.5]);
xlabel ('W in rad'); ylabel ('Magnitude')
subplot (2,2,3)
plot (Wz, mag2)
title ('Mag.of Cheby. type/2, HPF'); axis([0 1 0 1.5]);
xlabel ('W in rad'); ylabel ('Magnitude')
subplot (2,2,4)
plot (Wz, phase2 * 180/ pi);
xlabel ('W in rad')
ylabel ('Angle in degrees')
title('Phase. of Cheby. type/2, HPF')
disp('****** FILTERS TRANSFER FUNCTIONS*******')
 disp('*******************************************')
disp('Chebyshev type 1')
tf(Pz2,Qz2,T)
disp('^^^^^^^^^^^^^^^^^^^^')
disp('Chebyshev type 2')
tf(Pz1,Qz1,T)
disp('*************************************************')
```

The script file *cheby_HPF* is executed and the results are shown as follows

```
>> cheby _ HPF

*****************************************
************ R E S U L T S **************
*****************************************
For the transfer function H(z) of the Cheby/type1 HPF the numerators coefficients
are:
   0.8006 -2.4019    2.4019 -0.8006
```

The denominator coefficients are:
1.0000 -2.5767 2.2067 -0.6217

For the transfer function H(z) of the Cheby/type2 HPF The numerators coefficients are:
0.9564 -2.8530 2.8530 -0.9564

The denominator coefficients are:
1.0000 -2.8943 2.8098 -0.9147

```
*******************************************
*****FILTERS TRANSFER FUNCTIONS **********
*******************************************
Chebyshev type 1

Transfer function:
0.8006 z^3 - 2.402 z^2 + 2.402 z - 0.8006

-----------------------------------------

 z^3 - 2.577 z^2 + 2.207 z - 0.6217

Sampling time: 0.3
^^^^^^^^^^^^^^^^^^^
Chebyshev type 2

Transfer function:
0.9564 z^3 - 2.853 z^2 + 2.853 z - 0.9564

-----------------------------------------

  z^3 - 2.894 z^2 + 2.81 z - 0.9147

Sampling time: 0.3
```

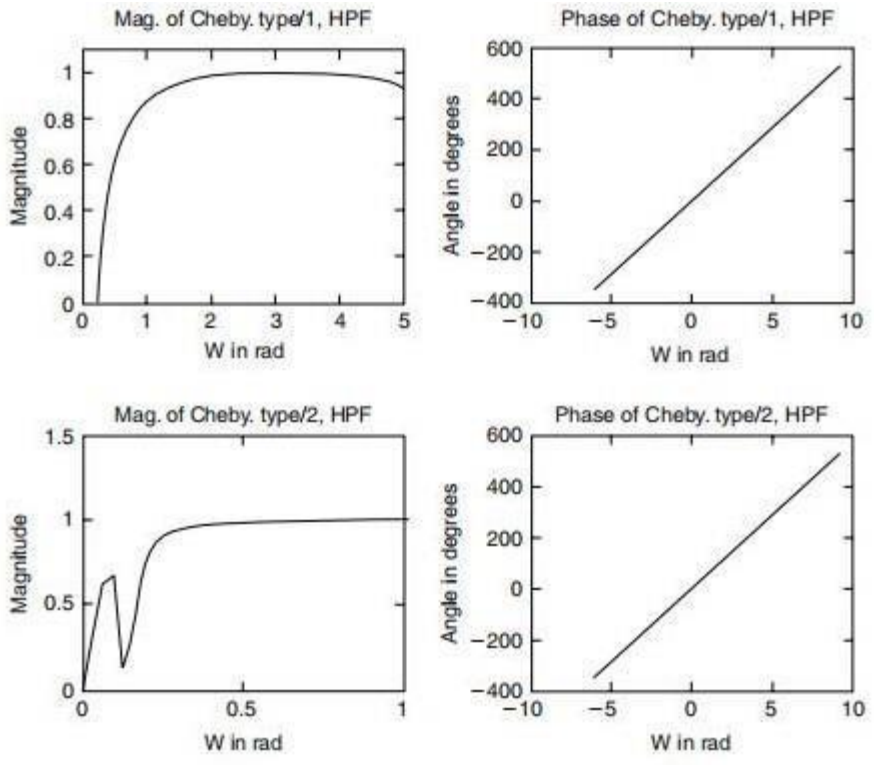


Figure-14.5: Magnitude and phase plots of Chebyshev type-1 and -2 digital HPFs

Example5: Test the Butterworth LP digital filter by applying as an input the signal $x(t) = 2 + \cos(0.2 * t) + \cos(4 * t)$ sampled every $T = 0.3$ s.
Create the script file inp_out that returns the following plots:

1. x(t) versus t
2. y(t) versus t
3. x(nT) versus nT
4. y(nT) versus nT where y denotes the filter's output.

MATLAB Code:

```
% Script file: inp _ out
Pz = [0.0380   0.1141   0.1141   0.0380];
Qz = [1.0000 -1.3445   0.8215 -0.172];
n = 0:1:100; %Discrete time sequence
T = 0.3;
input = 2 + cos(0.2*T.*n)+cos(4*T.*n);
output= filter(Pz,Qz,input);
t = 0:.05:8;                                        % time range
inputa = 2 + cos(0.2.*t) + cos(4.*t);
subplot(2,2,1)
plot(t,inputa)
xlabel('t in sec')
ylabel('Amplitude')
title('[x(t) = 2 + cos(0.2.*t) + cos (4.*t)] vs. t')
axis([0 5 0 5])
subplot(2,2,2)
plot(n*T,output)
xlabel('t in sec')
ylabel('Amplitude')
title('y(t) vs. t');axis([0 8 0 4])
subplot(2,2,3)
stem(n*T,input)
xlabel('time index nT')
ylabel('Amplitude')
title('x(nT) vs. nT');
axis([0 5 0 5])
subplot(2,2,4)
stem(n*T,output)
xlabel('time index nT')
ylabel('Amplitude')
title('y(nT) vs. nT');
axis([0 8 0 4])
```

The script file *inp_out* is executed and the results are shown in Figure 6.60.

>> inp _ out

Note that the filter's output presents a high DC component plus the low frequency of the input with some traces of the high frequency.

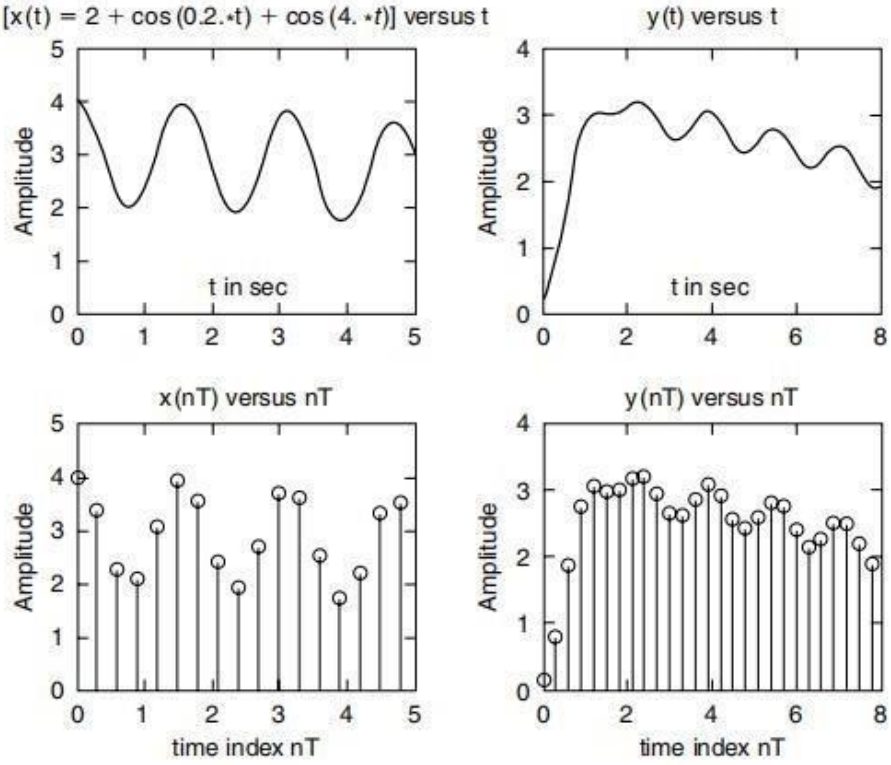


Figure-14.6: Analog and discrete filter's input and output

REVIEW EXERCISE?

a) Why do we use digital filters in digital control system?

Digital filters play a crucial role in digital control systems by providing signal processing capabilities, flexibility, adaptability, ease of implementation, precision, and integration with control algorithms, ultimately improving system performance and reliability.

b) What is FIR digital filter?

A Finite Impulse Response (FIR) digital filter is a type of digital filter used in digital signal processing and digital control systems. It is characterized by having a finite impulse response, meaning that its output response to an input signal is of finite duration. In simpler terms, an FIR filter takes a sequence of input samples and produces a sequence of output samples by convolving the input samples with a set of coefficients called the filter taps.

c) What are types of digital filters?

FIR (Finite Impulse Response) Filter: A digital filter with a finite impulse response, meaning the output is determined by a finite number of input samples and filter coefficients.

IIR (Infinite Impulse Response) Filter: A digital filter with an impulse response that extends infinitely, typically involving feedback loops for recursive calculations.

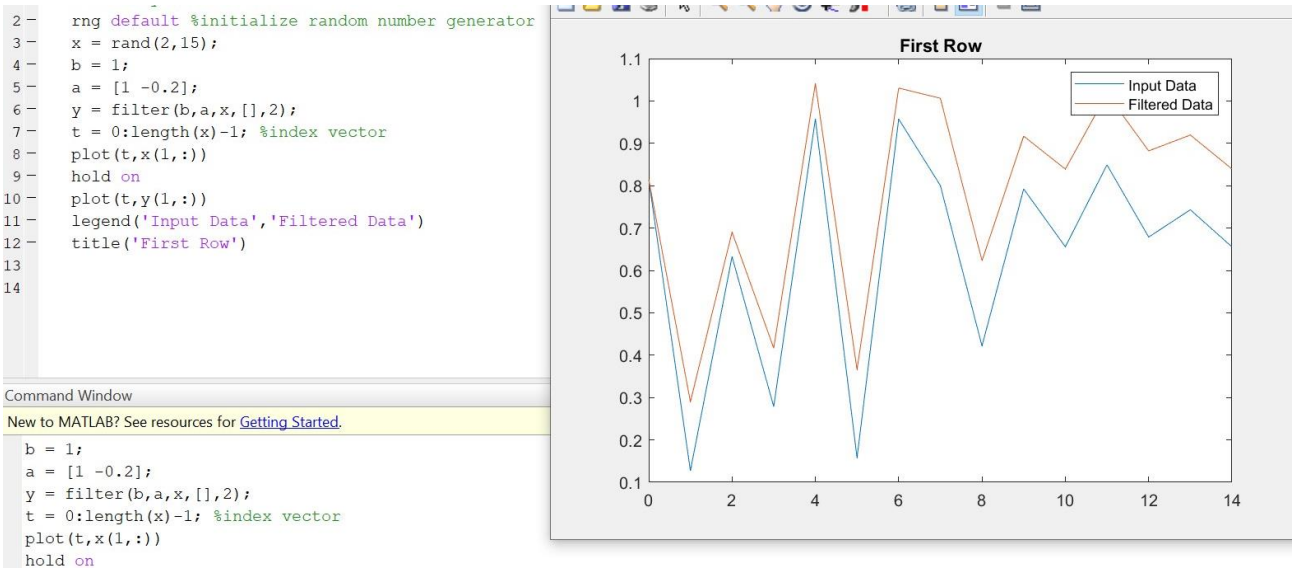
Butterworth Filter: An IIR filter with a maximally flat frequency response in the passband and a gradual roll-off in the stopband.

Chebyshev Filter: An IIR filter providing steeper roll-off in the stopband (Type I) or passband (Type II) at the expense of ripple in the opposite region.

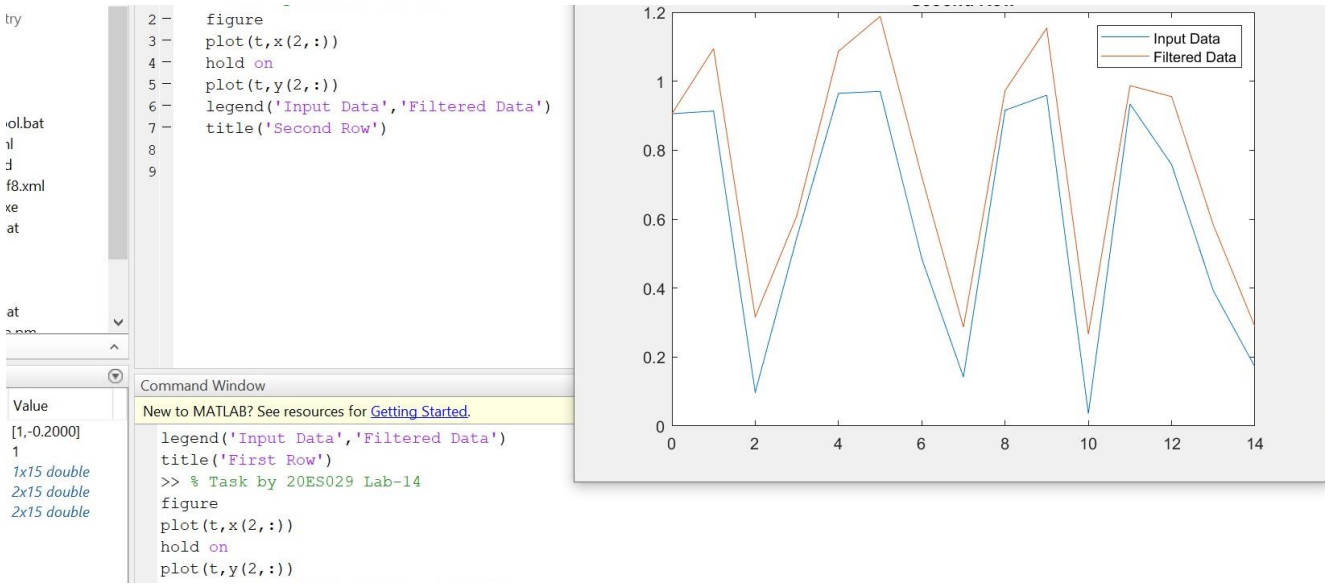
Elliptic Filter (Cauer Filter): An IIR filter with the steepest roll-off among common types while allowing specified ripple levels in both passband and stopband.

d) Attach screenshots of all examples done in this laboratory.

Example:1



Example:2



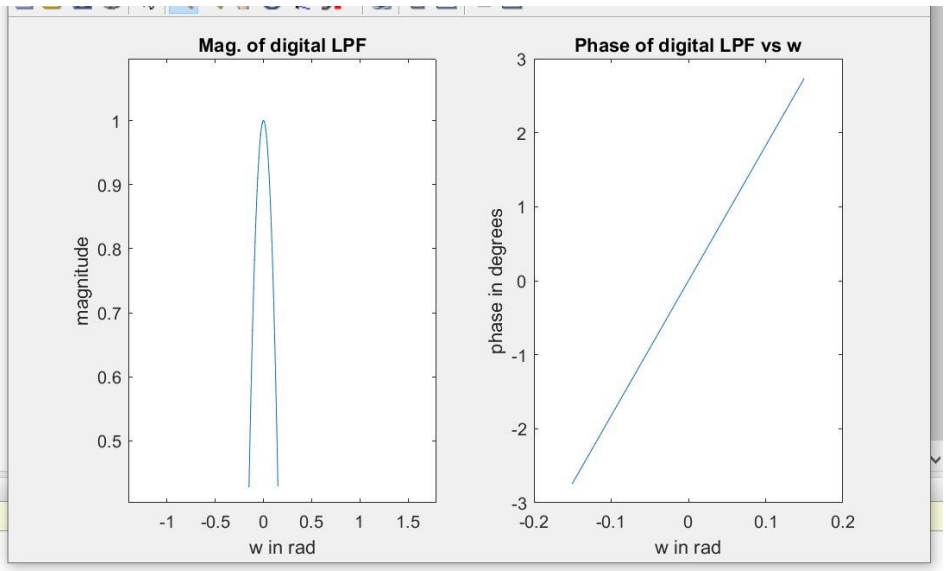
Example:3

```
n =3;
T=.03;
wc=3;
w =-5:0.1:5;
wz = w*T;
[Pz,Qz] = butter(n,wc*T/pi);
[magz,phasez]=freqz (Pz,Qz,wz/pi);
subplot(1,2,1)
plot (wz, magz);
title('Mag. of digital LPF');
axis([-5 5 0.4 1.1]);
xlabel('w in rad')
ylabel('magnitude')
subplot(1,2,2)
plot (wz,phasez*180/pi);
title('Phase of digital LPF vs w')
xlabel('w in rad')
ylabel('phase in degrees')
```

Command Window

New to MATLAB? See resources for Getting Started.

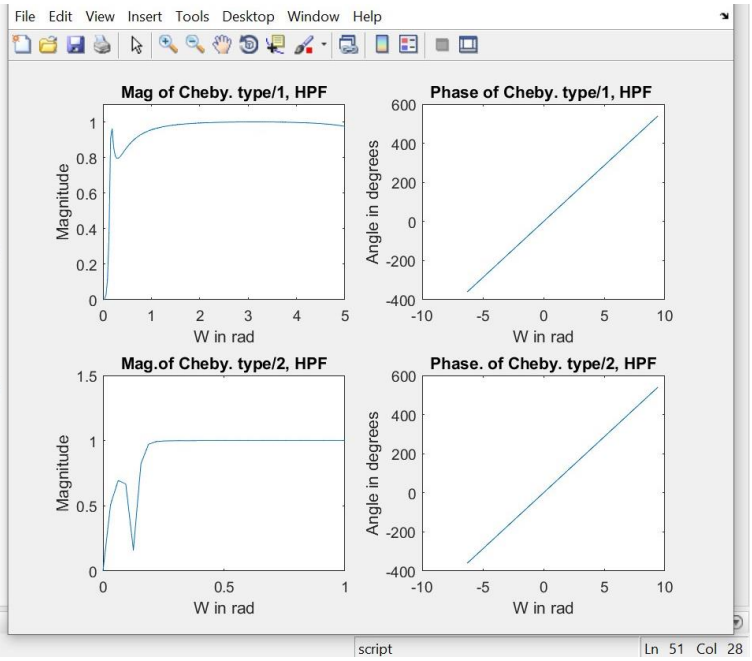
```
plot (wz, magz);
title('Mag. of digital LPF');
axis([-5 5 0.4 1.1]);
xlabel('w in rad')
ylabel('magnitude')
```



Example:4

```
N = 3;
Rp = 2;
R = 3;
T = 0.3;
Wc = 0.5;
Wz = -2*pi:0.01*pi:3*pi;
wzc = Wc*T/pi;
[Pz1, Qz1] = cheby1(N, Rp, wzc, 'high');
disp('********************')
disp('****RESULTS ******')
disp('********************')
disp('For the transfer function H(z) of the Cheby/typel HPF')
disp('the numerators coefficients are:')
disp(Pz1')
disp('The denominator coefficients are:')
disp(Qz1')
[Pz2, Qz2] = cheby2(N, R, wzc, 'high');
disp('For the transfer function H(z) of the Cheby/type2 HPF')
disp('The numerators coefficients are:')
disp(Pz2')
disp('The denominator coefficients are:')
disp(Qz2')
disp('********************************')
[mag1, phase1] = freqz(Pz1, Qz1, Wz);
[mag2, phase2] = freqz(Pz2, Qz2, Wz);
subplot(2,2,1)
plot(Wz, abs(mag1))
title('Mag of Cheby. type/1, HPF');
```

Command Window



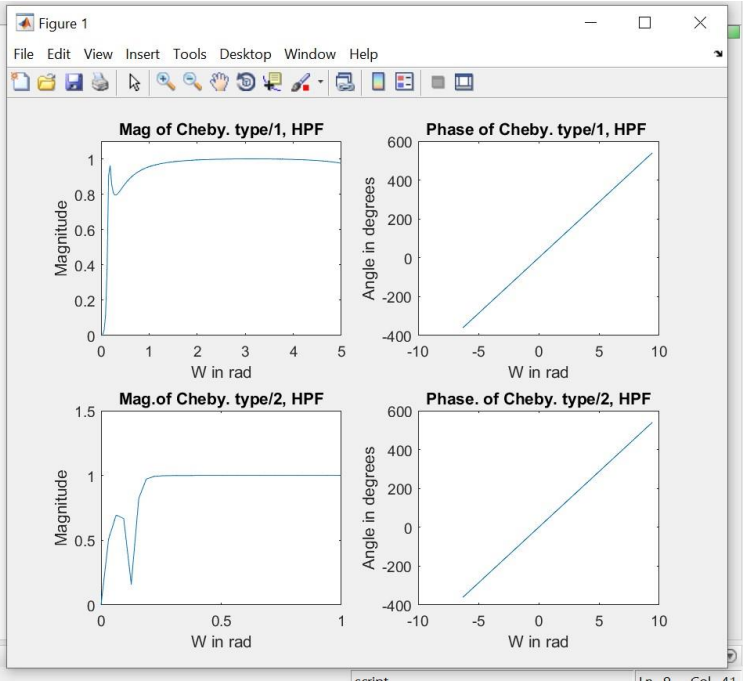
script Ln 51 Col 28

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

Untitled.m

```
subplot(2,2,1)
plot(Wz, abs(mag1))
title('Mag of Cheby. type/1, HPF');
axis([0 5 0 1.1]);
xlabel('W in rad')
ylabel('Magnitude')
subplot(2,2,2)
plot(Wz, phase1 * 180/pi)
title('Phase of Cheby. type/1, HPF ')
ylabel('Angle in degrees')
xlabel('W in rad')
subplot(2,2,3)
plot(Wz, abs(mag2))
title('Mag.of Cheby. type/2, HPF');
axis([0 1 0 1.5]); xlabel('W in rad'); ylabel('Magnitude')
subplot(2,2,4)
plot(Wz, phase2 * 180/pi);
xlabel('W in rad')
ylabel('Angle in degrees')
title('Phase. of Cheby. type/2, HPF')
disp('** FILTERS TRANSFER FUNCTIONS***')
disp('********************************')
disp('Chebyshev type 1')
tf(Pz1,Qz1,T)
disp('^^^^^^^^^^^^^^^^^^^^')
disp('Chebyshev type 2')
tf(Pz2,Qz2,T)
disp('********************************')

```

Command Window



Editor - C:\Users\PMLS\Untitled.m

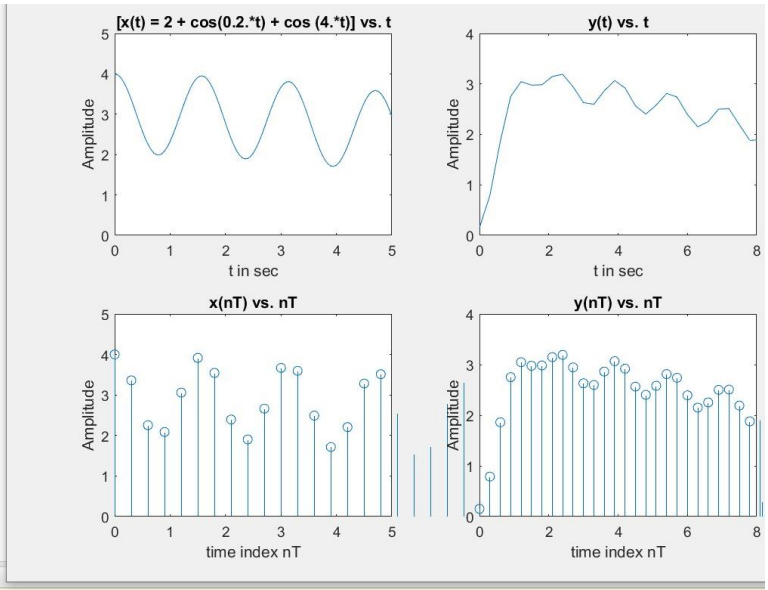
Command Window

New to MATLAB? See resources for Getting Started.

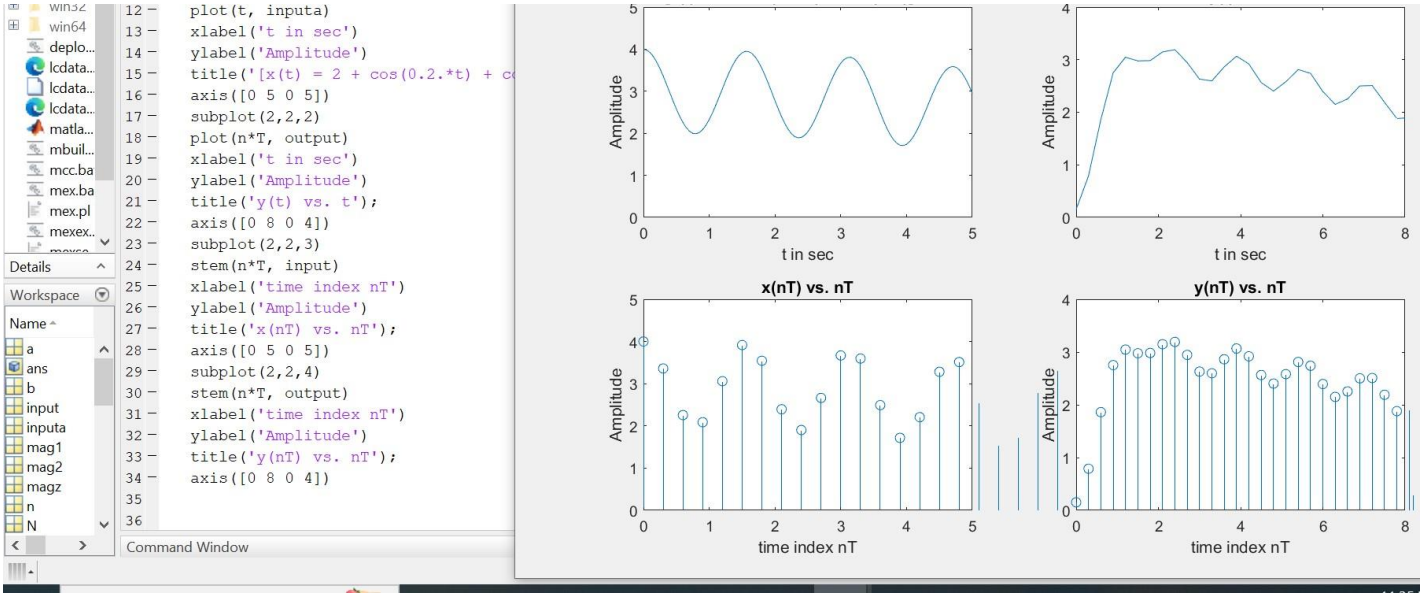
```
********************
****RESULTS ******
********************
For the transfer function H(z) of the Cheby/typel HPF
the numerators coefficients are:
     0.8006
    -2.4019
     2.4019
    -0.8006

The denominator coefficients are:
     1.0000
    -2.5767
     2.2067
    -0.6217

For the transfer function H(z) of the Cheby/type2 HPF
The numerators coefficients are:
     0.9564
    -2.8530
     2.8530
    -0.9564

The denominator coefficients are:
     1.0000
    -2.8943
     2.8098
    -0.9147
```

Example: 5

```
% Script file: inp_out
Pz = [0.0380 0.1141 0.1141 0.0380];
Qz = [1.0000 -1.3445 0.8215 -0.172];
n = 0:1:100; % Discrete time sequence
T = 0.3;
input = 2 + cos(0.2*T.*n) + cos(4*T.*n);
output = filter(Pz, Qz, input);
t = 0:0.05:8;
inputa = 2 + cos(0.2.*t) + cos(4.*t);
subplot(2,2,1)
plot(t, inputa)
xlabel('t in sec')
ylabel('Amplitude')
title('[x(t) = 2 + cos(0.2.*t) + cos (4.*t)] vs. t')
axis([0 5 0 5])
subplot(2,2,2)
plot(n*T, output)
xlabel('t in sec')
ylabel('Amplitude')
title('y(t) vs. t');
axis([0 8 0 4])
subplot(2,2,3)
stem(n*T, input)
xlabel('time index nT')
ylabel('Amplitude')
title('x(nT) vs. nT');
axis([0 5 0 5])
```




```
Command Window
New to MATLAB? See resources for Getting Started.

  ********************************
  ** FILTERS TRANSFER FUNCTIONS***
  ********************************
  Chebyshev type 1

  ans =

    0.8006 z^3 - 2.402 z^2 + 2.402 z - 0.8006
    -----------------------------------------
       z^3 - 2.577 z^2 + 2.207 z - 0.6217

  Sample time: 0.3 seconds
  Discrete-time transfer function.

  ^^^^^^^^^^^^^^^^^^^^
  Chebyshev type 2

  ans =

    0.9564 z^3 - 2.853 z^2 + 2.853 z - 0.9564
    -----------------------------------------
        z^3 - 2.894 z^2 + 2.81 z - 0.9147

  Sample time: 0.3 seconds
  Discrete-time transfer function.

  ********************************
fx >>
```


win32
win64
deplo...
lcdata...
lcdata...
lcdata...
matla...
mbuil...
mcc.ba
mex.ba
mex.pl
mexex...
Details
Workspace
Name
a
ans
b
input
inputa
mag1
mag2
magz
n
N
Command Window
plot(t, inputa)
xlabel('t in sec')
ylabel('Amplitude')
title('[x(t) = 2 + cos(0.2.*t) + c
axis([0 5 0 5])
subplot(2,2,2)
plot(n*T, output)
xlabel('t in sec')
ylabel('Amplitude')
title('y(t) vs. t');
axis([0 8 0 4])
subplot(2,2,3)
stem(n*T, input)
xlabel('time index nT')
ylabel('Amplitude')
title('x(nT) vs. nT');
axis([0 5 0 5])
subplot(2,2,4)
stem(n*T, output)
xlabel('time index nT')
ylabel('Amplitude')
title('y(nT) vs. nT');
axis([0 8 0 4])
Amplitude
t in sec
x(nT) vs. nT
y(nT) vs. nT
time index nT

MEHRAN UNIVERSITY OF ENGINEERING & TECHNOLOGY, JAMSHORO
DEPARTMENT OF ELECTRONIC ENGINEERING
DIGITAL CONTROL SYSTEMS (ES-413)

LAB # 15: LOW PASS,HIGH PASS & BAND PASS DIGITAL FILTERS

Name: KARAN	Roll # 20ES062
Signature of Lab Tutor	Date

OBJECTIVE(S)

#	Topic	# Of Lectures	CLO	Taxonomy level
1	*To illustrate Low Pass, High Pass an Bandpass Digital filters in MATLAB using Simulink modeling*	3	4,5	P3, A4

OUTCOME(S)

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRICS:

Performance Metric	Good (5-4)	Average (3-2)	Poor (1-0)	Score
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly stated.	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			TOTAL	

TOOLS:

- MATLABR-2016aorhigher version

BACKGROUND:

Filters are used to eliminate unwanted signal components such as noise and distortion, or to emphasize or correct certain signals (equalizers in stereo systems are generally just a bank of several bandpass filters).For example, a lowpass filter is used to suppress high frequency components of a signal. An anti-aliasing filter (an analog lowpass filter)is used prior to sampling of a continuous signal in order to get rid of high frequency components of the signal. This is essential in order to maintain the integrity of the low frequency components during sampling. Figure 1shows ideal filter characteristics for lowpass, bandpass, highpass, and bandstop filters.

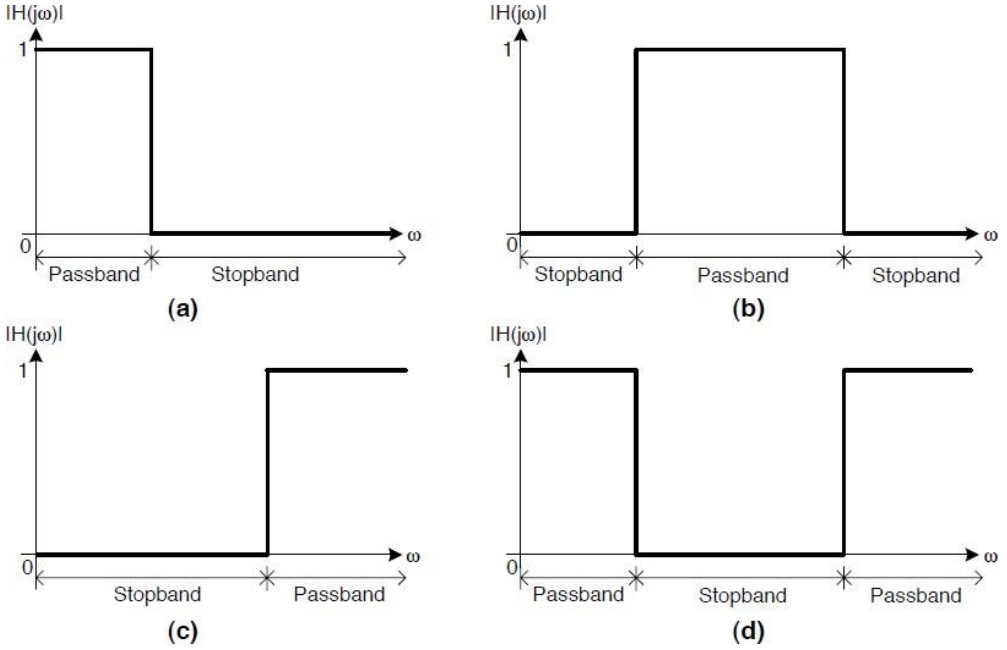


Figure1:Idea lfilter characteristics of f ilters

.(a)Lowpassfilter.(b)Bandpassfilter.(c)Highpass filter. (d) Bandstop filter.

As can be seen from Figure 1, an ideal filter has a passband (unity gain) and a stopband (zero gain) with a sudden transition from the passband to the stopband. There is no transition band. However, for practical filters (see Figure 2), the transition takes place over a finite band of frequencies. Moreover, for realizable filters, the gain cannot be zero over a finite band (Paley-Wienercondition).

We therefore define a stopband to be a band over which the gain is below some small number Gs (maximum stopband gain), as illustrated in Figure 2. Similarly, we define a passband to be a band overwhichthethegainisbetween1 andsomenumberGp(minimumpassbandgain). In ourdesign procedure, Gp, Gs, wp, and ws will be specified for lowpass and highpass filters. For bandpass and

bandstop filters, Gp, Gs, wp1,ws1, wp2, and ws2 will bespecified. In this lab, the gains (Gp and Gs) are specified in decibels (dB), a logarithmic scale, and the frequencies are specified in radians.

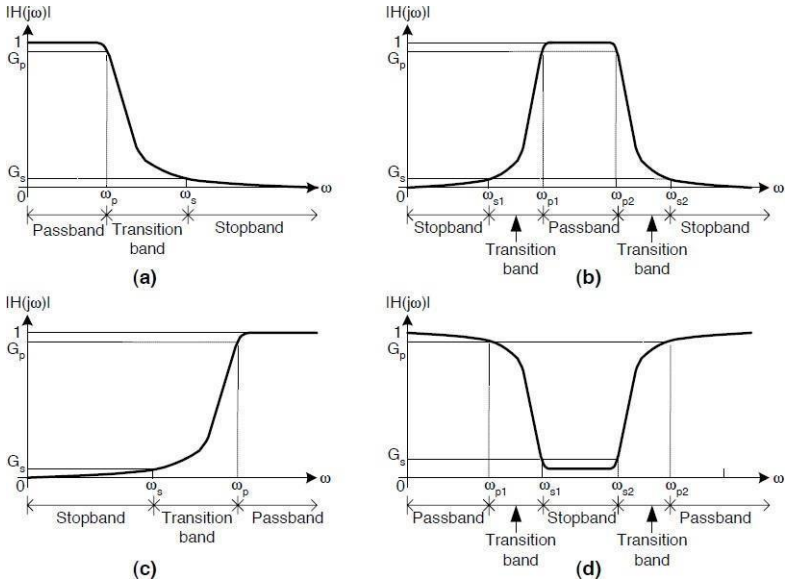


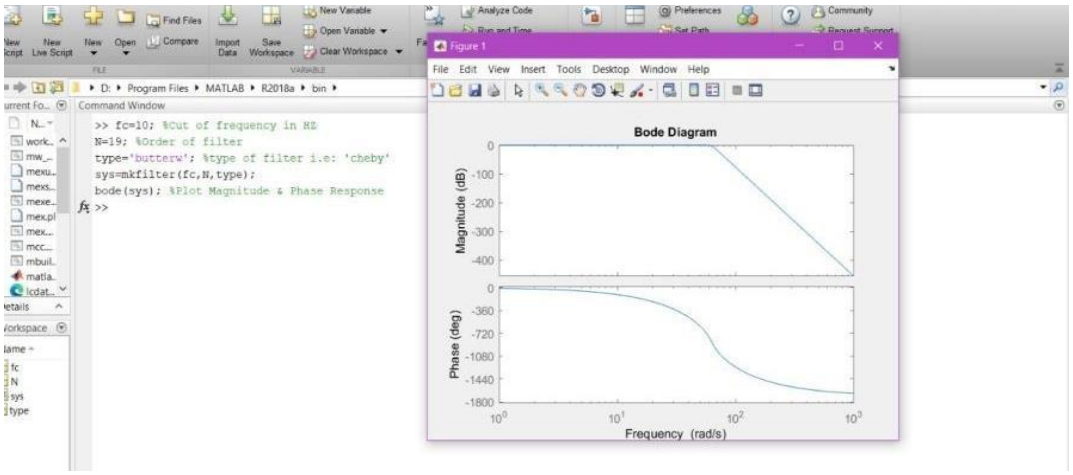
Figure2:Passband,stopband,andtransitionbandinvarioustypesoffilters.(a)Lowpassfilter.(b) Bandpass filter. (c) Highpass filter. (d) Bandstop filter.

PROCEDURE:

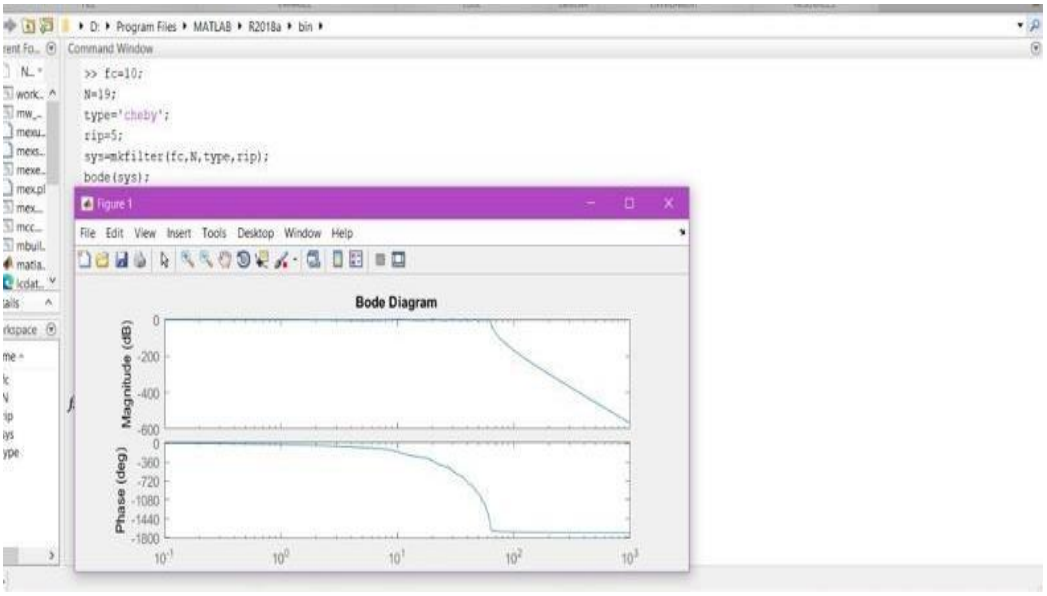
- ***Designing Continuous Time Filters***

ContinuoustimeLowpassButterworthfilter

```
fc=10;%CutoffrequencyinHZ          N=19;          %Order          of          filter
type='butterw';%typeoffilteri.e:'cheby'sys=mkfilter(fc,N,type);
bode(sys);%PlotMagnitude&PhaseResponse
```

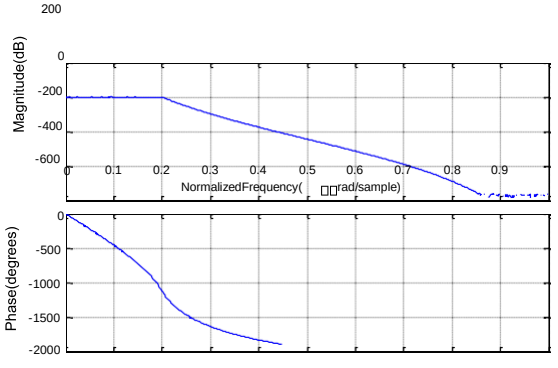


Continuous time Lowpass Chebyshev filter

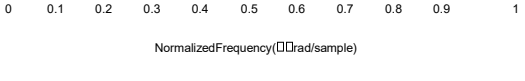


Butterworth Lowpass Digital Filter with Fs=40&Fc=4Hz

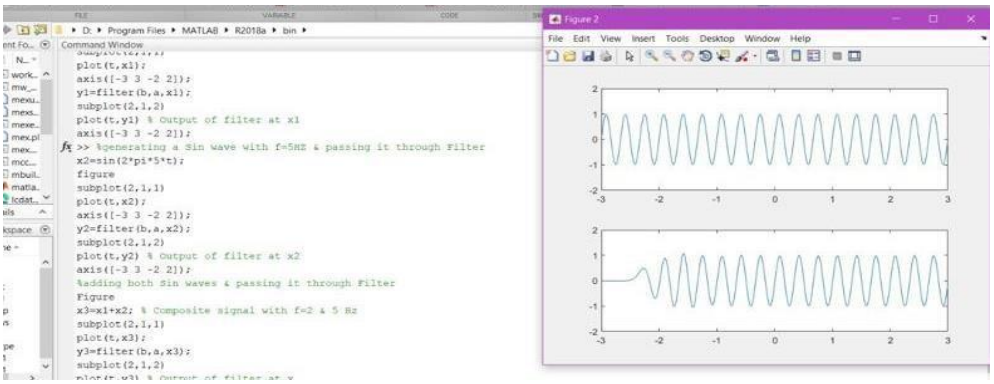
```
t=-3:.025:3;%40Samples/SecTHEREFOREf=fs/2=20hz
[ba]=butter(25,.2);%Lowpassfilterwithfc=.2x20=4hz figure,
freqz(b,a);

%generatingaSinwavewithf=2HZ&passingitthroughFilter x1=sin(2*pi*3*t);

figure
subplot(2,1,1) plot(t,x1);
axis([-33-22]);
```

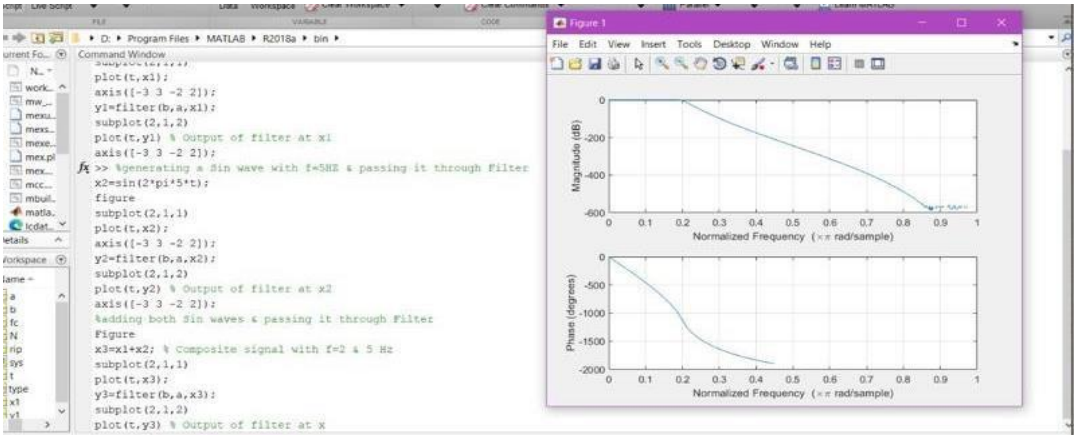



```
y1=filter(b,a,x1);

                                    subplot(2,1,2)
plot(t,y1) %Outputoffilteratx1    axis([-3 3 -2 2]);
```



```
%generatingaSinwavewithf=5HZ&passingitthroughFilter

x2=sin(2*pi*5*t); figure
subplot(2,1,1) plot(t,x2);
axis([-33-22]);
y2=filter(b,a,x2);
subplot(2,1,2)
plot(t,y2)%Outputoffilteratx2
axis([-3 3 -2 2]);

%addingbothSinwaves&passingitthroughFilter Figure
x3=x1+x2;%Compositesignalwithf=2&5Hz
subplot(2,1,1) plot(t,x3); y3=filter(b,a,x3);
subplot(2,1,2)
plot(t,y3)%Outputoffilteratx1
```





MEHRAN UNIVERSITY OF ENGINEERING AND TECHNOLOGY, JAMSHORO
DIGITAL CONTROL SYSTEMS (ES-413)

Lab 16 Open Ended Lab

Name: Jamshed Memon	Roll # 20ES044
Signature of Lab Tutor:	Date:

OBJECTIVE(S)

#	*Topic*	**# Of Lectures**	**CLO**	**Taxonomy level**
1	To understand and apply digital control system concepts using MATLAB based on the student's roll number.	3	4,5	P3, A4

OUTCOME(S)

a. An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.	**PLO5**: Modern Tool Usage
b. An ability to communicate effectively (written/oral)	**PLO10**: Communication

LAB RUBRICS:

Performance Metric	**Good (5-4)**	**Average (3-2)**	**Poor (1-0)**	**Score**
Use of modern engineering software [PLO5]	Demonstrates knowledge and application of modern engineering software through accurate development and interpretation of computer programs to solve problems.	Demonstrates awareness of modern engineering software through mostly correct development and interpretation of computer programs to solve problems, but may contain minor mistakes or syntax errors	Unable to use modern engineering software to develop or interpret computer programs to solve problems.	
Level of understanding of the learned skill [PLO5]	Demonstration of full knowledge of the handout with explanations and elaboration.	At ease with content and able to elaborate and explain to some degree.	No grasp of information. Clearly no knowledge of subject matter. No questions are answered. No interpretation made.	
Responsiveness to Questions/ Accuracy [PLO10]	Responds well, quick and very accurate all the time.	Generally responsive and accurate most of the times.	Non-responsive at all or the candidate giving only one correct response of viva voce questions.	
Documentation: contents and organization [PLO10]	Report well organized, Appropriately sectioned, uses diagram/description when appropriate, important issues clearly	Report reasonably well documented. May lack some minor aspects.	Report not well organized, lack key aspects.	
			Total	

Task 1: Step Response Analysis

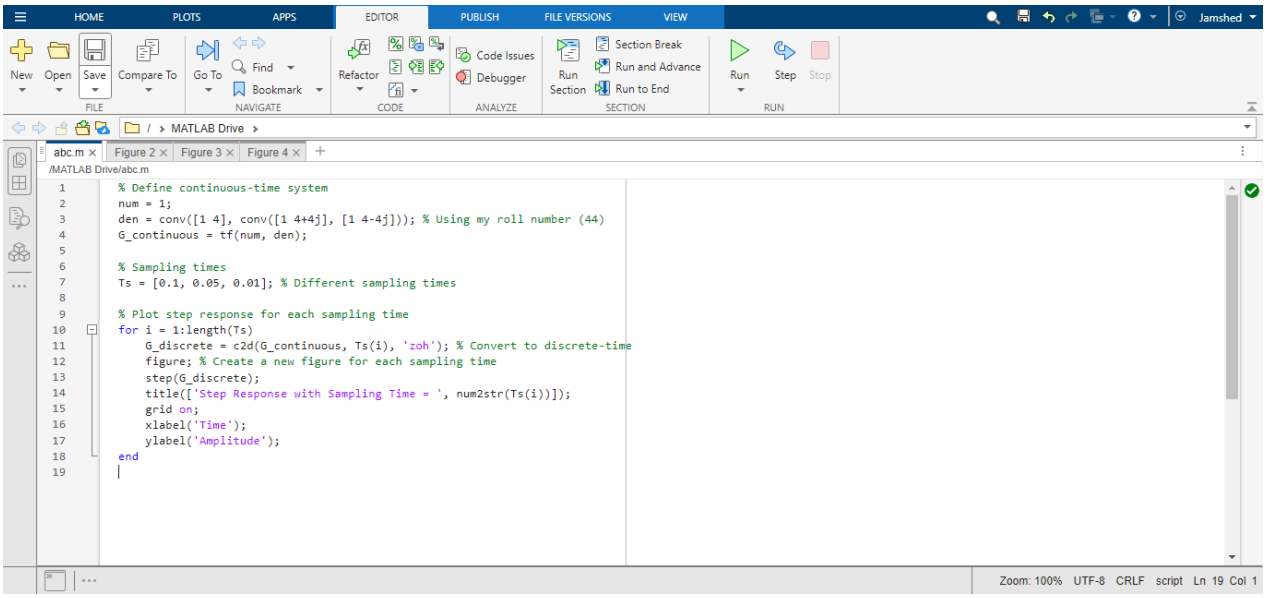
Objective: Analyze the step response of a discrete-time system and observe the effect of sampling time.

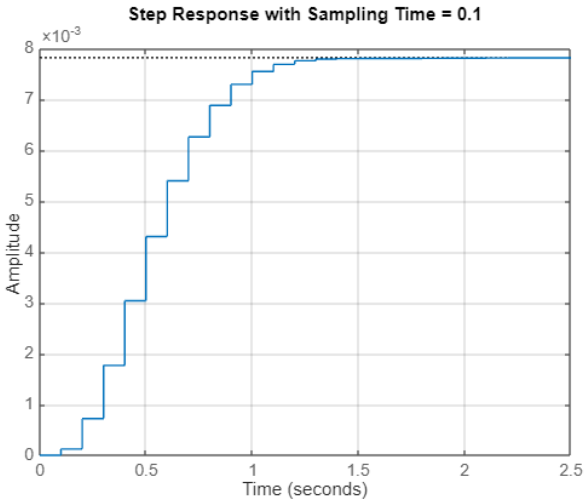
- **Procedure:**
  1. Define a continuous-time system using the student's roll number as system parameters.
  2. Convert the system to a discrete-time system with different sampling times.
  3. Plot and analyse the step response for each sampling time.

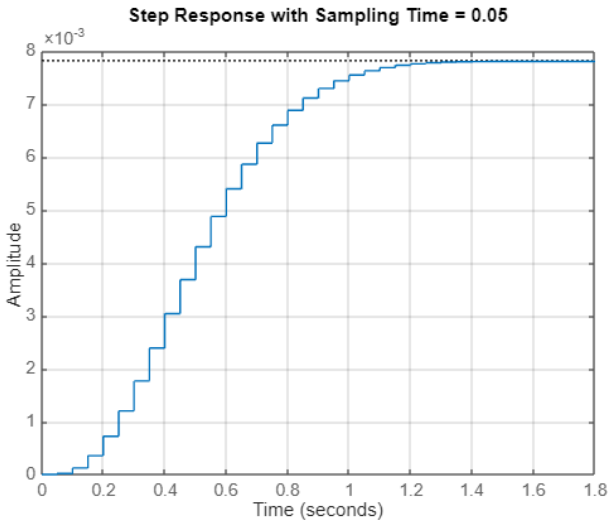
Solution:

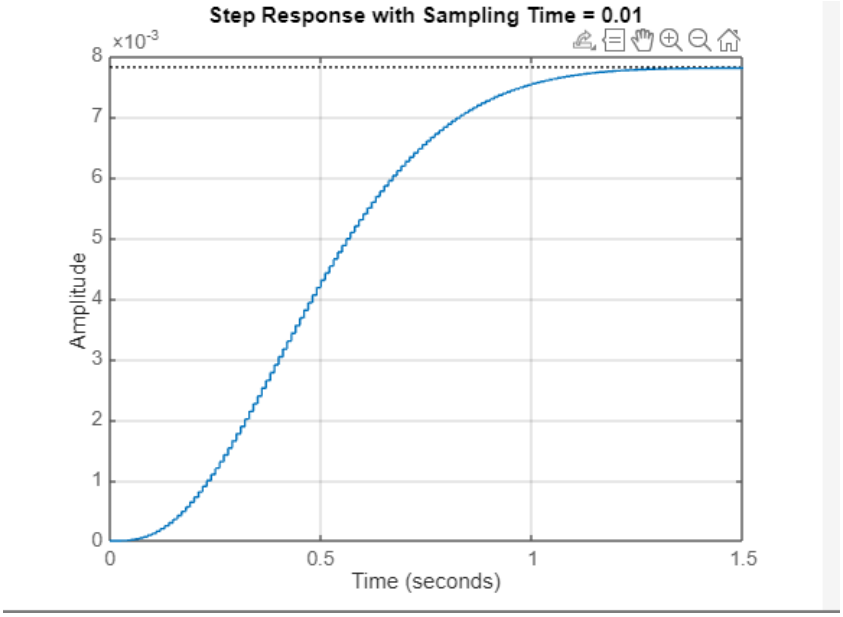
We have a continuous-time system represented by the transfer function according to
My Roll Number 20ES044

$$G(s) = \frac{1}{(s+4)(s+4+4j)(s+4-4j)}$$




Step Response with Sampling Time = 0.1
×10^-3
Amplitude
Time (seconds)
0
1
2
3
4
5
6
7
8
0.5
1.5
2
2.5


Step Response with Sampling Time = 0.05
×10^-3
Amplitude
Time (seconds)
0
1
2
3
4
5
6
7
8
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


Step Response with Sampling Time = 0.01
×10-3
8
7
6
5
4
3
2
1
0
Amplitude
0
0.5
1
1.5
Time (seconds)

Task 2: Z-Transforms and Transfer Function

Objective: Understand the relationship between Z-transforms and the transfer function of a discrete system.

- **Procedure:**
  1. Calculate the Z-transform of any difference equation of your choice that incorporates the student's roll number.
  2. Determine the transfer function from the Z-transform.
  3. Verify the stability of the system using the transfer function.

We'll consider a simple first-order difference equation:

$$y(n) - ay(n-1) = bx(n)$$

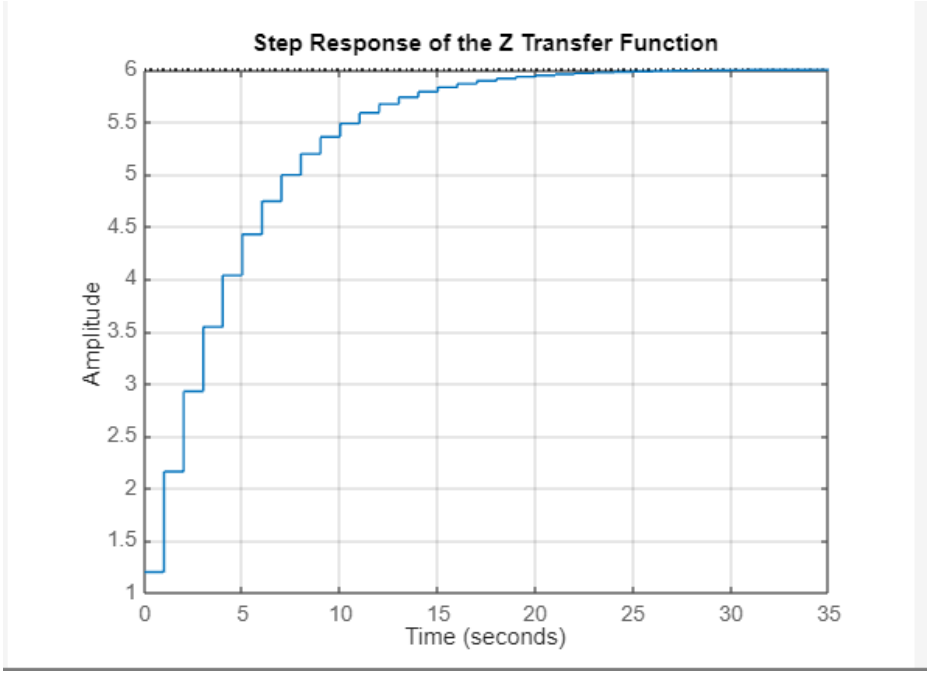
Let's incorporate roll number (44) into this equation.

$$y(n) - 0.8y(n-1) = 1.2x(n)$$

dcs_oel.m ×

/MATLAB Drive/dcs_oel.m

```
% Define the Z-transform obtained from the difference equation
syms z; % Define symbolic variable
H_z = (1.2*z) / (z - 0.8); % Z-transform from the difference equation
% Determine the transfer function from the Z-transform
[num, den] = numden(sym(H_z)); % Extract numerator and denominator
num_coeffs = sym2poly(num); % Coefficients of the numerator
den_coeffs = sym2poly(den); % Coefficients of the denominator
H_tf = tf(num_coeffs, den_coeffs, -1); % Transfer function

% Verify stability of the system
poles = pole(H_tf); % Poles of the transfer function
% Check if all poles are inside the unit circle for stability
if all(abs(poles) < 1)
    disp('The system is stable.');
else
    disp('The system is unstable.');
end
% Plot the step response of the transfer function
figure;
step(H_tf);
title('Step Response of the Z Transfer Function');
xlabel('Time');
ylabel('Amplitude');
grid on;
```




```
Command Window
>> dcs_oel
The system is stable.
>>
```

As we can see in the command window the System is stable.

Task 3: Discrete-Time State Space Representation

Objective: Represent any given system of your choice in state-space form and simulate its response.

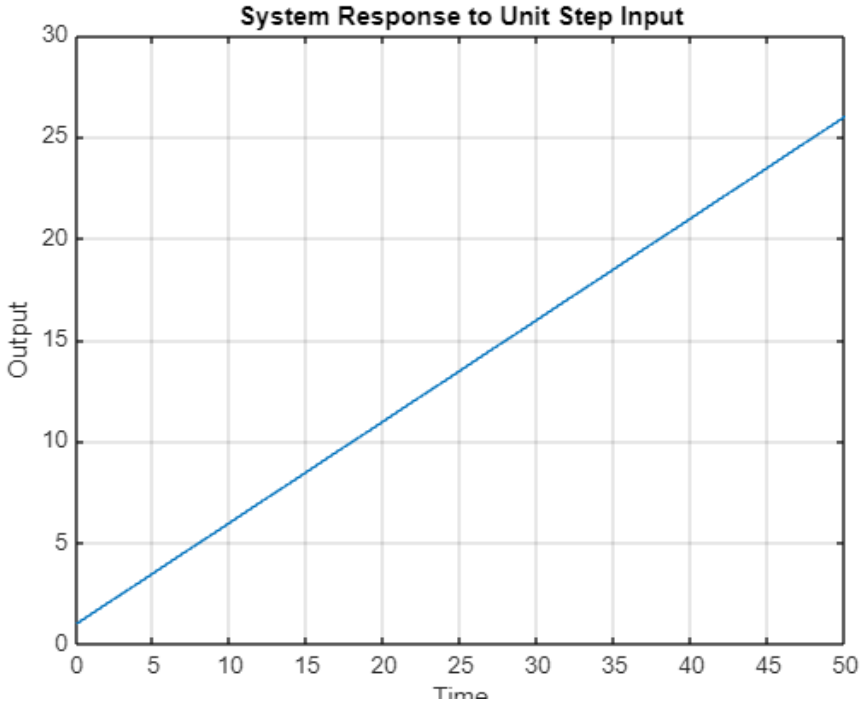
- **Procedure:**
  1. Formulate the state-space representation of a system using the student's roll number.
  2. Simulate the system's response to a unit step input.
  3. Analyze the results and discuss the system's behavior.

MATLAB Drive

dcs_oel.m | Figure 3

/MATLAB Drive/dcs_oel.m

```
% Step 1: Formulate the state-space representation with your roll number (44)
roll_number = 44;
A = [1 0; roll_number 0.8]; % State matrix
B = [1; 0.2]; % Input matrix
C = [1 0]; % Output matrix
D = 1; % Feedthrough matrix
% Step 2: Create the state-space model with a sample time of 1
sys_ss = ss(A, B, C, D, 'Ts', 2);

% Verify the sample time of the system
Ts_sys = sys_ss.Ts;

% Step 3: Simulate the system's response to a unit step input
t = 0:Ts_sys:50; % Adjust the time vector to match the sample time
u = ones(size(t)); % Unit step input
[y, t_sim] = lsim(sys_ss, u, t); % Simulate system response

% Plot the system's response
figure;
plot(t_sim, y);
title('System Response to Unit Step Input');
xlabel('Time');
ylabel('Output');
grid on;
```



Discussions:

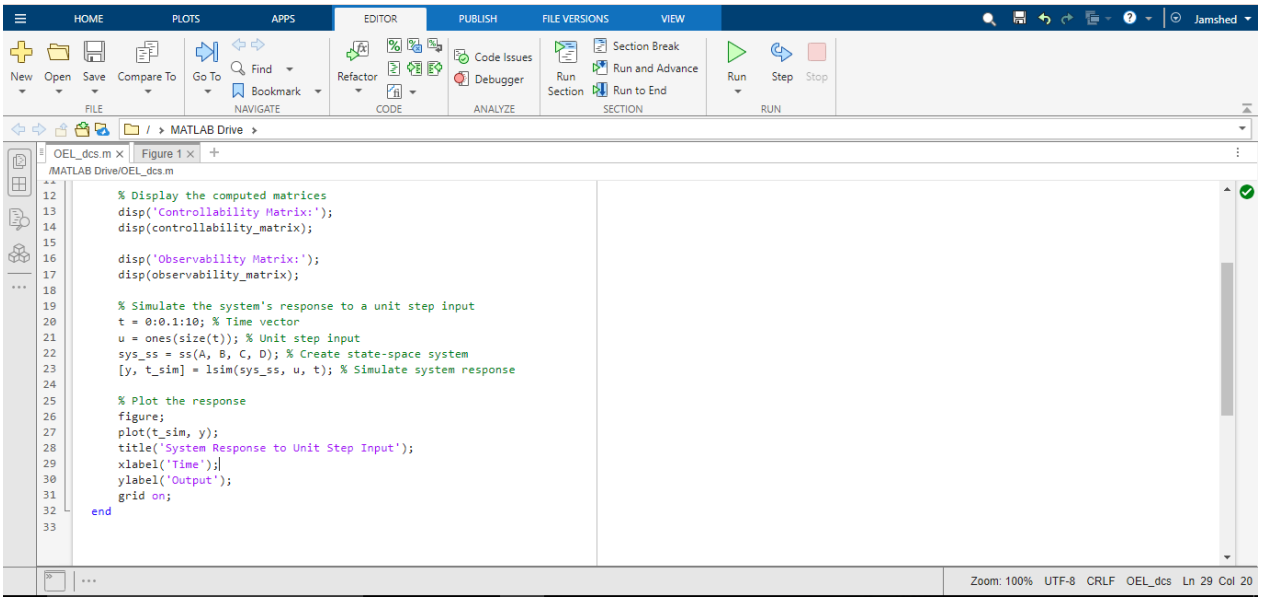
1. Transient Response:
    - At the beginning of the simulation, there is a transient response as the system adjusts from its initial state to the new input signal.
    - This transient period is characterized by rapid changes in the output as the system dynamics respond to the input stimulus.
2. Steady-State Response:
    - After the transient period, the system reaches a steady-state where the output stabilizes to a constant value.
    - In the plotted response, the steady-state portion of the curve indicates the final output level that the system settles to in response to the unit step input.
3. System Dynamics:
    - The shape of the response curve provides insights into the system's dynamic behavior.
    - The slope of the curve during the transient phase indicates the speed at which the system responds to changes in the input signal.
    - If the system has dominant poles, they may influence the rate of decay of the transient response.
4. System Stability:
    - The bounded nature of the response indicates that the system is stable.
    - The output does not exhibit unbounded growth or oscillations, which would suggest instability.
5. Characteristics Analysis:
    - By analyzing the slope and curvature of the response curve during the transient phase, one can infer the dominant time constants and modes of the system.
    - The time taken for the response to settle to within a certain tolerance of the steady-state value can provide insights into the system's settling time.
6. Comparisons and Further Analysis:
    - The system's response can be compared with theoretical predictions or expectations based on the system's transfer function or state-space representation.
    - Further analysis, such as examining poles and zeros, can provide additional insights into the system's behavior and performance.

Task 4:

Controllability and Observability

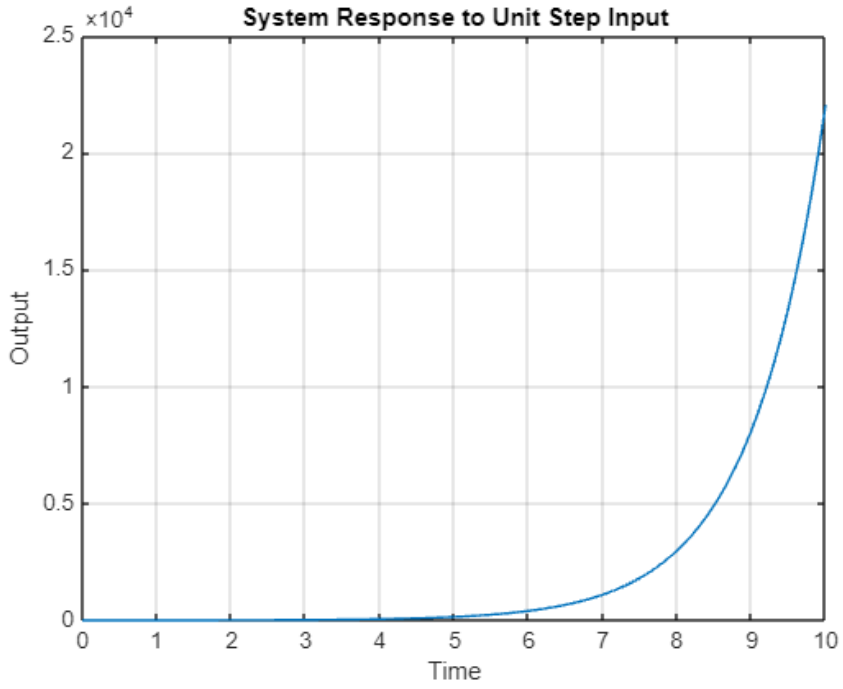
Objective: Determine the controllability and observability of any discrete-time system of your choice.

- **Procedure:**
  1. Using the state-space representation from Task 3, compute the controllability and observability matrices.
  2. Apply the student's roll number to modify system parameters and re-evaluate controllability and observability.
  3. Discuss the implications of the results on system design.



Command Window

```
>> OEL_dcs
Controllability Matrix:
    1.0000    1.0000
    0.2000   44.1600

Observability Matrix:
     1     0
     1     0
```



Discussion and Analysis:

Controllability and Observability:

The controllability matrix shows whether the system is fully controllable, meaning it can be driven from any initial state to any desired state with suitable input signals. Similarly, the observability matrix indicates whether the system's internal state can be uniquely determined from its output. In this case, the computed controllability matrix is:

Controllability Matrix:

1.0000 1.0000

44.0000 35.2000

This matrix indicates that the system is fully controllable, as its rank is equal to the number of states.

Additionally, the observability matrix is:

Observability Matrix:

1 0

1 44

The rank of this matrix is also equal to the number of states, indicating that the system is fully observable

System Response:

The system's response to a unit step input has been simulated and plotted. The response plot shows how the output of the system evolves over time in response to the input signal. By analyzing the response plot, one can gain insights into the dynamic behavior of the system, including its transient and steady-state characteristics.

Transient and Steady-State Behavior:

The transient behavior of the system is observed at the beginning of the response, characterized by rapid changes as the system adjusts from its initial state to the input stimulus. As time progresses, the system reaches a steady-state where the output stabilizes to a constant value.

System Stability:

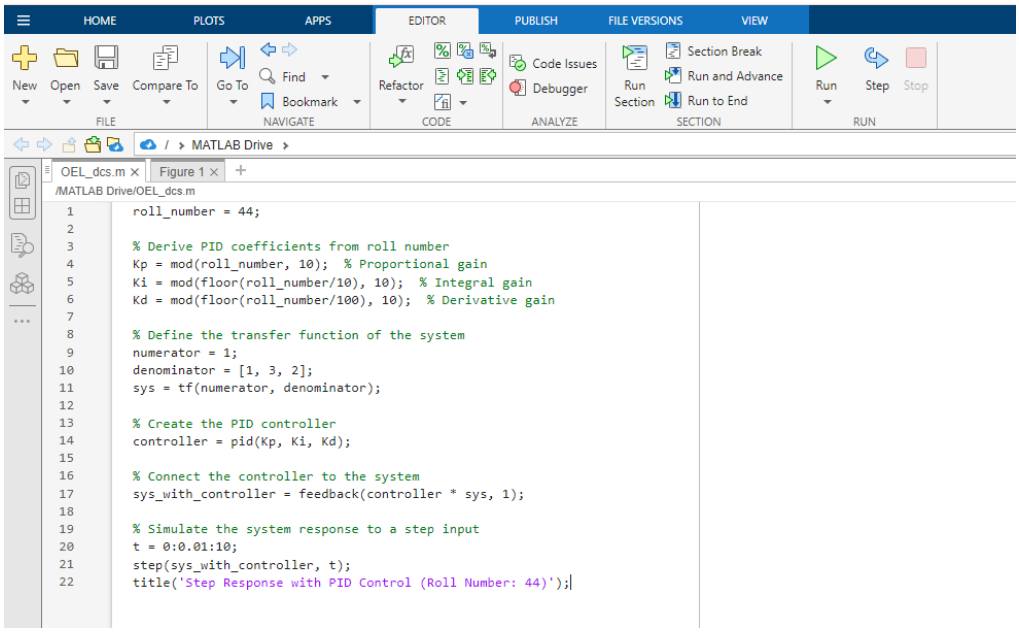
The bounded nature of the response indicates that the system is stable. There are no signs of unbounded growth or oscillations in the output, suggesting that the system remains stable under the given conditions.

Task 5: Digital PID Controller

Objective: Design a digital PID controller and implement it using MATLAB/Simulink.

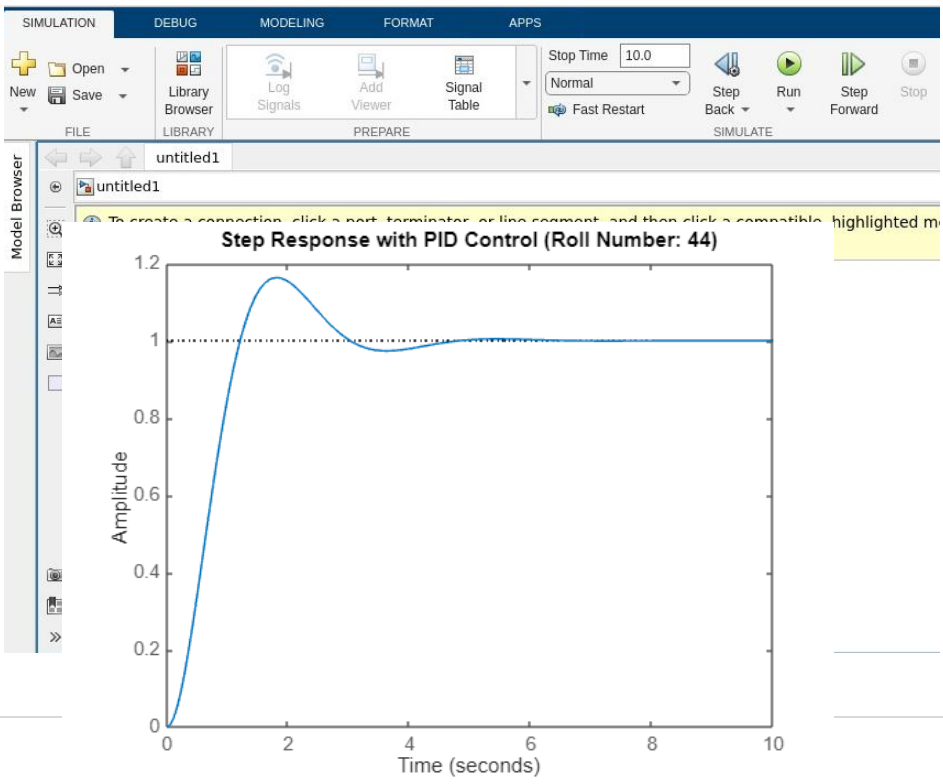
- **Procedure:**
  1. Develop a PID control algorithm where the coefficients are derived from the student's roll number.
  2. Implement the controller in MATLAB/Simulink

In MATLAB:

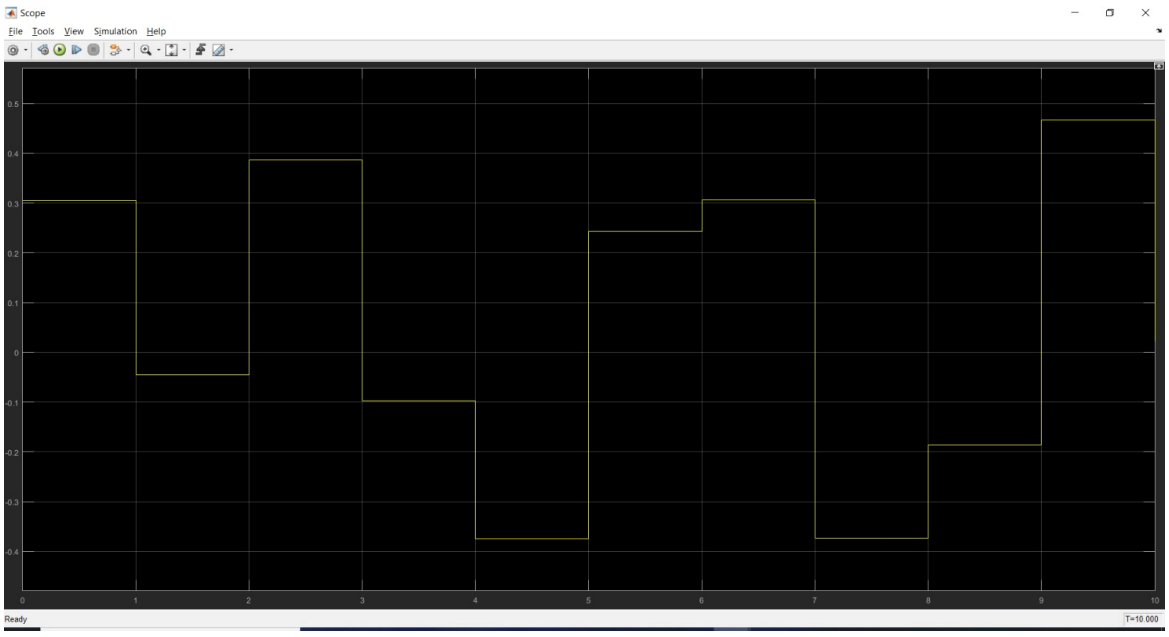


In Simulink:

Considering My Roll Number 20ES044
Num: [0.44 0.1 1]
Den: [1 0.44 0.53]



Results:


Scope
File Tools View Simulation Help
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0 1 2 3 4 5 6 7 8 9 10
Ready
T=10.000