Maximum correntropy unscented filter

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State space function.

$$\begin{aligned} \mathbf{x}(k) &= \mathrm{f}(k-1, \mathbf{x}(k-1)) + \mathbf{q}(k-1) \\ \mathbf{y}(k) &= \mathrm{h}(k, \mathbf{x}(k)) + \mathbf{r}(k) \end{aligned} \tag{1}$$

Expectation of kernel.

$$V(X,Y) = \mathrm{E}[\kappa(X,Y)] = \int \kappa(x,y) \mathrm{dF}_{XY}(x,y) \tag{2}$$

Kernel function.

$$\kappa(x,y) = \mathrm{G}_\sigma(e) = \exp\left(-\frac{e^2}{2\sigma^2}\right) \tag{3}$$

Sigma points.

$$\begin{array}{l} \chi^0(k-1 \mid k-1) = \widehat{\mathbf{x}}(k-1 \mid k-1) \\ \chi^i(k-1 \mid k-1) = \widehat{\mathbf{x}}(k-1 \mid k-1) \\ \quad +(\sqrt{(n+\lambda)\mathbf{P}(k-1 \mid k-1)})_i, \text{ for } i = 1 \ldots n \\ \chi^i(k-1 \mid k-1) = \widehat{\mathbf{x}}(k-1 \mid k-1) \\ -(\sqrt{(n+\lambda)\mathbf{P}(k-1 \mid k-1)})_{i-n}, \text{ for } i = n+1 \ldots 2n.(9) \end{array} \tag{4}$$

Parameter of UT.

$$\lambda = \alpha^2(n+\phi) - n \tag{5}$$

Sigma points after UT.

$$\chi^{i*}(k \mid k-1) = \mathrm{f}\left(k-1, \chi^i(k-1 \mid k-1)\right), \text{ for } i = 0 \ldots 2n \tag{6}$$

Mean of UTed sigma points, which will be the priori of state.

$$\widehat{\mathbf{x}}(k \mid k-1) = \sum_{i=0}^{2n} w_m^i \chi^{i*}(k \mid k-1) \tag{7}$$

Covariance matrix of $\chi^{i*}$and$\widehat{\mathbf{x}}(k \mid k-1)$, which should be the Predicted estimate covariance of KF.

$$\begin{aligned} \mathbf{P}(k \mid k-1) = \textstyle\sum_{i=0}^{2n} w_c^i \left[\chi^{i*}(k \mid k-1) - \widehat{\mathbf{x}}(k \mid k-1)\right] \\ \times \left[\chi^{i*}(k \mid k-1) - \widehat{\mathbf{x}}(k \mid k-1)\right]^T + \mathbf{Q}(k-1) \end{aligned} \tag{8}$$

UTed observation.

$$\gamma^i(k) = \mathrm{h}\left(k, \chi^i(k \mid k-1)\right), \text{ for } i = 0 \ldots 2n \tag{9}$$

With weight, like a mean of UTed observation.

$$\widehat{\mathbf{y}}(k) = \sum_{i=0}^{2n} w_m^i \gamma^i(k) \tag{10}$$

Covariance matrix ofχ^{i*}andγ^i, this shows the relationship between χ^{i*}andγ^i.

$$\mathbf{P}_{\mathbf{xy}}(k) = \sum_{i=0}^{2n} w_c^i \left[\chi^i(k \mid k-1) - \widehat{\mathbf{x}}(k \mid k-1)\right] \left[\gamma^i(k) - \widehat{\mathbf{y}}(k)\right]^T \tag{11}$$

$x(k)$is the real state, means $\eta$is observation error.

$$\eta(\mathbf{x}(k)) = \mathbf{x}(k) - \widehat{\mathbf{x}}(k \mid k-1) \tag{12}$$

The measurement slope matrix, I don't know what is this.

$$\mathbf{H}(k) = \left(\mathbf{P}^{-1}(k \mid k-1)\mathbf{P}_{\mathbf{xy}}(k)\right)^{T} \tag{13}$$

Because of Wang et al. (2010):

$$\mathbf{y}(k) \approx \widehat{\mathbf{y}}(k) + \mathbf{H}(k)(\mathbf{x}(k) - \widehat{\mathbf{x}}(k \mid k-1)) + \mathbf{r}(k) \tag{14}$$

For (7), (10), (14), the statistical linear regression model will be

$$\left[\begin{array}{c} \widehat{\mathbf{x}}(k \mid k-1) \\ \mathbf{y}(k) - \widehat{\mathbf{y}}(k) + \mathbf{H}(k)\widehat{\mathbf{x}}(k \mid k-1) \end{array}\right] = \left[\begin{array}{c} \mathbf{I} \\ \mathbf{H}(k) \end{array}\right] \mathbf{x}(k) + \xi(k) \tag{15}$$

Where

$$\xi(k) = \left[\begin{array}{c} \eta(\mathbf{x}(k)) \\ \mathbf{r}(k) \end{array}\right]$$

With

$$\begin{aligned} \Xi(k) &= \mathrm{E}\left[\xi(k)\xi^{T}(k)\right] \\ &= \left[\begin{array}{cc} \mathbf{P}(k \mid k-1) & 0 \\ 0 & \mathbf{R}(k) \end{array}\right] \\ &= \left[\begin{array}{cc} \mathbf{S}_p(k \mid k-1)\mathbf{S}_p^{T}(k \mid k-1) & 0 \\ 0 & \mathbf{S}_r(k)\mathbf{S}_r^{T}(k) \end{array}\right] \\ &= \mathbf{S}(k)\mathbf{S}^{T}(k) \end{aligned} \tag{16}$$

Transform (15) to

$$\mathbf{D}(k) = \mathbf{W}(k)\mathbf{x}(k) + \mathbf{e}(k) \tag{17}$$

Where

$$\begin{aligned} \mathbf{D}(k) &= \mathbf{S}^{-1}(k)\left[\begin{array}{c} \widehat{\mathbf{x}}(k \mid k-1) \\ \mathbf{y}(k) - \widehat{\mathbf{y}}(k) + \mathbf{H}(k)\widehat{\mathbf{x}}(k \mid k-1) \end{array}\right] \\ \mathbf{W}(k) &= \mathbf{S}^{-1}(k)\left[\begin{array}{c} \mathbf{I} \\ \mathbf{H}(k) \end{array}\right] \\ \mathbf{e}(k) &= \mathbf{S}^{-1}(k)\xi(k) \end{aligned}$$

With $\mathrm{E}\left[\mathbf{e}(k)\mathbf{e}^{T}(k)\right] = \mathbf{I}$. Define a cost function based on the MCC:

$$J_L(\mathbf{x}(k)) = \sum_{i=1}^{L} \mathrm{G}_{\sigma}\left(d_i(k) - \mathbf{w}_i(k)\mathbf{x}(k)\right) \tag{18}$$

The optimal solution is:

$$\frac{\partial J_L(\mathbf{x}(k))}{\partial \mathbf{x}(k)} = 0 \tag{19}$$

Which should able to be transformed to:

$$\begin{aligned} \mathbf{x}(k) = &\left(\sum_{i=1}^{L}\left(\mathrm{G}_{\sigma}\left(e_i(k)\right)\mathbf{w}_i^{T}(k)\mathbf{w}_i(k)\right)\right)^{-1} \\ &\times\left(\sum_{i=1}^{L}\left(\mathrm{G}_{\sigma}\left(e_i(k)\right)\mathbf{w}_i^{T}(k)d_i(k)\right)\right) \end{aligned} \tag{20}$$

But I'm not sure how. Since $e_i(k) = d_i(k) - \mathbf{w}_i(k)\mathbf{x}(k)$, obtain x(k) with a fixed-point iterative algorithm

$$\widehat{\mathbf{x}}(k)_{t+1} = g\left(\widehat{\mathbf{x}}(k)_t\right) \tag{21}$$

For x(k) is a matrix, (20) can also be expressed as

$$\mathbf{x}(k)=\left(\mathbf{W}^T(k)\mathbf{C}(k)\mathbf{W}(k)\right)^{-1}\mathbf{W}^T(k)\mathbf{C}(k)\mathbf{D}(k) \tag{22}$$

$$\begin{aligned} &\text{where } \mathbf{C}(k)=\left[\begin{array}{cc} \mathbf{C}_x(k) & 0 \\ 0 & \mathbf{C}_y(k) \end{array}\right], \text{ with} \\ &\mathbf{C}_x(k)=\operatorname{diag}\left(\mathbf{G}_\sigma\left(e_1(k)\right),\ldots,\mathbf{G}_\sigma\left(e_n(k)\right)\right) \\ &\mathbf{C}_y(k)=\operatorname{diag}\left(\mathbf{G}_\sigma\left(e_{n+1}(k)\right),\ldots,\mathbf{G}_\sigma\left(e_{n+m}(k)\right)\right) \end{aligned} \tag{23}$$

So the correct function of KF is

$$\mathbf{x}(k)=\widehat{\mathbf{x}}(k\mid k-1)+\overline{\mathbf{K}}(k)(\mathbf{y}(k)-\widehat{\mathbf{y}}(k)) \tag{24}$$

where

$$\left\{\begin{array}{l} \overline{\mathbf{K}}(k)=\overline{\mathbf{P}}(k\mid k-1)\mathbf{H}^T(k)\left(\mathbf{H}(k)\overline{\mathbf{P}}(k\mid k-1)\mathbf{H}^T(k)+\overline{\mathbf{R}}(k)\right)^{-1} \\ \overline{\mathbf{P}}(k\mid k-1)=\mathbf{S}_p(k\mid k-1)\mathbf{C}_x^{-1}(k)\mathbf{S}_p^T(k\mid k-1) \\ \overline{\mathbf{R}}(k)=\mathbf{S}_r(k)\mathbf{C}_y^{-1}(k)\mathbf{S}_r^T(k) \end{array}\right. \tag{25}$$

And the corresponding covariance matrix is updated by

$$\begin{aligned} \mathbf{P}(k\mid k)=&(\mathbf{I}-\overline{\mathbf{K}}(k)\mathbf{H}(k))\mathbf{P}(k\mid k-1)(\mathbf{I}-\overline{\mathbf{K}}(k)\mathbf{H}(k))^T \\ &+\overline{\mathbf{K}}(k)\mathbf{R}(k)\overline{\mathbf{K}}^T(k) \end{aligned} \tag{26}$$