MCKF Summarize

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The Common KF

Predict:

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= \mathbf{F}_k\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k\mathbf{u}_k \\ \mathbf{P}_{k|k-1} &= \mathbf{F}_k\mathbf{P}_{k-1|k-1}\mathbf{F}_k^\top + \mathbf{Q}_k\end{aligned}$$

Update:

$$\begin{aligned}\tilde{\mathbf{y}}_k &= \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} \\ \mathbf{S}_k &= \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^\top + \mathbf{R}_k \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1}\mathbf{H}_k^\top\mathbf{S}_k^{-1} \\ \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\tilde{\mathbf{y}}_k \\ \mathbf{P}_{k|k} &= \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)\mathbf{P}_{k|k-1} \\ \tilde{\mathbf{y}}_{k|k} &= \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k}\end{aligned}$$

The MCKF

By fixed point iteration

Predict is exactly same with KF.

Update:

$$c = \widehat{\mathbf{x}}(k \mid k-1) + \widetilde{\mathbf{K}}(k)(\mathbf{y}(k) - \mathbf{H}(k)\widehat{\mathbf{x}}(k \mid k-1))$$

with

$$\begin{aligned}&\tilde{\mathbf{K}}(k) = \tilde{\mathbf{P}}(k \mid k-1)\mathbf{H}^T(k)\left(\mathbf{H}(k)\tilde{\mathbf{P}}(k \mid k-1)\mathbf{H}^T(k) + \widetilde{\mathbf{R}}(k)\right)^{-1} \\ &\widetilde{\mathbf{P}}(k \mid k-1) = \mathbf{B}_p(k \mid k-1)\tilde{\mathbf{C}}_x^{-1}(k)\mathbf{B}_p^T(k \mid k-1) \\ &\widetilde{\mathbf{R}}(k) = \mathbf{B}_r(k)\widetilde{\mathbf{C}}_y^{-1}(k)\mathbf{B}_r^T(k) \\ &\widetilde{\mathbf{C}}_x(k) = \operatorname{diag}\left(G_\sigma\left(\tilde{e}_1(k)\right), \ldots, G_\sigma\left(\widetilde{e}_n(k)\right)\right) \\ &\widetilde{\mathbf{C}}_y(k) = \operatorname{diag}\left(G_\sigma\left(\widetilde{e}_{n+1}(k)\right), \ldots, G_\sigma\left(\widetilde{e}_{n+m}(k)\right)\right) \\ &\widetilde{e}_i(k) = d_i(k) - \mathbf{w}_i(k)\widehat{\mathbf{x}}(k \mid k)_{t-1}\end{aligned}$$

By differential

Predict is exactly same with KF.

Update:

$$\begin{gathered}L_k = \frac{G_\sigma\left(\left\|y_k - H\hat{x}_k^-\right\|_{R_k^{-1}}\right)}{G_\sigma\left(\left\|\hat{x}_k^- - F\hat{x}_{k-1}\right\|_{P_{k|k-1}^{-1}}\right)} \\ K_k = \left(P_{k|k-1}^{-1} + L_k H^T R_k^{-1} H\right)^{-1} L_k H^T R_k^{-1} \\ \hat{x}_k = \hat{x}_k^- + K_k\left(y_k - H\hat{x}_k^-\right) \\ P_{k|k} = \left(I - K_k H\right)P_{k|k-1}\left(I - K_k H\right)^T + K_k R_k K_k^T\end{gathered}$$

The UKF

Predict:

$$
\begin{aligned}
&\mathcal{X}_{t-1}=\left(\mu_{t-1} \quad \mu_{t-1}+\gamma \sqrt{\Sigma_{t-1}} \quad \mu_{t-1}-\gamma \sqrt{\Sigma_{t-1}}\right) \\
&\overline{\mathcal{X}}_t^*=g\left(u_t, \mathcal{X}_{t-1}\right) \\
&\bar{\mu}_t=\sum_{i=0}^{2n} w_m^{[i]} \overline{\mathcal{X}}_t^{*[i]} \\
&\bar{\Sigma}_t=\sum_{i=0}^{2n} w_c^{[i]}\left(\bar{X}_t^*[i]-\bar{\mu}_t\right)\left(\overline{\mathcal{X}}_t^*[i]-\bar{\mu}_t\right)^T+Q_t \\
&\bar{X}_t=\left(\bar{\mu}_t \quad \bar{\mu}_t+\gamma \sqrt{\bar{\Sigma}_t} \quad \bar{\mu}_t-\gamma \sqrt{\bar{\Sigma}_t}\right) \\
&\overline{\mathcal{Z}}_t=h\left(\overline{\mathcal{X}}_t\right) \\
&\hat{z}_t=\sum_{i=0}^{2n} w_m^{[i]} \overline{\mathcal{Z}}_t^{[i]}
\end{aligned}
$$

$$
S_t=\sum_{i=0}^{2n} w_c^{[i]}\left(\bar{z}_t^{[i]}-\hat{z}_t\right)\left(\overline{\mathcal{Z}}_t^{[i]}-\hat{z}_t\right)^T+R_t
$$

$\bar{\Sigma}_t$ can also showed as $P^-1(k|k-1)$ or $P_x x$, S_t can also showed as P_{zz}.

Update:

$$
\begin{aligned}
&\bar{\Sigma}_t^{x,z}=\sum_{i=0}^{2n} w_c^{[i]}\left(\overline{\mathcal{X}}_t^{[i]}-\bar{\mu}_t\right)\left(\overline{\mathcal{Z}}_t^{[i]}-\hat{z}_t\right)^T \\
&K_t=\bar{\Sigma}_t^{x,z} S_t^{-1} \\
&\mu_t=\bar{\mu}_t+K_t\left(z_t-\hat{z}_t\right) \\
&\Sigma_t=\bar{\Sigma}_t-K_t S_t K_t^T
\end{aligned}
$$

$\bar{\Sigma}_t^{x,z}$ can also showed as P_{xz}

The MCUKF

By fixed point iteration

Predict is exactly same with UKF.

Update:

$$
\begin{aligned}
\widehat{\mathbf{x}}(k \mid k)_t &=\widehat{\mathbf{x}}(k \mid k-1)+\widetilde{\mathbf{K}}(k)(\mathbf{y}(k)-\widehat{\mathbf{y}}(k)) \\
\widetilde{\mathbf{K}}(k) &=\widetilde{\mathbf{P}}(k \mid k-1) \mathbf{H}^T(k) \\
&\times\left(\mathbf{H}(k) \widetilde{\mathbf{P}}(k \mid k-1) \mathbf{H}^T(k)+\widetilde{\mathbf{R}}(k)\right)^{-1} \\
\widetilde{\mathbf{P}}(k \mid k-1) &=\mathbf{S}_p(k \mid k-1) \widetilde{\mathbf{C}}_x^{-1}(k) \mathbf{S}_p^T(k \mid k-1) \\
\widetilde{\mathbf{R}}(k) &=\mathbf{S}_r(k) \widetilde{\mathbf{C}}_y^{-1}(k) \mathbf{S}_r^T(k) \\
\widetilde{\mathbf{C}}_x(k) &=\operatorname{diag}\left(\mathrm{G}_\sigma\left(\widetilde{e}_1(k)\right), \ldots, \mathrm{G}_\sigma\left(\widetilde{e}_n(k)\right)\right) \\
\widetilde{\mathbf{C}}_y(k) &=\operatorname{diag}\left(\mathrm{G}_\sigma\left(\widetilde{e}_{n+1}(k)\right), \ldots, \mathrm{G}_\sigma\left(\widetilde{e}_{n+m}(k)\right)\right) \\
\widetilde{e}_i(k) &=d_i(k)-\mathbf{w}_i(k) \widehat{\mathbf{x}}(k \mid k)_{t-1}
\end{aligned}
$$

where:

$$
\begin{aligned}
\mathbf{D}(k) &=\mathbf{S}^{-1}(k)\left[\begin{array}{c}
\widehat{\mathbf{x}}(k \mid k-1) \\
\mathbf{y}(k)-\widehat{\mathbf{y}}(k)+\mathbf{H}(k) \widehat{\mathbf{x}}(k \mid k-1)
\end{array}\right] \\
\mathbf{W}(k) &=\mathbf{S}^{-1}(k)\left[\begin{array}{c}
\mathbf{I} \\
\mathbf{H}(k)
\end{array}\right] \\
\mathbf{e}(k) &=\mathbf{S}^{-1}(k) \xi(k)
\end{aligned}
$$

$$
\mathbf{H}(k)=\left(\mathbf{P}^{-1}(k \mid k-1) \mathbf{P}_{\mathbf{xy}}(k)\right)^T
$$

By differential

Predict is exactly same with UKF.

Update:

$$\begin{aligned}
\mathbf{K}(k) &= (P_{xx}(k) + (P_{zz}(k) - R(k))L^T)^{-1}\widetilde{H}R^{-1} \\
x_{k|k} &= \bar{x} + K(k)(y - \bar{y}) \\
P_{k|k} &= (I - K\widetilde{H})P_{xx}(I - K\widetilde{H})^T + KRK^T
\end{aligned} \tag{1}$$

with:

$$\begin{aligned}
L &= \frac{G(\left\| y - \widetilde{H}\bar{x} \right\|_{R_k^{-1}})}{G(\|\bar{x} - f(x_{k-1})\|_{p_{xx}^{-1}})} \\
\widetilde{H} &= P_{xx}^{-1}P_{xz}
\end{aligned} \tag{2}$$