

Maximum correntropy unscented filter

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State space function.

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{f}(k-1, \mathbf{x}(k-1)) + \mathbf{q}(k-1) \\ \mathbf{y}(k) &= \mathbf{h}(k, \mathbf{x}(k)) + \mathbf{r}(k)\end{aligned}\quad (1)$$

Expectation of kernel.

$$V(X, Y) = \mathbb{E}[\kappa(X, Y)] = \int \kappa(x, y) dF_{XY}(x, y) \quad (2)$$

Kernel function.

$$\kappa(x, y) = G_\sigma(e) = \exp\left(-\frac{e^2}{2\sigma^2}\right) \quad (3)$$

Sigma points.

$$\begin{aligned}\chi^0(k-1 | k-1) &= \widehat{\mathbf{x}}(k-1 | k-1) \\ \chi^i(k-1 | k-1) &= \widehat{\mathbf{x}}(k-1 | k-1) \\ &\quad + (\sqrt{(n+\lambda)\mathbf{P}(k-1 | k-1)})_i, \text{ for } i = 1 \dots n \\ \chi^i(k-1 | k-1) &= \widehat{\mathbf{x}}(k-1 | k-1) \\ &\quad - (\sqrt{(n+\lambda)\mathbf{P}(k-1 | k-1)})_{i-n}, \text{ for } i = n+1 \dots 2n.\end{aligned}\quad (4)$$

Parameter of UT.

$$\lambda = \alpha^2(n + \phi) - n \quad (5)$$

Sigma points after UT.

$$\chi^{i*}(k | k-1) = \mathbf{f}(k-1, \chi^i(k-1 | k-1)), \text{ for } i = 0 \dots 2n \quad (6)$$

Mean of UTed sigma points, which will be the priori of state.

$$\widehat{\mathbf{x}}(k | k-1) = \sum_{i=0}^{2n} w_m^i \chi^{i*}(k | k-1) \quad (7)$$

Covariance matrix of χ^{i*} and $\widehat{\mathbf{x}}(k | k-1)$, which should be the Predicted estimate covariance of KF.

$$\begin{aligned}\mathbf{P}(k | k-1) &= \sum_{i=0}^{2n} w_c^i [\chi^{i*}(k | k-1) - \widehat{\mathbf{x}}(k | k-1)] \\ &\quad \times [\chi^{i*}(k | k-1) - \widehat{\mathbf{x}}(k | k-1)]^T + \mathbf{Q}(k-1)\end{aligned}\quad (8)$$

UTed observation.

$$\gamma^i(k) = \mathbf{h}(k, \chi^i(k | k-1)), \text{ for } i = 0 \dots 2n \quad (9)$$

With weight, like a mean of UTed observation.

$$\widehat{\mathbf{y}}(k) = \sum_{i=0}^{2n} w_m^i \gamma^i(k) \quad (10)$$

Covariance matrix of χ^{i*} and γ^i , this shows the relationship between χ^{i*} and γ^i .

$$\mathbf{P}_{\mathbf{xy}}(k) = \sum_{i=0}^{2n} w_c^i [\chi^i(k | k-1) - \widehat{\mathbf{x}}(k | k-1)] [\gamma^i(k) - \widehat{\mathbf{y}}(k)]^T \quad (11)$$

$x(k)$ is the real state, means η is observation error.

$$\eta(\mathbf{x}(k)) = \mathbf{x}(k) - \hat{\mathbf{x}}(k | k-1) \quad (12)$$

The measurement slope matrix, I don't know what is this.

$$\mathbf{H}(k) = (\mathbf{P}^{-1}(k | k-1) \mathbf{P}_{\mathbf{xy}}(k))^T \quad (13)$$

Because of Wang et al. (2010):

$$\mathbf{y}(k) \approx \hat{\mathbf{y}}(k) + \mathbf{H}(k)(\mathbf{x}(k) - \hat{\mathbf{x}}(k | k-1)) + \mathbf{r}(k) \quad (14)$$

For (7), (10), (14), the statistical linear regression model will be

$$\begin{bmatrix} \hat{\mathbf{x}}(k | k-1) \\ \mathbf{y}(k) - \hat{\mathbf{y}}(k) + \mathbf{H}(k)\hat{\mathbf{x}}(k | k-1) \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{H}(k) \end{bmatrix} \mathbf{x}(k) + \xi(k) \quad (15)$$

Where

$$\xi(k) = \begin{bmatrix} \eta(\mathbf{x}(k)) \\ \mathbf{r}(k) \end{bmatrix}$$

With

$$\begin{aligned} \Xi(k) &= \mathbb{E} [\xi(k)\xi^T(k)] \\ &= \begin{bmatrix} \mathbf{P}(k | k-1) & 0 \\ 0 & \mathbf{R}(k) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{S}_p(k | k-1)\mathbf{S}_p^T(k | k-1) & 0 \\ 0 & \mathbf{S}_r(k)\mathbf{S}_r^T(k) \end{bmatrix} \\ &= \mathbf{S}(k)\mathbf{S}^T(k) \end{aligned} \quad (16)$$

Transform (15) to

$$\mathbf{D}(k) = \mathbf{W}(k)\mathbf{x}(k) + \mathbf{e}(k) \quad (17)$$

Where

$$\begin{aligned} \mathbf{D}(k) &= \mathbf{S}^{-1}(k) \begin{bmatrix} \hat{\mathbf{x}}(k | k-1) \\ \mathbf{y}(k) - \hat{\mathbf{y}}(k) + \mathbf{H}(k)\hat{\mathbf{x}}(k | k-1) \end{bmatrix} \\ \mathbf{W}(k) &= \mathbf{S}^{-1}(k) \begin{bmatrix} \mathbf{I} \\ \mathbf{H}(k) \end{bmatrix} \\ \mathbf{e}(k) &= \mathbf{S}^{-1}(k)\xi(k) \end{aligned}$$

With $\mathbb{E} [\mathbf{e}(k)\mathbf{e}^T(k)] = \mathbf{I}$. Define a cost function based on the MCC:

$$J_L(\mathbf{x}(k)) = \sum_{i=1}^L G_\sigma(d_i(k) - \mathbf{w}_i(k)\mathbf{x}(k)) \quad (18)$$

The optimal solution is:

$$\frac{\partial J_L(\mathbf{x}(k))}{\partial \mathbf{x}(k)} = 0 \quad (19)$$

Which should be able to be transformed to:

$$\begin{aligned} \mathbf{x}(k) &= \left(\sum_{i=1}^L (G_\sigma(e_i(k)) \mathbf{w}_i^T(k) \mathbf{w}_i(k)) \right)^{-1} \\ &\quad \times \left(\sum_{i=1}^L (G_\sigma(e_i(k)) \mathbf{w}_i^T(k) d_i(k)) \right) \end{aligned} \quad (20)$$

But I'm not sure how. Since $e_i(k) = d_i(k) - \mathbf{w}_i(k)\mathbf{x}(k)$, obtain $\mathbf{x}(k)$ with a fixed-point iterative algorithm

$$\hat{\mathbf{x}}(k)_{t+1} = g(\hat{\mathbf{x}}(k)_t) \quad (21)$$

For $\mathbf{x}(k)$ is a matrix, (20) can also be expressed as

$$\mathbf{x}(k) = (\mathbf{W}^T(k)\mathbf{C}(k)\mathbf{W}(k))^{-1} \mathbf{W}^T(k)\mathbf{C}(k)\mathbf{D}(k) \quad (22)$$

$$\begin{aligned} \text{where } \mathbf{C}(k) &= \begin{bmatrix} \mathbf{C}_x(k) & 0 \\ 0 & \mathbf{C}_y(k) \end{bmatrix}, \text{ with} \\ \mathbf{C}_x(k) &= \text{diag}(\mathbf{G}_\sigma(e_1(k)), \dots, \mathbf{G}_\sigma(e_n(k))) \\ \mathbf{C}_y(k) &= \text{diag}(\mathbf{G}_\sigma(e_{n+1}(k)), \dots, \mathbf{G}_\sigma(e_{n+m}(k))) \end{aligned} \quad (23)$$

So the correct function of KF is

$$\mathbf{x}(k) = \hat{\mathbf{x}}(k | k-1) + \bar{\mathbf{K}}(k)(\mathbf{y}(k) - \hat{\mathbf{y}}(k)) \quad (24)$$

where

$$\begin{cases} \bar{\mathbf{K}}(k) = \bar{\mathbf{P}}(k | k-1)\mathbf{H}^T(k) (\mathbf{H}(k)\bar{\mathbf{P}}(k | k-1)\mathbf{H}^T(k) + \bar{\mathbf{R}}(k))^{-1} \\ \bar{\mathbf{P}}(k | k-1) = \mathbf{S}_p(k | k-1)\mathbf{C}_x^{-1}(k)\mathbf{S}_p^T(k | k-1) \\ \bar{\mathbf{R}}(k) = \mathbf{S}_r(k)\mathbf{C}_y^{-1}(k)\mathbf{S}_r^T(k) \end{cases} \quad (25)$$

And the corresponding covariance matrix is updated by

$$\begin{aligned} \mathbf{P}(k | k) &= (\mathbf{I} - \bar{\mathbf{K}}(k)\mathbf{H}(k))\mathbf{P}(k | k-1)(\mathbf{I} - \bar{\mathbf{K}}(k)\mathbf{H}(k))^T \\ &\quad + \bar{\mathbf{K}}(k)\mathbf{R}(k)\bar{\mathbf{K}}^T(k) \end{aligned} \quad (26)$$