КАРАСЕВА Домашняя работа

$\boxed{№1}$

$2x^2yy' + y^2 = 2$

$2x^2yy' = 2 - y^2$

$\frac{2x^2y\,dy}{dx} = 2 - y^2 \quad \Big| \cdot \frac{dx}{x^2(2-y^2)}$

1. $2 - y^2 = 0$

   $y = \pm\sqrt{2}$

   $y' = 0$

   $y = \pm\sqrt{2}$ - решение

2. $x^2 = 0$

   $x = 0$

   $2 - y^2 = 0$

   $y =$

$$\int \frac{2y\,dy}{(2-y^2)} = \int \frac{dx}{x^2}$$

$$-\int \frac{dy^2}{y^2-2} = \int \frac{dx}{x^2}$$

$$-\ln|y^2-2| = -\frac{1}{x} + C$$

$$\ln|y^2-2| = \frac{1}{x} + C$$

$$|y^2-2| = e^{1/x} \cdot C$$

Ответ $|y^2-2| = e^{1/x} \cdot C$, $y = \pm\sqrt{2}$

$\boxed{№2}$ $y' = \cos(y-x)$

$\exists\ t = y - x \iff y = t + x$

$dt = d(y-x) = dy - dx \Rightarrow dy = dt + dx$

$$\frac{dt+dx}{dx} = \cos t$$

$$\frac{dt}{dx} + 1 = \cos t$$

$$\frac{dt}{dx} = \cos t - 1 \quad \Big| : \cos t - 1$$

1) $\cos t - 1 = 0$

$t = 2\pi k,\ k \in Z$

$t'(x) = 0$ решение $\Rightarrow y = 2\pi k + x,\ k \in Z$

$0 = 0$

$$\int \frac{dt}{\cos t - 1} = \int dx$$

$$ctg\, t/2 = x + c$$

$$t = 2arcctg(x+c)$$

$$y = 2arcctg(x+c) + x$$

Ответ: $y = 2arcctg(x+c) + x$

$y = 2\pi k + x,\ k \in Z$

$\boxed{N3}$ $y' - y = 2x - 3$

$\frac{dy}{dx} = y + 2x - 3$

$t = y + 2x - 3 \Leftrightarrow y = t - 2x - 3$

$dt = dy + 2dx \Leftrightarrow dy = dt - 2dx$

$\frac{dt}{dx} - 2 = t$

$\frac{dt}{dx} = t + 2 \quad | \cdot \frac{dx}{t+2}$

① $t + 2 = 0$

$t = -2$ - решение $\Rightarrow y = -2x - 5$

$t'(x) = 0$

$\int \frac{dt}{t+2} = \int dx$

$\ln|t+2| = x + C$

$|t+2| = C \cdot e^x$

② $t < -2$

$-y - 2x + 3 - 2 = C \cdot e^x$

$y = -2x + 1 - C \cdot e^x$

③ $t > 2$

$y + 2x - 3 + 2 = C \cdot e^x$

равносильны

$y = -2x + 1 + C \cdot e^x$

Ответ: $y = -2x + 1 + C \cdot e^x$

$y = -2x - 5$

$\boxed{N 4}$ $y' = \sqrt{4x+2y-1}$

$t = 4x+2y-1$

$dt = 4dx + 2dy \iff dy = \frac{1}{2}\cdot(dt - 4dx)$

$\frac{1}{2}\frac{dt}{dx} - 2 = \sqrt{t}$

$\frac{dt}{dx} = 2\sqrt{t} + 4 \quad \Big| \cdot \frac{dx}{2\sqrt{t}+4}$

$\int \frac{dt}{2\sqrt{t}+4} = \int dx$

$\int \frac{dt}{2\sqrt{t}+4} = \frac{1}{2}\int \frac{dt}{\sqrt{t}+2} = \left[\begin{array}{l} u = \sqrt{t}+2 \\ dt = 2\sqrt{t}\,du = 2(u-2)du \end{array}\right] =$

$= \int \frac{(u-2)du}{u} = u - 2\ln|u| + C = \sqrt{\cancel{t}} + 2 - 2\ln|\sqrt{t}+2| + C$

$\sqrt{t} - 2\ln|\sqrt{t}+2| + C = x$

Ответ: $\sqrt{4x+2y-1} - 2\ln|\sqrt{4x+2y-1}+2| + C = x$

N5

$$x^2y' - \cos 2y = 1, \quad y(+\infty) = \frac{9\pi}{4}$$

$$\frac{x^2 dy}{dx} = \cos 2y + 1 \ \Big| \cdot \frac{dx}{\cos 2y + 1}$$

(1) $\cos 2y + 1 = 0$

$2y = \pi + 2\pi k, \ k \in Z$

$y = \frac{\pi}{2} + \pi k, \ k \in Z$

$y' = 0$ не решение, $\lim_{x\to\infty} y \neq \frac{9\pi}{4}$

$0 = 0$

$$\int \frac{dy}{\cos 2y + 1} = \int \frac{dx}{x^2}$$

$$\int \frac{dy}{2\cos^2 y - 1 + 1} = \int \frac{dx}{x^2}$$

$$1/2 \ tg\, y = \frac{-1}{x} + C$$

$$y = arctg\left(\frac{-2}{x} + C\right) + \pi k, \ k \in Z$$

$$\lim_{x\to+\infty} arctg\left(\frac{-2}{x} + C\right) + \pi k = \frac{9\pi}{4}$$

$$arctg\, C + \pi k = \frac{9\pi}{4}$$

$C = 1, \ k = 2$

$\boxed{№6}$ $3y^2y' + 16x = 2xy^3$, $y(x)$ ограничено при $x \to +\infty$

$3y^2y' = 2xy^3 - 16x$

$3y^2y' = x(2y^3 - 16) \quad \Big| \cdot \dfrac{dx}{(2y^3-16)}$

(1) $2y^3 - 16 = 0$

$y = 2$
$y' = 0$ ← решение ограничено при $\forall x$

$$\int \frac{3y^2\,dy}{(2y^3-16)} = \int x\,dx$$

$$\frac{1}{2}\int \frac{dy^3}{(y^3-8)} = \int x\,dx$$

$$\tfrac{1}{2}\ln|y^3-8| = \frac{1}{2}x^2 + C$$

$$\ln|y^3-8| = x^2 + C$$

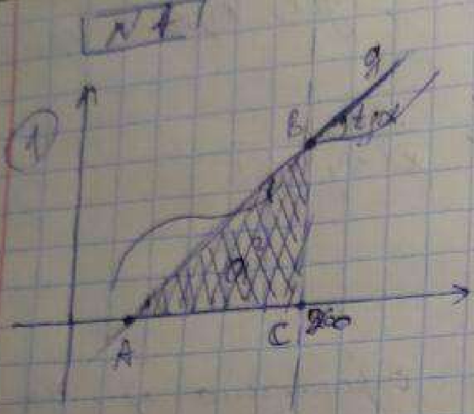
$$|y^3-8| = C \cdot e^{x^2}$$

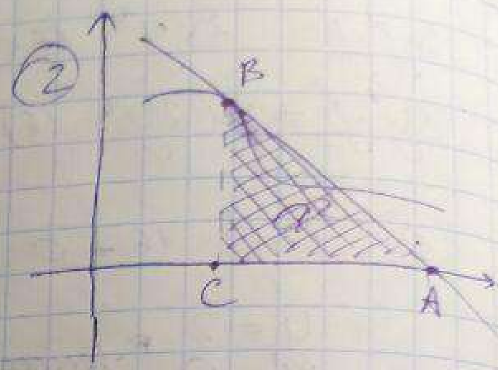
$$y^3 - 8 = C \cdot e^{x^2}$$

$y = \sqrt[3]{C \cdot e^{x^2} + 8} \xrightarrow[x \to +\infty]{} +\infty$ – не решение

Ответ: $y = 2$

№1





$y = f(x)$

$g(x_0, x) = f'(x_0)X + f(x_0) - f'(x_0) \cdot x_0$

$S = 1/2\, AC \cdot BC$

$A_x(x_0) = \dfrac{f'(x_0) \cdot x_0 - f(x_0)}{f'(x_0)}$

$C_x(x_0) = x_0. \quad C_y(x_0) = 0.$

$B_y(x_0) = f(x_0)$

$AC(x_0) = \left| x_0 - \dfrac{f'(x_0) \cdot x_0 - f(x_0)}{f'(x_0)} \right| = \pm \dfrac{f(x_0)}{f'(x_0)}$

$BC(x_0) = f(x_0).$

$\pm \dfrac{f^2(x_0)}{2f'(x_0)} = a^2, \quad \forall x_0$

$\pm \dfrac{y^2}{2y'} = a^2 \; \cancel{\forall x} \Rightarrow \pm dx = \dfrac{a^2\, dy}{y^2}$

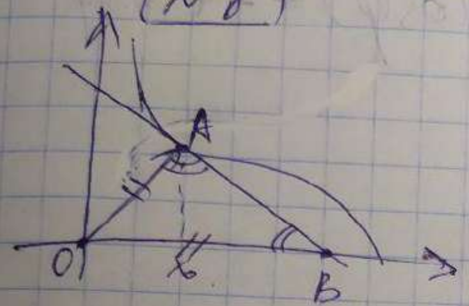
$\pm x = -\dfrac{a^2}{y} + c$

$y = \dfrac{-a^2}{\pm x + c}$

$y = 0$ решение

$S = 0 = const$

$\boxed{№8}$



$y = f(x)$

$A_x(x_0) = x_0$

$A_y(x_0) = f(x_0)$

$$\left.\begin{matrix} A_x(x_0) = x_0 \\ A_y(x_0) = f(x_0) \end{matrix}\right\} OA = \sqrt{x_0^2 + f^2(x_0)}$$

$$B_x(x_0) = \frac{f'(x_0)x_0 - f(x_0)}{f'(x_0)} = OB$$

$$\frac{f'(x_0)\cdot x_0 - f(x_0)}{f'(x_0)} = \sqrt{x_0^2 + f^2(x_0)}$$

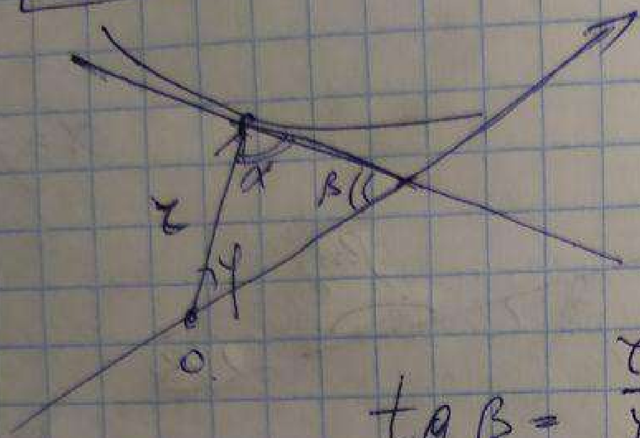
$$\frac{y'x - y}{y'} = \sqrt{x^2 + y^2}$$

$$x - y/y' = \sqrt{x^2 + y^2}$$

$$y/y' = x - \sqrt{x^2 + y^2}$$

$$\frac{y\,dx}{dy} = x - \sqrt{x^2 + y^2}$$ — не могу это решить. попробую в полярных координатах

$\boxed{N8}$



$$tg\beta = \frac{r'\sin\varphi + r\cos\varphi}{r'\cos\varphi - r\sin\varphi}$$

$$tg\,\alpha = \frac{r}{r'}$$

$$\frac{r'\sin\varphi + r\cos\varphi}{r'\cos\varphi - r\sin\varphi} = \frac{r}{r'}$$

$$r'(r\sin\varphi + r\cos\varphi - r\cos\varphi) = -r^2\sin\varphi$$

$$r'(r\sin\varphi) = -r^2\sin\varphi$$

$$\frac{dr}{d\varphi}\cdot r\sin\varphi = -r^2\sin\varphi$$

$$\frac{dr}{d\varphi} = -r$$

$$\frac{dr}{r} = -d\varphi$$

$$\ln|r| = -\varphi + c$$

$$r = e^{-\varphi}\cdot C$$