Контрольная работа

$\boxed{N 1}$

$V_{обу} = 200 м^3$

$V_{CO_2}(0) = 200 \cdot 0{,}0015 = 0{,}3\ м^3$

за Δt : $\Delta V_{CO_2} = (\underbrace{20 \cdot 0{,}0004}_{+\text{от вентилятора}} - \underbrace{20 \cdot \frac{V_{CO_2}}{200}}_{\text{выходит}}) \Delta t$

$y' = 0{,}008 - 0{,}1y \quad \Big| \cdot \frac{dt}{0{,}008 - 0{,}1y}$

по $y = 0{,}08$ - решение

$\int \frac{dy}{0{,}008 - 0{,}1y} = \int dt \iff -10 \int \frac{dy}{y - 0{,}08} = \int dt$

$-10 \ln|y - 0{,}08| = t + C$

$\ln|y - 0{,}08| = -\frac{1}{10}t + C$

$y - 0{,}08 = A \cdot e^{-0{,}1t}$

$y = A \cdot e^{-0{,}1t} + 0{,}08$

входит в общее

$y(0) = 0{,}3 = A \cdot e^{-0{,}1 \cdot 0} + 0{,}08 \Rightarrow A = 0{,}22$

$y = 0{,}22 e^{-0{,}1t} + 0{,}08$

$y(t) = \frac{y(0)}{3} = 0{,}1 \iff 0{,}22 e^{-0{,}1t} = 0{,}1 - 0{,}08$

$t = 23{,}97$

$t \approx 24$ мин.

$\boxed{N2}$ $x^2+y^2+xy-x^2y'=0$ $y(e)=0$

однородное $y=tx \Rightarrow y'=t'x+t$

$x^2+t^2x^2+tx^2-x^2(t'x+t)=0$

- $x=0$ - не подходит.
- $1+t^2+t-t'x-t=0$

$1+t^2=t'x$

$$\int\frac{dx}{x}=\int\frac{dt}{t^2+1} \Rightarrow \ln|x|=\operatorname{arctg} t+C$$

$t=\frac{y}{x}$

Ответ: $\ln|x|=\operatorname{arctg}\frac{y}{x}+C$

$y(e)=0 \Rightarrow \ln|e|=\operatorname{arctg}0+C$

$1=C$

$\frac{y(1)}{\pi}$: $\ln|1|=\operatorname{arctg}y+1 \Rightarrow \operatorname{arctg}y=$

$y(1)=tg(-1)$

$\frac{y(1)}{\pi}=\frac{tg(-1)}{\pi}=-0,50$

$\boxed{N3}$ $y' + \frac{y}{x} = \sin x \qquad y\left(\frac{\pi}{2}\right) = \frac{4}{\pi}$

$y' = -\frac{1}{x} y + \sin x. \qquad y\left(\frac{\pi}{4}\right) = ?$

$y = C \cdot e^{\int -\frac{1}{x} \cdot dx} = C \cdot e^{-\ln|x|} = C \cdot \frac{1}{|x|}$

$y' = C' \frac{1}{|x|} + \frac{-1}{x^2} \cdot \frac{x}{|x|} \cdot C = \frac{-1}{x|x|} \cdot C + \sin x.$

$C' = |x| \cdot \sin x \qquad C = \sin|x| - \cos x \cdot |x| + C$

$y = \frac{\sin|x|}{|x|} - \cos x + \frac{C}{|x|}$

$y\left(\frac{\pi}{2}\right) = \frac{\sin \pi/2}{\pi/2} - \cos \pi/2 + \frac{C}{\pi/2} = \frac{4}{\pi}$

$\frac{2}{\pi} - 0 + \frac{2C}{\pi} = \frac{4}{\pi} \to C = 1$

$y\left(\frac{\pi}{4}\right) = \frac{\sin \pi/4}{\pi/4} - \cos \pi/4 + \frac{1}{\pi/4} = \frac{4\sqrt{2}}{2\pi} - \frac{\sqrt{2}}{2} + \frac{4}{\pi}$

$= \frac{2\sqrt{2}}{\pi} - \frac{\sqrt{2}}{2} + \frac{4}{\pi} = 1.4664\ldots$

$\boxed{N4}$ $y' - y + y^2\cos x = 0$ $y(-\frac{\pi}{4}) = -2e^{3\pi/4}$

$y(0) = ?$

$y' = y - (\cos x)\, y^2$

$z = \frac{1}{y}$ $z' = \frac{-1}{y^2}\cdot y'$

$y' = y - \cos x \cdot y^2 \;\big|: -y^2$ но $y = 0$ - решение

$\frac{-y'}{y^2} = \frac{-1}{y} + \cos x$

$z' = -z + \cos x$ - линейное

$z = C\cdot e^{\int -1\,dx} = C(x)\cdot e^{-x}$

$z' = C'e^{-x} - \cancel{e^{-x}\cdot C} = -\cancel{e^{-x}\cdot C} + \cos x$

$C' = \frac{\cos x}{e^{-x}} = e^{x}\cos x$ $C = \frac{e^x(\sin x + \cos x)}{2} + A$

$z = \frac{\sin x + \cos x}{2} + A\cdot e^{-x}$

$z = \frac{1}{y}$ $\frac{1}{y} = \frac{\sin x + \cos x}{2} + A\cdot e^{-x}$

$y(-\frac{\pi}{4}) = -2\, e^{-3\pi/4}$

$\frac{1}{-2\cdot e^{3\pi/4}} = \frac{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{2} + A\cdot e^{\pi/4}$

$A\cdot e^{\pi/4} = \frac{-e^{3\pi/4}}{2} \Rightarrow A = \frac{-e^{\pi/2}}{2}$

$y(0) = \frac{1}{y} = \frac{0+1}{2} - \frac{e^{\pi/2}}{2} = \frac{1-e^{\pi/2}}{2}$

$y(0) = \frac{2}{1-e^{\pi/2}} = -0{,}524868\ldots$

№5

$$\underbrace{(y\cdot x\cdot e^{x^2y} + \cos 2x + x^2)}_{P}dx + \underbrace{\left(\frac{x^2}{2}\cdot e^{x^2y} + y\right)}_{q}dy = 0$$

$$P'_y = x\cdot e^{x^2y} + x\cdot e^{x^2y}\cdot x^2\cdot y = x\cdot e^{x^2y} + x^3y\cdot e^{x^2y}$$

$$q'_x = x\cdot e^{x^2y} + e^{x^2y}\cdot 2yx\cdot\frac{x^2}{2} = x\cdot e^{x^2y} + x^3y\cdot e^{x^2y}$$

Это УПД

$\exists\ u(x,y) = C,\ C\in\mathbb{R}$ — это решение

$$u'_y = \frac{x^2}{2}\cdot e^{x^2y} + y \Leftrightarrow u = \frac{e^{x^2y}+y^2}{2} + C(x)$$

$$u'_x = \cancel{\frac{1}{2}\cdot e^{x^2y}\cdot 2xy} + C'(x) = \cancel{y\cdot x\cdot e^{x^2y}} + \cos 2x + x^2$$

$$C' = \cos 2x + x^2 \Rightarrow C = \cos x\cdot\sin x + \frac{x^3}{3} + A$$

Ответ $u = \frac{e^{x^2y}+y^2}{2} + \cos x\cdot\sin x + \frac{x^3}{3} = C,\ C\in\mathbb{R}$

$$u(2,2) = \frac{e^8+4}{2} + \cos 2\cdot\sin 2 + \frac{8}{3} = 1494{,}7672$$

$\boxed{N6}$ $y'' x \ln x = y'$ $(x>0)$ $y(e)=0$ $y'(e)=1$

$z = y'$ $z(e)=1$

$y(e^2) = ?$

$\frac{z'}{z} = \frac{1}{x\ln(x)}$ (замет. $x = 1$ - не решение)

$\int \frac{dz}{z} = \int \frac{dx}{x \ln x} \iff \int \frac{dz}{z} = \int \frac{d\ln(x)}{\ln(x)}$

$\ln|z| = \ln|\ln(x)| + C$

$z = \pm \ln(x) \cdot A$

$y' = A \cdot \ln(x)$

$y = A \cdot x \cdot (\ln(x) - 1) + A_2$

$y'(e) = A \cdot \ln e = 1 \Rightarrow A = 1$

$y(e) = 0 = e \cdot (\ln(e) - 1) + A_2 \Rightarrow A_2 = 0$

$y = x(\ln x - 1)$

$y(e^2) = e^2 \cdot (\ln(e^2) - 1) = e^2 \approx 7{,}39$

$\boxed{N7}$ $y'' = \frac{1}{4\sqrt{y}}$ $\begin{cases} y(0) = 1 \\ y'(0) = 1 \end{cases}$ $y\left(\frac{28}{3}\right) = ?$

$z(y) = y'$ $y = 0$ – не решение

$z'z = \frac{1}{4\sqrt{y}} \Leftrightarrow \int z\,dz = \int \frac{dy}{4\sqrt{y}}$

$\int z\,dz = 4\int y^{-1/2}\,dy \Leftrightarrow \frac{z^2}{2} = \frac{\sqrt{y}}{2} + C$

$z^2 = \sqrt{y} + C \Leftrightarrow (y')^2 = \sqrt{y} + C$

З. Коши: $x = 0$ $y = 1$ $y' = 1$

$1^2 = \sqrt{1} + C \Rightarrow C = 0$

$(y')^2 = \sqrt{y}$ $y' = y^{1/4}$

$\int \frac{dy}{y^{1/4}} = \int dx \Leftrightarrow \frac{4}{3} y^{3/4} = x + A$

З. Коши: $\frac{4}{3} \cdot 1 = 0 + A$ $A = \frac{4}{3}$

$\frac{4}{3} \cdot y^{3/4} = x + \frac{4}{3}$

$y^{3/4} = \frac{3}{4}x + 1 \Rightarrow y = \left(\frac{3}{4}x + 1\right)^{4/3}$

$y\left(\frac{28}{3}\right) = (7 + 1)^{4/3} = 16.$

N8

$$\begin{cases} \dot{x} = x^2 - 1 \\ \dot{y} = t + y \end{cases}$$

$x(0) = 0$

$y(0) = 1$

$x_0 = 0$

$y_0 = 1$

$$x_k = x_0 + \int_0^t \left((x_{k-1}(\xi))^2 - 1 \right) d\xi$$

$$y_k = y_0 + \int_0^t (\xi + y_{k-1}(\xi)) d\xi$$

$$x_1 = 0 + \int_0 -1\, d\xi = -t$$

$$y_1 = 1 + \int_0^t (\xi + 1) d\xi = 1 + \frac{t^2}{2} + t$$

$$x_2 = 0 + \int_0^t (\xi^2 - 1) d\xi = \frac{t^3}{3} - t$$

$$y_2 = 1 + \int_0^t \left(\xi + \frac{\xi^2}{2} + \xi + 1 \right) d\xi = 1 + \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^2}{2} + t$$

$$x_2(1) = \frac{1}{3} - 1 = \frac{-2}{3} = -0{,}666\ldots$$

$$y_2(1) = 1 + 1 + 1 + \frac{1}{6} = 3\frac{1}{6} = 3{,}1666\ldots$$