№1 КАРАСЕВА M3235

$$x^2y'' - 2xy' + (x^2+2)y = 0$$

$\exists$ $y = ax \Leftrightarrow y' = a'x + a\cdot 1 \Leftrightarrow$

$\Leftrightarrow y'' = a''x + 2a'$

$$x^2(a''x + 2a') - 2x(a'x + a) + (x^2+2)ax = 0$$

$$x^3a'' + \cancel{2x^2a'} - \cancel{2x^2a'} - \cancel{2xa} + x^3a + \cancel{2xa} = 0$$

$$x^3a'' + x^3a = 0.$$

№2

$$xy'' - y' - 4x^3y = 0.$$

$\exists$ $x = f(t) \Leftrightarrow y'_x = y'_t f'$

$y''_x = (y'_t f')'_x = (y'_t f')'_t f'_t = (y''_{tt} f' + y' f'') f'$

$$f(f')^2y'' - y'f' - 4f^3y + y'f''f' = 0.$$

$$f''f' - f' = 0.$$

$$f'(f''-1) = 0.$$

$f' \neq 0$ тк $x \neq const$.

$\Rightarrow f'' = 1 \Rightarrow f' = tC_1$

$f = \frac{1}{2}t^2 + C_1t + C_2$ $\quad\exists$ $C_1 = 0$ $C_2 = 0$

$$\frac{1}{2}t^2y''_t - 4f^3y = 0.$$

№ 3 $(1-x^2)y'' - 4xy' - 2y = 0$

$\exists \; y = a_0 + a_1 x + a_2 x^2 \ldots \infty = \sum\limits_{i=0}^{\infty} a_i x^i$

$y' = \sum\limits_{i=1}^{\infty} i a_i x^{i-1}$ $\quad y'' = \sum\limits_{i=2}^{\infty} i(i-1) a_i x^{i-2}$

Подставим в ур-ние.

$(1-x^2) \cdot \sum\limits_{i=2}^{\infty} i(i-1) a_i x^{i-2} - 4x \sum\limits_{i=1}^{\infty} i a_i x^{i-1} - 2 \sum\limits_{i=0}^{\infty} a_i x^i = 0.$

$\sum\limits_{i=2}^{\infty} i(i-1) a_i x^{i-2} - \sum\limits_{i=2}^{\infty} i(i-1) a_i x^i - 4 \sum\limits_{i=1}^{\infty} i a_i x^i - 2 \sum\limits_{i=0}^{\infty} a_i x^i = 0$

$\sum\limits_{i=0} (i+2)(i+1) a_{i+2} x^i - \; // \; - \; // \; - \; // = 0$

$2a_2 + 6a_3 x - 4a_i x - 2a_0 - 2a_1 x + \sum\limits_{i=2}^{\infty} \big((i+2)(i+1) a_{i+2} - i(i-1) a_i -$

$- 4 i a_i - 2 a_i \big) x^i = 0.$

$a_0 = a_2 \quad a_1 = a_3 \quad , \; a_i = a_{i+2} \quad i \geq 2$

$\exists \; a_0 = 1 \quad a_1 = 0 \quad \Rightarrow \quad a_{2k} = 1 \quad a_{2k+1} = 0$

$y = 1 + x^2 + x^4 + \ldots = \dfrac{1}{1-x^2}$

$\exists \; a_0 = 1 \quad a_1 = 0 \quad \Rightarrow \quad a_{2k} = 0 \quad a_{2k+1} = 1$

$y = x + x^3 + x^5 \ldots = \dfrac{x}{1-x^2}$

№5 $2(y')^2(y - xy') = 1$

] $z = y' = \frac{dy}{dx}$

$2z^2(y - xz) = 1$

$y = xz + \frac{1}{2z^2}$ $\qquad dy = z\,dx.$

$z\,dx = d\left(xz + \frac{1}{2z^2}\right) = x\,dz + z\,dx - \frac{1}{z^3}dz$

$\left(x - \frac{1}{z^3}\right)dz = 0 \Rightarrow z = C$

$x = \frac{1}{z^3}$

Подставим.

$8y^3 = 27x^2$ $\qquad y = Cx + \frac{1}{2C^2}$

$y(1/9) = \sqrt[3]{\frac{3}{8}} = \frac{\sqrt[3]{3}}{2}$ (4)

$\frac{55}{18} = C + \frac{1}{2C^2} \quad | \cdot 2C^2$

$\frac{55}{9}C^2 = 2C^3 + 1$