№1

$$\begin{cases} \dot{x} = t + x & x(0) = 1 \\ \dot{y} = xyt & y(0) = -1 \end{cases}$$

$x_0 = 1$

$$x_k = 1 + \int_0^t \left(\xi + x_{k-1}(\xi)\right) d\xi$$

$$x_1 = 1 + \frac{t^2}{2} + t$$

$$x_2 = 1 + \frac{t^3}{6} + t^2 + t$$

$y_0 = -1$

$$y_k = -1 + \int_0^t x_{k-1}(\xi)\cdot \xi \cdot y_{k-1}(\xi)\cdot d\xi$$

$$y_1 = -1 - \frac{t^2}{2}$$

$$y_2 = -1 - \int_0^t \left(\frac{\xi^2}{2} + \xi + 1\right)\cdot \xi \cdot \left(\frac{\xi^2}{2} + 1\right) d\xi =$$

$$= -1 - \int_0^t \left(\frac{\xi^5}{4} + \frac{\xi^3}{2} + \frac{\xi^4}{2} + \xi^2 + \frac{\xi^3}{2} + \xi\right) d\xi =$$

$$= -1 - \int_0^t \left(\frac{\xi^5}{4} + \frac{\xi^4}{2} + \xi^3 + \xi^2 + \xi\right) d\xi =$$

$$= -1 - \frac{t^6}{24} - \frac{t^5}{10} - \frac{t^4}{4} - \frac{t^3}{3} - \frac{t^2}{2}$$

$$y_2(1) = -1 - \frac{1}{24} - \frac{1}{10} - \frac{1}{4} - \frac{1}{3} - \frac{1}{2} = -2\frac{9}{40} = -2{,}225$$

$x'_y = \frac{1}{f(y)}$

области

N2

$$y'' - y'\sin x - x^2 = 0 \qquad y(0) = 1, \quad y'(0) = 0$$

$$y' = z \qquad \begin{cases} z' = y'' = y'\sin x + x^2 = z\sin x + x^2 \\ y' = z \end{cases}$$

$$z_0 = 0$$

$$z_k = \int_0^x \left(z_{k-1}(\xi)\cdot\sin\xi + \xi^2\right)d\xi$$

$$z_1 = \int_0^x \left(0\cdot\sin\xi + \xi^2\right)d\xi = \frac{x^3}{3}$$

$$\cancel{z_2 = \int_0^x \left(\frac{-\xi^3\cdot\sin\xi}{3} - \xi^2\right)d\xi =}$$

$$\cancel{= -\frac{(3x^2-6)\sin x + (6x - x^3)\cos x + x^3}{3}}$$

$$y_0 = 1$$

$$y_k = 1 + \int_0^x z_{k-1}(\xi)\,d\xi$$

$$y_1 = 1$$

$$y_2 = 1 + \int_0^x \frac{\xi^3}{3}\,d\xi = 1 + \frac{x^4}{12}$$

$$y_2(1) = 1 + \frac{1}{12} = \frac{13}{12} \approx 1{,}08$$

X_k

X_1

X_2

X … области.

№3 $$\underbrace{(x+y+1)}_{p(x,y)}dx + \underbrace{(2x+2y+1)}_{q(x,y)}dy = 0$$

$$p'_y = 1 \qquad q'_x = 2$$

Ищем интегр множитель

$\exists\, m(z)$: $$m'_y(x+y+1) + m = m'_x(2x+2y+1) + 2m$$

$$m'_z(z'_y(x+y+1) - z'_x(2x+2y+1)) = m$$

Заметим, что хорошо бы если $z'_y = 2$, $z'_x = 1$

тогда сократятся все x и y

Тогда $\exists\; z = x+2y$

$$m'_z(\,2(x+y+1) - (2x+2y+1)) = m$$

$$m'_z = m \Leftrightarrow m = e^{z} = e^{x+2y}$$

$$e^{x+2y}(x+y+1)dx + e^{x+2y}(2x+2y+1)dy = 0$$ ← это УПД

$\exists\; u(x,y) = C,\ C\in\mathbb{R}$ ← это решение, тогда

$$u'_x = e^{x+2y}(x+y+1) \Rightarrow u = (x+y)\cdot e^{x+2y} + C(y)$$

$$u'_y = e^{x+2y}(2x+2y+1) = e^{x+2y} + e^{x+2y}\cdot 2\cdot(x+y) + C'(y)$$

$$= e^{x+2y}(1 + 2(x+y)) + C'(y) \Rightarrow C'(y) = 0$$

$$C = const$$

Решение $(x+y)\cdot e^{x+2y} = C,\ C\in\mathbb{R}$

Зад. Коши $y(0) = 0 \Rightarrow C = 0 \qquad x+y = 3 \Rightarrow 3\cdot e^{3+y} = 0$

Ука?

$\boxed{N4}$ $m = 0{,}4\ кг = 400г$ $h = S = 16{,}3\ м$

$F_{сопрот.\ воздуха} = K \cdot v^2 = F_8$; $F_8(1) = 0{,}48$

$F_8 = 0{,}48 \cdot v^2$

$$a = \frac{mg - 0{,}48v^2}{m} = \frac{4000 - 0{,}48v^2}{400} =$$

$$= 10 - \frac{12}{10^4}v^2$$

$$a = v'$$

$$v' = 10 - \frac{12}{10^4}v^2$$

$$\int \frac{dv}{10 - \frac{12}{10^4}v^2} = \int dt \iff \frac{-10^4}{12}\int \frac{dv}{v^2 - \frac{10^5}{12}} = \int dt$$

$$\frac{-10^4}{12} \cdot \frac{1}{2\sqrt{\frac{10^5}{12}}} \cdot \ln\left|\frac{v - \sqrt{\frac{10^5}{12}}}{v + \sqrt{\frac{10^5}{12}}}\right| = t + C$$

$$\sqrt{\frac{10^5}{12}} = R$$

$$-\frac{R}{20} \cdot \ln\left|\frac{v - R}{v + R}\right| = t + C$$

$$\ln\left|\frac{v-R}{v+R}\right| = \frac{-20t}{R} + C$$

$$\frac{v-R}{v+R} = e^{-20t/R} \cdot C \iff v - R = e^{-20t/R} \cdot C \cdot v + e^{-20t/R} \cdot C \cdot R$$

$$v = \frac{R + e^{-20t/R} \cdot C \cdot R}{1 - e^{-20t/R} \cdot C} = S'$$

$$S = \frac{R\left(R\ln\left|e^{\frac{20t}{R}} - C\right| - 10t\right)}{10} + C_2$$