Дифф. уравнение

КАРАСЕВА
М3235

№1 $R = \frac{1}{\cos y} = \sqrt{1+tg^2 y}$

$$\frac{\left(1+(y'(x))^2\right)^{3/2}}{y''(x)} = \sqrt{1+(y'(x))^2}$$

$$y'' = (y')^2 + 1$$

$\exists\ z(x) = y'(x) \Rightarrow z'(x) = y''(x)$

$$z' = z^2 + 1$$

$\varphi = tg\, x$ — частное решение

$$\boxed{\frac{1}{\cos^2 x} = tg^2 x + 1}$$

заменим $z = \varphi + u$

$$\varphi' + u' = (\varphi + u)^2 + 1$$

$$\cancel{\frac{1}{\cos^2 x}} + u' = \cancel{tg^2 x} + (2\, tg\, x)u + u^2 \cancel{+1}$$

$$u' = (2\, tg\, x)\, u + u^2$$

$\exists\ v = u^{-1}$

$v' = -1\, u^{-2} \cdot u' \Rightarrow \frac{u'}{u^2} = -v'$

$$\frac{u'}{u^2} = \frac{2\, tg\, x}{u} + 1 \Rightarrow -v' = (2\, tg\, x)v + 1$$

$v' = (-2\, tg\, x)\, v - 1$ — линейное

$$v = \left(c + \int -1 \cdot e^{-\int -2\, tg\, x\, dx}\, dx\right) \cdot e^{\int -2\, tg\, x\, dx}$$

$$v = \left(c + \int -1\, e^{-2\ln(\cos x)}\, dx\right) e^{2\ln\cos x} =$$

$$= \left(c + \int -1/\cos^2 x\, dx\right)\cos^2 x = (c - tg\, x)\cos^2 x$$

Полное решение $z = tg x + (c - tg x)\cos^2 x$

$y' = tg x + (c - tg x)\cos^2 x$

$y = -2\ln|\cos x| + \dfrac{\cos x\,(c\sin x + 2\cos x) + cx}{2} + C$

$\boxed{№3}$ $y' = e^{\frac{xy'}{y}}$

$\exists\ z = y' = \dfrac{dy}{dx} \Rightarrow dy = z\,dx$

$z = e^{\frac{xz}{y}}$

$\ln z = \dfrac{xz}{y} \Leftrightarrow y = \dfrac{\ln z}{xz} \Leftrightarrow$

$\Leftrightarrow dy = \underbrace{\dfrac{-\ln z}{x^2 z}}_{p(x,z)} dx + \underbrace{\dfrac{1 - \ln z}{xz^2}}_{q(x,z)} dz$ ← это полный дифф

$\left(-z + \dfrac{-\ln z}{x^2 z}\right) dx + \dfrac{1 - \ln z}{xz^2} dz = 0$

$R(x,z)$ тк у нас был уже полный дифф-ал $\Rightarrow$

$\Rightarrow p'_z = q'_x$

$R'_z = \left(-z + \dfrac{-\ln z}{x^2 z}\right)'_z = -1 + p'_z \qquad \left(\dfrac{1 - \ln z}{xz^2}\right)'_x = q'_x$

$\dfrac{\partial(mR)}{\partial z} = \dfrac{\partial(mq)}{\partial x}$

$m'_z \cdot R + m \cdot R'_z = m'_x q + m q'_x$

$m'_z R - m'_x q = (q'_x - p'_z + 1) m = m$

$\exists\ u = xz$

$m'_u (u'_z \cdot R - u'_x q) = m.$

$m'_u \left(x\left(-z - \dfrac{\ln z}{x^2 z}\right) - z\left(\dfrac{1 - \ln z}{xz^2}\right)\right) = m$

$$m'_u\left(-xz-\frac{\ln z}{xz}+\frac{\ln z-1}{xz}\right)=m$$

$$m'_u\,\frac{-(x^2z^2+1)}{xz}=m.$$

$$m'=\frac{-u}{u^2+1}\,m.$$

$$m=C\cdot e^{\int\frac{-u}{u^2+1}du}=C\cdot e^{-\frac12\ln(u^2+1)}=C\cdot\frac{1}{\sqrt{u^2+1}}$$

$$m=\frac{1}{\sqrt{x^2z^2+1}}$$

$$\frac{-x^2z^2-\ln z}{x^2z\sqrt{x^2z^2+1}}\,dx+\frac{1-\ln z}{xz^2\sqrt{x^2z^2+1}}\,d\cancel{y}z=0.$$

$\exists\ u(x,z)=C$ – решение.

$$u'_x=\frac{-x^2z^2-\ln z}{x^2z\sqrt{x^2z^2+1}}\Leftrightarrow u=\frac{\ln z\sqrt{x^2z^2+1}}{xz}-$$

$$-\ln\left(\sqrt{x^2z^2+1}+xz\right)+C(z)$$

$$u'_y=\frac{1-\ln z}{xz^2\sqrt{x^2z^2+1}}=\frac{1}{x}\left(\frac{1-\ln z}{z^2}\cdot\sqrt{x^2z^2+1}+\frac{2z}{2\sqrt{x^2z^2+1}}\cdot\frac{\ln z}{z}\right)$$

$$-\frac{\frac{2z}{2\sqrt{x^2z^2+1}}+x}{\sqrt{x^2z^2+1}+xz}+C'(z)=\frac{(1-\ln z)(x^2z^2+1)+z^2\ln z}{xz^2\sqrt{x^2z^2+1}}-$$

$$-\frac{z+x\sqrt{x^2z^2+1}}{\sqrt{x^2z^2+1}+xz}+C'(z)\Leftrightarrow\frac{-(x^2+\ln z)(\sqrt{x^2z^2+1}+xz)+xz\sqrt{x^2z^2+1}-x^2\sqrt{\ }}{x(x^2z^2+1)+x^2z\sqrt{x^2z^2+1}}$$

$$\neq C'(z)$$