Вариант 0 Кр. 24.12

№1 $(y'''')+y''=7x-3\cos x$

$$y''''+y''=7x-3\cos x$$

$\exists\ y''=z(x)\ \Rightarrow\ z'=y'''\ \Rightarrow\ z''=y''''$

$$z''+z=7x-3\cos x$$

$$\lambda^2+1=0 \qquad \lambda=\pm i$$

$$y_0=c_1e^{ix}+c_2e^{-ix}=c_1(\cos x+i\sin x)+c_2(\cos(-x)+i\sin(-x))=$$

$$=\tilde{c}_1\cos x+\tilde{c}_2\sin x$$

$$\begin{cases}\tilde{c}_1'(x)\cdot\cos x+\tilde{c}_2'(x)\sin(x)=0\\ \tilde{c}_1'(x)-\sin x+\tilde{c}_2'(x)\cos x=7x-3\cos x\end{cases}$$

$$\begin{vmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{vmatrix}=\cos^2x+\sin^2x=1$$

$$\begin{vmatrix}0 & \sin x\\ 7x-3\cos x & \cos x\end{vmatrix}=3\sin x\cdot\cos x-7x\sin x$$

$$\begin{vmatrix}\cos x & 0\\ -\sin x & 7x-3\cos x\end{vmatrix}=7x\cos x-3\cos^2x$$

$$\tilde{c}_1'(x)=\frac{3\sin x\cdot\cos x-7x\cdot\sin x}{1}\Rightarrow\tilde{c}_1=\int(3\sin x\cdot\cos x-7x\sin x)dx=$$

$$=\int3\sin x\cdot\cos x\,dx-\int7x\cdot\sin x\,dx=\int3\sin x\,d\sin x-$$

$$-\int7x\sin x\,dx=\frac{3}{2}\sin^2x+7x\cdot\cos x-7\sin x+c_1$$

$$\tilde{C}_2'(x) = \frac{7x\cos x - 3\cos^2 x}{1} \to \tilde{C}_2 = \int (7x\cos x - 3\cos^2 x)\,dx =$$

$$= 7(x\sin x + \cos x) - \frac{3}{2}(\cos x \cdot \sin x + x) + C_2$$

$$z = \left(\frac{3}{2}\sin^2 x + 7x\cos x - 7\sin x + C_1\right)\cos x + \sin x \cdot ($$

$$\cdot \left(7x\sin x + 7\cos x - \frac{3}{2}(\cos x \cdot \sin x + x) + C_2\right) =$$

$$= \cancel{\frac{3}{2}\sin^2 x \cdot \cos x} + 7x \cdot \cos^2 x \cancel{- 7\sin x \cdot \cos x} + C_1\cos x +$$

$$+ 7x\sin^2 x + \cancel{7\cos x \cdot \sin x} \cancel{- \frac{3}{2}\cos x \sin^2 x} - \frac{3}{2}\sin x \cdot x + C_2\sin x$$

$$y'' = 7x + C_1\cos x + C_2\sin x - \frac{3}{2}x\sin x$$

$$\int dy' = \int \left(7x + C_1\cos x + C_2\sin x - \frac{3}{2}x\sin x\right)dx$$

$$y' = \frac{7x^2 + (2C_1 - 3)\sin x + (3x - 2C_2)\cos x}{2} + C_3$$

$$y = \frac{(9x - 6C_2)\sin x + (18 - 6C_1)\cos x + 7x^3 + 6C_3 x}{6} + C_4$$

№2 $y'' + 2y' + y = 3e^{-x}\sqrt{x+1}$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda = -1 \quad \alpha = 2$

$y = (C_1 x + C_2)e^{-x}$

$y = C_1(x)\, x \cdot e^{-x} + C_2(x) e^{-x}$

$$\begin{cases} C_1'(x) x e^{-x} + C_2'(x) e^{-x} = 0 \\ C_1'(x)(e^{-x} - x e^{-x}) - C_2'(x) e^{-x} = 3e^{-x}\sqrt{x+1} \end{cases}$$

$\Rightarrow C_1'(x) = 3\sqrt{x+1}$

$C_1(x) = \int 3(x+1)^{1/2} dx = 2(x+1)^{3/2} + C_3$

$C_2' = -3x\sqrt{x+1}$

$C_2 = \int -3x (x+1)^{1/2} dx = -\frac{2(x+1)^{3/2}(3x-2)}{5} + C_4$

Ответ:

$y = (2(x+1)^{3/2} + C_3) \cdot e^{-x} \cdot x + \left(\frac{-2(x+1)^{3/2}(3x-2)}{5} + C_4\right) e^{-x} =$

$= C_3 e^{-x} + C_4 x \cdot e^{-x} + 4/5 \cdot e^{-x} (x+1)^{5/2}$

N3

$$\begin{cases} \dot{x} = x - y - z \\ \dot{y} = x + y \\ \dot{z} = 3x + z \end{cases} \qquad \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & -1 & -1 \\ 1 & 1-\lambda & 0 \\ 3 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 + 1 - \lambda + 3 - 3\lambda = -(\lambda-1)(\lambda^2 - 2\lambda + 5)$$

$\lambda_1 = 1 \qquad \lambda_2 = 1 - 2i \qquad \lambda_3 = 1 + 2i$

1) $\lambda_1 = 1$

$$\begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} \Rightarrow x_1 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

2) $\lambda_2 = 1 - 2i$

$$\begin{pmatrix} -2i & -1 & -1 \\ 1 & -2i & 0 \\ 3 & 0 & -2i \end{pmatrix} \Rightarrow x_2 \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} \sim$$

для $\lambda_3 = 1 - 2i$

$$X = \begin{pmatrix} -2i \\ 1 \\ 3 \end{pmatrix}$$

Базис из с.в.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} e^{(1+2i)i} + C_3 \begin{pmatrix} -2i \\ 1 \\ 3 \end{pmatrix} e^{(1-2i)t}$$

$$\Delta \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} e^{(1+2i)t} = \left(\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2i \\ 0 \\ 0 \end{pmatrix} \right) e^t (\cos 2t + i \sin 2t) =$$

$=$

$$= \left(\begin{pmatrix}0\\1\\3\end{pmatrix} e^t \cos 2t - \begin{pmatrix}2\\0\\0\end{pmatrix} e^t \sin 2t \right) +$$

$$i\left(\begin{pmatrix}0\\1\\3\end{pmatrix} e^t \sin 2t + \begin{pmatrix}2\\0\\0\end{pmatrix} e^t \cos 2t \right) =$$

$$= \begin{pmatrix}-2\sin 2t\\ \cos 2t\\ 3\cos 2t\end{pmatrix} e^t + i\begin{pmatrix}2\cos 2t\\ \sin 2t\\ 3\sin 2t\end{pmatrix} e^t$$

Для сопряженного $\begin{pmatrix}-2i\\1\\3\end{pmatrix}$ будет $-i$

Ответ:

$$\begin{pmatrix}x(t)\\y(t)\\z(t)\end{pmatrix} = C_1\begin{pmatrix}0\\1\\-1\end{pmatrix}e^t + C_2\begin{pmatrix}-2\sin 2t\\ \cos 2t\\ 3\cos 2t\end{pmatrix}e^t + C_3\begin{pmatrix}2\cos 2t\\ \sin 2t\\ 3\sin 2t\end{pmatrix}$$

№4 $\begin{cases}\dot{x} = y - 5\cos t\\ \dot{y} = 2x + y\end{cases}$ $\begin{cases}\dot{x} = y - 5\cos t\\ \ddot{y} = 2\dot{x} + \dot{y}\end{cases}$

$$\ddot{y} = \dot{y} + 2y - 10\cos t$$

$$\ddot{y} - \dot{y} - 2y = -10\cos t$$

$$\lambda^2 - \lambda - 2 = 0. \qquad \lambda_1 = 2 \quad \lambda_2 = -1$$

$$y = C_1 e^{2t} + C_2 e^{-t}$$

$$\begin{cases} C_1'(t)e^{2t} + C_2'(t)e^{-t} = 0\\ C_1'(t)2e^{2t} - C_2'(t)e^{-t} = -10\cos t\end{cases}$$

$$\begin{vmatrix} e^{2t} & e^{-t} \\ 2e^{2t} & -e^{-t} \end{vmatrix} = -e^{t} - 2e^{t} = -3e^{t}$$

$$\begin{vmatrix} 0 & e^{-t} \\ -10\cos t & -e^{-t} \end{vmatrix} = 10\cos t \cdot e^{-t}$$

$$\begin{vmatrix} e^{2t} & 0 \\ 2e^{2t} & -10\cos t \end{vmatrix} = -10\cos t \cdot e^{2t}$$

$$C_1' = \frac{10\cos t \cdot e^{-t}}{-3e^{t}} \Rightarrow C_1 = \int -\frac{10}{3}\cos t \cdot e^{-2t}\,dt =$$

$$= -\frac{2}{3}e^{-2t}(\sin t - 2\cos t) + C_3$$

$$C_2' = \frac{-10\cos t \cdot e^{2t}}{-3e^{t}} \Rightarrow C_2 = \int \frac{10}{3}\cos t \cdot e^{t}\,dt =$$

$$= \frac{5}{3}e^{t}(\sin t + \cos t) + C_4$$

$$y = \left(-\frac{2}{3}e^{-2t}(\sin t - 2\cos t) + C_3\right)e^{2t} +$$
$$+ \left(\frac{5}{3}e^{t}(\sin t + \cos t) + C_4\right)\cdot e^{-t} =$$
$$= \sin t + 3\cos t + C_3 e^{2t} + C_4 e^{-t}$$

$$y' = \cos t - 3\sin t + 2C_3 e^{2t} - C_4 e^{-t}$$

$$\cos t - 3\sin t + 2C_3 e^{2t} - C_4 e^{-t} = 2x + \sin t + 3\cos t + C_3 e^{2t} + C_4 e^{-t}$$

$$x = -2\sin t - 2\cos t + \tfrac{1}{2}C_3 e^{2t} - C_4 e^{-t}$$