Дифф уравнения Карреева М3235

$\boxed{№1}$ $y'' - 3y' + 2 = \frac{1}{1+e^x}$

$y'' - 3y' = \frac{1}{1+e^x} - 2$

Решим однородное $y'' - 3y' = 0$

$\exists\ y = e^{\lambda x}$ $\quad y' = \lambda e^{\lambda x}$ $\quad y'' = \lambda^2 e^{\lambda x}$

$\underbrace{e^{\lambda x}}_{\neq 0}(\lambda^2 - 3\lambda) = 0$

$\lambda(\lambda - 3) = 0 \Leftrightarrow \left[\begin{array}{l}\lambda = 0\\ \lambda = 3\end{array}\right.$

$y = c_1 \cdot e^0 + c_2 \cdot e^{3x} = c_1 + c_2 e^{3x}$

$y' = \underbrace{c_1' + c_2' \cdot e^{3x}}_{=0} + 3 \cdot e^{3x} \cdot c_2 = 3c_2 \cdot e^{3x}$

$y'' = 3c_2' e^{3x} + 9c_2 e^{3x}$

$3c_2' e^{3x} + \cancel{9c_2 e^{3x}} - \cancel{9c_2 e^{3x}} + 2 = \frac{1}{1+e^x}$

$3c_2' e^{3x} + 2 = \frac{1}{1+e^x}$

$c_2' = \frac{-1-2e^x}{3e^{3x}(1+e^x)}$

$c_2 = \frac{1}{18}\left(6\ln(e^{-x}+1) - 6e^{-x} + 3e^{-2x} + 2(e^{-3x} - 3x)\right) + a_1$

$c_1' = \frac{1+2e^x}{3(1+e^x)}$

$c_1 = \frac{\ln(e^{-x}+1) + 2x}{3} + a_2$

$y = \frac{\ln(e^{-x}+1)}{3} + \frac{2}{3}x + \frac{\ln(e^x+1)\cdot e^{3x}}{3} - \frac{e^{2x}}{3} + \frac{e^x}{6} - \frac{x \cdot e^{3x}}{3} + a_1 e^{3x} + a_2$

$\boxed{N2}$

$$y'' - y' = \frac{-(x+1)}{x^2}$$

Решение линейного $y'' - y' = 0$

$\exists\ y = e^{\lambda x} \quad y' = \lambda e^{\lambda x} \quad y'' = \lambda^2 e^{\lambda x}$

$\lambda^2 e^{\lambda x} - \lambda e^{\lambda x} = 0 \Leftrightarrow \lambda(\lambda - 1) = 0 \quad \left[\begin{array}{l}\lambda = 0\\ \lambda = 1\end{array}\right.$

$y = C_1 + C_2 e^x$

$y' = \underbrace{C_1' + C_2' e^x}_{=0} + C_2 e^x = C_2 e^x$

$y'' = C_2' e^x + C_2 e^x$

$C_2' e^x \cancel{+ C_2 e^x - C_2 e^x} = \frac{-(x+1)}{x^2}$

$C_2' = \frac{-(x+1)}{x^2 e^x} \qquad C_2 = \frac{1}{x e^x} + a_1$

$C_1' = \frac{x+1}{x^2} \qquad C_1 = \ln|x| - \frac{1}{x} + a_2$

Ответ: $y = \ln|x| \cancel{- \frac{1}{x}} + a_2 \cancel{+ \frac{1}{x}} + a_1 e^x =$

$= \ln|x| + a_1 e^x + a_2$

$\boxed{N3}$ $x^2 y'' + 2xy' - 12y = 0$

$x = e^t \qquad y'_x = \frac{dy}{dx} = \frac{dy}{dt} : \frac{dx}{dt} = e^{-t}\frac{dy}{dt} = e^{-t} \cdot y'_t$

$y''_{xx} = (e^{-t} \cdot y'_t)'_x = e^{-t} \cdot (e^{-t} \cdot y'_t)'_t = e^{-t} \cdot (-e^{-t} \cdot y'_t + e^{-t} \cdot y''_{tt})$

$= e^{-2t}(y''_{tt} - y'_t)$

$e^{2t} \cdot y''_{xx} + 2e^t \cdot y'_x - 12y = 0$

$y''_{tt} - y'_t + 2y'_t - 12y = 0$

$y'' + y' - 12y = 0$

$\exists \; y = e^{\lambda t} \quad y' = \lambda e^{\lambda t} \quad y'' = \lambda^2 e^{\lambda t}$

$e^{\lambda t}(\lambda^2 + \lambda - 12) = 0 \qquad \left[\begin{array}{l} \lambda = 3 \\ \lambda = -4 \end{array}\right.$

$y = C_1 e^{3t} + C_2 e^{-4t} = C_1 x^3 + \frac{C_2}{x^4}$

$\boxed{N\,4}$ $x^2y'' - 2y = \frac{4}{x^2}$

$x = e^t$ $y'_x = \frac{dy}{dx} = e^{-t}y'_t$

$y''_{xx} = e^{-2t}(y''_{tt} - y'_t)$

$e^{2t}\cdot e^{-2t}(y''_{tt} - y'_t) - 2y = \frac{4}{x^2}$

$y''_{tt} - y'_t - 2y = \frac{4}{x^2}$

Решим однородное $y'' - y' - 2y = 0$

$\exists$ $y = e^{\lambda t}$; $y' = \lambda e^{\lambda t}$; $y'' = \lambda^2 e^{\lambda t}$

$\lambda^2 - \lambda - 2 = 0$ $\left[\begin{array}{l}\lambda = -1\\ \lambda = 2\end{array}\right.$

$y = c_1 e^{-t} + c_2 \cdot e^{2t}$

$y' = \underbrace{c_1' e^{-t} + c_2' \cdot e^{2t}}_{=0} - c_1 e^{-t} + 2c_2 e^{2t}$

$y'' = -c_1' \cdot e^{-t} + c_1 \cdot e^{-t} + 2c_2' \cdot e^{2t} + 4 \cdot c_2 e^{2t}$

$-c_1' \cdot e^{-t} \cancel{+ c_1 \cdot e^{-t}} + 2c_2' e^{2t} \cancel{+ 4c_2 e^{2t}} \cancel{+ c_1 \cdot e^{-t}} \cancel{- 2c_2 e^{2t}}$

$\cancel{-2c_1 \cdot e^{-t}} \cancel{- 2c_2 \cdot e^{2t}} = 4/x^2$

$\underbrace{(-c_1' \cdot e^{-t} - c_2' \cdot e^{2t})}_{=0} + 3c_2' \cdot e^{2t} = 4/x^2$

$c_2' = \frac{4}{3x^2e^{2t}} = \frac{4}{3x^4} \Rightarrow c_2' = \frac{-4}{9x^3} + a_1$

$c_1' = \frac{-4e^t}{3x^2} = \frac{-4}{3x} \Rightarrow c_1' = \frac{-4}{3}\ln|x| + a_2$

Ответ: $\frac{-4\ln|x|}{3x} + \frac{a_2}{x} - \frac{4}{9x} + a_1 x^2$