Домашняя работа 7 Карасева M3135

$\boxed{\text{Задача 1}}$

а) $\frac{2i}{x}+4-4i=-\frac{8}{x}+2y+3i$

$4+\left(\frac{2}{x}-4\right)i=\left(-\frac{8}{x}+2y\right)+3i$

$\begin{cases} 2y-\frac{8}{x}=4 \\ \frac{2}{x}-4=3 \end{cases} \Rightarrow x=\frac{2}{7}$

$2y-\frac{56}{2}=4 \Rightarrow y=16$

б) $(-y+xi)^2=6-8i+(x+yi)^2$

$y^2-2xyi-x^2=6-8i+x^2+2xyi-y^2$

$y^2-x^2-2xyi=6+x^2-y^2+(2xy-8)i$

$\begin{cases} y^2-x^2=6+x^2-y^2 \\ -2xy=2xy-8 \end{cases}$ $\begin{cases} y^2-x^2=3 \\ xy= \end{cases} \Rightarrow y=\frac{2}{x}$

$\frac{4}{x^2}-x^2=3$ ] $t=x^2$ $(t>0) \Rightarrow \frac{4}{t}-t=3$

$t^2+3t-4=0$ $t_1=1$ $t>0$ $t_2=-4$ не удов $t>0$.

$x^2=1 \Rightarrow \boxed{\begin{matrix} x_1=1 \\ y_1=2 \end{matrix}}$ $\boxed{\begin{matrix} x_2=-1 \\ y_2=-2 \end{matrix}}$

$\boxed{\text{Задача №2}}$

а) $(5+i)(15-3i)+(34i)(6-5i)=75-15i+15i+3+$

$+204i+170=248+204i$

б) $\sqrt{-7-24i}=\sqrt{(a+8i)^2}$

$a^2+2abi-b^2=-7-24i$

$\begin{cases} a^2-b^2=-7 \\ 2ab=-24 \end{cases}$ $\begin{cases} a^2-b^2=-7 \\ ab=-12 \end{cases} \rightarrow b=\frac{-12}{a}$

$a^2-\frac{144}{a^2}=-7$ ] $t=a^2$ $(t>0)$

$t^2+7t-144=0$

$t_1=-16$ не удов $t>0$ $\quad t_2=9$

$a^2=9 \rightarrow a_1=3 \quad a_2=-3$

$b_1=-4 \quad b_2=4$

$\sqrt{-7-24i}=\pm(3-4i)$

c) $(1+i)^6=\left(\sqrt{2}\cdot\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^6=(\sqrt{2})^6\cdot\left(\cos\frac{6\pi}{4}+i\sin\frac{6\pi}{4}\right)$

$|z|=\sqrt{2} \quad tg\varphi=\frac{1}{1}=1 \rightarrow \varphi=\frac{\pi}{4}$

$(\sqrt{2})^6\cdot\left(\cos\frac{6\pi}{4}+i\sin\frac{6\pi}{4}\right)=8\cdot\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$

$=8(0+i\cdot(-1))=-8i$

d) $i^{17}-3i^{12}+5\cdot i^{11}-i^6=i\cdot(i^2)^8-3(i^2)^6+5\cdot i\cdot(i^2)^5-$

$-(i^2)^3=i-3-5i+1=-2-4i$

e) $\left(\frac{2i-1}{1+4i}+\frac{5-2i}{4-i}\right)\cdot\frac{34}{(1-i)}=$

$\frac{2i-1}{1+4i}=\frac{(2i-1)(1-4i)}{(\sqrt{1^2+4^2})^2}=\frac{2i+8-1+4i}{17}=\frac{7}{17}+\frac{6}{17}i$

$\frac{5-2i}{4-i}=\frac{(5-2i)(4+i)}{(\sqrt{4^2+1^2})^2}=\frac{20+5i-8i+2}{17}=\frac{22}{17}-\frac{3}{17}i$

$\left(\frac{29}{17}+\frac{3}{17}i\right)\cdot\frac{34}{1-i}=\frac{2(29+3i)}{1-i}=2\cdot\frac{(29+3i)(1+i)}{(\sqrt{1^2+1^2})^2}=$

$(29+3i)(1+i)=29+29i+3i-3=26+32i$

f) $\sqrt[3]{-1+i} = z$ ?

$$-1+i = \sqrt{1^2+1^2}\cdot\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

$$z_k = \sqrt[3]{\sqrt{2}}\cdot\left(\cos\frac{\frac{3\pi}{4}+2\pi k}{3}+i\sin\frac{\frac{3\pi}{4}+2\pi k}{3}\right)$$

$$z_0 = \sqrt[6]{2}\cdot\left(\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}\right)$$

$$z_1 = \sqrt[6]{2}\cdot\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)$$

$$z_2 = \sqrt[6]{2}\cdot\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$$

g) $\sqrt[4]{-8-i8\sqrt{3}} = z$ ?

$$-8-8\sqrt{3}\,i = 16\cdot\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$$

$$z_0 = 2\cdot\left(\cos\frac{\pi}{3}+i\cdot\sin\frac{\pi}{3}\right)$$

$$z_1 = 2\cdot\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$$

$$z_2 = 2\cdot\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$$

$$z_3 = 2\cdot\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)$$

h) $\sqrt[4]{(-1+i)^3(\sqrt{3}-i)^5}$ ...

Задание 3

а) $\frac{\sqrt{3}+i}{1-i} = \frac{2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})}{\sqrt{2}(\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4}))} = \frac{2}{\sqrt{2}}\cdot(\cos(\frac{\pi}{6}+\frac{\pi}{4})+$

$+ i\cdot\sin(\frac{\pi}{6}+\frac{\pi}{4})) = \sqrt{2}\cdot(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12})$

б) $\frac{(1-i)^7(-\sqrt{3}-i)^{12}}{(1+i)^{15}} = \ldots$

1. $(1-i)^7 = (\sqrt{2}(\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4}))^7 = 8\sqrt{2}\cdot(\cos\frac{-7\pi}{4}+i\sin\frac{-7\pi}{4}) =$

$= 8\sqrt{2}\cdot(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

2. $(-\sqrt{3}-i)^{12} = (2\cdot(\cos -\frac{\pi}{6}+i\sin\frac{-\pi}{6}))^{12} = 2^{12}\cdot(\cos -2\pi + i\sin -2\pi) =$

$= 2^{12}\cdot(\cos 0 + i\sin 0)$

3. $(1+i)^{15} = (\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}))^{15} = 2^7\cdot\sqrt{2}(\cos\frac{15\pi}{4}+i\sin -\frac{15\pi}{4}) =$

$= 2^7\cdot\sqrt{2}\cdot(\cos -\frac{\pi}{4}+i\sin -\frac{\pi}{4})$

4. $(1-i)^7\cdot(-\sqrt{3}-i)^{12} = 8\sqrt{2}\cdot(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})\cdot 2^{12}(\cos 0 + i\sin 0) =$

$= 2^{15}\cdot\sqrt{2}\cdot(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$

5. $\frac{(1-i)^7\cdot(-\sqrt{3}-i)^{12}}{(1+i)^{15}} = \frac{2^{15}\cdot\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}{2^7\cdot\sqrt{2}(\cos -\frac{\pi}{4}+i\sin -\frac{\pi}{4})} =$

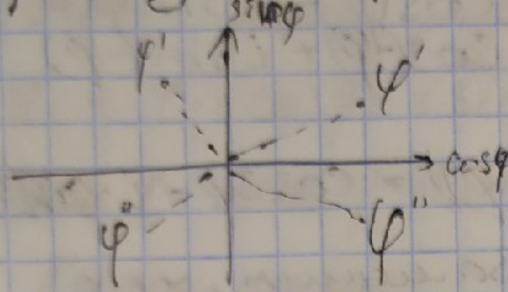
$= 2^8\cdot(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$

Ответ: $256\cdot(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})$

c) $\frac{1+\cos\varphi+i\sin\varphi}{1+\cos\varphi-i\sin\varphi}=$

$\operatorname{tg}\varphi'=\operatorname{tg}\left(\frac{\sin\varphi}{1+\cos\varphi}\right)\to\varphi'=\operatorname{arctg}\left(\frac{\sin\varphi}{1+\cos\varphi}\right)$ (для числителя

$\varphi''=\operatorname{arctg}\left(\frac{\sin\varphi}{1+\cos\varphi}\right)$ (для знаменателя




заметим, что $\varphi''$ - симметрично $\varphi'$ относительно оси абсцисс.

$\to\varphi''=\pi-\varphi'$

$\frac{1+\cos\varphi+i\sin\varphi}{1+\cos\varphi-i\sin\varphi}=\frac{\cos\varphi'+i\sin\varphi'}{\cos\varphi''+i\sin\varphi''}=\frac{\cos\varphi'+i\sin\varphi'}{\cos(\pi-\varphi')+i\sin(\pi-\varphi')}$

$=1\cdot(\cos(\varphi'-\pi+\varphi')+i\sin(\varphi'-\pi+\varphi'))=\cos(2\varphi'-\pi)+i\sin(2\varphi'-\pi)$

$=-\cos2\varphi'-i\cdot\sin2\varphi'=-\cos\left(2\cdot\operatorname{arctg}\left(\frac{\sin\varphi}{1+\cos\varphi}\right)\right)-\sin\left(2\cdot\operatorname{arctg}\left(\frac{\sin\varphi}{1+\cos\varphi}\right)\right)$

d) $(1+\cos\varphi+i\sin\varphi)^8=$

$\operatorname{tg}\varphi'=\operatorname{tg}\frac{\sin\varphi}{1+\cos\varphi}\to\varphi'=\operatorname{arctg}\frac{\sin\varphi}{1+\cos\varphi}$

$((1+\cos\varphi)+i\sin\varphi)^8=1\cdot(\cos8\varphi'+i\sin8\varphi')=$

$=\cos\left(8\cdot\operatorname{arctg}\frac{\sin\varphi}{1+\cos\varphi}\right)+i\cdot\sin\left(8\cdot\operatorname{arctg}\frac{\sin\varphi}{1+\cos\varphi}\right)$

Задача 4

$\exists$ $z$ и $\bar{z}$ - сопряженные числа.

$z = \cos\varphi + i\cdot\sin\varphi$ ; $\bar{z} = \cos\varphi - i\sin\varphi =$
$= \cos(-\varphi) + i\sin(-\varphi)$

$(z)^3 = \cos 3\varphi + i\cdot\sin 3\varphi$

$(\bar{z})^3 = \cos(3(-\varphi)) + i\cdot\sin(3\cdot(-\varphi)) = \cos 3\varphi + i\cdot\sin(3(-\varphi)) =$

$= \cos 3\varphi - i\cdot\sin 3\varphi \longrightarrow$ заменим, что

оно сопряжено $z^3 = \cos 3\varphi + i\cdot\sin 3\varphi$

Задача 5. Дано: $|z| = 1$

$\exists$ $z = (\cos\varphi + i\sin\varphi)\cdot 1$

$1 = 1\cdot(\cos 0 + i\cdot\sin 0)$

$\frac{1}{z} = \frac{\cos 0 + i\cdot\sin 0}{\cos\varphi + i\cdot\sin\varphi} = \cos(-\varphi) + i\cdot\sin(-\varphi) =$

$= \cos\varphi - i\sin\varphi = \bar{z}$ чтд.

Задача 6

$z_1 = a + bi \qquad z_2 = a + bi$

$$\frac{z_1}{z_2} = \frac{a+bi}{a+bi} = \frac{(a+bi)(a-bi)}{(a+bi)(a-bi)} = \frac{a^2+b^2}{a^2+b^2} = 1$$