Домашнее задание КАРАСЕВА М3123

1-й способ

Приведение матрицы к нормальной форме Жордана.

Задание 1

1) $A = \begin{pmatrix} 4 & 1 & -1 \\ -2 & 4 & 5 \\ 1 & 0 & 1 \end{pmatrix}$

$\begin{vmatrix} 4-t & 1 & -1 \\ -2 & 4-t & 5 \\ 1 & 0 & 1-t \end{vmatrix} = 0 \iff (4-t)^2(1-t) + 5 + 4 - t + 2 - 2t = 0$

$(t^2 - 8t + 16)(1-t) + 11 - 3t = 0$

$t^3 - 9t^2 + 27t - 27$

$(t-3)(t^2 - 6t + 9) = 0$

$(t-3)^3 = 0$

$\lambda = 3 \quad \alpha(\lambda) = 3$

$\begin{pmatrix} 1 & 1 & -1 \\ -2 & 1 & 5 \\ 1 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & 3 \\ 0 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$x_1 = 2x_3$
$x_2 = -x_3$
$x_3 = x_3$

$x = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \alpha, \ \alpha \in R$

Найти ещё 2 присоединённых вектора

$$\begin{pmatrix} 1 & 1 & -1 \\ -2 & 1 & 5 \\ 1 & 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2\alpha \\ -\alpha \\ \alpha \end{pmatrix}.$$

Эта система должна быть совместна

$$rg\begin{pmatrix} 1 & 1 & -1 \\ -2 & 1 & 5 \\ 1 & 0 & -2 \end{pmatrix} = rg\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2\alpha \\ -2 & 1 & 5 & -\alpha \\ 1 & 0 & -2 & \alpha \end{array}\right)$$

$$rg\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} = rg\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2\alpha \\ 0 & 1 & 1 & \alpha \\ 0 & 0 & 0 & 0 \end{array}\right)$$

всегда совместна.

$y_1 = \alpha + 2\beta$

$y_2 = \alpha - \beta$

$y_3 = \beta$

$y = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ y ← это вектор, присоединённый к $x = \begin{pmatrix} 2\alpha \\ -\alpha \\ \alpha \end{pmatrix}$

$$rg\begin{pmatrix} 1 & 1 & -1 \\ -2 & 1 & 5 \\ 1 & 0 & -2 \end{pmatrix} = rg\left(\begin{array}{ccc|c} 1 & 1 & -1 & \alpha + 2\beta \\ -2 & 1 & 5 & \alpha - \beta \\ 1 & 0 & -2 & \beta \end{array}\right)$$

$$rg\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} = rg\left(\begin{array}{ccc|c} 1 & 1 & -1 & \alpha + 2\beta \\ 0 & 1 & 1 & \alpha + \beta \\ 0 & 0 & 0 & 0 \end{array}\right)$$

всегда совместна

$\exists\ \alpha = 1 \quad \beta = 0$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{aligned} z_1 &= 2z_3 \\ z_2 &= 1 - z_3 \\ z_3 &= z_3 \end{aligned} \qquad \exists\ z_3 = 1 \quad z = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

Жорданов базис:

$$e_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad e_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad e_3 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$A' = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix} \quad . \quad P = \begin{pmatrix} 2 & 1 & 2 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$P \cdot A' = \begin{pmatrix} 6 & 5 & 7 \\ -3 & 2 & 1 \\ 3 & 1 & 3 \end{pmatrix} \quad A \cdot P = \begin{pmatrix} 6 & 5 & 7 \\ -3 & 2 & 7 \\ 3 & 1 & 10 \end{pmatrix}$$

2) $A = \begin{pmatrix} 1 & 4 & -8 & 4 \\ -1 & 5 & -6 & 4 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & -1 & 3 \end{pmatrix}$

$$\begin{vmatrix} 1-t & 4 & -8 & 4 \\ -1 & 5-t & -6 & 4 \\ 0 & 0 & -1-t & 4 \\ 0 & 0 & -1 & 3-t \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} -1 & (5-t) & -6 & 4 \\ 0 & 4+(t-1)(t-5) & -14+6t & 8-4t \\ 0 & 0 & -1-t & 4 \\ 0 & 0 & -1 & 3-t \end{vmatrix}$$

$$\begin{vmatrix} 4+(t-1)(t-5) & -14+6t & 8-4t \\ 0 & -1-t & 4 \\ 0 & -1 & 3-t \end{vmatrix} = 0$$

$$(4+(t-1)(t-5)) \begin{vmatrix} -1-t & 4 \\ -1 & 3-t \end{vmatrix} = 0$$

$$(t^2-6t+5+4) \cdot ((t-3)(t+1)+4) = 0$$

$$(t-3)^2 (t^2-2t-3+4) = 0$$

$$(t-3)^2 (t-1)^2 = 0$$

$\lambda_1 = 1 \qquad \lambda_2 = 3$

$\alpha = 2 \qquad \alpha = 2$

$\boxed{\lambda = 1}$

$$\begin{pmatrix} 0 & 4 & -8 & 4 \\ -1 & 4 & -6 & 4 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & -6 & 4 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_1 = 4x_4$

$x_2 = 3x_4$

$x_3 = 2x_4$

$x_4 = x_4$

$$x = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \alpha$$

$$\left(\begin{array}{cccc|c} 0 & 4 & -8 & 4 & 4\alpha \\ -1 & 4 & -6 & 4 & 3\alpha \\ 0 & 0 & -2 & 4 & 2\alpha \\ 0 & 0 & -1 & 2 & 1\alpha \end{array}\right) \sim \left(\begin{array}{cccc|c} -1 & 4 & -6 & 4 & 3\alpha \\ 0 & 4 & -8 & 4 & 4\alpha \\ 0 & 0 & -1 & 2 & \alpha \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

совместно при любом α

$\exists\ \alpha = 1$ $x = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$ $\left(\begin{array}{cccc|c} -1 & 4 & -6 & 4 & 3 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$y_1 = 4y_4 - 1$
$y_2 = 3y_4 - 1$
$y_3 = 2y_4 - 1$
$y_4 = y_4$

$\exists\ y = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$

$\boxed{\lambda = 3}$

$$\begin{pmatrix} -2 & 4 & -8 & 4 \\ -1 & 2 & -6 & 4 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & -6 & 4 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & -6 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_1 = 2x_2$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$x = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \alpha$

$$\left(\begin{array}{cccc|c} -2 & 4 & -8 & 4 & 2\alpha \\ -1 & 2 & -6 & 4 & \alpha \\ 0 & 0 & -4 & 4 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} -1 & 2 & -6 & 4 & \alpha \\ 0 & 0 & 4 & -4 & 0 \\ 0 & 0 & -4 & 4 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} -1 & 2 & -6 & 4 & \alpha \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

α - любое

$\exists\, \alpha = 1 \quad x = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{cccc|c} -1 & 2 & -6 & 4 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{matrix} y_1 = 2y_2 - 1 \\ y_2 = y_2 \\ y_3 = 0 \\ y_4 = 0 \end{matrix} \quad \exists\, y_2 = 1 \quad y = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Жорданов базис

$$e_1 = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad e_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} \quad e_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad e_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix} \quad P' \begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 2 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

$$PA' = \begin{pmatrix} 4 & 7 & 6 & 5 \\ 3 & 5 & 3 & 4 \\ 2 & 3 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{pmatrix} \quad AP = \begin{pmatrix} 4 & 7 & 6 & 5 \\ 5 & 5 & 3 & 4 \\ 2 & 3 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{pmatrix}$$

3) $A = \begin{pmatrix} 1 & 4 & 5 & -13 \\ -1 & 5 & 4 & -9 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 2 & -1 \end{pmatrix}$

$$\begin{vmatrix} 1-t & 4 & 5 & -13 \\ -1 & 5-t & 4 & -9 \\ 0 & 0 & 5-t & -4 \\ 0 & 0 & 2 & -1-t \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} \cancel{-1} & \cancel{(5-t)} & \cancel{4} & \cancel{-9} \\ 0 & 4+(5-t)(1-t) & 9-4t & -22+9t \\ 0 & 0 & 5-t & -4 \\ 0 & 0 & 2 & -1-t \end{vmatrix} = 0$$

$$\begin{vmatrix} \cancel{4+(t-5)(t-1)} & \cancel{9-4t} & \cancel{-22+9t} \\ 0 & 5-t & -4 \\ 0 & 2 & -1-t \end{vmatrix} = 0$$

$$(4+(t-5)(t-1))\cdot\begin{vmatrix}5-t & -4\\ 2 & -1-t\end{vmatrix}=0$$

$$(t-3)^2\cdot((t-5)(t+1)+8)=0$$

$$(t-3)^2\cdot(t^2-4t+3)=0$$

$$(t-3)^3\cdot(t-1)=0$$

$\lambda_1=3$ $\qquad\lambda_2=1$

$\alpha=3$ $\qquad\alpha=1$

$\boxed{\lambda=3}$

$$\begin{pmatrix}-2&4&5&-13\\-1&2&4&-9\\0&0&2&-4\\0&0&2&-4\end{pmatrix}\sim\begin{pmatrix}-1&2&4&-9\\0&0&-3&5\\0&0&2&-2\\0&0&0&0\end{pmatrix}\quad \begin{matrix}x_1=2x_2\\x_2=x_2\\x_3=0\\x_4=0\end{matrix}\qquad x=\begin{pmatrix}2\\1\\0\\0\end{pmatrix}\alpha$$

$$\left(\begin{matrix}-2&4&5&-13\\-1&2&4&-9\\0&0&2&-4\\0&0&2&-4\end{matrix}\middle|\begin{matrix}2\alpha\\\alpha\\0\\0\end{matrix}\right)\sim\left(\begin{matrix}-1&2&4&-9\\0&0&-3&5\\0&0&1&-2\\0&0&0&0\end{matrix}\middle|\begin{matrix}\alpha\\0\\0\\0\end{matrix}\right)$$

совместна при любом α

$\exists\ \alpha=1 \quad x=\begin{pmatrix}2\\1\\0\\0\end{pmatrix}$

$$\left(\begin{matrix}-1&2&4&-9\\0&0&-3&5\\0&0&1&-2\\0&0&0&0\end{matrix}\middle|\begin{matrix}1\\0\\0\\0\end{matrix}\right)\quad \begin{matrix}y_1=2y_2-1\\y_2=y_2\\y_3=0\\y_4=0\end{matrix}$$

$$\left(\begin{matrix}-2&4&5&-13\\-1&2&4&-9\\0&0&2&-4\\0&0&2&-4\end{matrix}\middle|\begin{matrix}2y_2-1\\y_2\\0\\0\end{matrix}\right)\sim\left(\begin{matrix}-1&2&4&-9\\0&0&-3&5\\0&0&1&-2\\0&0&0&0\end{matrix}\middle|\begin{matrix}y_2\\-1\\0\\0\end{matrix}\right)$$

совместна при любом y_2

$\exists\ y_2=1 \Rightarrow y=\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$

$$\left(\begin{matrix}-1&2&4&-9\\0&0&-3&5\\0&0&1&-2\\0&0&0&0\end{matrix}\middle|\begin{matrix}1\\-1\\0\\0\end{matrix}\right)\quad \begin{matrix}z_1=2z_2-2\\z_2=z_2\\z_3=2\\z_4=1\end{matrix}\qquad \exists\ z_2=1\Rightarrow z=\begin{pmatrix}0\\1\\2\\1\end{pmatrix}$$

$\boxed{\lambda = 1}$

$$\begin{pmatrix} 0 & 4 & 5 & -13 \\ -1 & 4 & 4 & -9 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & 4 & -9 \\ 0 & 4 & 5 & -13 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 4 & -4 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & 4 & -9 \\ 0 & 4 & 5 & -13 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 &= 3x_4 \\ x_2 &= 2x_4 \\ x_3 &= x_4 \\ x_4 &= x_4 \end{aligned} \qquad x = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} \alpha \qquad \exists\ \alpha = 1 \quad x = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}.$$

Жорданов базис:

$$e_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad e_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}. \quad e_4 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

$$A' = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$AP = \begin{pmatrix} 6 & 5 & 1 & 3 \\ 3 & 4 & 4 & 2 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 3 & 1 \end{pmatrix} \qquad P \cdot A' \begin{pmatrix} 6 & 5 & 1 & 3 \\ 3 & 4 & 4 & 2 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 3 & 1 \end{pmatrix}$$

Домашнее задание Карасева М3135

Приведение матрицы к нормальной форме Жордана

! 2й способ

Задача 1

1) $A = \begin{pmatrix} 4 & 1 & -1 \\ -2 & 4 & 5 \\ 1 & 0 & 1 \end{pmatrix}$

$\begin{vmatrix} 4-t & 1 & -1 \\ -2 & 4-t & 5 \\ 1 & 0 & 1-t \end{vmatrix} = 0 \Leftrightarrow (4-t)^2(1-t) + 5 + 4 - t + 2 - 2t = 0$

$(t^2 - 8t + 16)(1 - t) + 11 - 3t = 0$

$t^2 - 8t + 16 - t^3 + 8t^2 - 16t + 11 - 3t = 0$

$t^3 - 9t^2 + 27t - 27 = 0$

$$\begin{array}{l|l} t^3 - 9t^2 + 27t - 27 & t-3 \\ \underline{t^3 - 3t^2} & \overline{t^2 - 6t + 9} \\ \quad -6t^2 + 27t & \\ \quad \underline{-6t^2 + 18t} & \\ \qquad 9t - 27 & \\ \qquad \underline{9t - 27} & \\ \qquad\quad 0 & \end{array}$$

$(t-3)^3 = 0$

$\lambda_1 = 3 \quad \alpha(\lambda_1) = 3$

$$B = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 1 & 5 \\ 1 & 0 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & 3 \\ 0 & -1 & -1 \end{pmatrix} \sim \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \quad rgB = 2$$

$x_1 + x_2 - x_3 = 0 \Leftrightarrow x_1 = 2x_3$

$x_2 = -x_3$

$x_3 = x_3$

$\ker B = \mathrm{span}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\dim(\ker B) = 1$

$$B^2=\begin{pmatrix}-2&2&6\\1&-1&-3\\-1&1&3\end{pmatrix}\sim\begin{pmatrix}-2&2&6\\1&-1&-3\\0&0&0\end{pmatrix}\sim\begin{pmatrix}1&-1&-3\\0&0&0\\0&0&0\end{pmatrix}\quad rg\,B^2=1$$

$x_1-x_2-3x_3=0$ $\quad x_1=x_2+3x_3$ $\quad \ker B^2=\operatorname{span}\left(\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}3\\0\\1\end{pmatrix}\right)$

$x_2=x_2$ $\quad x_2=x_2$

$x_3=x_3$ $\quad x_3=x_3$ $\quad \dim(\ker B^2)=2$

$$B^3=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}\quad rg\,B^3=0\quad \dim(\ker B^3)=3.$$

$f\in\ker B^3 \quad f\notin\ker B^2 \qquad f\begin{pmatrix}0\\0\\1\end{pmatrix}$

$$Bf=\begin{pmatrix}-1\\5\\-2\end{pmatrix}\quad B^2f=\begin{pmatrix}6\\-3\\3\end{pmatrix}\in\ker B=\operatorname{span}\begin{pmatrix}2\\-1\\1\end{pmatrix}.$$

$n=2$ $\qquad n=1$

$\ker B^3$	f
$\ker B^2$	Bf
$\ker B$	B^2f

Жорданов базис:

$e_1=B^2f \quad e_2=Bf \quad e_3=f$

$$A'=\begin{pmatrix}3&1&0\\0&3&1\\0&0&3\end{pmatrix}$$

$$P=\begin{pmatrix}6&-1&0\\-3&5&0\\3&-2&1\end{pmatrix}\qquad P\cdot A'=\begin{pmatrix}18&3&-1\\-9&12&5\\9&-3&1\end{pmatrix}$$

$$A\cdot P=\begin{pmatrix}18&3&-1\\-9&12&5\\9&-3&1\end{pmatrix}$$

2) $A = \begin{pmatrix} 1 & 4 & -8 & 4 \\ -1 & 5 & -6 & 4 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & -1 & 3 \end{pmatrix}$

$$\begin{vmatrix} 1-t & 4 & -8 & 4 \\ -1 & 5-t & -6 & 4 \\ 0 & 0 & -1-t & 4 \\ 0 & 0 & -1 & 3-t \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} -1 & (5-t) & -6 & 4 \\ 0 & 4+(t-1)(t-5) & -14+6t & 8-4t \\ 0 & 0 & -1-t & 4 \\ 0 & 0 & -1 & 3-t \end{vmatrix}$$

$$\begin{vmatrix} 4+(t-1)(t-5) & -14+6t & 8-4t \\ 0 & -1-t & 4 \\ 0 & -1 & 3-t \end{vmatrix} = 0$$

$$(4+t^2-6t+5) \cdot \begin{vmatrix} -1-t & 4 \\ -1 & 3-t \end{vmatrix} = 0$$

$$(t^2-6t+9)\left((t-3)(t+1)+4\right) = 0$$

$$(t-3)^2(t^2-2t-3+4) = 0$$

$$(t-3)^2(t-1)^2 = 0$$

$\lambda_1 = 3 \quad \lambda_2 = 1$

$\alpha(\lambda_1) = 2 \quad \alpha(\lambda_2) = 2$

$$B_{\lambda_1} = \begin{array}{cccc} & & x_3 & x_4 \\ \end{array} \begin{pmatrix} -2 & 4 & -8 & 4 \\ -1 & 2 & -6 & 4 \\ 0 & 0 & -4 & 4 \\ 0 & 0 & -1 & 0 \end{pmatrix} \sim \begin{array}{cccc} & & x_4 & x_3 \\ \end{array} \begin{pmatrix} -2 & 4 & 4 & -8 \\ -1 & 2 & 4 & -6 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix} \sim$$

$$\sim \begin{array}{cccc} x_1 & x_2 & x_4 & x_3 \\ \end{array} \begin{pmatrix} 0 & 4 & 4 & -8 \\ 0 & 2 & 4 & -6 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$rg B_{\lambda_1} = 3$

$x_1 = x_1$
$x_2 = 0$
$x_3 = 0$
$x_4 = 0$

$\ker B_{\lambda_1} = span \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$dim \ker B_{\lambda_1} = 1$

$$B_{\lambda_1}^2 = \begin{pmatrix} 0 & 0 & 20 & -24 \\ 0 & 0 & 16 & -20 \\ 0 & 0 & 12 & -16 \\ 0 & 0 & 4 & -4 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & -4 & -24 \\ 0 & 0 & -4 & -20 \\ 0 & 0 & -4 & -16 \\ 0 & 0 & 0 & -4 \end{pmatrix} \quad rg\, B_{\lambda_1}^2 = 2$$

$x_1 = x_1$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$$\ker B_{\lambda_1}^2 = \mathrm{span}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right) \quad \dim \ker B_{\lambda_1}^2 = 2$$

$$B_{\lambda_1}^3 = \begin{pmatrix} 0 & 0 & -56 & 80 \\ 0 & 0 & -44 & 64 \\ 0 & 0 & -32 & 48 \\ 0 & 0 & -12 & 16 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 4 & 80 \\ 0 & 0 & 4 & 64 \\ 0 & 0 & 4 & 48 \\ 0 & 0 & 0 & 16 \end{pmatrix} \quad rg\, B_{\lambda_1}^3 = 2$$

$x_1 = x_1$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$$\ker B_{\lambda_1}^3 = \mathrm{span}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right) \quad \dim \ker B_{\lambda_1}^3 = 2$$

$$f_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad Bf_1 = \begin{pmatrix} -2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$ собственные вектора

$$B_{\lambda_2} = \begin{pmatrix} 0 & 4 & -8 & 4 \\ -1 & 4 & -6 & 4 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & -6 & 4 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & -6 & 4 \\ 0 & 4 & -8 & 4 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$rg\, B_{\lambda_2} = 3$

$x_1 = 4x_4$
$x_2 = 3x_4$
$x_3 = 2x_4$
$x_4 = x_4$

$$\ker B_{\lambda_2} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad \dim \ker B_{\lambda_2} = 1$$

$$B^2_{\lambda_2} = \begin{pmatrix} -4 & 16 & -12 & -8 \\ -4 & 12 & -8 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} -4 & 16 & -12 & -8 \\ 0 & -4 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad rg\, B^2_{\lambda_2} = 2$$

$x_1 = x_3 + 2x_4$
$x_2 = x_3 + x_4$
$x_3 = x_3$
$x_4 = x_4$

$$\ker B^2_{\lambda_2} = span\left(\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right) \quad \dim \ker B^2 = 2$$

$$B^3_{\lambda_2} = \begin{pmatrix} -16 & 48 & -32 & -16 \\ -12 & 32 & -20 & -8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} -16 & 48 & -32 & -16 \\ 0 & -4 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad rg\, B^3_{\lambda_2} = 2$$

$x_1 = x_3 + 2x$
$x_2 = x_3 + x_4$
$x_3 = x_3$
$x_4 = x_4$

$$\ker B^3_{\lambda_2} = span\left(\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right) \quad \dim \ker B^3_{\lambda} = 2$$

выберем $f_3 \in \ker B^2_{\lambda_2}$ $f_3 \notin \ker B_{\lambda_2}$

$$f_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \quad B f_3 = \begin{pmatrix} -4 \\ -3 \\ -2 \\ -1 \end{pmatrix}$$

Жорданов базис: $e_1 = \begin{pmatrix} -2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$e_3 = \begin{pmatrix} -4 \\ -2 \\ -3 \\ -1 \end{pmatrix}$ $e_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

$$A' = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P=\begin{pmatrix}-2&1&-4&1\\-1&0&-3&1\\0&0&-2&1\\0&0&-1&0\end{pmatrix}\qquad P\cdot A'=\begin{pmatrix}-6&1&-4&-3\\-3&-1&-3&-2\\0&0&-2&-1\\0&0&-1&-1\end{pmatrix}$$

$$A\cdot P=\begin{pmatrix}-6&1&-4&-3\\-3&-1&-3&-2\\0&0&-2&-1\\0&0&-1&-1\end{pmatrix}$$

3) $$A=\begin{pmatrix}1&4&5&-13\\-1&5&4&-9\\0&0&5&-4\\0&0&2&-1\end{pmatrix}$$

$$\begin{vmatrix}1-t&4&5&-13\\-1&5-t&4&-9\\0&0&5-t&-4\\0&0&2&-1-t\end{vmatrix}=0 \Leftrightarrow \begin{vmatrix}\cancel{-1}&\cancel{(5-t)}&\cancel{4}&\cancel{-9}\\0&4+(5-t)(1-t)&9-4t&-22+9t\\0&0&5-t&-4\\0&0&2&-1-t\end{vmatrix}=0$$

$$\begin{vmatrix}\cancel{4+(5-t)(1-t)}&9-4t&-22+9t\\0&5-t&-4\\0&2&-1-t\end{vmatrix}=0$$

$$(4+(5-t)(1-t))\cdot\begin{vmatrix}5-t&-4\\2&-1-t\end{vmatrix}=0$$

$$(4+t^2-6t+5)\cdot((t-5)(t+1)+8)=0$$

$$(t-3)^2\cdot(t^2-4t-5+8)=0$$

$$(t-3)^2\cdot(t^2-4t+3)=0$$

$$(t-3)^3\cdot(t-1)=0$$

$\lambda_1=1$ $\qquad\lambda_2=3$

$\alpha=1$ $\qquad\alpha=3$

$$B_{\lambda_1} = \begin{pmatrix} 0 & 4 & 5 & -13 \\ -1 & 4 & 4 & -9 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} -1 & 4 & 4 & -9 \\ 0 & 4 & 5 & -13 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \operatorname{rg} B_{\lambda_1} = 3$$

$x_1 = 3x_4$
$x_2 = 2x_4$
$x_3 = x_4$
$x_4 = x_4$

$$\ker B_{\lambda_2} = \operatorname{span}\begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} \quad \dim \ker B_{\lambda_1} = 1$$

$$B^2_{\lambda_1} = \begin{pmatrix} -4 & 16 & 10 & -30 \\ -4 & 12 & 9 & -21 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & 4 & -4 \end{pmatrix} \sim \begin{pmatrix} -4 & 16 & 10 & -30 \\ 0 & -4 & -1 & 9 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \operatorname{rg} B^2_{\lambda_1} = 3$$

$$f_1 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

$$B_{\lambda_2} = \begin{pmatrix} -2 & 4 & 5 & -13 \\ -1 & 2 & 4 & -9 \\ 0 & 0 & 2 & -4 \\ 0 & 0 & 2 & -4 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 4 & -9 \\ 0 & 0 & -3 & 5 \\ 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \operatorname{rg} B_{\lambda_2} = 3$$

$x_1 = 2x_2$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$$\ker B_{\lambda_2} = \operatorname{span}\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \dim \ker B_{\lambda_2} = 1$$

$$B^2_{\lambda_2} = \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \end{array}$$
$$\begin{pmatrix} 0 & 0 & -10 & 22 \\ 0 & 0 & -7 & 15 \\ 0 & 0 & -4 & 8 \\ 0 & 0 & -4 & 8 \end{pmatrix} \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_4 & x_3 \end{array} \\ \begin{pmatrix} 0 & 0 & 2 & -10 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -4 \end{pmatrix} \end{array} \quad \operatorname{rg} B^2_{\lambda_2} = 2$$

$x_1 = x_1$
$x_2 = x_2$
$x_3 = 0$
$x_4 = 0$

$$\ker B^2_{\lambda_2} = \operatorname{span}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}\right) \quad \dim \ker B^2_{\lambda_2} = 2$$

$$B^3_{\lambda_2} = \begin{pmatrix} 0&0&24&-48\\ 0&0&16&-32\\ 0&0&8&-16\\ 0&0&8&-16 \end{pmatrix} \sim \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&1&-2 \end{pmatrix} \quad \operatorname{rg} B^3_{\lambda_2} = 1$$

$x_1 = x_1$
$x_2 = x_2$
$x_3 = 2x_4$
$x_4 = x_4$

$$\ker B^3_{\lambda_2} = \operatorname{Span}\left(\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\\1\end{pmatrix} \right) \quad \dim\ker B^3_{\lambda_2} = 3$$

$$B^4_{\lambda_2} = \begin{pmatrix} 0&0&-48&96\\ 0&0&-32&64\\ 0&0&-16&32\\ 0&0&-16&32 \end{pmatrix} \sim \begin{pmatrix} 0&0&0&-48\\ 0&0&0&-32\\ 0&0&0&-16\\ 0&0&0&-16 \end{pmatrix} \quad \operatorname{rg} B^4_{\lambda_2} = 1$$

$$f_2 \in \ker B^3_{\lambda_2} \quad f_2 \notin B^2_{\lambda_1} \quad f_2 = \begin{pmatrix}0\\0\\2\\1\end{pmatrix}$$

$$B_{\lambda_2} f_2 = \begin{pmatrix}-3\\-1\\0\\0\end{pmatrix} \quad B^2_{\lambda_2} f_2 = \begin{pmatrix}2\\1\\0\\0\end{pmatrix}$$

Жорданов базис:

$$e_1 = f_1 = \begin{pmatrix}3\\2\\1\\1\end{pmatrix} \quad e_2 = \begin{pmatrix}2\\1\\0\\0\end{pmatrix} \quad e_3 = \begin{pmatrix}-3\\-1\\0\\0\end{pmatrix} \quad e_4 = \begin{pmatrix}0\\0\\2\\1\end{pmatrix}$$

$$A' = \begin{pmatrix} \boxed{1}&0&0&0\\ 0&3&1&0\\ 0&0&3&1\\ 0&0&0&3 \end{pmatrix} \quad P = \begin{pmatrix} 3&2&-3&0\\ 2&1&-1&0\\ 1&0&0&2\\ 1&0&0&1 \end{pmatrix}$$

$$P \cdot A' = \begin{pmatrix} 3&6&-7&-3\\ 2&3&-2&-1\\ 1&0&0&6\\ 1&0&0&3 \end{pmatrix}$$

$$A \cdot P = \begin{pmatrix} 3&6&-7&-3\\ 2&3&-2&-1\\ 1&0&0&6\\ 1&0&0&3 \end{pmatrix}$$