Домашнее задание №18 КАРАСЕВА М3135

Тензорная алгебра (часть 2)

Задача 1

$$a_{ijkl} = A = \left(\begin{array}{cc|cc} 1 & 1 & 2 & 2 \\ 3 & 3 & 4 & 4 \\ \hline 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 \end{array}\right)$$

$$b_{ijkl} = a_{kjil} = \left(\begin{array}{cc|cc} 1 & 1 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ \hline 3 & 3 & 4 & 4 \\ 5 & 4 & 3 & 2 \end{array}\right) \qquad c_{ijkl} = a_{ilkj} = \left(\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \\ \hline 2 & 0 & 1 & 1 \\ 5 & 3 & 4 & 2 \end{array}\right)$$

Задача 2

$$x^i a_{ik} = \begin{pmatrix} x^1 a_{11} + x^2 a_{21} \\ x^1 a_{12} + x^2 a_{22} \end{pmatrix}^T$$

$$x^k a^i_i = \begin{pmatrix} x^1 (a_{11} + a_{22}) \\ x^2 (a_{11} + a_{22}) \end{pmatrix}$$

$$a^i_i a^k_k = (a_{11} + a_{22}) \cdot (a_{11} + a_{22})$$

$$x^{(i} y^{k)} = \frac{1}{2!}(x^i y^k + x^k y^i) = \frac{1}{2}\begin{pmatrix} x^1 y^1 + x^1 y^1 & x^1 y^2 + x^2 y^1 \\ x^2 y^1 + x^1 y^2 & x^2 y^2 + x^2 y^2 \end{pmatrix}$$

$$x^{[k} a^{i]}_i = \frac{1}{2!}(x^k a^i_i - x^i a^k_i) = \frac{1}{2}\begin{pmatrix} x^1 (a^1_1 + a^2_2) - (x^1 a^1_1 + x^2 a^1_2) \\ x^2 (a^1_1 + a^2_2) - (x^1 a^2_1 + x^2 a^2_2) \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} x^1 a^2_2 - x^2 a^1_2 \\ x^2 a^1_1 - x^1 a^2_1 \end{pmatrix}$$

$$a^i_{[i} a^k_{k]} = \frac{1}{2!}\left(a^i_i a^k_k - a^i_k \cdot a^k_i\right) =$$

$$= \frac{1}{2}\left((a^1_1 + a^2_2)(a^1_1 + a^2_2) - (a^1_1 \cdot a^1_1 + a^1_2 \cdot a^2_1 + a^2_1 \cdot a^2_1 + a^2_2 a^2_2)\right) =$$

$$= \frac{1}{2}\left(2 a^1_1 a^2_2 - 2 a^1_2 a^2_1\right) = \left(a^1_1 a^2_2 - a^1_2 a^2_1\right)$$

Задача 3

$$A = \left(\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{array}\right) = a^i_{jkl}$$

$$a^i_{(ik)l} = \frac{1}{2}\left(a^i_{ikl} + a^i_{kil}\right) = \frac{1}{2}\begin{pmatrix} 12 & 16 \\ 16 & 12 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 8 & 6 \end{pmatrix}$$

$$a^i_{ikl} = \begin{pmatrix} 7 & 11 \\ 11 & 7 \end{pmatrix} \qquad a^i_{kil} = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$$

$$a^i_{[jkl]} = \frac{1}{6}\left(a^i_{jkl} + a^i_{klj} + a^i_{ljk} - a^i_{jlk} - a^i_{kjl} - a^i_{lkj}\right)$$

$$-a^i_{jlk} = \left(\begin{array}{cc|cc} 1 & 2 & 8 & 7 \\ 5 & 6 & 4 & 3 \\ \hline 3 & 4 & 6 & 5 \\ 7 & 8 & 2 & 1 \end{array}\right) \quad -a^i_{kjl} = \left(\begin{array}{cc|cc} 1 & 8 & 3 & 6 \\ 5 & 4 & 7 & 2 \\ \hline 2 & 7 & 4 & 5 \\ 6 & 3 & 8 & 1 \end{array}\right)$$

$$+a^i_{klj} = \left(\begin{array}{cc|cc} 1 & 8 & 2 & 7 \\ 5 & 4 & 6 & 3 \\ \hline 3 & 6 & 4 & 5 \\ 7 & 2 & 8 & 1 \end{array}\right) \quad +a^i_{ljk} = \left(\begin{array}{cc|cc} 1 & 3 & 8 & 6 \\ 5 & 7 & 4 & 2 \\ \hline 2 & 4 & 7 & 5 \\ 6 & 8 & 3 & 1 \end{array}\right)$$

$$-a^i_{lkj} = \left(\begin{array}{cc|cc} 1 & 3 & 2 & 4 \\ 5 & 7 & 6 & 8 \\ \hline 8 & 6 & 7 & 5 \\ 4 & 2 & 3 & 1 \end{array}\right)$$

$$a^{i}_{[jkl]} = \frac{1}{6}\cdot\left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Задача 4

$f(1\ 2\ 3)$ $g(1\ 1\ 1)$ $h = (1\ 0\ 1)$

$\dim \Lambda^p = C_3^3 = 1$

$f \wedge g \wedge h = 3!\, Alt(f \otimes g \otimes h)$

$C^{ijk} = f \otimes g \otimes h$

$C^{ij} = f \otimes g$ $\quad C^{ij} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}$

$$C^{ijk} = C^{ij} \otimes h = \left(\begin{array}{ccc|ccc|ccc} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 0 & 0 & 2 & 2 & 2 \\ 3 & 3 & 3 & 0 & 0 & 0 & 3 & 3 & 3 \end{array}\right)$$

$$C^{[ijk]} = \frac{1}{3!}\cdot\left(C^{ijk} + C^{jki} + C^{kij} - C^{ikj} - C^{jik} - C^{kji}\right)$$

$$-C^{ikj} = \left(\begin{array}{ccc|ccc|ccc} 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 2 & 2 & 0 & 2 & 2 & 0 & 2 \\ 3 & 0 & 3 & 3 & 0 & 3 & 3 & 0 & 3 \end{array}\right) \quad -C^{jik} = \left(\begin{array}{ccc|ccc|ccc} 1 & 2 & 3 & 0 & 0 & 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

$$+C^{jki} = \left(\begin{array}{ccc|ccc|ccc} 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 3 \end{array}\right) \quad +C^{kij} = \left(\begin{array}{ccc|ccc|ccc} 1 & 0 & 1 & 2 & 0 & 2 & 3 & 0 & 3 \\ 1 & 0 & 1 & 2 & 0 & 2 & 3 & 0 & 3 \\ 1 & 0 & 1 & 2 & 0 & 2 & 3 & 0 & 3 \end{array}\right)$$

$$-C^{kji} = \left(\begin{array}{ccc|ccc|ccc} 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \end{array}\right) \quad C^{[ijk]} = \left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Задача 5

$$A^{ij} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix} \qquad C^k = (6\ 3\ 3)$$

$$U \wedge V = \frac{3!}{2!} Alt(A \otimes C)$$

$$+B^{ijk} = \left(\begin{array}{ccc|ccc|ccc} 0 & -6 & 12 & 0 & -3 & 6 & 0 & -3 & 6 \\ 6 & 0 & -12 & 3 & 0 & -6 & 3 & 0 & -6 \\ -12 & 12 & 0 & -6 & 6 & 0 & -6 & 6 & 0 \end{array}\right)$$

$$-B^{ikj} \left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & -6 & -3 & -3 & 12 & 6 & 6 \\ 6 & 3 & 3 & 0 & 0 & 0 & -12 & -6 & -6 \\ -12 & -6 & -6 & 12 & 6 & 6 & 0 & 0 & 0 \end{array}\right) \quad -B^{jik} \left(\begin{array}{ccc|ccc|ccc} 0 & 6 & -12 & 0 & 3 & -6 & 0 & 3 & -6 \\ -6 & 0 & 12 & -3 & 0 & 6 & -3 & 0 & 6 \\ 12 & -12 & 0 & 6 & -6 & 0 & 6 & -6 & 0 \end{array}\right)$$

$$+B^{jki} \left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 6 & 3 & 3 & -12 & -6 & -6 \\ -6 & -3 & -3 & 0 & 0 & 0 & 12 & 6 & 6 \\ 12 & 6 & 6 & -12 & -6 & -6 & 0 & 0 & 0 \end{array}\right) \quad +B^{kij} \left(\begin{array}{ccc|ccc|ccc} 0 & 6 & -12 & -6 & 0 & 12 & 12 & -12 & 0 \\ 0 & 3 & -6 & -3 & 0 & 6 & 6 & -6 & 0 \\ 0 & 3 & -6 & -3 & 0 & 6 & 6 & -6 & 0 \end{array}\right)$$

$$-B^{kji} \left(\begin{array}{ccc|ccc|ccc} 0 & -6 & 12 & 6 & 0 & -12 & -12 & 12 & 0 \\ 0 & -3 & 6 & 3 & 0 & -6 & -6 & 6 & 0 \\ 0 & -3 & 6 & 3 & 0 & -6 & -6 & 6 & 0 \end{array}\right)$$

$$U \wedge V = \frac{3!}{2!} \cdot \frac{1}{3!} \cdot \left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 0 & 0 & 42 & 0 & -42 & 0 \\ 0 & 0 & -42 & 0 & 0 & 0 & 42 & 0 & 0 \\ 0 & 42 & 0 & -42 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$U \wedge V = \left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 0 & 0 & 21 & 0 & -21 & 0 \\ 0 & 0 & -21 & 0 & 0 & 0 & 21 & 0 & 0 \\ 0 & 21 & 0 & -21 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$