Контрольная работа Карасева

М3135

Вариант 4

1) $\int \frac{\sin x\,dx}{(1-\cos x)^2} = -\int \frac{d\cos x}{(1-\cos x)^2} = -\int \frac{dt}{(1-t^2)^2} =$

$= -\int \frac{dt}{(t-1)^2(t+1)^2}$

$\frac{1}{(t-1)^2(t+1)^2} = \frac{A}{(t-1)^2} + \frac{B}{(t-1)} + \frac{C}{(t+1)^2} + \frac{D}{t+1}$

$A(t+1)^2 + B(t+1)^2(t-1) + C(t-1)^2 + D(t-1)^2(t+1) = 1$

$t=1: A = \frac{1}{4}$. $\quad t=0: 1/4 - B + 1/4 + D = 1$

$t=-1: C = 1/4 \quad t=2: 9/4 + 9B + 1/4 + 3D = 1$

$D - B = 1/2 \Rightarrow D = B + 1/2$

$9/4 + 9B + 1/4 + 3B + 3/2 = 1$

$12B = -3 \Rightarrow B = -1/4. \Rightarrow D = 1/4$

Таким образом:

$-\int \frac{dt}{(t-1)^2(t+1)^2} = -\frac{1}{4}\int \frac{dt}{(t-1)^2} + \frac{1}{4}\int \frac{dt}{(t-1)} - \frac{1}{4}\int \frac{dt}{(t+1)^2} -$

$-\frac{1}{4}\int \frac{dt}{t+1} = \frac{1}{4}\cdot\frac{1}{t-1} + \frac{1}{4}\ln|t-1| + \frac{1}{4}\cdot\frac{1}{t+1} -$

$-\frac{1}{4}\ln|t+1| + C = \frac{1}{4}\cdot\frac{1}{\cos x - 1} + \frac{1}{4}\ln|\cos x - 1| +$

$+ \frac{1}{4}\cdot\frac{1}{\cos x + 1} - \frac{1}{4}\ln|\cos x + 1|$

(2) $$\int \frac{\sqrt[3]{tg\,x}\,dx}{\sin x \cos x} = \int \frac{dx}{\cos^2 x} \cdot \frac{\sqrt[3]{tg\,x}}{tg\,x} =$$

$$= \int (tg\,x)^{-2/3}\, d(tg\,x) = \frac{\sqrt[3]{tg\,x}}{3} + C$$

(3) $$\int \sqrt{3x^2 - 3x + 1}\, dx = \sqrt{3} \int \sqrt{x^2 - x + 1/3}\, dx =$$

$$= \sqrt{3} \int \sqrt{(x - 1/2)^2 + 1/12}\, dx = \{t = x - 1/2\} =$$

$$= \sqrt{3} \int \sqrt{t^2 + 1/12}\, dt$$

(8) $$\int \frac{(x^2 - 2)\,dx}{x^4 + x^2 + 4} = \int \frac{1 - 2/x^2}{x^2 + 1 + 4/x^2}\, dx =$$

$$= \int \frac{1 - 2/x^2}{\left(x + \frac{2}{x}\right)^2 - 3}\, dx =$$

$$t = x + \frac{2}{x}$$

$$t' = 1 - \frac{2}{x^2}$$

$$x^2 + 4 + 4/x^2 = (x + 2/x)^2$$

$$= \int \frac{t'}{(t^2 - 3)}\, dx = \int \frac{dt}{t^2 - 3} = -\int \frac{dt}{(\sqrt{3})^2 - t^2} =$$

$$= \frac{-1}{2\sqrt{3}} \cdot \ln\left|\frac{t + \sqrt{3}}{t - \sqrt{3}}\right| + C = \frac{-1}{2\sqrt{3}} \cdot \ln\left|\frac{x + \frac{2}{x} + \sqrt{3}}{x + 2/x - \sqrt{3}}\right| + C$$

⑤ $\int \frac{dx}{4-3\cos^2 x+5\sin^2 x} = \int \frac{dx}{8\sin^2 x+1}$

$\left[t = tg\, x,\quad \sin^2 x = \frac{t^2}{t^2+1},\quad dx = \frac{dt}{t^2+1} \right]$

$= \int \frac{(t^2+1)\,dt}{(9t^2+1)(t^2+1)} = \int \frac{dt}{(9t^2+1)} = \frac{1}{9}(3\, arctg(3t)) + c$

$= \frac{1}{3} arctg(3\, tg\, x) + c$

⑥

$\int \sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}\,dx = \int \sqrt{\frac{(1+\sqrt{x})^2}{1-x}}\,dx =$

$= \int \frac{1+\sqrt{x}}{\sqrt{1-x}}\,dx = \int \frac{dx}{\sqrt{1-x}} + \int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx =$

$= -\int \frac{d(1-x)}{\sqrt{1-x}} + \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} = 2\sqrt{1-x} +$

$+ \int \frac{\sqrt{x}\,dx}{\sqrt{1-x}} \overset{[u=\sqrt{x}]}{=} 2\sqrt{1-x} + \int \frac{u \cdot 2u}{\sqrt{1-u^2}}\,du =$

$= 2\sqrt{1-x} + 2\int \frac{u^2\,du}{\sqrt{1-u^2}} = \left[\begin{matrix} u = \sin t \\ du = \cos t\,dt \end{matrix} \right] =$

$= 2\sqrt{1-x} + 2\int \frac{\sin^2 t \cdot \cos t\,dt}{\cos t} =$

$= 2\sqrt{1-x} + 2\int \sin^2 t\,dt = 2\sqrt{1-x} +$

$2\int \frac{1-\cos 2t}{2}\,dt = 2\sqrt{1-x} + t - \frac{1}{2}\sin 2t + c$

$= 2\sqrt{1-x} + \arcsin\sqrt{x} - \frac{1}{2}\sin(2\arcsin\sqrt{x}) + C$

⑦ $\int \frac{dx}{x^2(x^2+1)} = \int \frac{dx}{x^2} - \int \frac{dx}{x^2+1} = \frac{-1}{x} - arctg\, x + c$

$\frac{A}{x^2} + \frac{B}{x^2+1} = \frac{Ax^2+A+Bx^2}{x^2(x^2+1)}$ $\quad \begin{cases} A+B=0 \\ A=1 \end{cases}$ $\quad \begin{cases} A=1 \\ B=-1 \end{cases}$