Карасева

$y^2 \le z \}$

сту чек

)

$= 1 \cdot / \cdot \lambda^2$

не верно

верно

III $L = x + 2y + 2z + \lambda(x^2 + y^2 - z)$

$$\begin{cases} L'_x = 1 + 2\lambda x = 0 & x = -1/4 \\ L'_y = 2 + 2\lambda y = 0 & y = -1/2 \\ L'_z = 2 - \lambda = 0 & \lambda = 2 \\ x^2 + y^2 = z & \frac{1}{16} + \frac{1}{4} = z = \frac{5}{16} \end{cases}$$

Проверим $x^2 + y^2 + 2z^2 \le 1$

$\frac{1}{16} + \frac{1}{4} + \frac{50}{256} \vee 1$

$\frac{16 + 64 + 50}{256} \vee 1$ $\frac{130}{256} \le 1$ - верно

$u(x, y, z) = \frac{-1}{4} + \frac{-2}{2} + \frac{2 \cdot 5}{16} = \frac{-1}{4} - 1 + \frac{5}{8} = \frac{-2 - 8 + 5}{8} = \frac{-5}{8}$

IV $L = x + 2y + 2z + \lambda_1(x^2 + y^2 + 2z^2 - 1) + \lambda_2(x^2 + y^2 - z)$

$$\begin{cases} L'_x = 1 + 2\lambda_1 x + 2\lambda_2 x = 0 & 1 + 2\lambda_1 x + (4\lambda_1 + 4)x = 0 \\ L'_y = 2 + 2\lambda_1 y + 2\lambda_2 y = 0 & 2 + 2\lambda_1 y + (4\lambda_1 + 4)y = 0 \\ L'_z = 2 + 4\lambda_1 z - \lambda_2 = 0 & \lambda_2 = 2\lambda_1 + 2 \\ \left.\begin{array}{l} x^2 + y^2 + 2z^2 = 1 \\ x^2 + y^2 = z \end{array}\right] & 2z^2 = 1 - z \quad 2z^2 + z - 1 = 0 \end{cases}$$

$D = 1 + 4 \cdot 2 = 9$

$z_{1,2} = \frac{-1 \pm 3}{4}$ $z_1 = -1$ не подх.

$z_2 = 1/2$

Отсюда $z = -1$ - не подходит.

$z = 1/2$

$x = \frac{-1}{2(3\lambda_1 + 2)}$ $y = \frac{-1}{(3\lambda_1 + 2)}$ $\frac{1}{4(3\lambda_1 + 2)^2} + \frac{1}{(3\lambda_1 + 2)^2} = \frac{1}{2}$ $(4(3\lambda_1+2)^2)$

$$u+1=2(3\lambda_1+2)^2$$
$$3\lambda_1+2=\pm\sqrt{\tfrac{5}{2}}$$

$$x=\frac{-1}{2(\pm\sqrt{\frac{5}{2}})} \qquad y=\frac{-1}{\pm\sqrt{\frac{5}{2}}}$$

$$u\left(\frac{-1}{2\sqrt{\frac{5}{2}}},\ \frac{-1}{\sqrt{\frac{5}{2}}},\ \frac{1}{2}\right)=\frac{-1-4+2\sqrt{\frac{5}{2}}}{2\sqrt{\frac{5}{2}}}$$

$$u\left(\frac{1}{2\sqrt{5/2}},\ \frac{1}{\sqrt{5/2}},\ \frac{1}{2}\right)=\frac{1+4+2\sqrt{5/2}}{2\sqrt{5/2}}$$

Итого точки: $\frac{-5+2\sqrt{5/2}}{2\sqrt{5/2}}$ $\frac{5+2\sqrt{5/2}}{2\sqrt{5/2}}$

$\frac{-5}{8}$ ← это наименьшее это наибольш.

$\boxed{N2}$

$$\begin{cases} x^2+tg(uy)=\ln(u+v) \\ x+y+u=2\ln v+1 \end{cases} \qquad u(0,1)=0 \quad v(0,1)=1$$

$$\Phi(xyuv)=\begin{pmatrix} x^2+tg(uy)-\ln(u+v) \\ x+y+u-2\ln v-1 \end{pmatrix}$$

$$\left(\begin{array}{cc|ccc} 2x & \frac{u}{\cos^2(uy)} & \frac{y}{\cos^2 uy}-\frac{1}{u+v} & & \frac{-1}{u+v} \\ 1 & 1 & 1 & & \frac{-2}{v} \end{array}\right)$$

$$\frac{y}{\cos^2 uy} - \frac{1}{u+v} = \frac{1}{\cos^2 0} - \frac{1}{1} = 0$$

$$\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = 0 - (-1)1 = 1 \neq 0$$

Отлично!

Сейчас подставим точки:

$0^2 + tg(0 \cdot 1) = \ln 1$ – верно

$0 + 1 + 0 = 2\ln 1 + 1$ – верно

Теперь разложим в Тейлора:

Первые по x:

$$\begin{cases} 2x + \dfrac{y u'_x}{\cos^2(yu)} = \dfrac{u'_x + v'_x}{u+v} \\ 1 + u'_x = \dfrac{2v'_x}{v} \end{cases} \qquad \begin{cases} 0 + u'_x = u'_x + v'_x \\ 1 + u'_x = 2v'_x \end{cases}$$

$u'_x = -1 \quad v'_x = 0$

по y:

$$\begin{cases} \dfrac{u'_y \cdot y + u}{\cos^2(uy)} = \dfrac{u'_y + v'_y}{u+v} \\ 1 + u'_y = \dfrac{2v'_y}{v} \end{cases} \qquad \begin{cases} u'_y = u'_y + v'_y \\ 1 + u'_y = 2v'_y \end{cases} \qquad \begin{matrix} v'_y = 0 \\ u'_y = -1 \end{matrix}$$

по xx:

$$\begin{cases} 2 + \dfrac{y u''_{xx} \cos^2(yu) - 2\cos(yu) \cdot \sin(yu) \cdot (y u'_x)^2}{\cos^4(yu)} = \dfrac{(u''_{xx} + v''_{xx})(u+v) - (u'_x + v'_x)^2}{(u+v)^2} \\ 0 + u''_{xx} = \dfrac{2v''_{xx} v - 2(v'_x)^2}{v^2} \end{cases}$$

$$\begin{cases} 2 + u''_{xx} - 0 = u''_{xx} + v''_{xx} - 1 \\ u''_{xx} = 2v''_{xx} \end{cases} \qquad \begin{cases} v''_{xx} = 3 \\ u''_{xx} = 6 \end{cases}$$

по yy:

$$\frac{(u''_{yy}\cdot y + u'_y + u'_y)\cos^2(yu) - 2\cos(yu)\cdot\sin(yu)\cdot(u'_y\cdot y+u)^2}{\cos^4(yu)} =$$

$$= \frac{(u''_{yy}+v''_{yy})(u+v) - (u'_y+v'_y)^2}{(u+v)^2}$$

$$u''_{yy} = \frac{2v''_{yy}\cdot v - 2(v'_y)^2}{v^2}$$

$$\frac{(u''_{yy}-1-1)-0}{1} = \frac{u''_{yy}+v'_{yy}-1}{1}$$

$u'_{yy} - 2 = u''_{yy} + v''_{yy} - 1 \qquad v''_{yy} = -1$

$u''_{yy} = -2$

по xy:

$$\frac{(u''_{xy}\cdot y + u'_x)\cdot\cos^2(yu) - 2\cos(yu)\cdot\sin(yu)(u'_y y + u'_y)yu'_x}{\cos^4(yu)} =$$

$$= \frac{(u''_{xy}+v''_{xy})(u+v) - (u'_y+v'_y)(u'_x+v'_x)}{(u+v)^2}$$

$$u''_{xy} = \frac{2v''_{xy}v - 2v'_y\cdot v'_x}{v^2}$$

$$\begin{cases} u'_{xy} - 1 \quad 0 = u''_{xy} + v'_{xy} - 1 \\ u''_{xy} = 2v'_{xy} \end{cases} \qquad \begin{cases} v'_{xy} = 0 \\ u'_{xy} = 0 \end{cases}$$

$v(x,y) = 0dx + 0dy + \frac{1}{2!}(3dx^2 + 0dxdy - 1dy^2)$

$= \frac{3}{2}dx^2 - \frac{1}{2}dy^2$

N5 $x^2 + 4y^2 - 32\ln(xy)$

$\begin{cases} 2x - \frac{32y}{xy} = 0 \\ 8y - \frac{32}{y} = 0 \end{cases}$ $\begin{cases} x^2 = 16 \\ y^2 = 4 \end{cases}$ $\begin{cases} x = \pm 4 \\ y = \pm 2 \end{cases}$

$\frac{d^2f}{dx^2} = 2 + \frac{32}{x^2}$, $\frac{d^2f}{dy^2} = 8 + \frac{32}{y^2}$

$\frac{d^2f}{dxdy} = 0$

$$\begin{pmatrix} 2 + \frac{32}{x^2} & 0 \\ 0 & 8 + \frac{32}{y^2} \end{pmatrix} = \left(\frac{32}{x^2} + 2\right)\left(\frac{32}{y^2} + 8\right) = \frac{1024}{x^2y^2} + \frac{64}{y^2} + \frac{8\cdot 32}{x^2} + 16$$

$(4,2)$: $\frac{2^{10}}{2^4 \cdot 2^2} + \frac{2^6}{2^2} + \frac{2^8}{2^4} + 16 > 0$ – Min.

$(-4,-2)$ – аналогично $(4,2)$

$(-4, 2)$, $(4, -2)$ — ф-ция недифференц.

$f((\pm 4, \pm 2)) = 32 - 32\ln 8$

$\boxed{N4}$
$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + m^2 z = 0$$

$2x = u^2 - v^2 \quad y = uv$

$x = \frac{u^2 - v^2}{2}$

$$\frac{\partial z}{\partial u} = z'_x \cdot x'_u + z'_y \cdot y'_u = z'_x \cdot u + z'_y \cdot v$$

$$\frac{\partial z}{\partial v} = -\frac{\partial z}{\partial x} \cdot v + \frac{\partial z}{\partial y} \cdot u$$

$$\frac{\partial^2 z}{\partial u^2} = \left(\frac{\partial^2 z}{\partial^2 x} u + \frac{\partial^2 z}{\partial x \partial y} v\right) u + \frac{\partial z}{\partial x} + \left(\frac{\partial^2 z}{\partial x \partial y} u + \frac{\partial z}{\partial y} v\right) \cdot v$$

$$\frac{\partial^2 z}{\partial v} = -\left(-\frac{\partial^2 z}{\partial x^2} v + \frac{\partial z}{\partial x \partial y} u\right) v - \frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial v^2} =$$

$$= (u^2 + v^2) \cdot \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

$\partial^2 z + \partial^2$