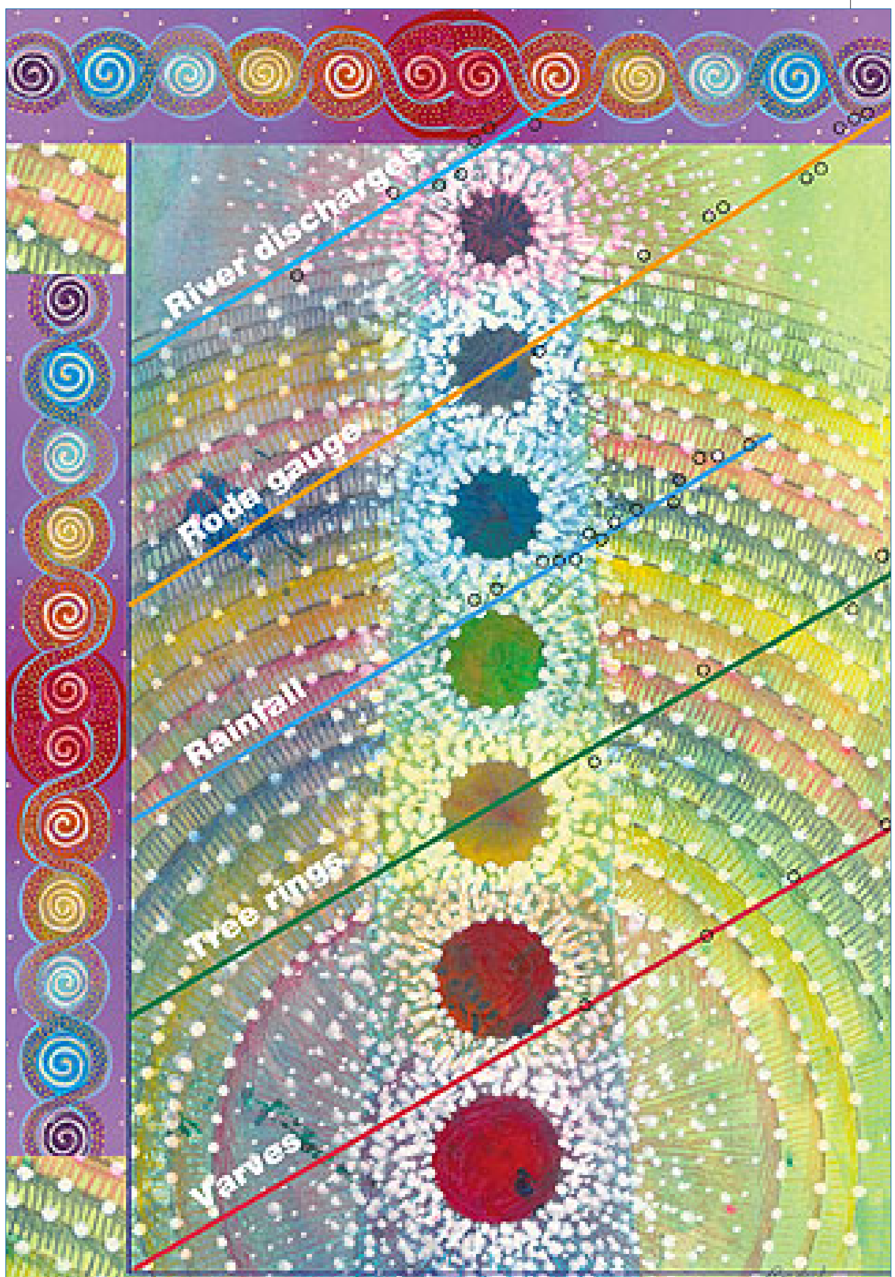
Welcome To The World Of Chaos

Making Sense Of Fractals

What is chaos? Here's a tool you can use to trade on the cutting edge of chaos and fractal theory.

by Erik Long



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Chaos is evident just about everywhere: in lightning, weather patterns, earthquakes, and financial markets, to name only a few. It may seem to appear in a random fashion, but it doesn't. In a nutshell, chaos is a nonlinear, dynamic system that appears to be random but actually is a higher form of order.

When you are studying the markets, *complexity* is the point at which the transition to chaos takes place. That should be the focus of your interest. Order and randomness exist simultaneously, and this gives us a degree of predictability. Social and natural systems, including private, governmental, and financial institutions, all fall within this category. People design these complex systems only to find those systems take on a life of their own. Each of these networks is sustained by complex feedback loops that reenter the system at unpredictable points in their cycles. This little-understood abstraction causes people to go about their business bewildered by the very systems they helped create.

Chaos is the realm of the nonlinear, and therefore is important to traders. Because financial markets are chaotic, it is necessary to use nonlinear tools to forecast market dynamics — tools that identify the hidden order in the apparently random process of the markets. But what underlies a nonlinear, chaotic system?

THE FRACTAL DIMENSION

Almost all chaotic systems have a quantifying measurement known as a *fractal dimension*. A fractal is an object in which individual parts are similar to the whole. The fractal dimension is a noninteger dimension that describes how an object takes up space. Objects in our space (and the systems that create them) are infinitely complex. If you examine any object with a microscope, more detail is revealed as the scale changes. In addition to levels of detail, most objects in nature demonstrate self-similarity, the organizing principle of fractals. Because of this, fractals will maintain their dimensions regardless of the scale used.

Regularity within irregularity is important, since each time frame in a market will have a similar fractal pattern. This goes to show that markets are natural phenomena rather than mechanical processes, which means you must use principles applicable to nonlinear, natural systems. Fractal geometry is an effective method for forecasting price movement accurately.

FRACTAL DIMENSION INDEX

The *fractal dimension index* (FDI) is a tool that applies the principles of chaos and fractal theory. This specialized indicator identifies the fractal dimension of the market by using rescaled range analysis and an estimated Hurst exponent. It does so by using all available data on the time/price chart to determine the volatility or "trendiness" of a given market.

FDI is the same type of tool used by eminent fractal scholars Benoit Mandelbrot, H.E. Hurst, and Edgar Peters in their examinations of time series analysis. With FDI you can determine the persistence or antipersistence of any equity or commodity that you display in your graphing program. A persistent time series will result in a chart that is less jagged, subject to fewer reversals, and resembles a straight line. An antipersistent time series will result in a chart that is more jagged and prone to more reversals. Essentially, FDI will tell you whether a market is a random, independent system or one with bias.

FRACTAL HISTORY

The history of the FDI begins with British dam builder and hydrologist H.E. Hurst (1900–78). He worked on the Nile River Dam Project in the early 20th century. Hurst searched for patterns in the Nile Delta as an effort to solve a hydrological problem.

The problem involved the storage capacity of the dam reservoir. This is important to hydrologists because if the dam is too high, resources are wasted. If the dam is too low, water will overflow. To solve this problem, Hurst considered the relationship between annual rainfall, the extremes of high/low water, and the reservoir's level.

Most hydrologists assumed that water inflow was a random process with no underlying order. Hurst came to a different conclusion after studying almost a millennium of Nile overflows. He found that large overflows tend to be followed by larger overflows. There appeared to be cycles, but their lengths were nonperiodic and standard statistical analysis revealed no patterns between observations.

Hurst developed his own analytical method to explain the nonperiodic cycles. To identify a nonrandom process, he tested the Nile using Albert Einstein's work on Brownian motion, a widely accepted model for a random walk. Einstein found that the distance a random particle travels increases with the square root of time used to measure it. This is called the $T^{1/2}$ *rule,* and is commonly used in finance and economics.

Hurst divided the Nile data into segments and examined the logarithmic range and scale of each segment in comparison to the total number of segments. This process is called *rescaled range analysis*. The range is rescaled because it has a zero mean and is expressed in terms of local standard deviation.

The rescaled range value scales with an increase in the time increment by a power-law value equal to H, or the Hurst exponent. Using rescaled range analysis, Hurst showed that water overflows tended to repeat, meaning that the natural overflows were partially predictable.

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The famous mathematician Benoit Mandelbrot used the Hurst exponent to experiment with time series found in cotton prices. He developed a method to measure irregular natural objects and named the measurement the *fractal dimension.* Thus, the FDI is based on the work of Mandelbrot and Hurst.

TRADING WITH FDI

Computing the FDI is not terribly complex. It can be programmed into a variety of trading platforms, including TradeStation, MetaStock, and CQG. To begin, you must compute the Hurst exponent. This can be derived with the formula displayed in the sidebar, "The fractal dimension."

Once you have computed the Hurst exponent, you must derive the fractal dimension of the time series. This is easily accomplished using the formula D=2–H. I use "2" here because you are using two dimensions for this computation. The value of "D" is the fraction of a dimension between 1 and 2 that your price data represents. Logarithmic returns are used to compute the FDI. Because logarithmic returns sum to cumulative returns, most analysts agree this is most appropriate for financial analysis. Although you may substitute price data for logarithmic returns, this is not recommended.

When experimenting with FDI, remember that a Hurst estimate is used, which may produce distorted results if you do not include enough data. The question of exactly how much data is needed is speculative at best, and is still debated among chaoticians and analysts.

Another postulate is the length of time necessary for each data period. Some researchers believe that shorter time periods, such as daily data, are subject to more noise from random information. If this is the case, FDI is less accurate with finer slices of sequential data. The individual length of each period will come into play as noise is filtered.

The FDI is useful because it determines the amount of market volatility. The easiest way to use this indicator is to understand that a value of 1.5 suggests the market is acting in a completely random fashion. As the market deviates from 1.5, the opportunity for earning profits is increased in proportion to the amount of deviation.

The entire scale is based on a range of 1 to 2, suggesting extreme linearity to extreme volatility. One example of this scale is its use in geography. If you examine an island and plot the fractal dimension, you will be able to determine how

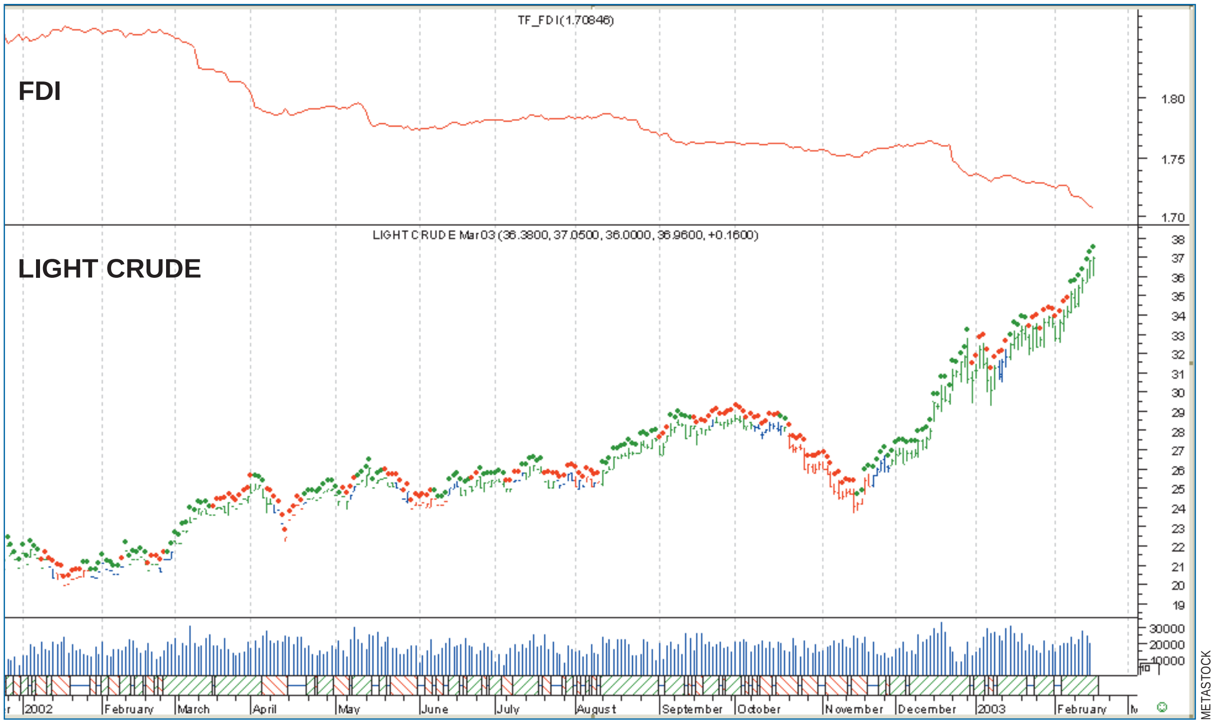
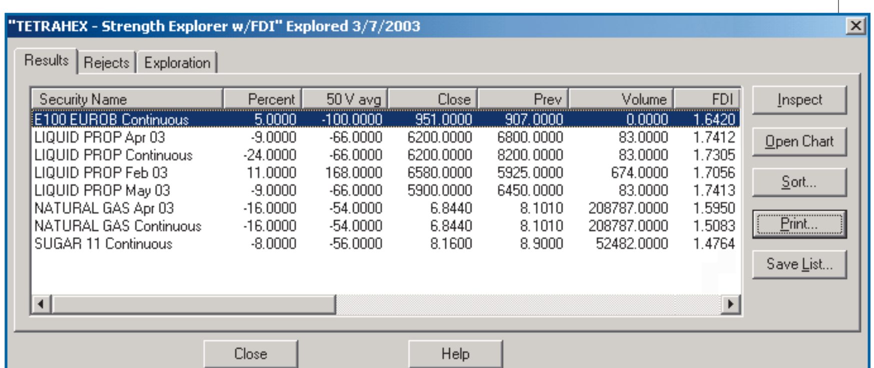


FIGURE 1: THE FRACTAL DIMENSION INDEX (FDI). In this daily chart of March 2003 crude oil contracts, notice the FDI beginning to decline as the market starts to trend.



"TETRAHEX - Strength Explorer w/FDI" Explored 3/7/2003

Security Name	Percent	50 V avg	Close	Prev	Volume	FDI
E100 EUROB Continuous	5.0000	-100.0000	951.0000	907.0000	0.0000	1.6420
LIQUID PROP Apr 03	-9.0000	-66.0000	6200.0000	6800.0000	83.0000	1.7412
LIQUID PROP Continuous	-24.0000	-66.0000	6200.0000	8200.0000	83.0000	1.7305
LIQUID PROP Feb 03	11.0000	168.0000	6580.0000	5925.0000	674.0000	1.7056
LIQUID PROP May 03	-9.0000	-66.0000	5900.0000	6450.0000	83.0000	1.7413
NATURAL GAS Apr 03	-16.0000	-54.0000	6.8440	8.1010	208787.0000	1.5950
NATURAL GAS Continuous	-16.0000	-54.0000	6.8440	8.1010	208787.0000	1.5083
SUGAR 11 Continuous	-8.0000	-56.0000	8.1600	8.9000	52482.0000	1.4764

FIGURE 2: IDENTIFYING NONRANDOM MARKETS. Notice the FDI suggests that liquid propane is far removed from a random market at 1.7056 FDI. You could consider this security for trading opportunities.

jagged the edges of the island are for a particular measurement scale. An island with a 1.7 fractal dimension is highly jagged, with many peaks and troughs on the periphery. An island with a fractal dimension of 1.3 is much more linear, approaching a single dimension or a straight line. If you examine this island on a map, the coastline will be straighter.

Applying FDI to the market is similar to studying an island on a map. The price plot is analogous to the periphery of the island. The FDI indicator thus determines how close the price plot is to two dimensions (a plane) or one dimension (a line).

Because the price plot will never be one extreme or the other, you need to measure the "fraction" of the dimension. That is why the FDI number is a fractal dimension. The further away this dimension is from 1.5, the more confident you can be that the market is not random. When a market is less random, it is more predictable. In the case of the FDI, a fractal dimension closer to 2 may provide substantial opportunities because of the high volatility and changes in market movement. An FDI closer to 1 signals a trending market that is moving in one direction.

An example of the FDI is displayed in Figure 1. As the market begins to trend, the FDI begins to decrease. Because a higher FDI identifies more volatile markets, all markets will peak in their fractal dimension before a new trend begins. You can use the FDI as a filtering tool to identify markets that are not acting randomly (Figure 2). You can then investigate these markets further to look for trading opportunities.

IN SUMMARY

The markets' apparent randomness makes them unpredictable and difficult to trade. The FDI is a great tool that gives you the ability to trade on the principles of nonlinearity. It helps you select markets that display nonrandomness, allowing you to take advantage of profitable trading opportunities.

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THE FRACTAL DIMENSION

$$X_t,N = \sum_{u=1}^{i} (E_u - M_N)$$

where X_t,N = cumulative deviation over N periods
E_u = influx in year u
M_N = average E_u over N periods

The range is the difference between the maximum and minimum levels.

$$R = Max(X_t,n) – Min(X_t,n)$$

where R = range of X
Max(X) = maximum value of X
Min(X) = minimum value of X

In order to compare different types of time series, H.E. Hurst divided this range by the standard deviation of the original observations. This "rescaled range" should increase with time. Hurst formulated the following relationship:

$$R/S = (a*N)^H$$

where R/S = rescaled range
N = number of observations
a = a constant
H = Hurst exponent

H should equal 0.5 if the series is a random walk. In other words, the range of cumulative deviations should increase with the square root of time. A deviation from 0.5 proves that each observation carries a memory of all the events that precede it.

You can estimate the Hurst exponent by:

$$Log(R/S) = H * log(N) + log(a)$$

You can also estimate the value of H from a single R/S value by:

$$H = log(R/S)/log(n/2)$$

Where n = number of observations.

The equation assumes that variable a equals 0.50. The fractal dimension is simply:

$$D = 2 – H$$

For the markets, I use logarithmic returns instead of percentage changes in prices:

$$S_t = ln(P_t/P_{(t-1)})$$

where S_t = logarithmic return a time t
P_t = price at time t

The range used in R/S analysis is the cumulative deviation from the average, and logarithmic returns sum to cumulative returns, while percentage changes do not.

—E.L.

SUGGESTED READING

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†See Traders' Glossary for definition

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