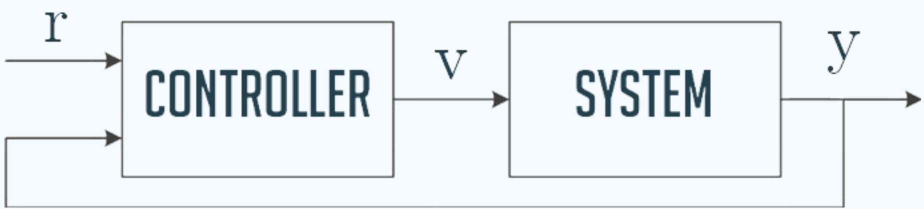
PID Neural Network

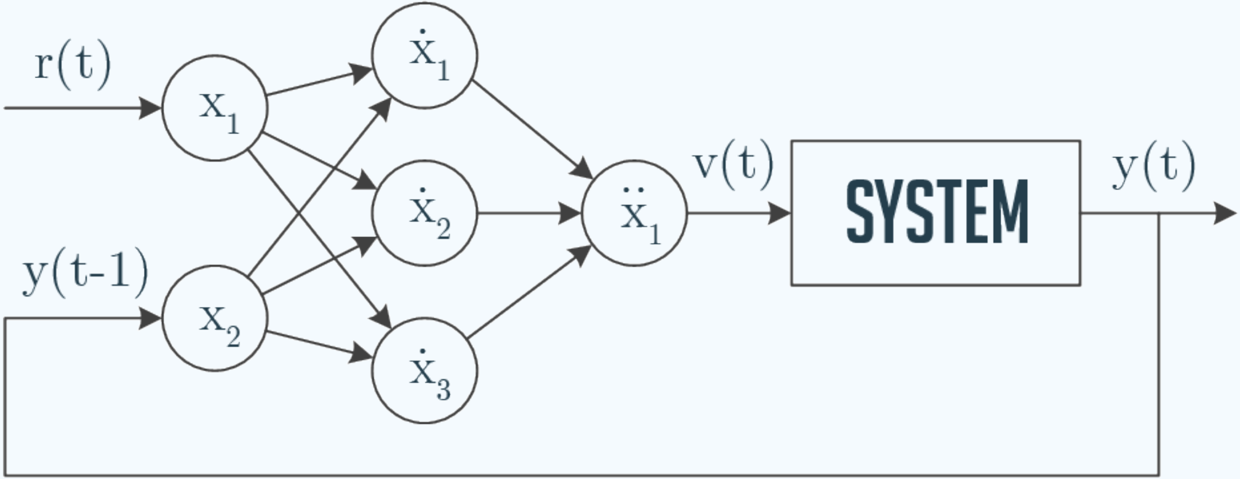
Classic Control-Feedback Problem

Controller takes target value $r(t)$ and feedback $y(t)$ and produces system input $v(t)$. System generates new output $y(t)$, which is used as a feedback.

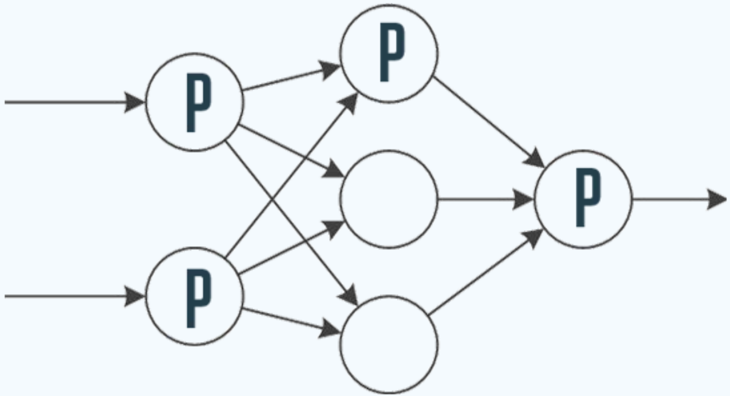


PID Neural Network Controller

Controller based on a neural network with 2-3-1 structure



P-neuron

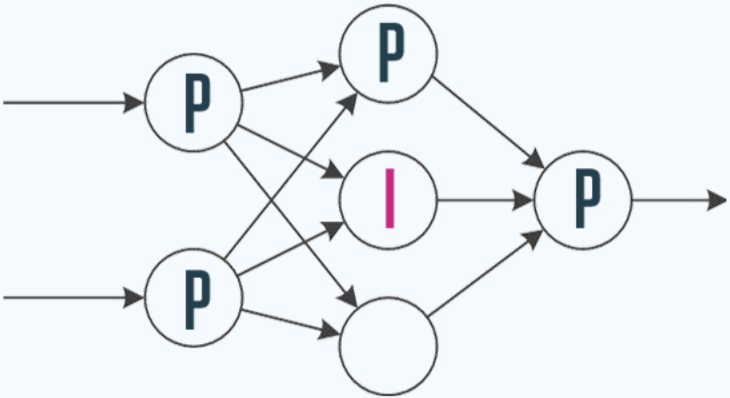


P-neuron transfer function:

$$x(t) = \begin{cases} -1 & u(t) < -1 \\ u(t) & -1 \le u(t) \le 1 \\ 1 & u(t) > 1 \end{cases}$$

$u(t)$ — neuron input, $x(t)$ — neuron output

I-neuron

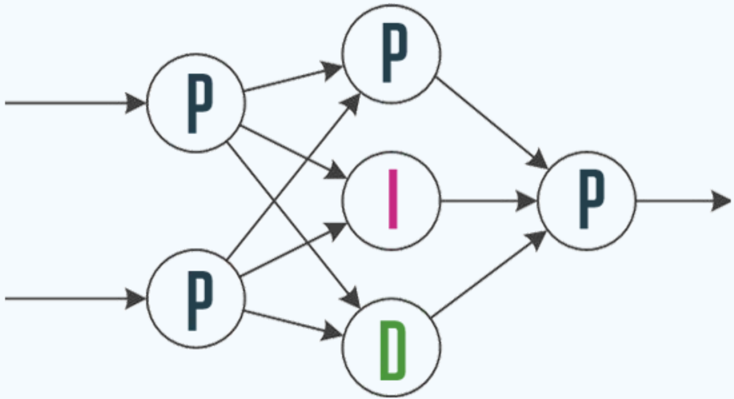


I-neuron transfer function:

$$x(t) = \begin{cases} -1 & x(t) < -1 \\ x(t-1) + u(t) & -1 \le x(t) \le 1 \\ 1 & x(t) > 1 \end{cases}$$

$x(t-1)$ — previous output

D-neuron



D-neuron transfer function:

$$x(t) = \begin{cases} -1 & x(t) < -1 \\ u(t) - u(t-1) & -1 \le x(t) \le 1 \\ 1 & x(t) > 1 \end{cases}$$

$u(t-1)$ — previous input

Learning Process

The aim of PIDNN controller is to minimize:

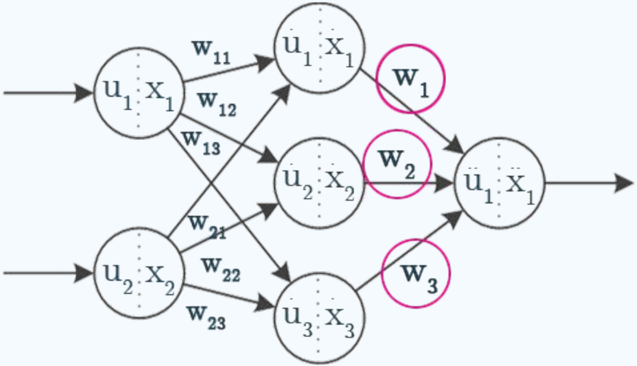
$$J = \sum_{t=1}^{N} E_t = \frac{1}{N} \sum_{t=1}^{N} (r(t) - y(t))^2$$

$r(t)$ — desired system output

$y(t)$ — real system output (feedback)

N — number of samples

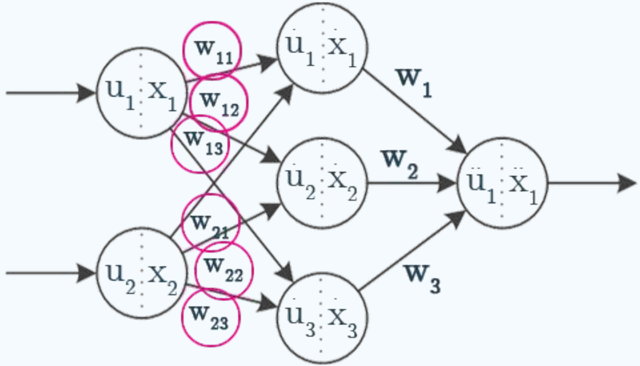
Error Propagation



$$w_i(n+1) = w_i(n) - \eta \frac{\partial J}{\partial w_i}$$

$$\frac{\partial J}{\partial w_i} \approx -\frac{2}{N} \sum_{t=1}^{N} (r(t) - y(t)) \frac{y(t) - y(t-1)}{v(t) - v(t-1)} \dot{x}_i(t)$$

Error Propagation



$$w_{ij}\,(n+1) = w_{ij}\,(n) - \eta \frac{\partial J}{\partial w_{ij}}$$

$$\frac{\partial J}{\partial w_i j} \approx -\sum_{t=1}^{N} \delta(t) \frac{\dot{x}_j\,(t) - \dot{x}_j\,(t-1)}{\dot{u}_j\,(t) - \dot{u}_j\,(t-1)} w_j\, x_i\,(t)$$

References

1. PID Neural Networks for Time-Delay Systems — H.L. Shu, Y. Pi (2000)
2. Decoupled Temperature Control System Based on PID Neural Network — H.L. Shu, Y. Pi (2005)
3. Adaptive System Control with PID Neural Networks — F. Shahrakia, M.A. Fanaeib, A.R. Arjomandzadeha (2009)
4. Control System Design (Chapter 6) — Karl Johan Åström (2002)