Introduction to Linear Algebra

Linear algebra is a field of study that focuses on understanding vectors, vector spaces, and mappings between these spaces. It plays a crucial role in solving systems of simultaneous equations and in fitting data using equations with adjustable parameters. A solid grasp of linear algebra is necessary to work with vectors effectively and perform calculus on them to find gradients and determine minima in data analysis. In the realm of data science, vectors are employed to describe objects, and they adhere to two fundamental rules: vector addition and multiplication by scalar numbers.

Vector Operations

Vectors support various operations:

- Addition
- Multiplication by scalar numbers
- Calculation of size (magnitude) of vectors
- Dot (inner) product
- Cosine rule and the geometric interpretation of the dot product
- Vector projection
- Scalar projection
- Basis vectors, which are used to express any vector in a space

A basis comprises n linearly independent vectors that span the space. When a vector space possesses a finite basis, it becomes a finite-dimensional vector space. Concepts like linear independence and linear combinations are central in linear algebra.

Matrices

Matrices form a mathematical framework for manipulating vectors within vector spaces. Columns in a matrix provide insights into the effects on unit vectors along each axis. Matrices facilitate various types of transformations, such as identity transformations, inversion, reflection, shearing, and rotation. Combining matrix transformations is associative, but not commutative. Understanding matrices' inverses and triangular structures is important.

Changing Bases and Orthogonal Matrices

Changing bases can be achieved using inverses or dot products. Orthogonal matrices are real square matrices with orthonormal rows and columns. Their transpose equals their inverse matrix. The Gram-Schmidt process converts a set of linearly independent vectors into an orthonormal set forming an orthonormal basis. Reflections on a plane and eigenproblems are also discussed, with a focus on eigenvalues and eigenvectors.

Special Eigen-Cases and Calculation of Eigenvectors

Eigenproblems involve minimizing the maximum eigenvalue of a matrix. Special cases include uniform scaling, pure rotation, vectors pointing in opposite directions, and combined shear and scaling. Calculating eigenvectors involves solving equations like $\det(A - \lambda I) = 0$. The change of eigenbasis involves the transformation matrix $T = CDC^{-1}$, resulting in a diagonal form.

PageRank and Other Concepts

The article concludes with a mention of the PageRank algorithm's significance in web analysis and search engine optimization. It touches upon the damping factor and its role in refining PageRank calculations. The article also introduces several additional concepts, including the span of vectors, grid lines, areas in relation to orientation-flipping, rank, cross product, dual vectors, determinants, eigenvectors, and axioms. These concepts are foundational in understanding linear algebra and its applications.