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Geometric Path Planning and Tracking Control with Bounded Steering Angle for the Parking Problem of Automatic Vehicles

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Abstract. In this paper, we deal with the problems of geometric collision-free path planning and feedback steering control for automatic perpendicular reverse parking maneuver of front wheel steering vehicles. An important condition to achieve effective collision-free parking is to determine a convenient start position of the vehicle depending on the widths of the parking aisle and the parking place, as well as, the dimensions and the steering capabilities of the car. Using Dubins strategy, geometric collision-free path planning based on admissible circular arcs within the available spot is proposed, in order to steer the vehicle moving backward with a given velocity profile and bounded steering angle in the direction of the parking place. A constrained feedback control is designed, which achieves quick steering in tiny spots and parks (practical stabilization) the vehicle in the parking place. The stability of the closed-loop system is analyzed using Lyapunov stability theory. Simulation results are presented to illustrate the effectiveness of the proposed approach.

INTRODUCTION

In recent years, the automatic parking assistance system is an important part of the autonomous vehicle capabilities and attracts considerable interest from research view point, as well, and from the automobile industry. Commercial products have been already available in the last decade. The perpendicular (garage) parking is very economical since more vehicles can be parked compared to parallel parking. However, the narrow operating space is one of the difficulties for collision-free maneuver of the vehicles during the parking. For this reason, the planning is an important component of the automatic parking system [1]. Different approaches have been proposed, such as a combination of an occupancy grid and optimal trajectory planning for collision avoidance [2], or a trajectory planning method based on forward path generation and backward tracking algorithm [3]. In [4], it has been proved that the shortest path connecting any two initial and terminal configurations of the vehicle is composed of not more than three connected segments, which are straight lines or circular arcs of minimum radius. Due to the nature of the nonholonomic constraints imposed on the generalized coordinates of the vehicle and limited space for the parking maneuver, the stabilization problem is challenging problem. Hybrid stabilizers for the parking problem has been proposed in [5, 6]. A trajectory tracking controller for the parking problem has been presentd in [7]. In [8], saturated feedback controllers have been reported. In this paper, a geometric collision-free path planning for reverse perpendicular parking in one trial based on the nonholonomic kinematic model of the vehicle is presented. Two feedback steering controllers – bang-bang controller and saturated controller are proposed, analyzed and evaluated under simulation.

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GEOMETRIC COLLISION-FREE PATH PLANNING

Vehicle Kinematic Model

In the two-dimensional plane, a plan view of the reverse perpendicular car parking considered in this paper is shown in Fig.1.

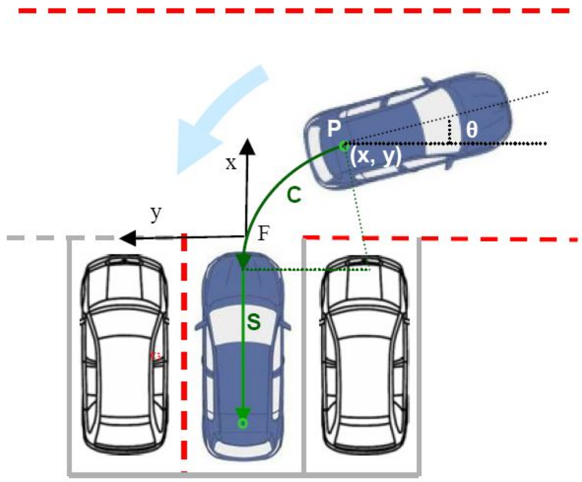


FIGURE 1. A top view of the parking vehicle and the shortest (Dubins) CR path

The slip angles of the wheels are considered to be equal to zero, which is a reasonable assumption for low speed motion during the parking maneuver. Referring to Fig. 1, the vehicle position is determined by the coordinates (x, y) of the mid-point P of the rear wheel axle assigned as a reference point with respect to an inertial frame Fxy. The orientation of the vehicle θ with respect to Fxy is denoted by θ. The kinematics of the car taking into account the nonholonomic constraints is described by the following system of nonlinear differential equations [9]

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{l}\tan\beta \end{aligned}, \tag{1}$$

where v is the velocity of point P and β is the front-wheel steering angle.

Geometric Considerations for Feasibility of the Reverse Perpendicular Parking in One Maneuver with a Constant Turning Radius

The Dubins car is often considered as an acceptable kinematic model for vehicle planning. Without considering obstacles, the shortest path for any fixed initial (x_0, y_0, θ_0) and final (x_f, y_f, θ_f) position has been obtained geometrically in [4]. It consists of a combination at most three sequentially connected parts, which are either arcs (C) or straight line (S) segments. In the case of perpendicular reverse parking scenario, the final position of the vehicle is fixed on the sentre line of the parking place (Fig. 1). If the straight line path (a straight line segment) is situated on right side of the vehicle (Fig. 1), the car will reach the desired final position and orientation if it takes a right turn with constant turning radius (a circular arc segment). Thus, for perpendicular parking, the shortest (Dubins) path is of the CS type.

In the presence of obstacles (Fig. 2), the vehicle moves initially backward with a constant steering angle β_c ($|\beta_c| \leq \beta_{max}$) from an initial position and orientation (x_0, y_0, θ_0), and has to enter into the parking place without collision with the boundary e_1 of parking lot 1 (conflict point K_1), boundaries e_2 of parking lot 2 (conflict point K_3), and the left border of the parking place e_3 (conflict point K_2) until a parallel position with respect to the parking place is reached. After that, the vehicle continues to move on the centre line of the parking place until the final position (x_f, y_f, θ_f) is attained.

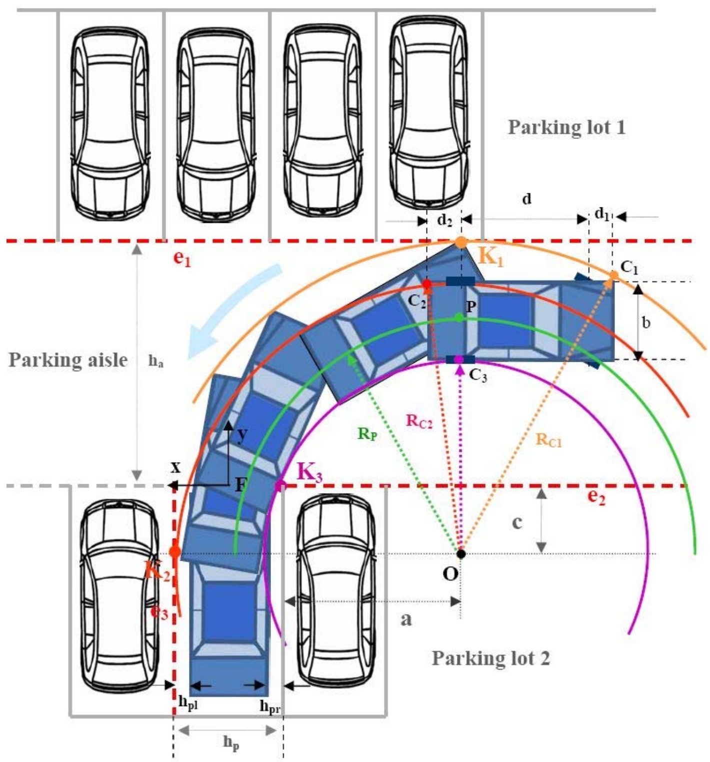


FIGURE 2. Geometry of a collision-free parking maneuver

In this paper, front-wheel steering vehicles are considered and a rectangular model of the vehicle is assumed The vehicle parameters which affect the parking maneuver, as well as the parameter values used in the simulations, are presented in Table I.

TABLE 1. Vehicle parameters

Vehicle Parameters	Notation	Value
Wheel Base	d	2.6m
Front Overhang	d_1	0.94m
Rear Overhang	d_2	0.74m
Overall Width	b	1.8m
Maximum Steering Angle	β_{max}	π/6rad

During a circular motion of the vehicle with constant steering angle β_c, the turn radius is calculated by the formula

$$R_P = \frac{d}{\tan \beta_c} \cdot \tag{2}$$

The boundaries of the turning path are determined by the left corner of the front bumper C_1 with radius R_{C1}, the left corner of the rear bumper C_2 with radius R_{C2}, and the right end of the rear wheel axle C_3, (Fig.2). The trajectories of points C_1, C_2, C_3, as well as of point P form arcs of concentric circles. The radii R_{C1}, and R_{C2} are determined by the expressions

$$R_{C_1} = OC_1 = \sqrt{(d + d_1)^2 + \left(R_P + \frac{b}{2}\right)^2} \tag{3}$$

$$R_{C_1} = OC_2 = \sqrt{d_2^2 + \left(R_P + \frac{b}{2}\right)^2}\,. \tag{4}$$

If point O denotes the vehicle centre of rotation for the first part of the reverse parking maneuver in one trial, without loss of generality, in this paper we consider that point O is situated in the Parking lot *2*, i.e., the distance c belongs to the following closed interval: $c \in \left[-\left(R_P - \frac{b}{2}\right), 0\right]$, as shown in Fig.2.

To avoid a collision between the left corner of the front bumper C_1 and the boundary e_1 of parking lot 1 (Fig. 2), the following expression for the width of the parking aisle h_a is derived

$$h_a = R_{C_1} - |c| = \sqrt{(d + d_1)^2 + \left(R_P + \frac{b}{2}\right)^2} - |c|\,. \tag{5}$$

To avoid a collision between the vertex K_3 of the parking place and the right side of the vehicle (point C_3 of the rear wheel axle), the following relationship must hold for the maximal distance a between point O and the right side of the parking place

$$a = \sqrt{\left(R_P - \frac{b}{2}\right)^2 - c^2}\,. \tag{6}$$

To avoid a collision between left side of the parking place e_3 and the left corner of rear vehicle bumper C_2, the following expression for the width of the parking place h_a is obtained

$$h_a = R_{C_2} - a = \sqrt{d_2^2 + \left(R_P + \frac{b}{2}\right)^2} - \sqrt{\left(R_P - \frac{b}{2}\right)^2 - c^2}\,. \tag{7}$$

From (3) and (5), it follows that

$$-|c|_{\max} = h_{as} - R_{C_1}\,, \tag{8}$$

where h_{as} is a specified in advance width of the parking place.

From (4) and (7), an expression for the minimum value of the of c is obtained

$$-|c|_{\min} = -\sqrt{\left(R_P - \frac{b}{2}\right)^2 - \left(R_{C_2} - h_{ps}\right)^2}\,, \tag{9}$$

where h_{ps} is specified in advance width of the parking place.

The expressions for the distances h_{pl} and h_{pr} between the vehicle and the parking place borders (Fig. 2) are given by the following expressions

$$h_{pr} = \left(R_P - \frac{b}{2}\right) - \sqrt{\left(R_P - \frac{b}{2}\right)^2 - c^2} \tag{10}$$

$$h_{pl} = h_{ps} - b - h_{pr}\,.$$

The minimum value of c for *symmetric* parking in the parking place is calculated as follows

$$-|c|_m = -\sqrt{\left(R_P - \frac{b}{2}\right)^2 - \left(R_P - \frac{h_{ps}}{2}\right)^2}\,. \tag{11}$$

Based on the expressions for $-|c|_m$ and $|c|_{max}$ given by (8) and (11), respectively, for symmetric reverse parking in one trial, it follows that the distance c has to take values in the closed interval: $-|c| \in \left[-|c_m|,\ -|c_{\max}|\right]$, where $-|c_m| > -|c_{\min}|$.

In conclusion, for symmetric parking with assigned steering angle $|\beta_c| \le |\beta_{\max}|$, the initial position of the vehicle can be on any one of the circle arcs of radius R_P with centre O (c, a), where c belongs to the aforementioned interval, $a = R_P$, and vehicle orientation is tangent to the arc at the initial position. In the particular case, when the initial vehicle position is perpendicular to the parking place, the coordinates of the reference point P have to be $(-|c|,\ -R_P)$, where $-|c| \in \left[-|c_m|,\ -|c_{\max}|\right]$. The shortest reference path consists of a 'CS' Dubins curve, where the 'C' segment is a circular arc, and the 'S' segment is a straight line along the x-coordinate of Fxy between the connecting point and the terminal position. The connecting point between the two segments has coordinates $\left(-|c|,\ \ 0\right)$.

BOUNDED FEEDBACK STEERING CONTROL

The procedure described in the previous Section, allows to determine the collision-free minimum length path (length optimal path) between the initial and final position of the vehicle for reverse perpendicular (garage) parking with constrained steering angle in one maneuver. The approach used in this paper for steering the vehicle from the initial position in the parking aisle to the terminal position into the parking place is based on a tracking strategy for controlling the vehicle motion along a straight line (the x-axis of an inertial frame Fxy attached to the parking place and passing through the terminal position), (Fig. 1). Two constraint feedback controllers are proposed for dealing with the problem of vehicle steering control.

Bang-Bang Steering Control

The bang-bang steering control presented in this paper was initially proposed for the forward parallel parking [5]. The design of the controller is based on a reduced-order system for (y, θ) in (1)

$$\begin{aligned} \dot{y} &= v \sin\theta \\ \dot{\theta} &= \frac{v}{l}\tan\beta \end{aligned}. \tag{12}$$

The control objective is to drive the errors y and θ to zero, i.e., $(y, \theta) \to 0$. Denoting

$$u_c = \frac{\tan\beta_c}{d}, \tag{13}$$

where $|\beta_c| = cte$ and $|\beta_c| \le \beta_{max}$, (TABLE 1), for a reverse perpendicular parking, when the velocity of the vehicle v is negative, the bang-bang controller takes the form

$$u_c = \begin{cases} u_c & if \quad y > 2\dfrac{d}{\tan\beta_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \\ & or \quad y = 2\dfrac{d}{\tan\beta_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \quad and \quad y > 0 \\ -u_c & if \quad y < 2\dfrac{d}{\tan\beta_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \\ & or \quad y = 2\dfrac{d}{\tan\beta_c}\sin\dfrac{\theta}{2}\left|\sin\dfrac{\theta}{2}\right| \quad and \quad y < 0 \end{cases}. \tag{14}$$

Steering Control with Saturation Constraints

Again, in the presence of control saturation constraints, the control objective is to drive the errors y and θ to zero. Supposing that $v = cte,$ the following differentiable steering control is proposed

$$\beta = a\tan\left[du_c \tanh(\lambda)\right], \tag{15}$$

where $\lambda = C(\theta - c_0 y)$, u_c is given by (13), and *C*, and c_0 are positive constants.

Applying (15) to the subsystem composed of the second and third equations of (1) given by (12), the resulting closed-loop takes the form

$$\begin{aligned} \dot{y} &= -|v|\sin\theta \\ \dot{\theta} &= -|v|u_c \tanh\lambda \end{aligned}. \tag{16}$$

The stability of system (16) is analyzed using Lyapunov techniques [10]. The system (16) has an equilibrium point at the origin. The following change of coordinates is introduced

$$\begin{aligned} y &= y \\ \eta &= \theta - c_0 y \end{aligned}. \tag{17}$$

The system (16) in the new coordinates can be written in the form

$$\begin{aligned} \dot{y} &= -v\sin(\eta + c_0 e) \\ \dot{\eta} &= -vh(C\eta) + c_0 v \sin(\eta + c_0 y) - C\eta \end{aligned}, \tag{18}$$

where $h(C\eta) = \lambda - u_c \tanh(C\eta)$.

The following positive-definite Lyapunov function candidate is considered

$$V = \frac{1}{2}\left(c_0^2 y^2 + \eta^2\right). \tag{19}$$

The derivative of *V* along the solutions of (18) is obtained in the form

$$\dot{V} = -c_0^3 vSy^2 + v\eta h(C\eta) + c_0 vS\eta^2 - Cv\eta^2, \tag{20}$$

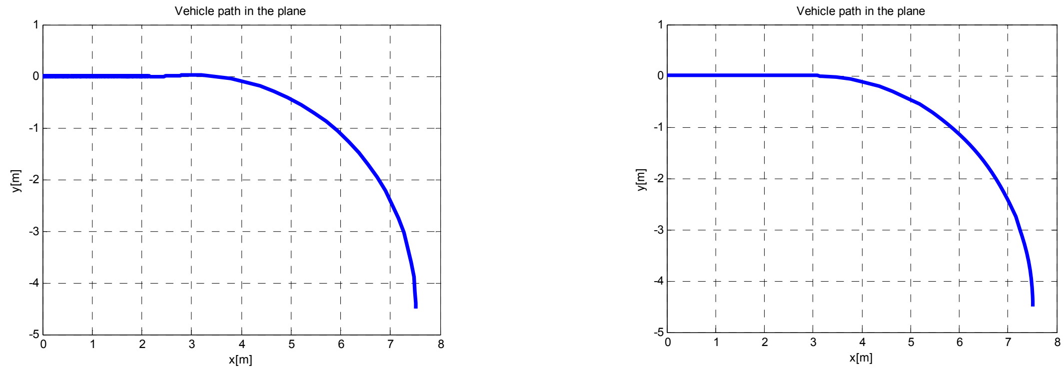
where $S = \dfrac{\sin(\eta + c_0 y)}{\eta + c_0 y}$. Based on the fact that $\forall \quad |u| \le u_c(1+\delta) \;\Rightarrow\; |h(C\eta)| \le \dfrac{\delta}{1+\delta}|u|$, $\delta = cte > 0$, it follows that

$$\dot{V} \le -c_0^3 vSy^2 - v\eta^2 \frac{C - c_0 S(1+\delta)}{1+\delta}. \tag{21}$$

Since $0 < S \le 1$, for $C \ge C^* = c_0(1+\delta)$, the derivative of *V* will be negative definite ($\dot{V} < 0$). Hence, local asymptotic stability of (18) is achieved using the proposed bounded control (15).

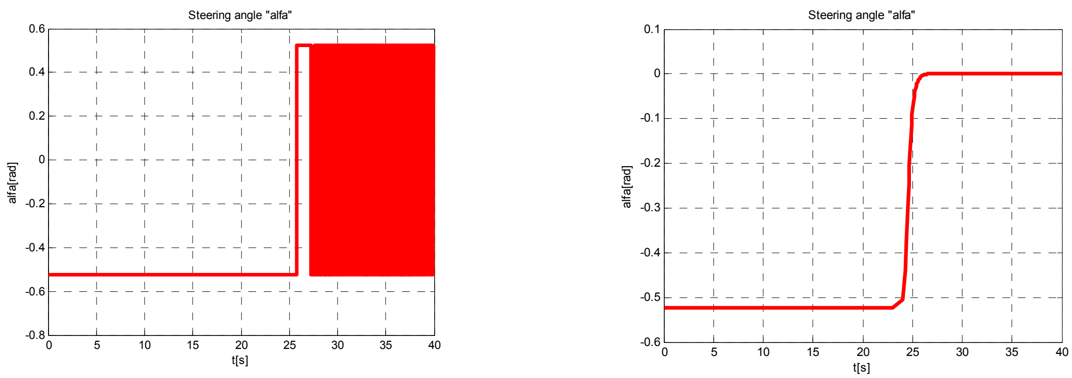
SIMULATION RESULTS

In order to evaluate the proposed geometric path planning and steering controls for reverse perpendicular parking in one trial, simulation results using MATLAB were conducted. The parameters of the test vehicle are listed in TABLE 1. The width of the parking place and parking aisle were assigned to be *2.4m* and *6m*, respectively. The initial position and orientation of the vehicle were (x_0 , y_0, θ_0) = (*3,5m, -4,5m, -π/2rad*). For the simulation, the coordinates of the vehicle with respect to the goal (the final position) are obtained to be (x_0, y_0) = (*7,5m, -4,5m*). The values of the bounded tanh-controller were $C = 5.85$ and $c_0 = 0.17$. Staring from the same initial position and orientation the planar path of the vehicle using bang-bang control and tanh-type (saturated) control are given in Fig. 3. As seen from Fig. 3, the trajectories are very similar. Evolution in time of the front wheel steering angle using bang-bang and tanh-type (saturated) controls are depicted in Fig. 4. It can be seen as expected the presence of chattering when using bang-bang control near the tracking line when the tracking error are small (Fig. 4(a)), and the smooth transition to zero of the steering angle when the lateral and orientation error become small near the tracking line when the tanh-type control is applied (Fig. 4(b)). The simulation results demonstrate the advantage of applying the tanh-type (saturated) control, which can be successfully used instead of the bang-bang control.



(a) (b)

FIGURE 3. Planar path of the vehicle: (a) bang-bang control; (b) saturated control



(a) (b)

FIGURE 4. Evolution in time of the front-wheel steering angle: (a) bang-bang control; (b) saturated control

The perpendicular reverse parking in one trial using saturated (tanh-type) control is shown in Fig.5.

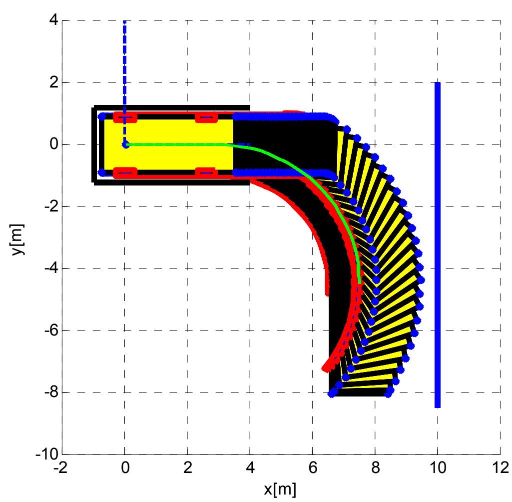


FIGURE 5. Reverse perpendicular parking in one maneuver using saturated control

CONCLUSION

In this paper, the problem of automatic perpendicular reverse parking of Ackermann steering vehicles has been considered. Based on Dubins strategy, geometric path planning for a feasible collision-free reverse perpendicular parking maneuver in one trial has been first presented. Relationships, connecting the widths of the parking place and parking aisle, as well as the vehicle dimensions and initial position have been determined, for achieving a collision-free maneuver. Using path tracking strategy, two types of constraint steering controllers – a bang-bang controller and saturated tanh-type controller have been proposed to solve the perpendicular parking problem. The stability properties have been analyzed using Lyapunov stability theory. The control performance has been demonstrated and evaluated by numerical simulation. It has been demonstrated that, the continuous saturated *tanh*-type controller has good control quality, and compared with the bang-bang controller avoids chattering. In addition, it also achieves quick steering and can be successfully used in solving parking problems in tinny spots.

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