Math 241 Sec 3

(T) **24.** Let

3×3 $\qquad$ 3×2 $\qquad$ $(AB)_{3\times 2}$

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 4 & 3 \\ 3 & 0 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 2 & 4 \end{bmatrix}.$$

Express the columns of AB as linear combinations of the columns of A.

Sol

$$AB = \begin{bmatrix} 1\times 1 - 2\times 3 - 1\times 2 & 1\times -1 - 2\times 2 - 1\times 4 \\ 2\times 1 + 4\times 3 + 3\times 2 & 2\times -1 + 4\times 2 + 3\times 4 \\ 3\times 1 + 0\times 3 - 2\times 2 & 3\times -1 + 0\times 2 - 2\times 4 \end{bmatrix}$$

$$Col_1(AB) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} * (1) + \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} * (3) + \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} * (2)$$

$$Col_2(AB) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} * (-1) + \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} * (2) + \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} * (4)$$

(T) 32. Write the following linear system in matrix form:

$$
\begin{aligned}
-2x_1 + 3x_2 &= 5 \\
x_1 - 5x_2 &= 4
\end{aligned}
$$

Sol

$$
\underbrace{\begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}}_{\text{Cefficent matrix}}
\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{unknow vector}}
=
\underbrace{\begin{bmatrix} 5 \\ 4 \end{bmatrix}}_{\text{rsult}}
$$

Cefficent matrix

مصفوفة المعاملات

unknow vector

المجاهيل

rsult

النواتج

T 33. Write the following linear system in matrix form:

$$
\begin{aligned}
2x_1 + 3x_2 &= 0 \\
3x_2 + x_3 &= 0 \\
2x_1 - x_2 &= 0
\end{aligned}
$$

Sol

$$
\begin{bmatrix} 2 & 3 & 0 \\ 0 & 3 & 1 \\ 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

is Homogenous system

is Consistant

trivial sol

$x_1 = x_2 = x_3 = 0$

infint Solution

36. Write each of the following linear systems as a linear combination of the columns of the coefficient matrix:

(a) $3x_1 + 2x_2 + x_3 = 4$
$x_1 - x_2 + 4x_3 = -2$

T (b) $-x_1 + x_2 = 3$
$2x_1 - x_2 = -2$
$3x_1 + x_2 = 1$

$$\begin{bmatrix} -1 & 1 \\ 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} x_2 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

(a) $3x_1 + 2x_2 + x_3 = 4$

$x_1 - x_2 + 4x_3 = -2$

5a

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ -1 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 4 \end{bmatrix} x_3 = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

37. Write each of the following linear combinations of columns as a linear system of the form in (4): matrix form

(a) $x_1\begin{bmatrix}2\\0\end{bmatrix}+x_2\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$

T **(b)** $x_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+x_2\begin{bmatrix}0\\1\\2\end{bmatrix}+x_3\begin{bmatrix}3\\4\\5\end{bmatrix}+x_4\begin{bmatrix}1\\3\\4\end{bmatrix}=\begin{bmatrix}2\\5\\8\end{bmatrix}$

Sol

b

$$\begin{bmatrix}1 & 0 & 3 & 1\\ 2 & 1 & 4 & 3\\ -1 & 2 & 5 & 4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}2\\5\\8\end{bmatrix}$$

(a) $x_1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Sol

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

T 3. Verify Theorem 1.2(a) for the following matrices:

$A(BC) = (AB)C$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}, \ 2 \times 3$$

$$\text{and} \quad C = \begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}. \ 3 \times 2$$

5a

$$BC = \begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}^{2\times 2}, \quad B = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}^{2\times 3},$$

$$\text{and} \quad C = \begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}. \ 3\times 2$$

$$AB = \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}$$

$$(AB)C = \overset{2\times 3}{\begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}} \overset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix}}$$

$$= \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix} = A(BC)$$

#

(T) 17. Verify Theorem 1.3(c) for $r = -3$, $\rightarrow r(A+B) = rA + rB$

$$A = \begin{bmatrix} 4 & 2 \\ 1 & -3 \\ 3 & 2 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 0 & 2 \\ 4 & 3 \\ -2 & 1 \end{bmatrix}.$$

$r = -3$

$$A+B = \begin{bmatrix} 4 & 4 \\ 5 & 0 \\ 1 & 3 \end{bmatrix}$$

$$r(A+B) = -3(A+B) = \begin{bmatrix} -12 & -12 \\ -15 & 0 \\ -3 & -9 \end{bmatrix}$$

$$-3A = \begin{bmatrix} -12 & -6 \\ -3 & 9 \\ -9 & -6 \end{bmatrix}, \quad -3B = \begin{bmatrix} 0 & -6 \\ -12 & -9 \\ 6 & -3 \end{bmatrix}$$

$$-3A + (-3B) = \begin{bmatrix} -12 & -12 \\ -15 & 0 \\ -3 & -9 \end{bmatrix}$$

$$= -3(A+B)$$

$$(A+B)^T = A^T + B^T$$

T 29. Verify Theorem 1.4(c) for $\longrightarrow (AB)^T = B^T A^T$

$$\underset{2\times 3}{A} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 1 & 2 \end{bmatrix}. \; 3\times 2$$

Sol

$$AB = \begin{bmatrix} 11 & 15 \\ 5 & -4 \end{bmatrix} \Rightarrow (AB)^T = \begin{bmatrix} 11 & 5 \\ 15 & -4 \end{bmatrix}$$

$$B^T A^T = \underset{2\times 3}{\begin{bmatrix} 3 & 2 & 1 \\ -1 & 4 & 2 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & -3 \end{bmatrix}} = \overset{2\times 2}{\begin{bmatrix} 11 & 5 \\ 15 & -4 \end{bmatrix}}$$

$$= (AB)^T$$

T 5. Verify Theorem 1.2(c) for the following matrices:

$C(A+B) = CA + CB$

$$A = \begin{bmatrix} 2 & -3 & 2 \\ 3 & -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & -2 \end{bmatrix},$$

$$\text{and} \quad C = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix}.$$

5a

$$A+B = \begin{bmatrix} 2 & -2 & 4 \\ 4 & 2 & -4 \end{bmatrix}$$

$$C(A+B) = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 & 4 \\ 4 & 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -8 & 16 \\ 10 & 14 & -28 \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -3 & 2 \\ 3 & -1 & -2 \end{bmatrix},$$

$$= \begin{bmatrix} -7 & 0 & 8 \\ 6 & 5 & -14 \end{bmatrix}$$

$$CB = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & -2 \end{bmatrix},$$

$$= \begin{bmatrix} -3 & -8 & 8 \\ 4 & 9 & -14 \end{bmatrix}$$

$$CA + CB = \begin{bmatrix} -10 & -8 & 16 \\ 10 & 14 & -28 \end{bmatrix}$$

T

34. Determine all 2×2 matrices A such that $AB = BA$ for any 2×2 matrix B.

Sol

$B \neq I$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$

$\boxed{AB = BA}$ $b = 0$, $c = 0$

a, d orbitery

$A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, $a, d \in \mathbb{R}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = BA$ $a, d \in \mathbb{R}$.

(T) 36. Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be solutions to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

(a) Show that $\mathbf{x}_1 + \mathbf{x}_2$ is a solution.

(b) Show that $\mathbf{x}_1 - \mathbf{x}_2$ is a solution.

Sol

let $AX = 0$ is Hom

x_1, x_2 is Solutn

$$Ax_1 = 0 \quad (1)$$
$$Ax_2 = 0 \quad (2)$$

(1) + (2)

$$Ax_1 + Ax_2 = 0$$

$$A(x_1 + x_2) = 0$$

$x_1 + x_2$ is Solutn for the Hom System $Ax = 0$

$$Ax_1 = 0 \quad (1)$$
$$Ax_2 = 0 \quad (2)$$

from (1) − (2)

$$Ax_1 - Ax_2 = 0$$
$$A(x_1 - x_2) = 0$$

$x_1 - x_2$ is solution for Hom $Ax = 0$