Math 241 Sec 2

the matrix

Condition to sum substract

A, B has same size

A_{mn}

m row

n column

2X3 3X2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix},$$

3X3 2X2

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix},$$

Sqvar matrix

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix},$$

3X3 2X2

$$C+E = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix}$$

6. If possible, compute the indicated linear combination:

(a) $C + E$ and $E + C$ T (b) $A + B$ not exist not same siz

T (c) $D - F$ (d) $-3C + 5O$

(e) $2C - 3E$ (f) $2B + F$

$$D - f = \begin{bmatrix} 3-(-4) & -2-5 \\ 2-2 & 4-3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -7 \\ 0 & 1 \end{bmatrix}$$

7. If possible, compute the indicated linear combination:

(a) $3D + 2F$ (b) $3(2A)$ and $6A$

T (c) $3A + 2A$ and $5A$

T (d) $2(D + F)$ and $2D + 2F$

~~(e)~~ $(2 + 3)D$ and $2D + 3D$

(f) $3(B + D)$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix},$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix},$$

Sol

(c)

$$3A = 3\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 6 & 3 & 12 \end{bmatrix}$$

$$2A = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$3A + 2A = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 20 \end{bmatrix}$$

(d)

$$D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}, \quad F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix},$$

$$(D + F) = \begin{bmatrix} -1 & 3 \\ 4 & 7 \end{bmatrix}$$

$$2(D + F) = \begin{bmatrix} -2 & 6 \\ 8 & 14 \end{bmatrix}$$

$$2D = \begin{bmatrix} 6 & -4 \\ 4 & 8 \end{bmatrix}$$

$$2F = \begin{bmatrix} -8 & 10 \\ 4 & 6 \end{bmatrix}$$

$$2D + 2F = \begin{bmatrix} -2 & 6 \\ 8 & 14 \end{bmatrix}$$

$$2(D + F) = 2D + 2F$$

$$c(A^T) = (cA)^T$$

$$(2A^T - 3B)^T$$

9. If possible, compute the following:

(a) $(2A)^T$ (b) $(A-B)^T$ not exist

(c) $(3B^T - 2A)^T$ (d) $(3A^T - 5B^T)^T$ not exist

(e) $(-A)^T$ and $-(A^T)$ (f) $(C + E + F^T)^T$ not exist

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$,

$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$,

$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$, $F = \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$,

So

(a)
$$2A = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$(2A)^T = \begin{bmatrix} 2 & 4 \\ 4 & 2 \\ 6 & 8 \end{bmatrix}$$

$$-A = \begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & -4 \end{bmatrix}$$

$$(-A)^T = \begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix}$$

(c)
$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}, \quad 2A = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}$$

$$3B^T = \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \end{bmatrix}$$

$$3B^T - 2A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 1 & -2 \end{bmatrix}$$

$$(3B^T - 2A)^T = \begin{bmatrix} 1 & -4 \\ 2 & 1 \\ 3 & -2 \end{bmatrix}$$

Multiplication of two matrix

$$A_{mn} \cdot B_{nk} = C_{mk}$$

to multiply two matrix
number of column of A = number of row of B

$$\boxed{AB \neq BA}$$

and $I_2 D$.

T **19.** Let

2X2 2X2

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}.$$

Show that $AB \neq BA$.

Sol

$$AB = \begin{bmatrix} 1\times 2 + 2\times -3 & 1\times -1 + 2\times 4 \\ 3\times 2 + 2\times -3 & 3\times -1 + 2\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 \\ 9 & 2 \end{bmatrix}$$

Consider the following matrices for Exercises 11 through 15

$$\underset{2\times 3}{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad \underset{3\times 2}{B} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix},$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad \text{and} \quad F = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}.$$

11. If possible, compute the following:

T **(a)** AB T **(b)** BA T **(c)** $F^T E$

(d) $CB + D$ T **(e)** $AB + D^2$, where $D^2 = DD$

$F^T_{2\times 3} \quad E_{3\times 3}$

$B_{3\times 2} \quad A_{2\times 3} = C_{3\times 3}$

$$AB = \begin{bmatrix} 14 & 8 \\ 16 & 9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 10 \\ 7 & 8 & 17 \end{bmatrix}$$

$$
\underset{}{\overset{3\times 3}{E}} = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad \textit{and} \quad \overset{3\times 2}{F} = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}
$$

(c) $F^T E$

See

$$
= \overset{2\times 3}{\begin{bmatrix} -1 & 0 & 3 \\ 2 & 4 & 5 \end{bmatrix}} \overset{3\times 3}{\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}} = 2\times 3
$$

$$
= \begin{bmatrix} 7 & 10 & -2 \\ 19 & 6 & 31 \end{bmatrix}
$$

$$D^n = D \cdot D^{n-1}$$

$$D^3 = D \cdot D^2$$

$$D^4 = D^2 \cdot D^2$$

(e) $AB + D^2$

$$D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix},$$

$$D^2 = D \cdot D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -16 \\ 16 & 21 \end{bmatrix}$$

<u>from (a)</u> $$AB = \begin{bmatrix} 4 & 8 \\ 16 & 9 \end{bmatrix}$$

$$AB + D^2 = \begin{bmatrix} 9 & -8 \\ 32 & 30 \end{bmatrix}$$

$Col_2(BA)$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 3 & -3 & 4 \\ 4 & 2 & 5 & 1 \end{bmatrix}.$$

T **21.** Using the method in Example 11, compute the following columns of AB:

$\longrightarrow col_j(AB) = A\, col_j(B)$

(a) the first column **(b)** the third column

Sol

(a) $$Col_1(AB) = \underset{4\times3}{\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}} = \underset{4\times1}{\begin{bmatrix} 8 \\ 25 \\ 10 \\ 25 \end{bmatrix}}$$

(b) $$Col_3(AB) = \underset{4\times3}{\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix}}$$

$$= \underset{4\times1}{\begin{bmatrix} 12 \\ 11 \\ 17 \\ 28 \end{bmatrix}}$$

$$\alpha_2(BA) = \underset{B_{3\times 4}}{\begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 3 & -3 & 4 \\ 4 & 2 & 5 & 1 \end{bmatrix}} \cdot \underset{4\times 1}{\begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix}}$$

$$= \underset{3\times 1}{\begin{bmatrix} 3 \\ 13 \\ -9 \end{bmatrix}}$$

$$E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}, \quad and \quad F = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$$

5a

$$F^T E = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 & -2 \\ 19 & 6 & 31 \end{bmatrix}$$