

$$T(n) = a T(n/b) + f(n)$$

1. if $f(n)$ is $O(n^{\log_b a - \epsilon})$, then $T(n)$ is $\Theta(n^{\log_b a})$
2. if $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$
3. if $f(n)$ is $\Omega(n^{\log_b a + \epsilon})$, then $T(n)$ is $\Theta(f(n))$,
provided $a f(n/b) \leq \delta f(n)$ for some $\delta < 1$.

a. $T(n) = 2T(n/2) + \log n$

$$\log_2 2 = 1$$

$$f(n) = \log n$$

Case 1: Is $f(n) = \log n = O(n^{1-e})$? True for $e = 1/2 > 0$

Therefore, $T(n) = \Theta(n)$

d. $T(n) = 7T(n/3) + n$

$$\log_3 7 = 1.xxxx$$

Case 1: Is $f(n) = n = O(n^{1.xxxx - e})$? True for $e \leq 0.xxxx$

Therefore, $T(n) = \Theta(n^{1.xxxx})$

If Dictionary D is binary search tree based, then the items are iterated in key-sorted order. Whereas, if D is hash table based, then the iteration is in a non-specified order. In the Java library, a LinkedHashMap iterates in the order the items were inserted into the Map.

What is the syntax for iterating through the items in a Dictionary D?

One way is using the for each syntax:

for each (k, e) in D.items() do

Another way is using an iterator and a while-loop as follows:

```
iter <- D.items()
```

```
while iter.hasNext() do
```

```
    (k, e) <- iter.nextObject()
```

Master Theorem:

$$T(n) = aT(n/b) + f(n)$$

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2. if $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$
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provided $af(n/b) \leq \delta f(n)$ for some $\delta < 1$.

b. $T(n) = 8T(n/2) + n^2$

$\log_2 8 = 3$, Is $f(n) = n^2 = O(n^{3-\epsilon})$, yes for $\epsilon = 1$ or less, so case 1 applies

Therefore, $T(n) = \Theta(n^3)$

c. $T(n) = 16T(n/2) + (n \log n)^4$

$\log_2 16 = 4$, so $n^{\log_2 16} = n^4$

Case 2: Is $f(n) = (n^4 \log^4 n) = \Theta(n^4 \log^k n)$ for some k ? Yes for $k = 4$.

Therefore, $\Theta(n^4 \log^5 n)$

e. $T(n) = 9T(n/3) + (n^3 \log n)$ Case 3 applies. Must solve for δ .