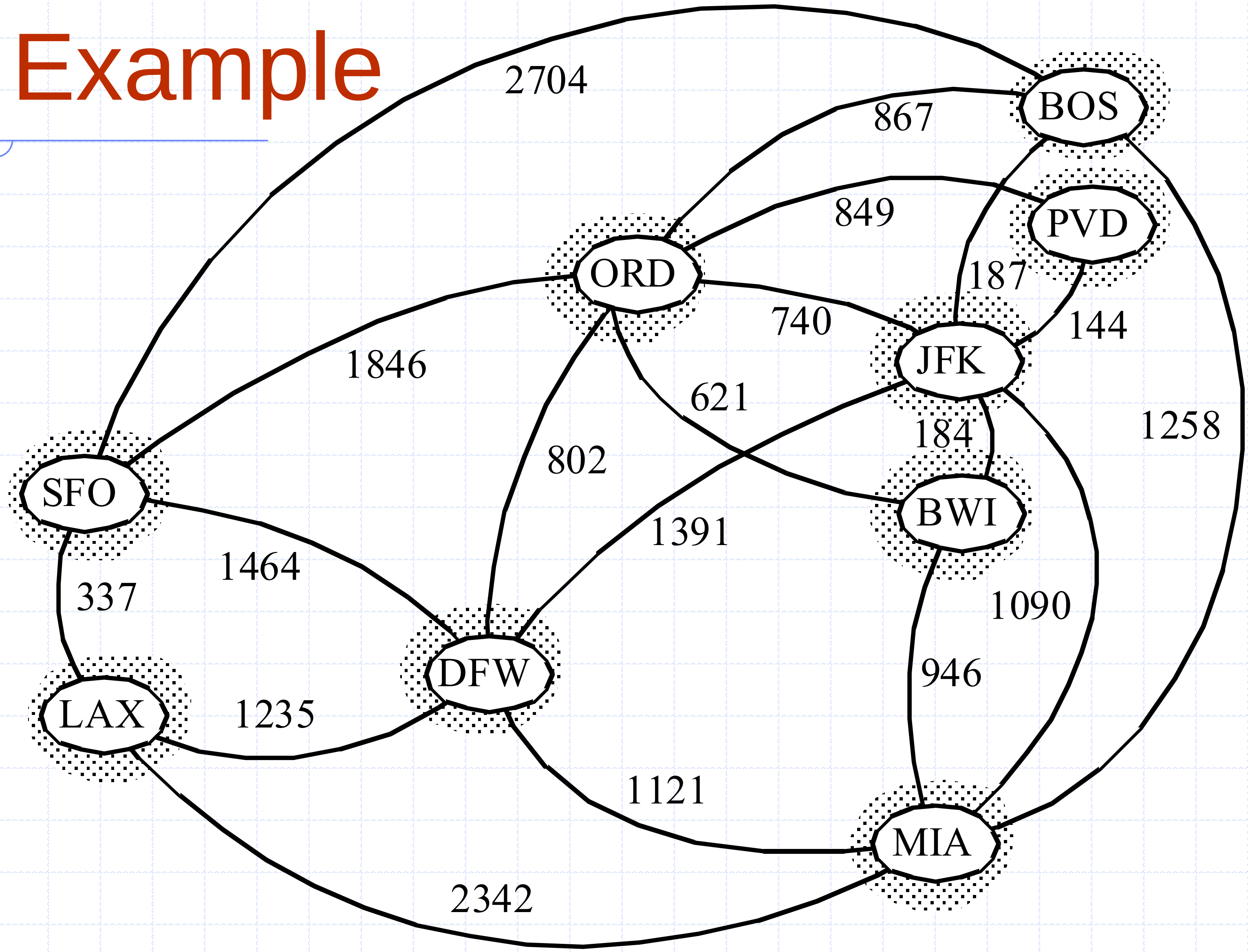
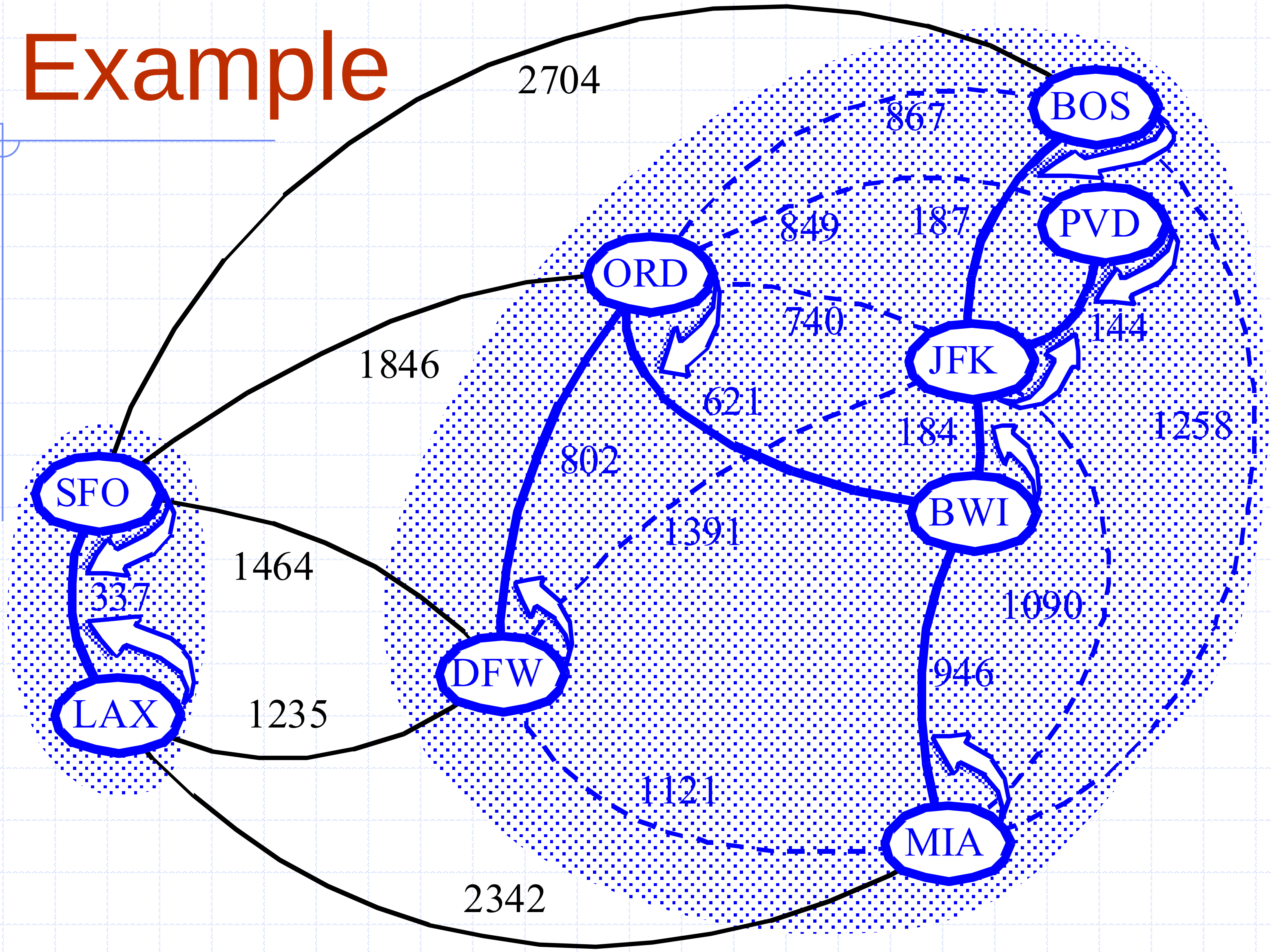
Baruvka's Algorithm (1926)

Template Method Solution <u>without</u> Making a Clone of G

Baruvka Example



Example



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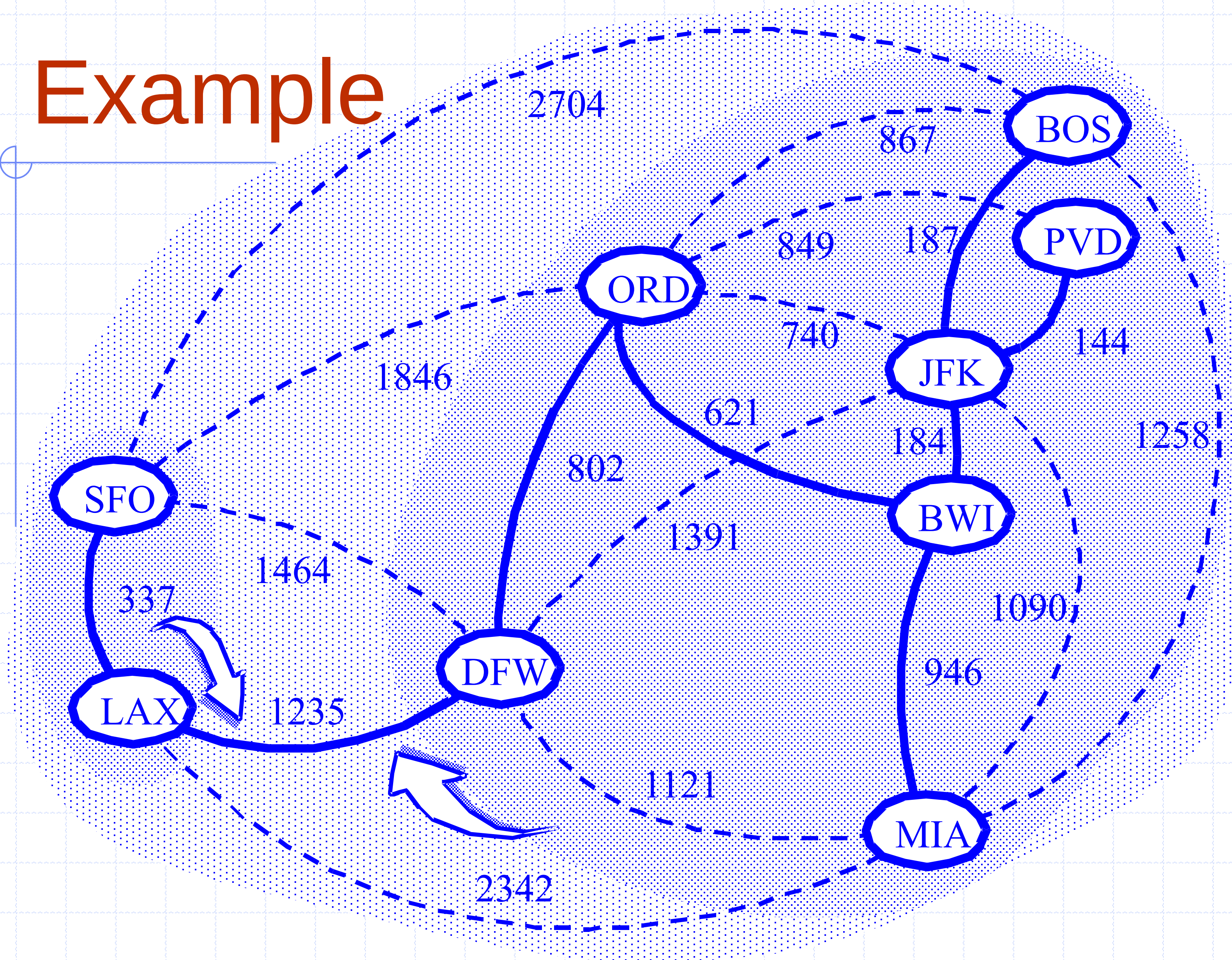
Example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Baruvka's Algorithm (from Lecture 14)

Algorithm ***BaruvkaMST***(***G***)
  for each e $\in$ ***G.edges()*** **do**
    setMSTLabel(e, ***NOT_IN_MST****)* {no edges in MST}
  numEdges $\leftarrow$ *0*   {numEdges is an instance variable}
  while *numEdges* < ***n***-1 **do**
    count $\leftarrow$ labelVerticesOfEachComponent(G) {BFS}
    insertSmallest-WeightEdges(G, count)
  return ***G***

Template Version of BFS

```
Algorithm BFS(G) {all components}
  Input graph G
  Output labeling of the edges of G as
    discovery edges and cross edges

    initResult(G)
    for all  u ∈ G.vertices() do
        setLabel(u, UNEXPLORED)
        postInitVertex(u)
    for all  e ∈ G.edges() do
        setLabel(e, UNEXPLORED)
        postInitEdge(e)
    for all  v ∈ G.vertices() do
        if  isNextComponent(G, v)
            preComponentVisit(G, v)
            BFScomponent(G, v)
            postComponentVisit(G, v)

    return result(G)

Algorithm isNextComponent(G, v)
    return getLabel(v) = UNEXPLORED
```

```
Algorithm BFScomponent(G, s) {1 component}
    setLabel(s, VISITED)
    Q ← new empty Queue
    Q.enqueue(s)
    startBFScomponent(G, s)
    while ¬Q.isEmpty()  do
        v ← Q.dequeue()
        preVertexVisit(G, v)
        for all  e ∈ G.incidentEdges(v)  do
            preEdgeVisit(G, v, e, w)
            if  getLabel(e) = UNEXPLORED
                w ← opposite(v, e)
                unexploredEdgeVisit(G, v, e, w)
                if getLabel(w) = UNEXPLORED
                    preDiscEdgeVisit(G, v, e, w)
                    setLabel(e, DISCOVERY)
                    setLabel(w, VISITED)
                    Q.enqueue(w)
                    postDiscEdgeVisit(G, v, e, w)
                else
                    setLabel(e, CROSS)
                    crossEdgeVisit(G, v, e, w)
        postVertexVisit(G, v)

    finishBFScomponent(G, s)
```

Label Vertices of Each Component (subclass methods of BFS template)

This algorithm runs in O(n+m) time. Why?

Algorithm **labelVerticesOfEachComponent(G)**
    return BFS(G)   {label vertices of G with component numbers}

Algorithm ***initResult***(***G***)
    count ← 0   {initialize count to zero}

Algorithm ***preEdgeVisit***(***G, v, e, w***)
    if getMSTLabel(e) = NOT_IN_MST **then**   {is e in the MST}
        setLabel(***e, SKIP***)   {so BFS only visits edges in MST}

Algorithm ***postComponentVisit***(***G, v***)
    count ← count + 1   {add one to count}

Algorithm ***preVertexVisit***(***G, v***)
    setComponentNum(***v,*** count)

Algorithm ***result***(***G***)
    return count   {return count which is the number of components}

Insert Minimum-Weight Edges Going Out from each Component

What is the running time?

```
Algorithm insertSmallest-WeightEdges(G, count)
    minEdges ← new array of size count  {could use hashtable based Dictionary}
    for i ← 0 to count - 1 do
        minEdges[i] ← ∅

    BFS(G)      {search for smallest edges connecting different components}
    for i ← 0 to count - 1 do
        e ← minEdges[i]
        if getMSTLabel(e) = NOT_IN_MST then       {is e already in MST}
            setMSTLabel(e, IN_MST)     {insert e into MST}
            numEdges ← numEdges + 1  {increase number of edges in MST}
```

{could be done by traversing G.edges() in a loop instead of during a BFS}

```
Algorithm preEdgeVisit(G, v , e , w)   {called during BFS(G) above}
    cv ← getComponentNum(v)
    cw ← getComponentNum(w)
    if cv ≠ cw then       {does e connect two different components of MST}
        if minEdges[cv] = ∅  then
            minEdges[cv] ← e       {insert new minimum}
        else
            min ← weight(minEdges[cv])    {current min weight for component cv}
            if min > weight(e)  then
                minEdges[cv] ← e         {insert new minimum}
```

An Issue Not Handled by the above algorithm

- Edges with the same weight
- Therefore,
  - Could insert more than n-1 edges
  - Or could create one or more cycles
- How could we fix this?

Insert Minimum-Weight Edges then remove cycles

What is the running time?

```
Algorithm insertSmallest-WeightEdges(G, count)
    minEdges ← new array of size count
    for i ← 0 to count - 1 do
        minEdges[i] ← ∅

    BFS(G)      {search for smallest edges connecting different components}
    for i ← 0 to count - 1 do
        e ← minEdges[i]
        if getMSTLabel(e) = NOT_IN_MST then      {is e already in MST}
            setMSTLabel(e, IN_MST)     {insert e into MST}
            numEdges ← numEdges + 1  {increase number of edges in MST}
    removeCycles(G)    {remove the edge with highest weight in each cycle}

Algorithm preEdgeVisit(G, v , e , w)   {called during BFS(G) above}
    cv ← getComponentNum(v)
    cw ← getComponentNum(w)
    if cv ¬= cw then       {does e connect two different components of MST}
        currentMinE ← minEdges[cv] {min edge for component cv}
        if currentMinE = ∅  then
            minEdges[cv] ← e       {insert new minimum}
        else
            min ← weight(currentMinE)    {current min weight for component cv}
            if min > weight(e)  then
                minEdges[cv] ← e        {insert new minimum}
```

DFS is better for Cycle Finding (Improved version)

```
Algorithm DFS(G) {top level}
    Input graph G
    Output the edges of G are labeled
           as discovery and back edges

  initResult(G)
  for all u ∈ G.vertices() do
      setLabel(u, UNEXPLORED)
      postInitVertex(u)
  for all e ∈ G.edges() do
      setLabel(e, UNEXPLORED)
      postInitEdge(e)
  for all v ∈ G.vertices() do
      if getLabel(v) = UNEXPLORED
          preComponentVisit(G, v)
          DFS(G, v)
          postComponentVisit(G, v)

  result(G)
```

```
Algorithm DFS(G, v)
  setLabel(v, VISITED)
  startVertexVisit(G, v)
  for all e ∈ G.incidentEdges(v)
      w ← opposite(v,e)
      preEdgeVisit(G, v, e, w)
      if getLabel(e) = UNEXPLORED
          if getLabel(w) = UNEXPLORED
              setLabel(e, DISCOVERY)
              preDiscoveryTraversal(G, v, e, w)
              DFS(G, w)
              postDiscoveryTraversal(G, v, e, w)
          else
              setLabel(e, BACK)
              backEdgeVisit(G, v, e, w)
      postEdgeVisit(G, v, e, w)

  finishVertexVisit(G, v)
```

Overriding template methods in subclass to Remove Cycles

```
Algorithm removeCycles(G)
        repeat
            DFS(G)
            if  cycleFound   then numEdges ← numEdges - 1  {decreased edges in MST}
        until ¬ cycleFound

Algorithm initResult(G)
   cycleFound ←  false
Algorithm postInitEdge(e)
   if  getMSTLabel(e) = NOT_IN_MST  then    setLabel(e, SKIP)
Algorithm startVertexVisit(G, v)
    if  ¬ cycleFound   then            S.push(v)
Algorithm finishVertexVisit(G, v)
    if  ¬ cycleFound   then            S.pop()
Algorithm preDiscoveryTraversal(G, v, e, w)
    if  ¬ cycleFound   then            S.push(e)
Algorithm postDiscoveryTraversal(G, v, e, w)
    if  ¬ cycleFound   then            S.pop()
Algorithm backEdgeTraversal(G, v, e, w)
    if  ¬ cycleFound   then
            max ← weight(e)
            maxE ← e
            u ← S.pop() {remove v from S}
            while u ¬ = w
                e ← S.pop()       {next edge}
                 if  weight(e) > max   then
                        max ← weight(e)
                        maxE ← e
                 u ← S.pop()       {next vertex}
            setMSTLabel(maxE, NOT_IN_MST)  {remove max weight edge}
            cycleFound ←  true   {cycleFound is a subclass field, initially set to false}
```

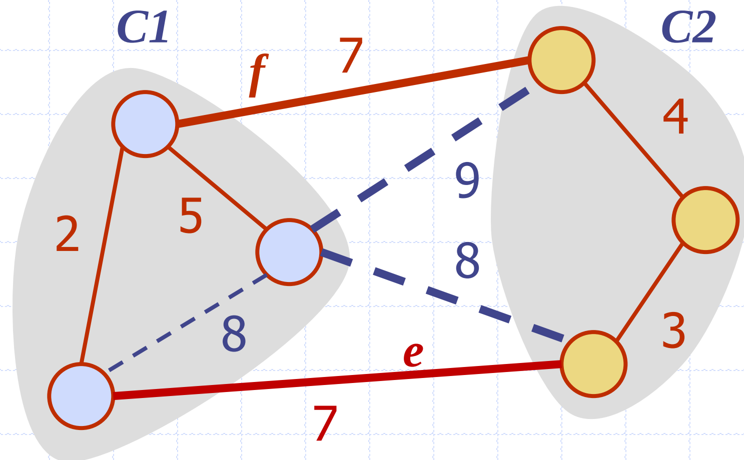
Another Possible Approach

Note that the problem occurs when:

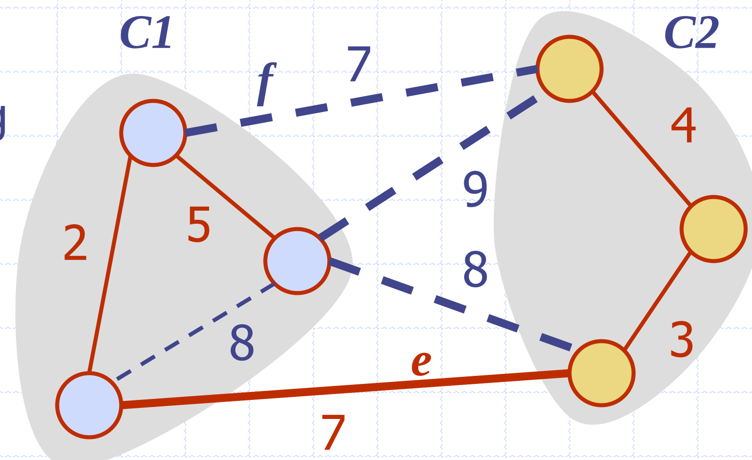
- There are two minimum, equal size edges connecting two components
- And each component chooses a different minimum edge
  - Say C1 chooses edge *f* and C2 chooses edge *e*
- Including *e* and *f* would create a cycle

Solution:

- Force both components C1 and C2 to choose the same edge connecting them
- Either edge *e* or *f* in the diagram



Choose either *f* or *e* for components C1 and C2



Baruvka's Algorithm (1926)

My Latest Version (version 6 at least, April & Aug. 2020)

Baruvka's Algorithm (another refinement, i.e., sorting edges)

Algorithm ***BaruvkaMST***(***G***)

  for each e $\in$ ***G.edges()*** **do**

    setMSTLabel(e, ***NOT_IN_MST****)* {no edges in MST}

  sortedEdges $\leftarrow$ MergeSort(G.edges()) {output is a list}

  components $\leftarrow$ *G.numVertices*()

  while *components* > 1 **do**

    components $\leftarrow$ labelVerticesOfEachComponent(G)

    if *components* > 1 **then**

      insertSmallestEdges(G, *components*, sortedEdges)

  return ***G***

Label Vertices of Each Component (subclass methods of BFS template)

Algorithm labelVerticesOfEachComponent(G)
    return BFS(G)   {label vertices of G with component numbers}

Algorithm ***initResult***(***G***)
    count ← 0   {initialize count to zero}

Algorithm ***preEdgeVisit***(***G, v, e, w***)
    if getMSTLabel(e) **=** NOT_IN_MST **then**   {is e in the MST}
        setLabel(***e, SKIP***)   {skips edges in not in MST}

Algorithm ***postComponentVisit***(***G, v***)
    count ← count + 1   {add one to count}

Algorithm ***preVertexVisit***(***G, v***)
    setComponentNum(***v,*** count)

Algorithm ***result***(***G***)
    return count   {return count which is the number of components}

Three refinements Apr., June, & Aug. 2020

```
Algorithm insertSmallestEdges
            (G, components, sortedEdges)
    minEdges ← new array of size components
    for i ← 0 to components - 1 do
        minEdges[i] ← ∅
    i ← 0
    p ← sortedEdges.first()
    while  i < components  do
       (c, isMax) ← insertEdge(G, minEdges,p)
       q ← p
       if  ! sortedEdges.isLast(p)  then
            p ← sortedEdges.after(p) {next edge}
 {Was edge added to MST or is it a max edge in
    a cycle? If yes, then no longer needed}
        if  c > 0 ∨ isMax then
             sortedEdges.remove(q)
        i ← i + c

{no need to look at edges after each component
    has added its smallest incident edge; loop
    terminates when smallest edge for each
    component has been added to MST}
```

```
Algorithm insertEdge(G, minEdges, p)
{Because edges are sorted, we don't have
    to be concerned with cycles because
    both components pick the same edge!!!}
     c ← 0
     e ← p.element()
     (v, w) ← G.endVertices(e)
     cv ← getComponentNum(v)
     cw ← getComponentNum(w)
 {does e connect two different components}
     if cv ≠ cw then
          {insert edge e if smallest}
        if minEdges[cv] = ∅  then
             minEdges[cv] ← e
             setMSTLabel(e, IN_MST)
             c ← c + 1
        if minEdges[cw] = ∅  then
             minEdges[cw] ← e
             setMSTLabel(e, IN_MST)
             c ← c + 1
    return (c, cv=cw)
       {cv=cw means e is max edge in a cycle}
```

Baruvka's Algorithm (another refinement, i.e., sorting edges)

```
Algorithm BaruvkaMST(G)
for each e ∈ G.edges() do
    setMSTLabel(e, NOT_IN_MST)  {no edges in MST}
sortedEdges ← MergeSort(G.edges()) {output is a list}
components ← G.numVertices()
while components > 1 do
    components ← labelVerticesOfEachComponent(G)
    if components > 1 then
        insertSmallestEdges(G, components, sortedEdges)
return G
```

Analysis of top level BaruvkaMST(G)

- Lines 1-2 runs in O(m) to initialize edges
- Line 3 runs in O(m log m) to sort edges i.e., O(m log n)
- Line 5 runs O(log n) times through the loop since number of components is divided at least in half each time through the loop
- Line 6 is O((m + n) log n) to run BFS log n times
- Line 8 is O(m log n) to search for smallest edges to insert into MST
- Thus this algorithm runs in

  O((m + n) log n + m log m)
- Why can we conclude that it's actual Big-O running time is O(m log n)?
  - That is, how did we eliminate the n log n and m log m terms?