Modelling and Simulation: The Biham-Middleton-Levine Model

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Abstract—Traffic jams are a common problem that can be found in many a country's infrastructure on a global scale. Within the current literature, a plethora of models exist that aim to simulate their formation both in an attempt to help us better understand and reinforce our understanding of their generation as well as to potentially uncover previously unconsidered reasons associated with their emergence. This paper seeks to take a closer look at one such model, namely the Biham-Middleton-Levine traffic model, in order to get a baseline of understanding as to how and why traffic jams occur.

Index Terms—Traffic Jams, Biham-Middleton-Levine

1 Introduction

As our reliance on automobiles for travelling continues to grow so too does the increase in the congestion that occurs on our freeways. This congestion, oftentimes referred to as a *traffic jam*, is a widespread phenomenon that occurs for any one of a number of reasons including, but not limited to, an increase in traffic demand that exceeds the capacity of the freeway, a reduction in freeway operation speed as a result of geometric constraints with respect to the layout of the roads of said freeway itself or the occurrence of events that are not entirely predictable such as a traffic accident or adverse weather conditions. Regardless of the cause, the resulting economic losses as a result of delays in traffic are enormous and are cause for concern for major governing bodies [1]. Given the increased attention that problems with traffic have accrued; various approaches have been applied throughout the years in order to describe the collective properties of traffic flow [2] with the intent to optimize and improve on what has become an urgently deteriorating situation [3]. Of these approaches is a self-organizing cellular automaton traffic flow model called the Biham-Middleton-Levine (BML) traffic model. Throughout the scope of this project, and subsequently this report, we will be exploring how the BML traffic model simulates traffic flow and traffic jams by means of implementing the model from scratch and performing experiments over a variable set of hyperparameters. The following sections of the paper will be organized as follows: Section 2 serves to provide a brief overview of the history of the BML traffic model while Section 3 serves to outline the details of our experiments. Finally, Sections 4 and 5 serve to present the results of our experiments alongside a short discussion and concluding thoughts.

2 The BML Traffic Model

The Biham-Middleton-Levine (BML) traffic model was first formulated in 1992 by Ofer Biham, A. Alan Middleton and Dov Levine [4]. It consists of a variable number of cars, represented as points on a lattice that are initialized with random starting positions, where each car on said lattice belongs to either one of two classes. The first class, generally represented by cars that take on a red color, can only move towards the right on the lattice while the second class, generally represented by cars that take on a blue color, can only move downwards on the lattice. During each turn of the simulation all the cars advance by one step if and *only if* they are not currently being blocked by another car i. e., there exists a car in the space directly in front of them.

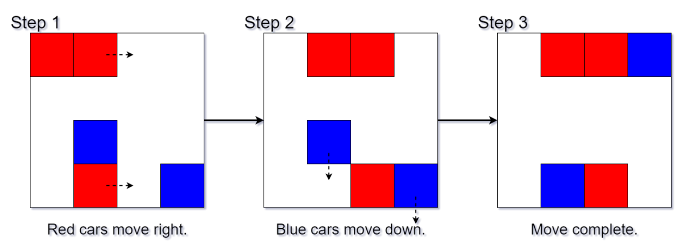

Fig. 1: Steps of the BML traffic model.

The lattice space that the cars are placed on varies in shape with the most common shapes simulated being either a square lattice, as seen in Figure 2, or a rectangular lattice. Regardless of shape, the topology of the lattice is equivalent to a torus; that is in the sense that cars that move off the right edge would reappear on the left edge and cars that move off the bottom edge would reappear on the top edge in the case of a lattice with 4 edges.

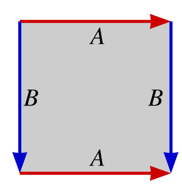

Fig. 2: The fundamental torus of the polygon on which the cars of both class A (red) and class B (blue) move on. Image source: [5], licensed under public domain.

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- *The GitHub repository associated with this project can be found here.*

The most interesting aspects of the BML traffic model are its two highly distinguishable phases: the jammed phased and the free-flowing phase and, as the names suggest, pertain to when the cars on the lattice organize themselves to either achieve a smooth flow of traffic or reach a globally jammed state in which not a single car can make a move. With all of that said, and in spite of the BML traffic model's simplicity, it has been subject to rigorous mathematical analysis with Angel, Holroyd, and Martin [6] noting that with densities $\rho \sim 1$ or $lim_{\rho \to 1}$ the system will reach a globally jammed phase infinitely often and Austin and Benjamini [7] noting that the model will always reach the free-flowing phase if the total number of cars (n) is less than half the length of the edge of a square lattice $(\frac{N}{2})$. In this paper we will be exploring the relationship of ρ, or otherwise the number of the cars on the lattice, with the state of the traffic flow and attempt to recreate the findings of both Angel, Holroyd, and Martin and Austin and Benjamini.

3 Methodology

To ascertain the assumptions made in Section 2 of this paper we will be implementing the BML traffic model from scratch and attempt to recreate the simulations under a similar set of hyperparameters.

3.1 Implementation

Implementing the BML traffic model is rather straightforward – we initialize an $X \times Y$ grid that serves to act as the lattice that the cars will move on. Following that, we populate the grid by placing $\rho \times X \times Y$ (with $0 < \rho < 1$) cars at random positions throughout the grid making sure that no two cars are occupying the same cell. Once the grid has been populated the cars take alternating turns moving depending on which class they belong to. In our configuration, blue cars move downwards at odd time periods $(t = 1, 3, 5, ...,)$ while red cars move rightwards at even time periods $(t = 2, 4, 6, ...,)$. When a car gets to the edge of the grid it "wraps" around, e. g., when a blue car gets to the bottom edge, we reset its position to the top of the grid and, similarly, when a red car gets to the right edge of the grid, we reset its position to the leftmost position on the grid. Finally, before moving, the cars have to consider the following 3 options:

1) The considered location is empty $\rightarrow$ we can move.
2) The considered location is occupied by a car of the opposite color $\rightarrow$ we can't move.
3) The considered location is occupied by a car of the same color $\rightarrow$ we check to see if the "blocking" car can move and act accordingly as per steps 1 & 2.

To wrap things up, we direct the reader to our GitHub repository located here that contains the relevant source code as well as further documentation on our (Python) implementation of the BML traffic model. The relevant hyperparameters for our experiments are listed in Table 1.

Hyperparameter	Explanation
X	The size of the x-axis of the grid.
Y	The size of the y-axis of the grid.
$p(red)$	The probability of a car being red.
ρ	Density of the cars on the grid $\rho = \frac{n}{X \times Y}$
t	Number of iterations.

TABLE 1: Hyperparameters of our implementation of the BML traffic model.

3.2 Experiment 1

The first experiment we will be performing is to ascertain that for a square lattice $(X = Y)$ if the total number of cars is less than or equal to half the length of one of the edges of lattice $(n \leq \frac{X}{2})$ then we will *always* achieve a free-flowing state of traffic flow. To do this we will run the simulation under the set of hyperparameters outlined in Table 2 and document the averaged results over 5 runs under the same value for ρ, predominantly noting the percentage of cars that moved at the last time step recorded and checking to see whether, for any of our simulations, the model jammed.

Hyperparameter	Value
X	50
Y	50
$p(red)$	0.5
ρ	$0.05 \rightarrow 0.25$ (in increments of 0.05)
t	2500

TABLE 2: Hyperparameters for experiment 1.

3.3 Experiment 2

The second experiment that we will be performing is done to ascertain that as ρ approaches 1 the model will reach a globally jammed phase infinitely often.

3.4 Experiment 3

The final experiment is done to explore the intermediate phase, frequently occurring close to the transition density, which combines features from both the jammed as well as the free-flowing phases. The two principal intermediate phases are **disordered** and **periodic**. We'll specifically be looking at whether we can achieve periodic orbits on rectangular lattices with coprime dimensions as per the findings of D'Souza [8].

4 Results and Discussion

5 Conclusion

References

[1] D. Yin and T. Z. Qiu, "Traffic Jam Modeling and Simulation," in *2012 15th International IEEE Conference on Intelligent Transportation Systems*, 2012, pp. 1423–1428. DOI: 10.1109/ITSC.2012.6338916.

[2] T. Nagatani, "Effect of Car Acceleration on Traffic Flow in 1D Stochastic CA Model," *Physica A: Statistical Mechanics and its Applications*, vol. 223, no. 1-2, pp. 137–148, 1996. DOI: 10.1016/0378-4371(95)00292-8.

[3] D. Helbing, “Improved Fluid-Dynamic Model for Vehicular Traffic,” *Physical Review E*, vol. 51, no. 4, pp. 3164–3169, 1995. DOI: 10.1103/PHYSREVE.51.3164.

[4] Ofer Biham, A. Alan Middleton and Dov Levine, “Self-organization and a Dynamical Transition in Traffic-flow Models,” *Phys. Rev. A*, vol. 46, pp. 6124–6127, 10 Nov. 1992. DOI: 10.1103/PHYSREVA.46.R6124.

[5] (Apr. 2021). “TorusAsSquare,” [Online]. Available: https://commons.wikimedia.org/wiki/File:TorusAsSquare.svg.

[6] O. Angel, A. Holroyd, and J. Martin, “The Jammed Phase of the Biham-Middleton-Levine Traffic Model,” *Electronic Communications in Probability*, vol. 10, pp. 167–178, 2005. DOI: 10 . 1214 / ECP . V10 - 1148. [Online]. Available: https://doi.org/10.1214/ECP.v10-1148.

[7] T. Austin and I. Benjamini, “For What Number of Cars Must Self Organization Occur in the Biham-Middleton-Levine Traffic Model From Any Possible Starting Configuration?,” Aug. 2006. [Online]. Available: https://arxiv.org/abs/math/0607759.

[8] R. M. D’Souza, “Coexisting Phases and Lattice Dependence of a Cellular Automaton Model for Traffic Flow,” *Physical Review E*, vol. 71, no. 6, 2005. DOI: 10.1103/PHYSREVE.71.066112.

Appendix

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Glossary

BML Biham-Middleton-Levine. 1, 2, 4