

COMP 5970/6970 HW 1: 5 questions 5 points 5% Credit
Due before 11:59 PM Tuesday January 29

Instructions:

1. This is an individual assignment. You should do your own work. Any evidence of copying will result in a zero grade and additional penalties/actions.
2. Enter your answers in this Word file. Submissions must be uploaded **as a single file** (Word or PDF preferred, but other formats acceptable as long as your work is LEGIBLE) to Canvas before the due date and time. Don't turn in photos of illegible sheets. If an answer is unreadable, it will earn zero points. Cleanly handwritten submissions (print out this assignment and write answers in the space provided, with additional sheets used if needed) scanned in as PDF and uploaded to Canvas are acceptable.
3. **Submissions by email or late submissions (even by minutes) will receive a zero grade.** No makeup will be offered unless prior permission to skip the assignment has been granted, or there is a valid and verifiable excuse.

Multiple Choice Questions (5 points)

In the following questions, circle the correct choice. If more than one answer is correct, circle all that apply. In those cases, partial credit will be given to partially correct answers. No explanation needed. Incorrect answers or unanswered questions are worth zero points.

1. "Any problem that can be solved with a greedy algorithm can also be solved with dynamic programming." The statement is:

- [a] True
- [b] False

Answer: [a]

2. "Dynamic programming can be used to find an approximate solution to an optimization problem, but cannot be used to find a solution that is guaranteed to be optimal." The statement is:

- [a] True
- [b] False

Answer: [b]

3. In dynamic programming algorithm, we drive a recurrence relation for the solution to one subproblem in terms of solution to others and reuse the solutions to smaller subproblems in order to solve a larger problem. Suppose the recurrence relation for a dynamic programming algorithm is of the form:

$$A(i, j) = f(A(i, j-1), A(i-1, j-1), A(i-1, j+1))$$

The number of subproblems is:

- [a] $3n$
- [b] $2n$
- [c] n^2
- [d] none of the above

Answer: [c]

Solve $A(i, j)$ for (i from 0 to n: for (j from 0 to n))

4. In bottom-up dynamic programming algorithm, we need a traversal order such that all needed subproblems are solved before solving the original problem. Suppose the recurrence relation for a dynamic programming algorithm is of the form:

$$A(i, j) = f(A(i-2, j-2), A(i+2, j+2))$$

A valid traversal order is:

- [a] Solve $A(i, j)$ for (i from 0 to n: for (j from 0 to n))
- [b] Solve $A(i, j)$ for (i from 0 to n-2: for (j from i+2 to n))
- [c] Solve $A(i, j)$ for (i from 0 to n-2: for (j from i to n))
- [d] none of the above

Answer: [d]

Impossible. Cyclic.

5. Consider two vertices, s and t, in some directed acyclic graph $G = (V, E)$. Let's assume that a dynamic programming algorithm is developed to determine the number of paths in G from s to t. The running time of the algorithm is:

- [a] $O(V E)$
- [b] $O(V + E)$
- [c] $O(V \lg V + E)$
- [d] none of the above

Answer: [b]

Produce a topological ordering v_1, v_2, \dots, v_n of the vertices of G. Without loss of generality, let $v_1 = s$ and $v_n = t$ (because that is the only part of the graph we care about).

Define subproblems as follows: let $S[i]$ be the number of paths from v_i to $v_n = t$. We can define a recurrence expressing the solution to these subproblems in terms of smaller subproblems:

$$S[i] = \sum_{j \in \text{adj}(i)} S[j]$$

where $\text{adj}(i)$ is the set of all vertices v s.t. $(i, v) \in E$. (That is, all vertices to which there is an edge from i .) The base case is $S[n] = 1$.

The solution to the general problem is the number of paths from $v_1 = s$ to $v_n = t$, or $S[1]$. The total time required to solve the $O(V)$ subproblems is $O(E)$, and the topological sort requires $O(V + E)$ time, so the total running time is $O(V + E)$.