COMP 5970/6970 HW 3: 5 questions 5 points 5% Credit
Due before 11:59 PM Friday March 8

Instructions:

1. This is an individual assignment. You should do your own work. Any evidence of copying will result in a zero grade and additional penalties/actions.
2. Enter your answers in this Word file. Submissions must be uploaded **as a single file** (Word or PDF preferred, but other formats acceptable as long as your work is LEGIBLE) to Canvas before the due date and time. Don't turn in photos of illegible sheets. **If an answer is unreadable, it will earn zero points.** Cleanly handwritten submissions (print out this assignment and write answers in the space provided, with additional sheets used if needed) scanned in as PDF and uploaded to Canvas are acceptable.
3. **Submissions by email or late submissions (even by minutes) will receive a zero grade.** No makeup will be offered unless prior permission to skip the assignment has been granted, or there is a valid and verifiable excuse.

Multiple Choice Questions (5 points)

In the following questions, circle the correct choice. If more than one answer is correct, circle all that apply. In those cases, partial credit will be given to partially correct answers. No explanation needed. Incorrect answers or unanswered questions are worth zero points.

1. "Two fair die are rolled together. Let the random variable S denote the sum of the numbers read from the two. Then probability that S = 11 is:

[a] 1/6
[b] 1/8
[c] 1/12
[d] 1/18

Answer: [d]

There are 36 different values S can take. Let's look at number of cases when S = 11. This is the set {<5,6>, <6,5>}. Hence P(S=11) = 2/36 = 1/18.

2. "MAP estimate converges to MLE estimate for infinite observed data no matter what the prior is." The statement is:

[a] True
[b] False

Answer: [b]

A simple counterexample is the prior that assigns probability 1 to a single choice of parameter θ.

Suppose X, Y, Z are Boolean random variables and the following table lists the probability of the possible values:

	Z = 0		Z = 1	
	X = 0	X = 1	X = 0	X = 1
Y = 0	1/15	1/15	4/15	2/15
Y = 1	1/10	1/10	8/45	4/45

Probability of a joint assignment of X, Y, Z can be deduced from this table by simple look up. For example, $P(X = 0, Y = 1, Z = 0) = 1/10$ and $P(X = 1, Y = 1, Z = 0) = 4/45$.

Answer the following questions based on this table:

3. Is X independent of Y?

    [a] Yes
    [b] No

Answer: [b]

Using the facts that for any events A and B, (i.e., any subsets of the possible assignments of 0 & 1 to the variables X, Y, Z) we have $P(A) = \sum P(X = x, Y = y, Z = z)$ and $P(A|B) = P(A, B)/P(B)$, we have:

$P(X=0)=1/15+1/10+4/15+8/45=11/18$,
$P(Y=0)=1/15+1/15+4/15+2/15=8/15$,
$P(X=0|Y=0)= P(X=0,Y=0) / P(Y=0) = (1/15+4/15)/ (8/15) = 5/8$.

Since $P(X = 0)$ does not equal $P(X = 0|Y = 0)$, X is not independent of Y .

4. Is X conditionally independent of Y given Z?

    [a] Yes
    [b] No

Answer: [a]

For all pairs $y, z \in \{0,1\}$, we need to check that $P(X=0|Y=y, Z=z) = P(X=0|Z=z)$.
That the other probabilities are equal follows from the law of total probability.

First we have:

$P(X=0|Y=0,Z=0) = (1/15) / (1/15 + 1/15) = 1/2$
$P(X=0|Y=1,Z=0) = (1/10) / (1/10 + 1/10) = 1/2$
$P(X=0|Y=0,Z=1) = (4/15) /(4/15 + 2/15) = 2/3$
$P(X=0|Y=1,Z=1) = (8/45) /(8/45 + 4/45) = 2/3$

Second,

$P(X = 0|Z = 0) = (1/15+1/10) / (1/15 + 1/15 + 1/10 + 1/10) = 1/2$
$P(X = 0|Z = 1) = (4/15+8/45) / (4/15 + 2/15 + 8/45 + 4/45) = 2/3$

This shows that X is conditionally independent of Y given Z.

5. What is the value of $P(X = 0 | X + Y > 0)$?

    [a] 1/2
    [b] 2/3
    [c] 5/12

[d] 8/45

Answer: [c]
$P(X=0|X+Y>0) = (1/10+8/45) / (1/15+1/10+1/10+2/15+4/45+8/45) = 5/12$.