AUBURN UNIVERSITY

SAMUEL GINN COLLEGE OF ENGINEERING

Sorting

COMP 2210 – Dr. Hendrix

History

History

Herman Hollerith's tabulating machine and "sorting box"

1880 Census: 8 years to process
1890 Census: 1 year to process

"This apparatus works unerringly as the mills of the gods, but beats them hollow as to speed." The Electrical Engineer, 11 Nov 1891.

http://www.columbia.edu/cu/computinghistory/census-tabulator.html

History

John von Neumann's first program for the EDVAC

"I have also worked on sorting questions … We wish to formulate code instructions for sorting … and to see how much … time they require. … At any rate, the moral seems to be that the EDVAC … is definitely faster than the [IBM sorters]. It is legitimate to conclude … that the EDVAC is very nearly an "all purpose" machine." JvN, 1945

http://dl.acm.org/citation.cfm?id=356581

Sorting

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Input: List a of N elements $(a_1, a_2, \ldots, a_N)$

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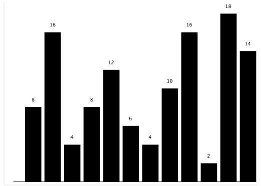
0	1	2	3	4	5	6	7	8	9	10	11



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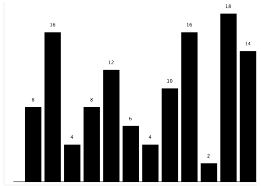

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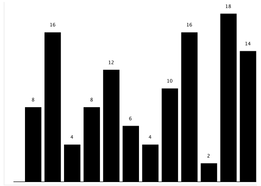

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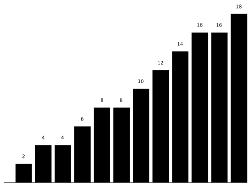

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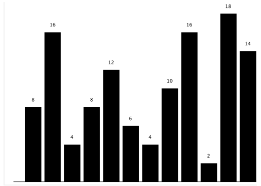

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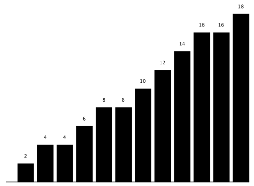

Lots of algorithms … selection sort, insertion sort, bubblesort, shaker sort, quicksort, merge sort, heapsort, samplesort, shellsort, solitaire sort, red-black sort, splaysort, psort, radix sort, counting sort, bucket sort, distribution sort, timsort, comb sort, …

Inversions and exchanges

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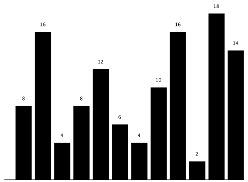

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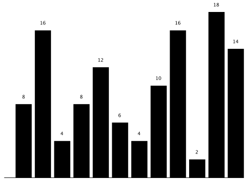

exchange = swap the locations of inverted elements

20 inversions: (8,4), (8,6), (8,4), (8,2), (16, 12), (4,2), (8,6), (8,4), (8,2), (12,6), (12,4), (12,10), (12,2), (6,4), (6,2), (4,2), (10,2), (16,2), (16,14), (18,14)

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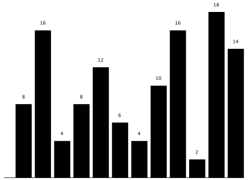

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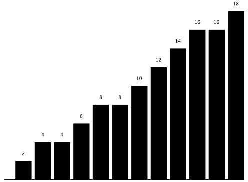

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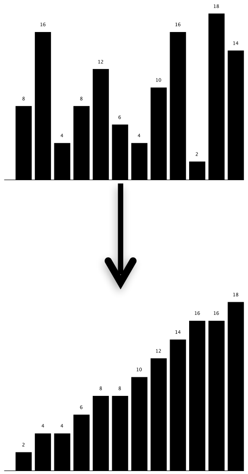

exchange = swap the locations of inverted elements

Sorting could be seen as a sequence of exchanges.

Inversions and exchanges

Randomly ordered data:

4	10	6	8	12	2
0	1	2	3	4	5

7 inversions

12	6	4	8	2	10
0	1	2	3	4	5

9 inversions

8	10	12	2	4	6
0	1	2	3	4	5

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A list in reverse order (non increasing) would have the maximum number of inversions.

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list	# inversions	
4, 3, 2, 1	3 + 2 + 1 + 0	6
5, 4, 3, 2, 1	4 + 3 + 2 + 1 + 0	10
6, 5, 4, 3, 2, 1	5 + 4 + 3 + 2 + 1 + 0	15
. . .		

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0	1	2	3	4	5

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. . .		

$$\sum_{i=0}^{N-1} i = \frac{(N-1)N}{2} = \frac{N^2 - N}{2}$$

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5, 4, 3, 2, 1	4 + 3 + 2 + 1 + 0	10
6, 5, 4, 3, 2, 1	5 + 4 + 3 + 2 + 1 + 0	15
. . .		

$$\sum_{i=0}^{N-1} i = \frac{(N-1)N}{2} = \frac{N^2 - N}{2}$$

In the worst case, there will be $O(N^2)$ inversions in the data.

Inversions and exchanges

Detecting an inversion:

Inversions and exchanges

Detecting an inversion:

```java
private boolean less(Comparable x, Comparable y) {
   return x.compareTo(y) < 0;
}
```

4	10	6	8	12	2
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`less(a[5], a[0]) -> true`

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Correcting an inversion:

Inversions and exchanges

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```

4	10	6	8	12	2
0	1	2	3	4	5

`less(a[5], a[0]) -> true`

Correcting an inversion:

```
private void swap(Comparable[] a, int i, int j) {
   T temp = a[i];
   a[i] = a[j];
   a[j] = temp;
}
```

4	10	6	8	12	2
0	1	2	3	4	5

`swap(a, 5, 0)`

2	10	6	8	12	4
0	1	2	3	4	5

Properties of sorts

Comparison sort: The only assumption about the data being sorted is that the data elements can be compared to each other on the basis of “less than” - that is, a total order exists.

In-place: The list itself is rearranged and only a constant amount of extra space is required.

Adaptive: Running time is affected by initial state of input.

Stable: Equal elements maintain the same relative order.

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	Selection	Insertion	Mergesort	Quicksort
In-place?	Yes	Yes	No	Yes
Stable?	No	Yes	Yes	No
Adaptive?	No	Yes	No	No

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(as presented)	Selection	Insertion	Mergesort	Quicksort
In-place?	Yes	Yes	No	Yes
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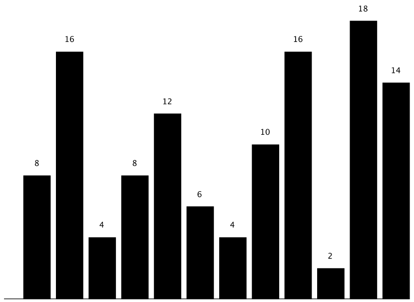
Selection sort

Walk from left to right through the array.

On each step, **select** the element that goes in the current location in sorted order and put it there.

After k steps, the first k elements are in sorted order and are in their final positions.



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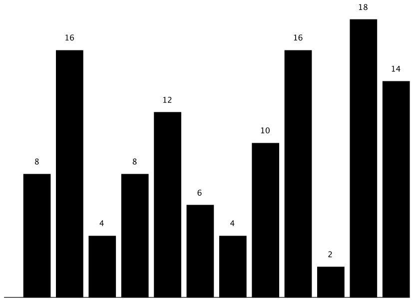

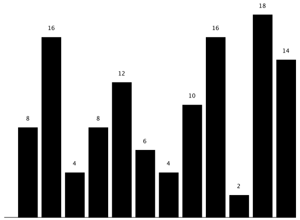

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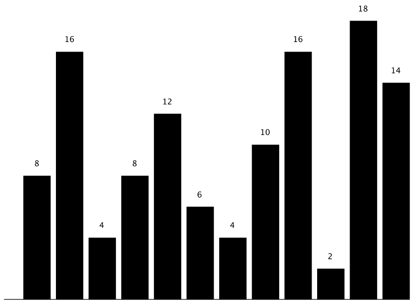

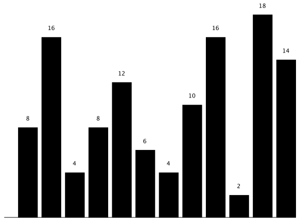

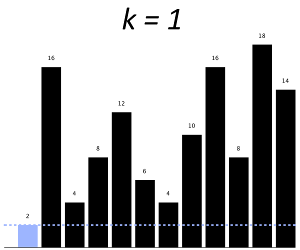

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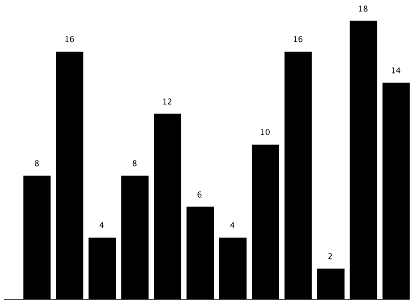

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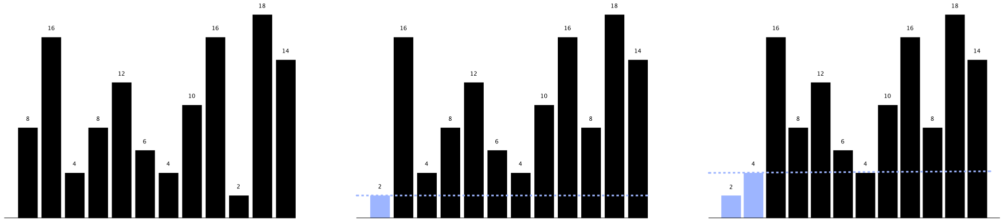

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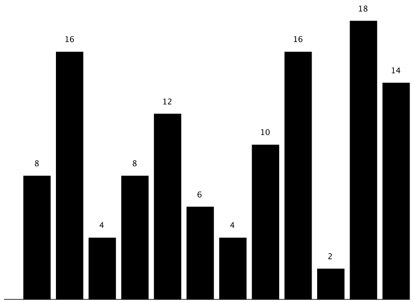

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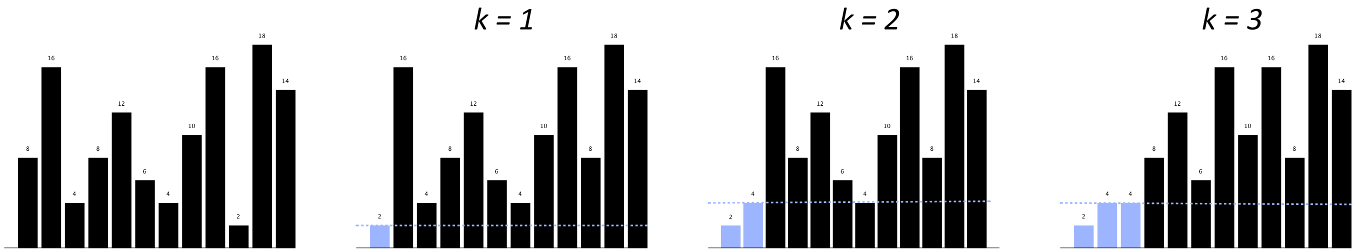

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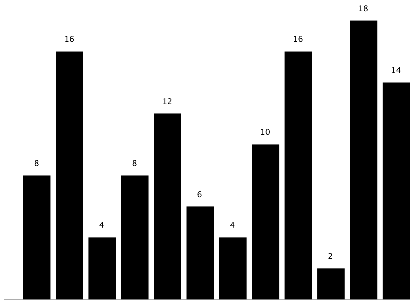

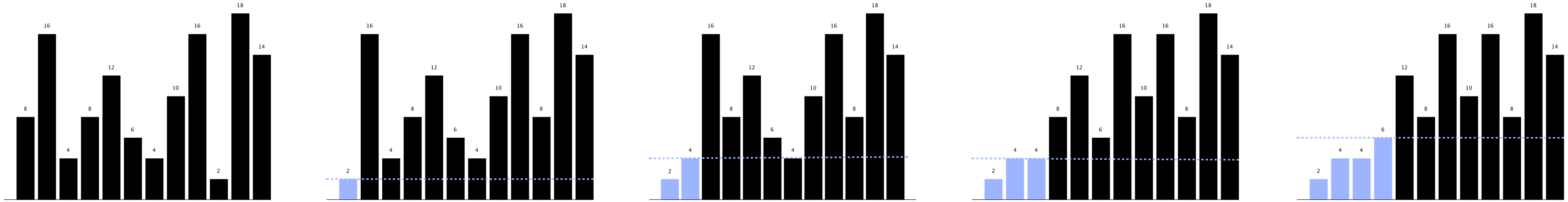

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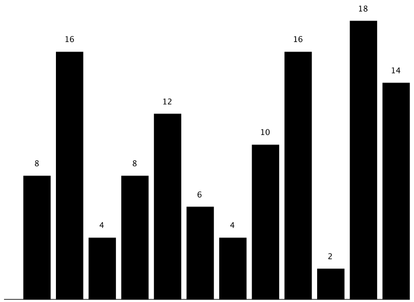

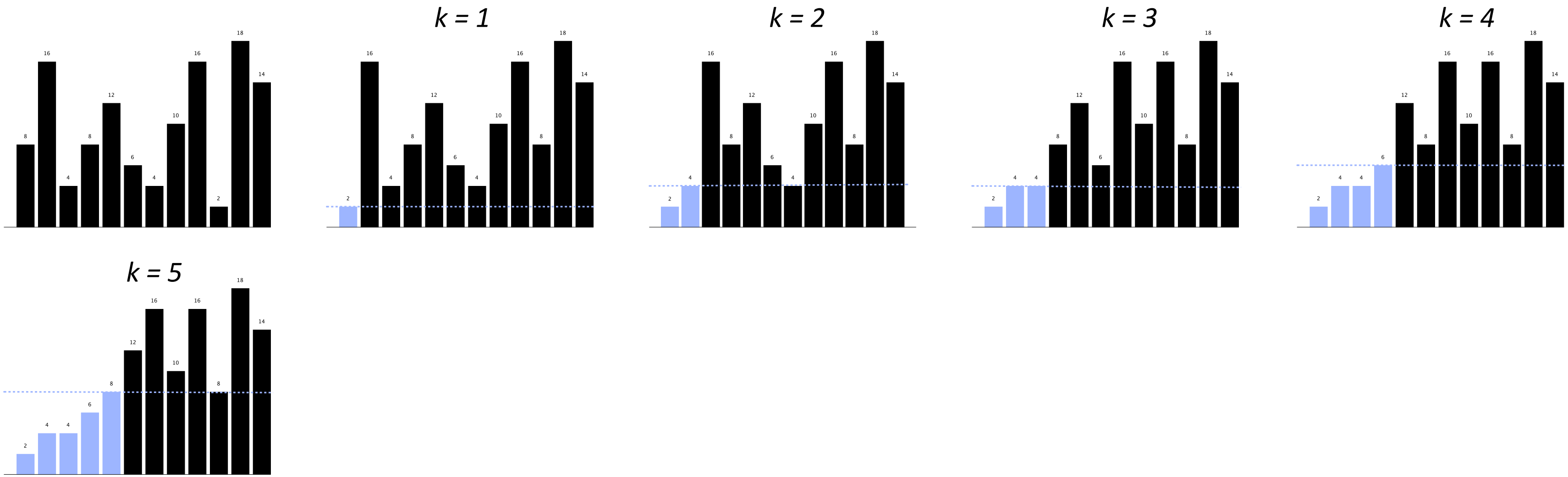

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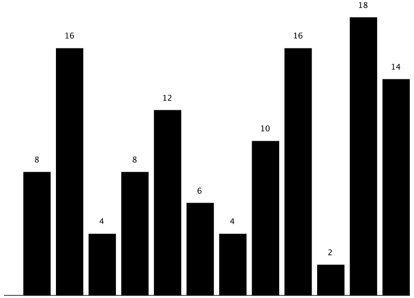

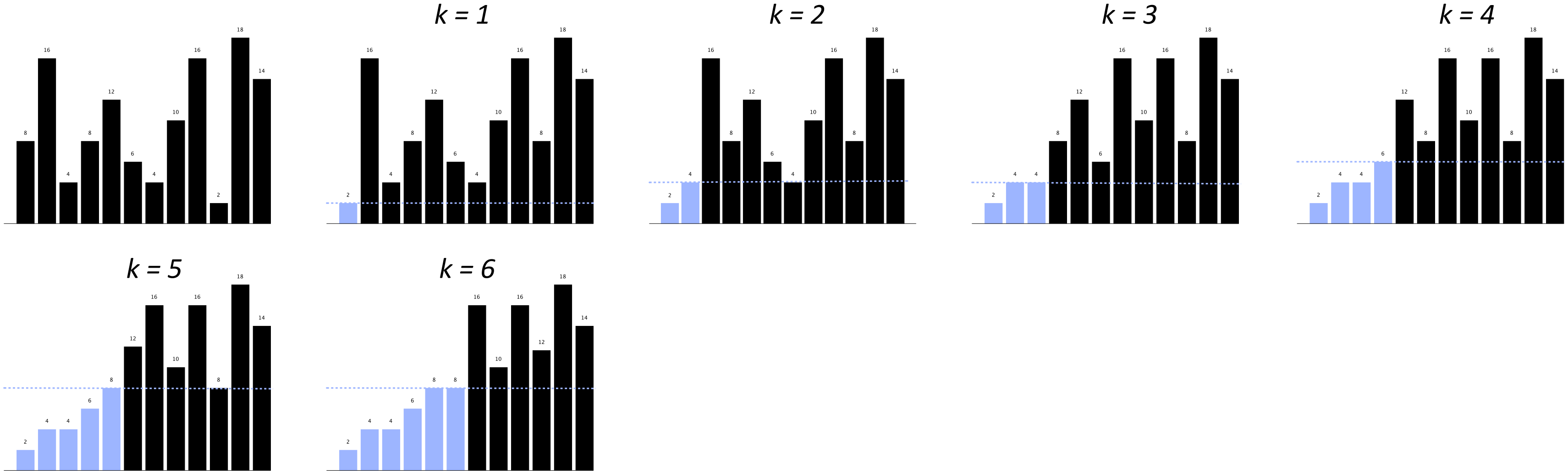

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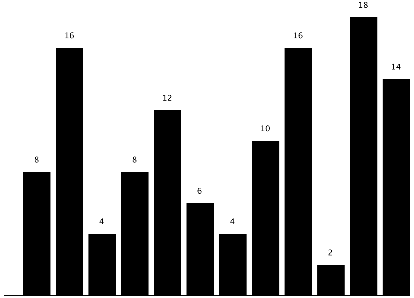

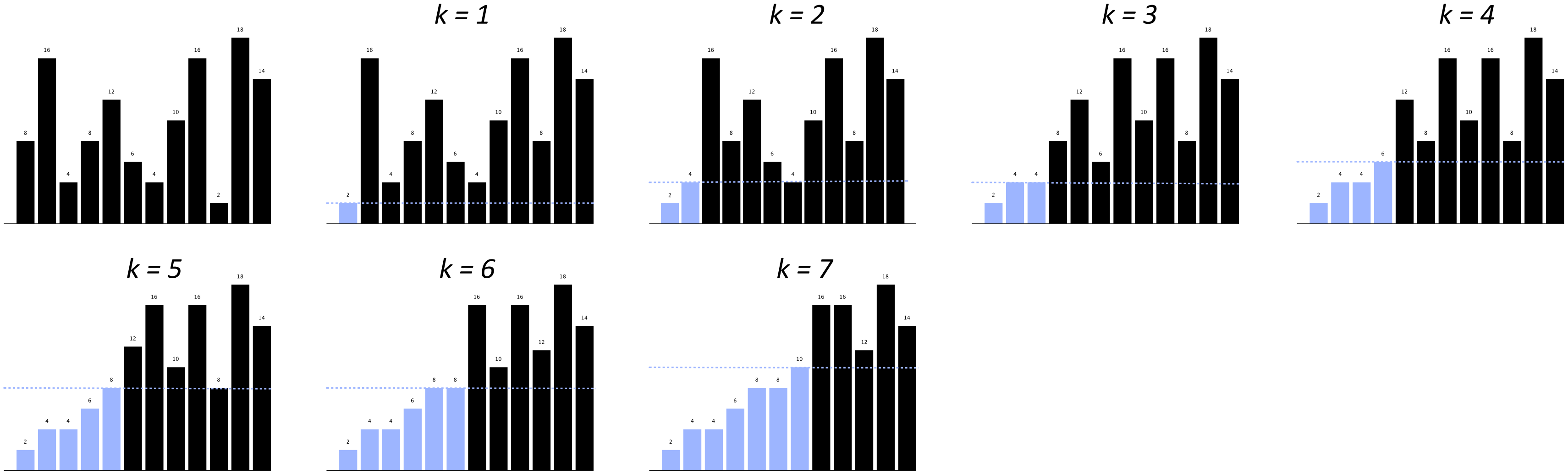

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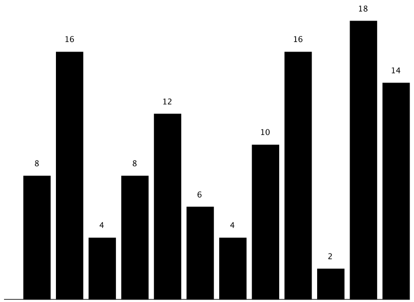

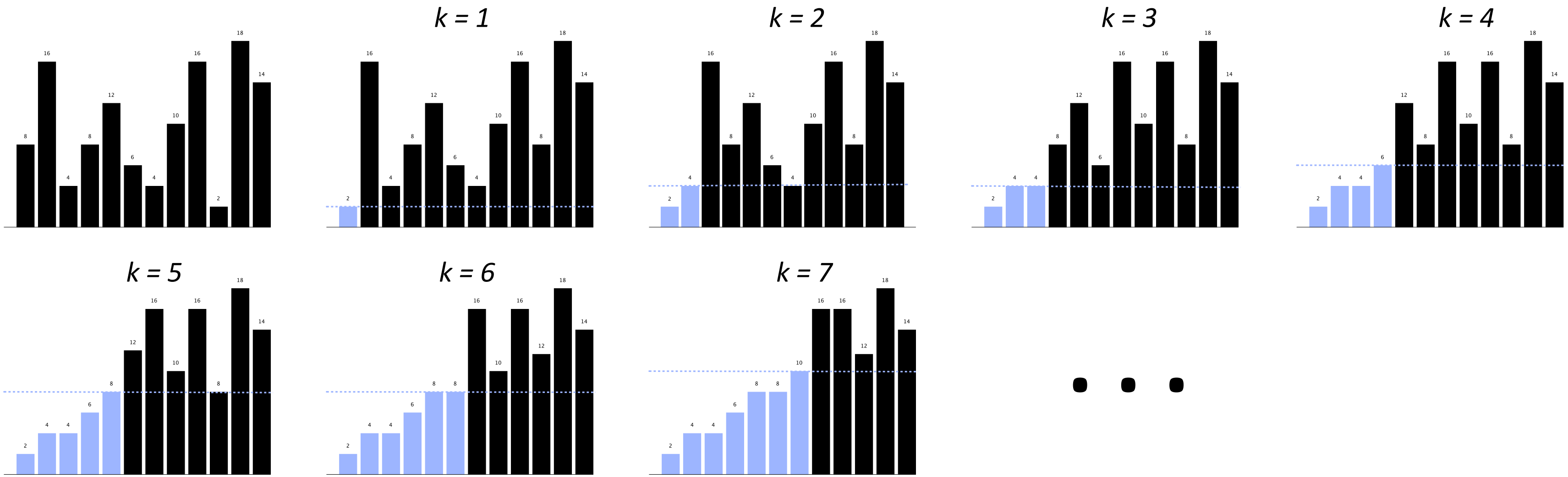

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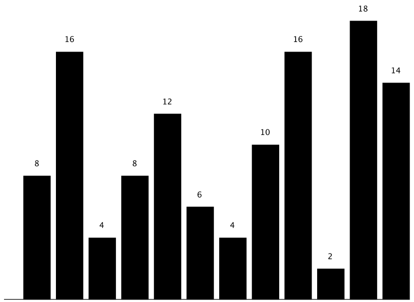

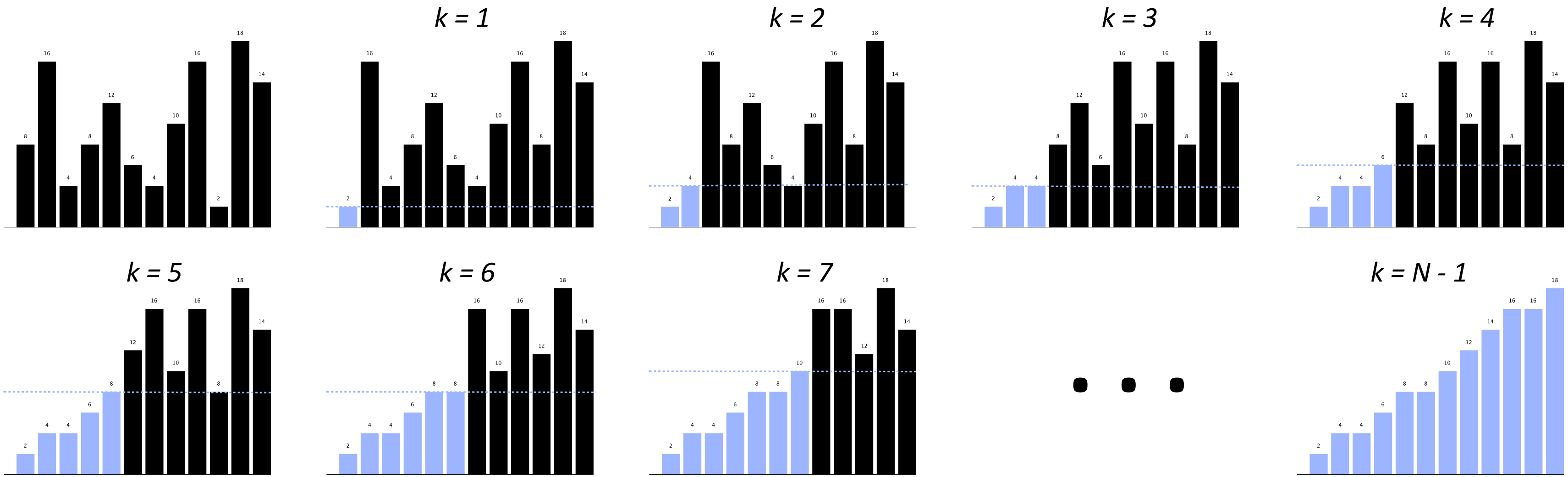

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```java
public void selectionSort(T[] a) {
   for (int i = 0; i < a.length; i++) {
      int min = i;
      for (int j = i + 1; j < a.length; j++) {
         if (less(a[j], a[min])) {
            min = j;
         }
      }
      swap(a, i, min);
   }
}
```

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```

For each index in the array …

Selection sort

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For each index in the array …

select the minimum value from the current index to the right …

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After any given iteration of the outer loop, the array is divided into two parts:

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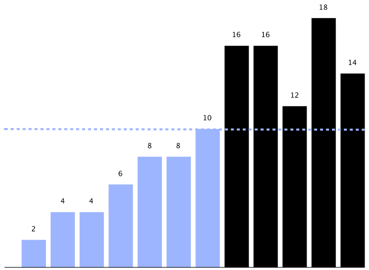

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```

8	16	4	8	12	6	4	10	16	2	18	14
0	1	2	3	4	5	6	7	8	9	10	11

Invariants

After any iteration of the outer loop:

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Time complexity

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$O(N^2)$
already sorted

2	4	6	8	10	12
0	1	2	3	4	5

$\sim N^2$ comparisons
$\sim N$ exchanges

$O(N^2)$
"almost" sorted

6	8	10	12	2	4
0	1	2	3	4	5

$\sim N^2$ comparisons
$\sim N$ exchanges

$O(N^2)$
in reverse order

12	10	8	6	4	2
0	1	2	3	4	5

$\sim N^2$ comparisons
$\sim N$ exchanges

$O(N^2)$
in random order

4	12	8	2	6	10
0	1	2	3	4	5

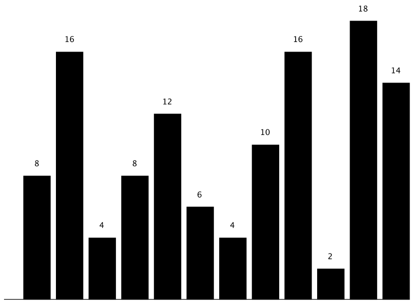
$\sim N^2$ comparisons
$\sim N$ exchanges

Insertion sort

Walk from left to right through the array.

On each step, **insert** the element in the current location in sorted order to its left.

After k steps, the first k + 1 elements are in sorted order relative to each other.



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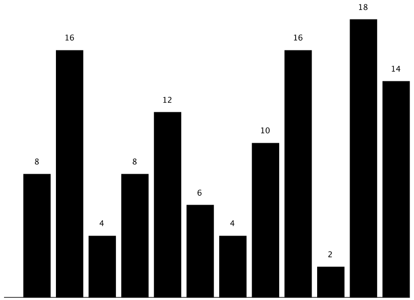

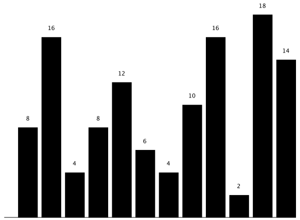

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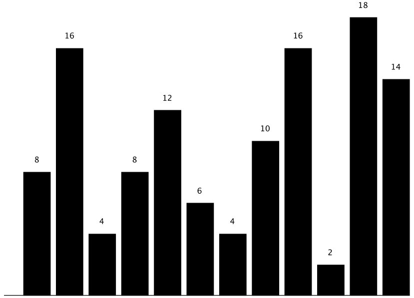
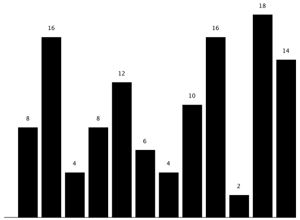
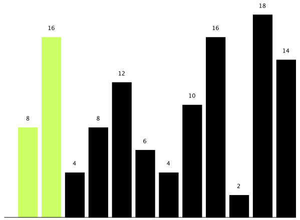




k = 1



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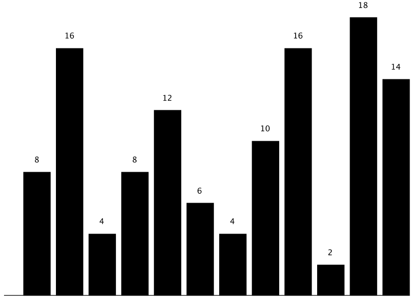

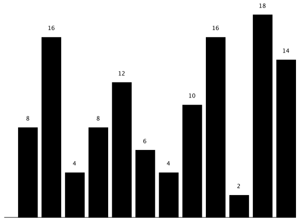

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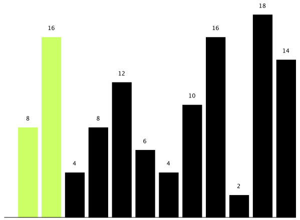
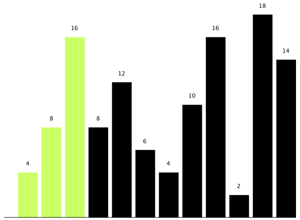


k = 2



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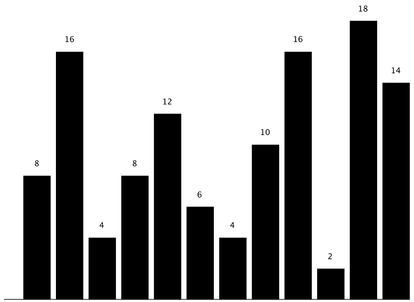

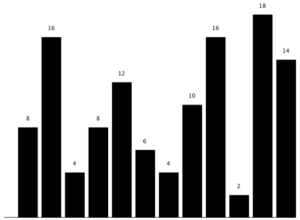

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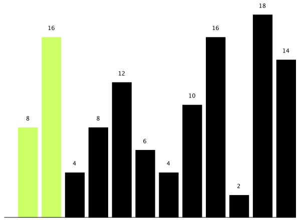

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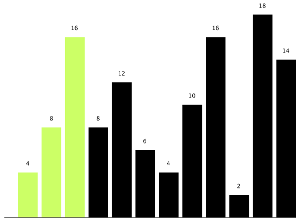

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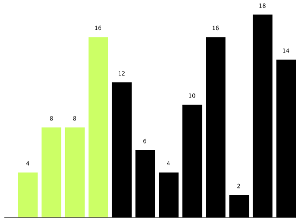

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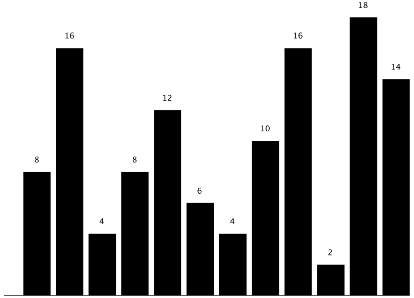

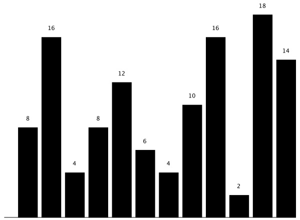

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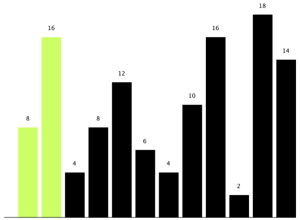
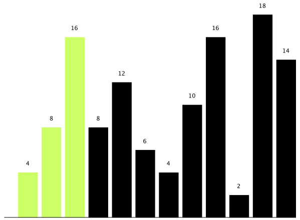
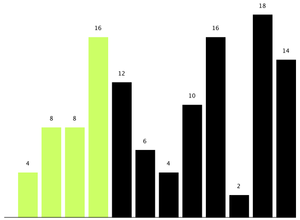


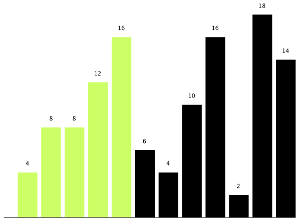
k = 2



k = 3



k = 4



Insertion sort

Walk from left to right through the array.

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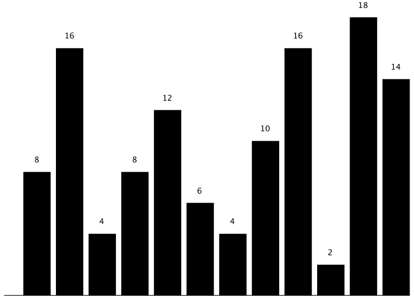
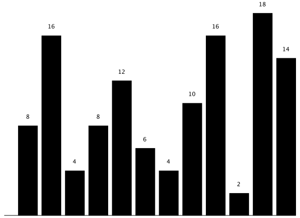
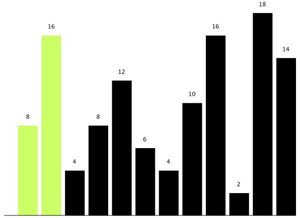
After k steps, the first k + 1 elements are in sorted order relative to each other.

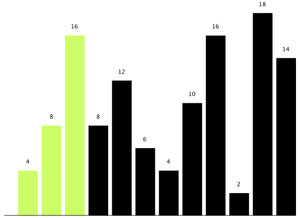




k = 1



k = 2



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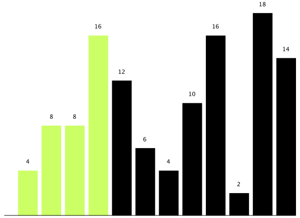

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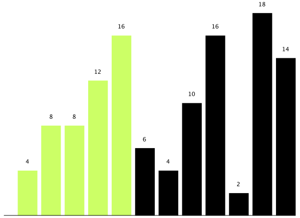
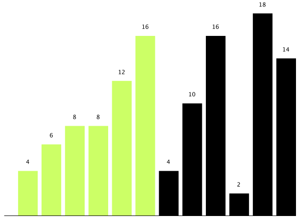


k = 5



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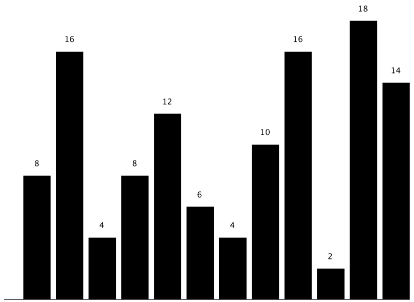

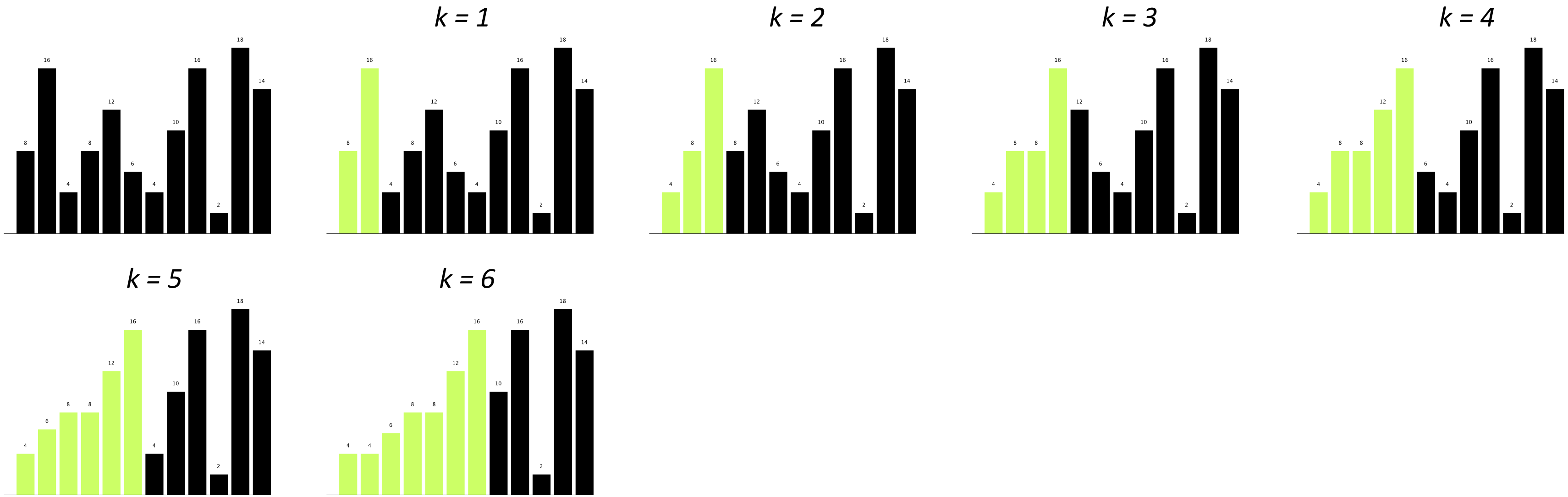

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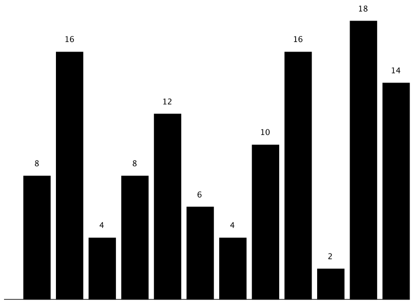

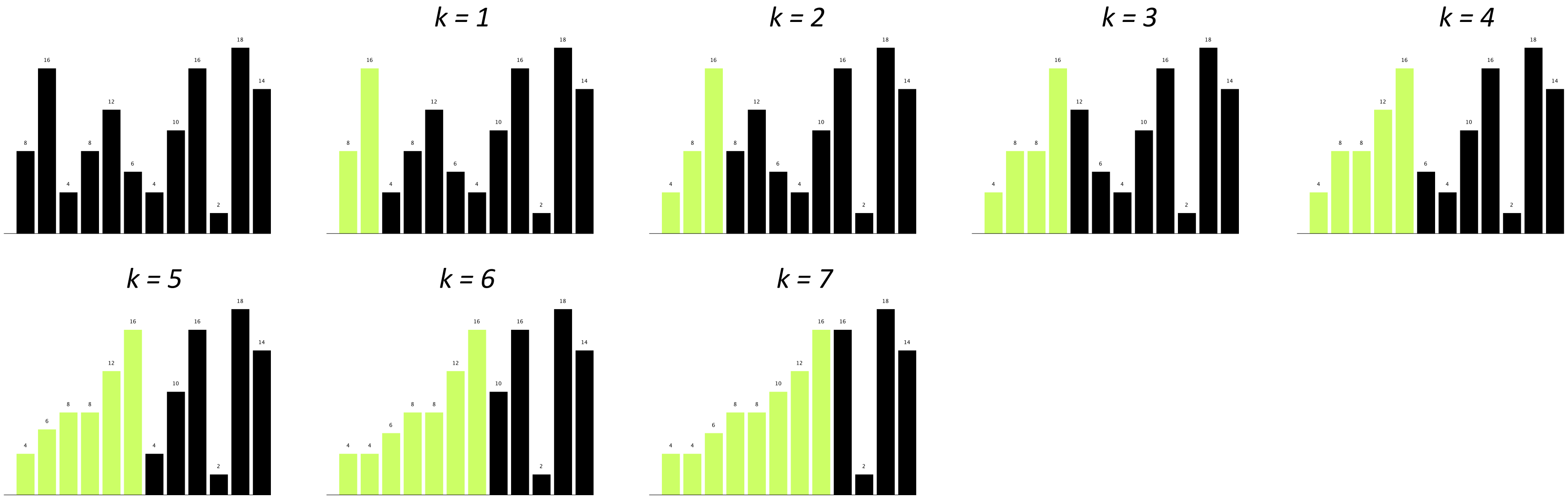

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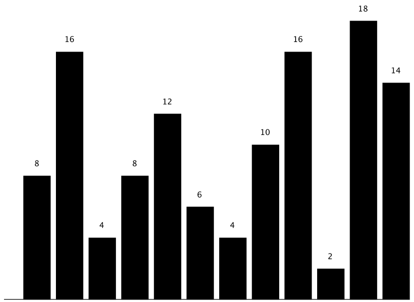

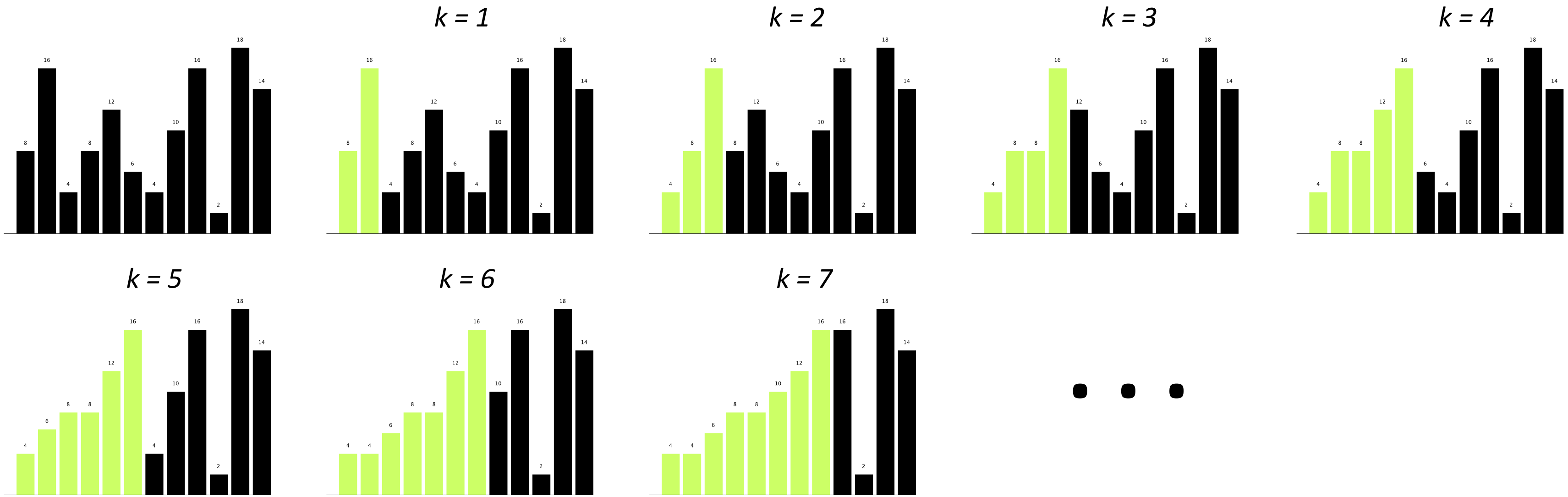

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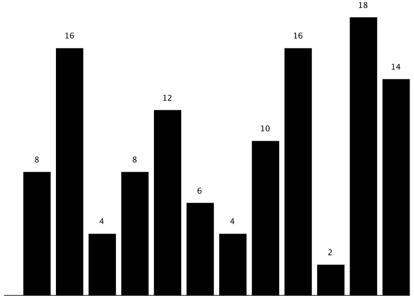

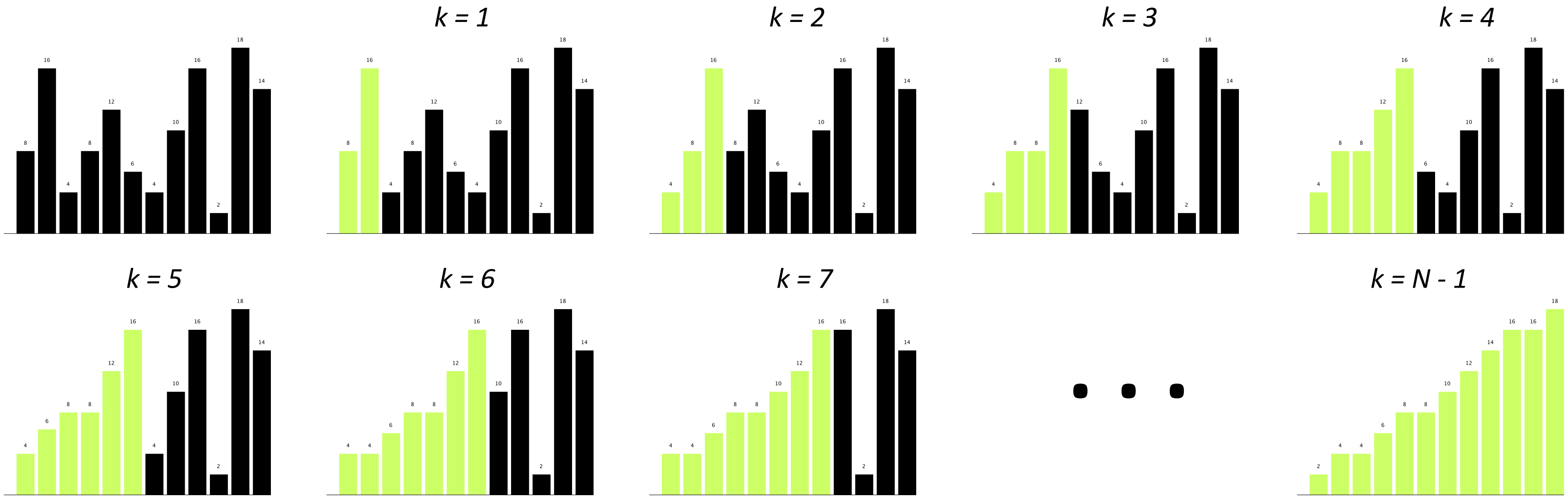

Insertion sort

Insertion sort

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insert it in sorted order relative to the values to its left …

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After any given iteration of the outer loop, the array is divided into two parts:

- values at indices 0 through i are in sorted order, but only with respect to each other
- values at indices i + 1 through a.length - 1 are unknown

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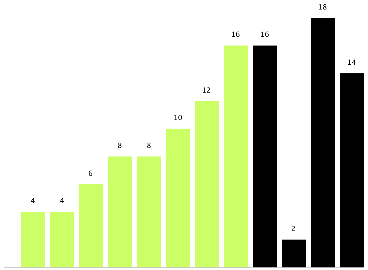

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}
```

8	16	4	8	12	6	4	10	16	2	18	14
0	1	2	3	4	5	6	7	8	9	10	11

Invariants:

After any iteration of the outer loop:

The first i elements are in sorted order relative to each other.

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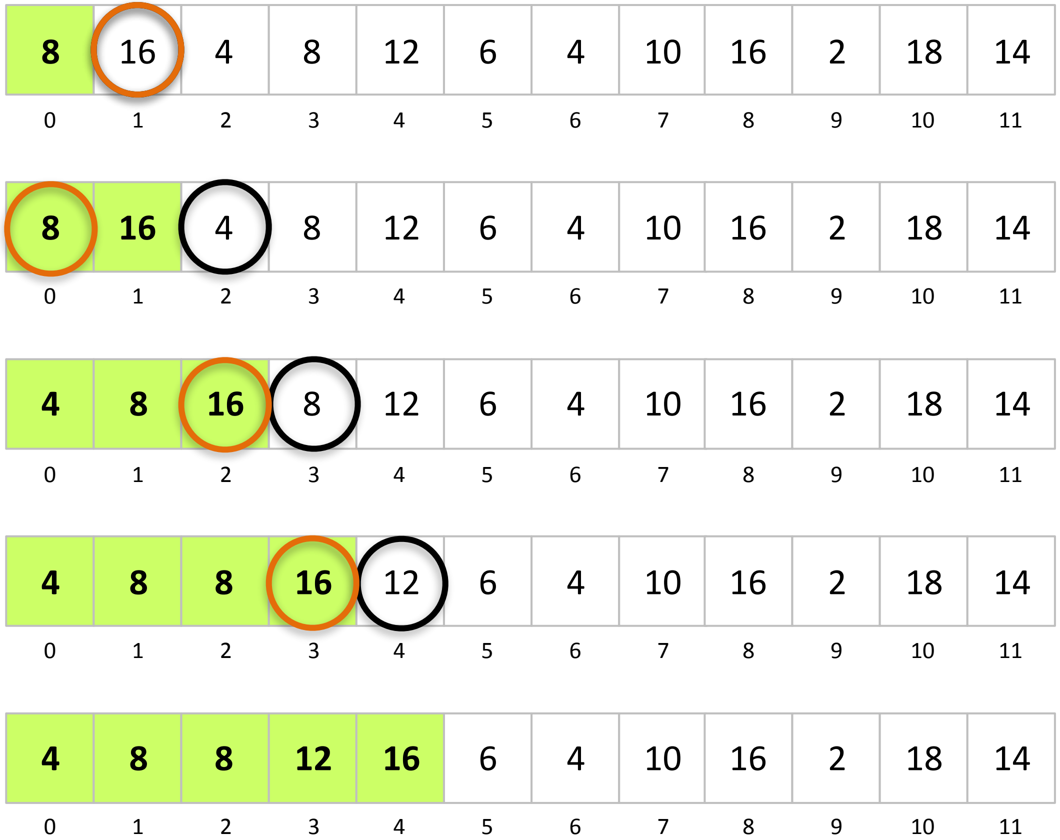

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Time complexity

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Insertion sort is $O(N^2)$.

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Insertion sort **is *adaptive*** to its input, so some arrangements of data in the array will require less work than others.

$O(N)$

already sorted

2	4	6	8	10	12
0	1	2	3	4	5

~N comparisons
0 exchanges

$O(N)$

"almost" sorted

6	8	10	12	2	4
0	1	2	3	4	5

~N comparisons
~N exchanges

$O(N^2)$

in reverse order

12	10	8	6	4	2
0	1	2	3	4	5

~N^2 comparisons
~N^2 exchanges

$O(N^2)$

in random order

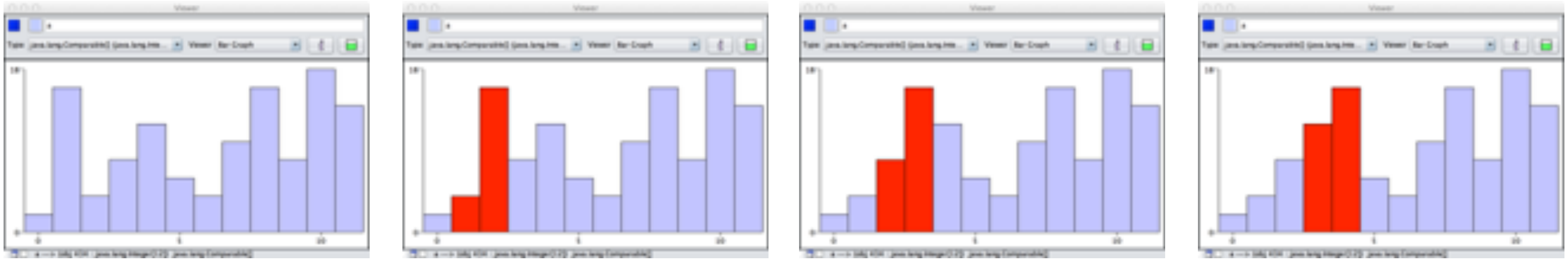
4	12	8	2	6	10
0	1	2	3	4	5

~N^2 comparisons
~N^2 exchanges

Participation

Q: Given the following sequence of images from a jGRASP viewer showing the successive changes in the elements of an array, what sorting algorithm is being used by the underlying program?

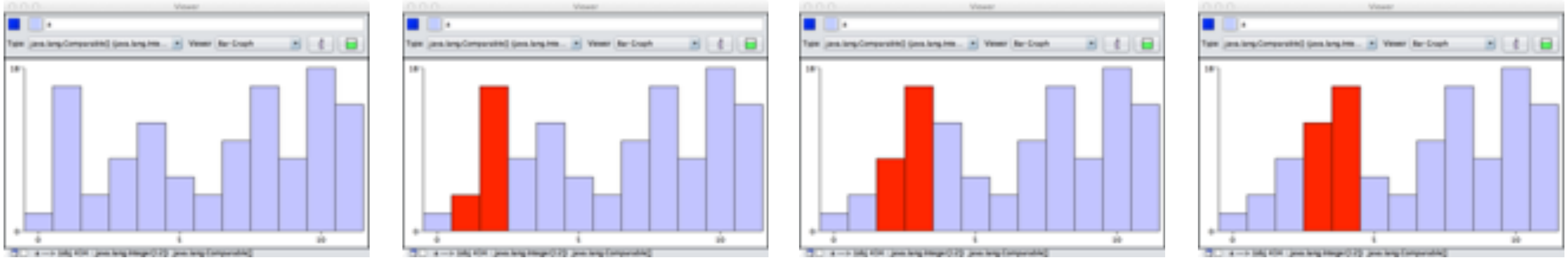
A. Insertion sort
B. Selection sort



Participation

Q: Given the following sequence of images from a jGRASP viewer showing the successive changes in the elements of an array, what sorting algorithm is being used by the underlying program?

A. Insertion sort
B. Selection sort



Sorting more efficiently

Insertion sort and selection sort both have $O(N^2)$ performance, so they don't scale very well.

	1,000	10,000	100,000	1,000,000	10,000,000
O(log N)	< 1 ns	10 ns	132 ns	166 ns	199 ns
O(N)	1 ns	10 ns	100 ns	1 ms	10 ms
O(NlogN)	10 ns	100 ns	2 ms	20 ms	0.2 sec
$O(N^2)$	**1 ms**	**0.1 sec**	**10 sec**	**17 min**	**28 hours**
$O(N^3)$	1 sec	17 min	12 days	32 yrs	32,000 yrs
$O(N^4)$	17 min	4 months	3,200 yrs	3.2 M yrs	3.17E15 yrs
$O(2^N)$	3.4E284 yrs	??	??	??	??
O(N!)	??	??	??	??	??

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How could we sort more efficiently?

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Impose additional constraints on the problem.

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Example: The values being sorted must be integers in a given range. => Counting sort O(N)

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← Counting sort, radix sort, etc. O(N)

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Use a divide-and-conquer algorithm

Example: Divide the array in half, sort each half, then combine the sorted halves.
=> Merge sort O(NlogN)

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← Merge sort, quicksort, O(NlogN)

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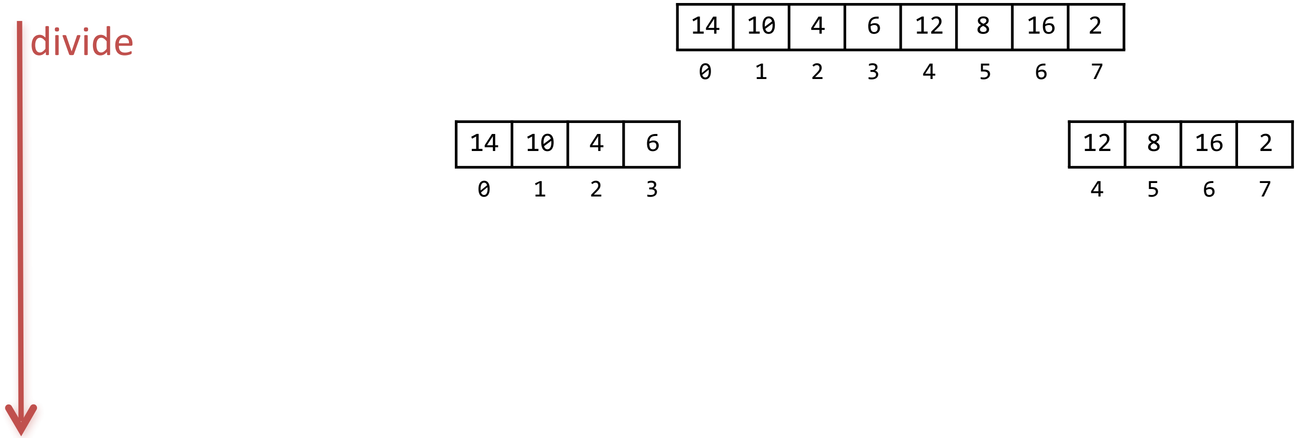

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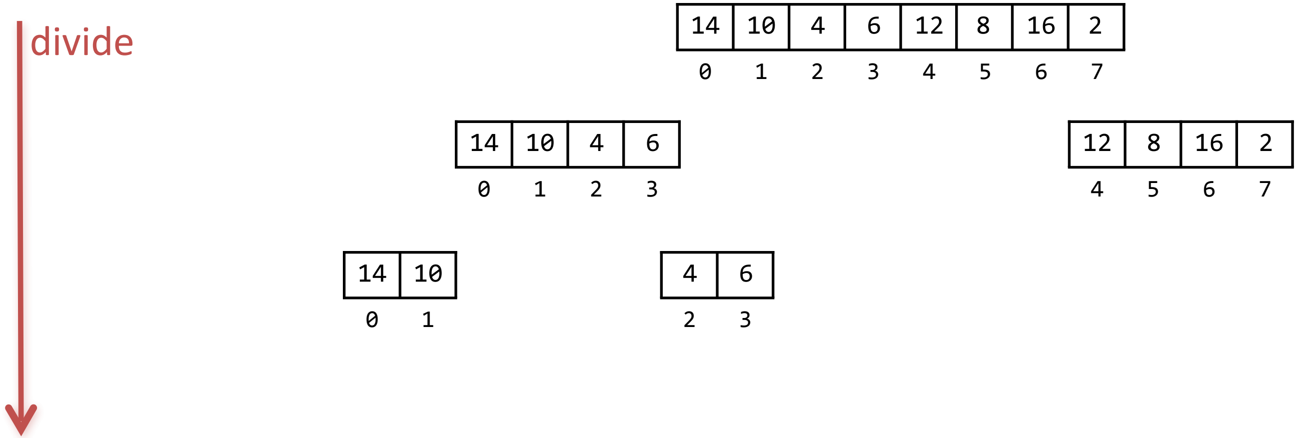

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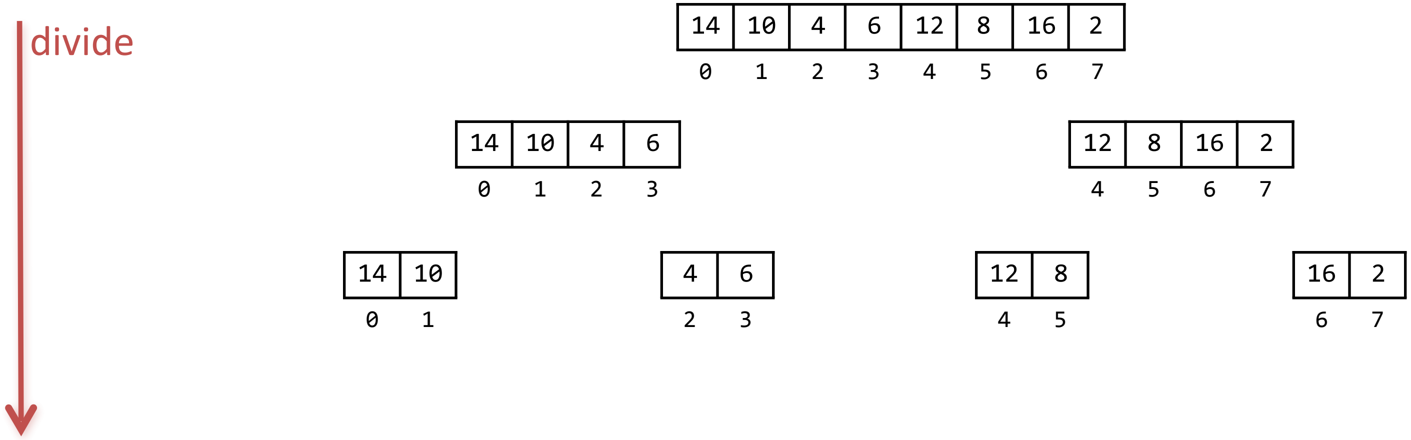

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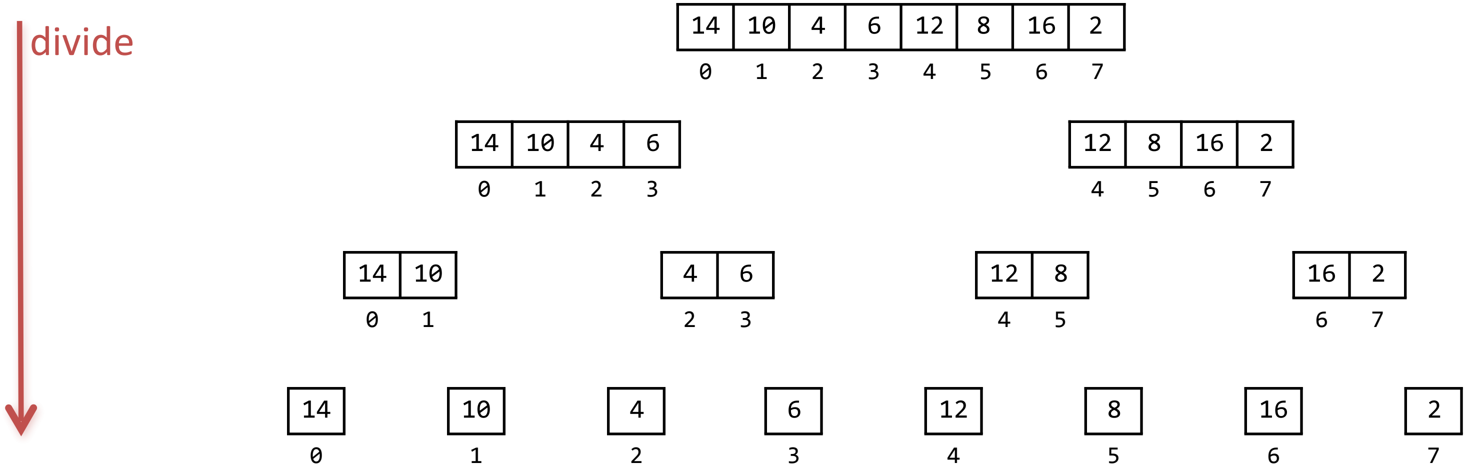

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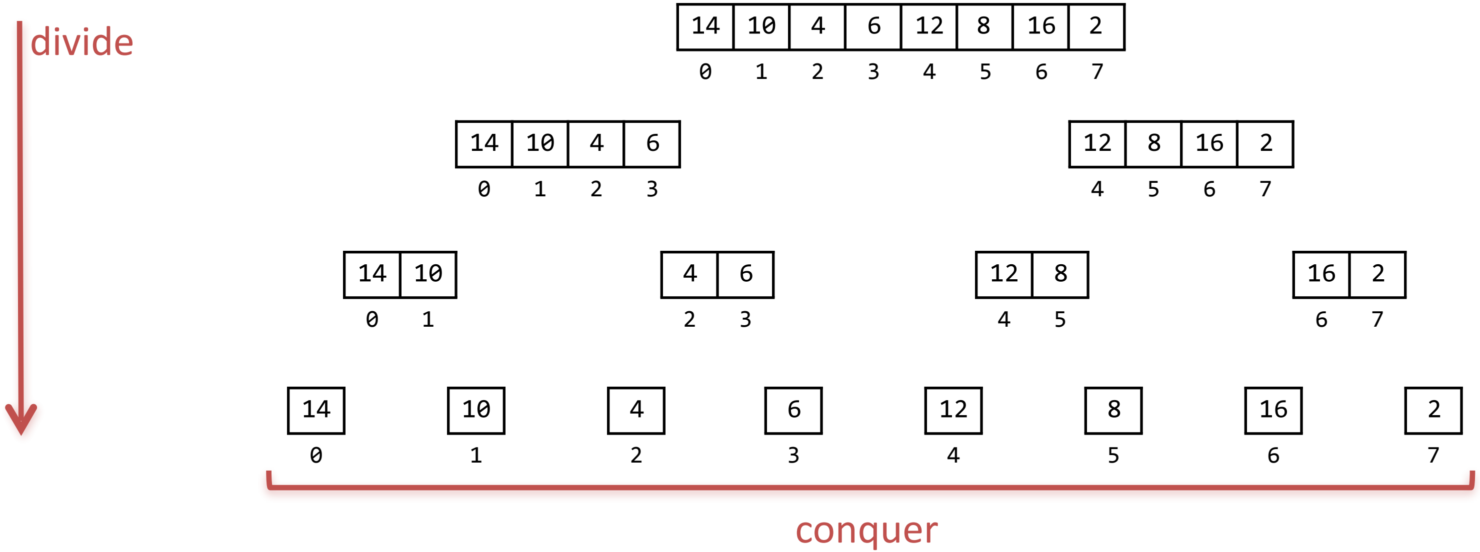

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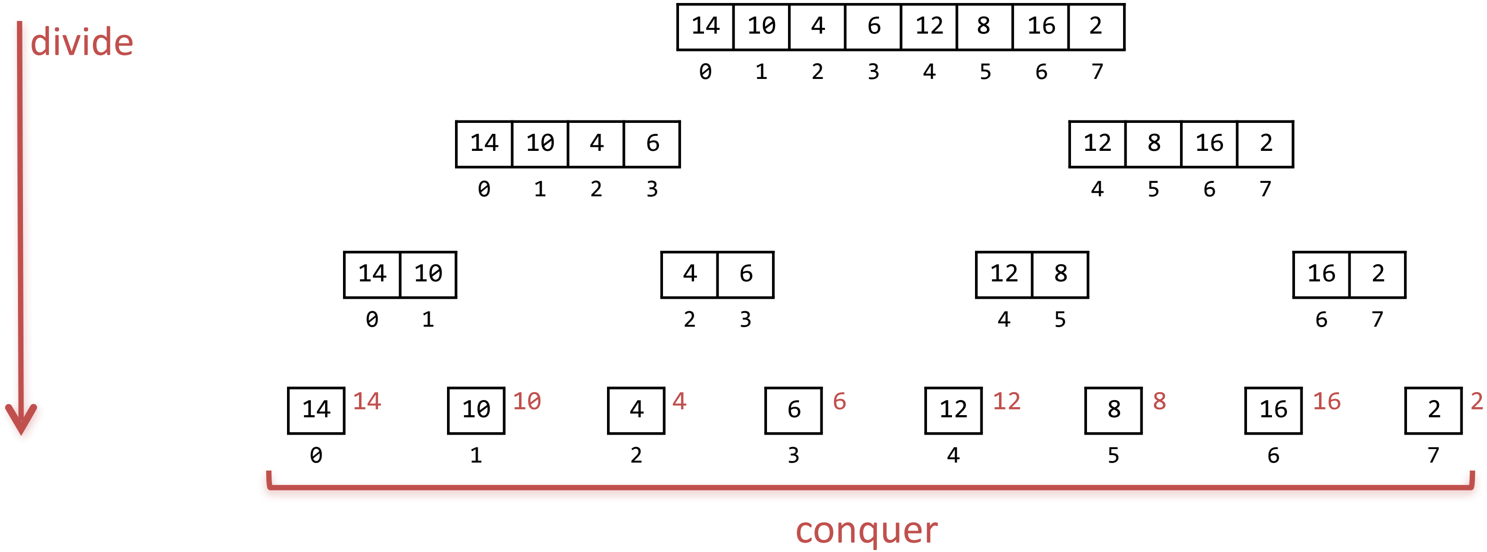

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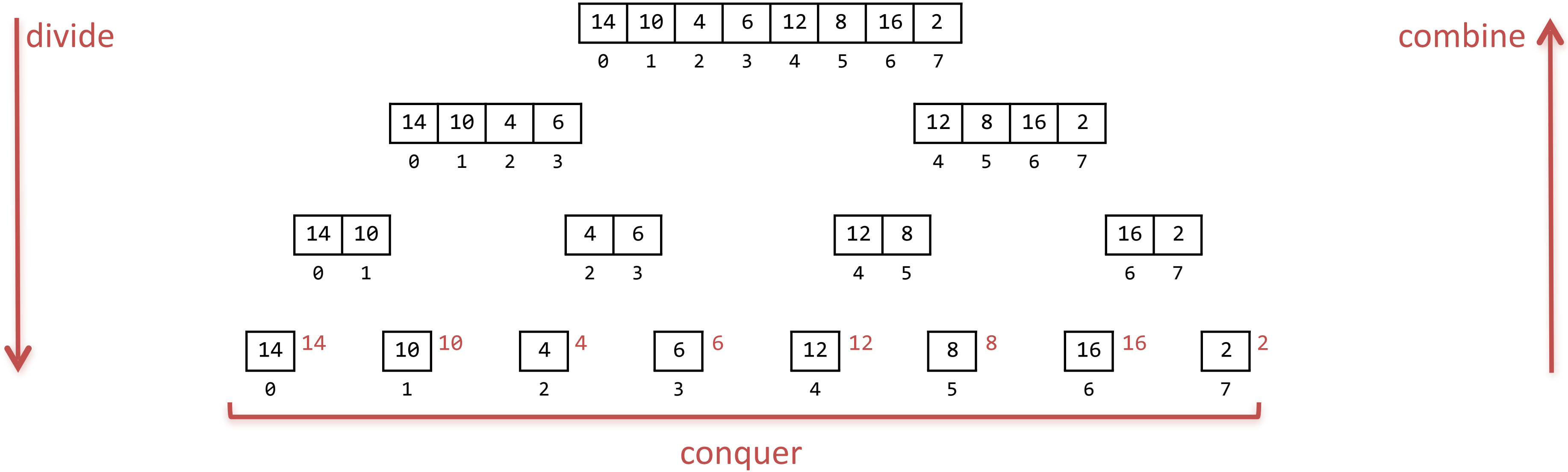

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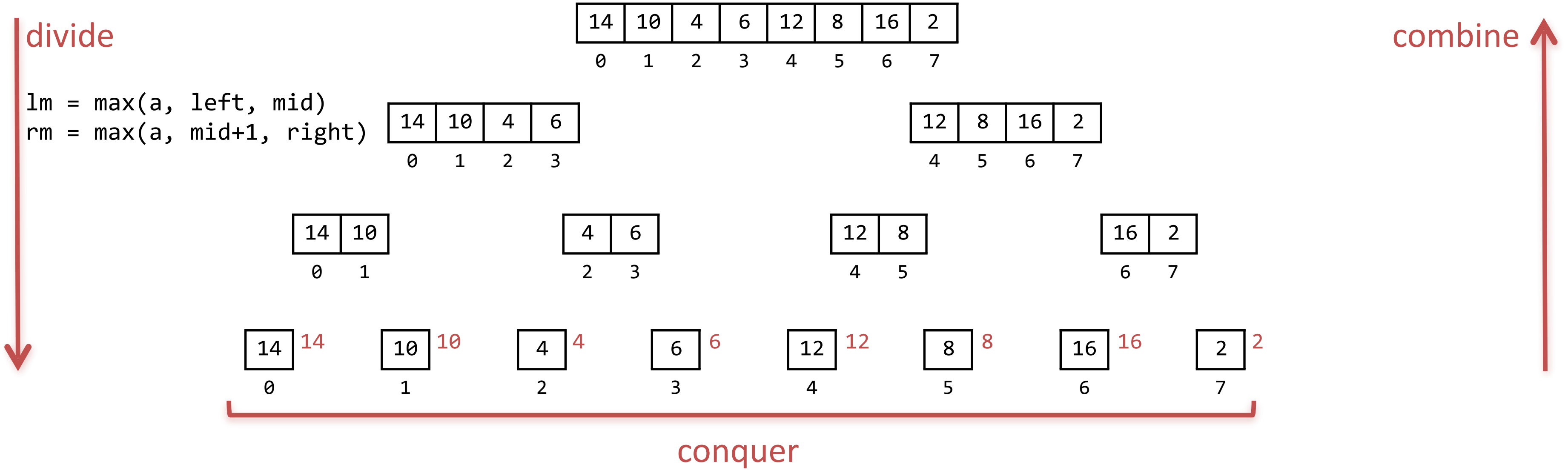

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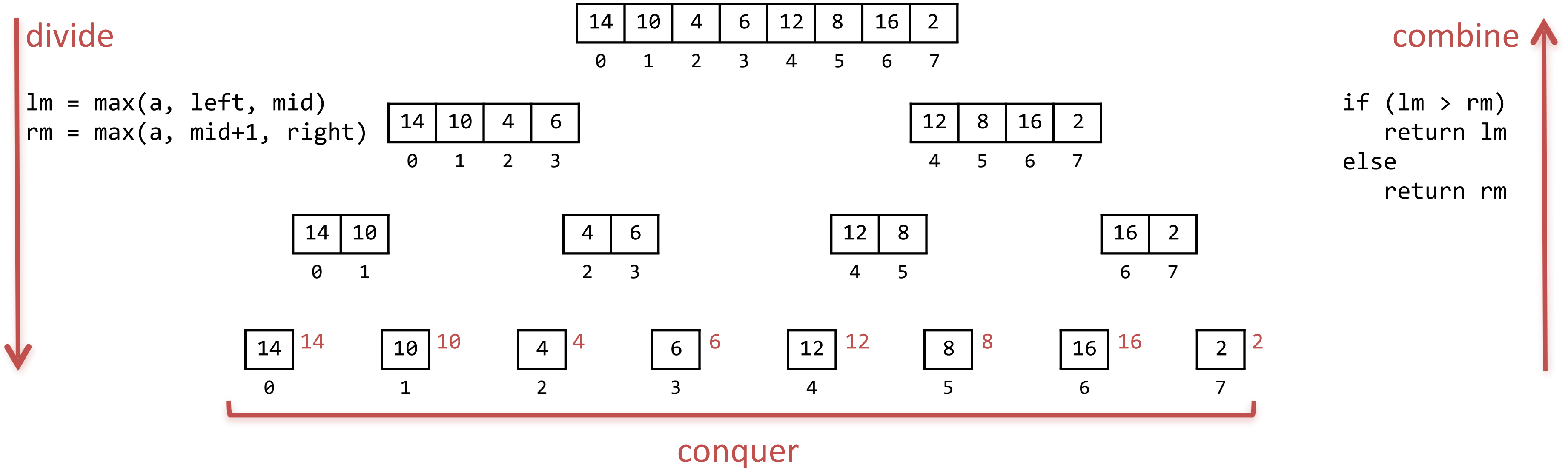

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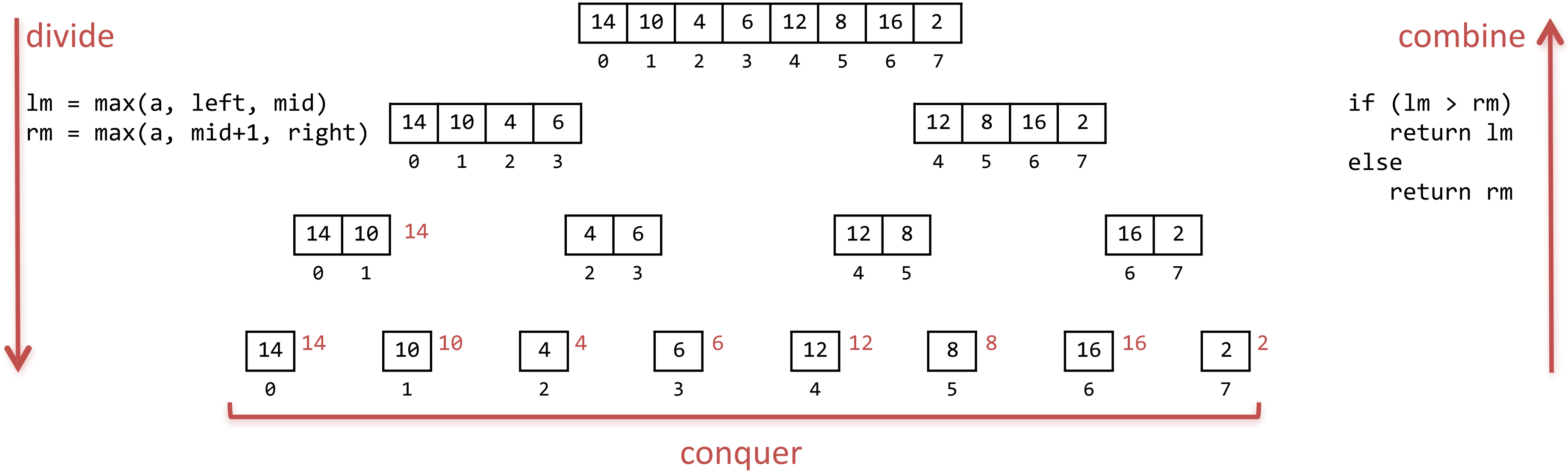

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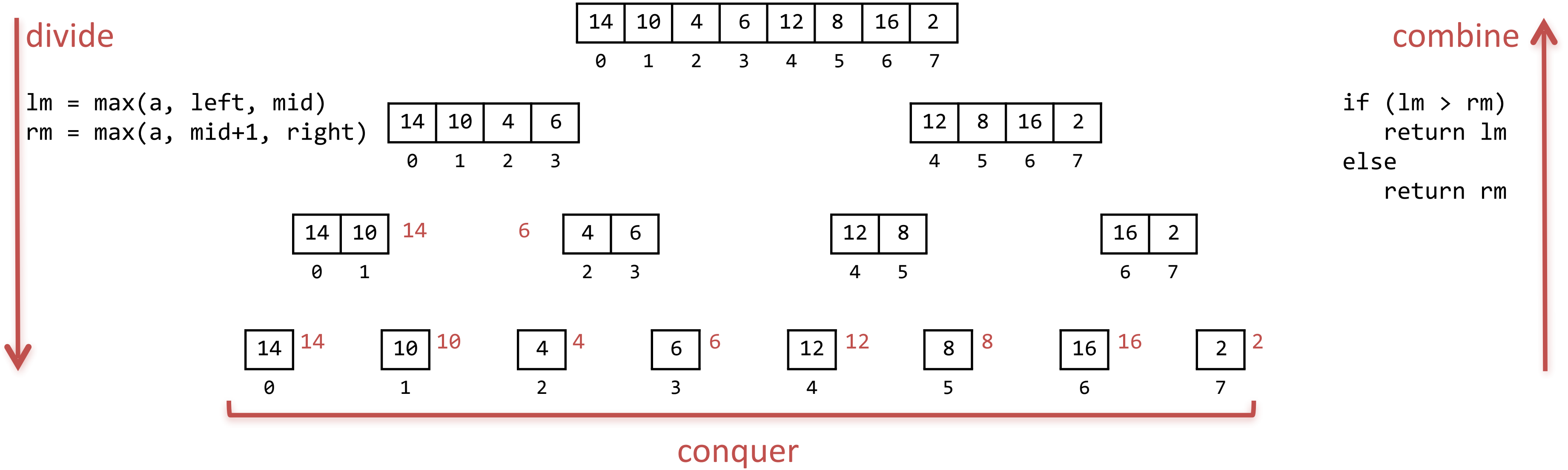
Divide and conquer algorithms are usually expressed **recursively**, and the division is repeated until each part is small enough to be solved directly or trivially. *(more to come . . .)*

```
public int max(int[] a, int left, int right) { . . . }
```



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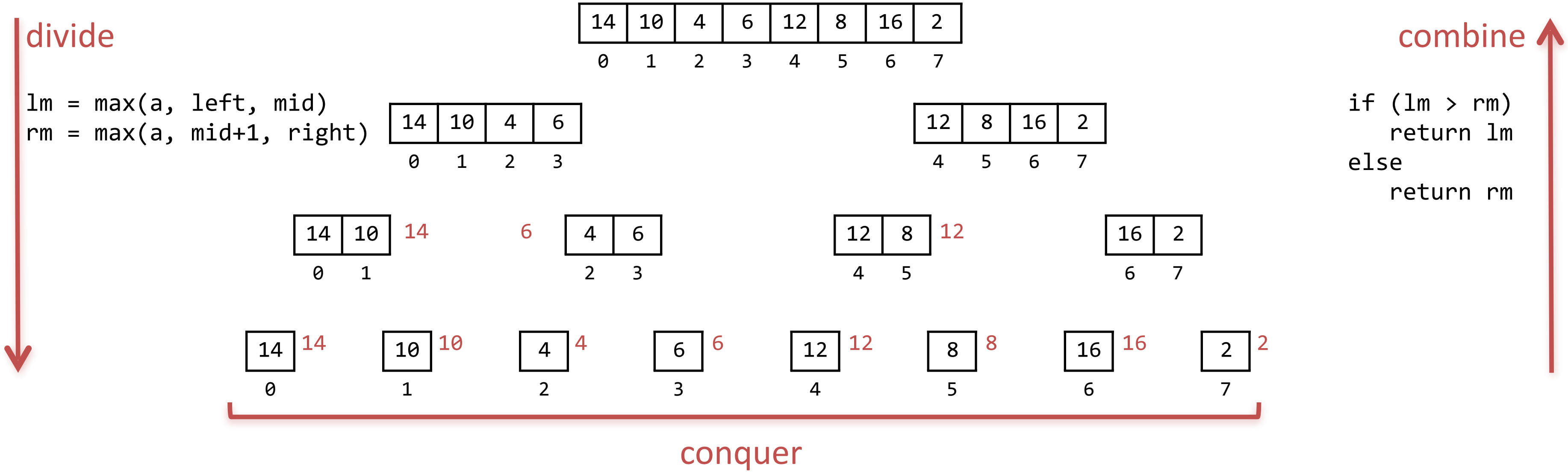

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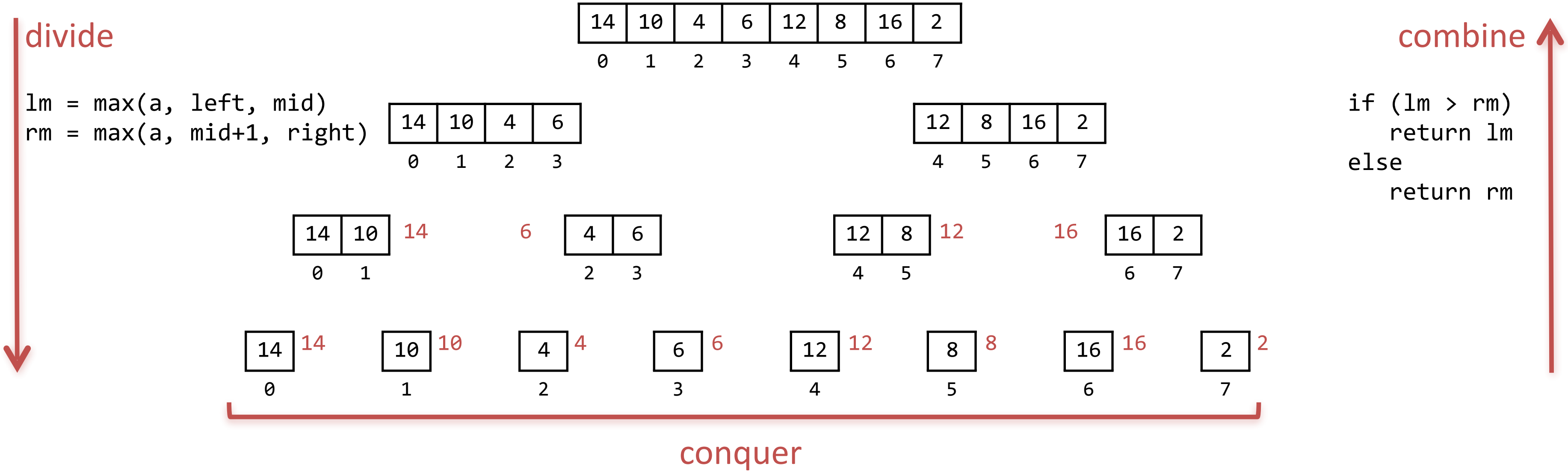

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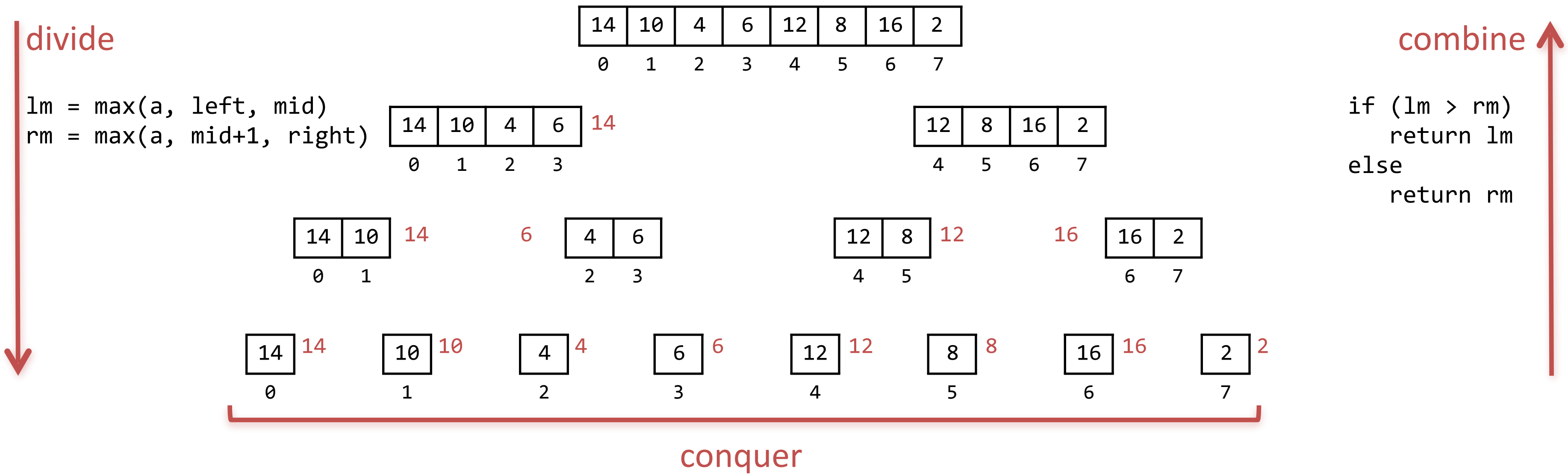

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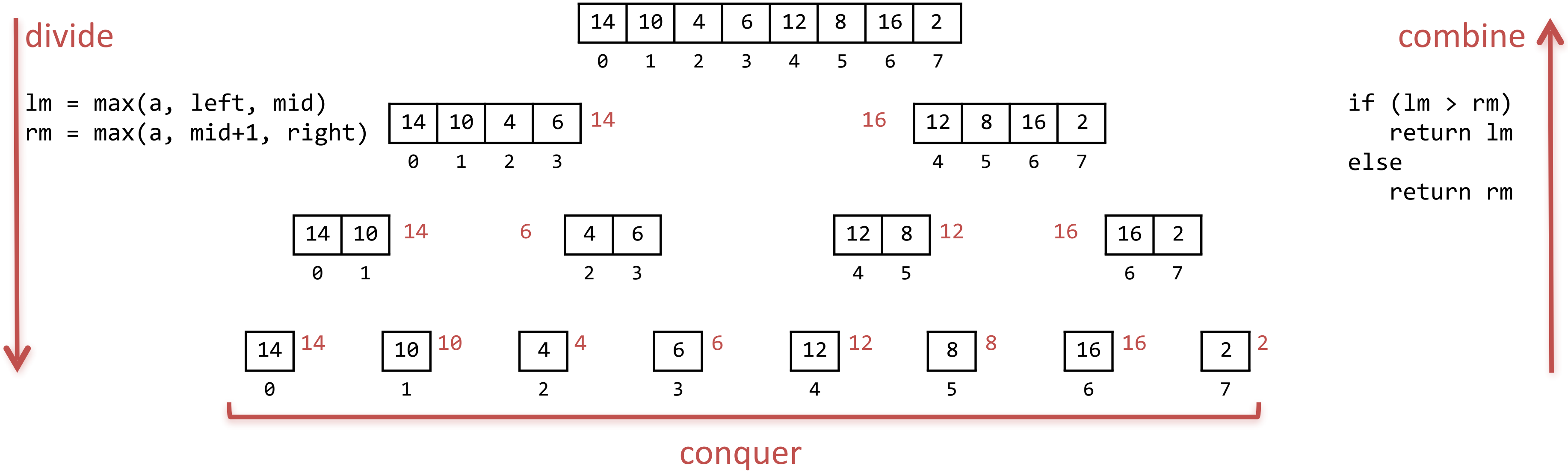

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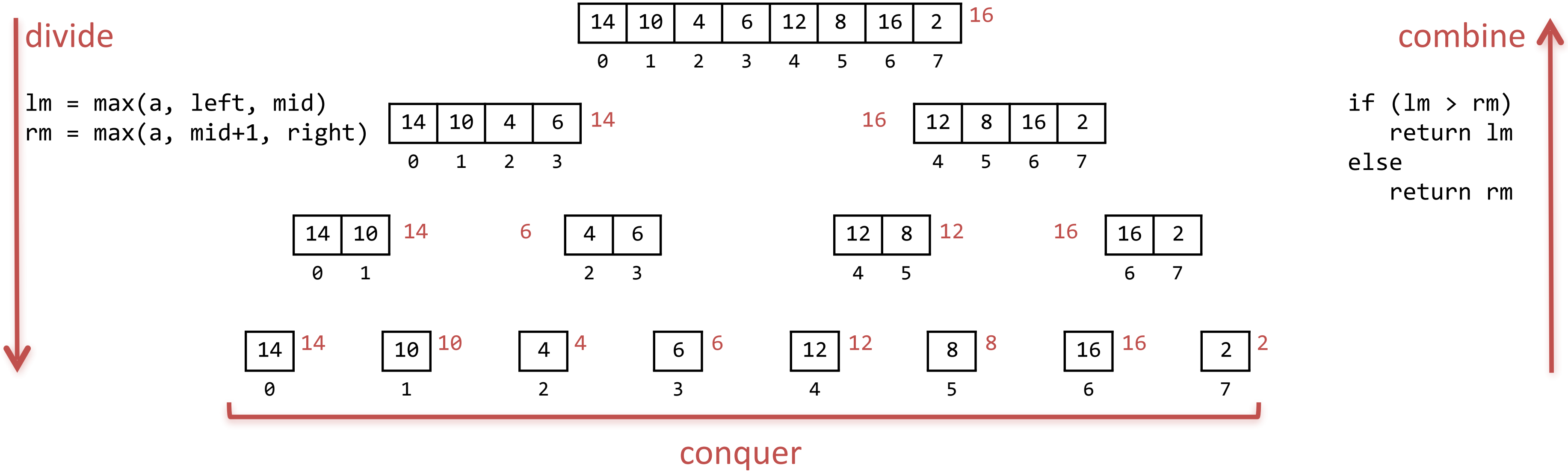

Divide and conquer

A divide-and-conquer solution for the maximum element in an array:

Just for illustration!

```
public static int max(int[] a, int l, int r) {
    if (l == r) {
        return a[l];
    }

    int mid = (l + r) / 2;
    int lm = max(a, l, mid);
    int rm = max(a, mid + 1, r);

    if (lm > rm )
        return lm;
    else
        return rm;
}
```

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divide

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```

conquer

divide

Divide and conquer

A divide-and-conquer solution for the maximum element in an array:

Just for illustration!

```
public static int max(int[] a, int l, int r) {
    if (l == r) {
        return a[l];        conquer
    }

    int mid = (l + r) / 2;
    int lm = max(a, l, mid);                divide
    int rm = max(a, mid + 1, r);

    if (lm > rm )
        return lm;
    else                   combine
        return rm;
}
```

D&C sorting?

D&C sorting?

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

D&C sorting?

divide

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

D&C sorting?

divide

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

14	10	4	6
0	1	2	3

D&C sorting?

divide

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

14	10	4	6
0	1	2	3

12	8	16	2
4	5	6	7

D&C sorting?

divide

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

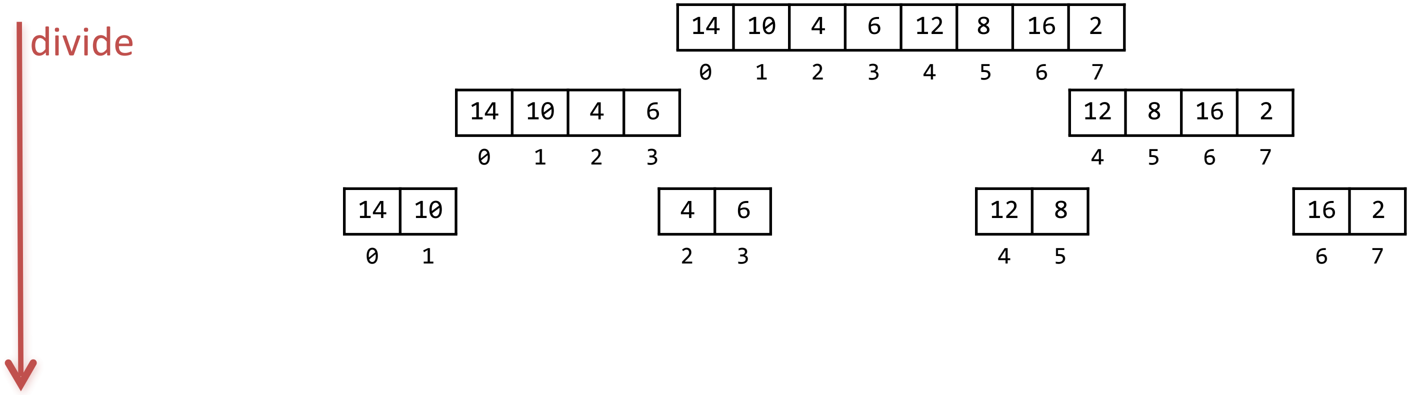
14	10	4	6
0	1	2	3

12	8	16	2
4	5	6	7

14	10
0	1

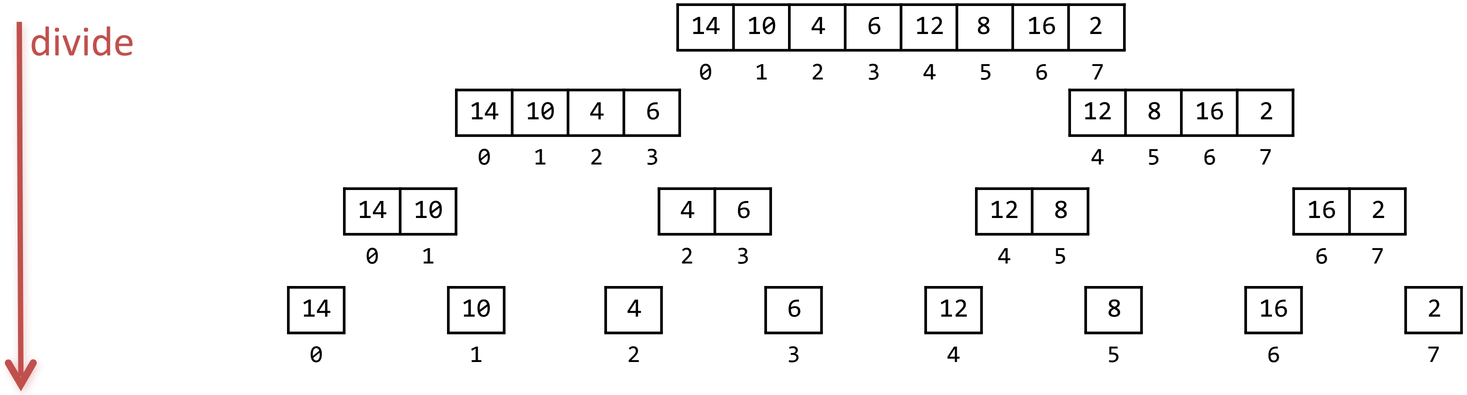
4	6
2	3

D&C sorting?

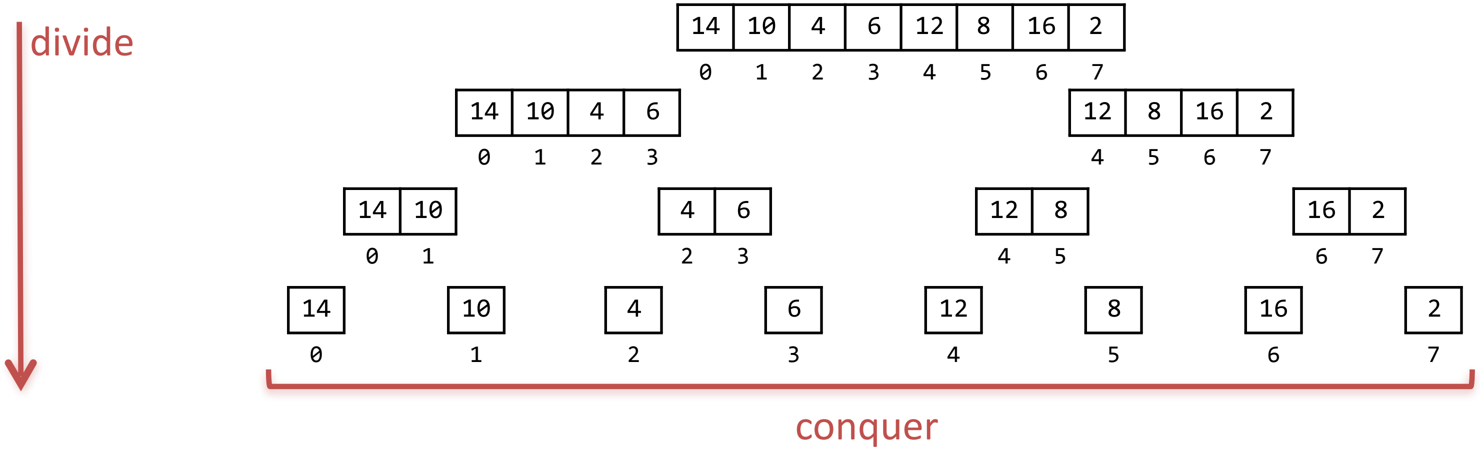


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7

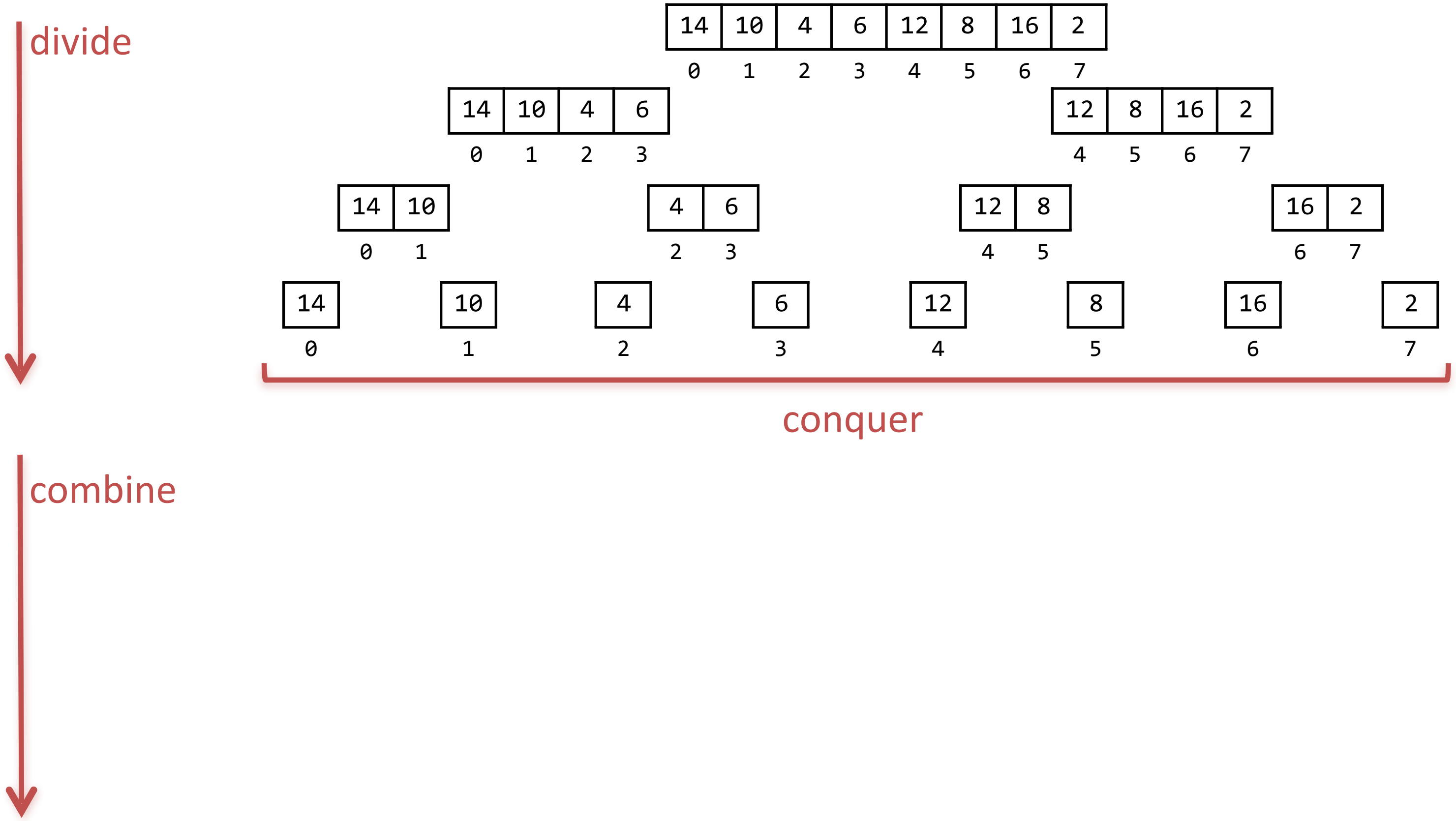
D&C sorting?


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14
0
10
1
4
2
6
3
12
4
8
5
16
6
2
7

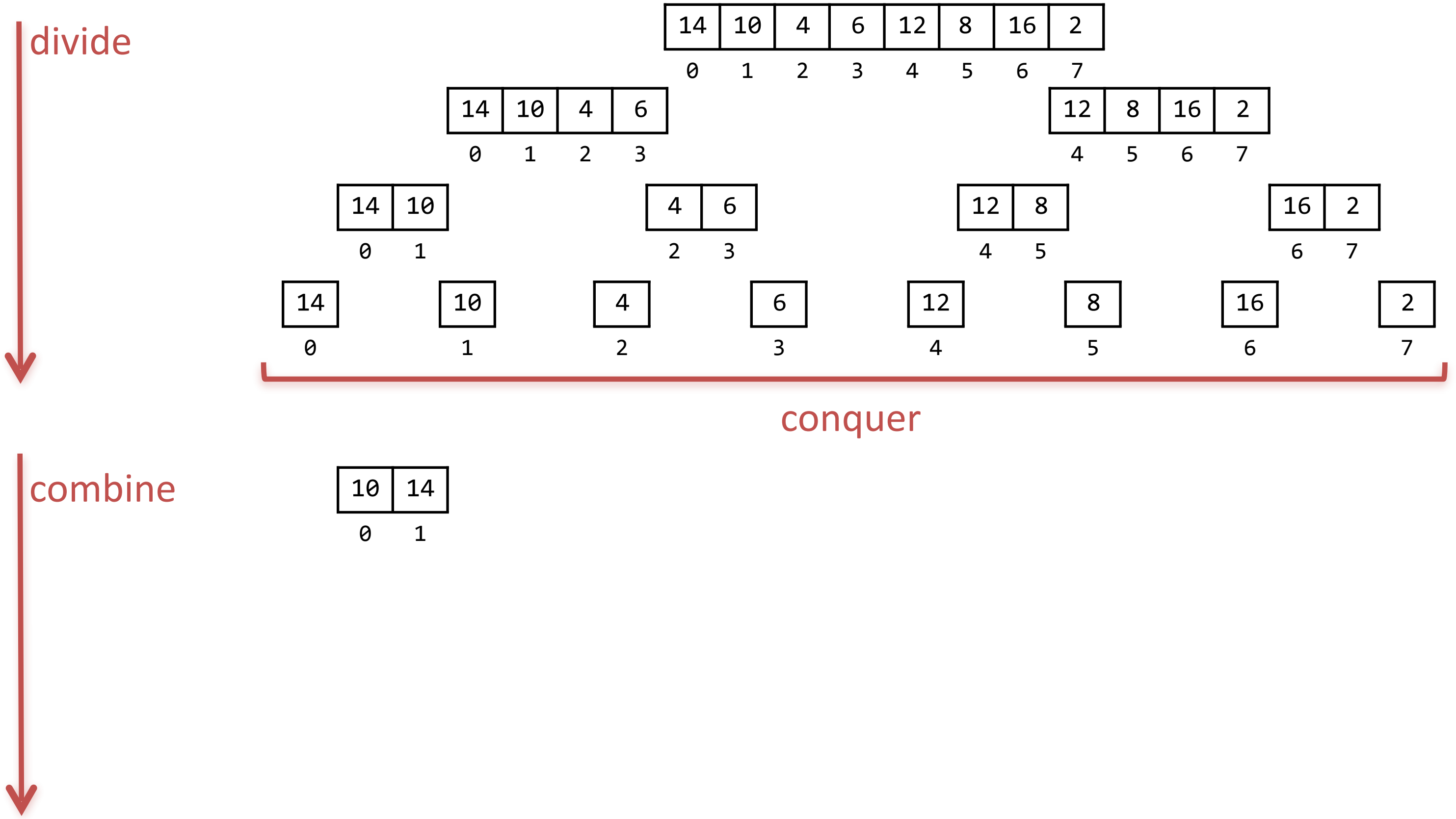
D&C sorting?


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
conquer

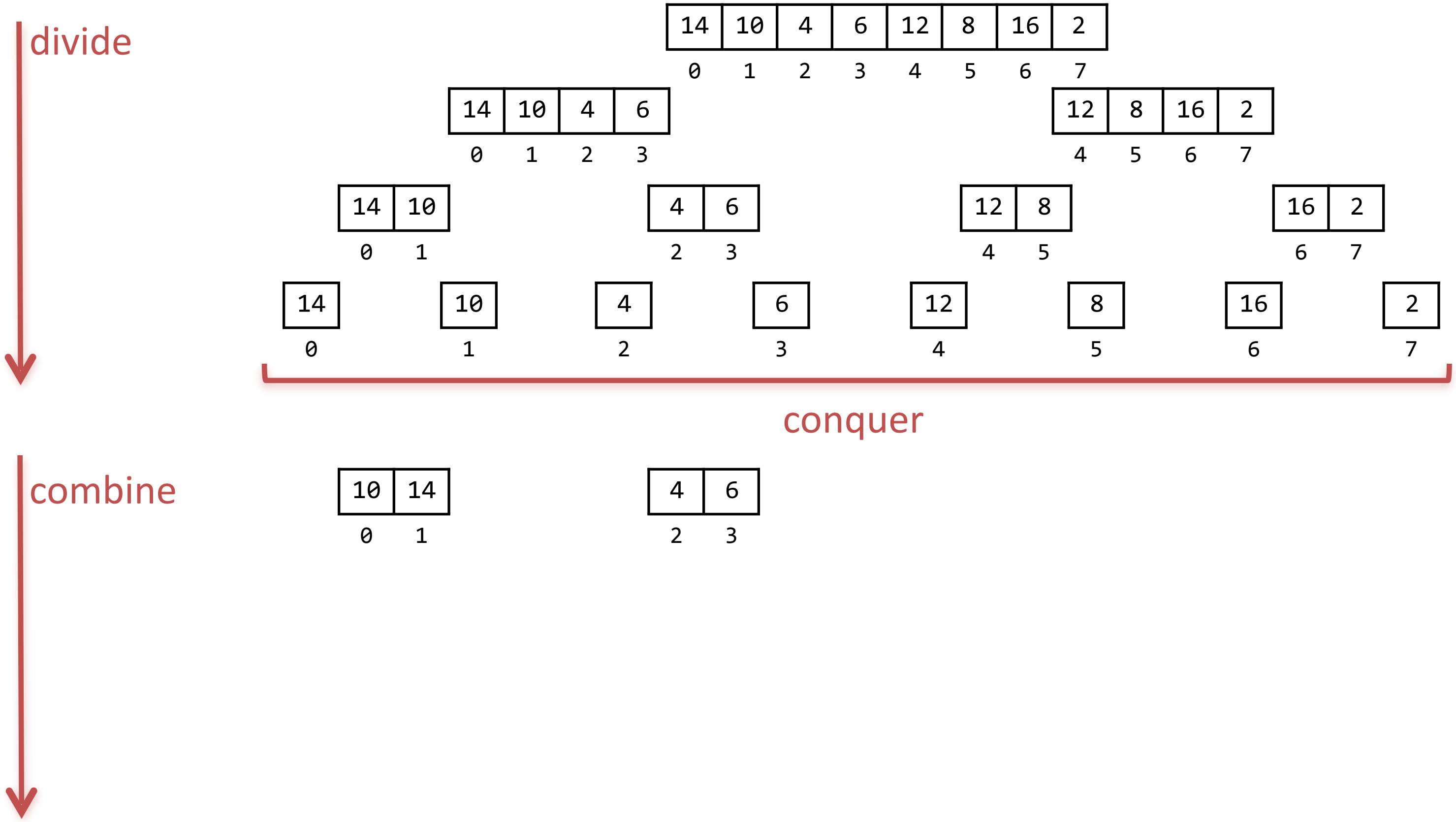
D&C sorting?


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
conquer
combine

D&C sorting?

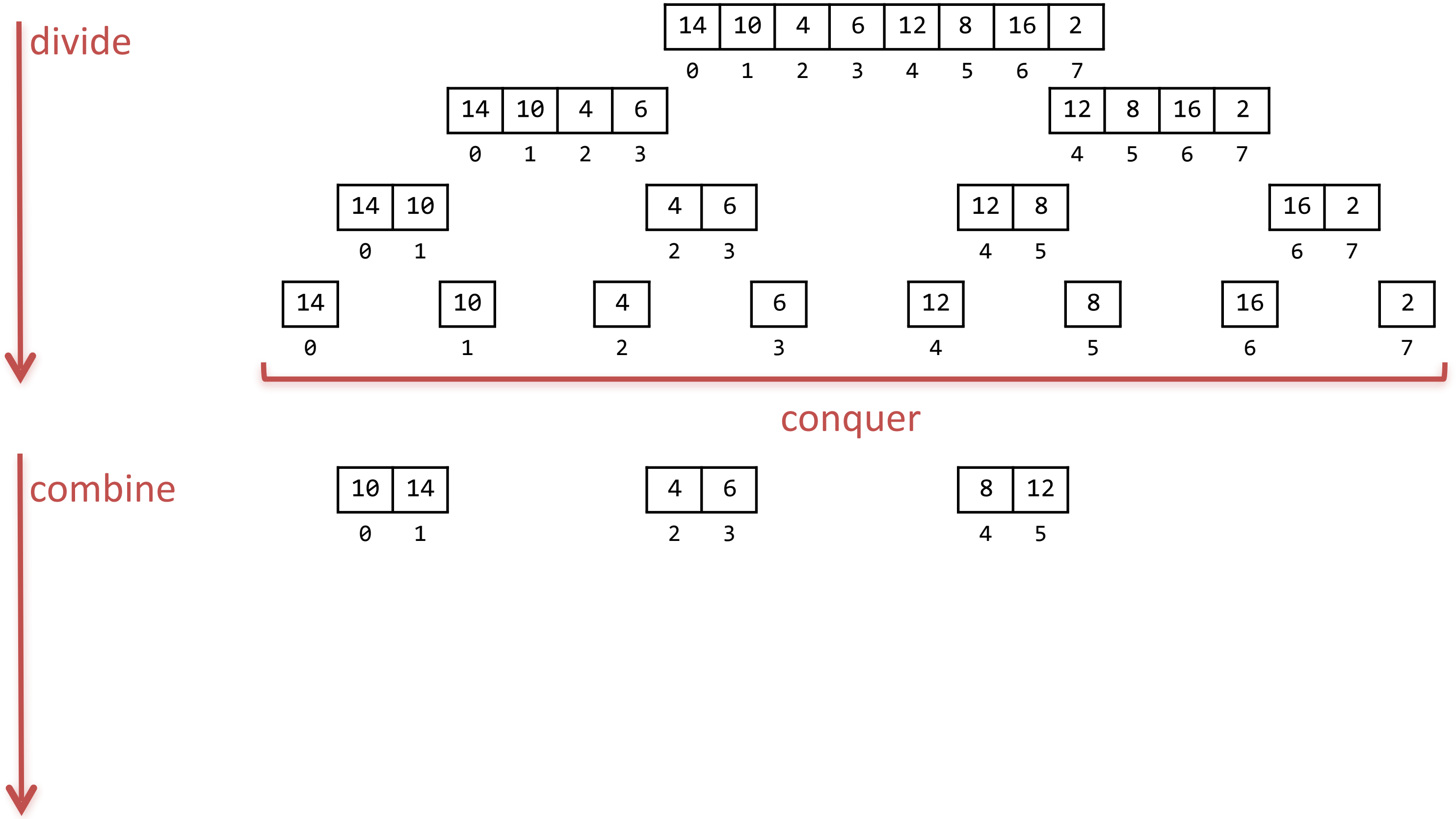

divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
conquer
combine
10 14
0 1

D&C sorting?

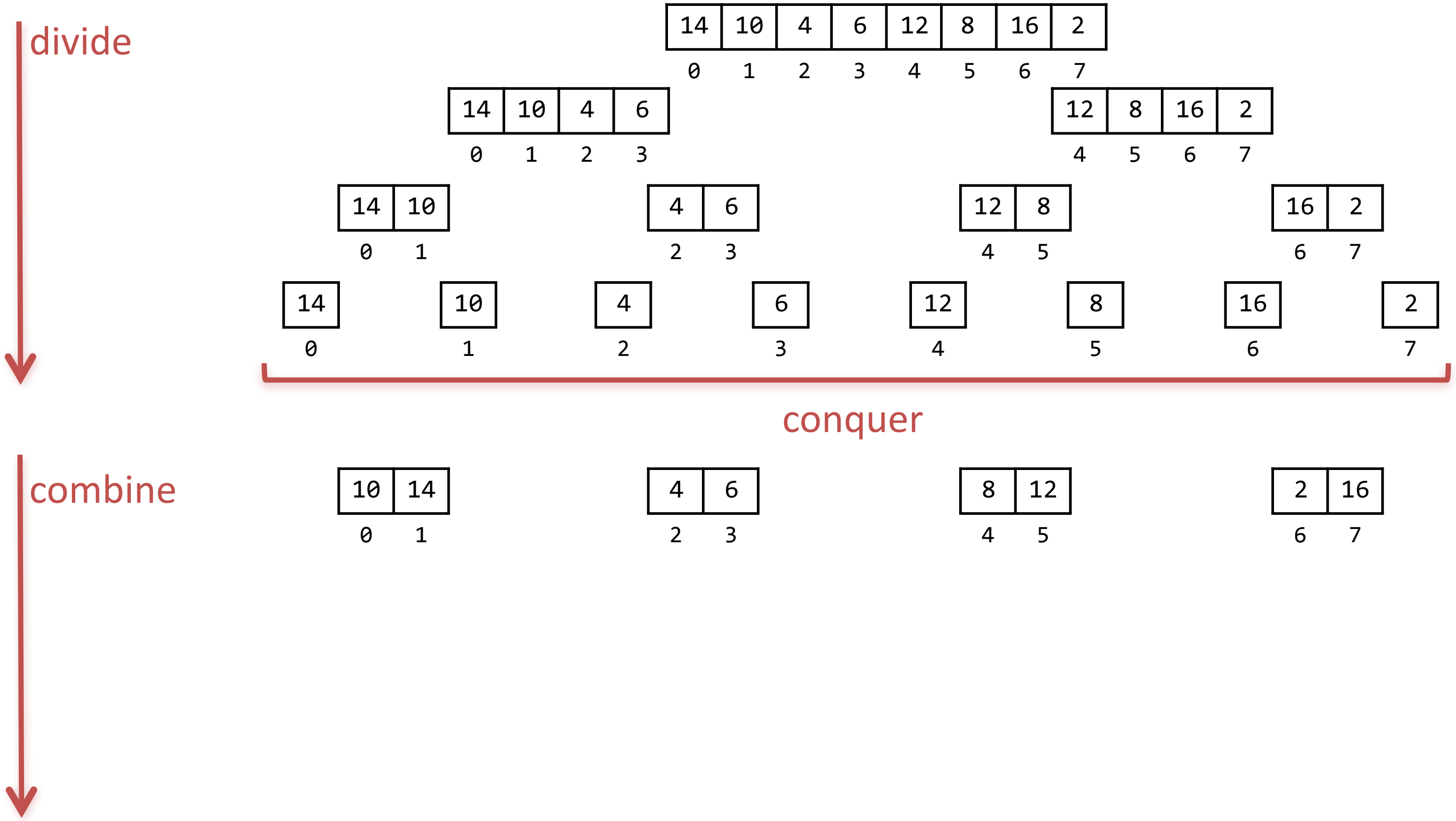


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14
0
10
1
4
2
6
3
12
4
8
5
16
6
2
7
conquer
combine
10 14
0 1
4 6
2 3

D&C sorting?


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
conquer
combine
10 14
0 1
4 6
2 3
8 12
4 5

D&C sorting?


divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
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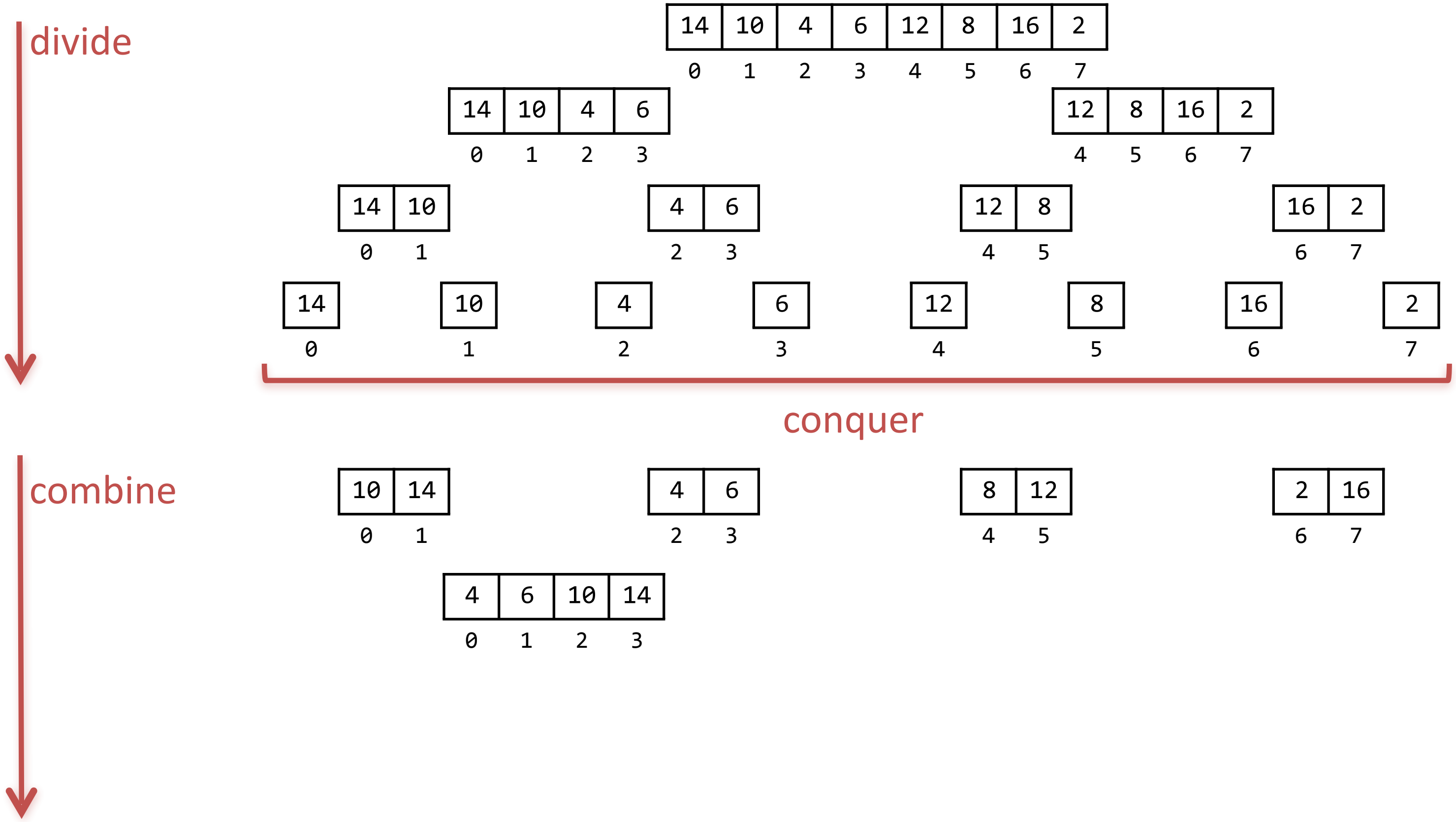
14 10
0 1
4 6
2 3
12 8
4 5
16 2
6 7
14
0
10
1
4
2
6
3
12
4
8
5
16
6
2
7
conquer
combine
10 14
0 1
4 6
2 3
8 12
4 5
2 16
6 7

D&C sorting?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14 10 4 6 12 8 16 2
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14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
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6 7
14
0
10
1
4
2
6
3
12
4
8
5
16
6
2
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0 1
4 6
2 3
8 12
4 5
2 16
6 7
4 6 10 14
0 1 2 3

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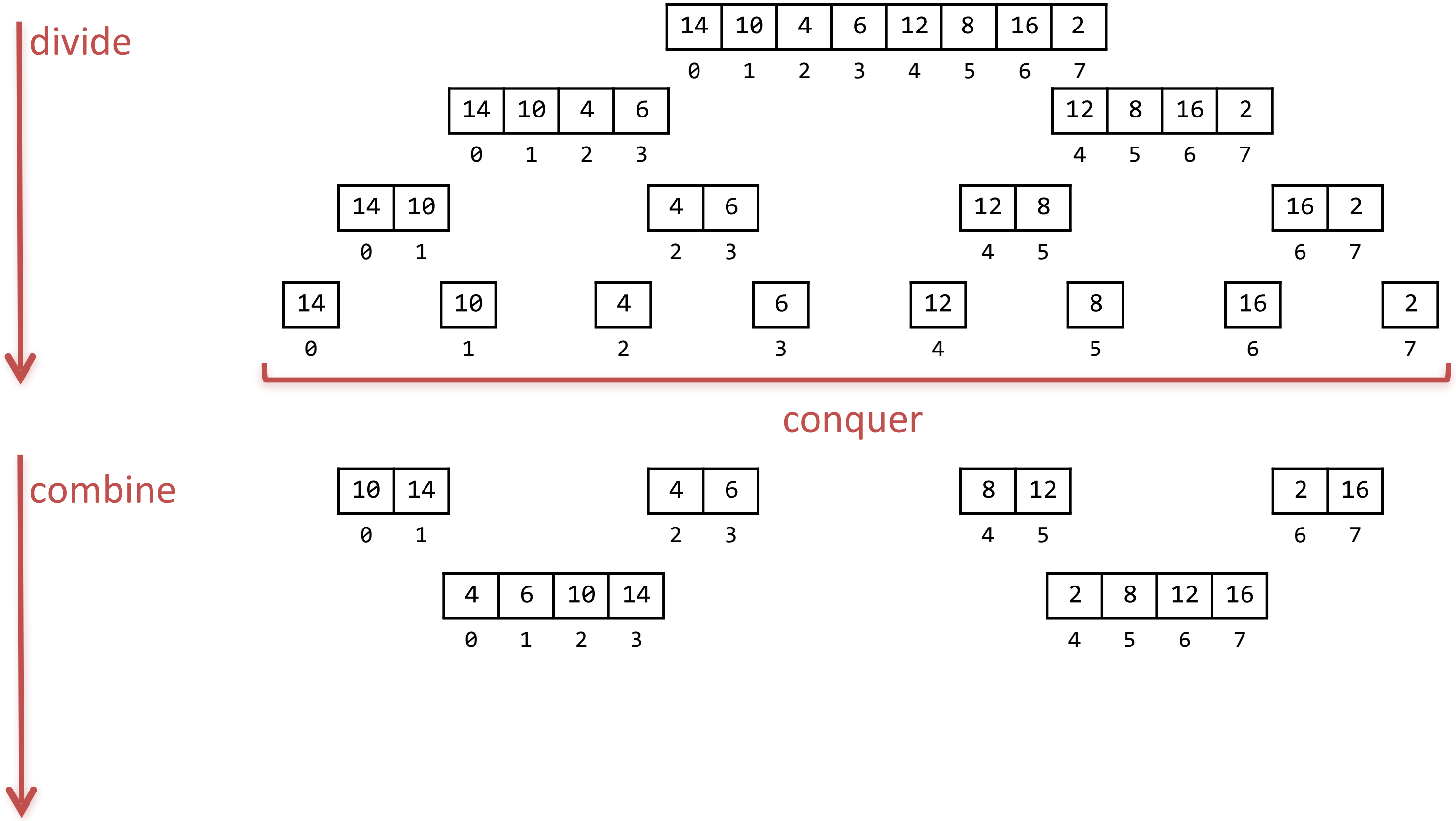
divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
14 10 4 6
0 1 2 3
12 8 16 2
4 5 6 7
14 10
0 1
4 6
2 3
12 8
4 5
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4
8
5
16
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conquer
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4 6 10 14
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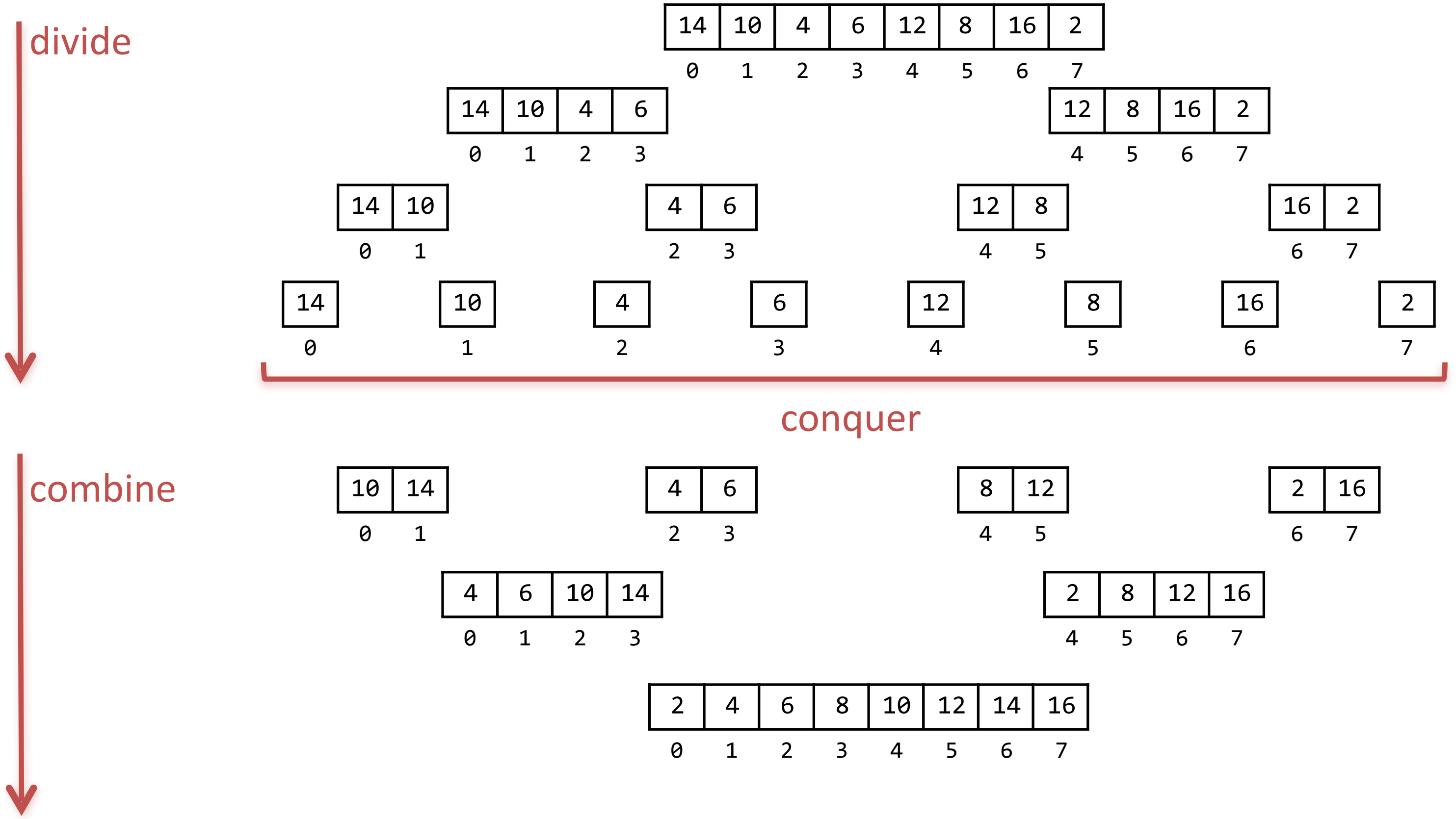

divide
14 10 4 6 12 8 16 2
0 1 2 3 4 5 6 7
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14 10
0 1
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12 8
4 5
16 2
6 7
14
0
10
1
4
2
6
3
12
4
8
5
16
6
2
7
conquer
combine
10 14
0 1
4 6
2 3
8 12
4 5
2 16
6 7
4 6 10 14
0 1 2 3
2 8 12 16
4 5 6 7
2 4 6 8 10 12 14 16
0 1 2 3 4 5 6 7

Merge sort

Merge sort is a comparison sort based on the **divide-and-conquer** strategy.

First described by John von Neumann in 1945 as part of his work on EDVAC.

Asymptotically optimal for comparison sorting – **O(NlogN)**, but requires ~N extra memory (it's **not** *in-place*).

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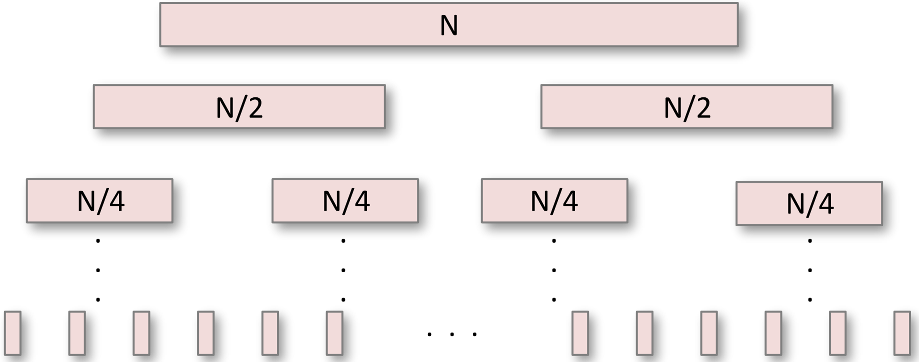
Divide: Divide the array in half.
Conquer: Sort each half (recursively).
Combine: Merge the sorted halves into a single array.



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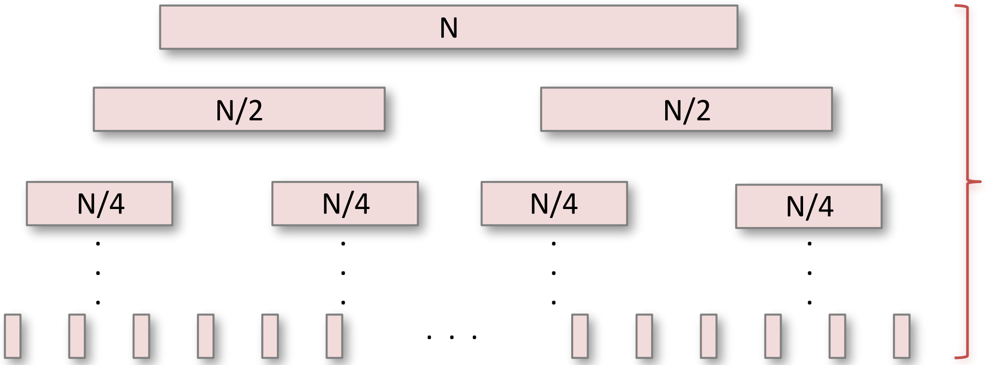

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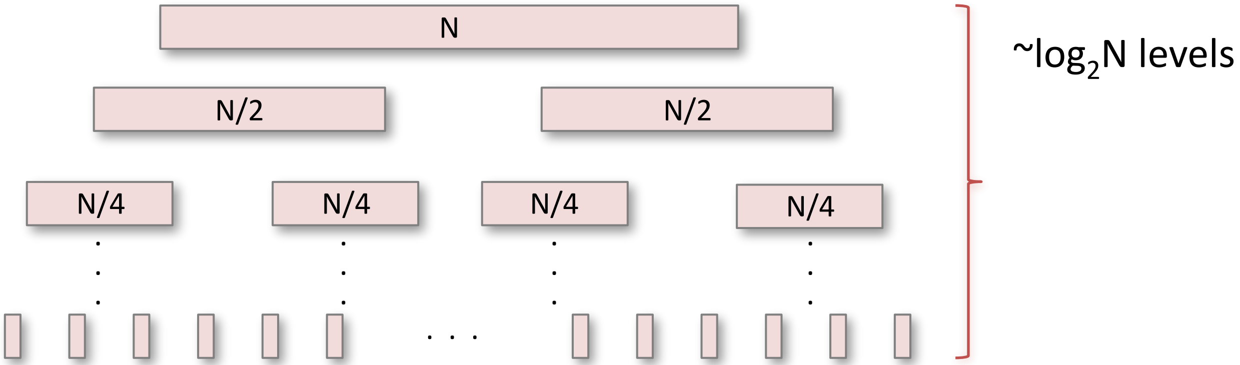

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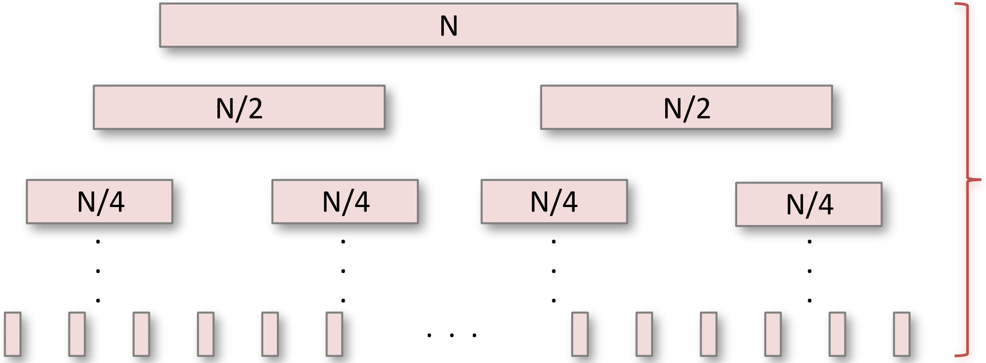

~$\log_2$N levels

N elements to compare, move per level

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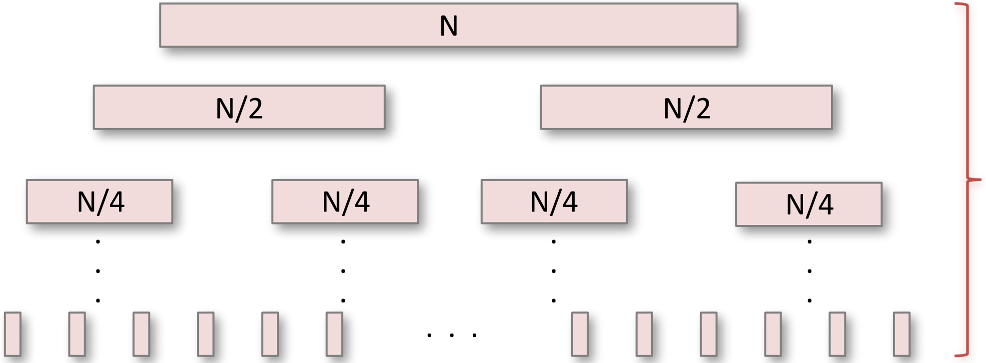

~$\log_2$N levels

N elements to compare, move per level

O(NlogN) work

Merge sort

```
public void mergeSort(Comparable[] a, int left, int right) {

   if (right <= left) return;

   int mid = left + (right – left) / 2;
   mergeSort(a, left, mid);
   mergeSort(a, mid + 1, right);

   merge(a, left, mid, right);
}
```

Merge sort

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divide

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    merge(a, left, mid, right);
}
```

conquer

divide

combine

Merge sort

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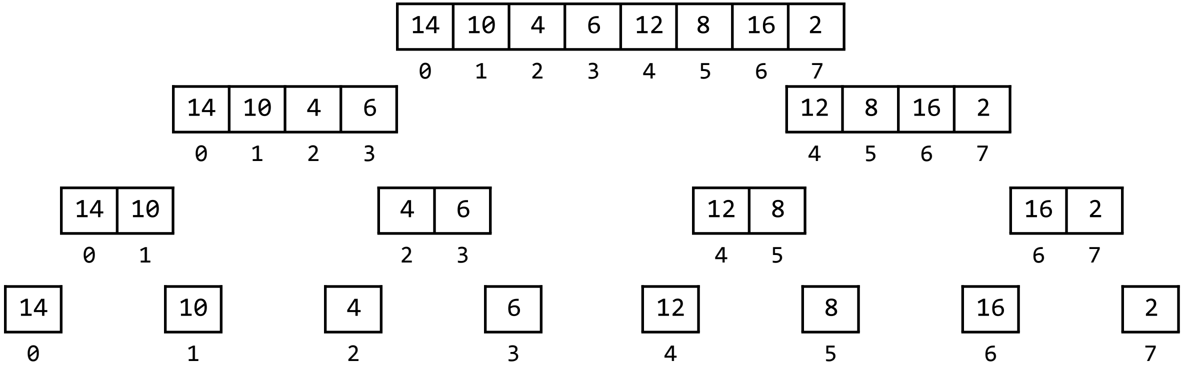

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```
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0	1	2	3	4	5	6	7

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}
```

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

Sorting a one-element array is trivial – it's already sorted so there's nothing to do.

Merge sort

```
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   int mid = left + (right - left) / 2;
   mergeSort(a, left, mid);
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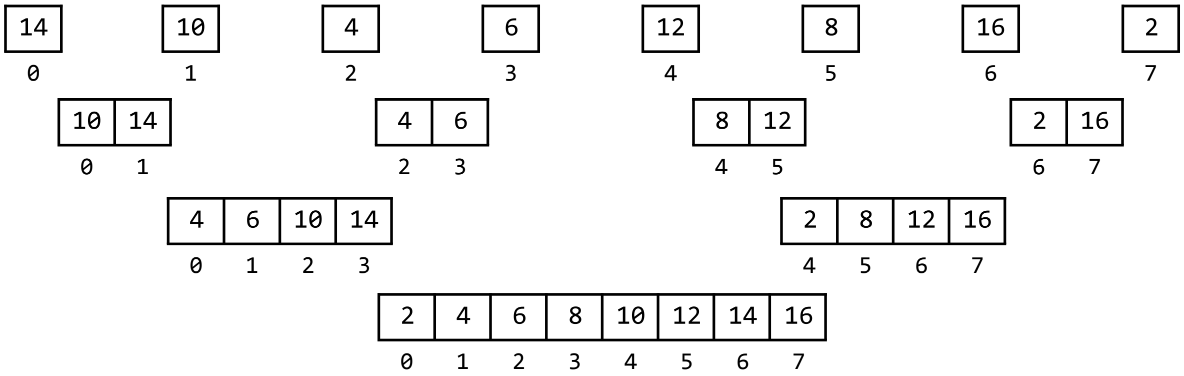
   merge(a, left, mid, right);  combine
}
```



Merge sort

Truth in advertising: order of execution

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0	1	2	3	4	5	6	7

Merge sort

Truth in advertising: order of execution

divide

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

Merge sort

Truth in advertising: order of execution

divide

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

14	10	4	6
0	1	2	3

Merge sort

Truth in advertising: order of execution

divide

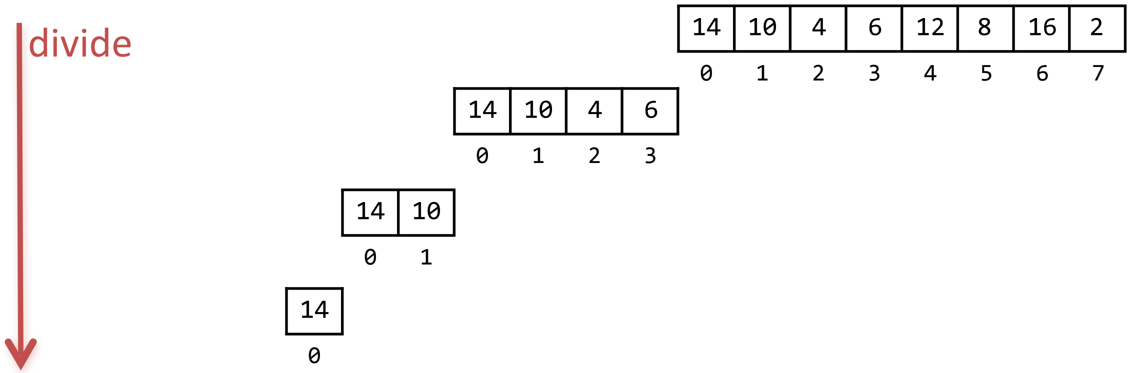
14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

14	10	4	6
0	1	2	3

14	10
0	1

Merge sort

Truth in advertising: order of execution



Merge sort

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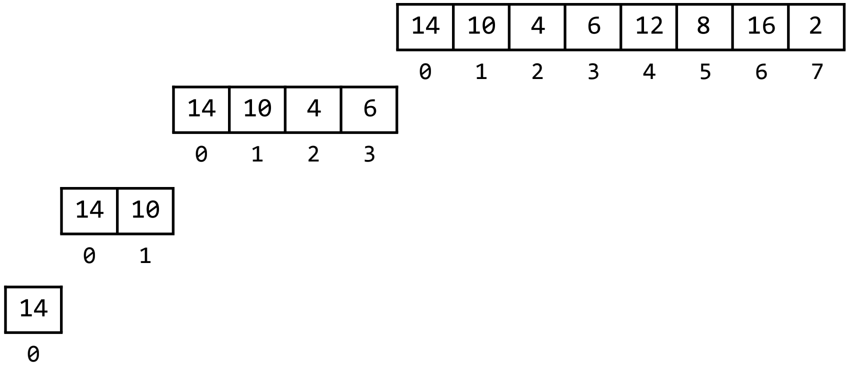

Merge sort

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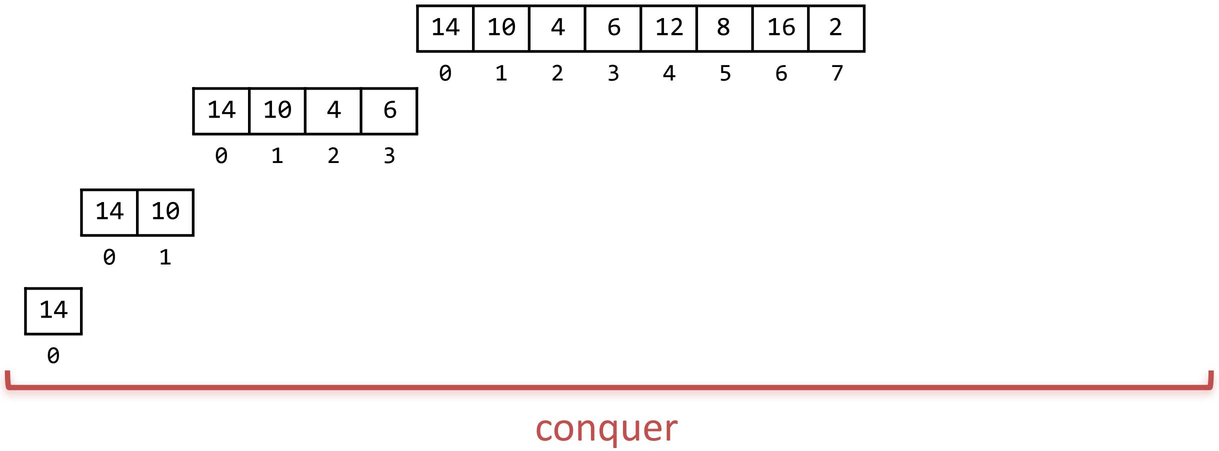
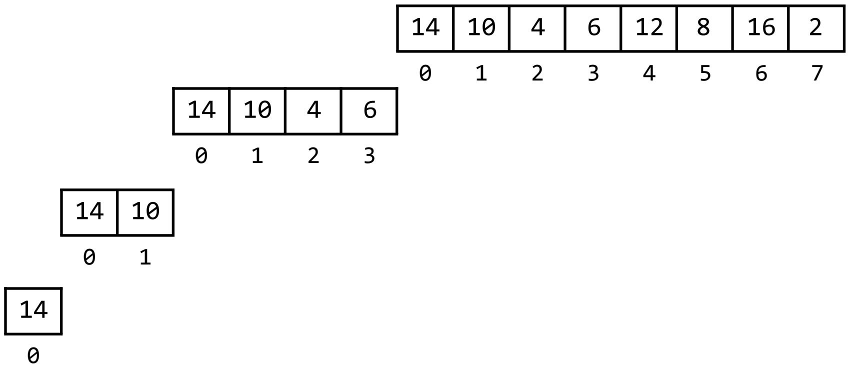


Merge sort

Truth in advertising: order of execution



Merge sort

Truth in advertising: order of execution

14	10	4	6	12	8	16	2
0	1	2	3	4	5	6	7

14	10	4	6
0	1	2	3

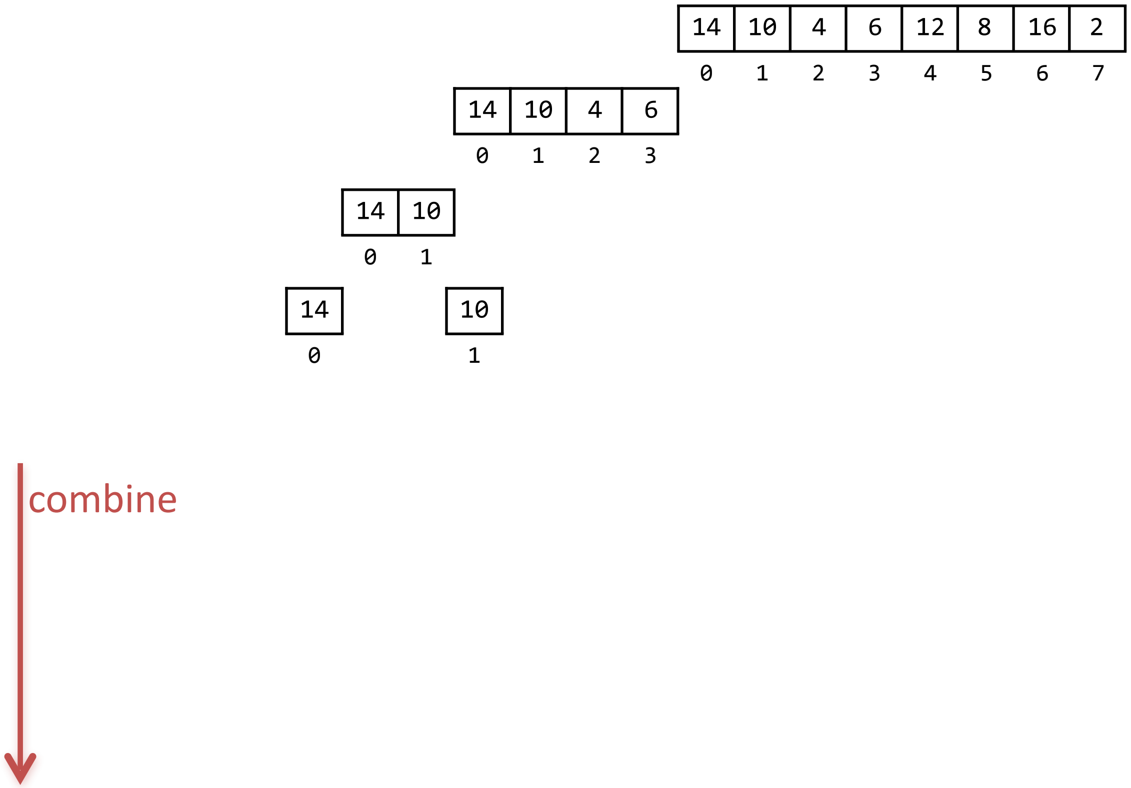
14	10
0	1

14
0

10
1

Merge sort

Truth in advertising: order of execution



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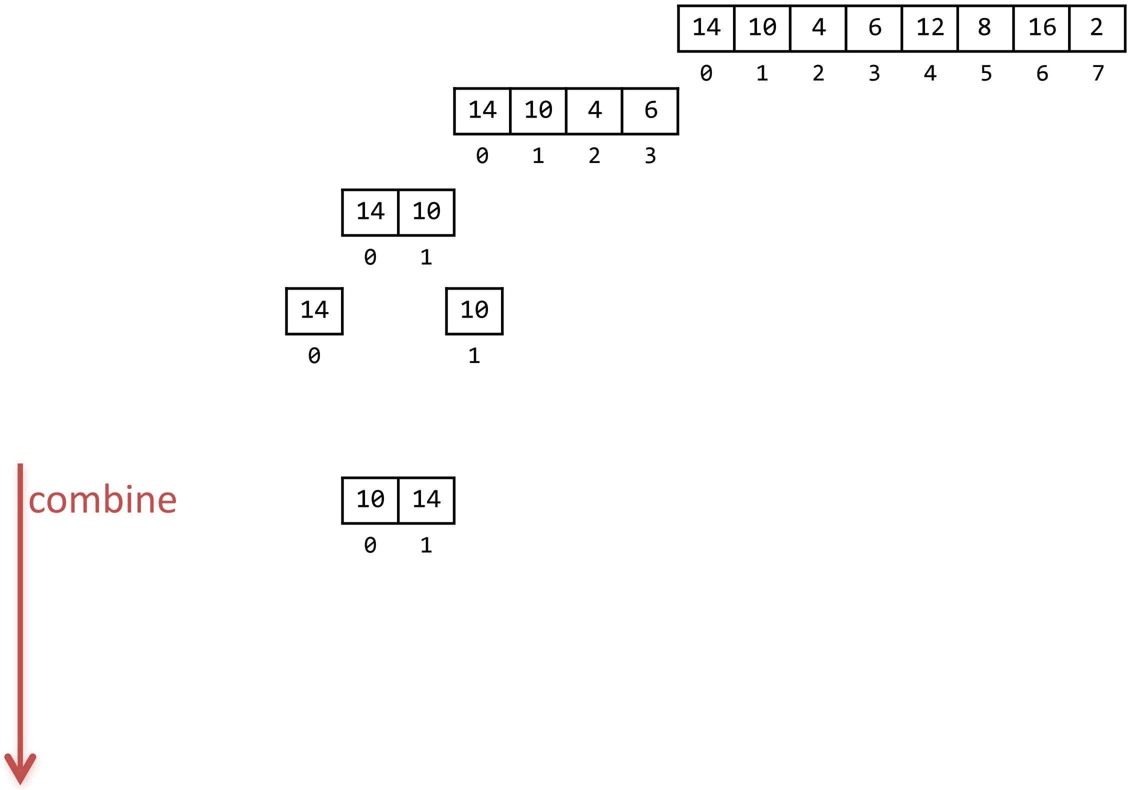

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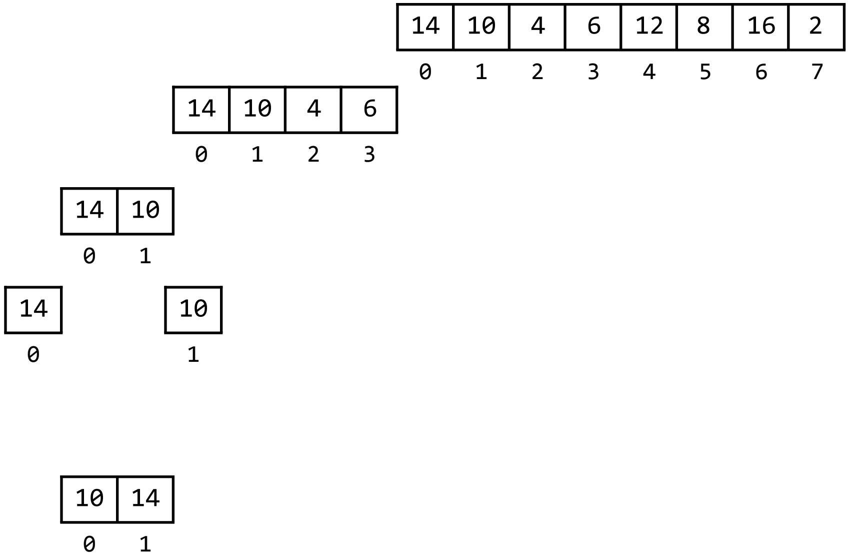

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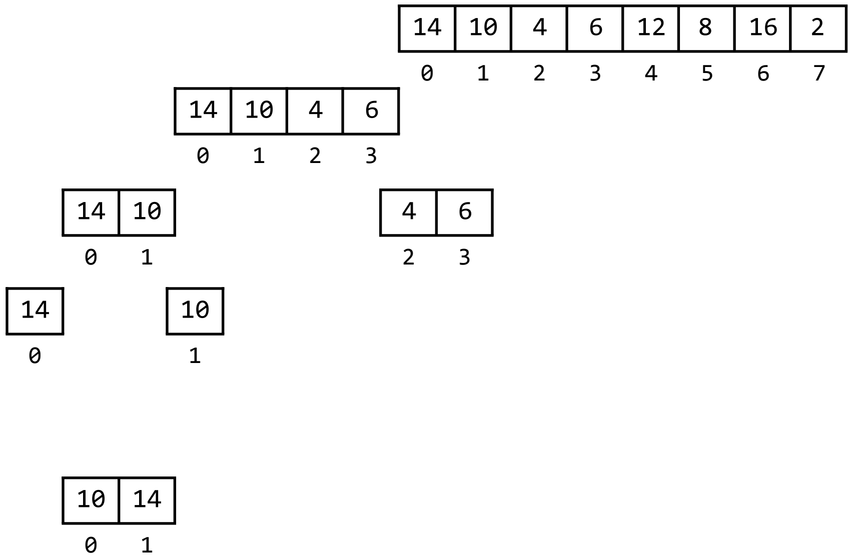

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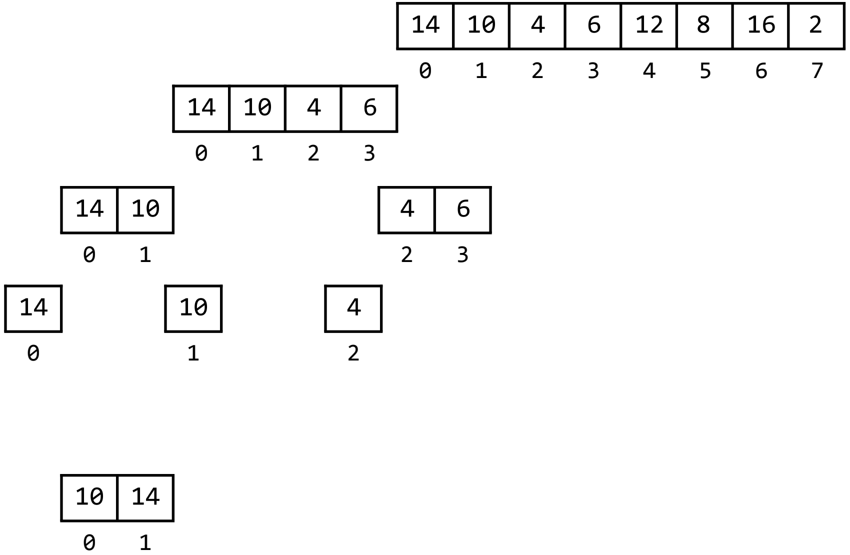

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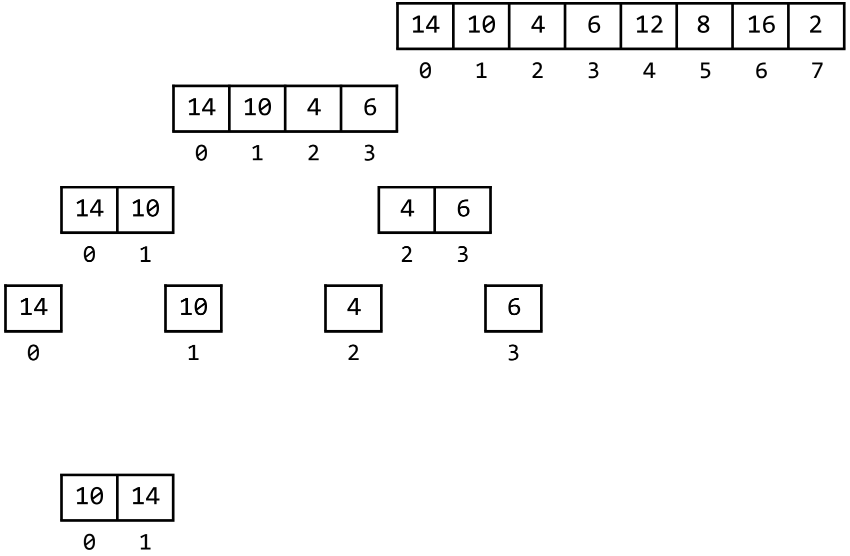

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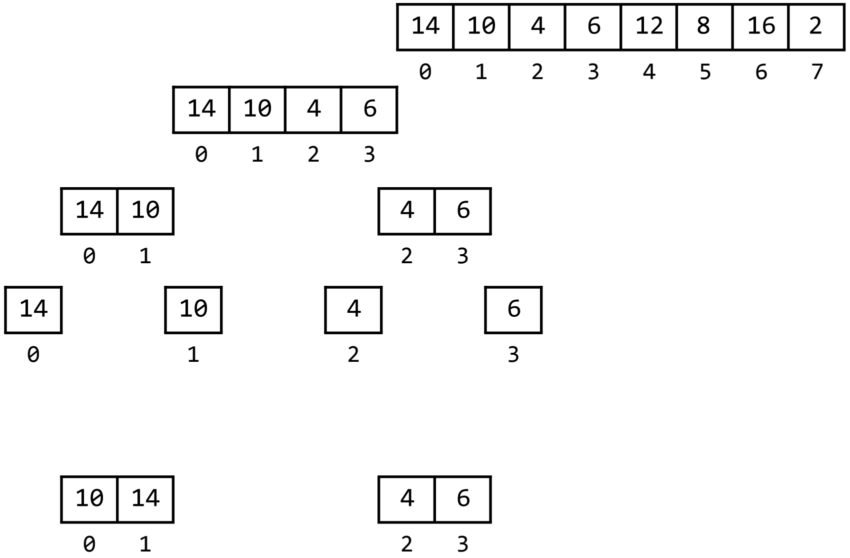

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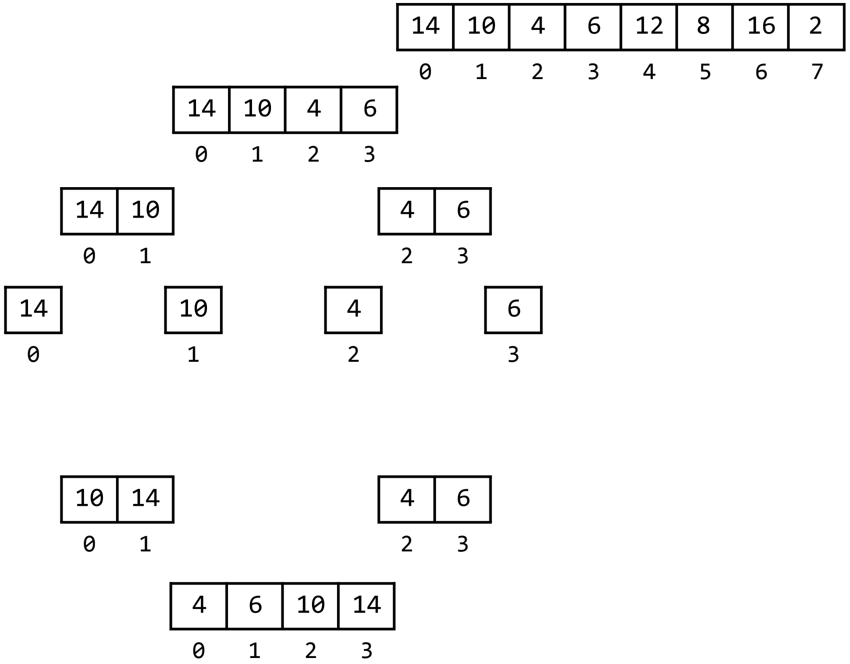

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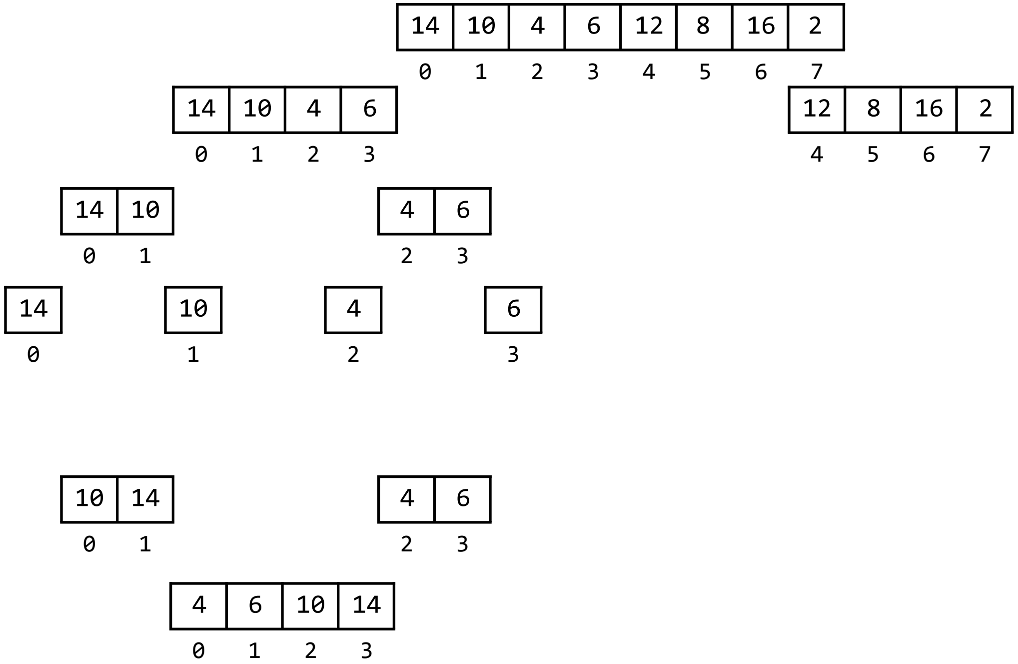

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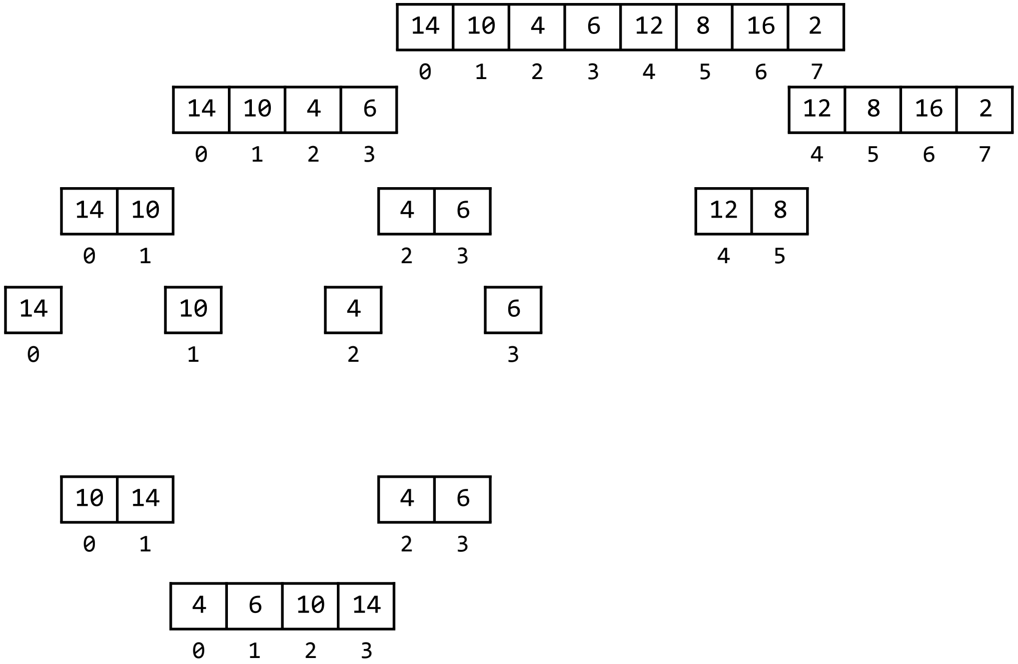

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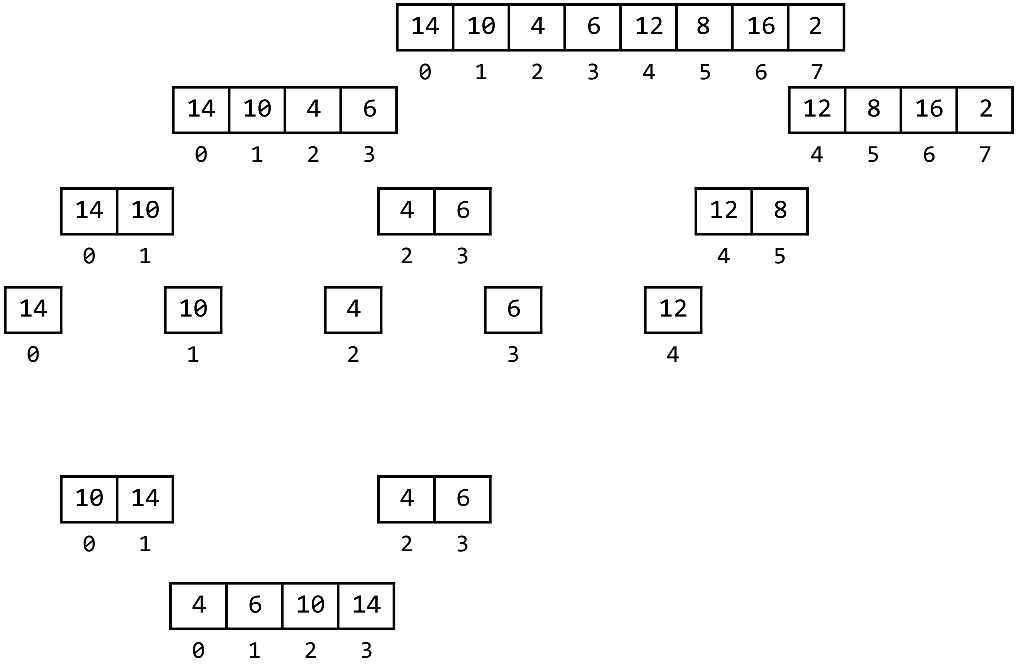

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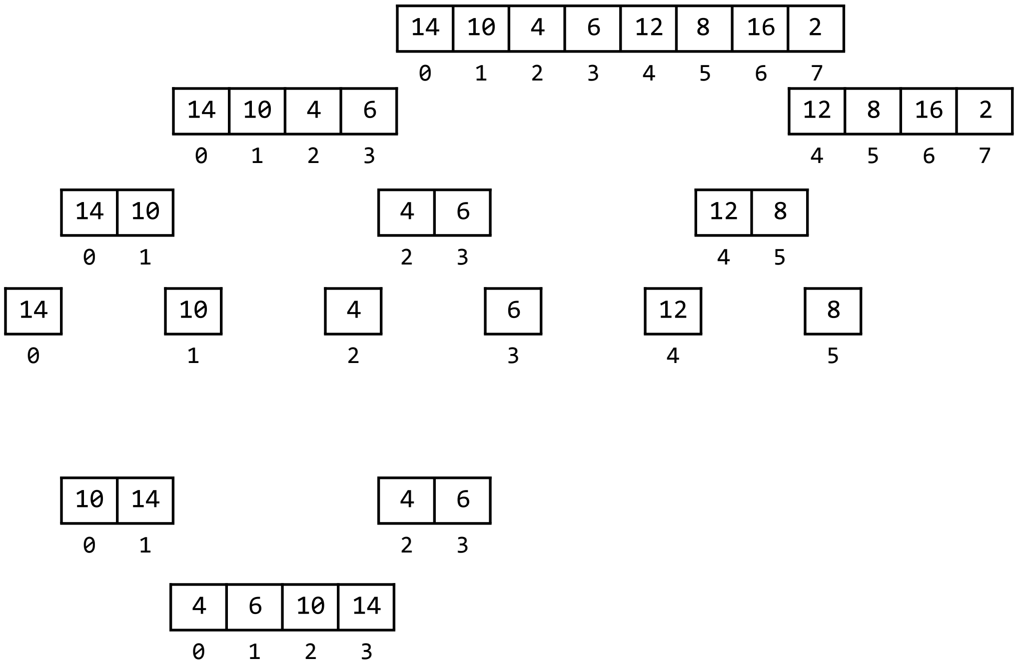

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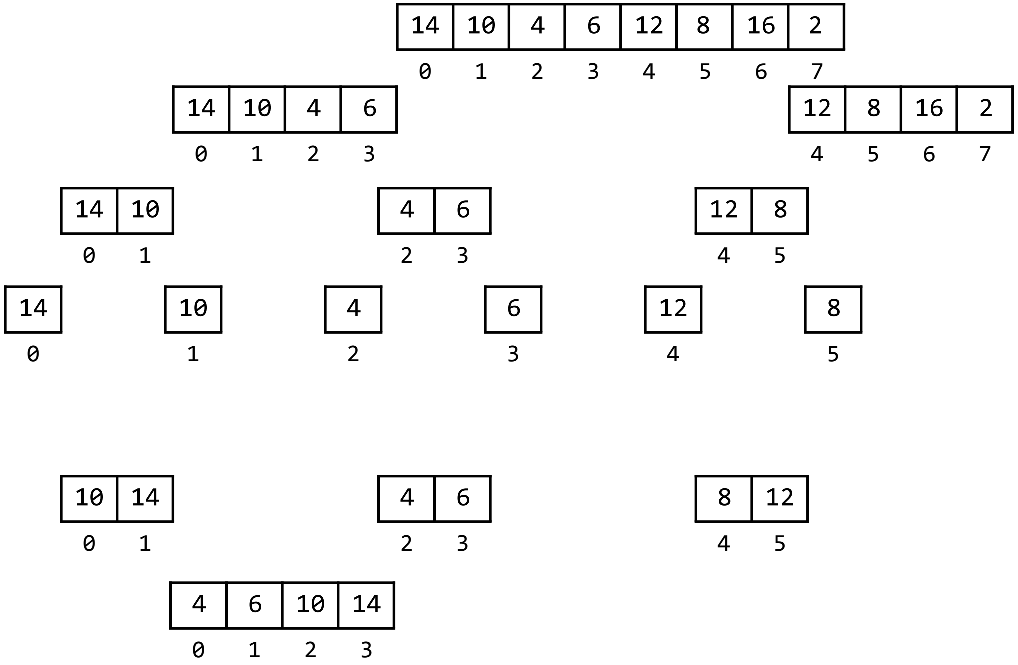

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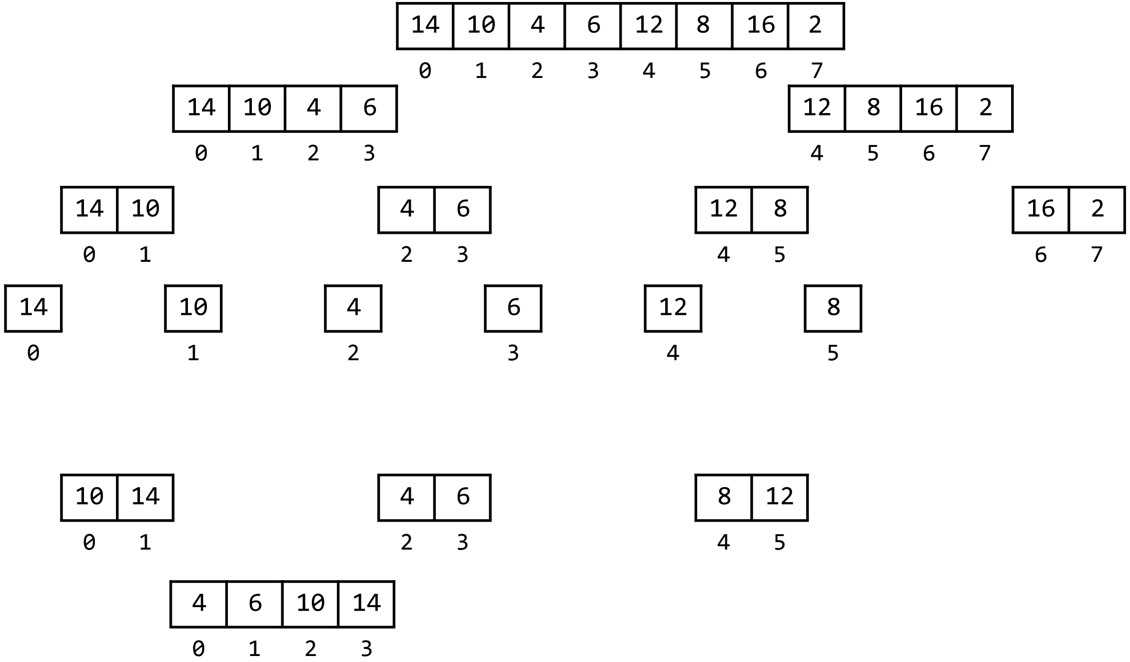

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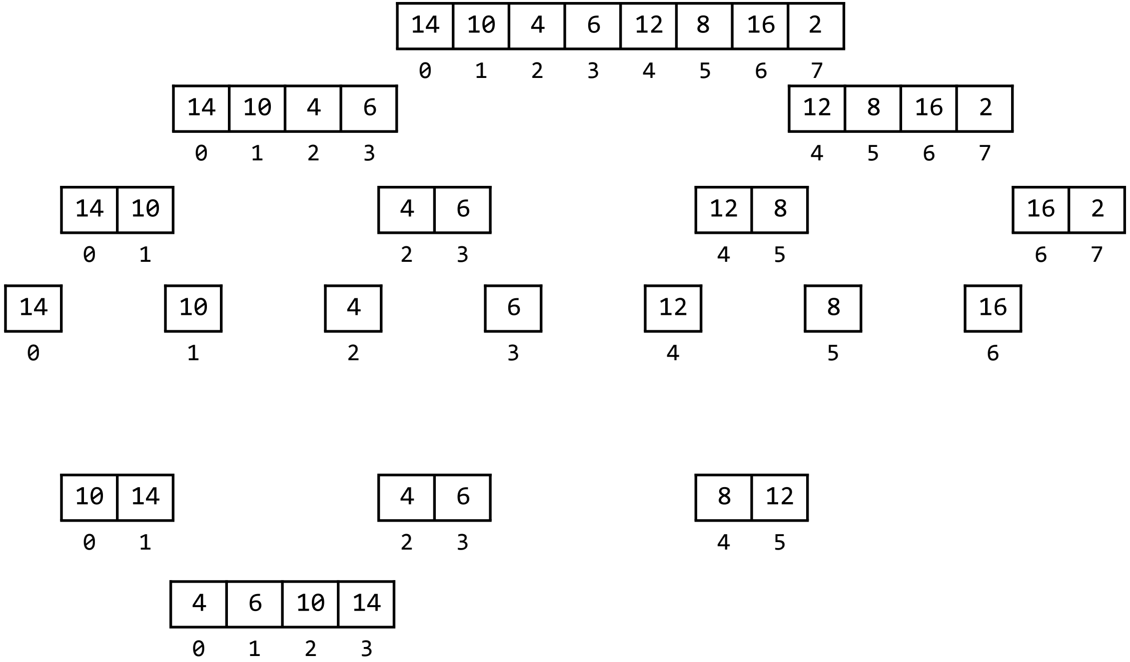

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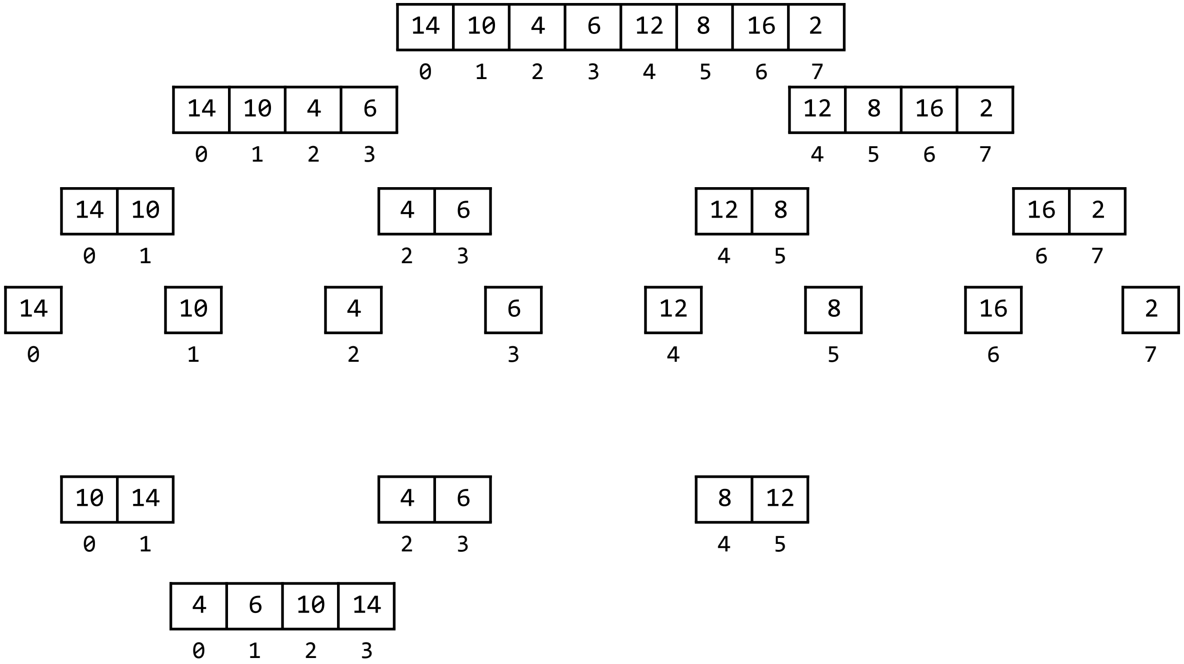

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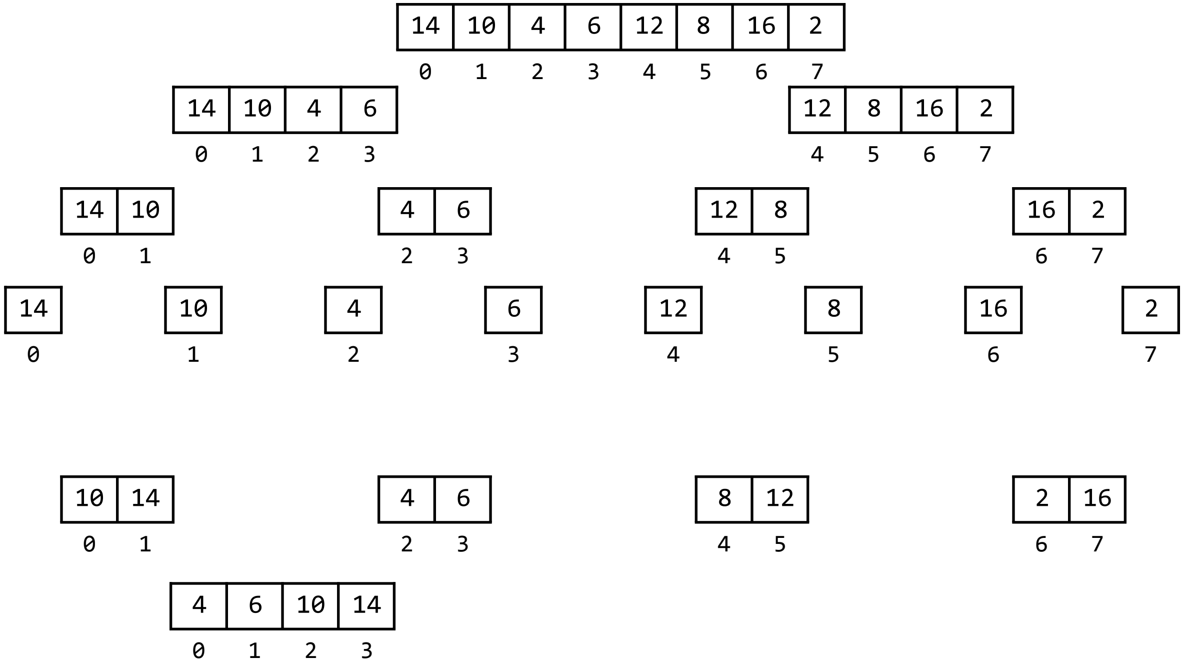

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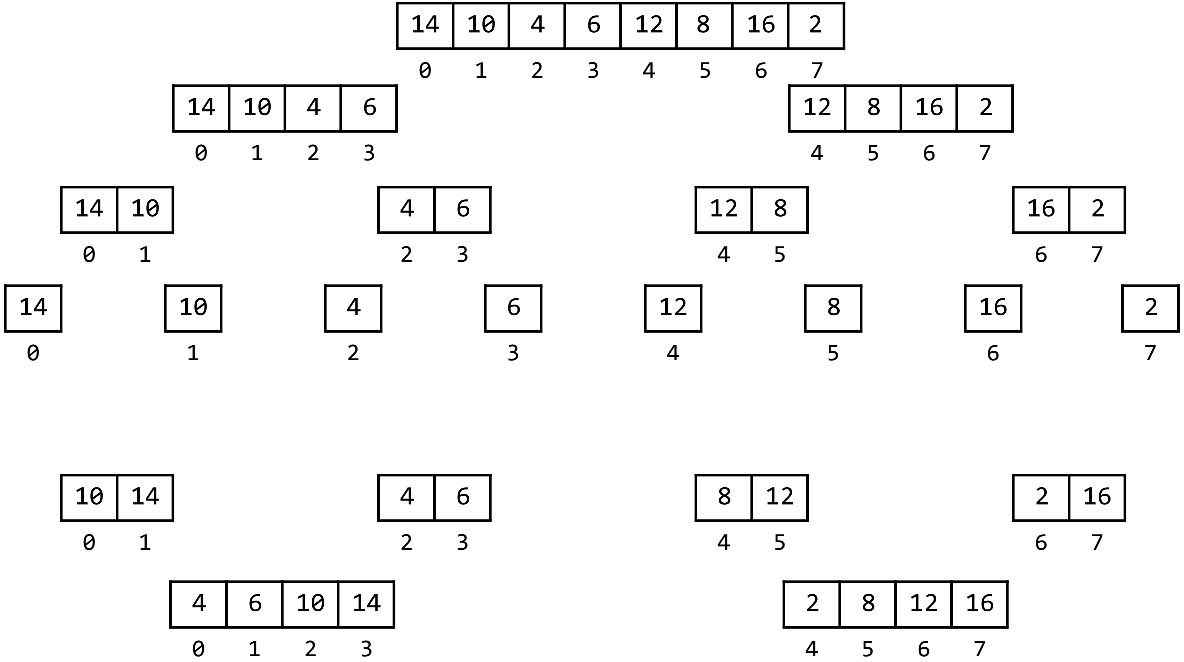

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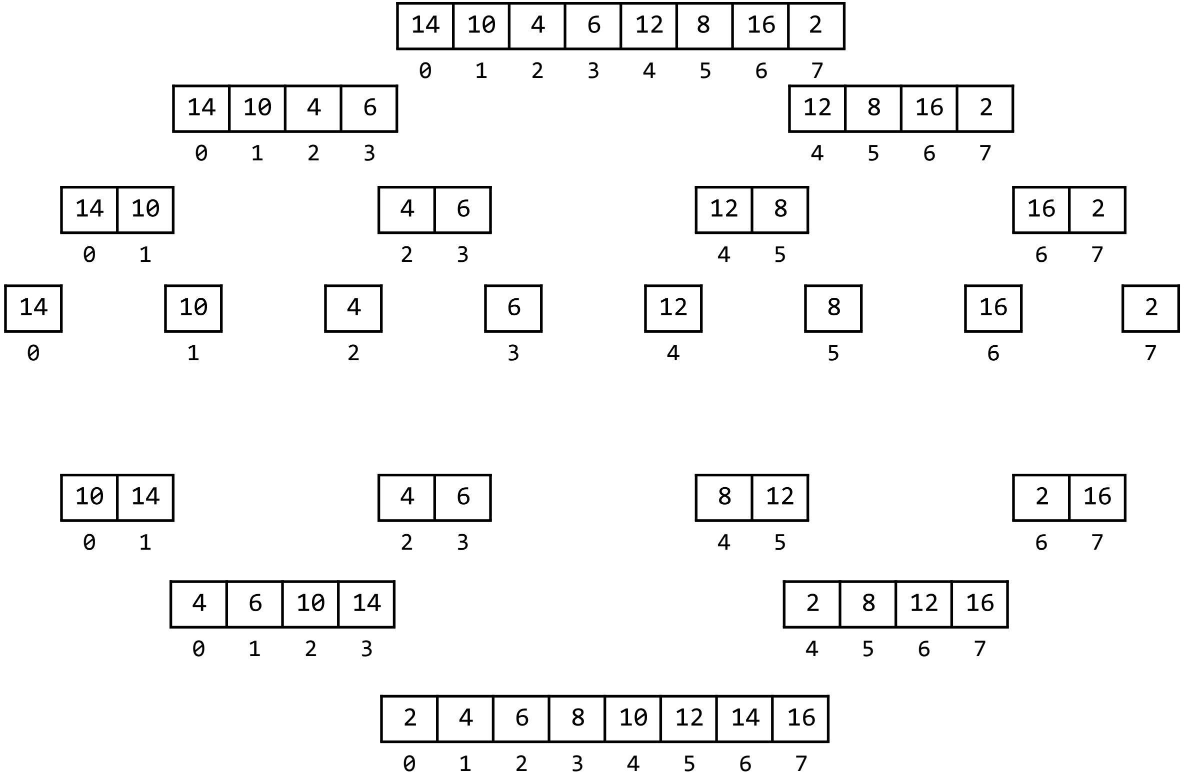

Merge sort

The real work in merge sort happens during the combine phase: merging two sorted arrays into one sorted array.

```
public static void mergeSort(Comparable[] a, int left, int right) {

   if (right <= left) return;

   int mid = left + (right – left) / 2;
   mergeSort(a, left, mid);
   mergeSort(a, mid + 1, right);

   merge(a, left, mid, right);
}
```

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```
public void merge(Comparable[] a, int left, int mid, int right) {

   for (int k = left; k <= right; k++)   aux[k] = a[k];

   int i = left; j = mid + 1;
   for (int k = left; k <= right; k++) {
      if         (i > mid)                a[k] = aux[j++];
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A field in the enclosing class.
O(N) extra space

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Time complexity:

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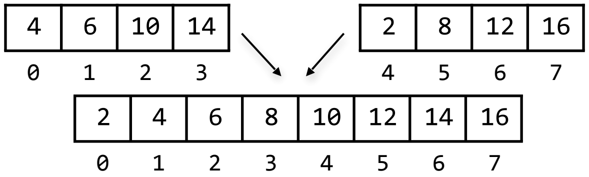

Merge sort

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```

	left							right
a	4	6	10	14	2	8	12	16
	0	1	2	3	4	5	6	7

Merge sort

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   }
}
```

	left							right
a	4	6	10	14	2	8	12	16
	0	1	2	3	4	5	6	7

aux	4	6	10	14	2	8	12	16
	0	1	2	3	4	5	6	7

Merge sort

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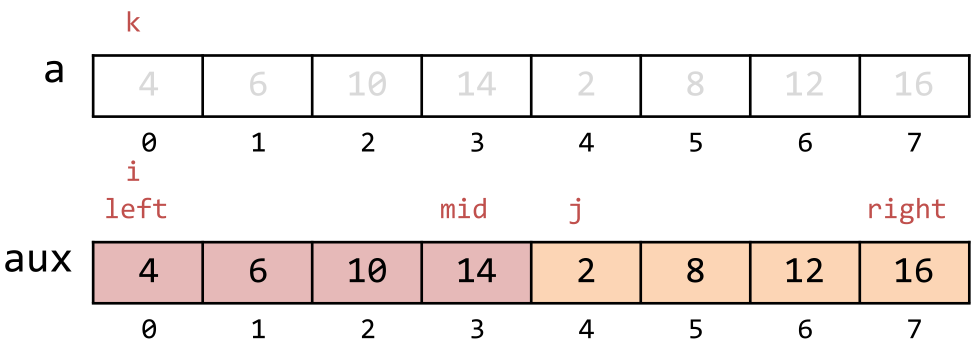

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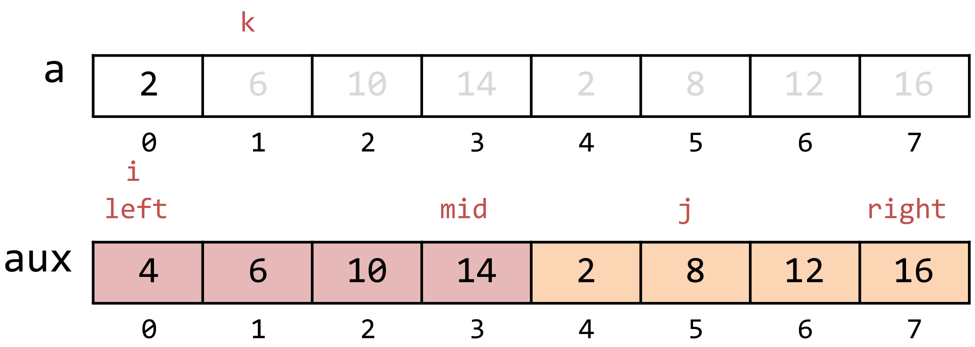

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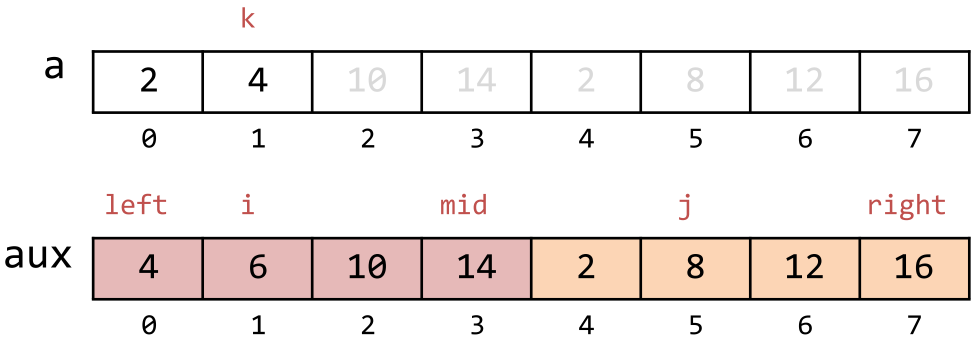

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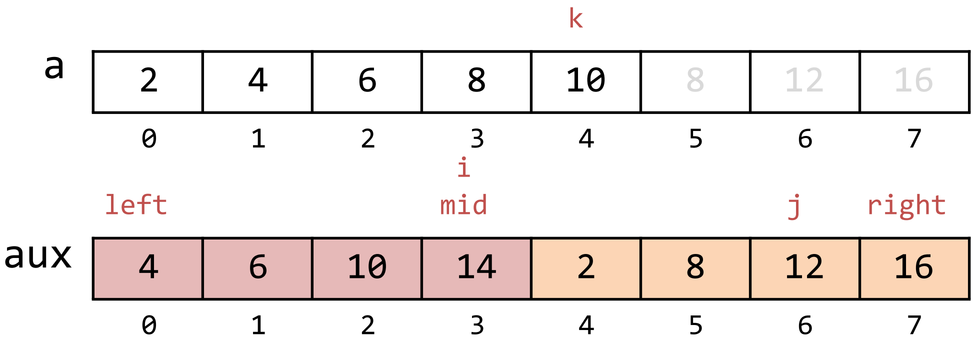

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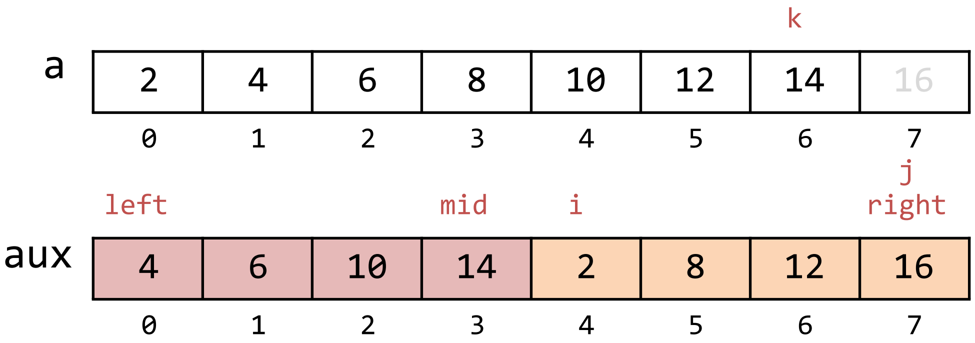

Merge sort

```
public void merge(Comparable[] a, int left, int mid, int right) {

   for (int k = left; k <= right; k++)   aux[k] = a[k];

   int i = left; j = mid + 1;
   for (int k = left; k <= right; k++) {
      if        (i > mid)              a[k] = aux[j++];
      else if   (j > right)            a[k] = aux[i++];
      else if   (less(aux[j], aux[i])) a[k] = aux[j++];
      else                             a[k] = aux[i++];
   }
}
```

								k
a	2	4	6	8	10	12	14	16
	0	1	2	3	4	5	6	7

	left			mid	i			j right
aux	4	6	10	14	2	8	12	16
	0	1	2	3	4	5	6	7

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   }
}
```

Note that this makes merge sort *stable*.

i						j					
2	4	6	8	12	16	4	8	10	14	16	18
0	1	2	3	4	5	6	7	8	9	10	11

Quicksort

Quicksort

Quicksort is a comparison sort based on the **divide-and-conquer** strategy.

First described by Tony Hoare in 1960 as part of his work on machine translation.

Asymptotically optimal for comparison sorting – **O(NlogN)**, but only in the average case. Has worst-case time that **is $O(N^2)$**.

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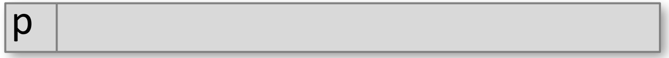

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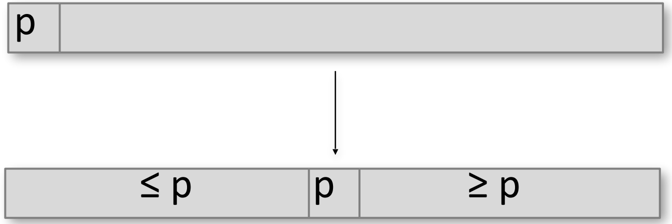

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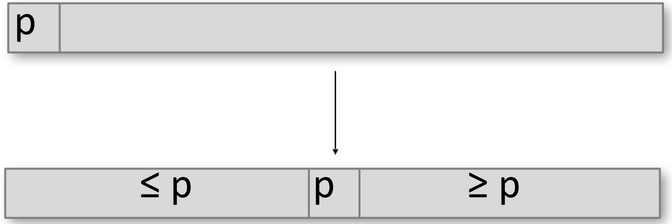

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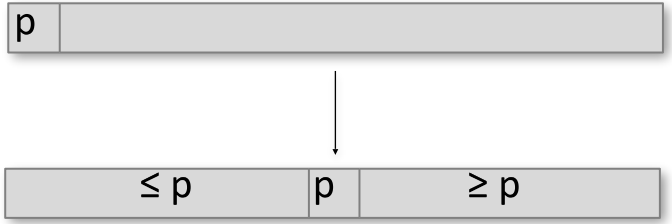

The partitioning can be done in linear time.

The choice of pivot determines the size of the partitions.

Quicksort

```
public void qsort(Comparable[] a, int left, int right)
{
    if (right <= left)
       return;

    int j = partition(a, left, right);
    qsort(a, left, j-1);
    qsort(a, j+1, right);
 }
```

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 }
```

conquer

divide

Quicksort

```
public void qsort(Comparable[] a, int left, int right)
{
    if (right <= left)          conquer
       return;

    int j = partition(a, left, right);      divide
    qsort(a, left, j-1);
    qsort(a, j+1, right);
 }
```

Partitioning does the real work of the sort, so there's nothing to do in the combine phase.

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2	16	4	8	12	6	4	10	16	8	18	14
0	1	2	3	4	5	6	7	8	9	10	11

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pivot = 10

Quicksort

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pivot = 10

≤ 10	10	≥ 10

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≤ 10	10	≥ 10

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The partition method takes the array and (at least) the left and right indexes of the current portion of the array, and returns the resulting index of the pivot value after partitioning.

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Notice that the partition method above chooses the pivot value internally. We could move that decision out to the qsort method, as the following example assumes.

Quicksort

There are several ways to approach partitioning. Here's one that's easy to follow and has the pivot location provided as a parameter:

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private int partition(T[] a, int left, int right, int pivotIndex) {
   T pivot = a[pivotIndex];
   swap(a, pivotIndex, right); // move pivot to the end
   int p = left; // p will become the final index of pivot
   for (int i = left; i < right; i++) {
      if (less(a[i], pivot)) {
         swap(a, i, p);
         p++;
      }
   }
   swap(a, p, right); // move pivot to its correct location
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left							pivotIndex				right
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i,p
left | | | | | | | | | | | right

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   for (int i = left; i < right; i++) {
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         swap(a, i, p);
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   return p;
}
```

left					p			i			right
2	4	8	6	4	16	12	14	16	8	18	10
0	1	2	3	4	5	6	7	8	9	10	11

Quicksort

```java
private int partition(T[] a, int left, int right, int pivotIndex) {
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Quicksort

```
public void qsort(Comparable[] a, int left, int right)
{
    if (right <= left)
       return;

    int j = partition(a, left, right);
    qsort(a, left, j-1);
    qsort(a, j+1, right);
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Quicksort

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public void qsort(Comparable[] a, int left, int right)
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The choice of pivot value determines the size of each partition, and therefore determines the number of divide steps that will be necessary.

Quicksort

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The choice of pivot value determines the size of each partition, and therefore determines the number of divide steps that will be necessary.

"It becomes evident that sorting on the basis of Quicksort is somewhat like a gamble in which one should be aware of how much one may afford to lose if bad luck were to strike." – Niklaus Wirth

Quicksort

The choice of pivot value determines the size of each partition, and therefore determines the number of divide steps that will be necessary.

Best case pivot choices at each step lead to partitions that are approximately the same size.

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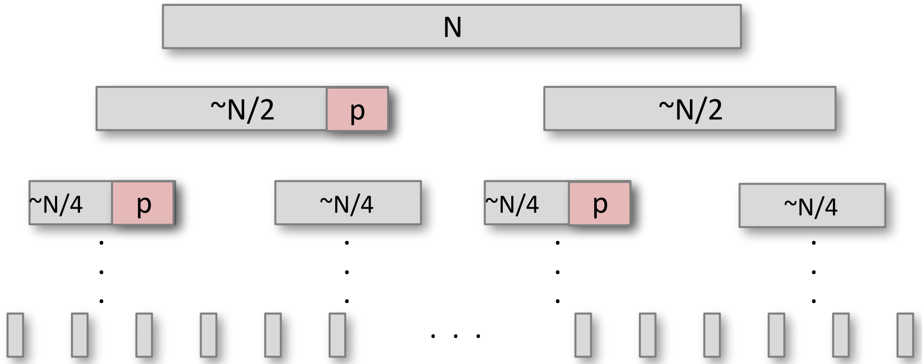

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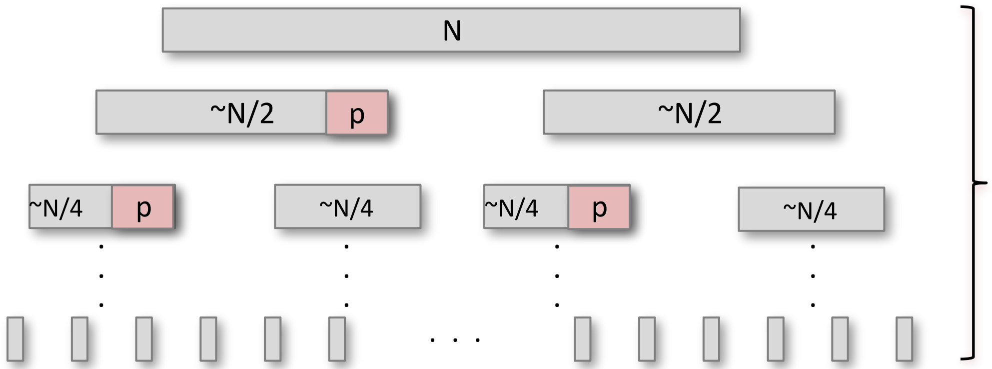

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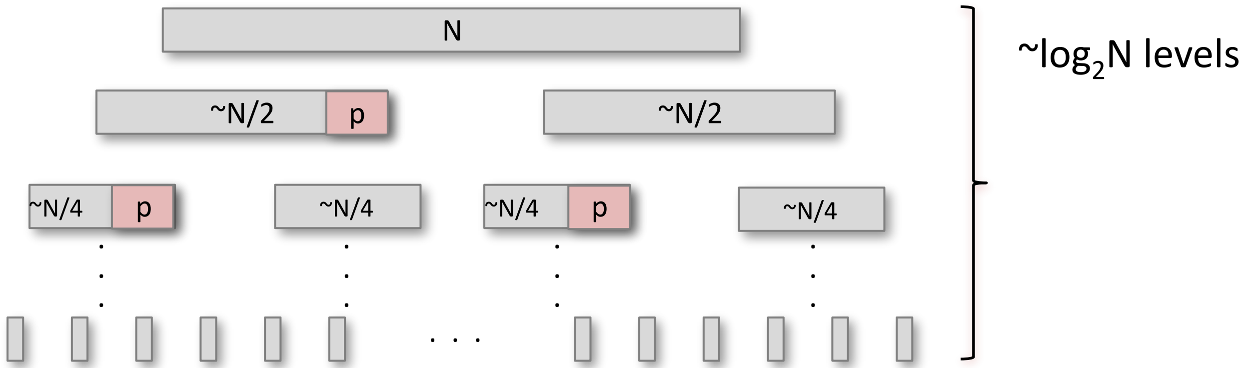

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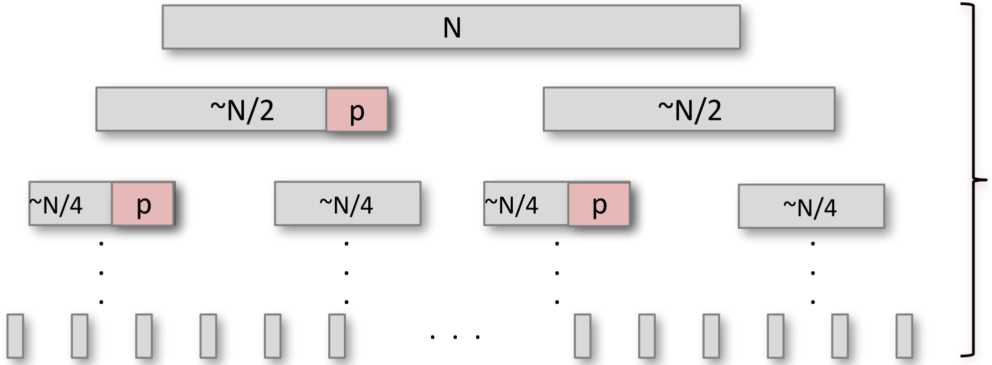

~$\log_2$N levels

N elements to compare, move per level

Quicksort

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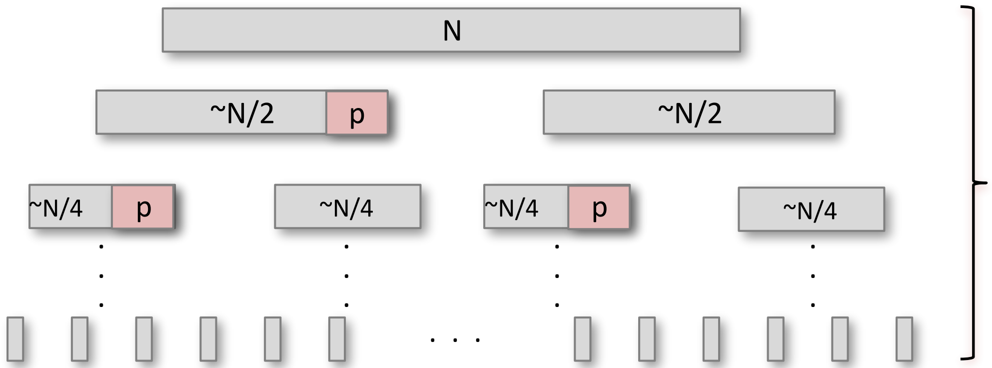

~$\log_2$N levels

N elements to compare, move per level

O(NlogN) work

Quicksort

The choice of pivot value determines the size of each partition, and therefore determines the number of divide steps that will be necessary.

Worst case pivot choices at each step lead to one partition that is empty.

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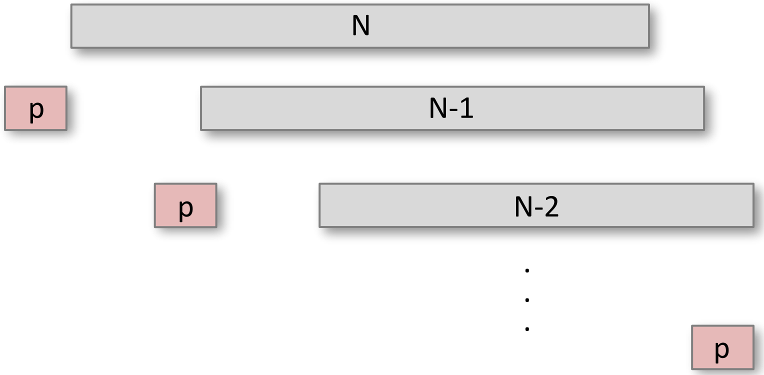

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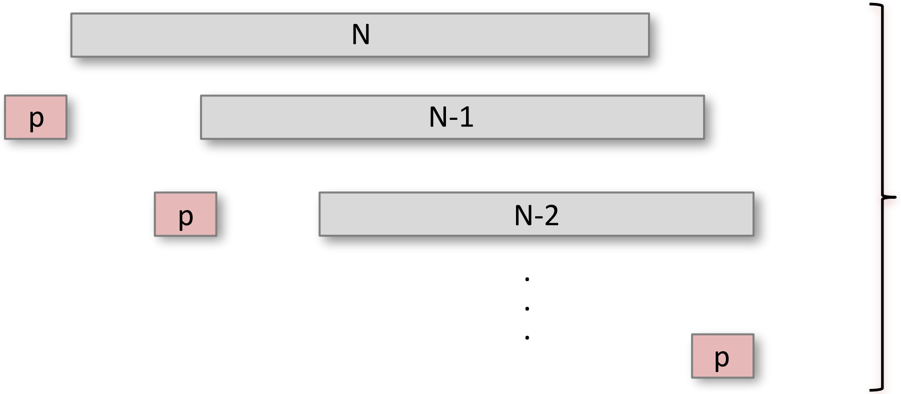

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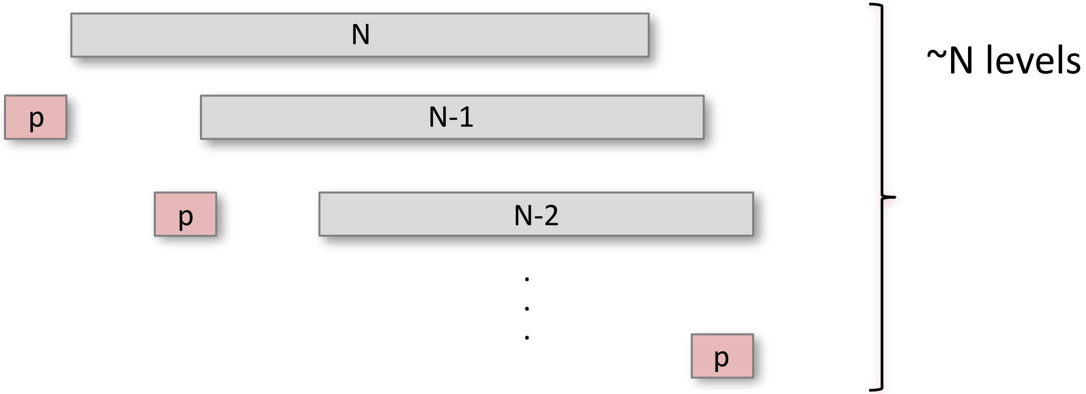

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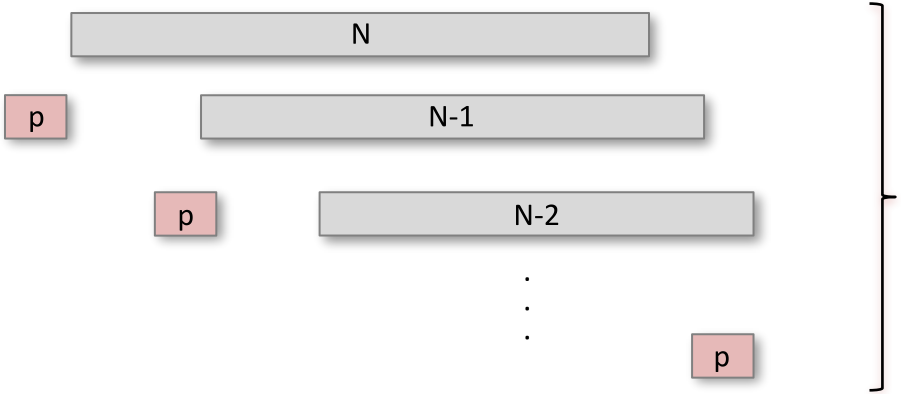

~N levels

N elements to compare, move per level

Quicksort

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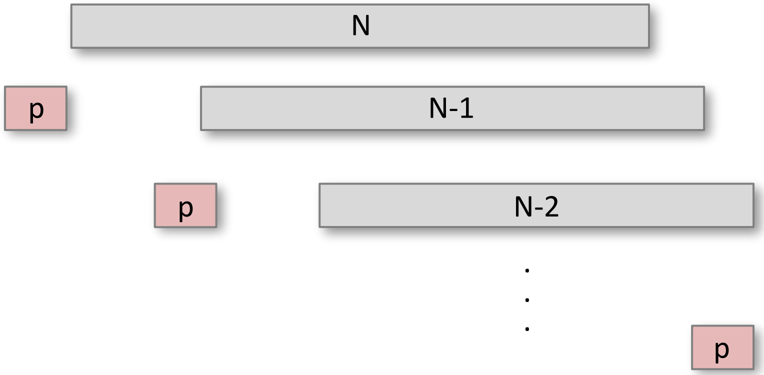


~N levels

N elements to compare, move per level

O(N²) work

Quicksort

Example partition based on a given pivot value for the following array:

2	16	4	8	12	6	4	10	16	8	18	14
0	1	2	3	4	5	6	7	8	9	10	11

≤ p	p	≥ p

pivot = 6

2	4	4	6	12	14	16	10	16	8	18	8
0	1	2	3	4	5	6	7	8	9	10	11

Quicksort

Example partition based on a given pivot value for the following array:

2	16	4	8	12	6	4	10	16	8	18	14
0	1	2	3	4	5	6	7	8	9	10	11

≤ p	p	≥ p

pivot = 10

2	4	8	6	4	8	10	14	16	16	18	12
0	1	2	3	4	5	6	7	8	9	10	11

Quicksort

Example partition based on a given pivot value for the following array:

2	16	4	8	12	6	4	10	16	8	18	14
0	1	2	3	4	5	6	7	8	9	10	11

≤ p	p	≥ p

pivot = 2

2	16	4	8	12	6	4	10	16	8	18	14
0	1	2	3	4	5	6	7	8	9	10	11

Quicksort

Example partition based on a given pivot value for the following array:

2	16	4	8	12	6	4	10	16	8	18	14
0	1	2	3	4	5	6	7	8	9	10	11

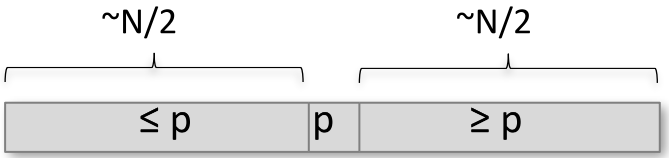
≤ p	p	≥ p

pivot = 18

2	16	4	8	12	6	4	10	16	8	14	18
0	1	2	3	4	5	6	7	8	9	10	11

Quicksort

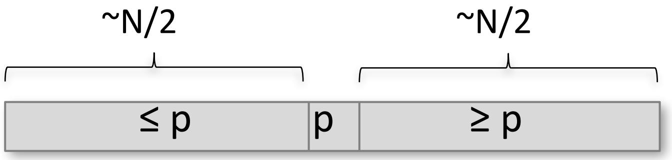
Choosing a pivot value



Quicksort

Choosing a pivot value

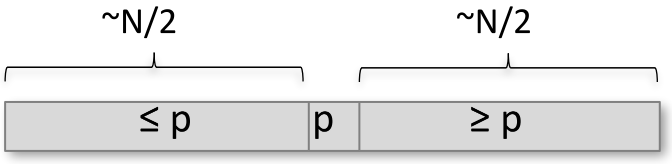
Median value



Quicksort

Choosing a pivot value

Median value

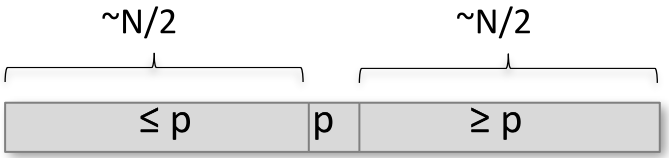


The median of a list of N items can be found in O(N) time using a *selection* algorithm.

Quicksort

Choosing a pivot value

Median value

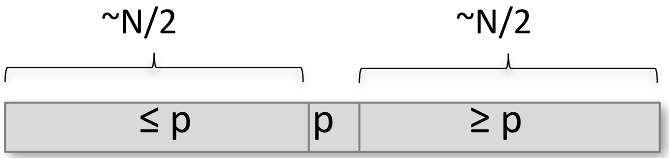


The median of a list of N items can be found in O(N) time using a *selection* algorithm.

But, this linear time overhead would be added to each divide step in quicksort.

Quicksort

Choosing a pivot value



Median value

The median of a list of N items can be found in O(N) time using a *selection* algorithm.

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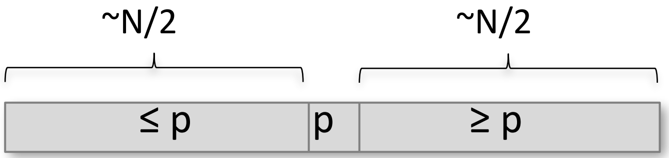
Median of three

Choosing a pivot value



Median value

The median of a list of N items can be found in O(N) time using a *selection* algorithm.

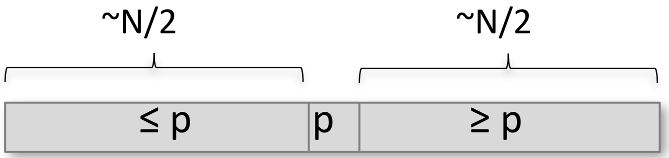
But, this linear time overhead would be added to each divide step in quicksort.

Median of three

Choose three elements of the array (usually first, middle, last) and then use the median of those three values as the pivot.

Quicksort

Choosing a pivot value



Median value

The median of a list of N items can be found in O(N) time using a *selection* algorithm.

But, this linear time overhead would be added to each divide step in quicksort.

Median of three

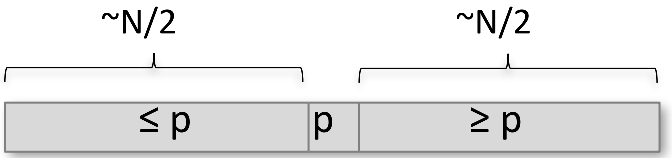
Choose three elements of the array (usually first, middle, last) and then use the median of those three values as the pivot.

This is a good compromise and a popular choice in quicksort implementations. The 3270 analysis of quicksort will show that even though the partitions may not be ~N/2 at any given divide step, the overall performance will be O(NlogN) on average.

Quicksort

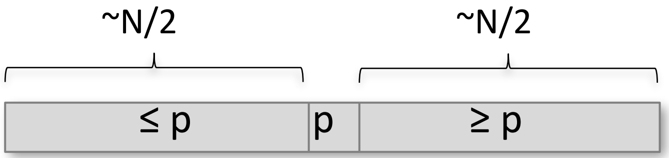
Choosing a pivot value

Randomly pick



Quicksort

Choosing a pivot value

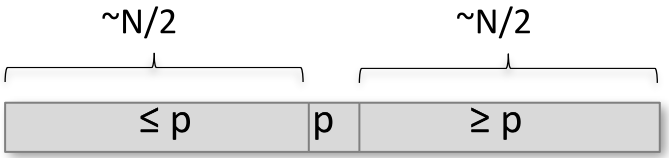


Randomly pick

Use a random number generator (PRNG) to pick an index and use the element at that index as the pivot. Again, 3270 will show that this will lead to O(NlogN) average case complexity.

Quicksort

Choosing a pivot value



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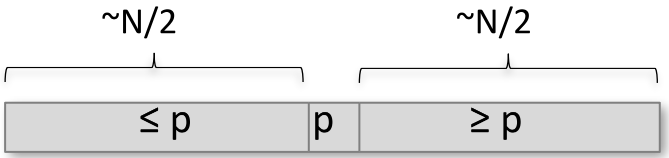
Use a random number generator (PRNG) to pick an index and use the element at that index as the pivot. Again, 3270 will show that this will lead to O(NlogN) average case complexity.

But, this makes the sort itself dependent on a PRNG and not portable or deterministic.

Quicksort

Choosing a pivot value



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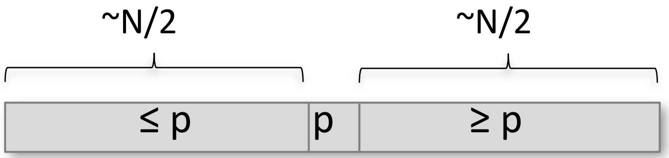
Shuffle once, pick first element

Quicksort

Choosing a pivot value



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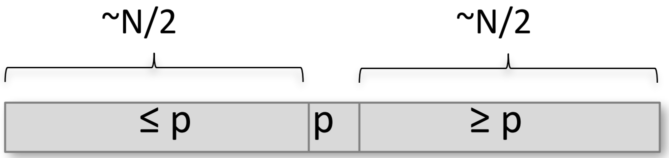
Shuffle once, pick first element

Randomize the order of elements in the array once up front (before the D&C part of the sort begins) and then just pick the first element as the pivot each time.

Quicksort

Choosing a pivot value



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Shuffle once, pick first element

Randomize the order of elements in the array once up front (before the D&C part of the sort begins) and then just pick the first element as the pivot each time.

This is really just a variation of the first approach, but it pulls all calls to the PRNG out of the sort and make the pivot choice trivial (and fast).

Quicksort

Randomize the order of elements in an array in $O(N)$ time.

Quicksort

Randomize the order of elements in an array in O(N) time.

```
public void shuffle(T[] a) {
    Random rng = new Random();
    for (int i = a.length - 1; i > 0; i--) {
        int j = rng.nextInt(i + 1);
        swap(a, i, j);
    }
}
```

[*The Knuth Shuffle*]

Quicksort

A “randomized” quicksort:

```
public void quicksort(Comparable[] a) {
   shuffle(a);
   qsort(a, 0, a.length - 1);
}
```

```
public void qsort(Comparable[] a, int left, int right) {
    if (right <= left)
       return;

    int j = partition(a, left, right);
    qsort(a, left, j-1);
    qsort(a, j+1, right);
}
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Quicksort

A "randomized" quicksort:

```
public void quicksort(Comparable[] a) {
   shuffle(a);
   qsort(a, 0, a.length - 1);
}
```

Shuffling happens once, before the sort begins.

```
public void qsort(Comparable[] a, int left, int right) {
    if (right <= left)
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    int j = partition(a, left, right);
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    qsort(a, j+1, right);
}
```

The partition method can use a[left] as the pivot – trivial and fast pivot selection.

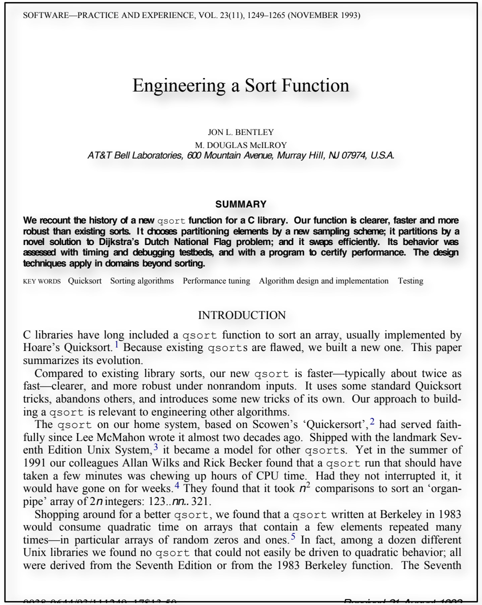
Quicksort

There are many variations on the implementation of quicksort, but when it's done well, quicksort is typically the sort-of-choice in many situations.

Quicksort has a worst case complexity of $O(N^2)$. However, a good pivot choice strategy makes the worst case highly unlikely.

Because of this, we categorize quicksort by its average case complexity – **O(NlogN)**.

Note that quicksort **is an in-place sort**, but is usually **not stable**.


SOFTWARE—PRACTICE AND EXPERIENCE, VOL. 23(11), 1249–1265 (NOVEMBER 1993)

Engineering a Sort Function

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M. DOUGLAS McILROY
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SUMMARY

We recount the history of a new `qsort` function for a C library. Our function is clearer, faster and more robust than existing sorts. It chooses partitioning elements by a new sampling scheme; it partitions by a novel solution to Dijkstra's Dutch National Flag problem; and it swaps efficiently. Its behavior was assessed with timing and debugging testbeds, and with a program to certify performance. The design techniques apply in domains beyond sorting.

KEY WORDS Quicksort Sorting algorithms Performance tuning Algorithm design and implementation Testing

INTRODUCTION

C libraries have long included a `qsort` function to sort an array, usually implemented by Hoare's Quicksort.[1] Because existing `qsorts` are flawed, we built a new one. This paper summarizes its evolution.

Compared to existing library sorts, our new `qsort` is faster—typically about twice as fast—clearer, and more robust under nonrandom inputs. It uses some standard Quicksort tricks, abandons others, and introduces some new tricks of its own. Our approach to building a `qsort` is relevant to engineering other algorithms.

The `qsort` on our home system, based on Scowen's 'Quickersort',[2] had served faithfully since Lee McMahon wrote it almost two decades ago. Shipped with the landmark Seventh Edition Unix System,[3] it became a model for other `qsorts`. Yet in the summer of 1991 our colleagues Allan Wilks and Rick Becker found that a `qsort` run that should have taken a few minutes was chewing up hours of CPU time. Had they not interrupted it, it would have gone on for weeks.[4] They found that it took n^2 comparisons to sort an 'organ-pipe' array of $2n$ integers: 123..nn..321.

Shopping around for a better `qsort`, we found that a `qsort` written at Berkeley in 1983 would consume quadratic time on arrays that contain a few elements repeated many times—in particular arrays of random zeros and ones.[5] In fact, among a dozen different Unix libraries we found no `qsort` that could not easily be driven to quadratic behavior; all were derived from the Seventh Edition or from the 1983 Berkeley function. The Seventh

[Jon Bentley]

[A great interview]

[A great video]

Participation

Q: Suppose an array with its elements in reverse (descending) order is passed to a sorting method and you observe that the method takes time proportional to N^2. Which sorting algorithm is definitely NOT implemented in this method?

A. Insertion sort
B. Selection sort
C. Quicksort
D. Merge sort

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A. Insertion sort
B. Selection sort
C. Quicksort
D. Merge sort

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> *"The sorting algorithm is a Dual-Pivot* ***Quicksort*** *by Vladimir Yaroslavskiy, Jon Bentley, and Joshua Bloch. This algorithm offers O(n log(n)) performance on many data sets that cause other quicksorts to degrade to quadratic performance, and is typically faster than traditional (one-pivot) Quicksort implementations."*

Merge sort is used for references.

> *Implementation note: This implementation is a stable, adaptive, iterative* ***mergesort*** *that requires far fewer than n lg(n) comparisons when the input array is partially sorted, while offering the performance of a traditional mergesort when the input array is randomly ordered. If the input array is nearly sorted, the implementation requires approximately n comparisons. Temporary storage requirements vary from a small constant for nearly sorted input arrays to n/2 object references for randomly ordered input arrays. The implementation takes equal advantage of ascending and descending order in its input array, and can take advantage of ascending and descending order in different parts of the the same input array. It is well-suited to merging two or more sorted arrays: simply concatenate the arrays and sort the resulting array. The implementation was adapted from Tim Peters's list sort for Python (TimSort). It uses techiques from Peter McIlroy's "Optimistic Sorting and Information Theoretic Complexity", in Proceedings of the Fourth Annual ACM-SIAM Symposium on Discrete Algorithms, pp 467-474, January 1993.*

Stability

```
private static class Data implements Comparable<Data> {
   private Integer field1;
   private Integer field2;

   public Data(Integer f1, Integer f2) { . . . }

   public int compareTo(Data that) {
      if (this.field1 < that.field1)
         return -1;
      else if (this.field1 > that.field1)
         return 1;
      else
         return 0;
   }
 . . .
}
```

Stability

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private static class Data implements Comparable<Data> {
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         return 1;
      else
         return 0;
   }
 . . .
}
```

```
Data d1 = new Data(4, 1);
Data d2 = new Data(3, 9);
```

Stability

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private static class Data implements Comparable<Data> {
   private Integer field1;
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}
```

```java
Data d1 = new Data(4, 1);
Data d2 = new Data(3, 9);

d1.compareTo(d2) ==
```

Stability

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         return -1;
      else if (this.field1 > that.field1)
         return 1;
      else
         return 0;
   }
 . . .
}
```

Compares only on field1

```
Data d1 = new Data(4, 1);
Data d2 = new Data(3, 9);

d1.compareTo(d2) ==
```

Stability

```
Data[] a = {new Data(5, 1), new Data(4, 2), new Data(3, 3), new Data(5, 4),
            new Data(2, 5), new Data(1, 6), new Data(5, 7)};
```

Stability

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Data[] a = {new Data(5, 1), new Data(4, 2), new Data(3, 3), new Data(5, 4),
            new Data(2, 5), new Data(1, 6), new Data(5, 7)};
```

(5,1)	(4,2)	(3,3)	(5,4)	(2,5)	(1,6)	(5,7)
0	1	2	3	4	5	6

Three duplicates according to natural order.

Stability

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$\downarrow$ `mergesort(a)`

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(1,6)	(2,5)	(3,3)	(4,2)	(5,1)	(5,4)	(5,7)
0	1	2	3	4	5	6

The duplicates are in the same relative order as before the sort.

Stability

```
Data[] a = {new Data(5, 1), new Data(4, 2), new Data(3, 3), new Data(5, 4),
            new Data(2, 5), new Data(1, 6), new Data(5, 7)};
```

(5,1)	(4,2)	(3,3)	(5,4)	(2,5)	(1,6)	(5,7)
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Three duplicates according to natural order.

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0	1	2	3	4	5	6

The duplicates are in the same relative order as before the sort.

Merge sort is stable.

Stability

```
Data[] a = {new Data(5, 1), new Data(4, 2), new Data(3, 3), new Data(5, 4),
            new Data(2, 5), new Data(1, 6), new Data(5, 7)};
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(5,1)	(4,2)	(3,3)	(5,4)	(2,5)	(1,6)	(5,7)
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(5,1)	(4,2)	(3,3)	(5,4)	(2,5)	(1,6)	(5,7)
0	1	2	3	4	5	6

Three duplicates according to natural order.

$\downarrow$ `quicksort(a)`

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Three duplicates according to natural order.

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(1,6)	(2,5)	(3,3)	(4,2)	(5,1)	(5,7)	(5,4)
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0	1	2	3	4	5	6

Three duplicates according to natural order.

quicksort(a)

(1,6)	(2,5)	(3,3)	(4,2)	(5,1)	(5,7)	(5,4)
0	1	2	3	4	5	6

The duplicates are <u>not</u> in the same relative order as before the sort.

Stability

```
Data[] a = {new Data(5, 1), new Data(4, 2), new Data(3, 3), new Data(5, 4),
            new Data(2, 5), new Data(1, 6), new Data(5, 7)};
```

(5,1)	(4,2)	(3,3)	(5,4)	(2,5)	(1,6)	(5,7)
0	1	2	3	4	5	6

Three duplicates according to natural order.

↓ `quicksort(a)`

(1,6)	(2,5)	(3,3)	(4,2)	(5,1)	(5,7)	(5,4)
0	1	2	3	4	5	6

The duplicates are not in the same relative order as before the sort.

Quicksort is not stable.

Stability

```
Data[] a = {new Data(5, 1), new Data(4, 2), new Data(3, 3), new Data(5, 4),
            new Data(2, 5), new Data(1, 6), new Data(5, 7)};
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(5,1)	(4,2)	(3,3)	(5,4)	(2,5)	(1,6)	(5,7)
0	1	2	3	4	5	6

Three duplicates according to natural order.

`quicksort(a)`

(1,6)	(2,5)	(3,3)	(4,2)	(5,1)	(5,7)	(5,4)
0	1	2	3	4	5	6

The duplicates are not in the same relative order as before the sort.

Quicksort is not stable.

Quicksort could be stable, but typical implementations aren't in the interest of efficiency.

Stability

```
class Student implements Comparable<Student> {
   private String fname;
   private String lname;
   private int    section;

   . . .

   public String toString() {
      return lname + ", " + fname + ", " + section;
   }

   public int compareTo(Student s) {
      return this.toString().compareTo(s.toString());
   }
}
```

Stability

```
Student[] roll = new Student[7];
// new Student(“last name”, “first name”, section)
roll[0] = new Student("Wei", "Patricia", 1);
roll[1] = new Student("Dipaolo", "Jordan", 1);
roll[2] = new Student("Cullins", "Janna", 2);
roll[3] = new Student("Center", "Wallace", 2);
roll[3] = new Student("Trojacek", "Evelyn", 3);
roll[4] = new Student("Buffum", "Tawana", 3);
roll[5] = new Student("Laber", "Sharyl", 3);
roll[6] = new Student("Hageman", "Rachel", 2);

shuffle(roll);

Comparator<Student> orderByName = new CompareStudentsByName();
Comparator<Student> orderBySection = new CompareStudentsBySection();
```

Stability

```
print(a);

quicksort(roll, orderByName);
print(a);

quicksort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample

Laber,    Sharyl,   3
Hageman,  Rachel,   2
Cullins,  Janna,    2
Dipaolo,  Jordan,   1
Buffum,   Tawana,   3
Trojacek, Evelyn,   3
Wei,      Patricia, 1
```

Stability

```
print(a);

quicksort(roll, orderByName);
print(a);

quicksort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample


. . .

Buffum,   Tawana,   3
Cullins,  Janna,    2
Dipaolo,  Jordan,   1
Hageman,  Rachel,   2
Laber,    Sharyl,   3
Trojacek, Evelyn,   3
Wei,      Patricia, 1
```

Stability

```
print(a);

quicksort(roll, orderByName);
print(a);

quicksort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample

. . .

. . .

Wei,      Patricia, 1
Dipaolo,  Jordan,   1
Hageman,  Rachel,   2
Cullins,  Janna,    2
Laber,    Sharyl,   3
Buffum,   Tawana,   3
Trojacek, Evelyn,   3
```

Stability

```
print(a);

quicksort(roll, orderByName);
print(a);

quicksort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample

. . .

. . .

Wei,      Patricia, 1
Dipaolo,  Jordan,   1
Hageman,  Rachel,   2
Cullins,  Janna,    2
Laber,    Sharyl,   3
Buffum,   Tawana,   3
Trojacek, Evelyn,   3
```

Stability

```
print(a);

mergesort(roll, orderByName);
print(a);

mergesort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample

Laber,    Sharyl,   3
Hageman,  Rachel,   2
Cullins,  Janna,    2
Dipaolo,  Jordan,   1
Buffum,   Tawana,   3
Trojacek, Evelyn,   3
Wei,      Patricia, 1
```

Stability

```
print(a);

mergesort(roll, orderByName);
print(a);

mergesort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample


. . .

Buffum,   Tawana,   3
Cullins,  Janna,    2
Dipaolo,  Jordan,   1
Hageman,  Rachel,   2
Laber,    Sharyl,   3
Trojacek, Evelyn,   3
Wei,      Patricia, 1
```

Stability

```
print(a);

mergesort(roll, orderByName);
print(a);

mergesort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample

. . .

. . .

Dipaolo,  Jordan,   1
Wei,      Patricia, 1
Cullins,  Janna,    2
Hageman,  Rachel,   2
Buffum,   Tawana,   3
Laber,    Sharyl,   3
Trojacek, Evelyn,   3
```

Stability

```
print(a);

mergesort(roll, orderByName);
print(a);

mergesort(roll, orderBySection);
print(a);
```

```
% java SortStabilityExample

. . .

. . .

Dipaolo,  Jordan,   1
Wei,      Patricia, 1
Cullins,  Janna,    2
Hageman,  Rachel,   2
Buffum,   Tawana,   3
Laber,    Sharyl,   3
Trojacek, Evelyn,   3
```

Summary

Here's a summary of the four sorting algorithms we've talked about – as illustrated by the given implementations – with respect to time complexity and basic properties of sorting.

	Selection	Insertion	Mergesort	Quicksort
Worst case	$O(N^2)$	$O(N^2)$	$O(N \log N)$	$O(N^2)$
Average case	$O(N^2)$	$O(N^2)$	$O(N \log N)$	$O(N \log N)$
Best case	$O(N^2)$	$O(N)$	$O(N \log N)$	$O(N \log N)$
In-place?	Yes	Yes	No	Yes
Stable?	No	Yes	Yes	No
Adaptive?	No	Yes	No	No

Quicksort example

pivot = 57, left = 0, right = 20

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
57	70	97	38	63	21	85	68	76	9	81	36	55	79	74	85	16	61	77	49	24

Swap pivot with a[right]

pivot = 57, left = 0, right = 20

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
24	70	97	38	63	21	85	68	76	9	81	36	55	79	74	85	16	61	77	49	57
	i,p																			

Scan forward, find 38 < 57.

pivot = 57, left = 0, right = 20

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
24	70	97	38	63	21	85	68	76	9	81	36	55	79	74	85	16	61	77	49	57
	p		i																	

Swap the border value (70) with 38.

pivot = 57, left = 0, right = 20

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
24	38	97	70	63	21	85	68	76	9	81	36	55	79	74	85	16	61	77	49	57
		p		i																