

Implementing Collections – LinkedSet

Problem Overview

This assignment requires you to implement a *set* collection using a doubly-linked list as the underlying data structure. You are provided with the `Set` interface and a shell of the `LinkedSet` implementing class. You must not change anything in the `Set` interface, but you must create correct implementations of the methods in the `LinkedSet` class. In doing so you are allowed to add any number of private methods and nested classes that you need, but you may not create or use any other top-level class and you may not create any public method. You must also use without modification the existing fields of the `LinkedSet` class.

The `Set` interface is generic and makes no restrictions on the type that it can contain. The `LinkedSet` class is also generic, but it *does* make a restriction on the type variable—any type bound by a client to `T` must be a class that implements the `Comparable` interface for that type. Thus, there is a *natural order* on the values stored in an `LinkedSet`, but not (necessarily) on those stored in a `Set`. This is an important distinction, so pause here long enough to make sure you understand this point.

The following sections describe each method to be implemented, organized according to the methods appearing in the `Set` interface and those specific to the `LinkedSet` class.

Methods of the Set Interface

Each method from the `Set` interface that you must implement in the `LinkedSet` class is described below and in comments in the provided source code. You must read both. Note that in addition to correctness, your code will also be graded on the stated performance requirements.

`add(T element)`

The `add` method ensures that this set contains the specified element. Neither duplicates¹ nor null values are allowed. The method returns true if this set was modified (i.e., the element was added) and false otherwise. Note that the constraint on the generic type parameter `T` of the `LinkedSet` class ensures that there is a *natural order* on the values stored in an `LinkedSet`. You must maintain the internal doubly-linked list in *ascending natural order* at all times. The time complexity of the `add` method must be $O(N)$, where N is the number of elements in the set.

`remove(T element)`

The `remove` method ensures that this set does not contain the specified element. The method returns true if this set was modified (i.e., an existing element was removed) and false otherwise. The `remove` method must maintain the ascending natural order of the doubly-linked list. The time complexity of the `remove` method must be $O(N)$, where N is the number of elements in the set.

¹This is an example of when it's important for `compareTo` and `equals` to be consistent in the underlying data class.

contains(T element)

The **contains** method searches for the specified element in this set, returning true if the element is in this set and false otherwise. The time complexity of the **contains** method must be $O(N)$, where N is the number of elements in the set.

size()

The **size** method returns the number of elements in this set. *This method is provided for you and must not be changed.* The time complexity of the **size** method is $O(1)$.

isEmpty()

The **isEmpty** method returns true if there are no elements in this set and false otherwise. *This method is provided for you and must not be changed.* The time complexity of the **isEmpty** method is $O(1)$. Any set for which **isEmpty()** returns true is considered the “empty set” (\emptyset) for purposes of union, intersection, and complement described below.

equals(Set<T> s)

Two sets are equal if and only if they contain exactly the same elements, regardless of order². If $A = \{1, 2, 3\}$, $B = \{3, 1, 2\}$, and $C = \{1, 2, 3, 4\}$, then $A = B$ and $A \neq C$. The **equals** method returns true if this set contains exactly the same elements as the parameter set, and false otherwise. The time complexity of the **equals** method must be $O(N^2)$ where N is the size of each set.

union(Set<T> s)

The *union* of set A with set B , denoted $A \cup B$, is defined as $\{x \mid x \in A \text{ or } x \in B\}$. Note that $A \cup B = B \cup A$ and $A \cup \emptyset = A$. The **union** method returns a set that is the *union* of this set and the parameter set; that is, the set that contains the elements of both this set and the parameter set. The result set must be in ascending natural order. The time complexity of the **union** method must be $O(N \times M)$ where N is the size of this set and M is the size of the parameter set.

intersection(Set<T> s)

The *intersection* of set A with set B , denoted $A \cap B$, is defined as $\{x \mid x \in A \text{ and } x \in B\}$. Note that $A \cap B = B \cap A$ and $A \cap \emptyset = \emptyset$. The **intersection** method returns a set that is the *intersection* of this set and the parameter set; that is, the set that contains the elements of this set that are also in the parameter set. The result set must be in ascending natural order. The time complexity of the **intersection** method must be $O(N \times M)$ where N is the size of this set and M is the size of the parameter set.

complement(Set<T> s)

The *relative complement* of set B with respect to set A , denoted $A \setminus B$, is defined as $\{x \mid x \in A \text{ and } x \notin B\}$. Note that $A \setminus B \neq B \setminus A$, $A \setminus \emptyset = A$, and $\emptyset \setminus A = \emptyset$. The **complement** method returns a set that is the *relative complement* of the parameter set with respect to this set; that is, the set that contains the elements of this set that are not in the parameter set. The result set must be in ascending natural order. The time complexity of the **complement** method must be $O(N \times M)$ where N is the size of this set and M is the size of the parameter set.

²Since the parameter s is typed as the interface **Set**, we don't know what the implementing class is. Therefore, we can't assume that the parameter set is in any particular order at all. This is true for every other method with a parameter typed as **Set**.

iterator()

The **iterator** method returns an **Iterator** over the elements in this set. Although the interface specifies that no particular order can be assumed (by a client), the **LinkedSet** implementation must ensure that the resulting iterator returns the elements in ascending natural order. The associated performance constraints are as follows: **iterator()**: $O(1)$; **hasNext()**: $O(1)$; **next()**: $O(1)$; required space: $O(1)$.

LinkedSet Methods

In addition to the methods from the **Set** interface, the **LinkedSet** class also implements its own methods. Most of these methods are designed to take advantage of the underlying representation, while some of them simply provide added functionality. Each of these class-specific methods is described below, as well as in comments in the provided source code. You must read both. Note that in addition to correctness, your code will also be graded on the stated performance requirements.

Constructors

The only public constructor that is allowed has been provided for you and you must not change it in any way. You may, however, find it helpful to write your own *private* constructor; one that offers direct support for building an **LinkedSet** from an existing doubly-linked list. Such a constructor will be helpful but it is not required and will not be graded.

equals(LinkedSet<T> s)

The semantics of this overloaded method is identical to the **equals** method described above. However, since the parameter is typed as an **LinkedSet**, this method can directly access the doubly-linked list in this set as well as in the parameter set. Having access to the underlying representation of both sets allows a more efficient algorithm for this method. The time complexity of this **equals** method must be $O(N)$ where N is the size of this set.

union(LinkedSet<T> s)

The semantics of this overloaded method is identical to the **union** method described above. However, since the parameter is typed as an **LinkedSet**, this method can directly access the doubly-linked list in this set as well as in the parameter set. Having access to the underlying representation of both sets allows a more efficient algorithm for this method. The time complexity of this **union** method must be $O(\max(N, M))$ where N is the size of this set and M is the size of the parameter set.

intersection(LinkedSet<T> s)

The semantics of this overloaded method is identical to the **intersection** method described above. However, since the parameter is typed as an **LinkedSet**, this method can directly access the doubly-linked list in this set as well as in the parameter set. Having access to the underlying representation of both sets allows a more efficient algorithm for this method. The time complexity of this **intersection** method must be $O(\max(N, M))$ where N is the size of this set and M is the size of the parameter set.

complement(LinkedSet<T> s)

The semantics of this overloaded method is identical to the **complement** method described above. However, since the parameter is typed as an **LinkedSet**, this method can directly access the doubly-linked list in this set as well as in the parameter set. Having access to the underlying representation of both sets allows a more

efficient algorithm for this method. The time complexity of this **complement** method must be $O(\max(N, M))$ where N is the size of this set and M is the size of the parameter set.

descendingIterator()

The **descendingIterator** method returns an Iterator over the elements in this set in *descending* natural order. The associated performance constraints are as follows: **descendingIterator()**: $O(1)$; **hasNext()**: $O(1)$; **next()**: $O(1)$; required space: $O(1)$.

powerSetIterator()

The *power set* of a set S , denoted $\mathcal{P}(S)$, is defined as $\{T \mid T \subseteq S\}$; that is, the set of all subsets of S . There are 2^N members of $\mathcal{P}(S)$ where N is the number of elements in S . For example, if $S = \{A, B, C\}$, then $\mathcal{P}(S) = \{\emptyset, \{A\}, \{B\}, \{C\}, \{A, B\}, \{B, C\}, \{A, C\}, \{A, B, C\}\}$. (Note that the empty set \emptyset is a member of every set.) The **powerSetIterator** method returns an Iterator over the elements in the *power set* of this set. The iterator makes no guarantees regarding the order in which the elements of $\mathcal{P}(S)$ will be returned. The associated time complexities are as follows: **powerSetIterator()**: $O(N)$; **hasNext()**: $O(1)$; **next()**: $O(N)$; required space: $O(N)$, where N is the size of this set.