

A Unified Three-Phase Transformer Model for Distribution Load Flow Calculations

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Abstract—This paper provides a unified method to model three-phase transformers for distribution system load flow calculations, especially when the matrix singularity caused by the transformer configuration arises. This paper shows that the singularity appears only in certain transformer admittance submatrices and only in certain transformer configurations. The unified method presented in this paper can solve the voltage/current equations in the forward/backward sweep algorithm for various types of transformer configurations, whether or not the corresponding admittance submatrices are singular. Comprehensive comparisons have been made between the proposed approach and other methods. Test results demonstrate the validity and effectiveness of the proposed method.

Index Terms—Admittance matrix, load flow analysis, power distribution, power transformers.

I. INTRODUCTION

As the power distribution networks become more and more complex, there is a higher demand for efficient and reliable system operation. Consequently, two of the most important system analysis tools, power flow and short-circuit studies, must have the capability to handle various system configurations with adequate accuracy and speed. Of the several dedicated distribution system load flow methods used in the power industry, the forward/backward sweep algorithm [1], with its low memory and computation requirements and robust convergence characteristic, has gained the most popularity in recent years. The algorithm is also known to be the branch current-based feeder analysis. Based on the ladder theory for linear circuit analysis, the forward/backward sweep algorithm can fully utilize the radial structure of most distribution networks. With minor modifications, several methods [2], [3] have been proposed to extend its application in weakly meshed distribution systems.

To take into account the existence of multiphase load and unsymmetrical feeders in distribution systems, three-phase system representation is generally used. The modeling of three-phase transformers is a vital step in distribution system analysis. Due to unbalanced system operations, a complete and accurate three-phase model is desirable for distribution and inline transformers of various core and winding configurations.

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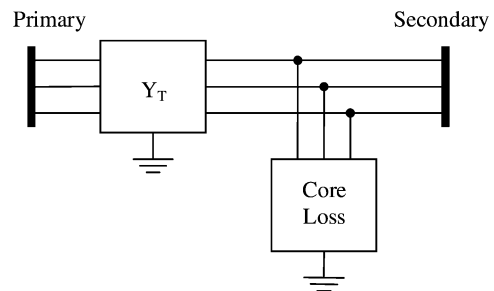


Fig. 1. General three-phase transformer model.

A two-block three-phase transformer model was presented in [4]. As shown in Fig. 1, a series block represents winding connections and leakage impedance, and a shunt block models real and reactive power losses in the transformer core. A similar model was proposed in [5], where the shunt block is connected to the primary side.

The core loss of a transformer is approximated by shunt core loss functions on each phase of the secondary terminal of the transformer. Generally, the functions are nonlinear and the coefficients should be determined by experiments. Since transformer winding connections have little effect on core loss, this paper focuses mainly on the series block, while the core loss block can be treated as a three-phase load on either side of the transformer.

To include such a transformer model into the forward/backward sweep algorithm, specific voltage/current relationships should be derived. Several approaches have been developed in the past several years. In [4], fictitious injection current sources were used to resolve the coupling between the primary and secondary sides, which greatly simplified the admittance matrix. However, this technique faces slow convergence problems when employed in the forward/backward sweep algorithm. In [5], voltage and current update equations were developed for three of the most commonly used transformer connections based on their equivalent circuits. In [9], voltage/current equations were derived in matrix form for transformers of the ungrounded wye-delta connection. However, these methods are mainly based on circuit analysis with Kirchhoff's voltage and current laws. Different set of equations are needed to handle each connection type. Transformers of various winding connections can only be analyzed on a case-by-case basis, which is not convenient for efficient implementation and incorporation of new connection types.

In this paper, a unified method is proposed to model the diverse distribution transformers into the forward/backward sweep load flow algorithm. A brief description of the forward/backward sweep algorithm is presented in Section II. In Section III, the proposed modeling procedure is explained in

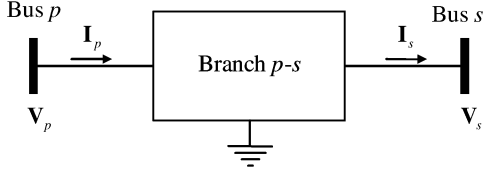


Fig. 2. One section of a distribution network.

detail. Extensive computation and comparisons have been done to verify the method, and the results are presented in Section IV.

II. FORWARD/BACKWARD SWEEP ALGORITHM

Although the forward/backward sweep algorithm can be extended to solve systems with loops and distributed generation buses, a radial network with only one voltage source is used here to depict the principles of the algorithm. Such a system can be modeled as a tree, in which the root is the voltage source, and the branches can be a segment of feeder, a transformer, or other components between two buses.

With the given voltage magnitude and phase angle at the root and known system load information, the power flow algorithm needs to determine the voltages at all other buses and currents in each branch. The forward/backward sweep algorithm employs an iterative method to update bus voltages and branch currents. During each iteration, a backward sweep is performed to update branch currents, and a forward sweep is performed to update bus voltages. The algorithm terminates when the voltages converge.

A brief description of the main steps of the forward/backward sweep algorithm is shown as follows.

A. Initialization

Before the first iteration, the voltage at each leaf node, i.e., the buses that do not have child buses, is given an initial guess value. The guess values should be as close to the true values as possible to reduce the number of iterations and to avoid divergence. Typically, the voltage magnitude is set to one per unit, while the voltage phase angle can be chosen by considering transformer phase shift between the root and the leaf nodes.

B. Backward Sweep

With voltages at all the leaf nodes given, the backward sweep procedure determines the currents in the branches that connect these nodes and their parent nodes and the voltages at the parent nodes. When the voltages at the parent nodes are calculated, these nodes can then be treated as “leaf” nodes and the calculation continues until the root is reached.

Fig. 2 shows one section of the system, in which bus p is the parent node and bus s is the leaf node. Each step of the backward sweep is to determine the currents flowing through the branch $p - s$ and the voltages at bus p . Therefore, in the backward sweep, the branch components need to be modeled in a way that V_p and I_p can be obtained when V_s and I_s are known.

C. Forward Sweep

After backward sweep, the currents in all branches are updated. However, since calculations of these currents are based

on estimated bus voltages, incorrect voltage values will result in incorrect branch currents. In forward sweep, the root voltage information, together with the branch currents obtained from the last backward sweep, are used to update the leaf node voltages.

Starting from the root node, the voltages at all the child nodes are calculated based on the parent bus voltages and the currents in the branches. The procedure continues until all the leaf node voltages are updated.

As indicated in Fig. 2, the forward sweep requires that the branch be modeled in a way that V_s can be obtained when V_p and I_s are known.

The proposed method for modeling distribution transformer satisfies the requirements of the forward/backward sweep algorithm and can be easily integrated into the algorithm, regardless of the types of transformer configurations.

III. TRANSFORMER MODELING PROCEDURE

A. Construction of the Primitive Admittance Matrix

The starting point for the proposed modeling approach is the primitive admittance matrix (\mathbf{Y}_{prim}) of the transformer, which can be determined according to the transformer winding connections. Depending on the system representation, both per-unit values or actual unit values can be used to form the matrix. A discussion of the advantages and disadvantages of both unit systems was presented in [6].

B. Conversion to Nodal Admittance Matrix

The conversion from the primitive admittance matrix \mathbf{Y}_{prim} to the nodal admittance matrix \mathbf{Y}_T was discussed in detail in [8]. Basically, the procedure involves using the node-to-branch incidence matrix \mathbf{A} with

$$\mathbf{Y}_{\text{node}} = \mathbf{A}\mathbf{Y}_{\text{prim}}\mathbf{A}^T. \quad (1)$$

The matrix \mathbf{Y}_{node} can then be reduced to 6×6 by eliminating the neutral point nodes with Kron reduction. The resulted matrix is the nodal admittance matrix \mathbf{Y}_T .

In [10], a systematic approach utilizing symbolic mathematical tools was proposed to establish the nodal admittance matrices for various transformer connection types. Minor modifications are necessary to take into account the off-nominal tap ratio between the primary and secondary sides of the transformer. The \mathbf{Y}_T matrices for the most common transformer configurations were proposed in [4].

Following the same transformer modeling procedures, it is straightforward to build the nodal admittance matrices for the more particular transformer configurations such as Open Wye/Open Delta. Once these matrices are determined, they can be used in the forward/backward algorithm. However, this paper does not discuss these more specialized transformer configurations due to space limitations.

C. Characteristics of the \mathbf{Y}_T Submatrices

With the nodal admittance matrix \mathbf{Y}_T , the transformer voltage-current relationship can be expressed as

$$\begin{bmatrix} \mathbf{I}_p \\ \mathbf{I}_s \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{pp} & \mathbf{Y}_{ps} \\ \mathbf{Y}_{sp} & \mathbf{Y}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{V}_p \\ \mathbf{V}_s \end{bmatrix} \quad (2)$$

TABLE I

 \mathbf{Y}_T SUBMATRICES FOR COMMON STEP-DOWN TRANSFORMER CONNECTIONS

Primary	Secondary	\mathbf{Y}_{pp}	\mathbf{Y}_{ss}	\mathbf{Y}_{ps}	\mathbf{Y}_{sp}
Y_g	Y_g	\mathbf{Y}_I	\mathbf{Y}_I	$-\mathbf{Y}_I$	$-\mathbf{Y}_I$
Y_g	Y	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y_g	Δ	\mathbf{Y}_I	\mathbf{Y}_{II}	\mathbf{Y}_{III}	\mathbf{Y}_{III}^T
Y	Y_g	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Y	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Δ	\mathbf{Y}_{II}	\mathbf{Y}_{II}	\mathbf{Y}_{III}	\mathbf{Y}_{III}^T
Δ	Y_g	\mathbf{Y}_{II}	\mathbf{Y}_I	\mathbf{Y}_{III}	\mathbf{Y}_{III}^T
Δ	Y	\mathbf{Y}_{II}	\mathbf{Y}_{II}	\mathbf{Y}_{III}	\mathbf{Y}_{III}^T
Δ	Δ	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$

where the \mathbf{Y}_T matrix is divided into four 3×3 submatrices: \mathbf{Y}_{pp} , \mathbf{Y}_{ps} , \mathbf{Y}_{sp} , and \mathbf{Y}_{ss} . Vectors \mathbf{V}_p , \mathbf{I}_p , and \mathbf{V}_s , \mathbf{I}_s are the three-phase line-to-neutral bus voltages and injection currents at the primary and secondary sides of the transformer, respectively.

As described in the last section, in backward sweep procedure, \mathbf{V}_s and \mathbf{I}_s are known, while \mathbf{V}_p and \mathbf{I}_p are to be calculated. From (2), the following can be derived:

$$\mathbf{V}_p = \mathbf{Y}_{sp}^{-1}(\mathbf{I}_s - \mathbf{Y}_{ss} \mathbf{V}_s) \quad (3)$$

$$\mathbf{I}_p = \mathbf{Y}_{pp} \mathbf{V}_p + \mathbf{Y}_{ps} \mathbf{V}_s. \quad (4)$$

In forward sweep, \mathbf{V}_p and \mathbf{I}_s are known, and \mathbf{V}_s needs to be calculated. Similarly

$$\mathbf{V}_s = \mathbf{Y}_{ss}^{-1}(\mathbf{I}_s - \mathbf{Y}_{sp} \mathbf{V}_p). \quad (5)$$

According to (3)–(5), the implementation of the forward/backward sweep algorithm requires the inversion of submatrices \mathbf{Y}_{sp} and \mathbf{Y}_{ss} . However, a close examination of the \mathbf{Y}_T matrices for common transformer configurations shows that these submatrices are often singular. Table I shows the submatrices of \mathbf{Y}_T for the nine most common step-down transformer connection types, and Table II shows the matrices for step-up transformers, where

$$\mathbf{Y}_I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} y_t \quad (6)$$

$$\mathbf{Y}_{II} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} y_t \quad (7)$$

$$\mathbf{Y}_{III} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} y_t \quad (8)$$

and y_t is the per-unit transformer leakage admittance. For simplification, the leakage admittances of each phase are assumed to be identical. For transformers with unbalanced admittances, their nodal admittance matrices are more complex and do not take the forms shown in Tables I or II. However, it is proved that the singularity of the transformer submatrices remains the same.

TABLE II

 \mathbf{Y}_T SUBMATRICES FOR COMMON STEP-UP TRANSFORMER CONNECTIONS

Primary	Secondary	\mathbf{Y}_{pp}	\mathbf{Y}_{ss}	\mathbf{Y}_{ps}	\mathbf{Y}_{sp}
Y_g	Y_g	\mathbf{Y}_I	\mathbf{Y}_I	$-\mathbf{Y}_I$	$-\mathbf{Y}_I$
Y_g	Y	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y_g	Δ	\mathbf{Y}_I	\mathbf{Y}_{II}	\mathbf{Y}_{III}^T	\mathbf{Y}_{III}
Y	Y_g	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Y	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$
Y	Δ	\mathbf{Y}_{II}	\mathbf{Y}_{II}	\mathbf{Y}_{III}^T	\mathbf{Y}_{III}
Δ	Y_g	\mathbf{Y}_{II}	\mathbf{Y}_I	\mathbf{Y}_{III}^T	\mathbf{Y}_{III}
Δ	Y	\mathbf{Y}_{II}	\mathbf{Y}_{II}	\mathbf{Y}_{III}^T	\mathbf{Y}_{III}
Δ	Δ	\mathbf{Y}_{II}	\mathbf{Y}_{II}	$-\mathbf{Y}_{II}$	$-\mathbf{Y}_{II}$

From (7) and (8), it is obvious that both \mathbf{Y}_{II} and \mathbf{Y}_{III} are singular. Hence, \mathbf{Y}_{sp} is invertible only for Y_g/Y_g connection, and \mathbf{Y}_{ss} is invertible only for Y_g/Y_g and Δ/Y_g connections. For Y_g/Y_g connection, (3)–(5) can be directly used for forward and backward sweep calculations. For Δ/Y_g connection, only (5) can be used in forward sweep. For all other connection types, since the matrices are singular, there is no unique solution to the above equations. In essence, the singularity in those transformer configurations arises due to the lack of voltage reference point on one or both sides of the transformer.

D. Solving the Singularity Problem

To circumvent the singularity issue, it is noted that although the three-phase line-to-neutral voltages \mathbf{V}_p and \mathbf{V}_s cannot be obtained by solving (3) and (5), the nonzero-sequence components of the voltages can be uniquely determined. To illustrate this point in the backward sweep, rewrite (3) as

$$\mathbf{Y}_{sp} \mathbf{V}_p = \mathbf{I}_s - \mathbf{Y}_{ss} \mathbf{V}_s. \quad (9)$$

Let \mathbf{V}'_p represent the nonzero-sequence components of \mathbf{V}_p , i.e.,

$$\mathbf{V}'_p = \mathbf{V}_p - \mathbf{V}_p^0 \quad (10)$$

where vector \mathbf{V}_p^0 is the zero-sequence voltage on the primary side; thus

$$\mathbf{Y}_{sp} (\mathbf{V}'_p + \mathbf{V}_p^0) = \mathbf{I}_s - \mathbf{Y}_{ss} \mathbf{V}_s. \quad (11)$$

The product of \mathbf{Y}_{sp} and \mathbf{V}_p^0 is always zero for transformer configurations other than Y_g/Y_g . This is because \mathbf{Y}_{sp} is represented by \mathbf{Y}_{II} or \mathbf{Y}_{III} in all other transformer configurations except Y_g/Y_g . From (7) and (8), it can be seen that

$$\mathbf{Y}_{II} \mathbf{V}_p^0 = \mathbf{Y}_{III} \mathbf{V}_p^0 = \mathbf{0} \quad (12)$$

so (11) can be reduced to

$$\mathbf{Y}_{sp} \mathbf{V}'_p = \mathbf{I}_s - \mathbf{Y}_{ss} \mathbf{V}_s. \quad (13)$$

Equation (13) indicates that the zero-sequence component of \mathbf{V}_p does not affect the backward sweep calculation for transformers with a singular \mathbf{Y}_{sp} matrix. The above analysis shows

that (3) can be used to calculate both \mathbf{V}_p and its nonzero-sequence components \mathbf{V}'_p . However, if \mathbf{Y}_{sp} is singular, (13) still cannot uniquely determine \mathbf{V}'_p , and a modification is needed. Since \mathbf{V}'_p does not contain zero-sequence component, it satisfies

$$[1 \ 1 \ 1]\mathbf{V}'_p = 0. \quad (14)$$

Equations (13) and (14) can be combined as

$$\mathbf{Y}'_{sp} \mathbf{V}'_p = \mathbf{I}'_s - \mathbf{Y}'_{ss} \mathbf{V}_s \quad (15)$$

where \mathbf{Y}'_{sp} is obtained by replacing the last row of \mathbf{Y}_{sp} with $[1 \ 1 \ 1]$, while \mathbf{I}'_s and \mathbf{Y}'_{ss} are the same as \mathbf{I}_s and \mathbf{Y}_{ss} , except that elements in their last row are set to 0 so that (14) is satisfied.

Now that \mathbf{Y}'_{sp} is not singular, the nonzero-sequence components of the voltages on the primary side can be determined by

$$\mathbf{V}'_p = \mathbf{Y}'_{sp}{}^{-1}(\mathbf{I}'_s - \mathbf{Y}'_{ss} \mathbf{V}_s). \quad (16)$$

Similar results can be obtained for forward sweep calculation

$$\mathbf{V}_s'' = \mathbf{Y}_{ss}''^{-1}(\mathbf{I}_s'' - \mathbf{Y}_{sp}'' \mathbf{V}_p) \quad (17)$$

where \mathbf{V}_s'' is the nonzero-sequence component of \mathbf{V}_s , \mathbf{Y}_{ss}'' is the same as \mathbf{Y}_{ss} , except that the last row is replaced with $[1 \ 1 \ 1]$, \mathbf{I}_s'' , and \mathbf{Y}_{sp}'' are obtained by setting the elements in the last row of \mathbf{I}_s and \mathbf{Y}_{sp} to 0, respectively.

Once the nonzero-sequence components of \mathbf{V}_p or \mathbf{V}_s are calculated, zero-sequence components are added to them to form the line-to-neutral voltages so that the forward/backward sweep procedure can continue.

As an example, consider the backward sweep for an ungrounded Y/ Δ transformer. According to Table I

$$\mathbf{Y}_{sp} = \mathbf{Y}_{III}^T = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} y_t \quad (18)$$

and

$$\mathbf{Y}_{ss} = \mathbf{Y}_{II} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} y_t. \quad (19)$$

Thus, (13) becomes

$$\frac{y_t}{\sqrt{3}} \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V'_{ap} \\ V'_{bp} \\ V'_{cp} \end{bmatrix} = \begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} - \frac{y_t}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix}. \quad (20)$$

The singularity of matrix \mathbf{Y}_{sp} is evident. However, the matrix can be changed to nonsingular if one of its rows is replaced with $[1 \ 1 \ 1]y_t/\sqrt{3}$, i.e.,

$$\mathbf{Y}'_{sp} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} y_t. \quad (21)$$

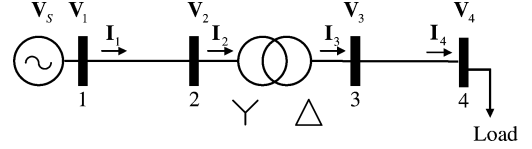


Fig. 3. Four-bus example system.

Equation (20) can then be transformed into

$$\begin{bmatrix} V'_{ap} \\ V'_{bp} \\ V'_{cp} \end{bmatrix} = \frac{1}{3} y_t \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \times \left(\begin{bmatrix} I_{as} \\ I_{bs} \\ 0 \end{bmatrix} - \frac{1}{3} y_t \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} \right). \quad (22)$$

E. Modified Forward/Backward Sweep Algorithm

With the above transformation, the equations for transformer voltage calculation are no longer singular. However, the resulted transformer voltages \mathbf{V}'_p and \mathbf{V}'_s only contain the positive and negative sequence components. Thus, zero-sequence voltages must be added to them to form line-to-neutral voltages. The primary-side zero-sequence voltage can be initialized to 0 and is updated during the forward sweep. Since voltages are also calculated when line currents are updated, the secondary-side zero-sequence voltage can be obtained directly from the backward sweep.

To illustrate the modified procedure, a four-bus example shown in Fig. 3 is used.

1) *Initialization*: The forward/backward sweep algorithm begins with all the load information and only the source voltage \mathbf{V}_s known. A guess value is given to the voltages on bus 4.

2) *Backward Sweep*: With \mathbf{V}_4 given, the load currents can be calculated. If the transformer core loss is modeled on the secondary side, core loss functions can be used to determine the absorbed power and current. Thus, the three-phase currents \mathbf{I}_4 that flows through feeder 3–4 can be obtained. Assume that line charging is neglected, then the currents flowing through the secondary side of the transformer (\mathbf{I}_3) are equal to \mathbf{I}_4 . The voltages on the secondary side of the transformer are

$$\mathbf{V}_3 = \mathbf{V}_4 + \mathbf{Z}_{\text{line34}} \mathbf{I}_4$$

where $\mathbf{Z}_{\text{line34}}$ is the line impedance matrix for feeder segment 3–4.

For the Y/ Δ -connected transformer, its \mathbf{Y}_{sp} matrix is not invertible. According to (16)

$$\mathbf{V}'_2 = \mathbf{Y}'_{sp}{}^{-1}(-\mathbf{I}'_3 - \mathbf{Y}'_{ss} \mathbf{V}_3).$$

Since \mathbf{V}'_2 only contains the positive-and negative-sequence components of \mathbf{V}_2 , an initial value of zero-sequence voltage V_2^0 is needed to get the transformer primary side voltage \mathbf{V}_2 ,

$$\mathbf{V}_2 = \mathbf{V}'_2 + V_2^0.$$

Next the currents at the ungrounded Wye side can be determined with

$$\mathbf{I}_2 = \mathbf{Y}_{pp} \mathbf{V}_2 + \mathbf{Y}_{ps} \mathbf{V}_3.$$

The backward sweep calculation continues until the source bus is reached. If the difference between the computed source voltage \mathbf{V}_1 and the actual source voltage \mathbf{V}_S is not within the required error limit, a forward sweep is performed.

3) *Forward Sweep*: The forward sweep begins by setting \mathbf{V}_1 to its actual value \mathbf{V}_S . Since the currents in feeder sections 1–2 have been calculated in the backward sweep, the voltage at bus 2 can be obtained by

$$\mathbf{V}_2 = \mathbf{V}_1 - \mathbf{Z}_{\text{line}12} \mathbf{I}_1$$

where $\mathbf{Z}_{\text{line}12}$ is the line impedance matrix for feeder segment 1–2. Note that by updating \mathbf{V}_2 , the zero-sequence component V_2^0 is also updated and will be used in the next backward sweep procedure.

The next step is to determine \mathbf{V}_3 based on the knowledge of \mathbf{V}_2 and \mathbf{I}_3 . Still, due to matrix singularity, only positive- and negative-sequence components can be determined

$$\mathbf{V}_3'' = \mathbf{Y}_{ss}''^{-1} (-\mathbf{I}_s'' - \mathbf{Y}_{sp}'' \mathbf{V}_2).$$

The voltage \mathbf{V}_3 can then be calculated by adding zero-sequence voltage V_3^0 , which is obtained from the last value of \mathbf{V}_3 in backward sweep. The forward sweep then continues until bus 4 is reached.

4) *Zero-Sequence Voltage Update*: As illustrated above, for transformers with singular \mathbf{Y}_{sp} or \mathbf{Y}_{ss} submatrices, zero-sequence voltage update calculation cannot be performed from one side to the other. This is due to the fact that zero-sequence equivalent circuit is interrupted at a transformer with ungrounded Y and/or Δ windings. Hence, the zero-sequence voltage on one side of the transformer cannot be determined based on line-to-neutral voltages on the other side, even when line current information on both sides is available. In these cases, the zero-sequence voltages on the primary side can be updated by the source voltage during the next forward sweep. However, the zero-sequence voltages on the secondary side with a Δ or ungrounded Y winding will not be updated due to the lack of voltage reference point on the secondary side, which makes determining the real line-to-neutral voltage impossible.

To avoid such difficulties, the method in [9] used the line-to-line voltage on the Δ or ungrounded Y side of the transformer and the line-to-neutral voltage in the remaining part of the system. The method in [5] introduced an arbitrary reference neutral point to convert the line-to-line voltages to the line-to-neutral voltages. The proposed method, using an initially guessed zero-sequence voltage on the primary side, does not require any special treatment in handling the Δ or ungrounded Y transformers. With this initial zero-sequence voltage, line-to-neutral voltages can be used throughout the system.

The primary-side zero-sequence voltages are updated by the source voltage. The zero-sequence voltage on the secondary side with a Δ or ungrounded Y winding are determined by

TABLE III
 \mathbf{V}_{LL4} DURING ITERATIONS

No.	V_{LL4}^a		V_{LL4}^b		V_{LL4}^c	
	V	θ	V	θ	V	θ
1	1.7321	30	1.7321	-90	1.7321	150
2	1.4273	-6.528	1.55	-128.56	1.4467	108.2
3	1.4331	-5.8093	1.5187	-130.04	1.3826	108.94
4	1.4267	-5.7707	1.5195	-130.24	1.375	108.57
5	1.4264	-5.7591	1.5182	-130.27	1.3732	108.59
6	1.4262	-5.7573	1.5182	-130.28	1.373	108.58

grounded elements that are connected to the same side of the transformer, such as grounded load, line charging, or other grounded transformers. However, due to the limitations of forward/backward sweep algorithm [11], zero-sequence voltage cannot be updated. Therefore, the secondary line-to-neutral voltages in these conditions are the assumed value. The common practice in the utility industry will seldom see a single phase-to-ground load connected to an ungrounded three-phase source or transformer, so the zero-sequence current is generally very small compared with load currents. Thus, even if the calculated line-to-neutral voltage is not real, the corresponding line-to-line voltage is still accurate.

IV. TEST EXAMPLES

To verify the proposed method, different transformer configurations were included in a three-phase distribution load flow program with a forward/backward sweep algorithm. Two different sized examples were implemented, and results were compared with other existing methods.

A. Model Validation

The IEEE four-node test feeder [12] was used to provide a simple system for the testing of various three-phase transformer connections. The one-line diagram of the example is shown in Fig. 3.

The two distribution feeder segments have unequal mutual coupling between the phases. The load is unbalanced with different kilovoltampere and power factor in each phase.

For an ungrounded Y/ Δ transformer connection, the program terminates after five iterations, and the difference between computed source voltage \mathbf{V}_1 and specified source voltage \mathbf{V}_S are within 0.0001 p.u. The a -phase line-to-line voltages at the load are listed in Table III.

The results match very well with those listed in [9], where a detailed step-by-step example was given. Tests have been conducted for all other transformer connections, and the results match those listed in [12].

However, the method mentioned in [9] and [12] requires different sets of equations for different transformer configurations, which is not convenient for efficient implementation and incorporation of new connection types. The unified method proposed in this paper does not require any special equations to handle the transformers with the Δ or ungrounded Y windings.

TABLE IV
ZERO-SEQUENCE VOLTAGES AT BUS 2

Connection	$V_{2init}^0 = 0$		$V_{2init}^0 = 0.1$	
	$ V_2^0 $	$\angle V_2^0$	$ V_2^0 $	$\angle V_2^0$
Y_g/Y_g	0.0058989	6.4284	0.0058989	6.4284
Y/Y	0.00047676	155.1	0.0005357	148.88
Y_g/Δ	0.00039628	175.65	0.00039628	175.65
Y/Δ	0.00059186	171.55	0.00059186	171.55
Δ/Y_g	0.0005409	156.28	0.0005409	156.28
Δ/Y	0.00038698	150.93	0.00036049	137.83
Δ/Δ	0.00054162	-179.93	0.00054162	-179.93

TABLE V
ZERO-SEQUENCE VOLTAGES AT BUS 4

Connection	$V_{4init}^0 = 0$		$V_{4init}^0 = 0.1$	
	$ V_4^0 $	$\angle V_4^0$	$ V_4^0 $	$\angle V_4^0$
Y_g/Y_g	0.084931	7.9887	0.084931	7.9887
Y/Y	0.020974	-24.727	0.13814	-0.91431
Y_g/Δ	0.012234	-75.804	0.1061	-6.7337
Y/Δ	0.012229	-75.763	0.10642	-6.699
Δ/Y_g	0.078316	-20.423	0.078316	-20.423
Δ/Y	0.048551	-43.331	0.14225	-6.296
Δ/Δ	0.014606	-110.75	0.098763	-8.1718

TABLE VI
LINE-TO-LINE VOLTAGES AT BUS 4

Connection	$V_{4init}^0 = 0$		$V_{4init}^0 = 0.1$	
	$ V_4^{ab} $	$\angle V_4^{ab}$	$ V_4^{ab} $	$\angle V_4^{ab}$
Y_g/Y_g	1.5004	22.6694	1.5004	22.6694
Y/Y	1.4903	23.2930	1.4903	23.2930
Y_g/Δ	1.4262	-5.7569	1.4262	-5.7569
Y/Δ	1.4262	-5.7569	1.4262	-5.7569
Δ/Y_g	1.4968	-7.3237	1.4968	-7.3237
Δ/Y	1.4972	-6.9034	1.4972	-6.9034
Δ/Δ	1.4284	24.2798	1.4284	24.2798

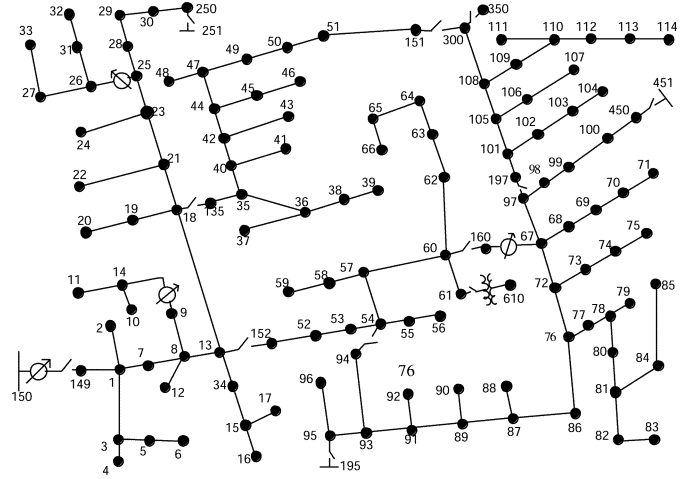


Fig. 4. IEEE 123-bus example system.

B. Zero-Sequence Voltage Update

Different transformer configurations were implemented in the four-bus example to further test the updating behavior of the zero-sequence voltage. Two different initial values (0 and 0.1 p.u.) were chosen for each configuration. If different initial values always converge to the same final value, then it means the zero-sequence voltage can be correctly updated during the iterations for the given type of transformer configuration.

Table IV shows the final zero-sequence voltages on the primary side of the transformer for different connection types. The results show that, for each transformer configuration, the primary side zero-sequence voltage always converges to a similar value, regardless of its initial value. These results indicate that the primary side zero-sequence voltage can be updated. This update takes place during the forward sweep, and it is due to the Y-grounded source voltage.

Table V shows the final zero-sequence voltages on the secondary side (bus 4) of the transformer. The results indicate that for some transformer configurations, different initial values will produce different final values. This is true for every transformer configuration except Y_g/Y_g and Δ/Y_g . The results further show that forward/backward sweep algorithm cannot produce unique secondary line-to-neutral voltages in these cases. It should be noted that the initial zero-sequence voltage V_4^0 is obtained from the initial leaf-node voltages V_4 .

For Y_g/Y_g and Δ/Y_g transformers, zero-sequence voltage on one side can be uniquely determined from voltages on the

other side together with current information. For other connections, a close examination reveals that the final values are in the vicinity of the initial value. In other words, zero-sequence voltages are not updated, even though the algorithm converges. This is due to the fact that without additional grounding devices on the secondary side, the subnetwork is isolated, and the zero-sequence voltage will not be affected by other part of the system.

Table VI shows the corresponding line-to-line voltages at bus 4 for different transformer configurations with different initial zero-sequence voltages. The results indicate that even though different initial values may produce different final zero-sequence voltages, the corresponding line-to-line voltages are correct when the algorithm converges, which meets the requirement for most load flow analysis.

C. Large System Tests

The IEEE 123-bus example shown in Fig. 4 is used to demonstrate the transformer models in large systems. The load flow analysis calculation was performed using per-unit values on a basis of 115 kV/4.16 kV/480 V and 10 MVA. There are two transformers in this system. One is located between nodes 150 and 149, and the other is between nodes 61 and 610.

Extensive tests have been performed on the system to verify the validity of the proposed method. Comparisons have been made against transformer modeling approaches developed in [4]

TABLE VII
ITERATION NUMBERS FOR DIFFERENT SYSTEM LOADING CONDITIONS

Case No.	Loading	Model 1	Model 2	Model 3
1	$0.25(P + jQ)$	5	5	7
2	$0.5(P + jQ)$	5	5	8
3	$0.75(P + jQ)$	5	6	10
4	$1.0(P + jQ)$	7	6	11
5	$1.25(P + jQ)$	7	6	12
6	$1.5(P + jQ)$	7	7	13
7	$1.75(P + jQ)$	10	9	18
8	$2.0(P + jQ)$	11	11	23

TABLE VIII
ITERATION NUMBERS FOR DIFFERENT R/X RATIOS

Case No.	Loading	Model 1	Model 2	Model 3
1	$R + jX$	5	5	7
2	$1.5R + jX$	5	5	7
3	$2R + jX$	7	7	10
4	$2.5R + jX$	9	10	15
5	$3R + jX$	11	12	22

and [5]. For convenience, the proposed modeling method is labeled model 1 in the tables, and methods in [5] and [4] are labeled model 2 and model 3, respectively.

Table VII compares the results under different loading conditions, and Table VIII compares the results under different R/X ratios. It appears that there is a strong agreement in terms of the resultant bus voltages and branch currents among the three models under all test conditions.

From the results, it is observed that as the load increases, the number of iterations for convergence increases irrespective of the models. However, model 1 and model 2 appear to be less sensitive than model 3. Results further reveal that the iteration numbers for model 3 are larger than that of model 1 and 2. The major reason is that in model 3, the injection currents of all the equivalent current sources are calculated based on voltages obtained in the previous iteration, instead of the updated voltages. Even though the computation time needed for each iteration is less for model 3, the smaller amount of voltage/current update makes the total iteration number much higher.

It can be seen that with increases in R/X ratios, all the models exhibit poor convergence, although model 1 and model 2 are less sensitive.

V. CONCLUSION

A unified method to incorporate three-phase transformers into the forward/backward sweep-based distribution load flow is presented. The singularity issues existing in certain transformer configurations were thoroughly examined. This paper indicates that the singularity appears only in certain transformer submatrices and only in certain transformer connections. This paper shows that when the singularity occurs,

the corresponding zero-sequence voltage cannot be updated. This paper also proves that the line-to-neutral voltage, excluding its zero-sequence component, can still be used in the forward/backward sweep. Based on these findings, the proposed technique separates the zero and nonzero sequence voltage during the forward/backward sweep, regardless of the transformer configuration to avoid the singularity problem. For certain transformer connections, the zero-sequence voltage on the secondary side cannot be updated during the sweeps. The proposed method was implemented in two different sized examples, and tests were conducted to compare with other existing approaches. The results show the validity and effectiveness of the proposed technique.

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