

# Effects of Non-transposed Lines and Unbalanced Loads on State Estimation

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**Abstract**— This paper investigates the errors introduced in the positive sequence state estimation due to the usual assumptions of having fully balanced bus loads/generations and continuously transposed transmission lines. A three-phase state estimator is first developed in order to verify the actual error free solution in the phase coordinates. Then, several tests are conducted using different assumptions regarding the availability of single and multi-phase measurements. It is demonstrated that incomplete metering of three-phase system quantities may lead to significant errors in the positive sequence state estimates for certain cases. Such cases may also lead to incorrect bad data detection and elimination, further deteriorating the quality of the state estimate. IEEE 30 bus test system is used to illustrate these cases.

**Index Terms** — State Estimation; Three-Phase Power Flow; Unbalanced Loads; Non-transposed Lines.

## I. INTRODUCTION

POWER systems are generally configured in three phases, and are designed to operate in an almost balanced manner. Balanced three-phase operation implies the following conditions to be met:

- Transposition of the transmission lines
- Even distribution of bus loads
- Maintaining balanced generator outputs

Analysis of balanced three-phase systems is relatively simple compared to the full detailed three-phase solution of the network equations. A symmetrical component transformation will decompose the balanced three-phase system into three independent systems, commonly referred to as the positive, negative and the zero sequence networks. Absence of negative and zero sequence signals under perfectly balanced three-phase operating conditions, allows the analysis to be carried out in the single phase, using only the positive sequence model. State estimators are no exception, making use of the positive sequence network model and the measurements in solving for the best estimate for the system state.

In practice, most high voltage systems are nearly balanced and depending on the system configuration and loading

conditions, they can be modeled and solved in the positive sequence. However, there may be cases where the balanced system assumptions no longer hold, when bus loads have an uneven distribution among the three phases, or relatively long but non-transposed transmission lines, carrying significant power flows exist in the system. Such lines will have different mutual coupling among the pairs of phase conductors and consequently the power flows through each of the three conductors of the lines will not be the same.

Unbalanced operating state of a power system can be obtained using a more detailed network model and measurement set containing all three-phase quantities of interest. The problem of three-phase state estimation for transmission and distribution systems operating under unbalanced conditions is described in several papers [1]-[9]. Some of them [1]-[2] describe the general three-phase state estimation algorithms, while others [3]-[8] focus on the application of three-phase state estimation in distribution systems utilizing special characteristics. However, any phase unbalances in loads and/or any existing non-transposed transmission lines are commonly ignored in state estimators which are used for power systems today. As indicated in [9], such simplifying assumptions may affect the accuracy and numerical robustness of the estimator.

This paper therefore studies the effects of such simplifying assumptions on the estimated state of the systems under varying operating conditions. A state estimator based on the full three-phase network model is developed first. This estimator is then utilized to evaluate cases of load unbalance as well as lack of line transposition. IEEE 30 bus test system is modified to generate these cases. A three-phase power flow program is used to generate the measurement data, which are then corrupted with Gaussian errors to simulate measurement deviations. True three-phase state of the system is compared against those obtained based on different assumptions on the measurements. Details of these cases will be described after an overview of the system modeling.

## II. ALGORITHM AND SYSTEM MODELING

The weighted least squares (WLS) algorithm is used in the implementation of the three-phase state estimator. The details of the measurement equations and Jacobian entries can be found in [1]. Sparse matrix techniques are used to improve the computational efficiency and memory savings. All system components such as transmission lines, loads, transformers and generators are modeled in three-phase as described below.

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### A. Three Phase Transmission Lines

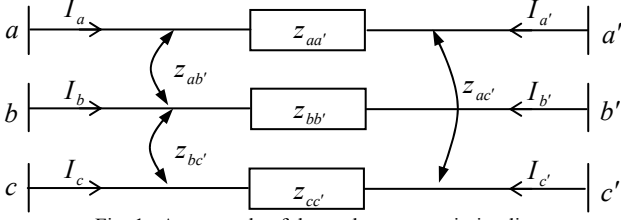


Fig. 1. An example of three-phase transmission line

A typical three-phase transmission line is given in Fig. 1. The network equations for this line can be written in compact form, according to the procedure described in [10]. The effect of the ground wire is included in the self and mutual impedance of the three-phase conductors. The primitive series impedance matrix of the line is given:

$$Z_P = \begin{bmatrix} z_{aa'} & z_{ab'} & z_{ac'} \\ z_{ba'} & z_{bb'} & z_{bc'} \\ z_{ca'} & z_{cb'} & z_{cc'} \end{bmatrix} \quad (1)$$

Defining the primitive admittance matrix  $Y_P = Z_P^{-1}$ , the nodal equations for the system of Fig. 1 can be written as:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_{a'} \\ I_{b'} \\ I_{c'} \end{bmatrix} = \begin{bmatrix} Y_P & -Y_P \\ -Y_P & Y_P \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_{a'} \\ V_{b'} \\ V_{c'} \end{bmatrix} \quad (2)$$

If the susceptances associated with the line charging exist, they will be added to the diagonal elements of the admittance matrix corresponding to the end nodes. The susceptances of all shunt elements in three phases are assumed equal.

### B. Three-Phase Loads and Generators

Each three-phase bus consists of three single-phase buses with loads connected in Wye and modeled by negative power injections in the state estimation measurement equations. Similarly, generated real and reactive power at each single-phase bus is modeled as a positive injection. Generator buses may have unbalanced injections assigned to them as measurements if the operating conditions are not balanced.

### C. Transformers

The winding-connection type of transformer becomes critically important in the three-phase study [10]. While all transformers considered in this study are assumed to be Wye connected at both sides, any other combination can be modeled as shown in [10]. Transformers with off-nominal tap settings are represented as shown in Fig. 2.

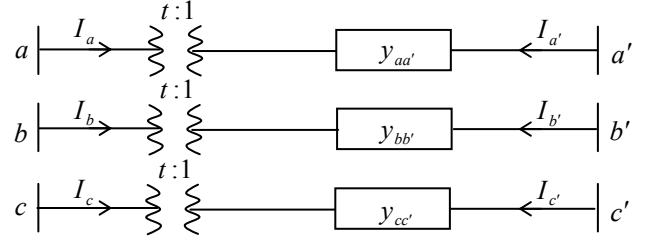


Fig. 2. Typical three-phase transformer model

Accordingly, the node equations of transformer can be described as (3).

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} A/t^2 & C/t \\ C^T/t & B \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix} \quad (3)$$

where:

$I_P$  and  $I_S$  are primary and secondary three-phase current;  
 $V_P$  and  $V_S$  are primary and secondary three-phase voltages;  
 $t$  is the off-nominal tap;

$$A = B = -C = \begin{bmatrix} y_{aa'} & 0 & 0 \\ 0 & y_{bb'} & 0 \\ 0 & 0 & y_{cc'} \end{bmatrix}.$$

For the admittance matrices corresponding to other kinds of winding connections, please refer to [2].

### D. Bus Shunts

Bus shunts are assumed to be decoupled in each phase and they are modeled by adding appropriate susceptance values to the diagonal elements corresponding to the buses.

## III. STUDIED CASES

All studied cases are built using the IEEE 30 bus system. Loading unbalances as well as the non-transposed line effects on the network model are studied. In order to create a three-phase network model for the IEEE 30 bus system, several assumptions are made regarding the sequence component data that are not readily available. This will be explained below.

### A. Convert Positive Sequence Model to Three-Phase Model

IEEE 30 bus system data are available only in the positive sequence. The following steps are followed to generate the three-phase network model based on the positive sequence model.

#### 1) Transmission lines:

We assume the relationship between negative, zero and positive sequence impedances of all the transmission lines is as follows:

$$Z_0 = 3Z_1; Z_2 = Z_1 \quad (4)$$

where:

$Z_0, Z_1, Z_2$  are the zero, positive and negative sequence impedances, respectively.

Then the three-phase impedance matrix will become:

$$Z_{abc} = K^{-1} \cdot \begin{Bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{Bmatrix} \cdot K \quad (5)$$

where:

$Z_{abc}$  is the  $3 \times 3$  three-phase impedance matrix;

$$K = \begin{Bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{Bmatrix}, \text{ and } a = e^{j120^\circ}$$

#### 2) Transformers:

All phase to phase coupling are ignored for the transformers as shown in Fig.2. The off-nominal taps and branch impedances are obtained directly from IEEE 30 bus data file.

#### B. Cases of unbalanced operation

Following cases are investigated. Each case involves a different type of unbalance and severity.

##### 1) Case T1:

In this case, all transmission lines are assumed to be non-transposed. The amount of coupling asymmetry among the three-phase conductors is chosen based on the mutual impedance between phase A and phase C. This quantity is set equal to 90% of the mutual impedances between the other two phases.

##### 2) Case T2:

This case is identical to Case 1, except for the severity of the coupling asymmetry. The mutual impedance between phase A and phase C is set equal to 60% of other two mutual impedances.

##### 3) Cases L1-L4:

These are a set of four cases where all bus loads in the system are assumed to be unbalanced. The amount of unbalance between phase loads, is varied by keeping the phase A and B loads equal, and changing phase C load to 90%, 80%, 70% and 60% of that of the other phase loads for the cases L1 through L4 respectively.

#### C. Generation of Three Phase Measurements

A three-phase power flow program is used to generate the measurements corresponding to all the cases described above. These perfect measurements are in turn corrupted by Gaussian distributed random errors with zero mean and standard deviation of 0.004, 0.01 and 0.008 for the voltage magnitude, power injection and power flow measurements respectively.

### IV. INVESTIGATION METHODOLOGY

The investigation concerning the above described cases is carried out by performing three state estimation solutions using different assumptions and available measurements, which are outlined below:

##### 1) Estimate1 (Three-phase):

A three-phase state estimation is performed using the full set of three-phase measurements, assuming that the necessary instrumentation is available to have access to the

measurements in all three phases. Once the three-phase estimates are obtained, their positive sequence components are evaluated and recorded.

##### 2) Estimate2 (Single-phase)

In this case, it is assumed that only phase A measurements are available at the control center where the state estimator is run. The positive sequence network model, similar to the one used by the common single-phase state estimators, is employed. The resulting state estimate, which is a single-phase result, is recorded.

##### 3) Estimate3 (Single-phase)

This is identical to the *Estimate2*, except for the measurement set, which now contains the positive sequence values of the three-phase measurements. This would correspond to a case where three-phase instrumentation and the corresponding measurements are available, yet a single-phase state estimator is to be run.

These three sets of estimated results will be referred respectively as *Estimate1*, *Estimate2* and *Estimate3* in the presented tables below. The estimated states in *Estimate2* and *Estimate3* are compared with *Estimate1* to quantify the effects of the assumptions involved. A flowchart of overall investigation procedure is summarized in Fig. 3.

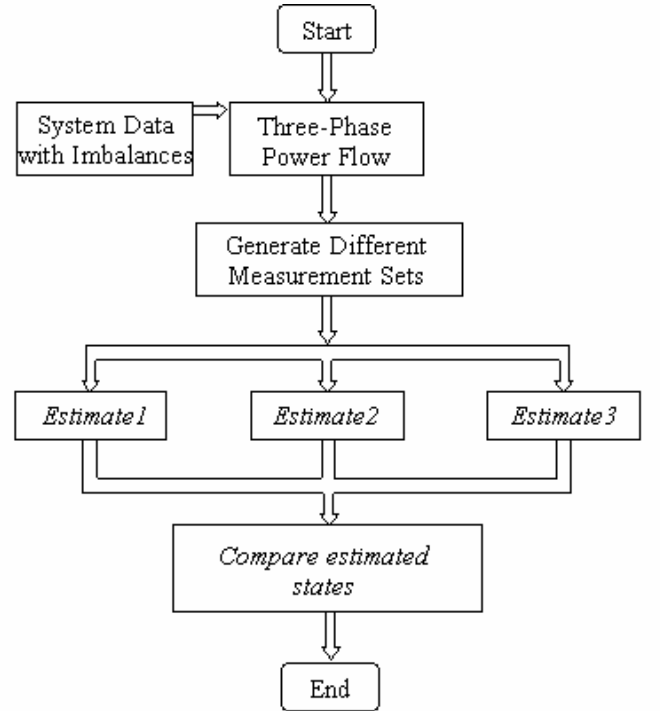


Fig. 3. Flowchart of investigation process

The following indices are used for the comparisons:

##### 1) Normalized residuals:

The normalized residuals for all measurements are computed for both kinds of state estimators. Those measurements with normalized residual great than 3.0 are recorded as the suspected bad data.

##### 2) Maximum Absolute State Mismatches:

Absolute mismatches between the states obtained as

*Estimate1* and as *Estimate2*, *Estimate3* are computed. Maximum absolute mismatches of voltage magnitude and phase angles are recorded.

### 3) Freq. of Relative Errors Greater than $3\sigma$ :

The relative errors given by (6) are computed for *Estimate2* and *Estimate3*.

$$err = \left| \frac{S_{estimated} - S_{true}}{S_{true}} \right| \quad (6)$$

where  $S_{estimated}$  is the estimated states and  $S_{true}$  is true states.

The number of times these relative errors exceed three times the corresponding measurement standard deviation will be referred to as *Freq. of  $err > 3\sigma$* .

### 4) Costs:

The values of objective functions evaluated after convergence are referred here as *Costs*. For comparing, the costs get from three-phase state estimator are divided by 3. The value of the cost is compared against the corresponding Chi-square test threshold in order to detect bad data. Chi-square test thresholds are looked up from Chi-square distribution table as 139 for the single phase and 127 (383/3) for the three-phase estimation cases. Those values greater than threshold are noted in the tables.

## V. RESULTS OF SIMULATIONS

All of the cases mentioned in section 3 are simulated and compared according to the procedure shown in Fig. 3. The detailed results of estimation will be presented for *case T1* (table I), and for brevity only the corresponding indices calculated for the other cases will be shown. In these tables, all the voltage magnitudes will be given in per-unit and phase angles in degrees.

### 1) Non-transposed Cases:

In the first two cases: *case T1* and *case T2*, the loads are balanced but the transmission lines are non-transposed. Table I shows all three estimation results for *case T1*. The indices of these two cases are presented in Table II and III, respectively.

Note that, the use of single phase (phase A) measurements to estimate the state of a system operating under unbalanced conditions, may lead to some bias in the state estimate. However, for both cases T1 and T2, Chi-square test thresholds are not hit, i.e. modeling errors due to the non-transposed lines are not detected by the estimator.

### 2) Unbalanced Cases:

Cases L1-L4 assume fully transposed transmission lines but unbalanced loads. Table IV shows the results for *case L1*. The results appear similar to the ones reported for case T2, which corresponds to a very extreme form of asymmetry as compared to a relatively mild unbalance (10%) of case L1. When the degree of unbalances increases, the errors will increase significantly. Table V shows the results for cases L1-L4, where the cost function for *case L4* exceeds the bad data detection threshold based on the Chi-square test. Hence, in this situation the state estimator may incorrectly identify some

measurements as bad and discard them to further deteriorate the accuracy of the estimator.

TABLE I  
ESTIMATED STATES OF CASE T1

Bus No.	Estimate1		Estimate2		Estimate3	
	[V]	Ang.	[V]	Ang.	[V]	Ang.
1	1.060	0.00	1.054	0.00	1.060	0.00
2	1.043	-5.40	1.039	-5.45	1.043	-5.40
3	1.020	-7.58	1.017	-7.67	1.020	-7.58
4	1.011	-9.33	1.009	-9.44	1.011	-9.33
5	1.009	-14.27	1.008	-14.39	1.009	-14.27
6	1.009	-11.11	1.007	-11.25	1.009	-11.11
7	1.001	-12.92	0.999	-13.08	1.001	-12.92
8	1.009	-11.87	1.008	-12.01	1.009	-11.87
9	1.049	-14.09	1.048	-14.22	1.049	-14.09
10	1.043	-15.64	1.041	-15.81	1.043	-15.64
11	1.080	-14.07	1.081	-14.26	1.080	-14.08
12	1.054	-14.91	1.054	-15.09	1.054	-14.90
13	1.068	-14.94	1.068	-15.16	1.068	-14.94
14	1.039	-15.82	1.038	-15.91	1.039	-15.82
15	1.035	-15.84	1.035	-16.03	1.035	-15.84
16	1.041	-15.46	1.040	-15.59	1.041	-15.46
17	1.037	-15.79	1.036	-15.94	1.037	-15.79
18	1.025	-16.45	1.025	-16.68	1.025	-16.45
19	1.023	-16.57	1.022	-16.79	1.023	-16.57
20	1.027	-16.37	1.026	-16.60	1.027	-16.37
21	1.030	-16.07	1.029	-16.24	1.030	-16.07
22	1.031	-16.06	1.030	-16.23	1.031	-16.06
23	1.025	-16.26	1.025	-16.40	1.025	-16.26
24	1.019	-16.43	1.018	-16.57	1.019	-16.43
25	1.015	-16.08	1.014	-16.09	1.015	-16.08
26	0.997	-16.35	0.999	-16.53	0.997	-16.35
27	1.020	-15.61	1.019	-15.58	1.020	-15.61
28	1.005	-11.73	1.004	-11.87	1.005	-11.73
29	0.999	-16.88	0.998	-16.73	0.999	-16.88
30	0.987	-17.75	0.985	-17.61	0.987	-17.75

TABLE II  
COMPARISON INDICES OF CASE T1

CaseT1	Bad Data No.	Maximum Mismatch		Freq. of err $>3\sigma$		Cost
		[V]	Ang.	[V]	Ang.	
Estimate1	0	-	-	-	-	101.3
Estimate2	0	0.005	0.238	0	9	117.0
Estimate3	0	4E-5	0.002	0	0	49.3

TABLE III  
COMPARISON INDICES OF CASE T2

CaseT2	Bad Data No.	Maximum Mismatch		Freq. of err $>3\sigma$		Cost
		[V]	Ang.	[V]	Ang.	
Estimate1	0	-	-	-	-	97.9
Estimate2	0	0.018	0.393	1	28	123.5
Estimate3	0	3E-4	0.031	0	0	41.5

TABLE IV  
COMPARISON INDICES OF CASE L1

CaseL1	Bad Data No.	Maximum Mismatch		Freq. of err >3 $\sigma$		Cost
		V	Ang.	V	Ang.	
Estimate1	0	-	-	-	-	90.6
Estimate2	0	0.011	0.455	0	6	125.9
Estimate3	0	4E-5	5E-4	0	0	42.5

TABLE V  
INFLUENCE OF UNBALANCED LOADS

Cases	Bad Data No.	Maximum Mismatch		Freq. of err >3 $\sigma$		Cost
		V	Ang.	V	Ang.	
L1	0	0.011	0.455	0	6	125.9
L2	1	0.017	0.868	12	29	127.5
L3	0	0.025	1.103	19	29	131.1
L4	4	0.030	1.243	24	29	159.1*

\*: This value exceeds Chi-square test threshold.

## VI. CONCLUSIONS

This paper investigates the effects of unbalanced loads and non-transposed transmission lines on the solution of the positive sequence state estimation problem. A number of simulations are carried out using varying degrees of unbalance among the three phases of bus loads as well as the mutual coupling between pairs of phase conductors. The simulation results indicate a higher sensitivity of the system state to loading unbalances than to asymmetries in the transmission line conductor configurations. It is also demonstrated that under certain cases, the use of single-phase state estimator may lead to significant biases in the solution due to existing asymmetries or load unbalances.

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## VIII. BIOGRAPHIES

**Shan Zhong** was born in Mashan, China, on September 25, 1977. He got his B. S. degree from Department of Electrical Engineering, HuaZhong University of Sci&Tech, China, in 1997. In 2000, he received his M. S. degree from Department of Electrical Engineering, Tsinghua University, China. Now he is pursuing his Ph.D. degree in Department of Electrical Engineering at Texas A&M University. His research field is power system monitoring and controlling.

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