A Branch-Estimation-Based State Estimation Method for Radial Distribution Systems

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***Abstract*—This paper presents a new branch-based state estimation method which is an estimation technique for radial distribution systems that can handle most kinds of real-time measurements. In contrast to the traditional weighted-least-square (WLS) method, the idea of this algorithm is to decompose WLS problem of a whole system into a series of WLS subproblems and each subproblem will deal with only single-branch state estimation. This approach can be implemented in a forward/backward sweep scheme for radial distribution systems and does not need the sparse matrix technique. Test results of a large-scale practical distribution system in China show that the proposed method is valid and efficient.**

***Index Terms*—Distribution-management system, distribution system, state estimation.**

I. Introduction

In a modern distribution-management system (DMS), state estimation (SE) plays a critical role to estimate the real-time system states that are unable to be obtained from the limited measuring instruments at the distribution system level. With distribution SE (DSE), the operators can calculate the theoretical power loss, implement voltage/var optimization, guide network reconfiguration, and prevent distribution lines from overloading, etc. Therefore, they can improve the capability of monitoring, controlling and economically dispatching distribution systems, and finally improve the power quality and reliability of distribution systems. DSE is a fundamental function in DMS.

State estimation techniques have been developed and applied at the generation and transmission levels for more than 30 years. The most commonly used approach is the weighted-least-square (WLS) method. The features of distribution networks include their wide range of resistance and reactance ratio values, low number of meshes, very limited measurement sets, and the large number of current measurements, which makes the problem of state estimation for distribution systems very challenging. Therefore, it is important for a distribution state estimator to consider these features.

Much encouraging work has been done in recent years [1]–[5]. A three-phase SE method, based on the conventional WLS approach and a three-phase node voltage formulation, has been developed in the past year. In [2], a novel branch-current-based SE method for three-phase unbalanced radial or weakly meshed feeders is presented. Compared with the conventional node-voltage-based methods, the method, using the branch currents as the system states, has superior performance in computation speed and memory requirements. In [3], a load adjustment model based on a custom-tailored Gauss–Seidel load-flow algorithm is presented. It incorporates all of the available measurements into the calculations and is simple to implement. A distribution state estimator based on a stochastic load model is presented in [4]. Test results of this method show that the estimated deviation of the system state is affected by several factors, such as load error correlation, real-time-measurement availability, pseudo measurement errors, and measurement placement. The authors in [5] present an alternative approach for the radial distribution circuit by a probabilistic extension of the load-flow algorithm, considering real-time measurements as solution constraints. The statistics of the states are calculated by a probabilistic method, in which the states are treated as random variables.

Most of the above algorithms share the common point that they are typically based on the WLS approach to obtain an estimate of the system operating point. The conventional Newton method is often used to solve the WLS problem, in which Jacobian matrix and gain matrix have to be recalculated during iterations.

A new method for real-time monitoring of radial distribution networks, known as branch-estimation-based state estimation method, is presented in this document. The difference between the proposed method and other methods is that it decomposes WLS problem of the whole system into a series of WLS subproblems. Each WLS subproblem is quite simple because only a single branch is considered. All kinds of measurements can be handled in each WLS subproblem. Moreover, this approach uses the efficient forward/backward sweep scheme similar to conventional branch-oriented power-flow algorithms, which means that sparse matrix/vector technique is not needed. In Section II, the algorithm is described in detail. Test results and discussions are presented in Section III. Section IV contains the conclusions.

II. Distribution State Estimation Algorithm

The proposed algorithm consists of two main parts: load allocation and state estimation. Due to the large number of customers, it is impractical to monitor distribution systems in real time at every customer location. A practical distribution SE scheme should be based on substation measurements, few critical circuit measurements, and a large number of customer load estimates. Thus, load demand estimation is indispensable to provide pseudo measurements to guarantee the observability of the distribution systems. We introduce the load allocation procedure first and then the distribution SE method.

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A. Load Allocation

A real-time load modeling technique [6], which incorporates the use of customer class curves and provides a measure of the uncertainty (statistics) in the estimates, is used in the proposed algorithm. It also incorporates the possibility of multiple line-flow measurements and makes use of customer information, such as transformer capacity, customer type, and load curve of a typical day, etc.

To illustrate the load allocation procedure, a small radial feeder (see Fig. 1), which has three real power measurements P_{m1}, P_{m2}, P_{m3}, is considered. We can define a zone for each meter in a radial feeder: a part of the feeder between the meter and its downstream neighbors. The meter values of each zone must be modified according to other associated zones. For example, assume that P'_{mi} is the modified value of meter zone i, $i = 1, 2, 3$, then $P'_{m1} = P_{m1} - P_{m2} - P_{m3}$, $P'_{m2} = P_{m2}$, $P'_{m3} = P_{m3}$.

For a specified meter zone, at time t, the real-time load estimates of transformer i, $P_{I,\,t}$ can be obtained as follows:

$$P_{i,\,t} = (P'_m - P_{loss}) \left[\frac{\sum_{j=1}^{C} LMF^*_{j,\,t} ADC_{i,\,j}}{\sum_{i=1}^{N} \sum_{j=1}^{C} LMF^*_{j,\,t} ADC_{i,\,j}} \right] \quad (1)$$

where

P'_m	modified meter value;
n	number of transformers;
P_{loss}	real power loss;
C	number of load classes;
ADC_{ij}	average daily customer demand at transformer i belonging to class j;
$LMF_{j,\,t}$	class-specific load model factor belonging to class j at time t;
tg$\phi_{i,\,t}$	power factor of transformer i at time t.

Before the state estimation, the real power loss of each zone cannot be determined. In this document, an approximate loss ratio of 2% is applied to get the approximate loss. The power factor of each transformer at time t is obtained by historical information.

The standard deviations of pseudo measurements of customer loads and real-time measurements are significant system parameters affecting the system state deviations. In the proposed algorithm, the standard deviation of telemeter values σ_{true} is assumed 1% and the standard deviation of pseudo measurements σ_{pseudo} varies from 20 to 50% with different type customers. Three types of customers are considered in the proposed algorithm. For instance, the industrial load can be estimated more accurately than that of residential or commercial loads, so it has lower variance. The residential load is difficult to be estimated so that the higher variances can be set. The commercial load is generally between the other two type loads. Since the reactive load is estimated by an approximate power factor, its variance is greater than that of the pseudo measurement of real load. For a bus without load demand, zero-injection measurement is set with variance 0.

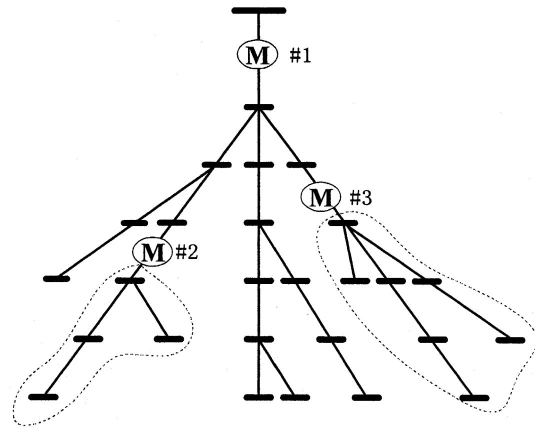

Fig. 1. Zones defined by meters in a typical radial feeder.

B. Branch-Estimation-Based State Estimation

For presentational convenience, we first consider a simple case, a two-bus system, and show the basic idea of single branch estimation, and the general case for any distribution system is considered next.

1) Simple Case: Consider a distribution system consisting of only one branch with sending bus i and receiving bus j, as shown in Fig. 2. In this figure, V_i represents the substation bus, whose magnitude is assumed constant. The real-time measurement set M of this case can be described as follows:

$$M = \{|V_i|,\, P_j,\, Q_j,\, |I_j|,\, |V_j|,\, P_{ij},\, Q_{ij},\, |I_{ij}|,\, P_{ji},\, Q_{ji},\, |I_{ji}|\}.$$

Obviously, due to economic reasons, it is impossible to place so many meters on a branch. Therefore, any practical measurement configuration of a single branch is just a subset of M.

Let z denote the vector containing the measurements, and w_i and $h_i(x)$ represent the weight and the measurement function associated with measurement z_i, respectively. Choosing V_i as reference bus and V_j as a state variable, state estimation of this case is to minimize the following function:

$$\min J(V_j) = \sum_{k=1}^{m} w_k (Z_k - h_k(V_j))^2 \quad (2)$$

where m is the number of real-time or pseudo measurements of this branch. For the solution of this problem, the conventional iterative method is adopted to obtain the correction ΔV_j^{k+1} by the following equations at iteration k:

$$[H^T(V_j^k) W H(V_j^k)] \Delta V_j^{k+1} = H^T(V_j^k) W [z - h(V_j^k)] \quad (3)$$

where H is the Jacobian matrix of the measurement function $h(x)$.

The sizes of Jacobian matrix and gain matrix $H^T(x)WH(x)$ are $m \times 2$ and 2×2, respectively. Since m is less than ten, we can process the above iteration quite easily, and do not need the sparse matrix technique that increases the complexity for the implementation of state estimation.

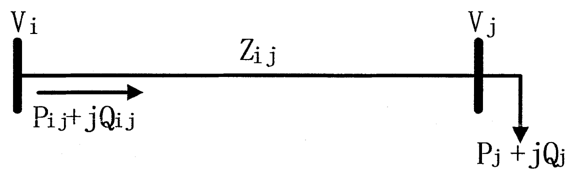

Fig. 2. Two-bus distribution system.

After obtaining the system state, we can get the equivalent power injection to the sending bus i by the following formula:

$$S_{equ-ij} = \frac{\left(V_i - \hat{V}_j\right)^*}{Z_{ij}^*} V_i \tag{4}$$

where Z_{ij} is the series impedance of this branch.

The variance of equivalent power injection is determined by the following rules:

Rule 1: if branch L_{ij} has real-time branch power measurements

$$\begin{aligned} \sigma^2(P_{equ-ij}) &= \min\left(\sigma_{true}^2, (P_{equ-ij} - P_{meas})^2\right) \\ \sigma^2(Q_{equ-ij}) &= \min\left(\sigma_{true}^2, (Q_{equ-ij} - Q_{meas})^2\right). \end{aligned} \tag{5}$$

Rule 2: if branch L_{ij} has real-time branch current measurements

$$\begin{aligned} \sigma^2(P_{equ-ij}) &= \min\left(\sigma_{true}^2, \left|\frac{S_{equ-ij}}{V_i}\right|^2 - |I_{meas}|^2\right) \\ \sigma^2(Q_{equ-ij}) &= \sigma^2(P_{equ-ij}). \end{aligned} \tag{6}$$

Rule 3: none of the above cases

$$\begin{aligned} \sigma^2(P_{equ-ij}) &= \sigma^2(P_j) \\ \sigma^2(Q_{equ-ij}) &= \sigma^2(Q_j). \end{aligned} \tag{7}$$

The procedure shown in (2)–(7) is called single branch estimation (SBE), which is useful for the next discussion.

2) General Case: Since the proposed algorithm is branch oriented, the branch numbering procedure must be done from the substation bus toward the feeder end before performing any calculations. Consider the system in Fig. 1, where we number the buses and branches in layers away from the root node (substation bus). The numbering of branches in one layer starts only after all of the branches in the previous layer have been numbered, as seen in Fig. 3.

Now, we extend the discussion to a general radial feeder to find the operating state.

Efficient power-flow algorithms based on the forward/backward sweep for solving single- and three-phase radial distribution networks have been extensively used by distribution engineers. In this paper, we try to use forward/backward sweep to solve SE problems for radial distribution systems.

Assume the radial feeder contains $n+1$ nodes, n branches, and a single voltage source at the root node. To apply the forward/backward sweep to distribution SE, one may divide all of the (real-time or pseudo) measurements into n groups that each group represents one branch, and reorganize the objective function as follows:

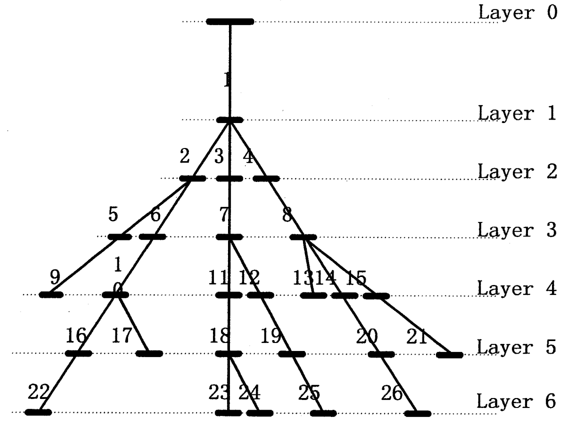

Fig. 3. Branch numbering of the radial distribution feeder.

$$\min J(x) = \sum_{l=1}^{n} J_l(x) = \sum_{l=1}^{n} \sum_{i=1}^{ml} w_{li}(z_{li} - h_{li}(x))^2 \tag{8}$$

where m_l is the number of measurements on branch l; z_{li} and $h_{li}(x)$ represents the ith measurement and relevant measurement function of branch l, respectively.

A novel forward/backward sweep schema is developed to solve (8). The backward sweep starts from the branches in the last layer, moves toward the branches connected to the root node, and applies SBE to every branch to obtain the system state. For example, for branch l with sending bus i and receiving bus j, choosing bus i as the reference bus, we can obtain the system state of bus j by SBE. The difference between the general case and the simple case is that the voltage magnitude of bus i is not a constant while the voltage magnitude of substation is a constant. It means that the acquired state of bus j is only a tentative result. After calculating all of the branches in the same layer as branch l, treat the sum of power injections emanating from bus i as the new pseudo measurement, as seen in Fig. 4

$$S_i' = S_i + \sum_l S_{equ-l} \tag{9}$$

where l is the branch emanating from bus i.

Assuming the customer loads are independent random variables, the variance of the new pseudo measurement is

$$\sigma^2(S_i') = \sigma^2(S_i) + \sum_l \sigma^2(S_{equ-l}). \tag{10}$$

The forward sweep, starting with the branches in the first layer, moving toward the last layer, corresponds to calculating the nodal voltages by

$$V_j = V_i - (S_{equ-ij}/V_i)^* Z_{ij} \tag{11}$$

where Z_{ij} is the series impedance of branch between sending bus i and receiving bus j, and S_{equ-ij} is the equivalent power injection of this branch obtained in the preceding backward sweep.

Repeat the backward/forward sweep until the maximum voltage mismatch is less than the tolerance.

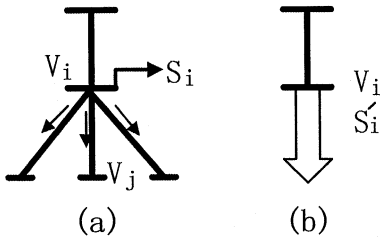

Fig. 4. Example of equivalent power injection.

Table I shows the difference of forward/backward sweep between the conventional distribution-power-flow (DPF) algorithm and the proposed method.

3) Adjustment of Pseudo Measurements: Due to the uncertain behavior of customer loads, the errors associated with allocated load demands are much greater than the telemeter values. Therefore, to satisfy the telemeter constraints, one allows the allocated load demands to be modified.

For meter zone k, the modification of allocated load demands can be achieved to minimize the following function:

$$\min J(r_P, r_Q) = \sum_{i=1}^{mk} w_{Pi} r_{Pi}^2 + w_{Qi} r_{Qi}^2$$
$$\text{s.t. } P'_{mk} - \sum_{i=1}^{mk} (P_i + r_{Pi}) - P_{loss}(r_P, r_Q) = 0$$
$$Q'_{mk} - \sum_{i=1}^{mk} (Q_i + r_{Pi}) - Q_{loss}(r_P, r_Q) = 0 \quad (12)$$

where P'_{mk} and Q'_{mk} are the modified telemeter values of zone k, r_P and r_Q represent the residue vector of allocated load demands, $P_{loss}(r_P, r_Q)$ and $Q_{loss}(r_P, r_Q)$ represent the system losses, and w_P and w_Q represent the weight of the allocated load.

Since the weights of residue are positive numbers, ignoring the partial differential of r_P, r_Q to $P_{loss}(r_P, r_Q)$, and $Q_{loss}(r_P, r_Q)$, we can obtain the minimal of (13) with Cauchy inequality:

$$J(r_P, r_Q) = \sum_{i=1}^{mk} w_{Pi} r_{Pi}^2 + w_{Qi} r_{Qi}^2 \geq \frac{\left(\sum_{i=1}^{mk} r_{Pi}\right)^2}{\sum_{i=1}^{mk} \frac{1}{w_{Pi}}} + \frac{\left(\sum_{k=1}^{mk} r_{Qi}\right)^2}{\sum_{i=1}^{mk} \frac{1}{w_{Qi}}}.$$

The optimal r_P and r_Q can be obtained from the following equations:

$$w_{P1}\hat{r}_{P1} = w_{P2}\hat{r}_{P2} = \cdots = w_{Pmk}\hat{r}_{Pmk}$$
$$w_{Q1}\hat{r}_{Q1} = w_{Q2}\hat{r}_{Q2} = \cdots = w_{Qmk}\hat{r}_{Qmk}.$$

TABLE I
DIFFERENCE BETWEEN ITERATION FORMULAS OF TWO METHODS

Backward sweep	DPF	$I_l^{(k)} = I^{(k)}_{injection\text{-}j} + \Sigma$ *(currents in branches emanating from bus j)*
	DSE	$V_j^{(k)} = \min J_l(V_1^{(k)}, V_2^{(k)}, \ldots V_j^{(k)}, V_{l+1}^{(k)}, \ldots V_n^{(k)})$ $S^{(k)}_{equ\text{-}l} = V_i^{(k)} (V_i^{(k)} - V_j^{(k)})^* / Z_l^*$ $S_i^{(k)\prime} = S_i^{(k)} + \Sigma$ *(equivalent power injection to bus i)*
Forward sweep	DPF	$V_j^{(k)} = V_I^{(k)} - I_l^{(k)} Z_l$
	DSE	$V_j^{(k)} = V_i^{(k)} - (S_{equ\text{-}l}^{(k)} / V_i^{(k)})^* Z_l$

Thus

$$\hat{r}_{Pi} = \frac{P'_{mk} - \sum_{j=1}^{mk} P_j - P_{loss}}{\left(\sum_{j=1}^{mk} \frac{1}{w_{Pj}}\right) w_{Pi}}$$
$$\hat{r}_{Qi} = \frac{Q'_{mk} - \sum_{j=1}^{mk} Q_j - Q_{loss}}{\left(\sum_{j=1}^{mk} \frac{1}{w_{Qj}}\right) w_{Qi}}. \quad (13)$$

With the solution of (12), each pseudo measurement is updated by

$$P_i^{(k+1)} = P_i^{(k)} + \hat{r}_{Pi}^k, \quad Q_i^{(k+1)} = Q_i^{(k)} + \hat{r}_{Qi}^k. \quad (14)$$

C. Algorithm Description

The proposed algorithm can be summarized as follows.

Allocating pseudo load measurements and assuming the substation bus voltage as the initial voltages at all of the buses, the following three-step iterative scheme is obtained:

At iteration k, do the following.

Step 1) Backward sweep: SBE is applied in the inverse sequence of branch number

$$\hat{x}_n^{(k)} = \left\{x_n | \min J_n\left(x_1^{(k-1)}, x_2^{(k-1)}, \ldots x_n^{(k-1)}\right)\right\}$$
$$\hat{x}_{n-1}^{(k)} = \left\{x_{n-1} | \min J_{n-1}\left(x_1^{(k-1)}, \ldots, x_{n-1}^{(k-1)}, \hat{x}_n^{(k-1)}\right)\right\}$$
$$\cdots$$
$$\hat{x}_i^{(k)} = \left\{x_i | \min J_i\left(x_1^{(k-1)}, \ldots, x_i^{(k-1)}, \hat{x}_{i+1}^{(k-1)}, \ldots, \hat{x}_n^{(k-1)}\right)\right\}$$
$$\cdots$$
$$\hat{x}_1^{(k)} = \left\{x_1 | \min J_1\left(x_1^{(k-1)}, \hat{x}_2^{(k-1)}, \ldots \hat{x}_n^{(k-1)}\right)\right\}.$$

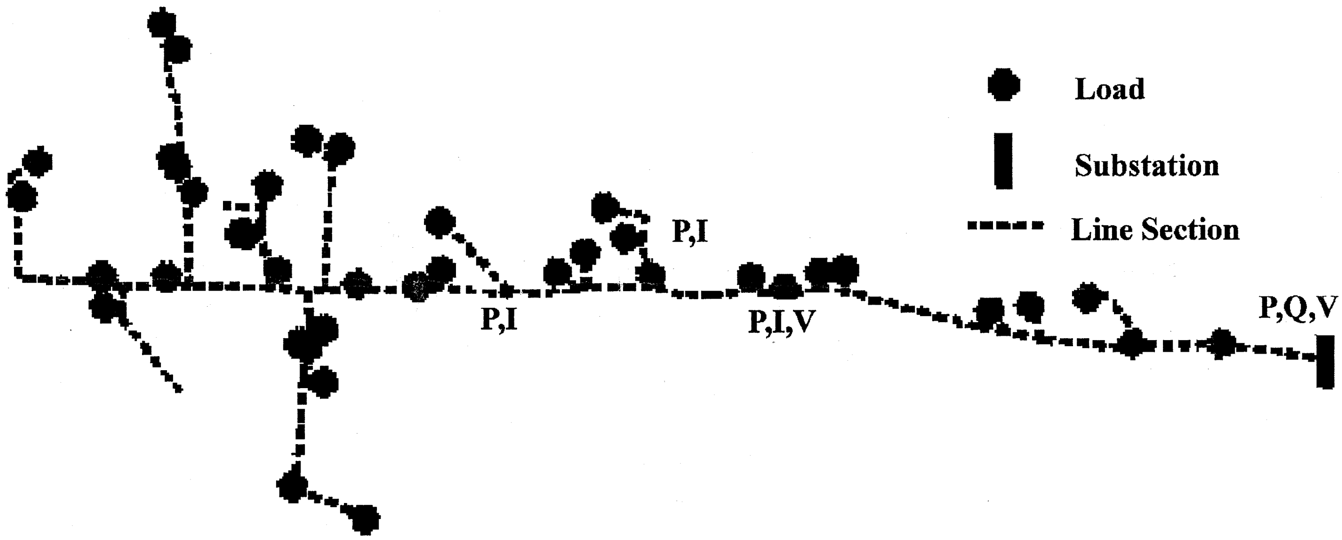

Fig. 5. Measurement placement of the test feeder.

Step 2) Nodal voltages are updated in a forward sweep starting from the first layer to those in the last

$$x_1^{(k)} = x_0 - \left(S_{equ,1}^{(k)} \Big/ x_0\right) * Z_{01}$$
$$x_2^{(k)} = \hat{x}_p^{(k)} - \left(S_{equ,2}^{(k)} \Big/ \hat{x}_p^{(k)}\right) * Z_{p2}, \quad p = \text{parent of bus } 2$$
$$\cdots$$
$$x_i^{(k)} = \hat{x}_p^{(k)} - \left(S_{equ,i}^{(k)} \Big/ \hat{x}_p^{(k)}\right) * Z_{pi}, \quad p = \text{parent of bus } i$$
$$\cdots$$
$$x_n^{(k)} = \hat{x}_p^{(k)} - \left(S_{equ,n}^{(k)} \Big/ \hat{x}_p^{(k)}\right) * Z_{pn}, \quad p = \text{parent of bus } n.$$

Step 3) Pseudo measurement adjustment: Calculate the mismatch of every meter zone, and update the pseudo measurements by (14).

Steps 1)–3) are repeated until power mismatches at all nodes satisfy the convergence tolerance.

III. Test Results and Discussions

The proposed method was implemented in commercial software and tested in a large-scale distribution network. The network consists of seven substations, 51 feeders, 834 transformers, and approximately 9000 nodes in China. The proposed algorithm converged in less than eight iterations in 0.67 CPU s on a PentiumII-400 computer.

A typical feeder is chosen from the system to illustrate the convergence characteristic of the proposed method. As seen in Fig. 5, ten real-time measurements are placed on the feeder: three of them are at the substation, the others are at critical circuits which create four telemeter zones. The r/x ratio of this feeder ranges from 0.43 to 2.30 and the average is approximately 1.15. A summary of the feeder information is given in Table II.

Table III shows the measured and calculated values of real-time measurements. As seen from the comparisons of the measured and calculated values, the proposed algorithm follows the real-time measurements quite closely despite the limited number of meters.

TABLE II
Test Feeder Topology Summary

Number of line sections	115
Number of nodes	116
Number of switches/fuses	47
Number of transformers	40

TABLE III
Measured and Calculated Values

	Zmeas	Zest.	(%)		Zmeas.	Zest.	(%)
P1	1330.1	1321.7	0.632	V2	10.24	10.27	0.298
Q1	460.0	455.9	0.891	P3	1014.2	1012.4	0.177
V1	10.32	10.32	0	I3	55.7	54.1	2.872
P2	1069.8	1045.7	2.25	P4	111.2	112.5	1.170
I2	60.1	58.8	2.16	I4	7.3	7.5	2.740

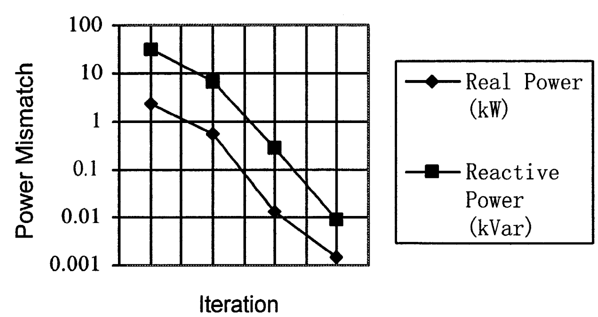

Fig. 6. Convergence characteristics of the proposed method.

Fig. 6 shows the values of the real and reactive power mismatches at the various iterations of this method for the test feeder. An excellent rate of convergence can be observed from the figure.

The previous discussions use a single-phase network model because the three-phase data were not available in practical application. However, the proposed algorithm is very straightforward to extend to three-phase network data. Since the proposed algorithm is branch oriented, the work we should do is to just extend the branch estimation procedure of the single-phase-to-three-phase network model to represent the unbalanced phase load and phase configuration. The difference between single-phase SBE and three-phase SBE is the number of state variables, the sizes of Jacobian matrix, and gain matrix of three phase. A similar procedure is used to find the state for a single branch and the same forward/backward sweep schema is applied to solve the SE problem. Of course, existing observability analysis algorithms, with small modifications, are used for the three-phase distribution network to satisfy the phase unbiased measurements (PUM) conditions [7].

IV. Conclusions

This paper has presented a novel and efficient approach, known as branch-estimation-based SE method, suitable for real-time monitoring of distribution systems. The advantages of the proposed method are listed as follows.

1) It incorporates all of the available measurements into the calculation to provide steady-state operating conditions of distribution systems.
2) It reduces significantly the computational complexity of SE for the radial distribution network due to its forward/backward sweep schema.
3) It is insensitive to wide-ranging resistance and reactance ratio values.
4) It is easy to implement and can be extended to a three-phase system straightforwardly.

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