College: S.M. Joshi College, Hadapsar

Department of Mathematics

Name: Pratiksha Goraksha Bhong Class: T.Y.B.Sc. Roll No.: 5021

Topic:/ Title: Turtle Graphics.

W.E. No.: 1. Date: Remarks: 'A'

Q. 1 Initialize the turtle canvas and draw the following commands

a) Move forward 150 pixel.

⇒
```
from turtle import *
t = Turtle ()
t = fd (150)
```

b) Rotate clockwise 45 degree.

⇒
```
from turtle import *
t = Turtle ()
t = right (45)
```

c) Move forward 180 pixel without drawing.

⇒
```
from turtle import *
t = Turtle ()
t = penup()
t = fd (180)
t = pendown.
```

d) Draw a circle with radius 200 pixel with green color.

⇒
```
from turtle import *
```

Teacher's Signature:

```
t = Turtle()
t = pencolor ('green')
t = circle (200)
```

e) Goto home:

```
from turtle import *
t = Turtle()
t = home()
```

f) Draw a circle in clockwise direction with fill color blue with radius 40 pixel:

⇒
```
from turtle import *
t = Turtle()
t = begin_fill()
t = fillcolor('blue')
t = circle(-40)
t = end_fill ()
```

g) Print the current position of turtle.

⇒
```
from turtle import *
t = Turtle()
t = position()
t
```

h) Print the current direction of turtle.

⇒
```
from turtle import *
t = Turtle()
```

```
t=heading()
t.
```

i) clear all screen :

⇒
```
from turtle import *
    t = Turtle()
    t= clear()
```

Q. 2 Write a python program to define a function, which plot a circle of a given radius starting with its current position.

⇒
```
from turtle import *
    t = Turtle()
    t = circle(100)
```

Q. 3 Draw a letter 'W' using turtle methods with width 3.

⇒
```
from turtle import *
    t = Turtle()
    t= pensize(3)
    t=pencolor('red')
    t = right(65)
    t = forward (200)
    t = left(130)
    t= forward(100)
    t= right(120)
```

```
t= forward (100)
t= left (130)
t= forward (200)
```

Q. 4 Draw a circle radius 250 pixel centered at (150,150).

⇒
```
from turtle import *
    t= Turtle()
    t= goto (150,150)
    t= forward (100)
    t= circle (250)
```

Q. 5 Using random function, draw a random walk with 60 steps which travels forward between 20 and 100 and moves right between 0 and 360 randomly.

⇒
```
from turtle import *
import random
Count =0
while count<61:
    count += 1
    if (turtle.xcor() >-300 and turtle.xcor() <300) and\
        (turtle.ycor() >-300 and turtle.ycor() <300):
        turtle.fd (random.randint (20,100))
        turtle.right(random.randint (0,360))
else:
    turtle.right(180)
```

```
    turtle.fd(300)
```

Q. 6 Draw a circles of different radius with random colors.

```
⇒ from turtle import *
    t=Turtle()
import random
    t=pencolor('red')
    t=circle(50)
    t=pencolor('green')
    t=circle(100)
    t=pencolor('blue')
    t=circle(150)
    t=pencolor('yellow')
    t=circle(200)
```

Q. 7 Draw a random walk with random color having 100 steps using random function.

```
⇒ from turtle import *
import random
    t=Turtle()
color=['Red','Green','Blue','Yellow','Gray']
for i in range(100):
    t=forward(40)
    t=right(random.randint(20,350))
    t=pencolor(color[i%5])
```

Q. 8 Use 'for' loops to make a turtle draw these regular polygons (regular means all sides the same lengths, all angles the same):

i) An equilateral triangle
ii) A square
iii) A hexagon (six sides)
iv) An octagon (eight sides).

⇒
```
from turtle import *
   t=Turtle()
for i in [3,4,6,8]:
   t= circle(200,360,i)
```

'C'

SBP

College: S.M. Joshi College, Hadapsar

Department of Mathematics

Name: Pratiksha Goraksha Bhong

Class: T.Y.B.Sc. Roll No.: 5081

Topic / Title: Data Visualization

W.E. No.: 2 Date:

Remarks: C' Sap

Q. 1. Plot the scatter plot pyplot interface for the data.

x=[0,1,2,3,4,5,6,7,8,9,10,11,12] &
y=[5,-1,6,-7,4,5,-6,4,2,9,1,-1,6]

⇒
```
from pylab import *
x=[0,1,2,3,4,5,6,7,8,9,10,11,12]
Y=[5,-1,6,-7,4,5,-6,4,2,9,1,-1,6]
plt.scatter(x,y,c='blue',marker='x',s=100)
plt.xlabel('x data')
plt.ylabel('y data')
plt.show()
```

Q. 2 Plot the scatter plot and line plot in pyplot interface for the data.

x=[-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6] and $y=x^3$

⇒
```
from pylab import *
x=[-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6]
y=[i**3 for i in x]
plt.scatter(x,y,c='blue',marker='x',s=100)
plt.plot(x,y,c='red',linewidth=2)
plt.xlabel('x data')
plt.ylabel('y data')
plt.title("an example plot")
plt.show()
```

Teacher's Signature:

Q. 3 Draw a set of vertical bar plots for the following data using seaborn.
x = [1, 2, 3, 4, 5, 6, 7, 8] , y = [101, 105, 114, 164, 120, 100, 200, 175].

⇒
```
import seaborn as sns
from pylab import *
sns.set_theme(style = "whitegrid")
ax=sns.barplot ([1,2,3,4,5,6,7,8],
               [101, 105, 114, 164, 120, 100, 200, 175])
show()
```

Q. 4 Plot a pie chart for the data.
Data = [83, 82, 10, 5, 120].

⇒
```
from pylab import *
import numpy as np.
y=np.array([83,82,10,5,120])
plt.pie(y)
plt.show()
```

Q. 5 Plot the random points in 3D, with marker and different sizes using mayaVI.

⇒
```
from mayaVI import mlab
import numpy as np.
x,y,z value = np.random.random((4,40))
mlab.points 3d (x,y,z, value)
```

Sd

College: S.M. Joshi College, Hadapsar
Department of Mathematics
Name: Pratiksha Goraksha Bhong
Topic:/ Title: Dictionary. Class: T.Y.B.Sc Roll No.: 5081
W.E. No.: 8. Date: Remarks: 'C'

Q. 1 Create dictionary with keys as {1,2,3,....,8} and corresponding values {'one', 'two', ..., 'eight'}.

⇒
```
d=dict()
d[1] = 'one'
d[2] = 'two'
d[3] = 'three'
d[4] = 'four'
d[5] = 'five'
d[6] = 'six'
d[7] = 'seven'
d[8] = 'eight'
d.
```

Q. 2 In the above dictionary d, delete the elements correspond to keys 3,5 and change the value for key 7 to 's'.

⇒
```
del d[3], d[5]
d[7] = 's'
d
```

Q. 3 Print all keys-value pairs from dictionary
D = {'a': 'one', 'b': 'two', 'c': 'three', 'd': 'four'}

Teacher's Signature:..........

⇒ D= dict()
D = {'a': 'one', 'b': 'two', 'c': 'three', 'd': 'four'}

for key,value in D.items ():
    print(key).

Q. 4 Consider the following dictionary
d={0: 'Peter', 2: 'Joseph', 3: 'Ricky', 'Emp-ages': (20,33,24)}
i) set default value 'Joseph2' to key '4'.
ii) Print value for key 4.

⇒
d_1 = {0: 'Peter', 2: 'Joseph', 3: 'Ricky', 'Emp-ages': (20,33,24)}
d_1

i) d_1.setdefault(4, 'Joseph2')
ii) d_1[4]
d_1

Q. 5 Merge the following dictionaries.
d_2 = {'a':1, 'b':2, 'c': 3}
d_3 = {'d':8, 'e':9, 'f': 6}

⇒
d_2 = {'a':1, 'b':2, 'c':3}
d_3 = {'d':8, 'e':9, 'f':6}
d_4 = {**d_2, **d_3}
d_4.

Expt.No............ Date: / /20 Page No.

Q. 6 Use ** to unpack the dictionary and define a function, which add values from dictionary:
d={'a': 2 , 'b': 3}.

⇒

```
d = {'a' : 2 , 'b' : 3}
def add(a=0 , b=0):
    return a + b
add (**d)
```

Q. 7 Create the ordered dictionary for the data.
keys = {0,1,4,5,6,3,2}.
values = {-9,-7,-4,-6,-3,3,4}

⇒

```
keys = [0,1,4,5,6,3,2]
values = [-9,-7,-4,-6,-3,3,4]
from collections import OrderdDict
d=OrderedDict()
for i in keys:
    d[i] = values[i]
d.
```

Teacher's Signature:..............................

College: S. M. Joshi College, Hadapsar
Department of Mathematics
Name: Pratiksha Goraksha Bhong Class: T.Y.B.Sc Roll No.: 5081
Topic:/ Title: Sorting, Minimum and Maximum in dictionary
W.E. No.: 4. Date: Remarks: 'C' SBP

Q. 1. Finding the minimum and maximum of a sequence of sequences.
L=[[5,3,4,7,8,-6,-8,-3,0],[6,5,6,3],[7,4,3,5]]

⇒

```
L=[[5,3,4,7,8,-6,-8,-3,0],[6,5,6,3],[7,4,3,5]]
min(L)
max(L)
```

Q. 2. Find minimum, maximum form the given dictionary.
adict = {'n':1, 's':2, 'r':2, 'u':2, 't':1}.
Also, sort the dictionary.

⇒

```
adict={'n':1,'s':2,'r':2,'u':2,'t':1}
min(adict)
max(adict)
sorted(adict)
```

Q. 3 Create the ordered dictionary for the data.
keys={0,1,4,5,6,3,2}
values={-9,-7,-4,-6,-3,3,4}.

⇒

```
keys=[0,1,4,5,6,3,2]
values=[-9,-7,-4,-6,-3,3,4]
```

Teacher's Signature:

```
from collections import OrderedDict
d=OrderedDict()
for i in keys :
    d[i] = values[i]
d.
```

Q. 4 Find minimum, maximum from the given dictionary
adict = {'a': 3 , 'b': 5 , 'c': 1}

⇒
```
adict = {'a': 3 , 'b': 5 , 'c': 1}
min(adict)
max (adict)
```

Q. 5 Find minimum, maximum from given dictionary.
adict = {'one': 'uno', 'three': 'tres', 'two': 'dos'}

⇒
```
adict = {'one': 'uno', 'three': 'tres', 'two': 'dos'}.
min(adict)
max(adict)
```

Q. 6 Finding the minimum and maximum of a sequence of sequences.
L = [[1,2], [6,5], [6,3], [7,4], [3,5]]

⇒
```
L = [[1,2], [6,5], [6,3], [7,4], [3,5]]
min(L)
max (L)
```

College: S.M.Joshi College, Hadapsar.
Department of Mathematics
Name: Pratiksha Goraksha Bhong Class: T.Y.B.Sc Roll No.: 5081
Topic:/ Title: Network Models
W.E. No.: 1. Date: 05-05-22 Remarks: 'C' SBP

Q. 1. A college Principal administrating infrastructures and classes for admissions by quoting tender from several agencies. The data is given as under:

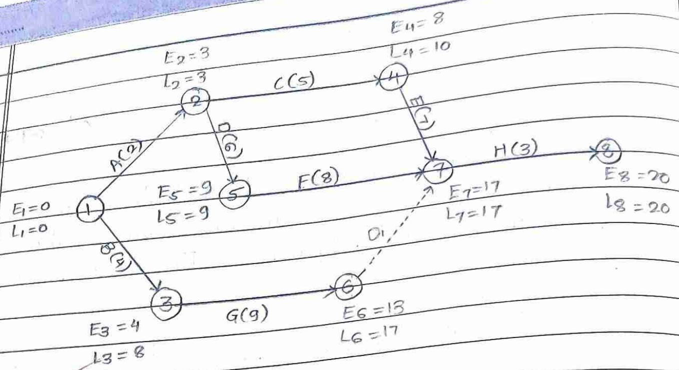
Activity	Predecessor(s)	Duration (Days)
A	-	3
B	-	4
C	A	5
D	A	6
E	C	7
F	D	8
G	B	9
H	E, F, G	3

i) Draw a project network.
ii) Number the events using Fulkerson's rule.
iii) Help the Principal to find the project completion time to participate in the tender.

⇒

* A network diagram for above project is given by,

Teacher's Signature:



* Forward Pass Method :-

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 3 = 3$

$E_3 = E_1 + t_{1,3} = 0 + 4 = 4$

$E_4 = E_2 + t_{2,4} = 3 + 5 = 8$

$E_5 = E_2 + t_{2,5} = 3 + 6 = 9$

$E_6 = E_3 + t_{3,6} = 4 + 9 = 13$

$E_7 = Max\{E_4 + t_{4,7},\ E_5 + t_{5,7},\ E_6 + t_{6,7}\}$

$= Max\{8 + 7,\ 9 + 8,\ 13 + 0\}$

$= Max\{15, 17, 13\} = 17$

$E_8 = E_7 + t_{7,8} = 17 + 3 = 20$.

* Backward Pass Method :-

$L_8 = E_8 = 20$

$L_7 = L_8 - t_{7,8} = 20 - 3 = 17$.

$L_6 = L_7 - t_{6,7} = 17 - 0 = 17$

$L_5 = L_7 - t_{5,7} = 17-8 = 9$

$L_4 = L_7 - t_{4,7} = 17-7 = 10$

$L_3 = L_6 - t_{3,6} = 17-9 = 8$

$L_2 = \text{Min}\{L_4 - t_{2,4}, L_5 - t_{2,5}\}$

$= \text{Min}\{10-5, 9-6\} = \text{Min}\{5,3\} = 3$

$L_1 = \text{Min}\{L_2 - t_{1,2}, L_3 - t_{1,3}\}$

$= \text{Min}\{3-3, 8-4\} = \text{Min}\{0,4\} = 0.$

∴ The critical path is, where E-values and L-values are same. The critical path is,

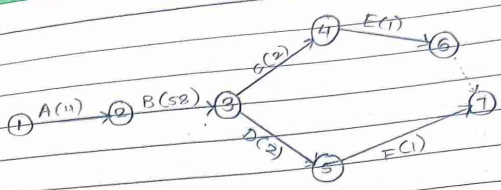
1 – 2 – 5 – 7 – 8.

Critical activities are, A,D, F, H.

The expected completion time for the project is 20 days.

Q. 2. Draw a network for a house construction project. The sequence of activities with their predecessors is given in following table :

Activity	Activity code	Predecessor(s)	Duration (days)
– Prepare house plan	A	–	4
– Construct the house	B	A	58
– Fix the door/windows	C	B	2
– Electrification	D	B	2
– Paint the house	E	C	1
– Polish the doors/ windows	F	D	1



Q 3. A multinational FMCG company (fast moving consumer goods) is in the process of preparing a budget for launching a new product to boost the immunity among human beings due to COVID-19 pandemic. The following table provides the associated activities and their durations. Construct the network.

Activity	Activity code	Predecessor	Duration (days)
- Forecast sales volume	A	–	10
- Study competetive market	B	–	7
- Design item & facilities	C	A	5
- Prepare production schedule	D	C	3
- Estimate cost of production	E	D	2
- Set sales price	F	B, E	1
- Prepare budget	G	F	14

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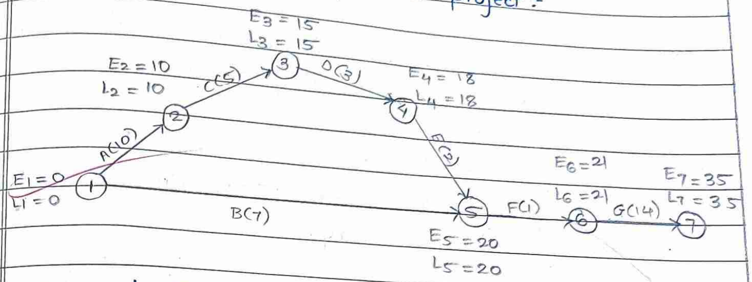
Also determine critical path and the project completion duration.

⇒

* Network diagram for above project :-



* Forward Pass Method :

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 10 = 10$

$E_3 = E_2 + t_{2,3} = 10 + 5 = 15$

$E_4 = E_3 + t_{3,4} = 15 + 3 = 18$

$E_5 = Max\{E_1 + t_{1,5}, E_4 + t_{4,5}\}$

$= Max\{0 + 7, 18 + 2\} = Max\{7, 20\} = 20$

$E_6 = E_5 + t_{5,6} = 20 + 1 = 21$

$E_7 = E_6 + t_{6,7} = 21 + 14 = 35$

* Backward Pass Method :-

$L_7 = E_7 = 35$

$L_6 = L_7 - t_{6,7} = 35 - 14 = 21$

$L_5 = L_6 - t_{5,6} = 21 - 1 = 20$

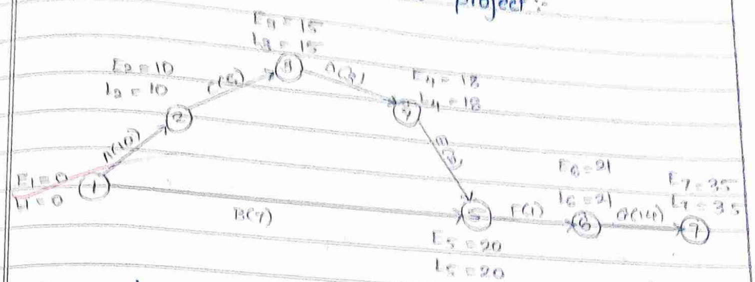
$L_4 = L_5 - t_{4,5} = 20 - 2 = 18$

Teacher's Signature:..............................

Also determine critical path and the project completion duration.

⇒

* Network diagram for above project :-



* Forward Pass Method :

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 10 = 10$

$E_3 = E_2 + t_{2,3} = 10 + 5 = 15$

$E_4 = E_3 + t_{3,4} = 15 + 3 = 18$

$E_5 = \text{Max}\{E_1 + t_{1,5}, E_4 + t_{4,5}\}$

$= \text{Max}\{0 + 7, 18 + 2\} = \text{Max}\{7, 20\} = 20$

$E_6 = E_5 + t_{5,6} = 20 + 1 = 21$

$E_7 = E_6 + t_{6,7} = 21 + 14 = 35$

* Backward Pass Method :-

$L_7 = E_7 = 35$

$L_6 = L_7 - t_{6,7} = 35 - 14 = 21$

$L_5 = L_6 - t_{5,6} = 21 - 1 = 20$

$L_4 = L_5 - t_{4,5} = 20 - 2 = 18$

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$L_3 = L_4 - t_{3,4} = 18 - 3 = 15$

$L_2 = L_3 - t_{2,3} = 15 - 5 = 10$

$L_1 = \text{Min}\{L_2 - t_{1,2}, L_5 - t_{1,5}\}$

$= \text{Min}\{10 - 10, 20 - 7\} = \text{Min}\{0, 13\} = 0$.

∴ The critical path is where E-values and L-values are same. The critical path are,

1 — 2 — 3 — 4 — 5 — 6 — 7

Critical activities are, A, C, D, E, F, G.

The expected completion time for the project is 35 days.

Q. 4 An examination department of university has a list of courses A, B, ..., I, J to be conducted whose time estimates are given in following table:

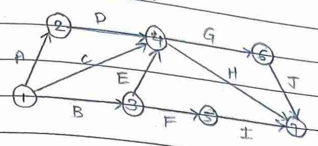
Activity code (i-j)	Activity	Time Estimates (in Days) Optimistic	Most Likely	Pessimistic
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	6	10

Teacher's Signature:..........

Expt. No.......... Date: / /20 Page No.

i) Draw the project network.
ii) Find the critical path.

⇒ * Project network :



The earliest and latest expected completion time for all events considering the expected completion time of each activity are below:

Here, te = expected completion time

$$= \frac{t_o + 4t_m + t_p}{6}$$

where, t_o = optimistic, t_m = most likely, t_p = pessimistic

Activity code (i-j)	Activity	t_o	t_m	t_p	$t_e = \frac{t_o+4t_m+t_p}{6}$
1-2	A	4	6	8	6
1-3	B	2	3	10	4
1-4	C	6	8	16	9
2-4	D	1	2	3	2
3-4	E	6	7	8	7
3-5	F	6	7	14	8
4-6	G	3	5	7	5
4-7	H	4	11	12	10
5-7	I	2	4	6	4
6-7	J	2	6	10	6

Teacher's Signature:..........

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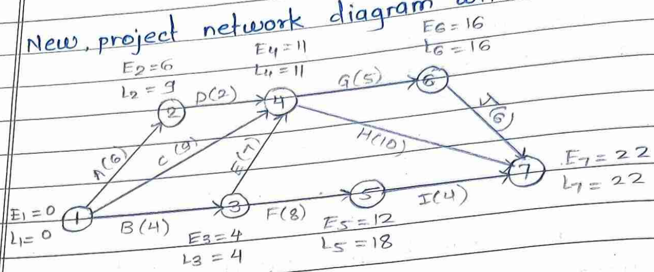
New, project network diagram with time duration.



* Forward Pass Method :-

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 6 = 6$

$E_3 = E_1 + t_{1,3} = 0 + 4 = 4$

$E_4 = Max\{E_1 + t_{1,4},\ E_2 + t_{2,4},\ E_3 + t_{3,4}\}$

$= Max\{0+9,\ 6+2,\ 4+7\} = Max\{9, 8, 11\} = 11$

$E_5 = E_3 + t_{3,5} = 4 + 8 = 12$

$E_6 = E_4 + t_{4,6} = 11 + 5 = 16$

$E_7 = Max\{E_4 + t_{4,7},\ E_5 + t_{5,7},\ E_6 + t_{6,7}\}$

$= Max\{11+10,\ 12+4,\ 16+6\} = Max\{21, 16, 22\} = 22.$

* Backward Pass Method :-

$L_7 = E_7 = 22$

$L_6 = L_7 - t_{6,7} = 22 - 6 = 16$

$L_5 = L_7 - t_{5,7} = 22 - 4 = 18$

$L_4 = Min\{L_6 - t_{4,6},\ L_7 - t_{4,7}\}$

$= Min\{22-10,\ 16-5\} = Min\{12, 11\} = 11$

$L_3 = Min\{L_4 - t_{3,4},\ L_5 - t_{3,5}\}$

Teacher's Signature:...

$= Min\{11-7, 18-8\} = Min\{4, 10\} = 4$

$L_2 = L_4 - t_{2,4} = 11-2 = 9$

$L_1 = Min\{L_2 - t_{1,2}, L_3 - t_{1,3}, L_4 - t_{1,4}\}$

$= Min\{9-6, 4-4, 11-9\} = \{3, 0, 2\} = 0$

The critical path is where E-values and L-values are same. The critical path are,

1 — 3 — 4 — 6 — 7

Critical activities are B, E, G, J.

The expected completion time for the project is 22 days.

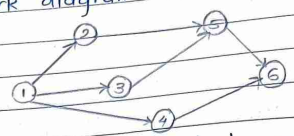
Q. 5 A small project is composed of seven activities whose time estimates are listed below. Activities are being identified by their beginning (i) and ending (j) node numbers.

Activities		Time in weeks		
i	j	to	tm	tp
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	2	14
4	6	2	5	8
5	6	3	6	15

i) Draw the network.

ii) Calculate the expected variances for each activity.
iii) Find the expected project completed time.

⇒ * Network diagram :-



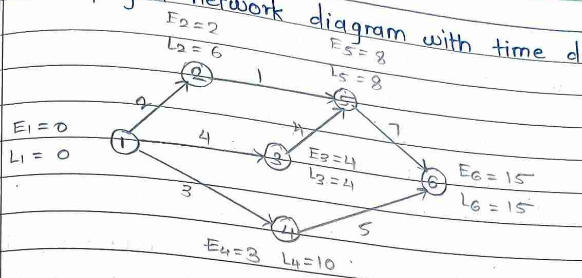
The earliest and latest expected time for each event is calculated by considering expected time of each activity, & calculate expected variances for each activity.

Expected time, $te = \frac{to + 4tm + tp}{6}$

Expected Variance, $\sigma^2 = \left[\frac{tp - to}{6}\right]^2$

Activities		Time in Weeks			$te = \frac{to+4tm+tp}{6}$	$\sigma^2 = \left[\frac{tp-to}{6}\right]^2$
i	j	to	tm	tp		
1	2	1	1	7	2	1
1	3	1	4	7	4	1
1	4	2	2	8	3	1
2	5	1	1	1	1	0
3	5	2	2	14	4	4
4	6	2	5	8	5	1
5	6	3	6	15	7	4

New project network diagram with time duration



* Forward Pass Method :-

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0+2 = 2$

$E_3 = E_1 + t_{1,3} = 0+4 = 4$

$E_4 = E_1 + t_{1,4} = 0+3 = 3$

$E_5 = Max\{E_2 + t_{2,5} , E_3 + t_{3,5}\}$

$= Max\{2+1 , 4+4\} = Max\{3,8\} = 8$

$E_6 = Max\{E_4 + t_{4,6} , E_5 + t_{5,6}\}$

$= Max\{3+5 , 8+7\} = Max\{8,15\} = 15$

* Backward pass Method :-

$L_6 = E_6 = 15$

$L_5 = L_6 - t_{5,6} = 15-7 = 8$

$L_4 = L_6 - t_{4,6} = 15-5 = 10$

$L_3 = L_5 - t_{3,5} = 8-4 = 4$

$L_2 = L_5 - t_{2,5} = 8-2 = 6$

$L_1 = Min\{L_2 - t_{1,2} , L_3 - t_{1,3} , L_4 - t_{1,4}\}$

$= Min\{6-2 , 4-4 , 10-3\} = Min\{4,0,7\} = 0$

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The critical path is where E-values & L-values are same. The critical path are.
1 — 3 — 5 — 6.
Critical activities are, 1 — 3 — 5 — 6
The expected completion time for the project is 15 weeks.

Q. 6 Construct the project network comprised of activities A to P that satisfies the following precedence relationships:

i) A, B and C the first activities of the project and can execute concurrently.
ii) D, E and F follow A.
iii) I and G follow both B and D.
iv) H follows both C and G.
v) K and L follow I.
vi) J suceeds both E and H.
vii) M and N succeed F, but cannot start until both E and H are completed.
viii) O suceeds M and I.
ix) P suceeds J, L and O.
x) K, N and P are the terminal activities of the project and can finish concurrently.

⇒

A network diagram of above project of activities A to P is given by.

Teacher's Signature:...

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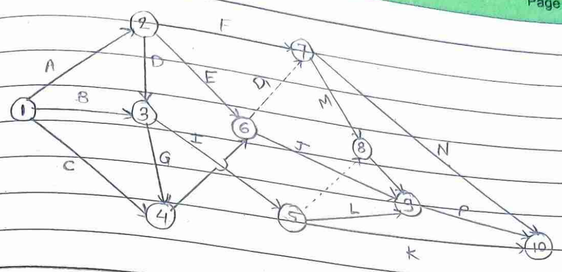

Q. 7 A small project has seven activities and the time in days for each activity is given below:

Activity	A	B	C	D	E	F	G
Duration in days	6	8	3	4	6	10	3

Given that, activities A and B can start at the beginning of the project. When A is completed C and D can start. An activity E can start only when B and D are finished. F can start when B, C and D are completed and it is the final activity. G can start when E is finished and it is the final activity. Draw the network and find the project completion time.

⇒

A network diagram of above project of above activities is given by,

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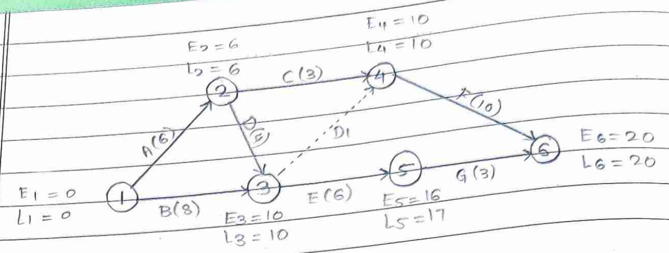

* Forward Pass Method :-

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 6 = 6$

$E_3 = Max\{E_1 + t_{1,3}, E_2 + t_{2,3}\}$

$= Max\{0+8, 6+4\} = Max\{8, 10\} = 10$

$E_4 = Max\{E_2 + t_{2,4}, E_3 + t_{3,4}\}$

$= Max\{6+3, 10+0\} = Max\{9, 10\} = 10$

$E_5 = E_3 + t_{3,5} = 10 + 6 = 16$

$E_6 = Max\{E_4 + t_{4,6}, E_5 + t_{5,6}\}$

$= Max\{10+10, 16+3\} = Max\{20, 19\} = 20$

* Backward Pass Method :-

$L_6 = E_6 = 20$

$L_5 = L_6 - t_{5,6} = 20 - 3 = 17$

$L_4 = L_6 - t_{4,6} = 20 - 10 = 10$

$L_3 = Min\{L_4 - t_{3,4}, L_5 - t_{3,5}\}$

$= Min\{10-0, 17-6\} = Min\{10, 11\} = 10$

$L_2 = Min\{L_3 - t_{2,3}, L_4 - t_{2,4}\}$

$= Min\{10-4, 10-3\} = Min\{6, 7\} = 6$

$L_1 = Min\{L_2 - t_{1,2}, L_3 - t_{1,3}\}$

$= Min\{6-6, 10-8\} = Min\{0, 8\} = 0$

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∴ The critical path is where E-values and L-values are same. The critical path are,
1 — 2 — 3 — 4 — 6
Critical activities are, A, D, D1, F.
The expected completion time for the project is 20 days.

Q. 8 A small project consists of seven activities for which the relevant data are given below:

Activity	Predecessor(s)	Duration (Days)
A	–	4
B	–	7
C	–	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5

i) Draw the project network and project completion time.
ii) Calculate the total, free and independent floats of each of the activities.

⇒

A network diagram for above project is given by,

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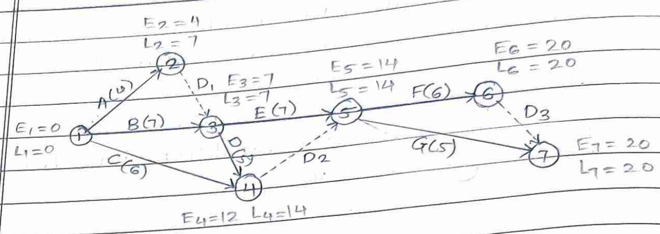

* Forward Pass Method :-

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 4 = 4$

$E_3 = \text{Max}\{E_1 + t_{1,3},\ E_2 + t_{2,3}\}$
$= \text{Max}\{0+7,\ 4+0\} = \text{Max}\{7, 4\} = 7$

$E_4 = \text{Max}\{E_1 + t_{1,4},\ E_3 + t_{3,4}\}$.
$= \text{Max}\{0+6,\ 7+5\} = \text{Max}\{6, 12\} = 12$

$E_5 = \text{Max}\{E_3 + t_{3,5},\ E_4 + t_{4,5}\}$
$= \text{Max}\{7+7,\ 12+0\} = \{14, 12\} = 14$

$E_6 = E_5 + t_{5,6} = 14 + 6 = 20$

$E_7 = \text{Max}\{E_5 + t_{5,7},\ E_6 + t_{6,7}\}$
$= \text{Max}\{14+5,\ 20+0\} = \text{Max}\{19, 20\} = 20$

* Backward Pass Method :

$L_7 = E_7 = 20$

$L_6 = L_7 - t_{6,7} = 20 - 0 = 20$

$L_5 = \text{Min}\{L_6 - t_{5,6},\ L_7 - t_{5,7}\}$
$= \text{Min}\{20-6,\ 20-5\} = \text{Min}\{14, 15\} = 14$

$L_4 = L_5 - t_{4,5} = 14 - 0 = 14$

Teacher's Signature:..

$L_3 = \text{Min}\{L_5 - t_{3,5},\ L_4 - t_{3,4}\}$
$= \text{Min}\{14-7,\ 14-5\} = \text{Min}\{7, 9\} = 7$
$L_2 = L_3 - t_{2,3} = 7 - 0 = 7$
$L_1 = \text{Min}\{L_2 - t_{1,2},\ L_3 - t_{1,3},\ L_4 - t_{1,4}\}$
$= \text{Min}\{7-4,\ 7-7,\ 14-6\} = \text{Min}\{3, 0, 8\} = 0.$

The critical path is where E-values and L-values are same. The critical path are.

1 — 3 — 5 — 6 — 7

Critical activities are, B, E, F, D_3

The expected completion time for the project is 20 days.

*Table for Non-critical activities :-

		Earliest		Latest		Float		
Activities	Duration	start	Finish	start	finish	Total	Free	Independent
(i, j)	Dij	Ei	Ei+tij	Lj-tij	Lj	Lj-Ei-tij	Ej-Ei-tij	Ej-Li-tij
(1,2)	4	0	4	3	7	3	0	0
(1,4)	6	0	6	8	14	8	6	6
(2,3)	0	4	4	7	7	3	3	0
(3,4)	5	7	12	9	14	2	0	0
(4,5)	0	12	12	14	14	2	2	0
(5,7)	5	14	19	15	20	1	1	1

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W.E. No.: 2. Date: 11-05-2022 Remarks: 'c' SBP

Q. 1 Solve the following game graphically. The payoff is for player A.

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	8	5	-7	9
	A_2	-6	6	4	-2

$\Rightarrow$ We have to calculate maxmin & minmax

		player B				
		B_1	B_2	B_3	B_4	maxmin
player A	A_1	8	5	-7	9	-7
	A_2	-6	6	4	-2	$\boxed{-6}$
minmax		8	6	4	9	

from above table,

Maxmin = -6 minmax = 4

maxmin $\neq$ minmax

$\therefore$ above payoff matrix has no saddle point.

$\therefore$ Player A has probabilities P_1 & P_2 with two strategies A_1 & A_2 s.t. $P_1 + P_2 = 1$ & $P_2 = 1 - P_1$ then expected payoff for A will be given by.

Teacher's Signature:

B's pure strategies	A's expected payoff
B_1	$8P_1 - 6P_2$
B_2	$5P_1 + 6P_2$
B_3	$-7P_1 + 4P_2$
B_4	$9P_1 - 2P_2$.

Player A must ~~b~~ choose his best possible strategies to get maximum expected gain i.e. the highest expected gain is found at point P. Where two strategy lines are

$$E_1 = 8P_1 - 6P_2 \quad \text{——} \ (1)$$

$$E_3 = -7P_1 + 4P_2 \quad \text{——} \ (2)$$

Now,

$E_1 = 8P_1 - 6(1 - P_1)$

$= 8P_1 - 6 + 6P_1$

$= 14P_1 - 6$

$E_3 = -7P_1 + 4(1 - P_1)$

$= -7P_1 + 4 - 4P_1$

$= -11P_1 + 4$

$\therefore$ Optimum strategy for player A is obtained by setting E_1 & E_3 are equal.

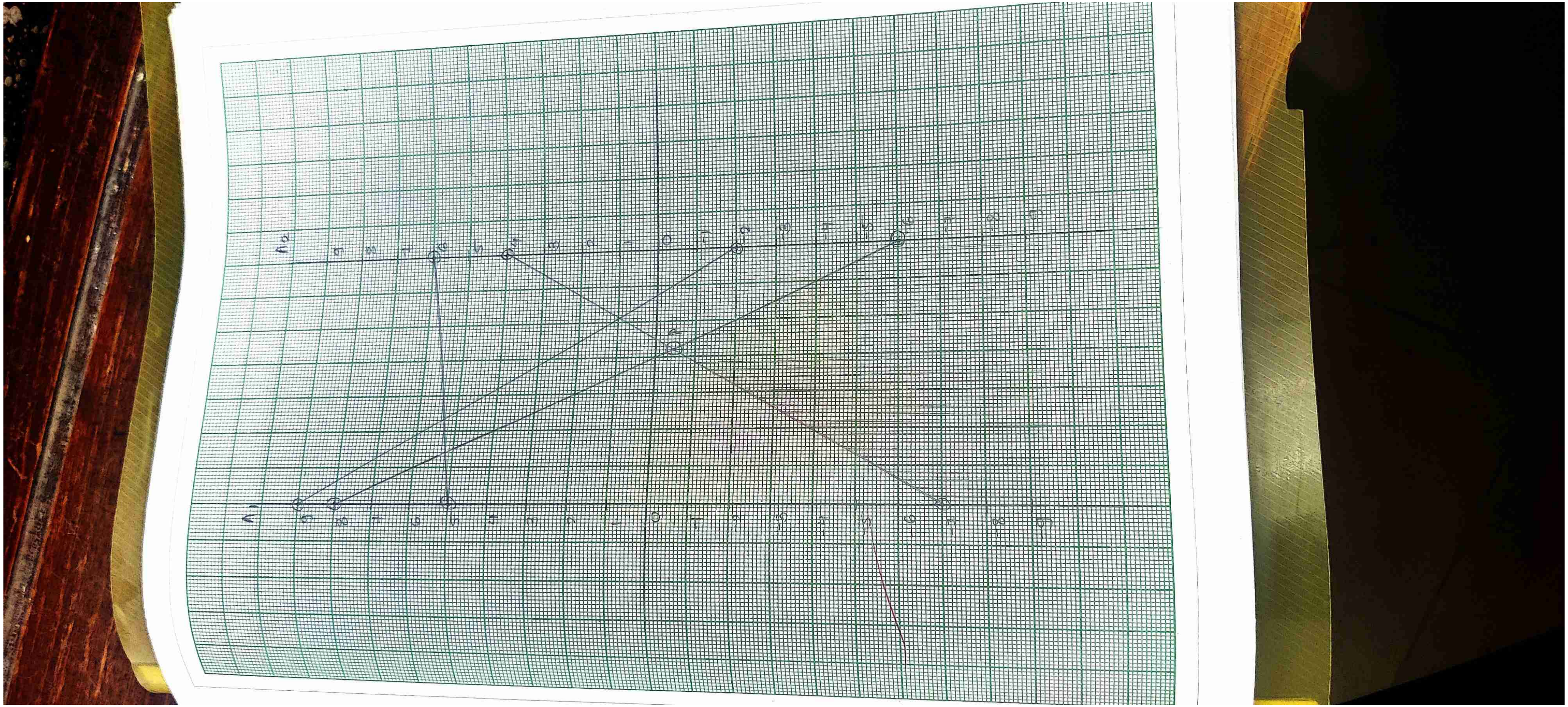
$\therefore \quad E_1 = E_3$

$14P_1 - 6 = -11P_1 + 4$

$14P_1 + 11P_1 = 4 + 6$

$25P_1 = 10$

$P_1 = \frac{10}{25}$



$P_2 = 1 - P_1 = 1 - \frac{10}{25} = \frac{15}{25}$

Substituting the values of P_1 & P_2 in equation ① i.e. in equation E_1, we get.

$V = E_1 = 8P_1 - 6P_2$

$= 8\left(\frac{10}{25}\right) - 6\left(\frac{15}{25}\right)$

$= \frac{80 - 90}{25}$

$= \frac{-10}{25}$

$\therefore$ value of game $= V = \frac{-10}{25}$

Similarly, player B we have to find optimal strategies with corresponding probabilities q_1 & q_2

$\therefore$ The expected loss of player B is,

	B_1	B_3
A_1	8	-7
A_2	-6	4

$L_1 = 8q_1 - 7q_2$ —— ③

$L_3 = -6q_1 + 4q_2$ —— ④

Now,

$L_1 = 8q_1 - 7q_2 = 8q_1 - 7(1-q_1) = 8q_1 - 7 + 7q_1 = 15q_1 - 7$

$L_3 = -6q_1 + 4(q_2) = -6q_1 + 4(1-q_1) = -6q_1 + 4 - 4q_1 = -10q_1 + 4$

$\therefore$ Optimum strategy for player B is obtained by setting L_1 & L_3 are equal

$\therefore$ $L_1 = L_3$

$15q_1 - 7 = -10q_1 + 4$

$15q_1 + 10q_1 = 4 + 7$

$25q_1 = 11$

$q_1 = \frac{11}{25}$

$\therefore$ $q_2 = 1 - q_1 = 1 - \left(\frac{11}{25}\right) = \frac{14}{25}$

$\therefore$ Substituting the values of q_1 & q_2 in equation (3) i.e. in equation L_1

$\therefore$ $V = 8q_1 - 7q_2$

$= 8\left(\frac{11}{25}\right) - 7\left(\frac{14}{25}\right)$

$= \frac{88 - 98}{25}$

$= \frac{-10}{25}$

Q. 2 Two companies, A and B sell two brands of flu medicine. Company A adverties in radio (A1), television (A2) and newspapers (A3). Company B, in addition to using radio (B1), television (B2) and newspapers (B3), also mails brochures (B4). Depending on the cleverness and the intensity of the advertisement campaign, each company can capture a portion of the market from the other. The following matrix summarizes the percentage of the market captured or lost by company A.

Players / Strategies		Player B			
		B_1	B_2	B_3	B_4
player A	A_1	8	-2	9	-3
	A_2	6	5	6	8
	A_3	-2	4	-9	5

Find the best strategy for each company. Also find the value of the game.

⇒

We have given payoff matrix.

		Player B				
		B_1	B_2	B_3	B_4	Maxmin
player A.	A_1	8	-2	9	-3	-3
	A_2	6	$\boxed{5}$	6	8	$\boxed{5}$
	A_3	-2	4	-9	5	-9
Min max		8	5	9	8	

Here, Maxmin $= 5$ & Minmax $= 5$

$\therefore$ Maxmin $=$ Minmax $= 5$

$\therefore$ Value of game is 5.

From the above table, we get,
The company A will always adopt strategy A_2

i.e. Company A adverties in television (A_2).
And Company B will adopt strategy B_2
i.e. Company B adverties in television (B_2).

From that, we see that, both companies A and B sell two brands of flu medicine by advertising in television.

Q. 3. Solve the following game by dominance principle whose payoff matrix is

$$M = [a_{ij}]_{4\times4}, \text{ where } a_{ij} = i-j \quad , \quad i \neq j$$
$$= 4-j \quad , \quad i = j$$

$\Rightarrow$ We know that, general matrix for 4x4 is,

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times4}$$

Now, we have the conditions,

$$M = [a_{ij}]_{4\times4}, \text{ where } a_{ij} = i-j \quad , \quad i \neq j$$
$$= 4-j \quad , \quad i = j$$

Now, let us construct the payoff matrix.
Let player A's strategies are A_1, A_2, A_3 & A_4
& for player B's strategies are B_1, B_2, B_3 & B_4.

∴ The matrix are,

		Player B			
Player A.		B_1	B_2	B_3	B_4
	A_1	3	-1	-2	-3
	A_2	1	2	-1	-2
	A_3	2	1	1	-1
	A_4	3	2	1	0

Now, solve the above matrix by dominance principle.

		player B				Maxmin
		B_1	B_2	B_3	B_4	
player A	A_1	3	-1	-2	-3	-3
	A_2	1	2	-1	-2	-2
	A_3	2	1	1	-1	-1
	A_4	3	2	1	0	$\boxed{0}$
Minmax		3	2	1	0	

Here, Minmax = 0 = Maxmin

∴ Value of game is 0.

But we have to solve it by dominance principle.

From above table, we see that.
A_3 is less than or equal to A_4.
$\therefore$ A_3 is dominated by A_4
$\therefore$ A_3 is deleted

$\therefore$ obtained matrix is.

	B_1	B_2	B_3	B_4
A_1	3	-1	-2	-3
A_2	1	2	-1	-2
A_4	3	2	1	0

from above we see that.
A_2 is less than or equal to A_4
$\therefore$ A_2 is dominated by A_4
$\therefore$ A_2 is deleted.

Now, obtained matrix is.

	B_1	B_2	B_3	B_4
A_1	3	-1	-2	-3
A_4	3	2	1	0

from above we see that.
A_1 is less than or equal to A_4
$\therefore$ A_1 is dominated by A_4
$\therefore$ A_1 is deleted

$\therefore$ obtained matrix is.

	B_1	B_2	B_3	B_4
A_4	3	2	1	0

from above we see that.
B_1 is greater than or equal to B_2
$\therefore$ B_1 is dominated by B_2
$\therefore$ B_1 is deleted

$\therefore$ obtained matrix is.

	B_2	B_3	B_4
A_4	2	1	0

from above we see that
B_2 is greater than or equal to B_3
$\therefore$ B_2 is dominated by B_3
$\therefore$ B_2 is deleted

$\therefore$ obtained matrix is,

	B_3	B_4
A_4	1	0

from above we see that
B_3 is greater than or equal to B_4
$\therefore$ B_3 is dominated by B_4
$\therefore$ B_3 is deleted

$\therefore$ Finally we get,

	B_4
A_4	0

$\therefore$ value of game is 0.

Q. 4 Solve the following game using dominance rule.

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3 & 5 & 4 & 2 \\ 5 & 6 & 2 & 4 \\ 2 & 1 & 4 & 0 \\ 3 & 3 & 5 & 2 \end{bmatrix} \end{array}$$

⇒ We have given matrix,

B

A		B_1	B_2	B_3	B_4
	A_1	3	5	4	2
	A_2	5	6	2	4
	A_3	2	1	4	0
	A_4	3	3	5	2

from above we see that,

A_3 is less than or equal to A_4

∴ A_3 is dominated by A_4

∴ A_3 is deleted.

∴ obtained matrix is,

B

A		B_1	B_2	B_3	B_4
	A_1	3	5	4	2
	A_2	5	6	2	4
	A_4	3	3	5	2

from above we see that,

B_2 is greater than or equal to B_4

∴ B_2 is dominated by B_4

$\therefore$ B_2 is deleted

$\therefore$ obtained matrix is,

		B		
		B_1	B_3	B_4
	A_1	3	4	2
A	A_2	5	2	4
	A_4	3	5	2

from above we see that,

B_1 is greater than or equal to B_4

$\therefore$ B_1 is dominated by B_4

$\therefore$ B_1 is deleted

$\therefore$ Obtained matrix is,

		B	
		B_3	B_4
	A_1	4	2
A	A_2	2	4
	A_4	5	2

from above we see that,

A_1 is less than or equal to A_4

$\therefore$ A_1 is dominated by A_4

$\therefore$ A_1 is deleted

$\therefore$ Obtained matrix is,

		B		Maxmin
		B_3	B_4	
A	A_2	2	4	$\boxed{2}$
	A_4	5	2	2
Minmax		5	4	

Here, Maxmin = 2 Minmax = 4

$\therefore$ Maxmin $\neq$ Minmax

$\therefore$ $2 \leq v \leq 4$

$\therefore$ For player A has optimal strategy are P_1 & P_2. Similarly, for player B has optimal strategy are q_1 & q_2.

$$\therefore P_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{2-5}{(2+2)-(4+5)} = \frac{-3}{4-9} = \frac{-3}{-5} = \frac{3}{5}$$

$$\therefore P_2 = 1 - P_1 = 1 - \left(\frac{3}{5}\right) = \frac{2}{5}$$

Now,

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{2-4}{4-9} = \frac{-2}{-5} = \frac{2}{5}$$

$$\therefore q_2 = 1 - q_1 = 1 - \left(\frac{2}{5}\right) = \frac{3}{5}$$

Value of game :

$$V = \frac{a_{22} \cdot a_{11} - a_{12} \cdot a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{2 \cdot 2 - 4 \cdot 5}{4-9}$$

$$= \frac{4-20}{4-9} = \frac{-16}{-5} = \frac{16}{5}$$

$\therefore$ Value of game is $\frac{16}{5}$.

Player A having strategies $(0, 3/5, 0, 2/5)$

& player B having strategies $(0, 0, 2/5, 3/5)$

Q. 5 Solve the following game using graphical approach.

$$\begin{array}{cc} & B \\ & \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ A \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{bmatrix} \end{array}$$

Does the game have alternate solution exists, if so, find it?

⇒ We have to calculate Maxmin & Minmax.

		Player B				
		B_1	B_2	B_3	B_4	Maxmin
player A	A_1	2	2	3	-1	-1
	A_2	4	3	2	6	$\boxed{2}$
Minmax		4	3	3	6	

Here,

Maxmin = 2 Minmax = 3

∴ Maxmin ≠ Minmax

∴ above payoff matrix has no saddle point.

∴ Player A has probabilities P_1 & P_2 with two strategies A_1 & A_2 s.t.

$P_1 + P_2 = 1$

∴ $P_2 = 1 - P_1$

then expected payoff for A will be given by,

B's pure strategies	A's expected payoff
B_1	$2P_1 + 4P_2$
B_2	$2P_1 + 3P_2$
B_3	$3P_1 + 2P_2$
B_4	$-P_1 + 6P_2$

Player A must choose his best possible strategies to get maximum expected gain i.e. the highest expected gain is found at point P. Where two strategy lines are.

$E_2 = 2P_1 + 3P_2$ ——— (1)

$E_3 = 3P_1 + 2P_2$ ——— (2)

Now,

$E_2 = 2P_1 + 3P_2 = 2P_1 + 3(1-P_1) = 2P_1 + 3 - 3P_1 = -P_1 + 3$

$E_3 = 3P_1 + 2P_2 = 3P_1 + 2(1-P_1) = 3P_1 + 2 - 2P_1 = P_1 + 2$.

$\therefore$ Optimum strategy for player A is obtained by setting E_2 & E_3 are equal

$\therefore \quad E_2 = E_3$

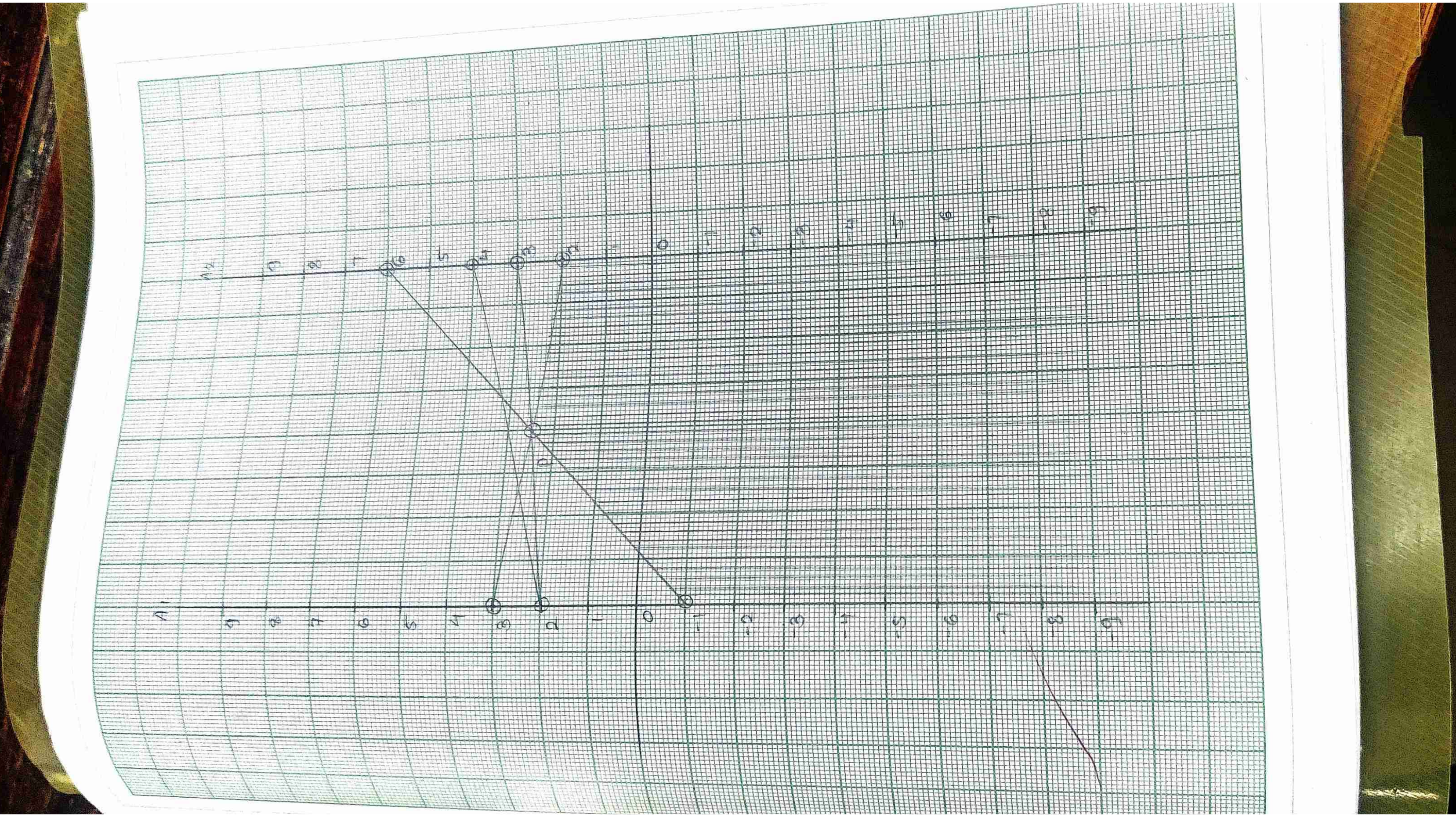
$-P_1 + 3 = P_1 + 2$

$3 - 2 = 2P_1$

$1 = 2P_1$

$P_1 = 1/2$.

$\therefore P_2 = 1 - P_1 = 1 - \left(\frac{1}{2}\right) = \frac{1}{2}$



Substituting the values of P_1 & P_2 in equation ① i.e. in equation E_2, we get,

$$V = E_2 = 2P_1 + 3P_2$$
$$= 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)$$
$$= \frac{2+3}{2}$$
$$\therefore V = \frac{5}{2}$$

Similarly, player B, we have to find optimal strategies with corresponding probabilities q_1 & q_2.

$\therefore$ The expected loss of player B is,

	B_2	B_3
A_1	2	3
A_2	3	2

$L_2 = 2q_1 + 3q_2$ ——— ③

$L_3 = 3q_1 + 2q_2$ ——— ④

Now,

$L_2 = 2q_1 + 3q_2 = 2q_1 + 3(1-q_1) = 2q_1 + 3 - 3q_1 = -q_1 + 3$

$L_3 = 3q_1 + 2q_2 = 3q_1 + 2(1-q_1) = 3q_1 + 2 - 2q_1 = q_1 + 2$

$\therefore$ Optimum strategy for player B is obtained by setting L_2 & L_3 are equal.

$\therefore L_2 = L_3$

$-q_1 + 3 = q_1 + 2$

$$3-2 = 2q_1$$
$$1 = 2q_1$$
$$q = \frac{1}{2}$$

$\therefore\ q_2 = 1 - q_1 = 1 - \left(\frac{1}{2}\right) = \frac{1}{2}$

$\therefore$ substituting the values of q_1 & q_2 in equation ③ i.e. in equation L_2

$\therefore\ V = 2q_1 + 3q_2$

$= 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)$

$= \frac{2+3}{2}$

$\therefore\ V = \frac{5}{2}$

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