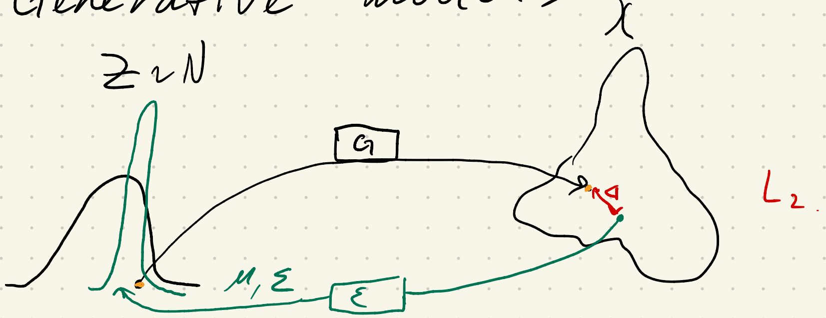
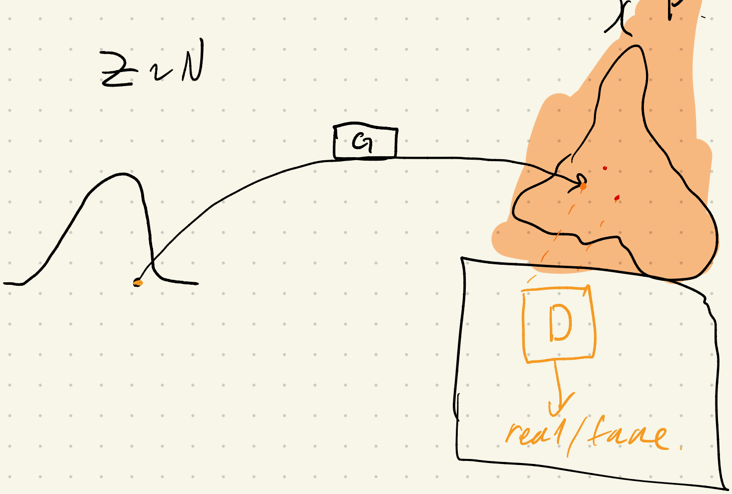
Generative models



$$G : z, \theta_G \to X$$

VAE

$$D : X, \theta_D \to [0, 1]$$



$$\underbrace{-\mathop{\mathbb{E}}_{x\sim p}\log_2(D(x)) - \mathop{\mathbb{E}}_{z\sim N}\log_2(1-D(G(z)))}_{L_D(\theta_D)}$$

$$L_G(\theta_G) = \mathop{\mathbb{E}}_{z\sim N}\log_2(1-D(G(z))) = \mathop{\mathbb{E}}_{x\sim q}\log_2(1-D(x))$$

$$V(\Theta_G, \Theta_D) = \mathbb{E}_{x \sim P} \log_2(D(x)) + \mathbb{E}_{z \sim N} \log_2(1 - D(G(z)))$$

$$\boxed{\min_{\Theta_G} \max_{\Theta_D} V(G, D)}$$

$$\int_x p(x) \log_2 D(x)\, dx + \int p_z(z) \log_2(1 - D(G(z)))\, dz$$

q s $G(z)$, $z \sim N$

$$= \int_X \left[p(x) \log_2 D(x) + q(x) \log_2(1 - D(x)) \right] dx$$

$$\underbrace{a \log m + b \log_2(1-m)}$$

$$\boxed{D_G^*(x) = \frac{p(x)}{p(x) + q(x)}}$$

$$\boxed{\mathbb{E}_{x \sim p} \log_2 \frac{p(x)}{p(x) + q(x)} + \mathbb{E}_{x \sim q} \log_2 \frac{q(x)}{p(x) + q(x)}}$$

$$-\log_2 2 - \log_2 2 = -\log_2 4$$

$$\underbrace{\log_2 4 + KL\left(p \,\|\, \frac{p+q}{2}\right) + KL\left(q \,\|\, \frac{p+q}{2}\right)}$$

Jensen - Shannon divergence.

$$KL(p \,\|\, q) = \mathbb{E}_{x \sim p} \log_2 \frac{p(x)}{q(x)}$$

$$KL(q \,\|\, p) = \mathbb{E}_{x \sim q} \log_2 \frac{q(x)}{p(x)},$$

$$JS(p, q) = \underbrace{\left(\log_2 4 + KL\left(p \,\|\, \frac{p+q}{2}\right) + KL\left(q \,\|\, \frac{p+q}{2}\right)\right)}_{4} \frac{1}{2}.$$

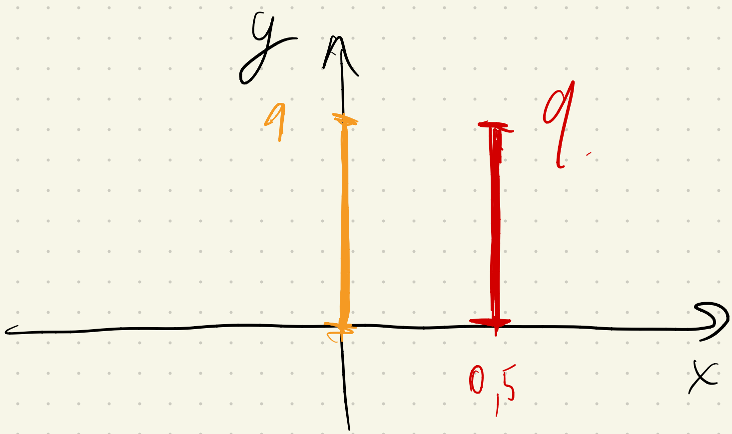
$$\boxed{\theta \neq 0 \quad \log 2.}$$

$z \sim U[0, 1]$

$P = (0, z)$

$q = (\theta, z)$

$$\boxed{\begin{array}{c} \theta \neq 0 \\ \theta = 0 \end{array}}$$



$$\frac{1}{2}\left(\log_2 4 + \mathbb{E}_{x \sim p} \log_2 \frac{2P(x)}{P(x) + \underbrace{\cancel{q(x)}}_{=0}} + \mathbb{E}_{x \sim q} \log_2 \frac{2q(x)}{\underbrace{\cancel{p(x)}}_{0} + q(x)} \right) =$$

$= 1$

$$= \frac{1}{2} \log_2 4 = \log_2 2 = 1$$

$p, q \qquad \forall \gamma \in \Pi \quad \int_x \gamma(x, y)\, dx = q(y)$

$$\boxed{\int_y \gamma(x, y)\, dy = p(x)}$$

$$\boxed{W(p, q) = \inf_{\gamma \in \Pi} \mathbb{E}_{(x,y)\sim\gamma} \|x - y\|_2}$$

$$W(p, q) = \sup_{\|f\|_L \le 1} \left(\mathbb{E}_{x \sim p} f(x) - \mathbb{E}_{x \sim q} f(x) \right)$$

$$|f(x) - f(y)| \le L \cdot |x - y|$$

$L = 1$

$$\boxed{\big| \|\nabla_x f\| - 1 \big|}$$

$D : \mathcal{X}, \theta_D \to \mathbb{R}$

$$L_D(\theta_D) = -\mathbb{E}_{x\sim p} D(x) + \mathbb{E}_{x\sim q} D(x) + \| \|\nabla_x D\| - 1 \|$$

$$L_G(\theta_G) = -\mathbb{E}_{x\sim q} D(x)$$