Generated data for Figure 3: first three dispersion curves for n=0, 1 and 2 PSSWs.

DMI on TOP only!
Single material (NiFe).

The Eigenfrequencies -- D = 4.2 mJ/m^2

Some parameters needed for the code

In[•]:=
```
B = 0.03;
Δ = 1;
Φ[0] = Table[0, {4}];
```

In[•]:=
```
LL = 16;
```

In[•]:=
```
a = 0.248 × 10^-9; (*0.25 nm atomic spacing*)
Ms = 835 563; (*A/m*)
AA = 1.355 × 10^-11; (*J/m*)
DD = 4.2 × 10^-3 × 1/1;
(* i.e. This is 4.2 mJ/m^2 for 1 layer and decreases for thicker films!*)
JJ = 2 AA / (a^2 Ms)
```

Out[•]=
```
527.335
```

In[•]:=
```
K[i_] := K[i] = Which[i < LL/2 + 0.5, 0, i > LL/2 + 0.5, 0]
J[i_] := J[i] = Which[i < LL/2 + 0.5, JJ, i > LL/2 + 0.5, JJ]
H[i_] := H[i] = Which[i < LL/2 + 0.5, B, i > LL/2 + 0.5, B]
HDMI[1] = 2 DD / (a Ms);
HDMI[LL] = 0
```

Out[•]=
```
0
```

In[•]:=
```
ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```
In[•]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
          H[i] Cos[ϕ[i]] + (4 π 10^-7) Ms + 2 K[i] (Cos[ϕ[i]]^2 - Sin[ϕ[i]]^2) + J[i]
            (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
        aplus[i_] := aplus[i] = -J[i] Cos[ϕ[i] - ϕ[i + 1]]
        aminus[i_] := aminus[i] = -J[i] Cos[ϕ[i] - ϕ[i - 1]]
        bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[ϕ[i]] - 2 K[i] Cos[ϕ[i]]^2 -
            J[i] (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]

In[•]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
          {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
        rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
          {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]
```

The 1st, (N/2)th, (N/2 +1)th and Nth planes all need individual codes sinse they have different exchange coupling to the planes on either side.
The codes are as follows:

```
In[•]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
          {-HDMI[1] Sin[y] i,
           H[1] Cos[ϕ[1]] + (4 π 10^-7) Ms + 2 K[1] (Cos[ϕ[1]]^2 - Sin[ϕ[1]]^2) +
            J[1] (0 + Cos[ϕ[1] - ϕ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
           0,
           aplus[1]},
          Table[0, {2 NN - 4}]];

In[•]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
          {-H[1] Cos[ϕ[1]] - 2 K[1] Cos[ϕ[1]]^2 -
            J[1] (0 + Cos[ϕ[1] - ϕ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
           -HDMI[1] Sin[y] i,
           J[2]},
          Table[0, {2 NN - 3}]];
```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

```
In[•]:= arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
          Table[0, {NN - 3}],
          {aminus[NN / 2],
           0,
           H[NN / 2] Cos[ϕ[NN / 2]] + (4 π 10^-7) Ms + 2 K[NN / 2] (Cos[ϕ[NN / 2]]^2 - Sin[ϕ[NN / 2]]^2) +
            J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
           0,
           -(J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]]},
          Table[0, {NN - 2}]]

In[•]:= brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
          Table[0, {NN - 4}],
          {J[NN / 2],
           0,
           -H[NN / 2] Cos[ϕ[NN / 2]] - 2 K[NN / 2] Cos[ϕ[NN / 2]]^2 -
            J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
           0,
           J[NN] + β (J[1] - J[NN])},
          Table[0, {NN - 1}]]

In[•]:= arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
          Table[0, {NN - 1}],
          {-(J[NN / 2] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]],
           0,
           H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] + (4 π 10^-7) Ms +
            2 K[NN / 2 + 1] (Cos[ϕ[NN / 2 + 1]]^2 - Sin[ϕ[NN / 2 + 1]]^2) + (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] + J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] +
            4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
           0,
           aplus[NN / 2 + 1]},
          Table[0, {NN - 4}]]
```

```
brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
    Table[0, {NN - 2}],
    {J[NN] + β (J[1] - J[NN]),
     0,
     -H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]]^2 -
      (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] -
      J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
      2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
     0,
     J[NN / 2 + 1]},
    Table[0, {NN - 3}]]
```

```
arow100[NN_, y_, z_] := Join[
    Table[0, {2 NN - 3}],
    {aminus[NN],
     -HDMI[NN] Sin[y] i,
     H[NN] Cos[ϕ[NN]] + (4 π 10^-7) Ms + 2 K[NN] (Cos[ϕ[NN]]^2 - Sin[ϕ[NN]]^2) +
      J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];
```

```
brow100[NN_, y_, z_] := Join[
    Table[0, {2 NN - 4}],
    {J[NN - 1],
     0,
     -H[NN] Cos[ϕ[NN]] - 2 K[NN] Cos[ϕ[NN]]^2 -
      J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
     -HDMI[NN] Sin[y] i}];
```

The dynamical matrix and eigenfrequencies

The dynamical matrix is:

```
big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
    {arow1[NN, y, z], brow1[NN, y, z]},
    Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
    {arow50[NN, y, z, β], brow50[NN, y, z, β],
     arow51[NN, y, z, β], brow51[NN, y, z, β]},
    Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
    {arow100[NN, y, z], brow100[NN, y, z]}]
```

The eigenfrequencies are given by (γ = 176 GHz rad/T):

```
freqs[NN_, y_, z_, β_] := freqs[NN, y, z, β] =
  176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 2}]
```

In[•]:=
```
freqs2[NN_, y_, z_, β_] := freqs2[NN, y, z, β] =
  176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 1}]
```

Dispersion plots

n = 0 (quasi-uniform)

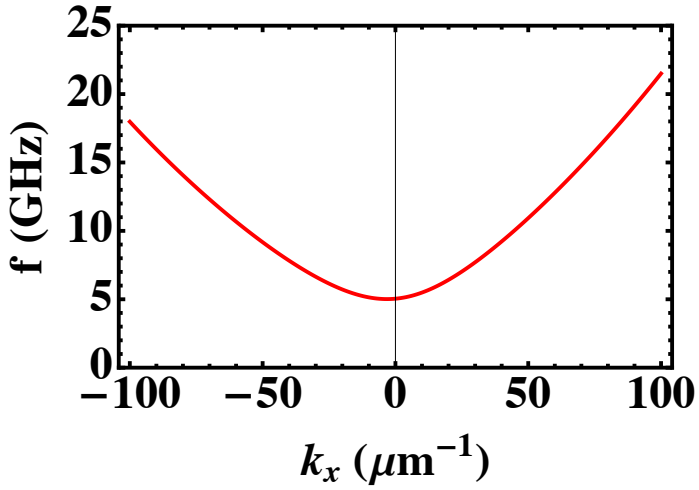
In[•]:=
```
Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
   freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}]
```

Out[ ]=

```
{{-100, 17.979}, {-99, 17.7711}, {-98, 17.5646}, {-97, 17.3593}, {-96, 17.1554},
 {-95, 16.9528}, {-94, 16.7516}, {-93, 16.5516}, {-92, 16.3529}, {-91, 16.1556},
 {-90, 15.9596}, {-89, 15.7648}, {-88, 15.5714}, {-87, 15.3792}, {-86, 15.1884},
 {-85, 14.9988}, {-84, 14.8105}, {-83, 14.6235}, {-82, 14.4377}, {-81, 14.2533},
 {-80, 14.0701}, {-79, 13.8882}, {-78, 13.7076}, {-77, 13.5282}, {-76, 13.3501},
 {-75, 13.1733}, {-74, 12.9977}, {-73, 12.8234}, {-72, 12.6504}, {-71, 12.4787},
 {-70, 12.3082}, {-69, 12.139}, {-68, 11.9711}, {-67, 11.8044}, {-66, 11.6391},
 {-65, 11.475}, {-64, 11.3122}, {-63, 11.1507}, {-62, 10.9905}, {-61, 10.8316},
 {-60, 10.674}, {-59, 10.5178}, {-58, 10.3628}, {-57, 10.2092}, {-56, 10.057},
 {-55, 9.90614}, {-54, 9.75664}, {-53, 9.60854}, {-52, 9.46184}, {-51, 9.31657},
 {-50, 9.17274}, {-49, 9.03038}, {-48, 8.88949}, {-47, 8.75012}, {-46, 8.61228},
 {-45, 8.47601}, {-44, 8.34132}, {-43, 8.20826}, {-42, 8.07686}, {-41, 7.94716},
 {-40, 7.81919}, {-39, 7.693}, {-38, 7.56863}, {-37, 7.44614}, {-36, 7.32558},
 {-35, 7.20699}, {-34, 7.09045}, {-33, 6.97601}, {-32, 6.86375}, {-31, 6.75373},
 {-30, 6.64604}, {-29, 6.54074}, {-28, 6.43794}, {-27, 6.33771}, {-26, 6.24016},
 {-25, 6.14539}, {-24, 6.0535}, {-23, 5.96459}, {-22, 5.8788}, {-21, 5.79624},
 {-20, 5.71702}, {-19, 5.64129}, {-18, 5.56917}, {-17, 5.50081}, {-16, 5.43633},
 {-15, 5.37589}, {-14, 5.31962}, {-13, 5.26766}, {-12, 5.22016}, {-11, 5.17726},
 {-10, 5.13908}, {-9, 5.10577}, {-8, 5.07745}, {-7, 5.05423}, {-6, 5.03621},
 {-5, 5.02351}, {-4, 5.01619}, {-3, 5.01434}, {-2, 5.018}, {-1, 5.02722},
 {0, 5.04203}, {1, 5.06242}, {2, 5.0884}, {3, 5.11994}, {4, 5.15699}, {5, 5.19951},
 {6, 5.24741}, {7, 5.30063}, {8, 5.35905}, {9, 5.42257}, {10, 5.49108},
 {11, 5.56446}, {12, 5.64256}, {13, 5.72526}, {14, 5.81242}, {15, 5.90389},
 {16, 5.99953}, {17, 6.09921}, {18, 6.20277}, {19, 6.31009}, {20, 6.42102},
 {21, 6.53543}, {22, 6.6532}, {23, 6.77419}, {24, 6.89829}, {25, 7.02539},
 {26, 7.15536}, {27, 7.28811}, {28, 7.42354}, {29, 7.56154}, {30, 7.70203},
 {31, 7.84493}, {32, 7.99015}, {33, 8.13761}, {34, 8.28725}, {35, 8.43899},
 {36, 8.59277}, {37, 8.74854}, {38, 8.90623}, {39, 9.06579}, {40, 9.22718},
 {41, 9.39035}, {42, 9.55525}, {43, 9.72185}, {44, 9.89011}, {45, 10.06},
 {46, 10.2315}, {47, 10.4045}, {48, 10.5791}, {49, 10.7552}, {50, 10.9327},
 {51, 11.1118}, {52, 11.2922}, {53, 11.4741}, {54, 11.6574}, {55, 11.8421},
 {56, 12.0282}, {57, 12.2156}, {58, 12.4044}, {59, 12.5945}, {60, 12.786},
 {61, 12.9788}, {62, 13.1729}, {63, 13.3683}, {64, 13.565}, {65, 13.7629},
 {66, 13.9622}, {67, 14.1628}, {68, 14.3646}, {69, 14.5678}, {70, 14.7722},
 {71, 14.9778}, {72, 15.1848}, {73, 15.393}, {74, 15.6025}, {75, 15.8132},
 {76, 16.0252}, {77, 16.2385}, {78, 16.4531}, {79, 16.6689}, {80, 16.886},
 {81, 17.1044}, {82, 17.3241}, {83, 17.545}, {84, 17.7672}, {85, 17.9907},
 {86, 18.2155}, {87, 18.4415}, {88, 18.6689}, {89, 18.8975}, {90, 19.1275},
 {91, 19.3587}, {92, 19.5912}, {93, 19.8251}, {94, 20.0602}, {95, 20.2967},
 {96, 20.5345}, {97, 20.7736}, {98, 21.014}, {99, 21.2558}, {100, 21.4989}}
```

In[ ]:=
```
ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
    freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}],
 Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → {0, 25},
 LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
 PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

Out[ ]=



n=1 (first PSSW)

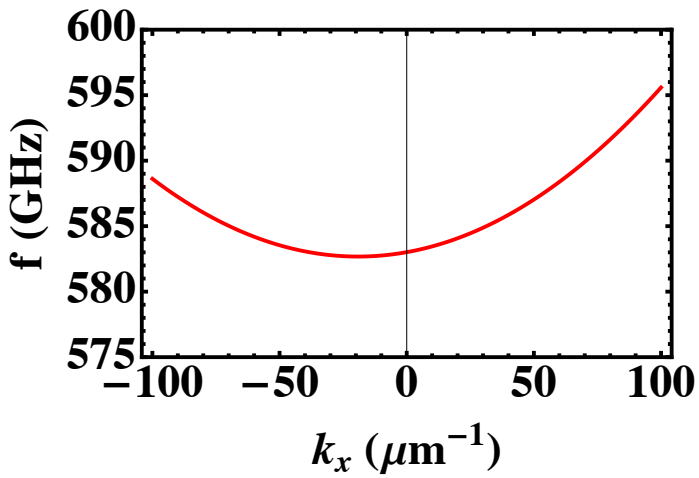
In[ ]:=
```
Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[3]] > 0, freqs2[LL, ky a, 0, 0.5][[3]],
    freqs2[LL, ky a, 0, 0.5][[4]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}]
```

Out[ ]=

```
{{-100, 588.614}, {-99, 588.468}, {-98, 588.324}, {-97, 588.182}, {-96, 588.041},
 {-95, 587.903}, {-94, 587.766}, {-93, 587.631}, {-92, 587.497}, {-91, 587.366},
 {-90, 587.236}, {-89, 587.109}, {-88, 586.983}, {-87, 586.859}, {-86, 586.736},
 {-85, 586.616}, {-84, 586.497}, {-83, 586.38}, {-82, 586.265}, {-81, 586.152},
 {-80, 586.041}, {-79, 585.931}, {-78, 585.823}, {-77, 585.717}, {-76, 585.613},
 {-75, 585.511}, {-74, 585.41}, {-73, 585.311}, {-72, 585.215}, {-71, 585.12},
 {-70, 585.026}, {-69, 584.935}, {-68, 584.845}, {-67, 584.757}, {-66, 584.671},
 {-65, 584.587}, {-64, 584.505}, {-63, 584.424}, {-62, 584.346}, {-61, 584.269},
 {-60, 584.194}, {-59, 584.121}, {-58, 584.049}, {-57, 583.979}, {-56, 583.912},
 {-55, 583.846}, {-54, 583.782}, {-53, 583.719}, {-52, 583.659}, {-51, 583.6},
 {-50, 583.543}, {-49, 583.488}, {-48, 583.435}, {-47, 583.383}, {-46, 583.334},
 {-45, 583.286}, {-44, 583.24}, {-43, 583.196}, {-42, 583.153}, {-41, 583.113},
 {-40, 583.074}, {-39, 583.037}, {-38, 583.002}, {-37, 582.969}, {-36, 582.937},
 {-35, 582.907}, {-34, 582.88}, {-33, 582.854}, {-32, 582.829}, {-31, 582.807},
 {-30, 582.787}, {-29, 582.768}, {-28, 582.751}, {-27, 582.736}, {-26, 582.722},
 {-25, 582.711}, {-24, 582.701}, {-23, 582.693}, {-22, 582.687}, {-21, 582.683},
 {-20, 582.681}, {-19, 582.68}, {-18, 582.682}, {-17, 582.685}, {-16, 582.689},
 {-15, 582.696}, {-14, 582.705}, {-13, 582.715}, {-12, 582.727}, {-11, 582.741},
 {-10, 582.757}, {-9, 582.774}, {-8, 582.794}, {-7, 582.815}, {-6, 582.838},
 {-5, 582.863}, {-4, 582.89}, {-3, 582.918}, {-2, 582.949}, {-1, 582.981},
 {0, 583.015}, {1, 583.05}, {2, 583.088}, {3, 583.127}, {4, 583.169}, {5, 583.212},
 {6, 583.257}, {7, 583.303}, {8, 583.352}, {9, 583.402}, {10, 583.454},
 {11, 583.508}, {12, 583.564}, {13, 583.621}, {14, 583.681}, {15, 583.742},
 {16, 583.805}, {17, 583.87}, {18, 583.937}, {19, 584.005}, {20, 584.075},
 {21, 584.147}, {22, 584.221}, {23, 584.297}, {24, 584.375}, {25, 584.454},
 {26, 584.535}, {27, 584.618}, {28, 584.703}, {29, 584.79}, {30, 584.878},
 {31, 584.968}, {32, 585.061}, {33, 585.154}, {34, 585.25}, {35, 585.348},
 {36, 585.447}, {37, 585.548}, {38, 585.651}, {39, 585.756}, {40, 585.863},
 {41, 585.971}, {42, 586.081}, {43, 586.194}, {44, 586.307}, {45, 586.423},
 {46, 586.541}, {47, 586.66}, {48, 586.781}, {49, 586.904}, {50, 587.029},
 {51, 587.156}, {52, 587.284}, {53, 587.414}, {54, 587.546}, {55, 587.68},
 {56, 587.816}, {57, 587.954}, {58, 588.093}, {59, 588.234}, {60, 588.377},
 {61, 588.522}, {62, 588.668}, {63, 588.817}, {64, 588.967}, {65, 589.119},
 {66, 589.273}, {67, 589.429}, {68, 589.586}, {69, 589.746}, {70, 589.907},
 {71, 590.07}, {72, 590.234}, {73, 590.401}, {74, 590.569}, {75, 590.74},
 {76, 590.912}, {77, 591.086}, {78, 591.261}, {79, 591.439}, {80, 591.618},
 {81, 591.799}, {82, 591.982}, {83, 592.167}, {84, 592.353}, {85, 592.542},
 {86, 592.732}, {87, 592.924}, {88, 593.118}, {89, 593.313}, {90, 593.511},
 {91, 593.71}, {92, 593.911}, {93, 594.114}, {94, 594.319}, {95, 594.526},
 {96, 594.734}, {97, 594.944}, {98, 595.156}, {99, 595.37}, {100, 595.586}}
```

In[•]:=
```
ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[3]] > 0, freqs2[LL, ky a, 0, 0.5][[3]],
    freqs2[LL, ky a, 0, 0.5][[4]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}],
 Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → {575, 600},
 LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
 PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

Out[•]=



n=2 (second PSSW)

In[•]:=
```
Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[5]] > 0, freqs2[LL, ky a, 0, 0.5][[5]],
    freqs2[LL, ky a, 0, 0.5][[6]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}]
```

Out[ ]=

```
{{-100, 2270.}, {-99, 2269.85}, {-98, 2269.71}, {-97, 2269.56}, {-96, 2269.42},
 {-95, 2269.28}, {-94, 2269.15}, {-93, 2269.01}, {-92, 2268.88}, {-91, 2268.74},
 {-90, 2268.61}, {-89, 2268.48}, {-88, 2268.36}, {-87, 2268.23}, {-86, 2268.11},
 {-85, 2267.99}, {-84, 2267.87}, {-83, 2267.75}, {-82, 2267.63}, {-81, 2267.52},
 {-80, 2267.41}, {-79, 2267.3}, {-78, 2267.19}, {-77, 2267.08}, {-76, 2266.97},
 {-75, 2266.87}, {-74, 2266.77}, {-73, 2266.67}, {-72, 2266.57}, {-71, 2266.48},
 {-70, 2266.38}, {-69, 2266.29}, {-68, 2266.2}, {-67, 2266.11}, {-66, 2266.02},
 {-65, 2265.94}, {-64, 2265.85}, {-63, 2265.77}, {-62, 2265.69}, {-61, 2265.62},
 {-60, 2265.54}, {-59, 2265.46}, {-58, 2265.39}, {-57, 2265.32}, {-56, 2265.25},
 {-55, 2265.19}, {-54, 2265.12}, {-53, 2265.06}, {-52, 2265.}, {-51, 2264.94},
 {-50, 2264.88}, {-49, 2264.82}, {-48, 2264.77}, {-47, 2264.72}, {-46, 2264.66},
 {-45, 2264.62}, {-44, 2264.57}, {-43, 2264.52}, {-42, 2264.48}, {-41, 2264.44},
 {-40, 2264.4}, {-39, 2264.36}, {-38, 2264.32}, {-37, 2264.29}, {-36, 2264.26},
 {-35, 2264.23}, {-34, 2264.2}, {-33, 2264.17}, {-32, 2264.15}, {-31, 2264.12},
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```

In[ ]:=
```
ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[5]] > 0, freqs2[LL, ky a, 0, 0.5][[5]],
    freqs2[LL, ky a, 0, 0.5][[6]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}],
 Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → {2260, 2285},
 LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
 PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

Out[ ]=