In this notebook, a bilayer is considered.
The exchange in the bottom half is larger than on the top half.

We apply the SAME DMI strength on the top and bottom, to show that the shift in the frequencies are not equal.
All the raw data is on the bottom. Figure 8 of the article is produced at the end.

The Eigenfrequencies -- Fe -- D = 3.9 mJ/m^2. **TOP**

Some parameters needed for the code

In[•]:=
```
B = 0.03;
Δ = 1;
Φ[0] = Table[0, {4}];
```

In[•]:=
```
LL = 20;
```

In[•]:=
```
a = 0.248 × 10^-9; (*0.25 nm atomic spacing*)
Ms = 1 750 000; (*A/m*)
AA = 1.88 × 10^-11; (*J/m*)
DD = 3.9 × 10^-3 × 1/1;
(* i.e. This is 4.2 mJ/m^2 for 1 layer and decreases for thicker films!*)
JJ = 2 AA / (a^2 Ms)
```

Out[•]=
```
349.338
```

In[•]:=
```
K[i_] := K[i] = Which[i < LL/2 + 0.5, 0, i > LL/2 + 0.5, 0]
J[i_] := J[i] = Which[i < LL/2 + 0.5, JJ, i > LL/2 + 0.5, 4 JJ]
H[i_] := H[i] = Which[i < LL/2 + 0.5, B, i > LL/2 + 0.5, B]
HDMI[1] = 2 DD / (a Ms);
HDMI[LL] = 0
```

Out[•]=
```
0
```

In[•]:=
```
ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```
In[•]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
          H[i] Cos[ϕ[i]] + (4 π 10^-7) Ms + 2 K[i] (Cos[ϕ[i]]^2 - Sin[ϕ[i]]^2) + J[i]
            (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
        aplus[i_] := aplus[i] = -J[i] Cos[ϕ[i] - ϕ[i + 1]]
        aminus[i_] := aminus[i] = -J[i] Cos[ϕ[i] - ϕ[i - 1]]
        bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[ϕ[i]] - 2 K[i] Cos[ϕ[i]]^2 -
           J[i] (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]

In[•]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
          {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
        rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
          {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]
```

The 1st, (N/2)th, (N/2 +1)th and Nth planes all need individual codes sinse they have different exchange coupling to the planes on either side.
The codes are as follows:

```
In[•]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
          {-HDMI[1] Sin[y] i,
           H[1] Cos[ϕ[1]] + (4 π 10^-7) Ms + 2 K[1] (Cos[ϕ[1]]^2 - Sin[ϕ[1]]^2) +
            J[1] (0 + Cos[ϕ[1] - ϕ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
           0,
           aplus[1]},
          Table[0, {2 NN - 4}]];

In[•]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
          {-H[1] Cos[ϕ[1]] - 2 K[1] Cos[ϕ[1]]^2 -
            J[1] (0 + Cos[ϕ[1] - ϕ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
           -HDMI[1] Sin[y] i,
           J[2]},
          Table[0, {2 NN - 3}]];
```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

```
In[•]:= arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
          Table[0, {NN - 3}],
          {aminus[NN / 2],
           0,
           H[NN / 2] Cos[ϕ[NN / 2]] + (4 π 10^-7) Ms + 2 K[NN / 2] (Cos[ϕ[NN / 2]]^2 - Sin[ϕ[NN / 2]]^2) +
            J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
           0,
           -(J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]]},
          Table[0, {NN - 2}]]

In[•]:= brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
          Table[0, {NN - 4}],
          {J[NN / 2],
           0,
           -H[NN / 2] Cos[ϕ[NN / 2]] - 2 K[NN / 2] Cos[ϕ[NN / 2]]^2 -
            J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
           0,
           J[NN] + β (J[1] - J[NN])},
          Table[0, {NN - 1}]]

In[•]:= arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
          Table[0, {NN - 1}],
          {-(J[NN / 2] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]],
           0,
           H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] + (4 π 10^-7) Ms +
            2 K[NN / 2 + 1] (Cos[ϕ[NN / 2 + 1]]^2 - Sin[ϕ[NN / 2 + 1]]^2) + (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] + J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] +
            4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
           0,
           aplus[NN / 2 + 1]},
          Table[0, {NN - 4}]]
```

```
brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
    Table[0, {NN - 2}],
    {J[NN] + β (J[1] - J[NN]),
     0,
     -H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]]^2 -
      (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] -
      J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
      2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
     0,
     J[NN / 2 + 1]},
    Table[0, {NN - 3}]]
```

```
arow100[NN_, y_, z_] := Join[
    Table[0, {2 NN - 3}],
    {aminus[NN],
     -HDMI[NN] Sin[y] i,
     H[NN] Cos[ϕ[NN]] + (4 π 10^-7) Ms + 2 K[NN] (Cos[ϕ[NN]]^2 - Sin[ϕ[NN]]^2) +
      J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];
```

```
brow100[NN_, y_, z_] := Join[
    Table[0, {2 NN - 4}],
    {J[NN - 1],
     0,
     -H[NN] Cos[ϕ[NN]] - 2 K[NN] Cos[ϕ[NN]]^2 -
      J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
     -HDMI[NN] Sin[y] i}];
```

The dynamical matrix and eigenfrequencies

The dynamical matrix is:

```
big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
    {arow1[NN, y, z], brow1[NN, y, z]},
    Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
    {arow50[NN, y, z, β], brow50[NN, y, z, β],
     arow51[NN, y, z, β], brow51[NN, y, z, β]},
    Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
    {arow100[NN, y, z], brow100[NN, y, z]}]
```

The eigenfrequencies are given by (γ = 176 GHz rad/T):

```
freqs[NN_, y_, z_, β_] := freqs[NN, y, z, β] =
  176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 2}]
```

```
In[•]:= freqs2[NN_, y_, z_, β_] := freqs2[NN, y, z, β] =
          176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 1}]
```

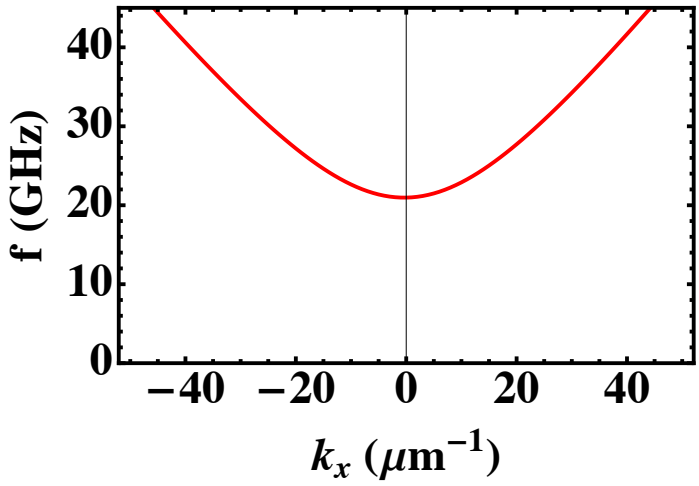
Dispersion plots

```
In[•]:= freqs2[LL, 100 × 10 a, 0, 0.5]
```

```
Out[•]=
{-20.9668, 20.9668, -476.439, 476.439, -2068.47, 2068.47, -4844.35, 4844.35,
 -6623.59, 6623.59, -10 767.4, 10 767.4, -15 472.7, 15 472.7, -18 491.3, 18 491.3,
 -23 359.9, 23 359.9, -28 566.6, 28 566.6, -32 331.4, 32 331.4, -35 309.2, 35 309.2,
 -38 058.7, 38 058.7, -43 351.7, 43 351.7, 60 179., -60 179., 82 442., -82 442.,
 -105 248., 105 248., 125 937., -125 937., -142 386., 142 386., -152 955., 152 955.}
```

```
In[•]:= ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
          freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -50 × 10^6, 50 × 10^6, 1 × 10^6}],
        Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → {0, 45},
        LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
        PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

```
Out[•]=
```



In[•]:=
```
Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
    freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -50 × 10^6, 50 × 10^6, 1 × 10^6}]
```

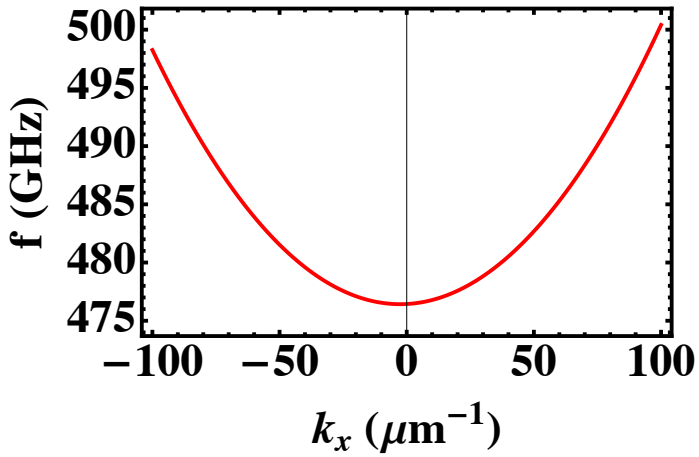
Out[•]=
```
{{-50, 48.2429}, {-49, 47.4648}, {-48, 46.6894}, {-47, 45.9169}, {-46, 45.1474},
 {-45, 44.3812}, {-44, 43.6184}, {-43, 42.8593}, {-42, 42.104}, {-41, 41.3528},
 {-40, 40.606}, {-39, 39.8639}, {-38, 39.1267}, {-37, 38.3947}, {-36, 37.6684},
 {-35, 36.9479}, {-34, 36.2338}, {-33, 35.5264}, {-32, 34.8262}, {-31, 34.1336},
 {-30, 33.4491}, {-29, 32.7733}, {-28, 32.1068}, {-27, 31.4501}, {-26, 30.8039},
 {-25, 30.169}, {-24, 29.5461}, {-23, 28.936}, {-22, 28.3395}, {-21, 27.7576},
 {-20, 27.1912}, {-19, 26.6413}, {-18, 26.1091}, {-17, 25.5957}, {-16, 25.1022},
 {-15, 24.6299}, {-14, 24.1801}, {-13, 23.7541}, {-12, 23.3533}, {-11, 22.979},
 {-10, 22.6326}, {-9, 22.3154}, {-8, 22.0288}, {-7, 21.774}, {-6, 21.5522},
 {-5, 21.3646}, {-4, 21.212}, {-3, 21.0954}, {-2, 21.0153}, {-1, 20.9724},
 {0, 20.9668}, {1, 20.9987}, {2, 21.068}, {3, 21.1744}, {4, 21.3174}, {5, 21.4963},
 {6, 21.7103}, {7, 21.9585}, {8, 22.2396}, {9, 22.5526}, {10, 22.8962},
 {11, 23.269}, {12, 23.6698}, {13, 24.0971}, {14, 24.5495}, {15, 25.0258},
 {16, 25.5245}, {17, 26.0445}, {18, 26.5845}, {19, 27.1433}, {20, 27.7197},
 {21, 28.3127}, {22, 28.9213}, {23, 29.5444}, {24, 30.1813}, {25, 30.8309},
 {26, 31.4926}, {27, 32.1655}, {28, 32.849}, {29, 33.5424}, {30, 34.2451},
 {31, 34.9565}, {32, 35.6761}, {33, 36.4033}, {34, 37.1378}, {35, 37.879},
 {36, 38.6266}, {37, 39.3802}, {38, 40.1394}, {39, 40.9038}, {40, 41.6733},
 {41, 42.4475}, {42, 43.2261}, {43, 44.0088}, {44, 44.7955}, {45, 45.5859},
 {46, 46.3798}, {47, 47.177}, {48, 47.9772}, {49, 48.7805}, {50, 49.5865}}
```

```
In[•]:= Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0,
     freqs2[LL, ky a, 0, 0.5][[1]], freqs2[LL, ky a, 0, 0.5][[2]]] +
    If[freqs2[LL, ky a, 0, 0.5][[1]] < 0, freqs2[LL, ky a, 0, 0.5][[1]],
     freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -50 × 10^6, 50 × 10^6, 1 × 10^6}]
```

```
Out[•]=
{{-50, -1.34355}, {-49, -1.31566}, {-48, -1.28784}, {-47, -1.26007},
 {-46, -1.23236}, {-45, -1.20471}, {-44, -1.17712}, {-43, -1.14958},
 {-42, -1.12209}, {-41, -1.09466}, {-40, -1.06727}, {-39, -1.03994},
 {-38, -1.01266}, {-37, -0.985424}, {-36, -0.958235}, {-35, -0.931092},
 {-34, -0.903993}, {-33, -0.876937}, {-32, -0.849922}, {-31, -0.822948},
 {-30, -0.796014}, {-29, -0.769117}, {-28, -0.742257}, {-27, -0.715432},
 {-26, -0.688642}, {-25, -0.661885}, {-24, -0.63516}, {-23, -0.608465},
 {-22, -0.581799}, {-21, -0.555162}, {-20, -0.528551}, {-19, -0.501965},
 {-18, -0.475404}, {-17, -0.448866}, {-16, -0.42235}, {-15, -0.395854},
 {-14, -0.369377}, {-13, -0.342918}, {-12, -0.316476}, {-11, -0.290049},
 {-10, -0.263636}, {-9, -0.237236}, {-8, -0.210847}, {-7, -0.184469},
 {-6, -0.1581}, {-5, -0.131738}, {-4, -0.105383}, {-3, -0.0790325},
 {-2, -0.0526862}, {-1, -0.0263425}, {0, 2.13163 × 10^-14}, {1, 0.0263425},
 {2, 0.0526862}, {3, 0.0790325}, {4, 0.105383}, {5, 0.131738}, {6, 0.1581},
 {7, 0.184469}, {8, 0.210847}, {9, 0.237236}, {10, 0.263636}, {11, 0.290049},
 {12, 0.316476}, {13, 0.342918}, {14, 0.369377}, {15, 0.395854},
 {16, 0.42235}, {17, 0.448866}, {18, 0.475404}, {19, 0.501965}, {20, 0.528551},
 {21, 0.555162}, {22, 0.581799}, {23, 0.608465}, {24, 0.63516}, {25, 0.661885},
 {26, 0.688642}, {27, 0.715432}, {28, 0.742257}, {29, 0.769117}, {30, 0.796014},
 {31, 0.822948}, {32, 0.849922}, {33, 0.876937}, {34, 0.903993}, {35, 0.931092},
 {36, 0.958235}, {37, 0.985424}, {38, 1.01266}, {39, 1.03994}, {40, 1.06727},
 {41, 1.09466}, {42, 1.12209}, {43, 1.14958}, {44, 1.17712}, {45, 1.20471},
 {46, 1.23236}, {47, 1.26007}, {48, 1.28784}, {49, 1.31566}, {50, 1.34355}}
```

In[ ]:=
```
ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[3]] > 0, freqs2[LL, ky a, 0, 0.5][[3]],
    freqs2[LL, ky a, 0, 0.5][[4]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}],
 Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → All,
 LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
 PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

Out[ ]=



In[ ]:=
```
Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[3]] > 0, freqs2[LL, ky a, 0, 0.5][[3]],
    freqs2[LL, ky a, 0, 0.5][[4]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}]
```

Out[ ]=

{{-100, 498.252}, {-99, 497.799}, {-98, 497.352}, {-97, 496.91}, {-96, 496.473},
{-95, 496.04}, {-94, 495.613}, {-93, 495.19}, {-92, 494.772}, {-91, 494.359},
{-90, 493.95}, {-89, 493.547}, {-88, 493.148}, {-87, 492.755}, {-86, 492.366},
{-85, 491.982}, {-84, 491.602}, {-83, 491.228}, {-82, 490.858}, {-81, 490.493},
{-80, 490.133}, {-79, 489.778}, {-78, 489.427}, {-77, 489.082}, {-76, 488.741},
{-75, 488.404}, {-74, 488.073}, {-73, 487.746}, {-72, 487.424}, {-71, 487.107},
{-70, 486.794}, {-69, 486.486}, {-68, 486.183}, {-67, 485.885}, {-66, 485.591},
{-65, 485.303}, {-64, 485.018}, {-63, 484.739}, {-62, 484.464}, {-61, 484.194},
{-60, 483.929}, {-59, 483.668}, {-58, 483.412}, {-57, 483.16}, {-56, 482.914},
{-55, 482.672}, {-54, 482.434}, {-53, 482.201}, {-52, 481.973}, {-51, 481.75},
{-50, 481.531}, {-49, 481.317}, {-48, 481.107}, {-47, 480.903}, {-46, 480.702},
{-45, 480.507}, {-44, 480.316}, {-43, 480.129}, {-42, 479.948}, {-41, 479.77},
{-40, 479.598}, {-39, 479.43}, {-38, 479.266}, {-37, 479.108}, {-36, 478.954},
{-35, 478.804}, {-34, 478.659}, {-33, 478.519}, {-32, 478.383}, {-31, 478.251},
{-30, 478.125}, {-29, 478.003}, {-28, 477.885}, {-27, 477.772}, {-26, 477.664},
{-25, 477.56}, {-24, 477.46}, {-23, 477.366}, {-22, 477.276}, {-21, 477.19},
{-20, 477.109}, {-19, 477.032}, {-18, 476.96}, {-17, 476.893}, {-16, 476.83},
{-15, 476.771}, {-14, 476.718}, {-13, 476.668}, {-12, 476.623}, {-11, 476.583},
{-10, 476.548}, {-9, 476.516}, {-8, 476.49}, {-7, 476.467}, {-6, 476.45},
{-5, 476.437}, {-4, 476.428}, {-3, 476.424}, {-2, 476.425}, {-1, 476.43},
{0, 476.439}, {1, 476.453}, {2, 476.472}, {3, 476.495}, {4, 476.522}, {5, 476.554},
{6, 476.591}, {7, 476.632}, {8, 476.678}, {9, 476.728}, {10, 476.783},
{11, 476.842}, {12, 476.906}, {13, 476.974}, {14, 477.047}, {15, 477.124},
{16, 477.206}, {17, 477.292}, {18, 477.383}, {19, 477.478}, {20, 477.578},
{21, 477.682}, {22, 477.791}, {23, 477.905}, {24, 478.023}, {25, 478.145},
{26, 478.272}, {27, 478.404}, {28, 478.54}, {29, 478.68}, {30, 478.825},
{31, 478.975}, {32, 479.129}, {33, 479.288}, {34, 479.451}, {35, 479.619},
{36, 479.791}, {37, 479.968}, {38, 480.15}, {39, 480.336}, {40, 480.526},
{41, 480.721}, {42, 480.921}, {43, 481.125}, {44, 481.334}, {45, 481.547},
{46, 481.765}, {47, 481.987}, {48, 482.215}, {49, 482.446}, {50, 482.682},
{51, 482.923}, {52, 483.168}, {53, 483.418}, {54, 483.673}, {55, 483.932},
{56, 484.196}, {57, 484.464}, {58, 484.737}, {59, 485.014}, {60, 485.296},
{61, 485.583}, {62, 485.874}, {63, 486.17}, {64, 486.471}, {65, 486.776},
{66, 487.086}, {67, 487.4}, {68, 487.72}, {69, 488.043}, {70, 488.372},
{71, 488.705}, {72, 489.042}, {73, 489.385}, {74, 489.732}, {75, 490.083},
{76, 490.44}, {77, 490.801}, {78, 491.167}, {79, 491.537}, {80, 491.912},
{81, 492.292}, {82, 492.676}, {83, 493.065}, {84, 493.459}, {85, 493.858},
{86, 494.261}, {87, 494.669}, {88, 495.082}, {89, 495.499}, {90, 495.921},
{91, 496.348}, {92, 496.78}, {93, 497.216}, {94, 497.658}, {95, 498.103},
{96, 498.554}, {97, 499.01}, {98, 499.47}, {99, 499.935}, {100, 500.405}}

The Eigenfrequencies -- Fe -- D = 3.9 mJ/m^2. **BOTTOM**

Some parameters needed for the code

```
In[ ]:= B = 0.03;
        Δ = 1;
        Φ[0] = Table[0, {4}];

In[ ]:= LL = 20;

In[ ]:= a = 0.248 × 10^-9; (*0.25 nm atomic spacing*)
        Ms = 1 750 000; (*A/m*)
        AA = 1.88 × 10^-11; (*J/m*)
        DD = 3.9 × 10^-3 × 1/1;
        (* i.e. This is 4.2 mJ/m^2 for 1 layer and decreases for thicker films!*)
        JJ = 2 AA / (a^2 Ms)
Out[ ]=
        349.338

In[ ]:= K[i_] := K[i] = Which[i < LL/2 + 0.5, 0, i > LL/2 + 0.5, 0]
        J[i_] := J[i] = Which[i < LL/2 + 0.5, JJ, i > LL/2 + 0.5, 4 JJ]
        H[i_] := H[i] = Which[i < LL/2 + 0.5, B, i > LL/2 + 0.5, B]
        HDMI[1] = 0;
        HDMI[LL] = 2 DD/(a Ms);

In[ ]:= ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```
In[ ]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
          H[i] Cos[ϕ[i]] + (4 π 10^-7) Ms + 2 K[i] (Cos[ϕ[i]]^2 - Sin[ϕ[i]]^2) + J[i]
            (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
        aplus[i_] := aplus[i] = -J[i] Cos[ϕ[i] - ϕ[i + 1]]
        aminus[i_] := aminus[i] = -J[i] Cos[ϕ[i] - ϕ[i - 1]]
        bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[ϕ[i]] - 2 K[i] Cos[ϕ[i]]^2 -
            J[i] (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]
```

```
In[•]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
          {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
        rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
          {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]
```

The 1st, (N/2)th, (N/2 +1)th and Nth planes all need individual codes sinse they have different exchange coupling to the planes on either side.
The codes are as follows:

```
In[•]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
          {-HDMI[1] Sin[y] ⅈ,
           H[1] Cos[ϕ[1]] + (4 π 10^-7) Ms + 2 K[1] (Cos[ϕ[1]]^2 - Sin[ϕ[1]]^2) +
            J[1] (0 + Cos[ϕ[1] - ϕ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
           0,
           aplus[1]},
          Table[0, {2 NN - 4}]];

In[•]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
          {-H[1] Cos[ϕ[1]] - 2 K[1] Cos[ϕ[1]]^2 -
            J[1] (0 + Cos[ϕ[1] - ϕ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
           -HDMI[1] Sin[y] ⅈ,
           J[2]},
          Table[0, {2 NN - 3}]];
```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

```
In[•]:= arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
          Table[0, {NN - 3}],
          {aminus[NN / 2],
           0,
           H[NN / 2] Cos[ϕ[NN / 2]] + (4 π 10^-7) Ms + 2 K[NN / 2] (Cos[ϕ[NN / 2]]^2 - Sin[ϕ[NN / 2]]^2) +
            J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
           0,
           -(J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]]},
          Table[0, {NN - 2}]]
```

```
In[•]:= brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
          Table[0, {NN - 4}],
          {J[NN / 2],
           0,
           -H[NN / 2] Cos[ϕ[NN / 2]] - 2 K[NN / 2] Cos[ϕ[NN / 2]]^2 -
            J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
           0,
           J[NN] + β (J[1] - J[NN])},
          Table[0, {NN - 1}]]

In[•]:= arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
          Table[0, {NN - 1}],
          {-(J[NN / 2] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]],
           0,
           H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] + (4 π 10^-7) Ms +
            2 K[NN / 2 + 1] (Cos[ϕ[NN / 2 + 1]]^2 - Sin[ϕ[NN / 2 + 1]]^2) + (J[NN] + β (J[1] - J[NN]))
             Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] + J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] +
            4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
           0,
           aplus[NN / 2 + 1]},
          Table[0, {NN - 4}]]

In[•]:= brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
          Table[0, {NN - 2}],
          {J[NN] + β (J[1] - J[NN]),
           0,
           -H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]]^2 -
            (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] -
            J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
            2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
           0,
           J[NN / 2 + 1]},
          Table[0, {NN - 3}]]

In[•]:= arow100[NN_, y_, z_] := Join[
          Table[0, {2 NN - 3}],
          {aminus[NN],
           -HDMI[NN] Sin[y] i,
           H[NN] Cos[ϕ[NN]] + (4 π 10^-7) Ms + 2 K[NN] (Cos[ϕ[NN]]^2 - Sin[ϕ[NN]]^2) +
            J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];
```

```
In[•]:= brow100[NN_, y_, z_] := Join[
          Table[0, {2 NN - 4}],
          {J[NN - 1],
           0,
           -H[NN] Cos[ϕ[NN]] - 2 K[NN] Cos[ϕ[NN]]^2 -
            J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
           -HDMI[NN] Sin[y] i}];
```

The dynamical matrix and eigenfrequencies

The dynamical matrix is:

```
In[•]:= big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
          {arow1[NN, y, z], brow1[NN, y, z]},
          Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
          {arow50[NN, y, z, β], brow50[NN, y, z, β],
           arow51[NN, y, z, β], brow51[NN, y, z, β]},
          Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
          {arow100[NN, y, z], brow100[NN, y, z]}]
```

The eigenfrequencies are given by (γ = 176 GHz rad/T):

```
In[•]:= freqs[NN_, y_, z_, β_] := freqs[NN, y, z, β] =
          176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 2}]

In[•]:= freqs2[NN_, y_, z_, β_] := freqs2[NN, y, z, β] =
          176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 1}]
```

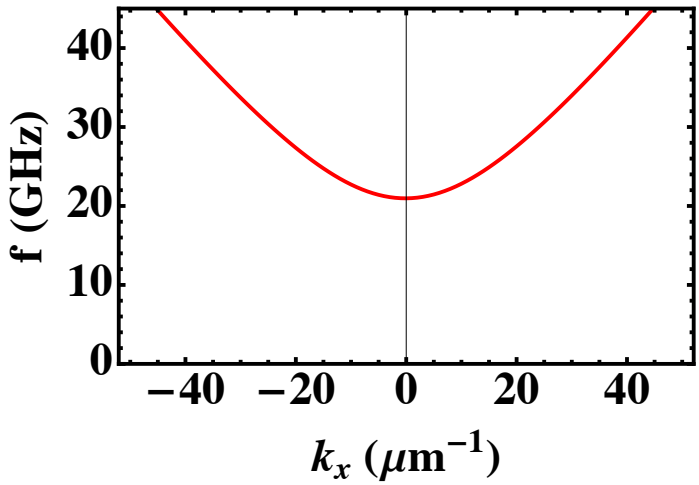
Dispersion plots

```
In[•]:= freqs2[LL, 100 × 10 a, 0, 0.5]
```

```
Out[•]=
{-20.9668, 20.9668, -476.439, 476.439, -2068.47, 2068.47, -4844.35, 4844.35,
 -6623.59, 6623.59, -10 767.4, 10 767.4, -15 472.7, 15 472.7, -18 491.3, 18 491.3,
 -23 359.9, 23 359.9, -28 566.6, 28 566.6, -32 331.4, 32 331.4, -35 309.2, 35 309.2,
 -38 058.7, 38 058.7, -43 351.7, 43 351.7, -60 179., 60 179., -82 442., 82 442.,
 -105 248., 105 248., -125 937., 125 937., -142 386., 142 386., -152 955., 152 955.}
```

In[ ]:=
```
ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
    freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -50 × 10^6, 50 × 10^6, 1 × 10^6}],
 Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → {0, 45},
 LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
 PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

Out[ ]=



```
In[•]:= Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
          freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -50 × 10^6, 50 × 10^6, 1 × 10^6}]
```

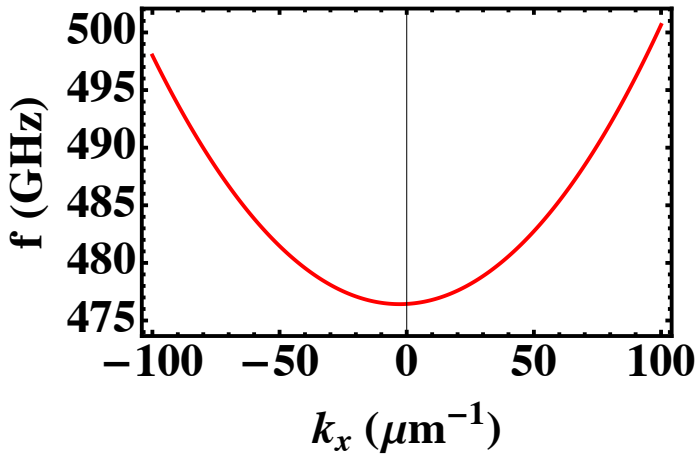
```
Out[•]=
{{-50, 48.6828}, {-49, 47.8951}, {-48, 47.1101}, {-47, 46.328}, {-46, 45.5491},
 {-45, 44.7734}, {-44, 44.0012}, {-43, 43.2327}, {-42, 42.4681}, {-41, 41.7077},
 {-40, 40.9517}, {-39, 40.2004}, {-38, 39.454}, {-37, 38.713}, {-36, 37.9775},
 {-35, 37.248}, {-34, 36.5249}, {-33, 35.8086}, {-32, 35.0994}, {-31, 34.3979},
 {-30, 33.7046}, {-29, 33.02}, {-28, 32.3446}, {-27, 31.6792}, {-26, 31.0243},
 {-25, 30.3807}, {-24, 29.7491}, {-23, 29.1303}, {-22, 28.5252}, {-21, 27.9346},
 {-20, 27.3596}, {-19, 26.8012}, {-18, 26.2605}, {-17, 25.7385}, {-16, 25.2365},
 {-15, 24.7558}, {-14, 24.2975}, {-13, 23.863}, {-12, 23.4538}, {-11, 23.071},
 {-10, 22.7162}, {-9, 22.3906}, {-8, 22.0956}, {-7, 21.8324}, {-6, 21.6023},
 {-5, 21.4063}, {-4, 21.2454}, {-3, 21.1204}, {-2, 21.032}, {-1, 20.9807},
 {0, 20.9668}, {1, 20.9904}, {2, 21.0514}, {3, 21.1494}, {4, 21.2841}, {5, 21.4547},
 {6, 21.6604}, {7, 21.9002}, {8, 22.173}, {9, 22.4777}, {10, 22.8129},
 {11, 23.1774}, {12, 23.5698}, {13, 23.9887}, {14, 24.4327}, {15, 24.9006},
 {16, 25.391}, {17, 25.9025}, {18, 26.4341}, {19, 26.9844}, {20, 27.5523},
 {21, 28.1368}, {22, 28.7369}, {23, 29.3515}, {24, 29.9797}, {25, 30.6208},
 {26, 31.2739}, {27, 31.9381}, {28, 32.613}, {29, 33.2977}, {30, 33.9916},
 {31, 34.6943}, {32, 35.4051}, {33, 36.1235}, {34, 36.8491}, {35, 37.5814},
 {36, 38.3201}, {37, 39.0647}, {38, 39.8149}, {39, 40.5703}, {40, 41.3307},
 {41, 42.0958}, {42, 42.8652}, {43, 43.6388}, {44, 44.4162}, {45, 45.1973},
 {46, 45.9819}, {47, 46.7697}, {48, 47.5606}, {49, 48.3543}, {50, 49.1508}}
```

In[•]:=
```
Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[1]] > 0,
    freqs2[LL, ky a, 0, 0.5][[1]], freqs2[LL, ky a, 0, 0.5][[2]]] +
   If[freqs2[LL, ky a, 0, 0.5][[1]] < 0, freqs2[LL, ky a, 0, 0.5][[1]],
    freqs2[LL, ky a, 0, 0.5][[2]]]}, {ky, -50 × 10^6, 50 × 10^6, 1 × 10^6}]
```

Out[•]=
```
{{-50, -0.467981}, {-49, -0.459248}, {-48, -0.450477}, {-47, -0.441669},
 {-46, -0.432826}, {-45, -0.423948}, {-44, -0.415035}, {-43, -0.406088},
 {-42, -0.397108}, {-41, -0.388096}, {-40, -0.379052}, {-39, -0.369978},
 {-38, -0.360873}, {-37, -0.351739}, {-36, -0.342576}, {-35, -0.333385},
 {-34, -0.324167}, {-33, -0.314922}, {-32, -0.305652}, {-31, -0.296356},
 {-30, -0.287037}, {-29, -0.277694}, {-28, -0.268328}, {-27, -0.25894},
 {-26, -0.249531}, {-25, -0.240102}, {-24, -0.230653}, {-23, -0.221185},
 {-22, -0.211699}, {-21, -0.202195}, {-20, -0.192675}, {-19, -0.183139},
 {-18, -0.173589}, {-17, -0.164023}, {-16, -0.154445}, {-15, -0.144854},
 {-14, -0.13525}, {-13, -0.125636}, {-12, -0.116012}, {-11, -0.106377},
 {-10, -0.0967346}, {-9, -0.0870838}, {-8, -0.0774259}, {-7, -0.0677616},
 {-6, -0.0580917}, {-5, -0.048417}, {-4, -0.0387384}, {-3, -0.0290566},
 {-2, -0.0193724}, {-1, -0.00968659}, {0, 2.13163 × 10^-14}, {1, 0.00968659},
 {2, 0.0193724}, {3, 0.0290566}, {4, 0.0387384}, {5, 0.048417}, {6, 0.0580917},
 {7, 0.0677616}, {8, 0.0774259}, {9, 0.0870838}, {10, 0.0967346},
 {11, 0.106377}, {12, 0.116012}, {13, 0.125636}, {14, 0.13525}, {15, 0.144854},
 {16, 0.154445}, {17, 0.164023}, {18, 0.173589}, {19, 0.183139}, {20, 0.192675},
 {21, 0.202195}, {22, 0.211699}, {23, 0.221185}, {24, 0.230653}, {25, 0.240102},
 {26, 0.249531}, {27, 0.25894}, {28, 0.268328}, {29, 0.277694}, {30, 0.287037},
 {31, 0.296356}, {32, 0.305652}, {33, 0.314922}, {34, 0.324167}, {35, 0.333385},
 {36, 0.342576}, {37, 0.351739}, {38, 0.360873}, {39, 0.369978}, {40, 0.379052},
 {41, 0.388096}, {42, 0.397108}, {43, 0.406088}, {44, 0.415035}, {45, 0.423948},
 {46, 0.432826}, {47, 0.441669}, {48, 0.450477}, {49, 0.459248}, {50, 0.467981}}
```

In[ ]:=
```
ListPlot[Table[{ky/10^6, If[freqs2[LL, ky a, 0, 0.5][[3]] > 0, freqs2[LL, ky a, 0, 0.5][[3]],
    freqs2[LL, ky a, 0, 0.5][[4]]]}, {ky, -100 × 10^6, 100 × 10^6, 1 × 10^6}],
 Frame → True, FrameLabel → {"k_x (μm^-1)", "f (GHz)"}, PlotRange → All,
 LabelStyle → Directive[Large, Black, Bold, FontFamily → Times], Joined → True,
 PlotStyle → Directive[Red, Thick], FrameStyle → Directive[Black, Thick]]
```

Out[ ]=



COMPARE TOP AND BOTTOM - the data

In[•]:=
```
bot0 = {{-50, 48.68280768259611`}, {-49, 47.89505935223889`},
    {-48, 47.110073254749636`}, {-47, 46.32801356341582`},
    {-46, 45.54905472559089`}, {-45, 44.773382242794106`},
    {-44, 44.00119351497889`}, {-43, 43.23269875350232`},
    {-42, 42.468121970168646`}, {-41, 41.70770204423754`},
    {-40, 40.95169387702442`}, {-39, 40.20036963747078`},
    {-38, 39.45402010538587`}, {-37, 38.71295611727694`},
    {-36, 37.97751012138618`}, {-35, 37.24803784487947`},
    {-34, 36.52492007840711`}, {-33, 35.80856457911833`},
    {-32, 35.09940809389675`}, {-31, 34.3979184998261`}, {-30, 33.70459705608631`},
    {-29, 33.01998075985965`}, {-28, 32.34464478749941`},
    {-27, 31.679205002095024`}, {-26, 31.024320494978117`},
    {-25, 30.38069611957227`}, {-24, 29.74908496589204`},
    {-23, 29.130290704460403`}, {-22, 28.525169715798246`},
    {-21, 27.934632897077716`}, {-20, 27.359647021598846`},
    {-19, 26.801235495642228`}, {-18, 26.260478338716098`},
    {-17, 25.738511187923876`}, {-16, 25.236523101969876`},
    {-15, 24.755752932392916`}, {-14, 24.297484014798478`},
    {-13, 23.863036943821726`}, {-12, 23.45376022302808`},
    {-11, 23.07101861906754`}, {-10, 22.716179135870096`},
    {-9, 22.3905946187083`}, {-8, 22.09558513805671`}, {-7, 21.832417460140743`},
    {-6, 21.60228308797822`}, {-5, 21.406275536308186`}, {-4, 21.24536767026954`},
    {-3, 21.120390061434833`}, {-2, 21.03201138699933`}, {-1, 20.980721886033063`},
    {0, 20.96682079255229`}, {1, 20.99040847650826`}, {2, 21.05138377176671`},
    {3, 21.149446648217854`}, {4, 21.284106070875538`}, {5, 21.454692566896497`},
    {6, 21.6603747694159`}, {7, 21.900179018448657`}, {8, 22.173011004944293`},
    {9, 22.4776784321969`}, {10, 22.81291374100669`}, {11, 23.17739606872729`},
    {12, 23.56977177880603`}, {13, 23.98867307701439`}, {14, 24.432734407474335`},
    {15, 24.900606478553183`}, {16, 25.390967908798032`}, {17, 25.90253457712563`},
    {18, 26.434066847949364`}, {19, 26.984374880035855`}, {20, 27.552322255362597`},
    {21, 28.136828175201458`}, {22, 28.73686845584164`}, {23, 29.351475548432933`},
    {24, 29.979737782223246`}, {25, 30.620798005173455`}, {26, 31.27385177739213`},
    {27, 31.938145241740045`}, {28, 32.612972780000646`}, {29, 33.29767453847492`},
    {30, 33.991633894219284`}, {31, 34.694274913633464`}, {32, 35.405059844985196`},
    {33, 36.12348667733382`}, {34, 36.84908678471542`}, {35, 37.581422674337276`},
    {36, 38.32008584620513`}, {37, 39.06469476997933`}, {38, 39.814892982051354`},
    {39, 40.57034730107642`}, {40, 41.33074616087664`}, {41, 42.09579805549371`},
    {42, 42.86523009345471`}, {43, 43.63878665462081`}, {44, 44.41622814470834`},
    {45, 45.1973298407841`}, {46, 45.98188082434512`}, {47, 46.769682992371756`},
    {48, 47.56055014444799`}, {49, 48.35430713858672`}, {50, 49.150789111223425`}};
```

In[•]:=
```
bot1 = {{-100, 498.0009766916759`}, {-99, 497.5544965205821`},
    {-98, 497.112819489913`}, {-97, 496.6759404521113`},
```

{-96, 496.2438542775265`}, {-95, 495.8165558555861`},
{-94, 495.3940400959019`}, {-93, 494.97630192926914`},
{-92, 494.5633363088086`}, {-91, 494.15513821101297`},
{-90, 493.7517026368197`}, {-89, 493.3530246126776`},
{-88, 492.95909919155235`}, {-87, 492.5699214540206`},
{-86, 492.1854865092888`}, {-85, 491.80578949622776`},
{-84, 491.43082558432826`}, {-83, 491.0605899748431`},
{-82, 490.6950779016694`}, {-81, 490.3342846323892`},
{-80, 489.9782054692617`}, {-79, 489.62683575017354`},
{-78, 489.2801708496696`}, {-77, 488.9382061798458`}, {-76, 488.6009371913`},
{-75, 488.26835937415126`}, {-74, 487.9404682588472`},
{-73, 487.61725941719965`}, {-72, 487.298728463233`},
{-71, 486.9848710540794`}, {-70, 486.67568289094316`},
{-69, 486.37115971984804`}, {-68, 486.07129733265697`},
{-67, 485.7760915677996`}, {-66, 485.4855383112435`},
{-65, 485.1996334972003`}, {-64, 484.91837310904504`},
{-63, 484.64175318011314`}, {-62, 484.3697697944818`},
{-61, 484.1024190877757`}, {-60, 483.8396972479469`},
{-59, 483.58160051604455`}, {-58, 483.3281251869495`},
{-57, 483.07926761016904`}, {-56, 482.8350241905136`},
{-55, 482.59539138885293`}, {-54, 482.36036572283086`},
{-53, 482.12994376753534`}, {-52, 481.90412215620813`},
{-51, 481.6828975808871`}, {-50, 481.4662667930799`},
{-49, 481.2542266044729`}, {-48, 481.04677388742294`},
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In[•]:=
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Out[•]=