DMI on top and bottom surfaces are equal here.

One may therefore expect the DMI to not change the spin wave frequencies because D_{eff} = 0. But it does!
Final data is at the end. We also played with the analytic expression for the frequency of the uniform mode here.

The Eigenfrequencies -- D==0. N=16 (restart kernel)

Some parameters needed for the code

In[∘]:=
```
B = 0.03;
Δ = 1;
Φ[0] = Table[0, {4}];
```

In[∘]:=
```
LL = 16;
```

In[∘]:=
```
a = 0.248 × 10^-9; (*0.25 nm atomic spacing*)
Ms = 835 563; (*A/m*)
AA = 1.355 × 10^-11; (*J/m*)
DD = 0.0 × 4.2 × 10^-3 × 1/1;
(* i.e. This is 4.2 mJ/m^2 for 1 layer and decreases for thicker films!*)
JJ = (2 AA)/(a^2 Ms)
```

Out[∘]=
```
527.335
```

```
K[i_] := K[i] = Which[i < LL/2 + 0.5, 0, i > LL/2 + 0.5, 0]
J[i_] := J[i] = Which[i < LL/2 + 0.5, JJ, i > LL/2 + 0.5, JJ]
H[i_] := H[i] = Which[i < LL/2 + 0.5, B, i > LL/2 + 0.5, B]
HDMI[1] = 2 DD/(a Ms);
HDMI[LL] = -2 DD/(a Ms);
```

Out[∘]=
```
0
```

In[∘]:=
```
ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```
In[•]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
         H[i] Cos[ϕ[i]] + (4 π 10^-7) Ms + 2 K[i] (Cos[ϕ[i]]^2 - Sin[ϕ[i]]^2) + J[i]
           (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
        aplus[i_] := aplus[i] = -J[i] Cos[ϕ[i] - ϕ[i + 1]]
        aminus[i_] := aminus[i] = -J[i] Cos[ϕ[i] - ϕ[i - 1]]
        bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[ϕ[i]] - 2 K[i] Cos[ϕ[i]]^2 -
          J[i] (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]
```

```
In[•]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
         {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
        rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
         {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]
```

The 1st, (N/2)th, (N/2 +1)th and Nth planes all need individual codes sinse they have different exchange coupling to the planes on either side.
The codes are as follows:

```
In[•]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
           {-HDMI[1] Sin[y] ⅈ,
            H[1] Cos[ϕ[1]] + (4 π 10^-7) Ms + 2 K[1] (Cos[ϕ[1]]^2 - Sin[ϕ[1]]^2) +
             J[1] (0 + Cos[ϕ[1] - ϕ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
            0,
            aplus[1]},
           Table[0, {2 NN - 4}]];
```

```
In[•]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
           {-H[1] Cos[ϕ[1]] - 2 K[1] Cos[ϕ[1]]^2 -
             J[1] (0 + Cos[ϕ[1] - ϕ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
            -HDMI[1] Sin[y] ⅈ,
            J[2]},
           Table[0, {2 NN - 3}]];
```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

In[∘]:=
```
arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
    Table[0, {NN - 3}],
    {aminus[NN / 2],
     0,
     H[NN / 2] Cos[ϕ[NN / 2]] + (4 π 10^-7) Ms + 2 K[NN / 2] (Cos[ϕ[NN / 2]]^2 - Sin[ϕ[NN / 2]]^2) +
      J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
        Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
     0,
     -(J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]]},
    Table[0, {NN - 2}]]
```

In[∘]:=
```
brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
    Table[0, {NN - 4}],
    {J[NN / 2],
     0,
     -H[NN / 2] Cos[ϕ[NN / 2]] - 2 K[NN / 2] Cos[ϕ[NN / 2]]^2 -
      J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
        Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
     0,
     J[NN] + β (J[1] - J[NN])},
    Table[0, {NN - 1}]]
```

In[∘]:=
```
arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
    Table[0, {NN - 1}],
    {-(J[NN / 2] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]],
     0,
     H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] + (4 π 10^-7) Ms +
      2 K[NN / 2 + 1] (Cos[ϕ[NN / 2 + 1]]^2 - Sin[ϕ[NN / 2 + 1]]^2) + (J[NN] + β (J[1] - J[NN]))
        Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] + J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] +
      4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
     0,
     aplus[NN / 2 + 1]},
    Table[0, {NN - 4}]]
```

In[ ]:=
```
brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
   Table[0, {NN - 2}],
   {J[NN] + β (J[1] - J[NN]),
    0,
    -H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]]^2 -
     (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] -
     J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
     2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
    0,
    J[NN / 2 + 1]},
   Table[0, {NN - 3}]]
```

In[ ]:=
```
arow100[NN_, y_, z_] := Join[
   Table[0, {2 NN - 3}],
   {aminus[NN],
    -HDMI[NN] Sin[y] i,
    H[NN] Cos[ϕ[NN]] + (4 π 10^-7) Ms + 2 K[NN] (Cos[ϕ[NN]]^2 - Sin[ϕ[NN]]^2) +
     J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];
```

In[ ]:=
```
brow100[NN_, y_, z_] := Join[
   Table[0, {2 NN - 4}],
   {J[NN - 1],
    0,
    -H[NN] Cos[ϕ[NN]] - 2 K[NN] Cos[ϕ[NN]]^2 -
     J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
    -HDMI[NN] Sin[y] i}];
```

The dynamical matrix and eigenfrequencies

The dynamical matrix is:

In[ ]:=
```
big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
   {arow1[NN, y, z], brow1[NN, y, z]},
   Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
   {arow50[NN, y, z, β], brow50[NN, y, z, β],
    arow51[NN, y, z, β], brow51[NN, y, z, β]},
   Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
   {arow100[NN, y, z], brow100[NN, y, z]}]
```

The eigenfrequencies are given by (γ = 176 GHz rad/T):

In[ ]:=
```
freqs[NN_, y_, z_, β_] := freqs[NN, y, z, β] =
  176/(2. π) Table[Reverse[Chop[i Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 2}]
```

In[ ]:=
```
freqs2[NN_, y_, z_, β_] := freqs2[NN, y, z, β] =
  176/(2. π) Table[Reverse[Chop[ⅈ Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 1}]
```

The Eigenfrequencies -- D = 9 mJ/m^2 on top and bottom. N=16 (restart kernel)

Some parameters needed for the code

In[ ]:=
```
B = 0.03;
Δ = 1;
Φ[0] = Table[0, {4}];
```

In[ ]:=
```
LL = 16;
```

In[ ]:=
```
a = 0.248 × 10^-9; (*0.25 nm atomic spacing*)
Ms = 835 563; (*A/m*)
AA = 1.355 × 10^-11; (*J/m*)
DD = 9 × 10^-3 × 1/1;
(* i.e. This is 4.2 mJ/m^2 for 1 layer and decreases for thicker films!*)
JJ = 2 AA/(a^2 Ms)
```

Out[ ]=
```
527.335
```

In[ ]:=
```
K[i_] := K[i] = Which[i < LL/2 + 0.5, 0, i > LL/2 + 0.5, 0]
J[i_] := J[i] = Which[i < LL/2 + 0.5, JJ, i > LL/2 + 0.5, JJ]
H[i_] := H[i] = Which[i < LL/2 + 0.5, B, i > LL/2 + 0.5, B]
HDMI[1] = 2 DD/(a Ms);
HDMI[LL] = -2 DD/(a Ms)
```

Out[ ]=
```
-86.8644
```

In[ ]:=
```
ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```
In[∘]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
          H[i] Cos[ϕ[i]] + (4 π 10^-7) Ms + 2 K[i] (Cos[ϕ[i]]^2 - Sin[ϕ[i]]^2) + J[i]
            (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
        aplus[i_] := aplus[i] = -J[i] Cos[ϕ[i] - ϕ[i + 1]]
        aminus[i_] := aminus[i] = -J[i] Cos[ϕ[i] - ϕ[i - 1]]
        bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[ϕ[i]] - 2 K[i] Cos[ϕ[i]]^2 -
           J[i] (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]
```

```
In[∘]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
          {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
        rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
          {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]
```

The 1st, (N/2)th, (N/2 +1)th and Nth planes all need individual codes sinse they have different exchange coupling to the planes on either side.
The codes are as follows:

```
In[∘]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
            {-HDMI[1] Sin[y] ⅈ,
             H[1] Cos[ϕ[1]] + (4 π 10^-7) Ms + 2 K[1] (Cos[ϕ[1]]^2 - Sin[ϕ[1]]^2) +
              J[1] (0 + Cos[ϕ[1] - ϕ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
             0,
             aplus[1]},
            Table[0, {2 NN - 4}]];
```

```
In[∘]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
            {-H[1] Cos[ϕ[1]] - 2 K[1] Cos[ϕ[1]]^2 -
              J[1] (0 + Cos[ϕ[1] - ϕ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
             -HDMI[1] Sin[y] ⅈ,
             J[2]},
            Table[0, {2 NN - 3}]];
```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

In[ ]:=
```
arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
    Table[0, {NN - 3}],
    {aminus[NN / 2],
     0,
     H[NN / 2] Cos[ϕ[NN / 2]] + (4 π 10^-7) Ms + 2 K[NN / 2] (Cos[ϕ[NN / 2]]^2 - Sin[ϕ[NN / 2]]^2) +
      J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
       Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
     0,
     - (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]]},
    Table[0, {NN - 2}]]
```

In[ ]:=
```
brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
    Table[0, {NN - 4}],
    {J[NN / 2],
     0,
     -H[NN / 2] Cos[ϕ[NN / 2]] - 2 K[NN / 2] Cos[ϕ[NN / 2]]^2 -
      J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
       Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
     0,
     J[NN] + β (J[1] - J[NN])},
    Table[0, {NN - 1}]]
```

In[ ]:=
```
arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
    Table[0, {NN - 1}],
    {- (J[NN / 2] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]],
     0,
     H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] + (4 π 10^-7) Ms +
      2 K[NN / 2 + 1] (Cos[ϕ[NN / 2 + 1]]^2 - Sin[ϕ[NN / 2 + 1]]^2) + (J[NN] + β (J[1] - J[NN]))
       Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] + J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] +
      4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
     0,
     aplus[NN / 2 + 1]},
    Table[0, {NN - 4}]]
```

```
In[∘]:= brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
    Table[0, {NN - 2}],
    {J[NN] + β (J[1] - J[NN]),
     0,
     -H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]]^2 -
      (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] -
      J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
      2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
     0,
     J[NN / 2 + 1]},
    Table[0, {NN - 3}]]
```

```
In[∘]:= arow100[NN_, y_, z_] := Join[
    Table[0, {2 NN - 3}],
    {aminus[NN],
     -HDMI[NN] Sin[y] ⅈ,
     H[NN] Cos[ϕ[NN]] + (4 π 10^-7) Ms + 2 K[NN] (Cos[ϕ[NN]]^2 - Sin[ϕ[NN]]^2) +
      J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];
```

```
In[∘]:= brow100[NN_, y_, z_] := Join[
    Table[0, {2 NN - 4}],
    {J[NN - 1],
     0,
     -H[NN] Cos[ϕ[NN]] - 2 K[NN] Cos[ϕ[NN]]^2 -
      J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
     -HDMI[NN] Sin[y] ⅈ}];
```

The dynamical matrix and eigenfrequencies

The dynamical matrix is:

```
In[∘]:= big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
    {arow1[NN, y, z], brow1[NN, y, z]},
    Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
    {arow50[NN, y, z, β], brow50[NN, y, z, β],
     arow51[NN, y, z, β], brow51[NN, y, z, β]},
    Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
    {arow100[NN, y, z], brow100[NN, y, z]}]
```

The eigenfrequencies are given by (γ = 176 GHz rad/T):

```
In[∘]:= freqs[NN_, y_, z_, β_] := freqs[NN, y, z, β] =
    176/(2. π) Table[Reverse[Chop[ⅈ Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 2}]
```

```
In[∘]:= freqs2[NN_, y_, z_, β_] := freqs2[NN, y, z, β] =
          176
         ───── Table[Reverse[Chop[ⅈ Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 1}]
         2. π
```

Analytic versus numerical - taken from “240913_TwoLayerAnalyticFrequency.nb”

We derived an analytic formula that works well for the uniform mode and small DMI constants:

```
In[∘]:= mu0 = 4. π 10^-7;
        AAA = 1.355 × 10^-11; (*J/m*)
        Msat = 835 563; (*A/m*)
        NN  = 16;
        aa = 0.248 × 10^-9; (* m *)
        BB = 0.03; (* T *)
```

```
In[∘]:= dI[DI_, kx_] := dI[DI, kx] = (2 DI)/(aa NN Msat) 2 Sin[kx aa]
        dII[DII_, kx_] := dII[DII, kx] = (2 DII)/(aa NN Msat) 2 Sin[kx aa]
        B1[kx_, kz_] :=
         B1[kx, kz] = (2 AAA)/(aa^2 Msat) (4 - 2 Cos[kx aa] - 2 Cos[kz aa]) + (2 AAA)/(aa^2 Msat (NN / 2)^2) + BB
        B2[kx_, kz_] :=
         B2[kx, kz] = (2 AAA)/(aa^2 Msat) (4 - 2 Cos[kx aa] - 2 Cos[kz aa]) + (2 AAA)/(aa^2 Msat (NN / 2)^2) + BB + mu0 Msat
        BexY = (2 AAA)/(aa^2 Msat (NN / 2)^2);
```

```
In[∘]:= freq2[DI_, DII_, kx_] :=
         176/(2. π) 1/2 (-dI[DI, kx] + dII[DII, kx] + √(4 B1[kx, 0] × B2[kx, 0] + 4 BexY^2 +
              (dI[DI, kx] + dII[DII, kx])^2 - 4 √(B1[kx, 0]^2 BexY^2 + B2[kx, 0]^2 BexY^2 +
                 B1[kx, 0] × B2[kx, 0] (2 BexY^2 + (dI[DI, kx] + dII[DII, kx])^2))))
```

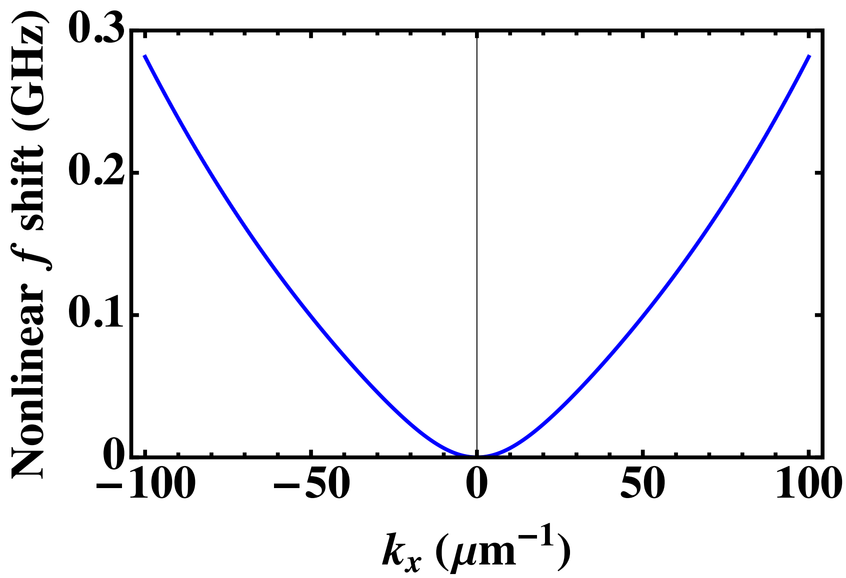
```
In[∘]:= freq4[DI_, DII_, kx_] :=
         176/(2. π) 1/2 (-dI[DI, kx] + dII[DII, kx] + √(4 B1[kx, 0] × B2[kx, 0] + 4 BexY^2 +
              (dI[DI, kx] + dII[DII, kx])^2 + 4 √(B1[kx, 0]^2 BexY^2 + B2[kx, 0]^2 BexY^2 +
                 B1[kx, 0] × B2[kx, 0] (2 BexY^2 + (dI[DI, kx] + dII[DII, kx])^2))))
```

The following plot is not appropriate as the analytic method gives results which are DIFFERENT from those from the numerical atomic layer method, at the resolution we are now looking at. (On the GHz level they match... not so much here.)

Actually... maybe it is appropriate... but ONLY for comparing to DMI=0 where the analytic is spot on.

In[ ]:=
```
ListPlot[Table[{ky/10^6, freq2[0, 0, ky] - (If[freqs2[LL, ky a, 0, 0.5][[1]] > 0,
      freqs2[LL, ky a, 0, 0.5][[1]], freqs2[LL, ky a, 0, 0.5][[2]]) },
   {ky, -100×10^6, 100×10^6, 1×10^6}], Frame → True, FrameLabel →
   {"kx (μm^-1)", "Nonlinear f shift (GHz)"}, PlotRange → {-0.0, 0.3},
  LabelStyle → Directive[Large, Black, Bold, FontFamily → Times],
  Joined → True, PlotStyle → Directive[Blue, Thick],
  FrameStyle → Directive[Black, Thick],
  FrameTicks → {{ {0, 0.1, 0.2, 0.3}, {{0, ""}, {0.1, ""}, {0.2, ""}, {0.3, ""}}},
    {Automatic, Automatic}}]
```

Out[ ]=



In[ ]:=
```
Table[{ky/10^6, (If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
      freqs2[LL, ky a, 0, 0.5][[2]]) }, {ky, -100×10^6, 100×10^6, 20×10^6}]
```

Out[ ]=
```
{{-100, 19.4773}, {-80, 15.2935}, {-60, 11.6098},
 {-40, 8.45701}, {-20, 6.04736}, {0, 5.04203}, {20, 6.04736},
 {40, 8.45701}, {60, 11.6098}, {80, 15.2935}, {100, 19.4773}}
```

In[ ]:=
```
Table[{ky/10^6, (If[freqs2[LL, ky a, 0, 0.5][[1]] > 0, freqs2[LL, ky a, 0, 0.5][[1]],
      freqs2[LL, ky a, 0, 0.5][[2]]) }, {ky, -100×10^6, 100×10^6, 1×10^6}]
```

Out[ ]=

```
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```

```
In[•]:= If[freqs2[LL, 100×10^6 a, 0, 0.5]〚1〛 > 0,
          freqs2[LL, 100×10^6 a, 0, 0.5]〚1〛, freqs2[LL, 100×10^6 a, 0, 0.5]〚2〛]
Out[•]=
        19.4773
```

```
In[•]:= ListPlot[Table[{ky/10^6, freq2[0, 0, ky] - (If[freqs2[LL, ky a, 0, 0.5]〚1〛 > 0,
              freqs2[LL, ky a, 0, 0.5]〚1〛, freqs2[LL, ky a, 0, 0.5]〚2〛])},
          {ky, -100×10^6, 100×10^6, 1×10^6}], Frame → True, FrameLabel →
          {"kx (μm^-1)", "f (D)-f (0)  (GHz)"}, PlotRange → {-0.0, 0.3},
         LabelStyle → Directive[Large, Black, Bold, FontFamily → Times],
         Joined → True, PlotStyle → Directive[Blue, Thick],
         FrameStyle → Directive[Black, Thick],
         FrameTicks → {{ {0, 0.1, 0.2, 0.3}, {{0, ""}, {0.1, ""}, {0.2, ""}, {0.3, ""}}},
           {Automatic, Automatic}}]
Out[•]=
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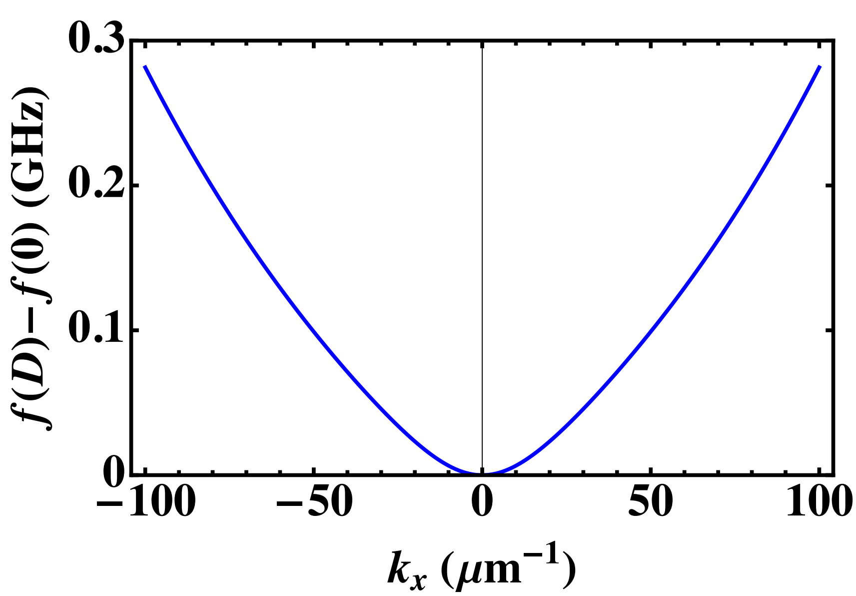

Compare D=0 and D=9 mJ/m^2 by running the code above for different values (Figure 6)

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           {-90, 17.56029984477575`}, {-89, 17.347668624288907`},
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In[ ]:=
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  Frame → True, FrameLabel → {"kx (rad/μm)", "f(D)-f(0) (GHz)"},
  PlotRange → {-0.31, 0.01},
  LabelStyle → Directive[Large, Black, Bold, FontFamily → Times],
  Joined → True, PlotStyle → Directive[Blue, Thickness[0.02]],
  FrameStyle → Directive[Black, Thick], FrameTicks → {{ {0, -0.1, -0.2, -0.3},
     {{0, ""}, {-0.1, ""}, {-0.2, ""}, {-0.3, ""}}}, {Automatic, Automatic}}]
```

Out[ ]=