

Let's look at how the quasi-uniform mode's dispersion is affected by DMI.  
The analytic expression by Moon et al will be put on top of the data.

DMI on TOP only!  
Single material (NiFe).

## The Eigenfrequencies

Some parameters needed for the code

```
In[*]:= B = 0.03;
        Δ = 1;
        ϕ[0] = Table[0, {4}];

In[*]:= LL = 4;

In[*]:= a = 0.248 × 10-9; (*0.25 nm atomic spacing*)
        Ms = 835 563; (*A/m*)
        AA = 1.355 × 10-11; (*J/m*)
        DD = 4.2 × 10-3 ×  $\frac{1}{1}$ ;
        (* i.e. This is 4.2 mJ/m2 for 1 layer and decreases for thicker films!*)
        JJ =  $\frac{2 AA}{a^2 Ms}$ 

Out[*]:=
527.335

In[*]:= K[i_] := K[i] = Which[i <  $\frac{LL}{2} + 0.5$ , 0, i >  $\frac{LL}{2} + 0.5$ , 0]

        J[i_] := J[i] = Which[i <  $\frac{LL}{2} + 0.5$ , JJ, i >  $\frac{LL}{2} + 0.5$ , JJ]

        H[i_] := H[i] = Which[i <  $\frac{LL}{2} + 0.5$ , B, i >  $\frac{LL}{2} + 0.5$ , B]

        HDMI[1] = 2  $\frac{DD}{a Ms}$ ;
        HDMI[4] = 0

Out[*]:=
0

In[*]:= ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```

In[*]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
  H[i] Cos[φ[i]] + (4 π 10-7) Ms + 2 K[i] (Cos[φ[i]]2 - Sin[φ[i]]2) + J[i]
  (Cos[φ[i] - φ[i - 1]] + Cos[φ[i] - φ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
aplus[i_] := aplus[i] = -J[i] Cos[φ[i] - φ[i + 1]]
aminus[i_] := aminus[i] = -J[i] Cos[φ[i] - φ[i - 1]]
bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[φ[i]] - 2 K[i] Cos[φ[i]]2 -
  J[i] (Cos[φ[i] - φ[i - 1]] + Cos[φ[i] - φ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]

In[*]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
  {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
  {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]

```

The 1st, (N/2)th, (N/2 + 1)th and Nth planes all need individual codes since they have different exchange coupling to the planes on either side.

The codes are as follows:

```

In[*]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
  {-HDMI[1] Sin[y]  $\frac{1}{2}$ ,
  H[1] Cos[φ[1]] + (4 π 10-7) Ms + 2 K[1] (Cos[φ[1]]2 - Sin[φ[1]]2) +
  J[1] (0 + Cos[φ[1] - φ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
  0,
  aplus[1]},
  Table[0, {2 NN - 4}]];

In[*]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
  {-H[1] Cos[φ[1]] - 2 K[1] Cos[φ[1]]2 -
  J[1] (0 + Cos[φ[1] - φ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
  -HDMI[1] Sin[y]  $\frac{1}{2}$ ,
  J[2]},
  Table[0, {2 NN - 3}]];

```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

```

In[*]:= arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
  Table[0, {NN - 3}],
  {aminus[NN / 2],
  0,
  H[NN / 2] Cos[φ[NN / 2]] + (4 π 10-7) Ms + 2 K[NN / 2] (Cos[φ[NN / 2]]2 - Sin[φ[NN / 2]]2) +
  J[NN / 2] Cos[φ[NN / 2] - φ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
  Cos[φ[NN / 2] - φ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
  0,
  - (J[NN] + β (J[1] - J[NN])) Cos[φ[NN / 2] - φ[NN / 2 + 1]]},
  Table[0, {NN - 2}]]

```

```

In[*]:= brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
  Table[0, {NN - 4}],
  {J[NN / 2],
  0,
  -H[NN / 2] Cos[φ[NN / 2]] - 2 K[NN / 2] Cos[φ[NN / 2]]2 -
  J[NN / 2] Cos[φ[NN / 2] - φ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
  Cos[φ[NN / 2] - φ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
  0,
  J[NN] + β (J[1] - J[NN])},
  Table[0, {NN - 1}]]

```

```

In[*]:= arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
  Table[0, {NN - 1}],
  {- (J[NN / 2] + β (J[1] - J[NN])) Cos[φ[NN / 2 + 1] - φ[NN / 2]],
  0,
  H[NN / 2 + 1] Cos[φ[NN / 2 + 1]] + (4 π 10-7) Ms +
  2 K[NN / 2 + 1] (Cos[φ[NN / 2 + 1]]2 - Sin[φ[NN / 2 + 1]]2) + (J[NN] + β (J[1] - J[NN]))
  Cos[φ[NN / 2 + 1] - φ[NN / 2]] + J[NN / 2 + 1] Cos[φ[NN / 2 + 1] - φ[NN / 2 + 2]] +
  4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
  0,
  aplus[NN / 2 + 1]},
  Table[0, {NN - 4}]]

```

```

In[*]:= brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
  Table[0, {NN - 2}],
  {J[NN] + β (J[1] - J[NN])},
  0,
  -H[NN / 2 + 1] Cos[φ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[φ[NN / 2 + 1]]2 -
  (J[NN] + β (J[1] - J[NN])) Cos[φ[NN / 2 + 1] - φ[NN / 2]] -
  J[NN / 2 + 1] Cos[φ[NN / 2 + 1] - φ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
  2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
  0,
  J[NN / 2 + 1]},
  Table[0, {NN - 3}]]

In[*]:= arow100[NN_, y_, z_] := Join[
  Table[0, {2 NN - 3}],
  {aminus[NN],
  -HDMI[NN] Sin[y] i,
  H[NN] Cos[φ[NN]] + (4 π 10-7) Ms + 2 K[NN] (Cos[φ[NN]]2 - Sin[φ[NN]]2) +
  J[NN] (Cos[φ[NN] - φ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];

In[*]:= brow100[NN_, y_, z_] := Join[
  Table[0, {2 NN - 4}],
  {J[NN - 1],
  0,
  -H[NN] Cos[φ[NN]] - 2 K[NN] Cos[φ[NN]]2 -
  J[NN] (Cos[φ[NN] - φ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
  -HDMI[NN] Sin[y] i}];

```

## The dynamical matrix and eigenfrequencies

The dynamical matrix is:

```

In[*]:= big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
  {arow1[NN, y, z], brow1[NN, y, z]},
  Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
  {arow50[NN, y, z, β], brow50[NN, y, z, β],
  arow51[NN, y, z, β], brow51[NN, y, z, β]},
  Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
  {arow100[NN, y, z], brow100[NN, y, z]}]

In[*]:= Eigenvalues[big[4, 0.1, 0, 0.5]]

Out[*]:=
{-5.68434 × 10-14 - 1806.56 i, 2.27374 × 10-13 + 1805.96 i,
5.70391 × 10-14 - 1061.51 i, 4.57801 × 10-13 + 1059.49 i, 1.3211 × 10-14 - 316.459 i,
-2.40424 × 10-17 + 313.005 i, 1.00582 × 10-14 - 6.80518 i, 8.88174 × 10-15 + 4.78167 i}

```

The eigenfrequencies are given by ( $\gamma = 176$  GHz rad/T):

```

In[*]:= freqs[NN_, y_, z_,  $\beta$ _] := freqs[NN, y, z,  $\beta$ ] =

$$\frac{176}{2 \cdot \pi} \text{Table}[\text{Reverse}[\text{Chop}[\text{Eigenvalues}[\text{big}[NN, y, z, \beta]]]] \llbracket k \rrbracket, \{k, 1, 2 NN, 2\}]$$


In[*]:= freqs2[NN_, y_, z_,  $\beta$ _] := freqs2[NN, y, z,  $\beta$ ] =

$$\frac{176}{2 \cdot \pi} \text{Table}[\text{Reverse}[\text{Chop}[\text{Eigenvalues}[\text{big}[NN, y, z, \beta]]]] \llbracket k \rrbracket, \{k, 1, 2 NN, 1\}]$$

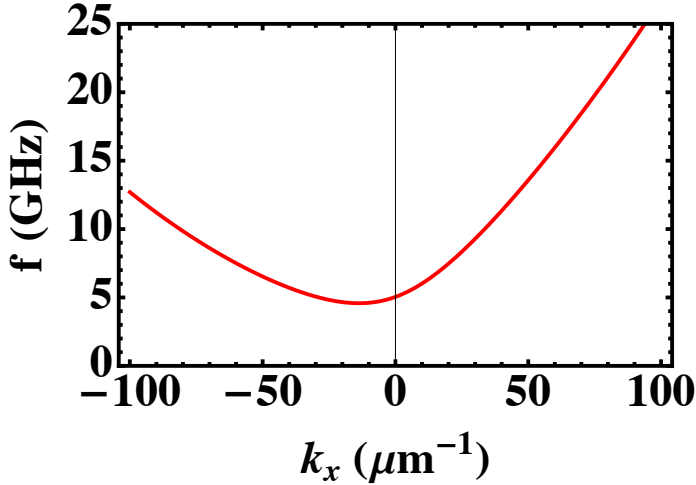

In[*]:= ListPlot[Table[ $\left\{ \frac{ky}{10^6}, \text{If}[\text{freqs2}[4, ky, a, 0, 0.5] \llbracket 1 \rrbracket > 0, \text{freqs2}[4, ky, a, 0, 0.5] \llbracket 1 \rrbracket, \right.$ 

$$\left. \text{freqs2}[4, ky, a, 0, 0.5] \llbracket 2 \rrbracket \right\}, \{ky, -100 \times 10^6, 100 \times 10^6, 1 \times 10^6\}],$$

Frame  $\rightarrow$  True, FrameLabel  $\rightarrow$  {" $k_x$  ( $\mu\text{m}^{-1}$ )", "f (GHz)"}, PlotRange  $\rightarrow$  {0, 25},
LabelStyle  $\rightarrow$  Directive[Large, Black, Bold, FontFamily  $\rightarrow$  Times], Joined  $\rightarrow$  True,
PlotStyle  $\rightarrow$  Directive[Red, Thick], FrameStyle  $\rightarrow$  Directive[Black, Thick]]

```

Out[\*] =



```

In[*]:= withDMI = {{-100, 12.704878253588868`}, {-99, 12.549679859861902`},
{-98, 12.395814815648043`}, {-97, 12.24327914721948`},
{-96, 12.092068934656018`}, {-95, 11.942180317836938`},
{-94, 11.793609502895011`}, {-93, 11.64635276889349`},
{-92, 11.50040647511958`}, {-91, 11.355767068580942`},
{-90, 11.212431092180948`}, {-89, 11.070395193239914`},
{-88, 10.929656132502656`}, {-87, 10.79021079390226`},
{-86, 10.652056194614001`}, {-85, 10.515189496049004`},
{-84, 10.379608015154519`}, {-83, 10.245309236755567`},
{-82, 10.112290826371847`}, {-81, 9.98055064399792`},
{-80, 9.850086758600419`}, {-79, 9.720897463624857`}, {-78, 9.59298129337003`},
{-77, 9.466337040370984`}, {-76, 9.340963773924`}, {-75, 9.216860859732149`},
{-74, 9.094027980707757`}, {-73, 8.972465159181798`},
{-72, 8.852172780434652`}, {-71, 8.733151617723149`},

```

```

{-70, 8.615402858873798`}, {-69, 8.498928134559046`},
{-68, 8.383729548394976`}, {-67, 8.269809708849046`},
{-66, 8.157171763290748`}, {-65, 8.045819434158895`},
{-64, 7.935757057390806`}, {-63, 7.826989623402514`},
{-62, 7.719522820588895`}, {-61, 7.613363081654602`},
{-60, 7.508517632867335`}, {-59, 7.404994546565838`},
{-58, 7.302802796818844`}, {-57, 7.20195231882442`}, {-56, 7.102454071908027`},
{-55, 7.004320106719225`}, {-54, 6.907563636332647`},
{-53, 6.812199112169584`}, {-52, 6.718242304284643`},
{-51, 6.625710386836316`}, {-50, 6.534622028586616`},
{-49, 6.444997488950504`}, {-48, 6.356858719580604`},
{-47, 6.2702294720397`}, {-46, 6.185135411307907`}, {-45, 6.101604235767747`},
{-44, 6.019665803482027`}, {-43, 5.939352265046507`},
{-42, 5.86069820289722`}, {-41, 5.783740777131579`}, {-40, 5.708519877735167`},
{-39, 5.635078282696119`}, {-38, 5.5634618220336005`},
{-37, 5.493719546673753`}, {-36, 5.4259039017653246`},
{-35, 5.360070903083551`}, {-34, 5.296280315511819`},
{-33, 5.234595831608407`}, {-32, 5.175085248350408`},
{-31, 5.117820639563471`}, {-30, 5.062878520884082`},
{-29, 5.010340003689603`}, {-28, 4.960290934003544`},
{-27, 4.912822010917046`}, {-26, 4.868028879425384`},
{-25, 4.82601219111913`}, {-24, 4.78687762525819`}, {-23, 4.750735862653864`},
{-22, 4.7177025035397575`}, {-21, 4.687897920204295`},
{-20, 4.661447034734035`}, {-19, 4.638479011921789`},
{-18, 4.619126857895889`}, {-17, 4.6035269148535765`},
{-16, 4.591818244086933`}, {-15, 4.58414189075401`}, {-14, 4.580640025873519`},
{-13, 4.581454964951905`}, {-12, 4.586728065298865`},
{-11, 4.596598509346653`}, {-10, 4.611201985986214`},
{-9, 4.6306692878008935`}, {-8, 4.655124847150824`}, {-7, 4.684685240110783`},
{-6, 4.719457692327537`}, {-5, 4.759538624229568`}, {-4, 4.805012276526547`},
{-3, 4.855949457266307`}, {-2, 4.912406450868137`}, {-1, 4.974424125908307`},
{0, 5.042027273532291`}, {1, 5.115224200524187`}, {2, 5.194006591574462`},
{3, 5.278349647012182`}, {4, 5.368212489735653`}, {5, 5.463538826800133`},
{6, 5.564257841632358`}, {7, 5.670285284997212`}, {8, 5.781524727940789`},
{9, 5.897868936290881`}, {10, 6.019201325447296`}, {11, 6.1453974545244625`},
{12, 6.2763265224136395`}, {13, 6.411852831698451`}, {14, 6.5518371914212405`},
{15, 6.696138235746905`}, {16, 6.844613640643603`}, {17, 6.99712122656725`},
{18, 7.153519939834387`}, {19, 7.313670710627577`}, {20, 7.477437188224208`},
{21, 7.644686357970584`}, {22, 7.815289046548551`}, {23, 7.98912032334621`},
{24, 8.166059807549809`}, {25, 8.345991890400411`}, {26, 8.528805882561423`},
{27, 8.71439609624764`}, {28, 8.9026618713432`}, {29, 9.093507554327552`},
{30, 9.286842437584303`}, {31, 9.482580666564685`}, {32, 9.680641121366111`},
{33, 9.880947277826866`}, {34, 10.083427053596123`}, {35, 10.288012643171554`},
{36, 10.494640345469724`}, {37, 10.703250387082093`}, {38, 10.913786743708283`},

```

```
{39, 11.1261969616744`}, {40, 11.34043198152922`}, {41, 11.556445964728459`},
{42, 11.77419612475893`}, {43, 11.99364256310997`}, {44, 12.214748111159151`},
{45, 12.43747817794539`}, {46, 12.661800604347967`}, {47, 12.887685523779092`},
{48, 13.115105229331254`}, {49, 13.344034047499422`}, {50, 13.574448218195903`},
{51, 13.806325781243173`}, {52, 14.03964646870142`}, {53, 14.274391603283744`},
{54, 14.51054400230685`}, {55, 14.748087887191325`}, {56, 14.987008797991127`},
{57, 15.22729351310685`}, {58, 15.468929973364222`}, {59, 15.711907210913054`},
{60, 15.956215282030634`}, {61, 16.20184520412356`}, {62, 16.448788896328495`},
{63, 16.6970391238531`}, {64, 16.946589445468195`}, {65, 17.19743416425437`},
{66, 17.44956828127106`}, {67, 17.702987452056476`}, {68, 17.95768794564736`},
{69, 18.213666606149793`}, {70, 18.47092081657192`}, {71, 18.729448464773316`},
{72, 18.989247911557214`}, {73, 19.25031796057279`}, {74, 19.51265783003897`},
{75, 19.77626712615113`}, {76, 20.041145818054108`}, {77, 20.307294214311415`},
{78, 20.574712940695832`}, {79, 20.843402919387024`}, {80, 21.113365349325854`},
{81, 21.384601687689518`}, {82, 21.65711363250853`}, {83, 21.930903106292252`},
{84, 22.20597224052226`}, {85, 22.482323361154968`}, {86, 22.75995897484152`},
{87, 23.03888175611082`}, {88, 23.319094535028054`}, {89, 23.600600285894178`},
{90, 23.88340211625241`}, {91, 24.167503256834355`}, {92, 24.452907051796004`},
{93, 24.739616949710513`}, {94, 25.02763649504661`}, {95, 25.316969319993692`},
{96, 25.607619136965027`}, {97, 25.899589731304474`}, {98, 26.192884954609372`},
{99, 26.48750871827662`}, {100, 26.783464987510786`}};
```

The following data was generated by re-running the program with the DMI strength set to zero:

```
In[*]:= noDMI = {{-100, 19.758783577662204`}, {-99, 19.532974592708243`},
{-98, 19.30850051700697`}, {-97, 19.085357370190305`},
{-96, 18.86354122544225`}, {-95, 18.643048215528193`},
{-94, 18.42387453921581`}, {-93, 18.206016467983655`},
{-92, 17.989470353275056`}, {-91, 17.774232634014087`},
{-90, 17.56029984477764`}, {-89, 17.347668624288335`},
{-88, 17.136335724480983`}, {-87, 16.92629802019321`},
{-86, 16.71755251926165`}, {-85, 16.510096373507974`},
{-84, 16.30392689002905`}, {-83, 16.099041543544235`},
{-82, 15.895437989210008`}, {-81, 15.693114076411625`},
{-80, 15.492067863226044`}, {-79, 15.292297631994153`},
{-78, 15.093801905627311`}, {-77, 14.896579465031662`},
{-76, 14.700629367626265`}, {-75, 14.505950967008566`},
{-74, 14.312543933730524`}, {-73, 14.120408277546874`},
{-72, 13.929544370929205`}, {-71, 13.739952974118967`},
{-70, 13.55163526171524`}, {-69, 13.364592850965904`},
{-68, 13.17882783187742`}, {-67, 12.994342799177266`},
{-66, 12.811140886290266`}, {-65, 12.629225801626953`},
{-64, 12.448601866966575`}, {-63, 12.269274058497455`},
```

```

{-62, 12.091248050304484`}, {-61, 11.91453026079999`},
{-60, 11.739127901947901`}, {-59, 11.565049031835366`},
{-58, 11.392302610395468`}, {-57, 11.220898558833406`},
{-56, 11.050847822676621`}, {-55, 10.882162439060881`},
{-54, 10.714855607922456`}, {-53, 10.548941767901194`},
{-52, 10.384436676841442`}, {-51, 10.22135749726364`},
{-50, 10.059722887057278`}, {-49, 9.899553095612381`},
{-48, 9.740870065527846`}, {-47, 9.583697540460317`},
{-46, 9.428061178833957`}, {-45, 9.273988673947873`}, {-44, 9.12150988055767`},
{-43, 8.97065694787662`}, {-42, 8.821464459176896`}, {-41, 8.673969577960037`},
{-40, 8.528212200379231`}, {-39, 8.384235113843998`},
{-38, 8.242084161341301`}, {-37, 8.101808410760016`},
{-36, 7.963460328657206`}, {-35, 7.827095957245708`}, {-34, 7.69277509333588`},
{-33, 7.560561467484032`}, {-32, 7.4305229214749735`},
{-31, 7.302731581328182`}, {-30, 7.177264023046039`},
{-29, 7.0542014273112965`}, {-28, 6.933629719095189`},
{-27, 6.815639687000045`}, {-26, 6.700327076882444`},
{-25, 6.587792653350304`}, {-24, 6.478142221717242`},
{-23, 6.371486602523424`}, {-22, 6.2679415502448546`},
{-21, 6.1676276063717035`}, {-20, 6.070669877708433`},
{-19, 5.9771977294578695`}, {-18, 5.887344383909062`},
{-17, 5.801246414870573`}, {-16, 5.71904313009359`}, {-15, 5.640875835038567`},
{-14, 5.566886973430798`}, {-13, 5.497219143944921`}, {-12, 5.43201399506022`},
{-11, 5.3714110054289845`}, {-10, 5.3155461616654085`},
{-9, 5.264550551737675`}, {-8, 5.2185488964550135`}, {-7, 5.177658048899456`},
{-6, 5.141985495163297`}, {-5, 5.111627894778227`}, {-4, 5.086669701102253`},
{-3, 5.067181904175333`}, {-2, 5.0532209353827`}, {-1, 5.044827772101612`},
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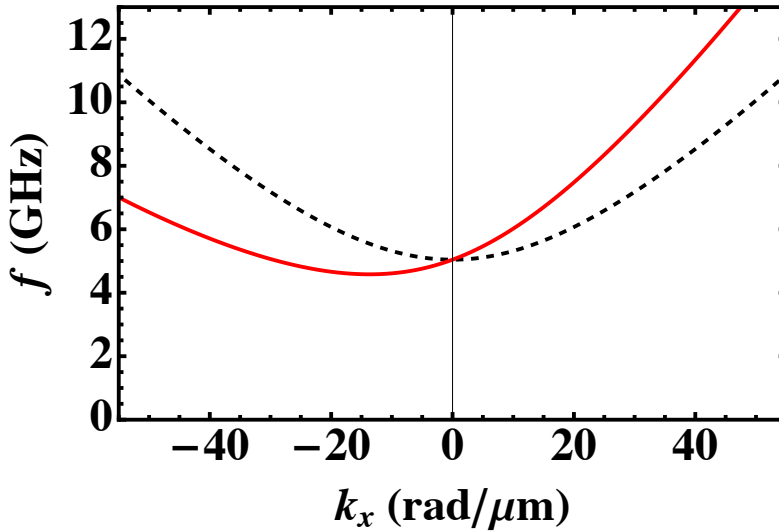
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```
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  Joined → True, PlotStyle → {Directive[Black, Dashed], Directive[Red, Thick]},
  FrameStyle → Directive[Black, Thick]]
```

Out[ ]:=



Now, the analytic expression by Moon gives:

```
In[*]:=  $\omega_{\text{Moon}}[\text{kx\_}] :=$ 

$$\frac{176}{2. \pi} \left( \text{Sqrt}[(\text{H}[1] + \text{J}[1] (2 - 2 \text{Cos}[\text{kx} \text{a}])) (\text{H}[1] + 1.05 + \text{J}[1] (2 - 2 \text{Cos}[\text{kx} \text{a}]))] + \right.$$


$$\left. 0.0 \frac{\text{HDMI}[1]}{4} \text{Sin}[\text{kx} \text{a}] \right)$$

```

Which in the long wavelength limit is:

```
In[*]:=  $\omega_{\text{MoonApprox}}[\text{kx\_}] :=$ 

$$\frac{176}{2. \pi} (\text{Sqrt}[(\text{H}[1] + \text{J}[1] (\text{kx}^2 \text{a}^2)) (\text{H}[1] + 1.05 + \text{J}[1] (\text{kx}^2 \text{a}^2))] + 0.0 \text{HDMI}[1] \text{kx} \text{a})$$

```

```
In[*]:=
```

```
In[*]:= Table[ $\left\{ \frac{\text{kx}}{10^6}, \omega_{\text{Moon}}[\text{kx}] \right\}$ , {kx,  $-100 \times 10^6$ ,  $100 \times 10^6$ ,  $0.5 \times 10^6$ }]
```

```
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```

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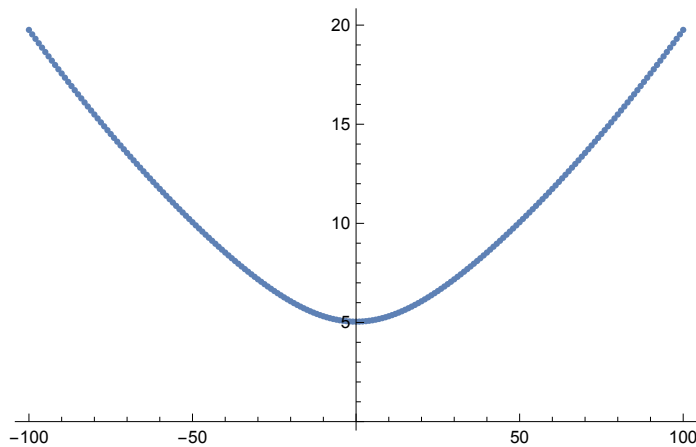
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```
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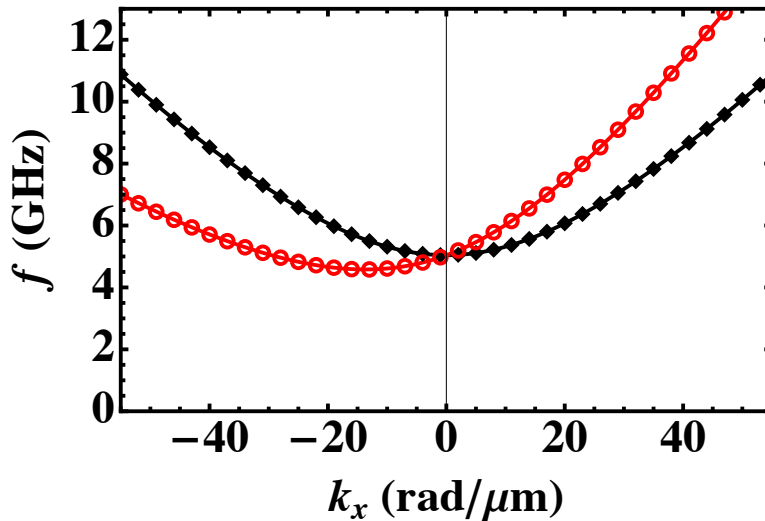
Now putting it all together (analytic by Moon as lines, and numerical results here as dots):

```

In[ ]:= fig2 = ListPlot[
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  Joined → {True, True, False, False},
  PlotStyle → {Directive[Black, Medium], Directive[Red, Thick],
    Directive[Black, PointSize[0.019]], Directive[Red, PointSize[0.019]]},
  FrameStyle → Directive[Black, Thick], PlotMarkers →
    {None, None, Automatic, Graphics[{Red, Thick, Circle[]}, ImageSize → 10]}]

```

Out[ ]:=



```

In[ ]:= Export["/Users/karen/Desktop/UON stuff/Students/Ellen/Spin
  waves DMI/Fig2_dispersion.jpg", fig2, ImageResolution → 1000]

```

Out[ ]:=

/Users/karen/Desktop/UON stuff/Students/Ellen/Spin waves DMI/Fig2\_dispersion.jpg

A great match!

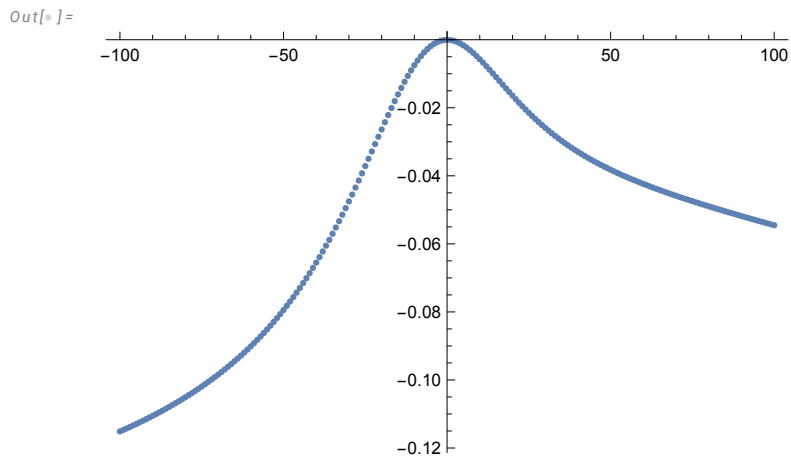
Export to ppt to add labels.

Looking at the percentage difference:

```

In[ ]:= ListPlot[Table[{withDMI[[i]][1],  $\frac{\text{withDMI}[[i]][2] - \text{withDMIanalytic}[[2\,i - 1]][2]]{\text{withDMI}[[i]][2]} 100$ },
  {i, 1, Length[withDMI]}]]

```



There is less than 0.15% difference between the analytic and numeric result, as discussed in the article, due to relaxation of the uniform mode assumption.