

In this notebook, the data for Figure 7 is generated.

This is for two ferromagnetic materials joined together, with the DMI being on the top, bottom or both interfaces. We consider only DMI on the top and bottom of a Ir/Fe/Co/Pt stack.

```
In [1]: 1 import numpy as np
        2 import matplotlib.pyplot as plt
        3 from numpy.linalg import eig
        4 from matplotlib.ticker import MaxNLocator
        5 from matplotlib.ticker import StrMethodFormatter
        6 from scipy.optimize import curve_fit
        7 import sympy as sp
        8 from sympy import expand, symbols
```

In [2]:

```

1 def MatrixM(H,A,Ms,a,gamma,laynum,D,k):
2     A1,A2,A12=A
3     gamma1,gamma2=gamma
4     a1,a2=a
5     M_s1,M_s2=Ms
6     N1,N2=laynum
7     D1,D2=D
8     kx,kz=k
9     SW=np.zeros((2*(N1+N2),2*(N1+N2)),dtype=np.complex128)
10    nn=np.linspace(0,N1+N2-1,N1+N2)
11    mu_0=4*np.pi*1e-7
12    a_ave=(a1+a2)/2
13    B_zee=mu_0*H
14    B_dem1=mu_0*M_s1
15    B_dem2=mu_0*M_s2
16    B_DM1=2*D1/(a1*M_s1)
17    B_DM2=2*D2/(a2*M_s2)
18    B_ex1=2*A1/((a1**2)*M_s1)
19    B_ex2=2*A2/((a2**2)*M_s2)
20    B_ex12_1=2*A12/((a_ave**2)*M_s1)
21    B_ex12_2=2*A12/((a_ave**2)*M_s2)
22    k_ex1=2*( np.cos(a1*kx)+np.cos(a1*kz) )
23    k_ex2=2*( np.cos(a2*kx)+np.cos(a2*kz) )
24    k_DM1=np.sin(a1*kx)
25    k_DM2=-np.sin(a2*kx)
26    iG1=-1j*gamma1
27    iG2=-1j*gamma2
28    for nn in range(0,1):
29        SW[0][0]=(1j*B_DM1*k_DM1)*iG1
30        SW[0][1]=(B_zee+B_ex1*(5-k_ex1)+B_dem1)*iG1
31        SW[0][3]=(-B_ex1)*iG1
32        SW[1][0]=( -(B_zee+B_ex1*(5-k_ex1)) )*iG1
33        SW[1][1]=( 1j*B_DM1*k_DM1 )*iG1
34        SW[1][2]=B_ex1*iG1
35        for nn in range(1,N1-1):
36            SW[2*nn][2*(nn-1)+1]=(-B_ex1)*iG1
37            SW[2*nn][2*nn+1]=(B_zee+B_ex1*(6-k_ex1)+B_dem1)*iG1
38            SW[2*nn][2*(nn+1)+1]=(-B_ex1)*iG1
39            SW[2*nn+1][2*(nn-1)]=B_ex1*iG1
40            SW[2*nn+1][2*nn]=( -(B_zee+B_ex1*(6-k_ex1)) )*iG1

```

```

41     SW[2*nn+1][2*(nn+1)]=B_ex1*iG1
42     for nn in range(N1-1,N1):
43         SW[2*nn][2*(nn-1)+1]=(-B_ex1)*iG1
44         SW[2*nn][2*nn+1]=(B_zee+B_ex1*(5-k_ex1)+B_dem1+B_ex12_1)*iG1
45         SW[2*nn][2*(nn+1)+1]=(-B_ex12_1)*iG1
46         SW[2*nn+1][2*(nn-1)]=B_ex1*iG1
47         SW[2*nn+1][2*nn]=(-(B_zee+B_ex1*(5-k_ex1)+B_ex12_1))*iG1
48         SW[2*nn+1][2*(nn+1)]=B_ex12_1*iG1
49         for nn in range(N1,N1+1):
50             SW[2*nn][2*(nn-1)+1]=(-B_ex12_2)*iG2
51             SW[2*nn][2*nn+1]=(B_zee+B_ex2*(5-k_ex2)+B_dem2+B_ex12_2)*iG2
52             SW[2*nn][2*(nn+1)+1]=(-B_ex2)*iG2
53             SW[2*nn+1][2*(nn-1)]=B_ex12_2*iG2
54             SW[2*nn+1][2*nn]=(-(B_zee+B_ex2*(5-k_ex2)+B_ex12_2))*iG2
55             SW[2*nn+1][2*(nn+1)]=B_ex2*iG2
56             for nn in range(N1+1,N1+N2-1):
57                 SW[2*nn][2*(nn-1)+1]=(-B_ex2)*iG2
58                 SW[2*nn][2*nn+1]=(B_zee+B_ex2*(6-k_ex2)+B_dem2)*iG2
59                 SW[2*nn][2*(nn+1)+1]=(-B_ex2)*iG2
60                 SW[2*nn+1][2*(nn-1)]=B_ex2*iG2
61                 SW[2*nn+1][2*nn]=(-(B_zee+B_ex2*(6-k_ex2)))*iG2
62                 SW[2*nn+1][2*(nn+1)]=B_ex2*iG2
63                 for nn in range(N1+N2-1,N1+N2):
64                     SW[2*nn][2*(nn-1)+1]=(-B_ex2)*iG2
65                     SW[2*nn][2*nn]=1j*B_DM2*k_DM2*iG2
66                     SW[2*nn][2*nn+1]=(B_zee+B_ex2*(5-k_ex2)+B_dem2)*iG2
67                     SW[2*nn+1][2*(nn-1)]=B_ex2*iG2
68                     SW[2*nn+1][2*nn]=(-(B_zee+B_ex2*(5-k_ex2)))*iG2
69                     SW[2*nn+1][2*nn+1]=1j*B_DM2*k_DM2*iG2
70     return SW
71 # Frequency -----
72 def FreqM(H,A,Ms,a,gamma,laynum,D,k,n):
73     A1,A2,A12=A
74     gamma1,gamma2=gamma
75     a1,a2=a
76     M_s1,M_s2=Ms
77     N1,N2=laynum
78     D1,D2=D
79     kx,kz=k
80     N1,N2=laynum
81     SW=MatrixM(H,A,Ms,a,gamma,laynum,D,k)

```

```

82     omegaGHz=[]
83     w,v=eig(SW)
84     omegaRaw = w
85     idx1 = np.argsort(omegaRaw)
86     omegaRaw = omegaRaw[idx1]
87     omega=np.zeros(N1+N2, dtype = 'complex_')
88     for ii in range (0,N1+N2):
89         omega[ii]=omegaRaw[ii+N1+N2]
90         omegaGHz=omega/1e9
91     return omegaGHz[n].real
92 # dispersion curve -----
93 def SWfreqkxM(H,A,Ms,a,gamma,laynum,D,MaxRe,n):
94     A1,A2,A12=A
95     gamma1,gamma2=gamma
96     a1,a2=a
97     M_s1,M_s2=Ms
98     N1,N2=laynum
99     D1,D2=D
100    N1,N2=laynum
101    Max,h=MaxRe
102    omega1=np.zeros(int(2*Max/(h*1e6)+1))
103    kx=np.linspace(-Max,Max, int(2*Max/(h*1e6)+1) )
104    for ii in range (0, int(2*Max/(h*1e6)+1) ):
105        omega1[ii]=FreqM(H,A,Ms,a,gamma,laynum,D,(kx[ii],0),n)
106    return omega1
107 def xSWfreqkxM(MaxRe):
108     Max,h=MaxRe
109     Max0=Max/1e6
110     return np.linspace(-Max0,Max0,int(2*Max0/h+1))
111 # Get eigenvector of SW matrix to obtain the mode profile -----
112 def ModAmplXM(H,A,Ms,a,gamma,laynum,D,k,n):
113     A1,A2,A12=A
114     gamma1,gamma2=gamma
115     a1,a2=a
116     M_s1,M_s2=Ms
117     N1,N2=laynum
118     D1,D2=D
119     kx,kz=k
120     N1,N2=laynum
121     SW=MatrixM(H,A,Ms,a,gamma,laynum,D,k)
122     w,v=eig(SW)

```

```

123     omegaRaw=w
124     idx1 = np.argsort(omegaRaw)
125     omegaRaw = omegaRaw[idx1]
126     v = v[:,idx1]
127     EigV=np.zeros((2*(N1+N2),N1+N2),dtype = 'complex_')
128     for ii in range (0,(N1+N2)):
129         for jj in range (0,2*(N1+N2)):
130             EigV[jj][ii]=v[jj][ii+(N1+N2)]
131     EigVx=np.zeros(((N1+N2),(N1+N2)),dtype=np.complex128)
132     for ii in range (0,(N1+N2)):
133         for jj in range (0,(N1+N2)):
134             EigVx[ii][jj]=EigV[2*ii][jj]
135             if abs(EigVx[ii][jj].real)>abs(EigVx[ii][jj].imag):
136                 EigVx[ii][jj]=EigVx[ii][jj].real
137             else:
138                 EigVx[ii][jj]=EigVx[ii][jj].imag
139     return EigVx[:,n].real
140 # layer number function -----
141 def xMobAmpM(laynum):
142     N1,N2=laynum
143     layers=np.linspace(1,(N1+N2),(N1+N2))
144     return layers
145 # convert back to film thickness
146 def xMobthicknessM(a,laynum):
147     a1,a2=a
148     N1,N2=laynum
149     layers=np.linspace(1,(N1+N2),(N1+N2))
150     thickness=np.zeros(len(layers))
151     for i in range (0,N1):
152         thickness[i]=a1*layers[i]
153         for i in range(N1,N1+N2):
154             thickness[i]=a1*N1+(layers[i]-N1)*a2
155     return thickness
156 def NModAmpXM(H,A,Ms,a,gamma,laynum,D,k,n):
157     return ModAmpXM(H,A,Ms,a,gamma,laynum,D,k,n)/np.max( abs(ModAmpXM(H,A,Ms,a,gamma,laynum,D,k,n)) )

```

Here are the material parameters (written in the main text of the article too):

```
In [3]: 1 H_1=300*(1e3/(4*np.pi)) # A/m
2 D_1=(3.9e-3,-1.5e-3) # J/m^2
3 a_1=(0.2866e-9,0.2506e-9) # m
4 A_1=(18.8e-12,32.5e-12,27e-12) # J/m
5 #A_1=(18.8e-12,32.5e-12,(18.8e-12+32.5e-12)/2)
6 Ms_1=(1.752e6,1.446e6) # A/m
7 gamma_1=(29e9,31.2e9) # Hz.rad/T
8 laynum_10_10=(10,10) # layer number
9 k_1=20e6 # example wave number in per m
10 MaxRe_1=(30e6,1)
11 A_2=(25.65e-12,25.65e-12,25.65e-12)
12 Ms_2=(1.599e6,1.599e6)
13 gamma_2=(30.1e9,30.1e9)
14 a_2=(0.2686e-9,0.2686e-9)
```

```
In [ ]: 1 MaxRe_100=(100e6,4)
2 x=xSwfreqkxM(MaxRe_100)
3 y0=SwfreqkxM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,MaxRe_100,0)
4 y1=SwfreqkxM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,MaxRe_100,1)
5 y2=SwfreqkxM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,MaxRe_100,2)
```

The k_x values probed (in units of μm^{-1}) are:

```
In [11]: 1 counter=0
2 x0 = []
3 while counter < len(x):
4     x0.append(x[counter])
5     counter = counter + 1
6
7 print(x0)
```

[-100.0, -96.0, -92.0, -88.0, -84.0, -80.0, -76.0, -72.0, -68.0, -64.0, -60.0, -56.0, -52.0, -48.0, -44.0, -40.0, -36.0, -32.0, -28.0, -24.0, -20.0, -16.0, -12.0, -8.0, -4.0, 0.0, 4.0, 8.0, 12.0, 16.0, 20.0, 24.0, 28.0, 32.0, 36.0, 40.0, 44.0, 48.0, 52.0, 56.0, 60.0, 64.0, 68.0, 72.0, 76.0, 80.0, 84.0, 88.0, 92.0, 96.0, 100.0]

The n=0 (quasiuniform) mode frequencies in GHz are:

In [12]:

```

1 counter=0
2 y00 = []
3 while counter < len(y0):
4     y00.append(y0[counter])
5     counter = counter + 1
6
7 print(y00)

```

[26.28218643453064, 25.192336846973806, 24.120850695701115, 23.067695331102183, 22.03289386462919, 21.016540188301327, 20.018817700841282, 19.040022769294843, 18.080594266259975, 17.141150927001714, 16.222538788365238, 15.325891614492251, 14.452707977897417, 13.60494948298836, 12.785165309556746, 11.996648375635754, 11.243627041343139, 10.531491593879819, 9.867043596079897, 9.258733733009747, 8.716814701904296, 8.253279216561728, 7.8813960193054085, 7.614651611679288, 7.465041133063337, 7.440982490201939, 7.54552426908115, 7.775629655221711, 8.12289251727552, 8.575329499068348, 9.11946589392493, 9.742044769926643, 10.431085243033055, 11.176346468913657, 11.969389642305728, 12.803425110759834, 13.673074531685845, 14.574121527358736, 15.503285203426625, 16.458028448461512, 17.43640177443053, 18.436918776413904, 19.458457912208612, 20.500185424236257, 21.561494916705282, 22.64195991943652, 23.741296533346915, 24.859333895665053, 25.995990720604077, 27.15125657534807, 28.325176862488185]

The n=1 (first exchange PSSW) mode frequencies/dispersion in GHz are:

In [13]:

```

1 counter=0
2 y10 = []
3 while counter < len(y1):
4     y10.append(y1[counter])
5     counter = counter + 1
6
7 print(y10)

```

[320.66869784138294, 319.99382809610063, 319.35050622022027, 318.7386778422274, 318.15829065418006, 317.6092944154997, 317.091640956095, 316.6052841790239, 316.1501800627057, 315.7262866626234, 315.3335641127073, 314.9719746262648, 314.64148249664083, 314.3420540975182, 314.0736578829342, 313.83626438708717, 313.62984622382675, 313.4543780860361, 313.3098367447296, 313.196201048043, 313.1134519200583, 313.06157235947114, 313.04054743815766, 313.05036429966754, 313.09101215752867, 313.16248229355824, 313.2647680560826, 313.39786485800784, 313.561770174904, 313.75648354305093, 313.9820065572681, 314.2383428688771, 314.52549818345756, 314.8434802585723, 315.1922989014662, 315.57196596660566, 315.98249535317143, 316.42390300245955, 316.89620689512566, 317.399427048362, 317.93358551287025, 318.49870636971326, 319.0948157270208, 319.72194171640643, 320.3801144892807, 321.06936621281903, 321.7897310657736, 322.5412452338976, 323.32394690512643, 324.1378762644122, 324.9830754881251]

The n=2 (second exchange PSSW) mode frequencies in GHz are:

In [14]:

```
1 counter=0
2 y20 = []
3 while counter < len(y2):
4     y20.append(y2[counter])
5     counter = counter + 1
6
7 print(y20)
```

```
[1280.1666857744235, 1279.5826685329127, 1279.025818430522, 1278.4961353386946, 1277.9936191548663, 1277.5182698007302, 1277.0700872204309, 1276.6490713789217, 1276.2552222603713, 1275.8885398666903, 1275.5490242161102, 1275.2366753417512, 1274.9514932904644, 1274.6934781215748, 1274.4626299057945, 1274.2589487241585, 1274.082434667072, 1273.933087833447, 1273.8109083298905, 1273.7158962699323, 1273.648051773427, 1273.607374965931, 1273.5938659782169, 1273.6075249458545, 1273.6483520088314, 1273.716347311282, 1273.8115110013125, 1273.933843230846, 1274.0833441555524, 1274.260013934943, 1274.4638527323866, 1274.6948607153263, 1274.9530380555218, 1275.238384929361, 1275.5509015182786, 1275.890588009209, 1276.2574445951448, 1276.6514714757657, 1277.0726688581358, 1277.5210369574645, 1277.9965759979784, 1278.4992862138163, 1279.0291678501053, 1279.586221163938, 1280.1704464255974, 1280.7818439197788, 1281.420413946894, 1282.0861568244327, 1282.779072888461, 1283.4991624951533, 1284.2464260224056]
```

Making plots of the dispersion relations:

In [7]:

```

1 MaxRe_100=(100e6,4)
2 x=xSwfreqkxM(MaxRe_100)
3 y0=SwfreqkxM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,MaxRe_100,0)
4 y1=SwfreqkxM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,MaxRe_100,1)
5 y2=SwfreqkxM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,MaxRe_100,2)
6
7 plt.rcParams["font.weight"] = "bold"
8 plt.rcParams["font.family"] = "Times New Roman"
9 plt.rcParams['mathtext.fontset'] = 'custom'
10 plt.rcParams['mathtext.it'] = 'STIXGeneral:italic'
11 plt.rcParams['mathtext.bf'] = 'STIXGeneral:italic:bold'
12 fig, (ax1,ax2,ax3) = plt.subplots(3, 1, sharex=True,figsize=(7, 14))
13 fig.subplots_adjust(hspace=0.15)
14 #fig.suptitle('Spin waves modes Ir/Fe/Co/Pt',fontsize=20)
15 ax1.plot(x,y0,'o',color='red', label='n=0',linewidth=1)
16 ax1.set_ylabel(r'$\mathbf{f}$ (GHz)',weight='bold',fontsize=38)
17 #ax1.set_xlabel('Normalized $S_x$',fontsize=15)
18 ax1.set_ylim([-1,31])
19
20 ax1.set_yticks([0,10,20,30])
21 ax1.tick_params(axis='y', right=True,direction='in',width=3,labels=32)
22 sec1 = ax1.secondary_yaxis(location=0)
23 sec1.set_yticks(np.linspace(0,30,16),labels=None)
24 sec1.tick_params(axis='y',right=True,direction='in',width=1,labelfleft=False)
25 thd1 = ax1.secondary_yaxis(location='right')
26 thd1.set_yticks(np.linspace(0,30,16),labels=None)
27 thd1.tick_params(axis='y',direction='in',width=1,labelright=False)
28 ax1.tick_params(axis='y',direction='in',labels=32)
29 ax1.set_xticks([-100,-50,0,50,100],labels=None)
30 ax1.tick_params(axis='x',direction='in', top=True,width=3,labelbottom=False)
31 sec1 = ax1.secondary_xaxis(location='bottom')
32 sec1.set_xticks(np.linspace(-100,100,21),labels=None)
33 sec1.tick_params(axis='x',direction='in',width=1,labelbottom=False)
34 thd1 = ax1.secondary_xaxis(location='top')
35 thd1.set_xticks(np.linspace(-100,100,21),labels=None)
36 thd1.tick_params(axis='x',direction='in',width=1,labeltop=False)
37
38 plt.setp(ax1.spines.values(), lw=3)
39 #ax1.legend(fontsize=15)
40 ax2.plot(x,y1,'o',color='red', label='n=1',linewidth=1)

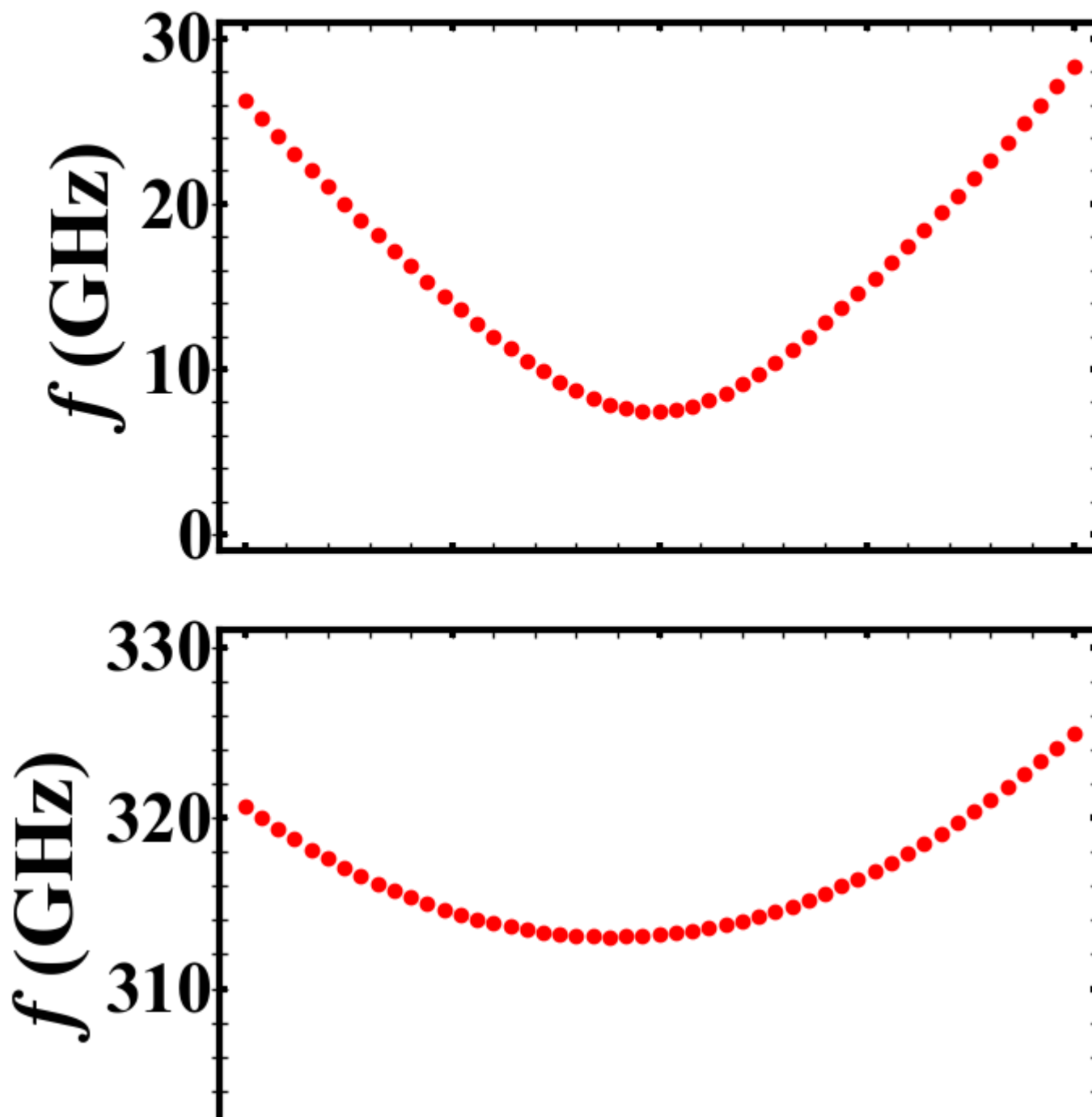
```

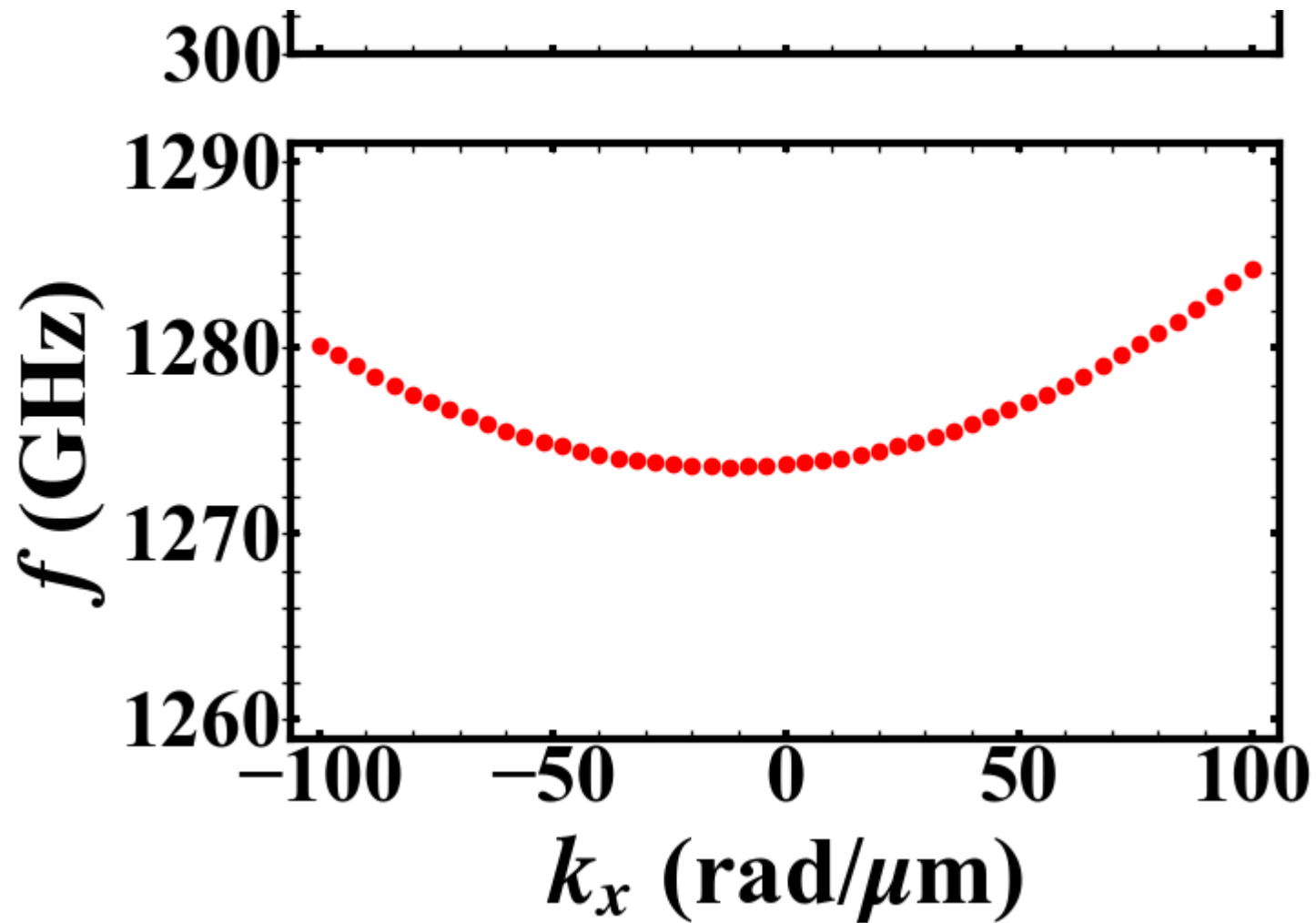
```

41 #ax2.set_ylabel('Layer index',fontsize=25)
42 #ax2.set_xlabel('Normalized $S_x$',fontsize=15)
43 ax2.set_ylim([329,331])
44 #ax2.set_xlim([0.99865,1.00005])
45 ax2.set_ylabel(r'$\mathbf{f}$ (GHz)',weight='bold',fontsize=38)
46
47 ax2.set_yticks([300,310,320,330])
48 ax2.tick_params(axis='y', right=True,direction='in',width=3,labels=32)
49 sec2 = ax2.secondary_yaxis(location=0)
50 sec2.set_yticks(np.linspace(300,330,16),labels=None)
51 sec2.tick_params(axis='y',right=True,direction='in',width=1,labelleft=False)
52 thd2 = ax2.secondary_yaxis(location='right')
53 thd2.set_yticks(np.linspace(300,330,16),labels=None)
54 thd2.tick_params(axis='y',width=1,direction='in',labelright=False)
55 ax2.tick_params(axis='y',direction='in',labels=32)
56 ax2.set_xticks([-100,-50,0,50,100],labels=None)
57 ax2.tick_params(axis='x', direction='in',top=True,width=3,labelbottom=False)
58 sec2 = ax2.secondary_xaxis(location='bottom')
59 sec2.set_xticks(np.linspace(-100,100,21),labels=None)
60 sec2.tick_params(axis='x',direction='in',width=1,labelbottom=False)
61 thd2 = ax2.secondary_xaxis(location='top')
62 thd2.set_xticks(np.linspace(-100,100,21),labels=None)
63 thd2.tick_params(axis='x',direction='in',width=1,labeltop=False)
64
65 plt.setp(ax2.spines.values(), lw=3)
66 #ax2.legend(fontsize=15)
67 ax3.plot(x,y2,'o',color='red',label='n=2',linewidth=1)
68 #ax3.set_ylabel('Layer index',fontsize=25)
69 #ax3.set_xlabel('Normalized $S_x$',fontsize=15)
70
71 ax3.set_ylim([1259,1291])
72 ax3.set_xlim([-106,106])
73
74 ax3.set_ylabel(r'$\mathbf{f}$ (GHz)',weight='bold',fontsize=38)
75 ax3.set_xlabel(r'$\mathbf{k_x}$ (rad/$\mathbf{\mu}$m)',weight='bold',fontsize=38)
76
77 ax3.set_yticks([1260,1270,1280,1290])
78 ax3.tick_params(axis='y',direction='in', right=True,width=3,labels=32)
79 sec3 = ax3.secondary_yaxis(location=0)
80 sec3.set_yticks(np.linspace(1260,1290,16),labels=None)
81 sec3.tick_params(axis='y',direction='in',right=True,width=1,labelleft=False)

```

```
82 thd3 = ax3.secondary_yaxis(location='right')
83 thd3.set_yticks(np.linspace(1260,1290,16), labels=None)
84 thd3.tick_params(axis='y', direction='in', width=1, labelright=False)
85 ax3.tick_params(axis='y', direction='in', labelsize=32)
86 ax3.set_xticks([-100,-50,0,50,100])
87 ax3.tick_params(axis='x', direction='in', top=True, width=3, labelsize=32)
88 sec3 = ax3.secondary_xaxis(location='bottom')
89 sec3.set_xticks(np.linspace(-100,100,21), labels=None)
90 sec3.tick_params(axis='x', direction='in', width=1, labelbottom=False)
91 thd3 = ax3.secondary_xaxis(location='top')
92 thd3.set_xticks(np.linspace(-100,100,21), labels=None)
93 thd3.tick_params(axis='x', direction='in', width=1, labeltop=False)
94
95 plt.setp(ax3.spines.values(), lw=3)
96 #ax2.legend(fontsize=15)
97 plt.show()
```





The corresponding mode profiles through the film thickness are drawn below:

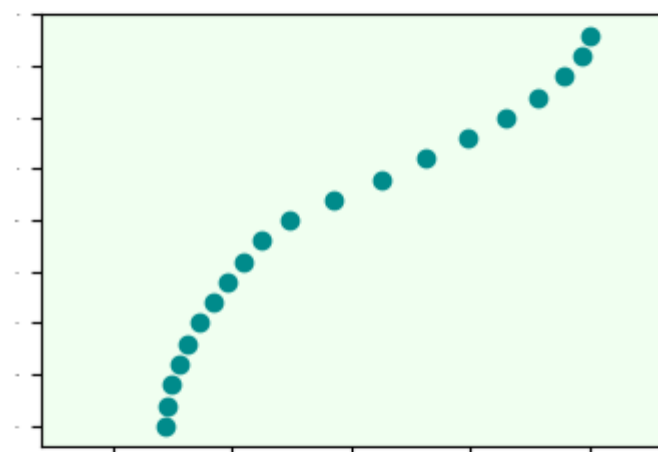
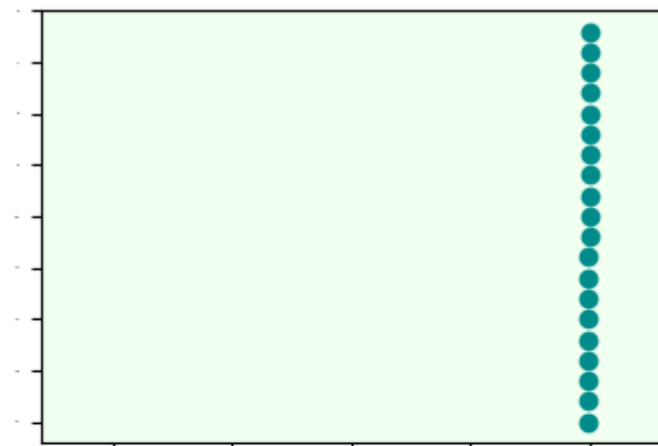
In [5]:

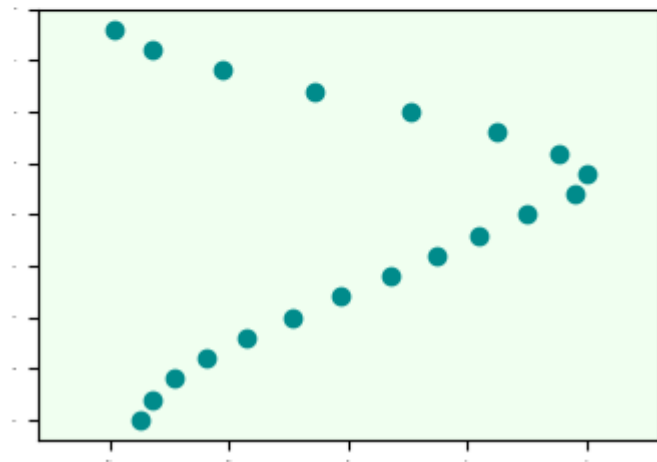
```

1 #NModAmpXM(H,A,Ms,a,gamma,laynum,D,k,n)
2 y=xMobAmpM(laynum_10_10)
3 x0=NModAmpXM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,(50e6,0),0)
4 x1=NModAmpXM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,(50e6,0),1)
5 x2=NModAmpXM(H_1,A_1,Ms_1,a_1,gamma_1,laynum_10_10,D_1,(50e6,0),2)
6
7 fig, (ax1,ax2,ax3) = plt.subplots(3, 1, sharex=True,figsize=(4, 14))
8 fig.subplots_adjust(hspace=1)
9 #fig.suptitle('Spin waves modes Ir/Fe/Co/Pt',fontsize=20)
10 ax1.plot(x0,y,'o',color='darkcyan', label='n=0',linewidth=1)
11 #ax1.plot(np.zeros(len(x)), y0,'--',color='black',linewidth=1)
12 #ax1.set_ylabel('$\Delta f$ (GHz)',fontsize=32)
13 #ax1.set_xlabel('Normalized $S_x$',fontsize=15)
14 ax1.set_ylim([21,0])
15 ax1.set_xlim([-1.1,1.1])
16 ax1.tick_params(axis='y', labelsz=0)
17 ax1.tick_params(axis='x', labelsz=0)
18 ax1.set_facecolor("honeydew")
19 #ax1.xaxis.set_ticks(np.arange(0.9985, 1.00061, 0.0006))
20 #ax1.legend(fontsize=15)
21 ax2.plot(x1,y,'o',color='darkcyan', label='n=1',linewidth=1)
22 #ax2.set_ylabel('Layer index',fontsize=25)
23 #ax2.set_xlabel('Normalized $S_x$',fontsize=15)
24 #ax2.xaxis.set_ticks(np.arange(0.9985, 1.00061, 0.0006))
25 ax2.set_ylim([21,0])
26 ax2.set_xlim([-1.1,1.1])
27 #ax2.set_ylabel('$\Delta f$ (GHz)',fontsize=32)
28 ax2.tick_params(axis='y', labelsz=0)
29 ax2.tick_params(axis='x', labelsz=0)
30 ax2.set_facecolor("honeydew")
31 #ax2.legend(fontsize=15)
32 ax3.plot(x2,y,'o',color='darkcyan',label='n=2',linewidth=1)
33 #ax3.set_ylabel('Layer index',fontsize=25)
34 #ax3.set_xlabel('Normalized $S_x$',fontsize=15)
35 #ax2.xaxis.set_ticks(np.arange(0.9985, 1.00061, 0.0006))
36 ax3.set_ylim([21,0])
37 ax3.set_xlim([-1.3,1.3])
38 #ax3.set_ylabel('$\Delta f$ (GHz)',fontsize=32)
39 ax3.tick_params(axis='y', labelsz=0)
40 ax3.tick_params(axis='x', labelsz=0)

```

```
41 #ax3.set_xlabel('$k_x$ (rad/$\mu m$)', fontsize=32)
42 #ax2.legend(fontsize=15)
43 ax3.set_facecolor("honeydew")
44 plt.show()
```





In []:

1