Here, we use the same data from Figure 3, but zoom in on the pinning of the (quasi)uniform mode.

This is used to make Figure 4 of the article.

The Eigenfrequencies -- D = 4.2 mJ/m^2 on top surface only

Some parameters needed for the code

In[∘]:=
```
B = 0.03;
Δ = 1;
Φ[0] = Table[0, {4}];
LL = 16; (* 16 atomic layers *)
```

In[∘]:=
```
a = 0.248 × 10^-9; (*0.25 nm atomic spacing*)
Ms = 835 563; (*A/m*)
AA = 1.355 × 10^-11; (*J/m*)
DD = 4.2 × 10^-3 × 1/1;
(* i.e. This is 4.2 mJ/m^2 for 1 layer and decreases for thicker films!*)
JJ = (2 AA)/(a^2 Ms)
```

Out[∘]=
```
527.335
```

In[∘]:=
```
K[i_] := K[i] = Which[i < LL/2 + 0.5, 0, i > LL/2 + 0.5, 0]
J[i_] := J[i] = Which[i < LL/2 + 0.5, JJ, i > LL/2 + 0.5, JJ]
H[i_] := H[i] = Which[i < LL/2 + 0.5, B, i > LL/2 + 0.5, B]
HDMI[1] = 2 DD/(a Ms);
HDMI[LL] = 0
```

Out[∘]=
```
0
```

In[∘]:=
```
ϕ[i_] := ϕ[i] = 0
```

Coding required to create dynamical matrix

For a typical plane of spins in the wall the code is:

```
In[ ]:= acomponent[i_, y_, z_] := acomponent[i, y, z] =
          H[i] Cos[ϕ[i]] + (4 π 10^-7) Ms + 2 K[i] (Cos[ϕ[i]]^2 - Sin[ϕ[i]]^2) + J[i]
            (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) + 4 J[i] - 2 J[i] Cos[y] - 2 J[i] Cos[z]
        aplus[i_] := aplus[i] = -J[i] Cos[ϕ[i] - ϕ[i + 1]]
        aminus[i_] := aminus[i] = -J[i] Cos[ϕ[i] - ϕ[i - 1]]
        bcomponent[i_, y_, z_] := bcomponent[i, y, z] = -H[i] Cos[ϕ[i]] - 2 K[i] Cos[ϕ[i]]^2 -
           J[i] (Cos[ϕ[i] - ϕ[i - 1]] + Cos[ϕ[i] - ϕ[i + 1]]) - 4 J[i] + 2 J[i] Cos[y] + 2 J[i] Cos[z]
```

```
In[ ]:= rowa[NN_, k_, y_, z_] := Join[Table[0, {2 k - 3}],
          {aminus[k], 0, acomponent[k, y, z], 0, aplus[k]}, Table[0, {2 NN - 2 - 2 k}]]
        rowb[NN_, k_, y_, z_] := Join[Table[0, {2 k - 4}],
          {J[k], 0, bcomponent[k, y, z], 0, J[k]}, Table[0, {2 NN - 1 - 2 k}]]
```

The 1st, (N/2)th, (N/2 +1)th and Nth planes all need individual codes sinse they have different exchange coupling to the planes on either side.
The codes are as follows:

```
In[ ]:= arow1[NN_, y_, z_] := arow1[NN, y, z] = Join[
            {-HDMI[1] Sin[y] ⅈ,
             H[1] Cos[ϕ[1]] + (4 π 10^-7) Ms + 2 K[1] (Cos[ϕ[1]]^2 - Sin[ϕ[1]]^2) +
              J[1] (0 + Cos[ϕ[1] - ϕ[2]]) + 4 J[1] - 2 J[1] Cos[y] - 2 J[1] Cos[z],
             0,
             aplus[1]},
            Table[0, {2 NN - 4}]];
```

```
In[ ]:= brow1[NN_, y_, z_] := brow1[NN, y, z] = Join[
            {-H[1] Cos[ϕ[1]] - 2 K[1] Cos[ϕ[1]]^2 -
              J[1] (0 + Cos[ϕ[1] - ϕ[2]]) - 4 J[1] + 2 J[1] Cos[y] + 2 J[1] Cos[z],
             -HDMI[1] Sin[y] ⅈ,
             J[2]},
            Table[0, {2 NN - 3}]];
```

Note that I have kept the ANGULAR dependence in this code, which was set up for dealing with an exchange spring. It is not needed here, but it is an interesting question to see how the DMI can change the modes on an exchange spring...

In[ ]:=
```
arow50[NN_, y_, z_, β_] := arow50[NN, y, z, β] = Join[
   Table[0, {NN - 3}],
   {aminus[NN / 2],
    0,
    H[NN / 2] Cos[ϕ[NN / 2]] + (4 π 10^-7) Ms + 2 K[NN / 2] (Cos[ϕ[NN / 2]]^2 - Sin[ϕ[NN / 2]]^2) +
     J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2]] + (J[NN] + β (J[1] - J[NN]))
      Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] + 4 J[NN / 2] - 2 J[NN / 2] Cos[y] - 2 J[NN / 2] Cos[z],
    0,
    -(J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]]},
   Table[0, {NN - 2}]]
```

In[ ]:=
```
brow50[NN_, y_, z_, β_] := brow50[NN, y, z, β] = Join[
   Table[0, {NN - 4}],
   {J[NN / 2],
    0,
    -H[NN / 2] Cos[ϕ[NN / 2]] - 2 K[NN / 2] Cos[ϕ[NN / 2]]^2 -
     J[NN / 2] Cos[ϕ[NN / 2] - ϕ[NN / 2 - 1]] - (J[NN] + β (J[1] - J[NN]))
      Cos[ϕ[NN / 2] - ϕ[NN / 2 + 1]] - 4 J[NN / 2] + 2 J[NN / 2] Cos[y] + 2 J[NN / 2] Cos[z],
    0,
    J[NN] + β (J[1] - J[NN])},
   Table[0, {NN - 1}]]
```

In[ ]:=
```
arow51[NN_, y_, z_, β_] := arow51[NN, y, z, β] = Join[
   Table[0, {NN - 1}],
   {-(J[NN / 2] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]],
    0,
    H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] + (4 π 10^-7) Ms +
     2 K[NN / 2 + 1] (Cos[ϕ[NN / 2 + 1]]^2 - Sin[ϕ[NN / 2 + 1]]^2) + (J[NN] + β (J[1] - J[NN]))
      Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] + J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] +
     4 J[NN / 2 + 1] - 2 J[NN / 2 + 1] Cos[y] - 2 J[NN / 2 + 1] Cos[z],
    0,
    aplus[NN / 2 + 1]},
   Table[0, {NN - 4}]]
```

```
In[•]:= brow51[NN_, y_, z_, β_] := brow51[NN, y, z, β] = Join[
          Table[0, {NN - 2}],
          {J[NN] + β (J[1] - J[NN]),
           0,
           -H[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]] - 2 K[NN / 2 + 1] Cos[ϕ[NN / 2 + 1]]^2 -
             (J[NN] + β (J[1] - J[NN])) Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2]] -
             J[NN / 2 + 1] Cos[ϕ[NN / 2 + 1] - ϕ[NN / 2 + 2]] - 4 J[NN / 2 + 1] +
             2 J[NN / 2 + 1] Cos[y] + 2 J[NN / 2 + 1] Cos[z],
           0,
           J[NN / 2 + 1]},
          Table[0, {NN - 3}]]

In[•]:= arow100[NN_, y_, z_] := Join[
          Table[0, {2 NN - 3}],
          {aminus[NN],
           -HDMI[NN] Sin[y] ⅈ,
           H[NN] Cos[ϕ[NN]] + (4 π 10^-7) Ms + 2 K[NN] (Cos[ϕ[NN]]^2 - Sin[ϕ[NN]]^2) +
             J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) + 4 J[NN] - 2 J[NN] Cos[y] - 2 J[NN] Cos[z]}];

In[•]:= brow100[NN_, y_, z_] := Join[
          Table[0, {2 NN - 4}],
          {J[NN - 1],
           0,
           -H[NN] Cos[ϕ[NN]] - 2 K[NN] Cos[ϕ[NN]]^2 -
             J[NN] (Cos[ϕ[NN] - ϕ[NN - 1]] + 0) - 4 J[NN] + 2 J[NN] Cos[y] + 2 J[NN] Cos[z],
           -HDMI[NN] Sin[y] ⅈ}];
```

The dynamical matrix and eigenfrequencies

The dynamical matrix is:

```
In[•]:= big[NN_, y_, z_, β_] := big[NN, y, z, β] = Join[
          {arow1[NN, y, z], brow1[NN, y, z]},
          Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, 2, NN / 2 - 1}], 1],
          {arow50[NN, y, z, β], brow50[NN, y, z, β],
           arow51[NN, y, z, β], brow51[NN, y, z, β]},
          Flatten[Table[{rowa[NN, j, y, z], rowb[NN, j, y, z]}, {j, NN / 2 + 2, NN - 1}], 1],
          {arow100[NN, y, z], brow100[NN, y, z]}]
```

The eigenfrequencies are given by (γ = 176 GHz rad/T):

```
In[•]:= freqs[NN_, y_, z_, β_] := freqs[NN, y, z, β] =
          176/(2. π) Table[Reverse[Chop[ⅈ Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 2}]
```

```
In[]:= freqs2[NN_, y_, z_, β_] := freqs2[NN, y, z, β] =
         176
        ----- Table[Reverse[Chop[ⅈ Eigenvalues[big[NN, y, z, β]]]][[k]], {k, 1, 2 NN, 1}]
        2. π
```

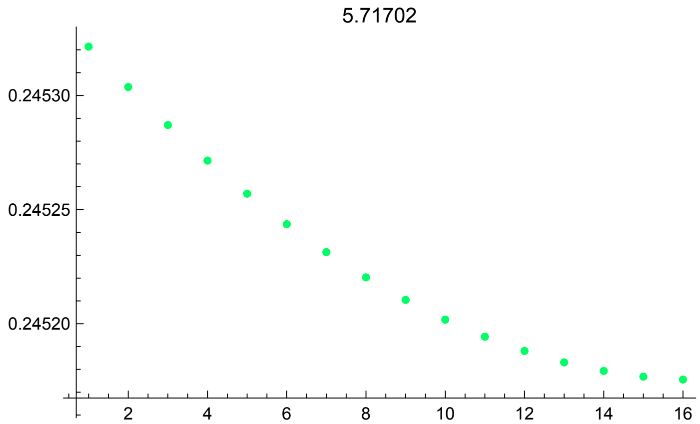
Mode profiles of PSSWs

Functions to extract and plot the PSSW mode profiles.

```
In[]:= modefunxions[NN_, y_, z_, β_] := modefunxions[NN, y, z, β] =
          Chop[Table[Reverse[Eigenvectors[big[NN, y, z, β]]][[k]], {k, 1, 2 NN, 1}]];

In[]:= xmotion[NN_, y_, z_, β_, k_] :=
        xmotion[NN, y, z, β, k] = Table[modefunxions[NN, y, z, β][[k]][[n]], {n, 1, 2 NN, 2}]
       ymotion[NN_, y_, z_, β_, k_] :=
        ymotion[NN, y, z, β, k] = Table[modefunxions[NN, y, z, β][[k]][[n]], {n, 2, 2 NN, 2}]

In[]:= px1[NN_, ky_, k_] :=
         px1[NN, ky, k] = ListPlot[xmotion[NN, ky a, 0, 0.5, k], PlotRange → All,
           PlotStyle → Hue[0.4], PlotLabel → Abs[freqs2[NN, ky a, 0, 0.5]][[k]]];
       py1[NN_, ky_, k_] :=
         py1[NN, ky, k] = ListPlot[ymotion[NN, ky a, 0, 0.5, k], PlotRange → All,
           PlotStyle → Hue[0], PlotLabel → Abs[freqs2[NN, ky a, 0, 0.5]][[k]]];
       pimag1[NN_, ky_, k_] :=
         pimag1[NN, ky, k] = ListPlot[ⅈ xmotion[NN, ky a, 0, 0.5, k], PlotRange → All,
           PlotStyle → Hue[0.7], PlotLabel → Abs[freqs2[NN, ky a, 0, 0.5]][[k]]];

In[]:= modefunxions[LL, 20 × 10^6, 0, 0.5];

In[]:= freqs2[LL, 20 × 10^6 a, 0, 0.5]

Out[]=
       {-5.71702, 6.42102, -582.681, 584.075, -2263.99, 2265.34, -4994.08, 4995.37,
        -8668.14, 8669.34, -13 145., 13 146.1, -18 252.6, 18 253.6, -23 794.7, 23 795.5,
        -29 558.2, 29 558.9, -35 321.8, 35 322.3, -40 863.8, 40 864.3, -45 971.4, 45 971.7,
        -50 448.3, 50 448.5, -54 122.3, 54 122.4, -56 852.4, 56 852.4, -58 533.6, 58 533.6}
```

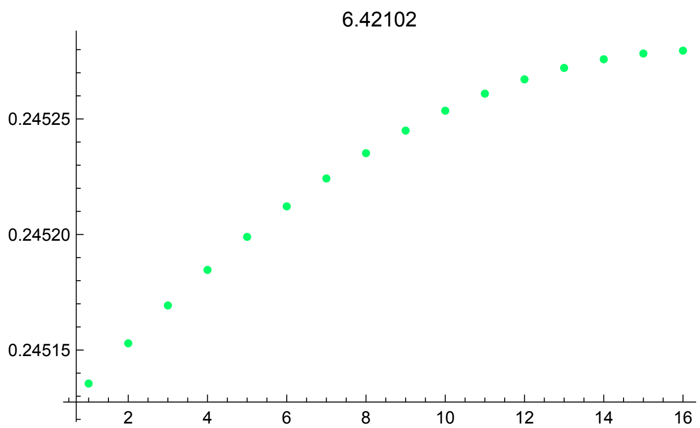
In[ ]:= px1[LL, 20 × 10^6, 1]

Out[ ]=



In[ ]:= px1[LL, 20 × 10^6, 2]

Out[ ]=



In[ ]:= 6.42102 - 5.71702

Out[ ]=

0.704

In[ ]:= xmotion[LL, 20 × 10^6 a, 0, 0.5, 1]

Out[ ]=

{0.245321, 0.245304, 0.245287, 0.245271, 0.245257, 0.245244, 0.245231, 0.24522, 0.24521, 0.245202, 0.245194, 0.245188, 0.245183, 0.245179, 0.245177, 0.245176}

Normalizing :

In[ ]:= uniformNegkx = xmotion[LL, 20 × 10^6 a, 0, 0.5, 1] / 0.24517555734108742`

Out[ ]=

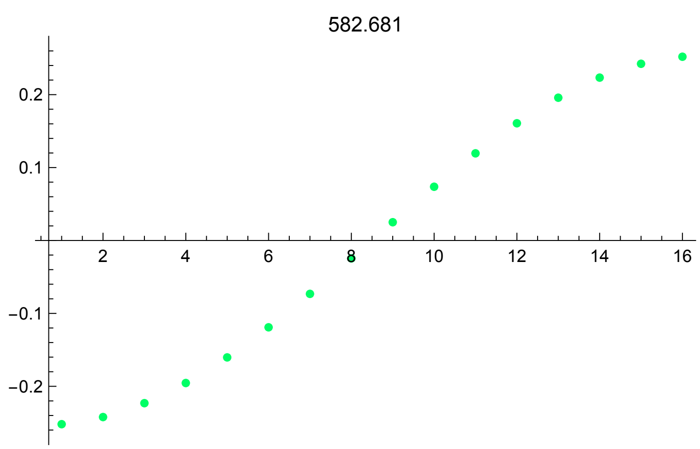
{1.00059, 1.00052, 1.00045, 1.00039, 1.00033, 1.00028, 1.00023, 1.00018, 1.00014, 1.00011, 1.00008, 1.00005, 1.00003, 1.00002, 1.00001, 1.}

Other mode:

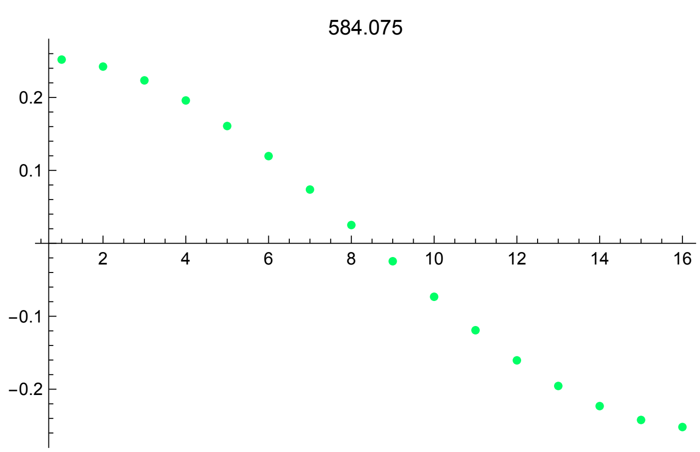
In[∘]:=
```
xmotion[LL, 20×10^6 a, 0, 0.5, 2]
```
Out[∘]=
```
{0.245135, 0.245153, 0.245169, 0.245185, 0.245199, 0.245212, 0.245224, 0.245235,
 0.245245, 0.245254, 0.245261, 0.245267, 0.245272, 0.245276, 0.245278, 0.24528}
```
In[∘]:=
```
uniformPoskx = xmotion[LL, 20×10^6 a, 0, 0.5, 2] / 0.2452795646287667`
```
Out[∘]=
```
{0.999413, 0.999484, 0.99955, 0.999613, 0.999671, 0.999725, 0.999774, 0.999819,
 0.999859, 0.999894, 0.999924, 0.999949, 0.99997, 0.999985, 0.999995, 1.}
```

Try the next mode (first exchange mode with one node, n=1):

In[∘]:=
```
px1[LL, 20×10^6, 3]
```
Out[∘]=



In[∘]:=
```
px1[LL, 20×10^6, 4]
```
Out[∘]=



Pinning of the uniform mode -- zoom in

By running the code above for the case of
(i) no DMI

(2) DMI with kx=+20 μm^{-1}
(3) DMI with kx=-20 μm^{-1}
we get the following mode profiles for the 16 atomic layers in the thickness direction, normalized to 1 on the bottom edge:

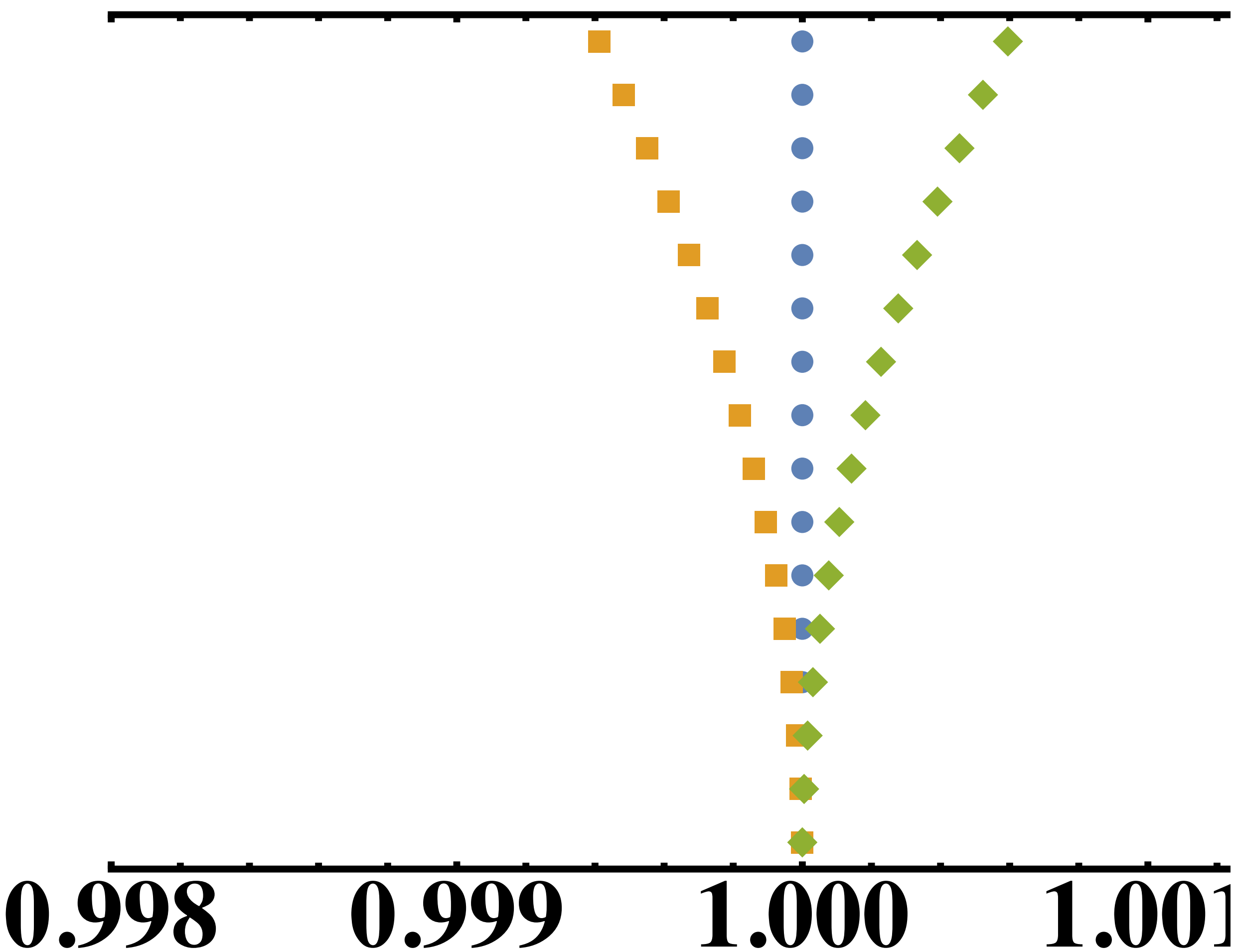
```
In[ ]:= uniform = {1.0000000000000078`, 1.0000000000000084`, 1.00000000000001`,
    1.0000000000000102`, 1.0000000000000113`, 1.0000000000000102`,
    1.0000000000000093`, 1.0000000000000073`, 1.000000000000006`,
    1.0000000000000042`, 1.000000000000003`, 1.0000000000000022`,
    1.0000000000000007`, 1.0000000000000002`, 0.9999999999999997`, 1.`};

In[ ]:= uniformPoskx = {0.9994126277726905`, 0.9994835793398625`, 0.9995504457161835`,
    0.9996130901747001`, 0.9996713869428284`, 0.9997252209326898`,
    0.9997744874943625`, 0.9998190921915322`, 0.9998589505990523`,
    0.9998939881219806`, 0.9999241398356828`, 0.9999493503466921`,
    0.9999695736739797`, 0.9999847731504135`, 0.9999949213441804`, 1.`};
uniformNegkx = {1.000594872803094`,
    1.0005227021836753`, 1.0004547714453609`, 1.0003912017790881`,
    1.000332104655793`, 1.0002775820677572`, 1.0002277267494373`,
    1.000182622378267`, 1.0001423437558634`, 1.0001069569700207`,
    1.0000765195378378`, 1.0000510805303007`, 1.0000306806785622`,
    1.00001535246217`, 1.000005120179421`, 0.9999999999999999`};
```

Now to plot these so they look like they are through the thickness:

```
In[ ]:= uniform2 = Table[{uniform[[i]], 17 - i}, {i, 1, Length[uniform]}];
uniformPoskx2 = Table[{uniformPoskx[[i]], 17 - i}, {i, 1, Length[uniform]}];
uniformNegkx2 = Table[{uniformNegkx[[i]], 17 - i}, {i, 1, Length[uniform]}];
```

In[ ]:=
```
pinninggg = ListPlot[{uniform2, uniformPoskx2, uniformNegkx2},
  PlotRange → {{0.998, 1.002}, {0.5, 16.5}},
  PlotMarkers → {Automatic, 0.04}, Frame → {{False, False}, {True, True}},
  FrameStyle → Directive[Black, Bold, Thickness[0.005]],
  LabelStyle → {FontFamily → "Times", FontSize → 50}, ImageSize → 1000]
```

Out[ ]=



In[ ]:=
```
Export["/Users/karen/Desktop/UON
  stuff/Students/Ellen/Spin waves DMI/Figures/Pinning.png", pinninggg]
```

Out[ ]=
```
/Users/karen/Desktop/UON stuff/Students/Ellen/Spin waves DMI/Figures/Pinning.png
```

Instead, I can normalize a different way (normalization is arbitrary) so that these look more like traveling waves with surface pinning:

In[∘]:=
```
pinninggg = ListPlot[{uniform2, uniformPoskx2/0.9994126277726905`, uniformNegkx2},
  PlotRange → {{0.998, 1.002}, {0.5, 16.5}},
  PlotMarkers → {Automatic, 0.04}, Frame → {{False, False}, {True, True}},
  FrameStyle → Directive[Black, Bold, Thickness[0.005]],
  LabelStyle → {FontFamily → "Times", FontSize → 50}, ImageSize → 1000]
```

Out[∘]=