Assignment and Exercises 6: ISYE 4133. Due November 6.

October 28, 2019

Instructions. The problems in the Section 1 are to be submitted by the due date. You are supposed to write your own solutions but you can discuss your solution with at most *one* other person. If you discuss with somebody, mention their name on your submission.

1 Assignment

1. (10 points) A small firm buys a new production machinery costing $12000. In order to decrease maintenance costs, it is possible to sell the machinery second-hand and buy a new one. The maintenance costs and possible gains derived from selling the machinery second-hand are given below (for the next 5 years):

Age(years)	costs(1000 Dollars)	gain (1000 dollars)
0	2	-
1	4	7
2	5	6
3	9	2
4	12	1

   Determine a renewal plan for the machinery which minimizes the total operation cost over a 5-year period by formulating a linear integer program. Solve the integer program in Gurobi as well its linear programming relaxation.

2. (10 points) Company Widget Maker makes widgets whose demand d_j for each of K time periods $j = 1, 2, \ldots, K$ is known. It can meet the demand by either producing some amount of widgets in period j and/or by drawing upon the inventory carried from the previous period. There is a cost c_j to make each widget in time period j that changes over time. Moreover, there is a holding cost of h_j per widget of holding from time period j to $j + 1$ for each $1 \leq j \leq K - 1$. Show how to devise a plan for the company that decides how many widgets to produce each time period.

3. (10 points) We have n items, each with weight $a_j, (j = 1, \ldots, n)$ and value $c_j (j = 1, \ldots, n)$ and an integer B. Assume all data is integral.

   (a) Formulate an integer linear program to find the subset of items of maximum value whose total weight is at most B.

   (b) Consider a variant in which it is allowed to increase B by an integral number which is at most 10. This comes with a cost, the cost for the first unit of increase 1 equals 2, the

second and third unit each cost 3, the fourth and fifth cost 6 per unit and the sixth to 10th unit cost 10 per unit. For example, increasing B by 6 costs $2 + 3 + 3 + 6 + 6 + 10 = 30$. Formulate the above problem as an integer linear programming problem. Give a description of the decision variables, objective and constraints.

(c) Suppose that the cost per unit increase are non-increasing, e.g. 2 for the first unit and 1 for the second unit. Check if your model is still correct. If not, give a modified integer linear programming model.

4. (20 points) Consider a cloud provider that is running a data center with hundreds of thousands of computers. Each of its computer has 64 cores. It receives a bunch of customer requests where each customer wants to run a program on some number cores. The following is a list of customers and number of programs it wants to run and number of cores required for each instance of these programs. The aim of the cloud provider is to satisfy the demand of all of its customers by using as few computers as possible to save electricity costs.

Customer ID	Number of Programs	Number of Cores per program
1	1000	20
2	1500	30
3	2000	32
4	2500	16
6	1200	5

(a) Based on the two formulations for the cutting stock problem, give two linear integer programming formulation for the problem. Solve both of them using Gurobi and use column generation to solve the second formulation.

(b) (Extra Credit 5 points). It turns out, that each computer comes with other resources that need to accounted for. Each computer has 32 GB of RAM and network bandwidth of 4 GB/s. Moreover, each customer requirement also depends in the following manner on these resources.

Customer ID	Number of Programs	Number of Cores per program	RAM per program	Bandwidth per program
1	1000	20	24	0
2	1500	30	8	2
3	2000	32	16	1
4	2500	16	20	0.5
6	1200	5	5	2.8

Update your model and the column generation algorithm to account for above resource requirement and constraints.