Karen Loscocco ISYE 4133 Assignment 1

①

ⓐ $\min\ x_1 + 2x_2$

s.t. $x_1 + x_2 \geq 10$
$2x_1 + 5x_2 \leq 40$
$x_1 \geq 0$
$x_2 \geq 0$

$\longrightarrow$

$\min\ x_1 + 2x_2$

s.t. $x_1 + x_2 - s_1 = 10$
$2x_1 + 5x_2 + s_2 = 40$
$x_1, x_2, s_1, s_2 \geq 0$

ⓑ $\max\ x_1 - x_2$

s.t. $x_1 + x_2 \leq 4$
$2x_1 + 5x_2 = 30$
$x_1 \leq 0$
$x_2 \geq 0$

$\longrightarrow$

$\min\ x_2 + x_1$

s.t. $-x_1 + x_2 + s_1 = 4$
$-2x_1 + 5x_2 = 30$
$x_1, x_2, s_1 \geq 0$

② Solved in Gurobi : Problem2.py

ⓐ Given Form

$x_1 = 10$
$x_2 = 0$

objective value = 10

Standard form

$x_1 = 10$ $s_1 = 0$
$x_2 = 0$ $s_2 = 20$

objective value = 10

Only changed the inequality constraints to equality constraints. Since we only changed the slack variables the solutions for x_1 and x_2 are the same

ⓑ Given Form

$x_1 = -3.3\overline{3}$
$x_2 = 7.3\overline{3}$

objective value = $-10.6\overline{6}$

Standard form

$x_1 = 3.3\overline{3}$ $s_1 = 0$
$x_2 = 7.33$

objective value = $10.6\overline{6}$

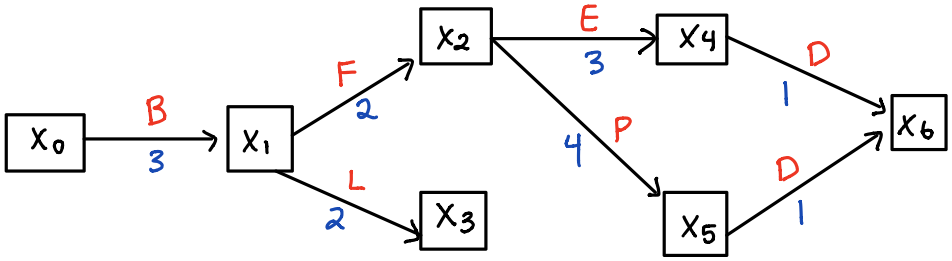
here the objective was changed from max to min this causes the objective value to be negated in standard form.
The value for x_1 is also negated because x_1 was negated ($x_1 \leq 0$ to $x_1 \geq 0$

③

Task	B	F	E	P	D	L
Duration	3	2	3	4	1	2

objective: minimize duration of project by scheduling starting times of tasks

F can start after B
L can start after B
E can start after F
P can start after F
D can start after E
D can start after P



X_i: starting time at beginning of task i

$$\min \quad x_0 + x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

s.t.

$$x_1 - x_0 \geq 3 \qquad x_5 - x_2 \geq 4$$
$$x_2 - x_1 \geq 2 \qquad x_6 - x_4 \geq 1$$
$$x_3 - x_1 \geq 2 \qquad x_6 - x_5 \geq 1$$
$$x_4 - x_2 \geq 3$$

$$x_0, x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Solved using Gurobi: Problem 3.py

optimal duration of project: 40

$x_0 = 0$ $\quad x_3 = 5$ $\quad x_6 = 10$

$x_1 = 3$ $\quad x_4 = 8$

$x_2 = 5$ $\quad x_5 = 9$

Task	B	F	L	E	P	D
Start	0	3	3	5	5	9
end	3	5	5	8	9	10

④ $m \in \{10, 20, 50, 100, 500, 1000, 10000\}$

$n \in \{10, 20, 50, 100, 1000, 10000\}$

$A \in \mathbb{R}^{m \times n}$ with entries are uniformly random $[0, 1]$

$b \in \mathbb{R}^{m}$ with entries randomly between $[0, 1000]$

$c \in \mathbb{R}^{n}$ with entries randomly between $[0, 1000]$

LP: $\min \; c^T x$
$\text{st. } Ax \geq b, \; x \geq 0$

Solved Using Gurobi : Problem4.py

Graphs attached at end: Problem4_Graphs.py

(5) 4 leg tables

.5 labor hour to produce tabletop
.3 labor hour to attach legs to tabletop

model	specs.	Labor hour	Profit
A	18" legs	.10	30
B	30" legs	.15	45

5000 feet = 60000 inches of leg stock availiable
800 labor hours
objective: maximize profit

X_1: # tables of model A produced

X_2: # tables of model B produced

model A costs in labor hours: $[4(.10) + .5 + .3] X_1$

model B costs in labor hours: $[4(.15) + .5 + .3] X_2$

} total $\leq$ 800 labor hours

model A costs in inches: $72 X_1$

model B costs in inches: $120 X_2$

} total $\leq$ 60000 inches

total profit: $30 X_1 + 45 X_2$

LP: max $30 x_1 + 45 x_2$

s.t. $1.2 x_1 + 1.4 x_2 \leq 800$

$72 x_1 + 120 x_2 \leq 60000$

$x_1, x_2 \geq 0$

Solved with Gurobi: Problem5.py

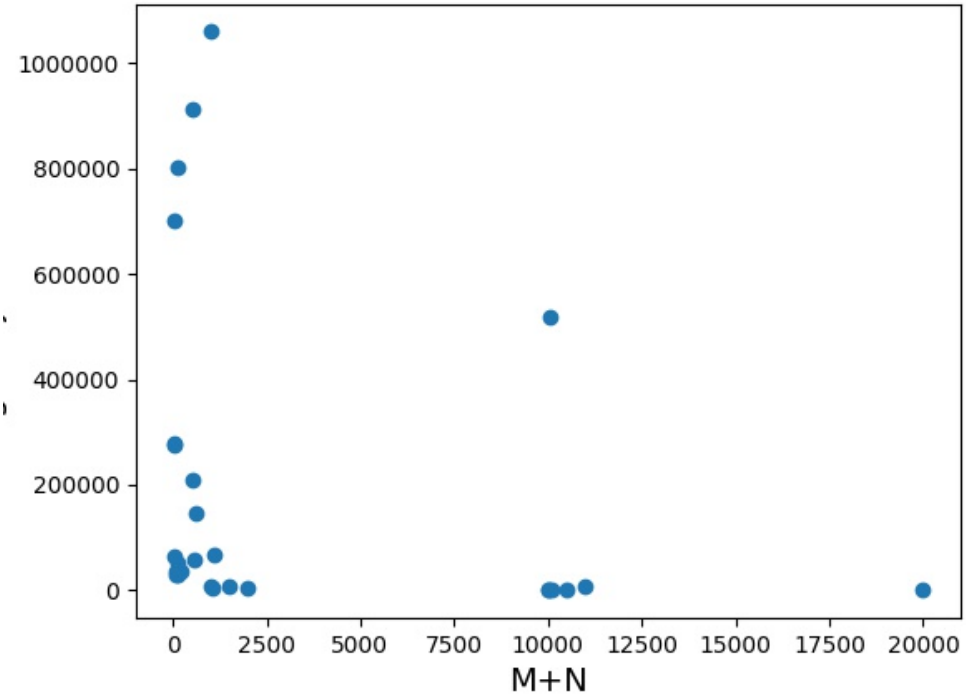
$x_1 = 277.7\overline{7}$

$x_2 = 333.3\overline{3}$

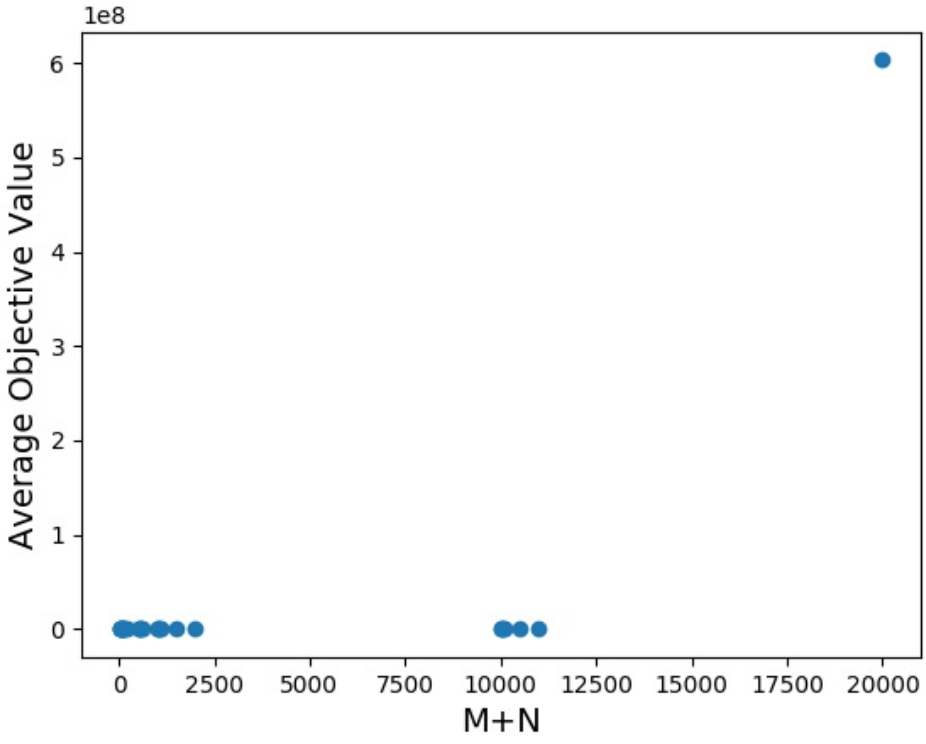
objective value: $23{,}333.3\overline{3}$

Assumption is not valid because a fraction of a table has no value.

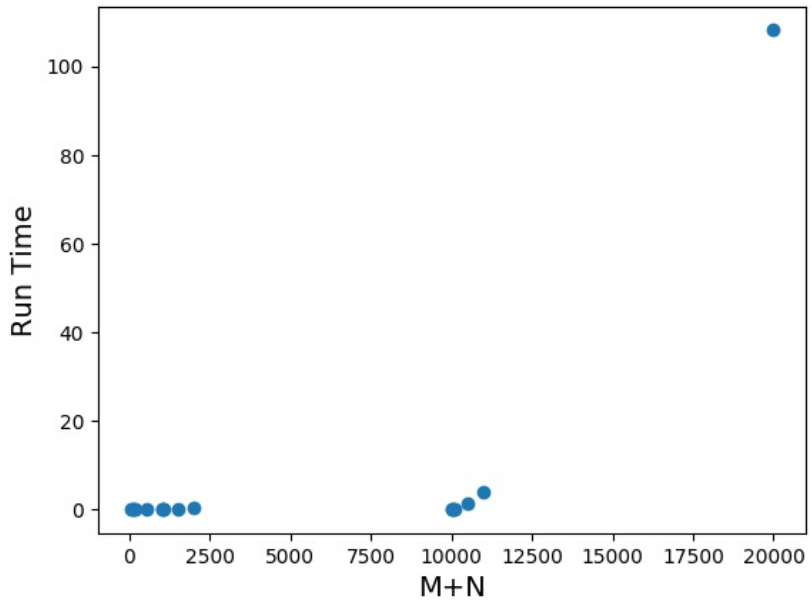
Average Objective Value



Average Objective Value (INTEGER variables)



Run Time



Run Time (INTEGER variables)