Assignment and Exercises 4: ISYE 4133. Due October 16.

October 9, 2019

Instructions. Exercises are for your practice. You need not submit them. The problems in the Section 2 are to be submitted by the due date. You are supposed to write your own solutions but you can discuss your solution with at most *one* other person. If you discuss with somebody, mention their name on your submission.

1 Exercises

1. We have n items, each with weight $a_j, (j = 1, \ldots, n)$ and value $c_j(j = 1, \ldots, n)$ and an integer B. Assume all data is integral.

   (a) Formulate an ILP to find the subset of items of maximum value whose total weight is at most B.

   (b) Consider a variant in which it is allowed to increase B by an integral number which is at most 10. This comes with a cost, the cost for the first unit of increase 1 equals 2, the second and third unit each cost 3, the fourth and fifth cost 6 per unit and the sixth to 10th unit cost 10 per unit. For example, increasing B by 6 costs $2 + 3 + 3 + 6 + 6 + 10 = 30$. Formulate the above problem as an integer linear programming problem. Give a description of the decision variables, objective and constraints.

   (c) Suppose that the cost per unit increase are non-increasing, e.g. 2 for the first unit and 1 for the second unit. Check if your model is still correct. If not, give a modified integer linear programming model.

2. Model the following problem as an integer linear program.

$$
\begin{aligned}
\min_x \, & c^T x \\
& s.t. \\
Ax &= b \\
x &\geq 0 \\
x_1 &\in \{r_1, \ldots, r_t\}
\end{aligned}
$$

   where $r_1, \ldots, r_t$ are some real numbers.

3. (10 points) Solve the following problem using cutting planes.

$$\begin{aligned} &\max \;\; 2x_1 + 9x_2 \\ &2x_1 + x_2 \leq 20 \\ &x_1 + 5x_2 \geq 24 \\ &x_1, x_2 \in \mathbb{Z}_+ \end{aligned}$$

4. Solve the following problem using Branch and bound.

$$\begin{aligned} &\max \;\; 3x_1 + 4x_2 \\ &2x_1 + x_2 \leq 6 \\ &2x_1 + 3x_2 \leq 9 \\ &x_1, x_2 \in \mathbb{Z}_+ \end{aligned}$$

5. Reformulate the following model as a integer linear program.

$$\begin{aligned} \min_{x_1, x_2} \; & 2x_1 - 7x_2 \\ & s.t. \\ & 0 \leq x_1 \leq 10 \\ & 0 \leq x_2 \leq 15 \end{aligned}$$

and at least one of the following holds.

$$\begin{aligned} -2x_1 + 3x_2 &\geq 0 \\ 5x_1 - 4x_2 &\geq 0 \end{aligned}$$

2 Assignment

1. (15 points) The following table shows the process times, release times, due dates and weights for three jobs that need to be done on a machine. The schedule is non-preemptive, i.e., once the job has been started, it must be finished before starting another. For each of these problems, formulate a model and solve using solver.

	Job 1	Job 2	Job 3
Process time	15	6	9
Release time	5	10	0
Due Date	20	27	38
Weight	6	10	40

   (a) Completion time of a job is the time the job is completed. Find a schedule that minimizes weighted completion time.

   (b) Find a schedule that minimizes the maximum completion time of any job.

(c) Now suppose that it is fine to not do a job by its deadline. Lateness of a job is the amount of time the job is late for, i.e., (completion time)- (due date). Find a schedule that has total tardiness of at most 100 but is of minimizes weighted completion time.

2. (10 points) Solve the following problem using cutting planes.

$$\begin{aligned}
&\min \ \ 4x_1 + 5x_2 \\
&x_1 + 4x_2 \geq 5 \\
&3x_1 + 2x_2 \geq 7 \\
&x_1, x_2 \in \mathbb{Z}_+
\end{aligned}$$

3. (10 points) Solve the following problem using Branch and bound.

$$\begin{aligned}
&\max \ \ 2x_1 + 3x_2 \\
&x_1 + 2x_2 \leq 3 \\
&6x_1 + 8x_2 \leq 15 \\
&x_1, x_2 \in \mathbb{Z}_+
\end{aligned}$$

4. (10 points) The university has 500 courses whose final exam it must schedule in 28 different slots. Since a student may be taking multiple courses, the exams for such courses should not be at the same time. You are also given the enrollment data detailing which students have registered for which courses.

   (a) Formulate a integer linear program that finds out a schedule so that every student can take exams for all their registered courses.

   (b) Suppose the formulation in the above part is infeasible. Update your model to minimize the number of re-exams that need to be held.

5. (10 points) The owner of a big motor company wants to build $k = 10$ new factories in different areas. All factories make the same product. The owner has $n = 15$ customers. Customer i demands d_i units of the product. The operating cost of the factory j is $f_j \geq 0$ and the maximum number of units it can make is M_j. The cost of delivering 1 unit from factory i to customer j is c_{ij}. Where should the owner build his new factories in order to minimise the delivery cost? Formulate the above problem as an integer program.

6. (15 points) The following diagram below shows the intersections at which automatic traffic monitoring devices might be installed. A station at any particular node can monitor all the road links meeting it. Number next to the nodes represent the monthly cost (in thousands of dollars) of operating a station at that location.

   (a) Formulate the problem of providing full coverage at minimum total cost and solve using a solver.

   (b) Revise your formulation to minimize the number of uncovered road links while using at most two stations that can't be placed on intersections connected by a link. Again solve your formulation.

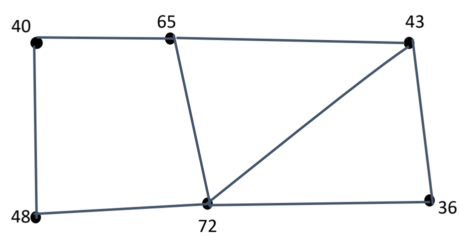

Figure 1: Road network with station costs.