



Purely Functional Data Structures

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GitHub Rep: <https://github.com/KarenKhou/progFct>

Introduction : What is a purely Funct. Data Structure?

A purely functional data structure is one that is designed to work within a functional programming paradigm, where **immutability** and **referential transparency** are key principles.

Immutability: Once created, it cannot be modified

Referential transparency: No Side Effects. An expression can be replaced with its value.

Advantages

No Hidden State: Easier to debug

Persistent: Previous versions remain accessible after updates

Memory: Structural Sharing

Concurrency: purely functional data structures are automatically thread safe

Structures Explored:

- Lists / Streams / Haskell Std lib
- Binomial Heap
- 23 Trees
- Zipper
- Finger Tree



Language Used:

- **Haskell**
- **Python**



Benchmarking

Criterion: To measure the execution time of Haskell code with high precision and reliability.

Deepseq: To force the FULL evaluation of data structures instead of lazily evaluation which is important for controlling memory usage and performance

PLEASE NOTE: Our benchmark is forcing haskell to fully evaluate our data structures. So in reality, the time measured and the efficacy of those data structures should be



Lists Implementation - Haskell

File: list.py and List1.hs

```
-- Custom List Definition
data List a = Empty | Cons a (List a) deriving (Show, Eq)

-- Making List an instance of NFData
instance NFData a => NFData (List a) where
    rnf Empty = ()
    rnf (Cons x xs) = x `deepseq` rnf xs
```

INSTANCE OF THE TYPE CLASS NFData -> can be fully evaluated using deepseq
rnf method recursively forces evaluation during benchmarking

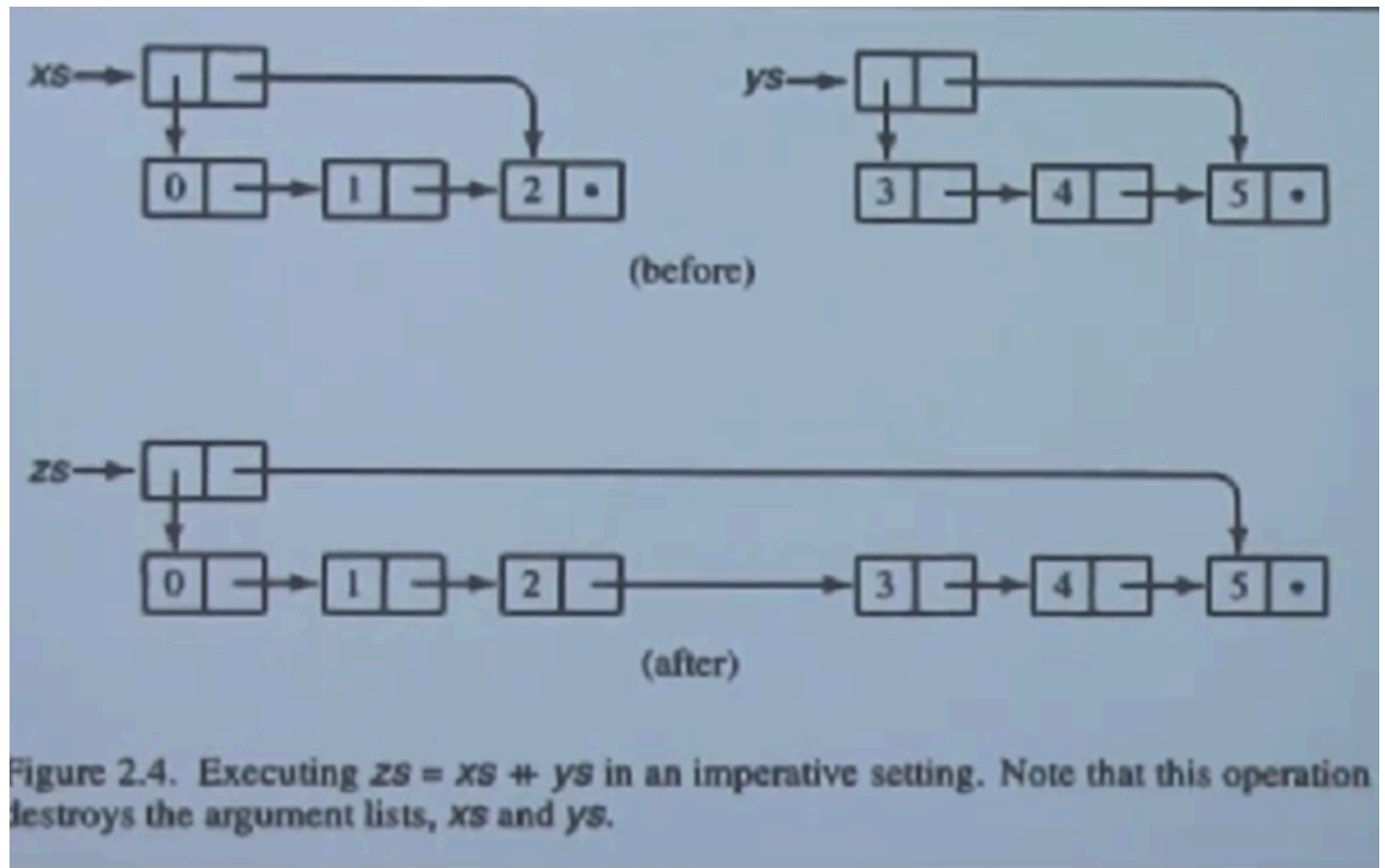
Implemented Functions: map,filter,fold,toList,fromList

- toList: Converts a standard Haskell list into a custom List (a list of Cons).
- fromList: Converts a custom List back into a standard Haskell list.

Complexity:

- append : $O(n)$
- Insert : $O(1)$
- AddFirst : $O(1)$
- Search : $O(n)$

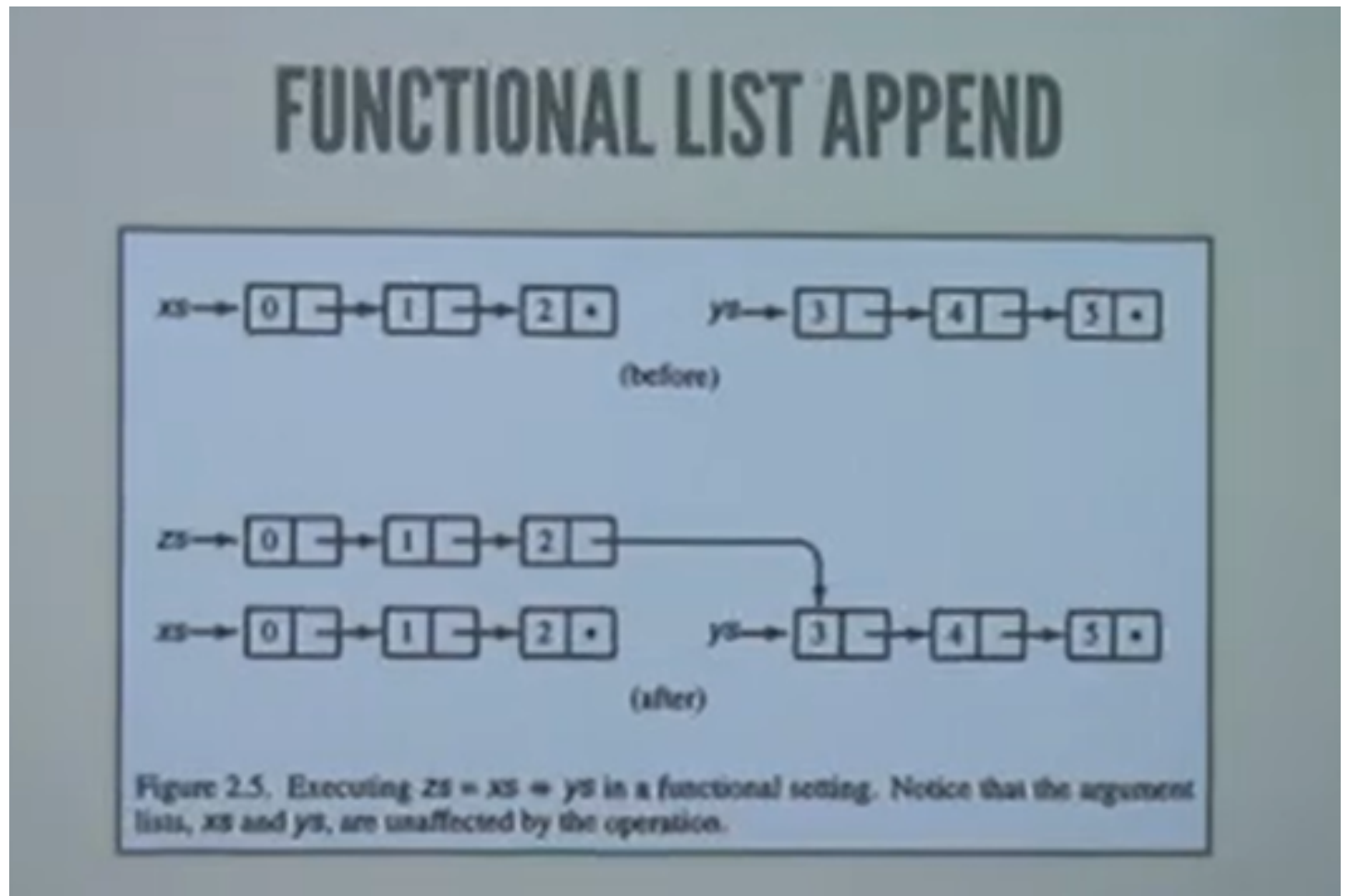
Mutable Lists: (normal concatenation)



y has not changed, but *x* is not only formed by 3 elements anymore, because the Node 2 has now a reference to node 3 => Mutation of *x*

```
def concatenate_lists(lst1, lst2):
    # If lst1 is empty, return lst2
    if lst1 is None:
        return lst2
    # Otherwise, find the tail of lst1 and attach lst2 to it
    current = lst1
    while current.next:
        current = current.next
    current.next = lst2
    return lst1
```

Immutable Lists Concatenation:



We copy xs because of immutability.

Increase Usage of memory? To compensate: Structural Sharing / Lazy Evaluation

```
-- Appending two Lists
append :: List a -> List a -> List a
append Empty ys = ys
append (Cons x xs) ys = Cons x (append xs ys)
```

Lists Implementation - Python

Linked List

```
class ListNode:
    def __init__(self, value, next_node=None):
        self.value = value
        self.next = next_node

    def __str__(self):
        values = []
        current = self
        while current is not None:
            values.append(str(current.value))
            current = current.next
        return " -> ".join(values)
```

Feature	Python Linked List	Haskell Linked List
Mutability	Mutable	Immutable
Ease of Implementation	Manual implementation required	Built-in with extensive support
Memory Efficiency	Higher overhead due to explicit nodes	Efficient due to structural sharing
Lazy Evaluation	Not supported	Supported, enabling infinite lists
Performance	Manual optimization needed	Optimized for immutability

Lists Implementation Haskell benchmark

benchmarking mapList/smallList

time	1.160 μ s	(1.149 μ s .. 1.172 μ s)
	0.999 R^2	(0.999 R^2 .. 1.000 R^2)
mean	1.164 μ s	(1.155 μ s .. 1.173 μ s)
std dev	30.66 ns	(25.41 ns .. 37.51 ns)
variance introduced by outliers: 35% (moderately inflated)		

benchmarking mapList/largeList

time	35.16 ms	(34.05 ms .. 36.35 ms)
	0.996 R^2	(0.993 R^2 .. 0.999 R^2)
mean	33.58 ms	(33.06 ms .. 34.26 ms)
std dev	1.213 ms	(937.7 μ s .. 1.573 ms)
variance introduced by outliers: 11% (moderately inflated)		

benchmarking reverseList/smallList

time	1.476 μ s	(1.411 μ s .. 1.547 μ s)
	0.988 R^2	(0.984 R^2 .. 0.994 R^2)
mean	1.411 μ s	(1.363 μ s .. 1.461 μ s)
std dev	168.1 ns	(140.1 ns .. 212.1 ns)
variance introduced by outliers: 92% (severely inflated)		

benchmarking reverseList/largeList

time	33.59 ms	(29.22 ms .. 40.50 ms)
	0.936 R^2	(0.880 R^2 .. 0.987 R^2)
mean	40.36 ms	(37.72 ms .. 46.51 ms)
std dev	7.922 ms	(3.297 ms .. 14.33 ms)
variance introduced by outliers: 72% (severely inflated)		

Memory in use: 77594624 bytes

benchmarking filterList/smallList

time	938.4 ns	(922.0 ns .. 958.9 ns)
	0.997 R ²	(0.996 R ² .. 0.999 R ²)
mean	924.0 ns	(909.6 ns .. 937.8 ns)
std dev	48.33 ns	(39.66 ns .. 59.62 ns)
variance introduced by outliers: 69% (severely inflated)		

benchmarking filterList/largeList

time	26.18 ms	(25.65 ms .. 26.89 ms)
	0.995 R ²	(0.988 R ² .. 0.999 R ²)
mean	25.82 ms	(25.37 ms .. 26.31 ms)
std dev	1.031 ms	(804.6 µs .. 1.332 ms)

benchmarking foldList/smallList

time	498.8 ns	(491.8 ns .. 505.1 ns)
	0.998 R ²	(0.997 R ² .. 0.999 R ²)
mean	506.2 ns	(498.2 ns .. 515.9 ns)
std dev	29.57 ns	(24.33 ns .. 37.50 ns)
variance introduced by outliers: 74% (severely inflated)		

benchmarking foldList/largeList

time	17.56 ms	(17.09 ms .. 18.07 ms)
	0.995 R ²	(0.989 R ² .. 0.998 R ²)
mean	17.05 ms	(16.71 ms .. 17.37 ms)
std dev	782.0 µs	(574.0 µs .. 1.040 ms)
variance introduced by outliers: 16% (moderately inflated)		

Lists Implementation

Benchmark Group	Benchmark Name	Time (Mean)
mapList	<code>smallList</code>	1.236 μ s
	<code>largeList</code>	33.81 ms
filterList	<code>smallList</code>	925.7 ns
	<code>largeList</code>	26.26 ms
foldList	<code>smallList</code>	472.7 ns
	<code>largeList</code>	16.43 ms
reverseList	<code>smallList</code>	1.177 μ s
	<code>largeList</code>	31.26 ms
append	<code>small + small</code>	1.408 μ s
	<code>large + small</code>	24.46 ms
insert	<code>insert at start</code>	819.3 ns
	<code>insert at middle</code>	1.193 μ s
	<code>insert at end</code>	1.746 μ s
listLookup	<code>lookup 2 in smallList</code>	377.9 ns
	<code>lookup 100000 in largeList</code>	15.68 ms

Streams - Lazy Evaluated lists

PS: In Haskell all lists are Steams.

Advantages:

- Good for defining sequences can be infinite
- Append is still $O(n)$ but paid as it's used

Side effect of Lazy Evaluation:

- In non-functional programming, lazy evaluation can lead to unintended consequences.
- log messages may not be written as expected.
- This behavior can result in hidden or missing logs, making debugging and monitoring more challenging.



Binary Tree

[File: binarytree.hs](#)

Binary Tree Data Structure

The binary tree is a fundamental structure used to store data hierarchically. Here's how it's defined:

`data Tree a = Empty | Node a (Tree a) (Tree a) deriving (Show)`

- Tree a represents a binary tree where each Node contains:
 - A value of type a.
 - A left subtree.
 - A right subtree.
- Empty represents a leaf or an empty tree.

For example, a tree like this:

```
  1
 / \
2   3
```

is represented as:

`Node 1 (Node 2 Empty Empty) (Node 3 Empty Empty)`

```

data BinaryTree a = Empty
                  | Node a (BinaryTree a) (BinaryTree a)
                  deriving (Show, Eq)

-- Insert an element into the binary search tree
insert :: (Ord a) => a -> BinaryTree a -> BinaryTree a
insert x Empty = Node x Empty Empty
insert x (Node v left right)
  | x < v      = Node v (insert x left) right
  | x > v      = Node v left (insert x right)
  | otherwise  = Node v left right -- No duplicates allowed

-- In-order traversal
inOrder :: BinaryTree a -> [a]
inOrder Empty = []
inOrder (Node v left right) = inOrder left ++ [v] ++ inOrder right

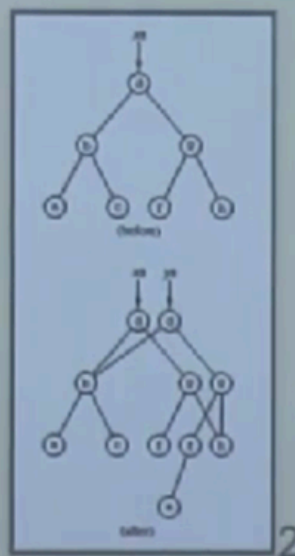
```

Complexity: (balanced tree)

- Insertion $O(\log n)$
- Search $O(\log n)$

INSERTION

As we insert a new value, we're able to reuse subtrees on the untraveled path.



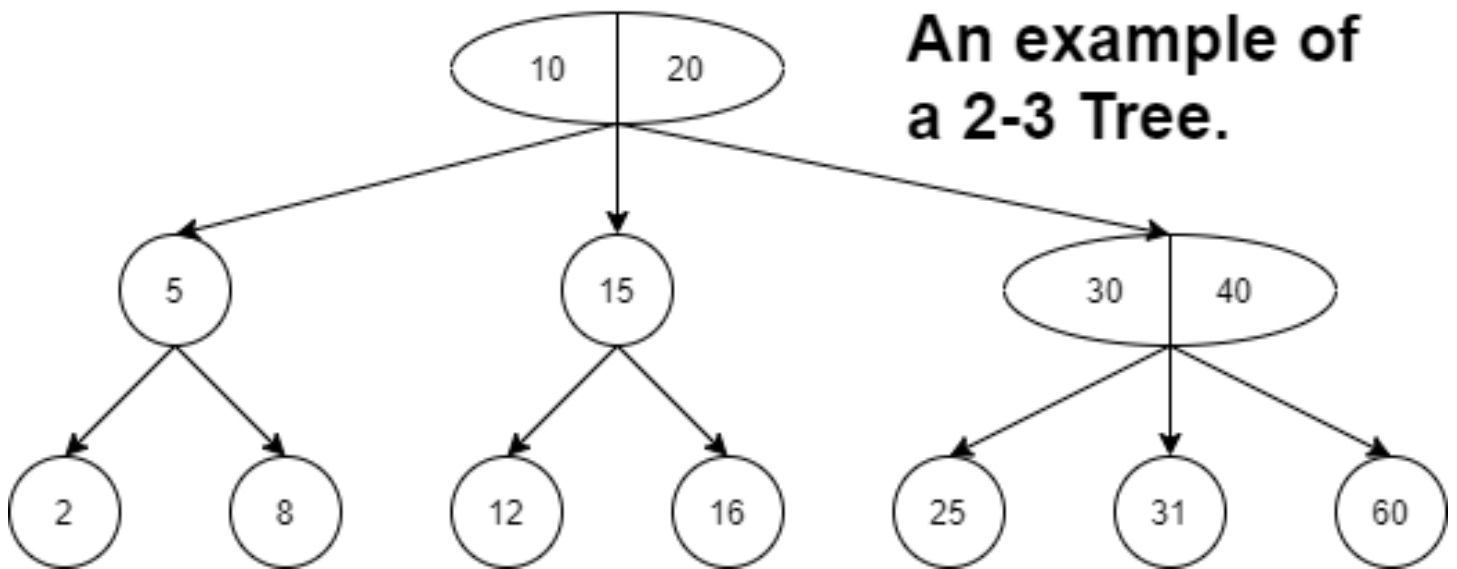
As we insert a new value, we are able to reuse subtrees

```
let tree1 = insert 10 tree
let tree2 = insert 5 tree1
let tree3 = insert 15 tree2
let tree4 = insert 7 tree3
let tree5 = insert 12 tree4
```



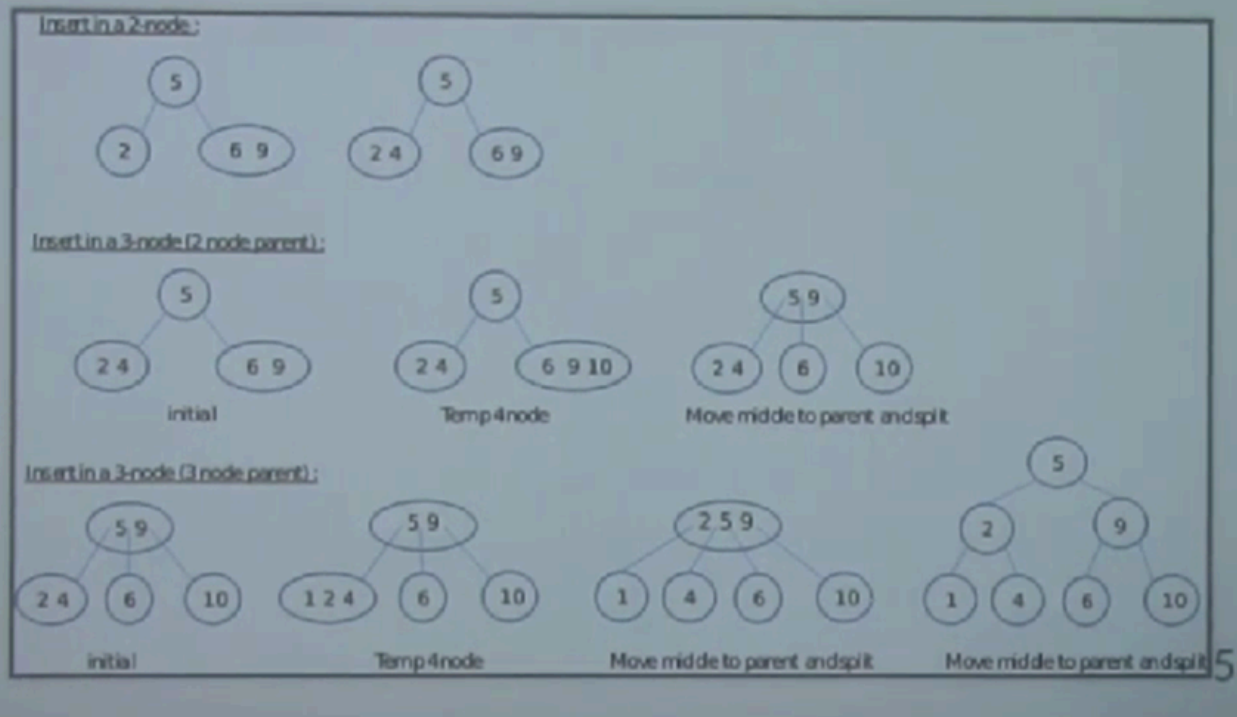
23 Tree

TwoThreeTree.hs and 2-3Tree.py



```
data Tree k v = Leaf
  | Node2 (Tree k v) k v (Tree k v)
  | Node3 (Tree k v) k v (Tree k v) k v (Tree k v)
  deriving (Show, Read, Eq)
```

COMPLEX INSERTION PROCESS



```
insert :: (Ord k) => k -> v -> (Tree k v) -> (Tree k v)
insert k v t =
  let add k v Leaf = Pushed Leaf k v Leaf
      {- First we handle all the corner cases, when a visited node is empty -}
      add k v (Node2 Leaf k' v' Leaf)
        | k < k' = Consumed (Node3 Leaf k v Leaf k' v' Leaf)
        | k == k' = Consumed (Node2 Leaf k v Leaf)
        | otherwise = Consumed (Node3 Leaf k' v' Leaf k v Leaf)
      add k v (Node3 Leaf k' v' Leaf k'' v'' Leaf)
        | k < k' = Pushed (singleton k v) k' v' (singleton k'' v'')
        | k == k' = Consumed (Node3 Leaf k v Leaf k'' v'' Leaf)
        | k' < k && k < k'' = Pushed (singleton k' v') k v (singleton k'' v'')
        | k == k'' = Consumed (Node3 Leaf k' v' Leaf k v Leaf)
        | otherwise = Pushed (singleton k' v') k'' v'' (singleton k v)
      {- Typical cases, when a visited node is full -}
      add k v (Node2 l k' v' r)
        | k < k' = case add k v l of
            Consumed newL -> Consumed (Node2 newL k' v' r)
            Pushed newL k'' v'' newR -> Consumed (Node3 newL k'' v'' newR k' v' r)
        | k == k' = Consumed (Node2 l k v r)
        | otherwise = case add k v r of
            Consumed newR -> Consumed (Node2 l k' v' newR)
            Pushed newL k'' v'' newR -> Consumed (Node3 l k' v' newL k'' v'' newR)
      add k v (Node3 l k' v' m k'' v'' r)
        | k < k' = case add k v l of
            Consumed newL -> Consumed (Node3 newL k' v' m k'' v'' r)
            Pushed newL x y newR -> Pushed (Node2 newL x y newR) k' v' (Node2 m k'' v'' r)
        | k == k' = Consumed (Node3 l k v m k'' v'' r)
        | k' < k && k < k'' = case add k v m of
            Consumed newM -> Consumed (Node3 l k' v' newM k'' v'' r)
            Pushed newL x y newR -> Pushed (Node2 l k' v' newL) x y (Node2 newR k'' v'' r)
        | k == k'' = Consumed (Node3 l k' v' m k v r)
        | otherwise = case add k v r of
            Consumed newR -> Consumed (Node3 l k' v' m k'' v'' newR)
            Pushed newL x y newR -> Pushed (Node2 l k' v' m) k'' v'' (Node2 newL x y newR)
      {- If the two subtrees have been pushed whole way up the tree,
         we create a new Node2 root with these subtrees as children. -}
      in case add k v t of
        Consumed newT -> newT
        Pushed newL x y newR -> Node2 newL x y newR
```

23Tree Benchmarking

Operation	Time (Mean)
Insert/Small Tree	471.3 ns
Insert/Large Tree	5.883 µs
Lookup/Small Tree	613.2 ns
Lookup/Large Tree	5.996 µs
fromList/Small List	1.489 µs
fromList/Large List	63.03 ms
toList/Small Tree	737.2 ns
toList/Large Tree	2.278 µs

Lists Benchmarking

Benchmark Group	Benchmark Name	Time (Mean)
insert	insert at start	819.3 ns
	insert at middle	1.193 µs
	insert at end	1.746 µs
listLookup	lookup 2 in smallList	377.9 ns
	lookup 100000 in largeList	15.68 ms

List vs Tree Benchmarking:

**23 Tree have a more complex insertion,
But are more efficient at lookup than lists**

23Tree- Python vs Haskell

The Python code uses a mutable object and modifies the tree in place, whereas the Haskell code creates new trees for every insertion due to immutability.

Haskell's style is more declarative, and its use of pattern matching makes it more concise,

After benchmarking, 23tree is better than normal linked lists for lookups even if insertion is more complex (5 micro sec VS 1.5 micro sec)

Haskell Std Libraries

File : liststd.hs

Data.List:

- Sequential traversal ($O(n)$).
- Prepending ($:$) is $O(1)$
- Appending ($++$) is $O(n)$.
- Indexing ($!!$) is $O(n)$.

It is Lightweight and has a simpler memory layout.

Data.Sequence:

- Prepending ($<|$) and appending ($|>$) $O(\log n)$.
- Indexing and updates are $O(\log n)$

It is A FINGER TREE structure that uses more memory but reduces redundant computations.

	type	[a]	Seq a
head	<code>:: Seq a -> a</code>	$O(1)$	$O(1)$
tail	<code>:: Seq a -> Seq a</code>	$O(1)$	$O(\log n)$
cons	<code>:: a -> Seq a -> Seq a</code>	$O(1)$	$O(\log n)$
last	<code>:: Seq a -> a</code>	$O(n)$	$O(1)$
init	<code>:: Seq a -> Seq a</code>	$O(n)$	$O(\log n)$
snoc	<code>:: Seq a -> a -> Seq a</code>	$O(n)$	$O(\log n)$
(++)	<code>:: Seq a -> Seq a -> Seq a</code>	$O(n)$	$O(\log n)$
(!!)	<code>:: Seq a -> Int -> a</code>	$O(n)$	$O(\log n)$

[link vid](#)

ACCESSING DATA

```
benchmarking Access/List Access
```

```
time          7.802 µs    (7.737 µs .. 7.859 µs)
              0.999 R²    (0.999 R² .. 1.000 R²)
mean          7.865 µs    (7.802 µs .. 7.959 µs)
std dev       273.8 ns    (198.7 ns .. 414.9 ns)
variance introduced by outliers: 43% (moderately inflated)
```

```
benchmarking Access/Seq Access
```

```
time          149.1 ns    (147.5 ns .. 150.9 ns)
              0.999 R²    (0.998 R² .. 0.999 R²)
mean          149.5 ns    (148.0 ns .. 151.3 ns)
std dev        5.405 ns    (4.414 ns .. 7.226 ns)
variance introduced by outliers: 55% (severely inflated)
```

Appending DATA

```
benchmarking Append/List Append
```

```
time          76.59 ns    (74.60 ns .. 78.35 ns)
              0.997 R²    (0.996 R² .. 0.998 R²)
mean          74.70 ns    (73.50 ns .. 75.91 ns)
std dev        4.018 ns    (3.388 ns .. 4.703 ns)
variance introduced by outliers: 74% (severely inflated)
```

```
benchmarking Append/Seq Append
```

```
time          273.7 ns    (263.4 ns .. 285.5 ns)
              0.990 R²    (0.986 R² .. 0.994 R²)
mean          281.7 ns    (274.6 ns .. 291.4 ns)
std dev        27.54 ns    (20.54 ns .. 43.08 ns)
variance introduced by outliers: 90% (severely inflated)
```

Weird right? Why in our benchmark Appending in SEQ is taking longer than append in LIST?

We just said that appending should be more efficient!

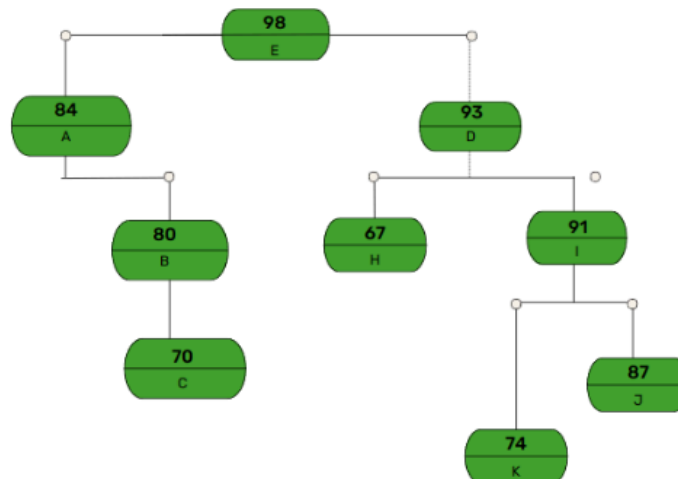
=>Amortized $O(\log n)$ means that most of the time, the operation will be fast, but there are occasional, more expensive operations due to the tree rebalancing that happens



Finger Tree

File : fingerTree.hs and fingerTree.py

Finger Search Tree



finger search trees can be a good choice for applications that require efficient searching and flexible data structures, but may not be the best choice for all situations. It is important to consider the specific requirements of your application and evaluate the performance and tradeoffs of different data structures before making a decision. Therefore, finger search trees are a powerful data structure that can provide fast searching, insertion, and deletion operations. While they can be complex to implement, they are well worth the effort for applications where searching is a critical operation.

A finger tree is a more specialized and optimized type of tree. It is a functional data structure used in functional programming (like Haskell) to represent sequences efficiently.

Finger trees keep fingers to both ends of the sequence to allow efficient access. These are like "shortcut" references to the first and last parts of the sequence

A finger is a point where one can access part of a data structure; in imperative languages, this is called a pointer.

```
data FingerTree a = Empty | Single a | Deep (FingerTree (Node a)) (Node a) (FingerTree (Node a))
```

Deep: A deep node with two parts:

- A left subtree (finger reference pointing to the left end),
- A middle node that holds data,
- A right subtree (finger reference pointing to the right end).

Accessing Elements

First Element

The first function retrieves the first element:

```
first :: FingerTree a -> Maybe a
```

```
first Empty = Nothing
```

```
first (Single x) = Just x
```

```
first (Deep _ (x:_) _) = Just x
```

- If the tree is Empty, it returns Nothing.
- If the tree is a Single node, it returns the value.
- If the tree is Deep, it extracts the first element from the middle section [x:_].

Last Element

The last function retrieves the last element:

```
last :: FingerTree a -> Maybe a
```

```
last Empty = Nothing
```

```
last (Single x) = Just x
```

```
last (Deep _ _ y) = last y
```

- If the tree is Empty, it returns Nothing.
- If the tree is a Single node, it returns the value.
- If the tree is Deep, it recursively calls last on the right subtree (y).

Constructing a FingerTree

The fromList function creates a FingerTree from a list:

```
fromList :: [a] -> FingerTree a
```

```
fromList [] = Empty
```

```
fromList [x] = Single x
```

```
fromList xs = Deep (fromList (take half xs)) (take half xs) (fromList (drop half xs))
```

```
where half = length xs `div` 2
```

- An empty list results in an Empty tree.
- A single-element list results in a Single node.
- For a longer list:
 - It divides the list into two halves.

- Recursively constructs left and right subtrees from the halves.
- Places the first half in the middle section of the Deep node.

Showing the FingerTree Structure

The `showFingerTree` function provides a string representation of the tree:

```
showFingerTree :: Show a => FingerTree a -> String
```

```
showFingerTree Empty = "Empty"
```

```
showFingerTree (Single x) = "Single " ++ show x
```

```
showFingerTree (Deep l xs r) = "Deep (" ++ showFingerTree l ++ ") " ++ show xs ++ " (" ++ showFingerTree r ++ ")"
```

- Displays the structure in a human-readable form, useful for debugging.

Inserting Elements

Insert at the Left

The `insertLeft` function inserts an element at the left end:

```
insertLeft :: a -> FingerTree a -> FingerTree a
```

```
insertLeft x Empty = Single x
```

```
insertLeft x (Single y) = Deep Empty [x] (Single y)
```

```
insertLeft x (Deep l xs r) = Deep (insertLeft x l) xs r
```

- If the tree is Empty, it creates a Single node.
- If the tree is a Single node, it creates a Deep node, placing the new element in the middle section.
- For a Deep node, it recursively inserts the element into the left subtree.

Insert at the Right

The `insertRight` function inserts an element at the right end:

```
insertRight :: a -> FingerTree a -> FingerTree a
```

```
insertRight x Empty = Single x
```

```
insertRight x (Single y) = Deep (Single y) [x] Empty
```

```
insertRight x (Deep l xs r) = Deep l xs (insertRight x r)
```

- Similar to `insertLeft`, but recursively inserts into the right subtree instead.

Example Usage

The main function demonstrates the operations:

```
main :: IO ()
```

```
main = do
```

```
let ft = fromList [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
```

```
putStrLn $ "FingerTree: " ++ showFingerTree ft
```

```
putStrLn $ "First element: " ++ show (first ft)
```

```
putStrLn $ "Last element: " ++ show (last ft)
```

```
let ft = fromList [1, 2, 3, 4, 5, 6]
```

```
putStrLn $ "Original FingerTree: " ++ showFingerTree ft
```

```
let ftLeft = insertLeft 0 ft
```

```
putStrLn $ "After inserting 0 at the left: " ++ showFingerTree ftLeft
```

```
let ftRight = insertRight 7 ft
```

```
putStrLn $ "After inserting 7 at the right: " ++ showFingerTree ftRight
```

1. A FingerTree is created from a list.
2. The first and last elements are accessed using first and last.
3. Elements are inserted at the left and right ends, and the modified trees are displayed.

Advantages of Finger Search Tree:

- **Efficient searching:** Finger trees support fast search operations with a worst-case time complexity of $O(\log n)$, making them suitable for applications that require frequent searching.
- **Flexible:** Finger trees can be used to implement a wide range of data structures, including priority queues, ordered sets, and sequences.
- **Persistent:** Finger trees support efficient persistent data structures, which allow multiple versions of a data structure to be maintained without affecting the original structure.
- **Easy to implement:** Finger trees have a simple and elegant recursive structure that makes them easy to implement and understand.

Disadvantages of Finger Search Tree:

- **Overhead:** Finger trees have a higher memory overhead than other search tree data structures because they require additional information to be stored at each node, including the size of the subtree.
- **Complex operations:** Some operations, such as concatenation and splitting, can be more complex and less efficient in finger trees than in other search tree data structures.
- **Lack of popularity:** Finger trees are not as widely used as other search tree data structures, so there may be less community support and fewer third-party libraries available.

```
data FingerTree a
  = Empty
  | Single a
  | Deep (FingerTree a) [a] (FingerTree a)

-- Access the first element
first :: FingerTree a -> Maybe a
first Empty = Nothing
first (Single x) = Just x
first (Deep _ (x:_) _) = Just x

-- Access the last element
last :: FingerTree a -> Maybe a
last Empty = Nothing
last (Single x) = Just x
last (Deep _ _ y) = last y
```

Example: Deep (Node [1,2]) (Node [3,4]) (Node [5,6])

Finger references: The left and right subtrees of the tree maintain fingers to the leftmost and rightmost nodes respectively.

Accessing First and Last Elements:

- First element: To get the first element, you start at the leftmost chunk (Node [1,2]) and access its leftmost element (here 1).
- Last element: To get the last element, you start at the rightmost chunk (Node [4,5]) and access its rightmost element (here 5).



Zipper

File: Zipper.hs and zipper.py

The Zipper

REFERENCE: LearnYouAHaskell Chapter 14 Zippers

The **Zipper** is a way to traverse and modify a tree **without rebuilding the entire structure** every time. It achieves this by keeping track of the current position in the tree (the focus) and a trail of breadcrumbs showing how to get back to the root.

Components:

1. **Crumbs (Breadcrumbs):** These keep track of the path taken to reach the current focus:
2. `data Crumb a = LeftCrumb a (Tree a) | RightCrumb a (Tree a) deriving (Show)`
 - LeftCrumb means you came from the left child of a node and stores:
 - The value of the parent node (a).
 - The right subtree of the parent (the part you didn't go down).
 - RightCrumb is similar but indicates coming from the right child.

1. **Zipper Type:**

4. `type Zipper a = (Tree a, Breadcrumbs a)`

- The first part is the current focus (Tree a).
- The second part is the trail (Breadcrumbs a) to retrace steps back to the root.

Tree Traversal and Modifications

The code provides operations to move around the tree, modify nodes, and attach new subtrees:

1. **Move Left (goLeft):** Moves focus to the left child of the current node and updates the breadcrumbs.
2. **Move Right (goRight):** Moves focus to the right child similarly.
3. **Go Back (goUp):** Moves the focus back to the parent node using breadcrumbs.
4. **Modify a Node (modify):** Changes the value at the current focus (e.g., applying a function like (+1) to increment).
5. **Attach a New Subtree (attach):** Replaces the current focus with a new subtree.

Why Zipper?

The Zipper allows us to:

- Traverse a tree efficiently without rebuilding it.

- Modify parts of a tree while maintaining immutability (a key property in functional programming).

For example:

- Without a zipper, modifying a node would require reconstructing the entire path back to the root.
- With a zipper, we can focus on one node at a time, apply changes, and seamlessly move around.

```
-- Binary tree data structure
data Tree a = Empty | Node a (Tree a) (Tree a) deriving (Show)

-- Breadcrumbs for navigation in the tree
data Crumb a = LeftCrumb a (Tree a) | RightCrumb a (Tree a) deriving (Show)
type Breadcrumbs a = [Crumb a]

-- Zipper type combining the current focus and breadcrumbs
type Zipper a = (Tree a, Breadcrumbs a)
```

```
lst = [1, 2, 3, 4]
zipper = ListZipper([], lst[0], lst[1:])
print(zipper)  # Left: [], Focus: 1, Right: [2, 3, 4]
zipper = zipper.move_right()
print(zipper)  # Left: [1], Focus: 2, Right: [3, 4]
zipper = zipper.update_focus(5)
print(zipper)  # Left: [1], Focus: 5, Right: [3, 4]
print(zipper.to_list())  # [1, 5, 3, 4]
```

Zippers Implementation

Operation	Time (Mean)
Zipper Operations/goLeft	135.1 ns
Zipper Operations/goRight	136.1 ns
Zipper Operations/goUp	116.5 ns
Zipper Operations/modify	131.0 ns



Binomial Heap

File: BinomialHeap.hs and binomialHeap.py

A binomial heap is a collection of binomial trees satisfying the min-heap property, where the key of a parent node is always less than or equal to the keys of its children. It is a type of priority queue data structure that supports efficient insertion, deletion, and finding the minimum element.

Merge (Union): Merging two binomial heaps is one of the most efficient operations and takes $O(\log n)$ time.

Binomial Tree Basics

1. Binomial Tree:

- A binomial tree of rank r is recursively defined:
 - A binomial tree of rank 0 is a single node.
 - A binomial tree of rank $r+1$ is formed by linking two binomial trees of rank r , where the smaller root becomes the parent.
- Example of a binomial tree of rank 2:

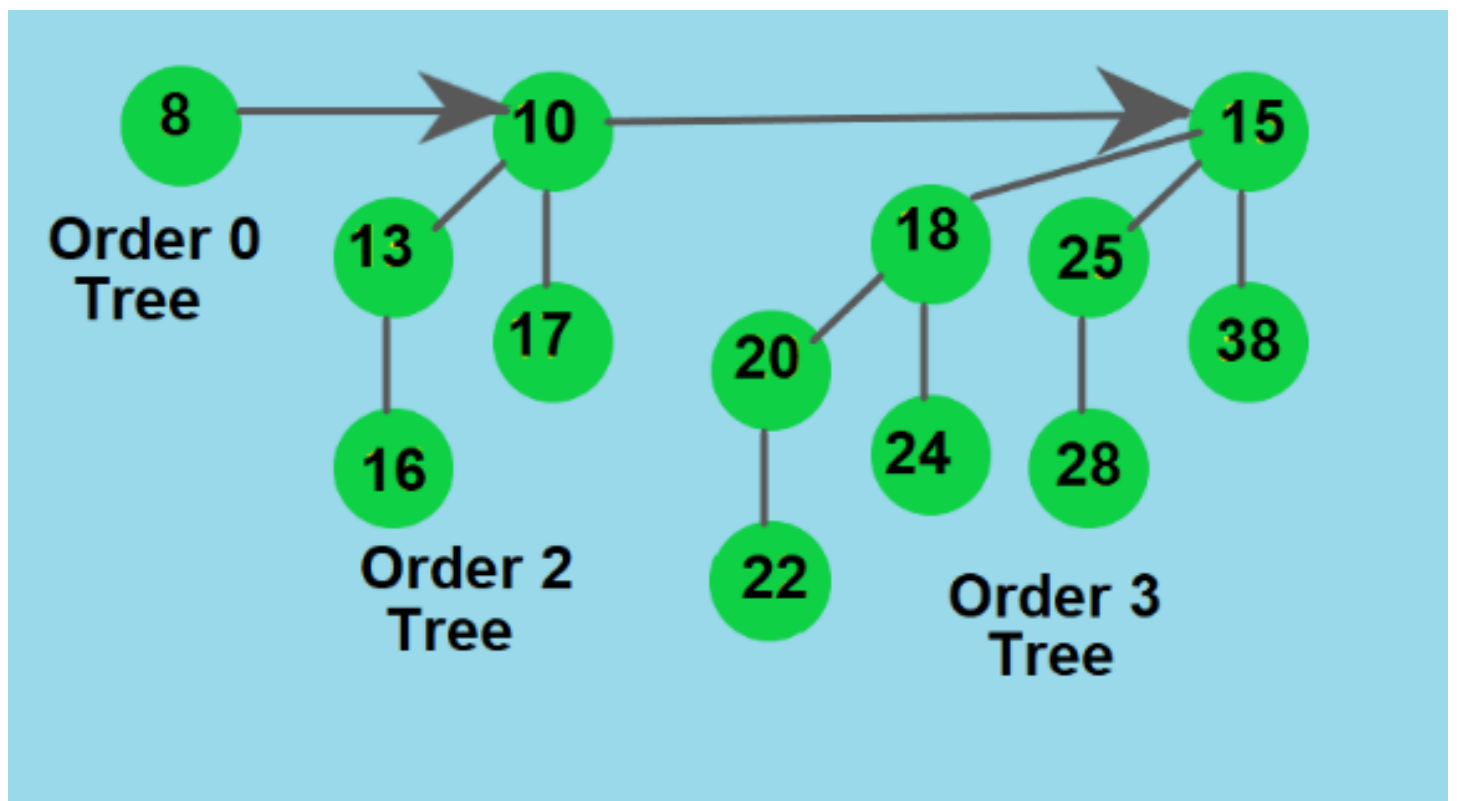
1

/ \

2 3

1. Heap Property:

- Every parent node has a value less than or equal to its children (min-heap).



```
benchmarking BinomialHeap Operations/insert
time          142.4 ns    (137.2 ns .. 147.4 ns)
              0.992 R²    (0.990 R² .. 0.997 R²)
mean          139.7 ns    (136.4 ns .. 143.7 ns)
std dev       11.48 ns    (9.090 ns .. 14.20 ns)
variance introduced by outliers: 87% (severely inflated)
```

```
benchmarking BinomialHeap Operations/merge
time          99.30 ns    (97.99 ns .. 100.7 ns)
              0.998 R²    (0.997 R² .. 0.999 R²)
mean          100.9 ns    (99.55 ns .. 102.9 ns)
std dev        5.272 ns    (4.386 ns .. 6.953 ns)
variance introduced by outliers: 72% (severely inflated)
```

```
benchmarking BinomialHeap Operations/deleteMin
time          589.4 ns    (582.8 ns .. 596.6 ns)
              0.999 R²    (0.998 R² .. 0.999 R²)
mean          588.9 ns    (581.8 ns .. 599.0 ns)
std dev       26.96 ns    (19.45 ns .. 35.68 ns)
variance introduced by outliers: 63% (severely inflated)
```

Use cases of binomial Heaps

- **Merging Efficiency:** Binomial heaps allow merging of two priority queues in $O(\log n)$ time, while normal priority queues typically require $O(n)$ time for merging.
- **Insertion Time:** Both binomial heaps and normal priority queues offer $O(\log n)$ insertion time, but binomial heaps handle merging operations more efficiently.
- **Memory Structure:** Binomial heaps are more memory-efficient than normal priority queues when handling large numbers of elements due to their tree-based structure, while normal priority queues often use arrays or linked lists. (The memory for these trees is more compact because each tree is linked in a way that reduces redundant storage, using pointers to represent relationships between nodes.)

Code Explanation

We implemented a binomial heap in both Haskell and Python. Now, let's dive into the Haskell implementation to explain its structure and functionality.

BinomialHeap.hs

Implementation Details

Data Structures

1.Tree

data Tree a = Node Int a [Tree a]

- Represents a binomial tree.
 - Int: Rank of the tree.
 - a: Value of the root element.
 - [Tree a]: List of child trees.

2.Heap

type Heap a = [Tree a]

- Represents a binomial heap as a list of binomial trees.

3.Element Typeclass

class Ord a => Element a where

leq :: a -> a -> Bool

- Defines a custom comparison method (leq) for elements in the heap.
- Ensures flexibility in handling different data types.

Heap Operations

1.Initialization

- empty: Represents an empty heap.

empty :: Heap a

- isEmpty: Checks whether a heap is empty.

isEmpty :: Heap a -> Bool

2.Tree Manipulation

- rank: Retrieves the rank of a tree.
rank :: Tree a -> Int
- root: Retrieves the root element of a tree.
root :: Tree a -> a
- link: Combines two trees of the same rank into one, preserving the heap property.
link :: (Element a) => Tree a -> Tree a -> Tree a

3.Insertion

- insTree: Inserts a tree into a heap, maintaining structural properties.
insTree :: (Element a) => Tree a -> Heap a -> Heap a
- insert: Adds a new element to the heap.
insert :: (Element a) => a -> Heap a -> Heap a

4.Merging

- merge: Combines two heaps into one while preserving the heap property.
merge :: (Element a) => Heap a -> Heap a -> Heap a

5.Finding and Deleting the Minimum

- removeMinTree: Removes the tree with the smallest root value.
removeMinTree :: (Element a) => Heap a -> (Tree a, Heap a)
- findMin: Retrieves the minimum element from the heap.
findMin :: (Element a) => Heap a -> a
- deleteMin: Deletes the minimum element and rebalances the heap.
deleteMin :: (Element a) => Heap a -> Heap a

Benchmarking

The Criterion library is used for performance measurement

Memory Profiling

The printMemoryUsage function reports memory usage using RTS statistics

<i>Operation</i>	<i>Time Complexity</i>
insert	$O(\log n)$
merge	$O(\log n)$
findMin	$O(\log n)$
deleteMin	$O(\log n)$

Class Definitions

Node Class:

- Represents a node in a Binomial Tree.
- Contains attributes for:
 - data: Value of the node.
 - degree: Number of children (subtrees).
 - child: Pointer to the leftmost child.
 - parent: Pointer to the parent node.
 - sibling: Pointer to the next sibling node.

Key Functions

1. mergeBinomialTrees(b1, b2)

- Merges two Binomial Trees of the same degree.
- Ensures the smaller data tree becomes the root.
- Adjusts the parent and sibling pointers.

Objective: Fundamental operation for building a Binomial Heap.

Time Complexity: $O(1)$.

2. unionBionomialHeap(l1, l2)

- Merges two binomial heaps to one, keeping sorted by their degree.

Steps:

- This function merges the two sorted lists of Binomial Trees
- Returns combined list of trees.

Objective: preparation for adjustment in heap

Time Complexity: $O(\log n)$.

3. adjust(_heap)

- Ensures the Binomial Heap maintains its properties:
 - Increasing order of tree degrees.
 - No two trees have the same degree.
- Merges trees with the same degree using mergeBinomialTrees.

Time Complexity: $O(\log n)$.

4. insertATreeInHeap(_heap, tree)

- Adds a single Binomial Tree to the heap.
- Calls unionBionomialHeap and adjust.

Objective: Allows insertion of nodes into the heap.

5. removeMinFromTreeReturnBHeap(tree)

- Removes the root (minimum element) of a Binomial Tree.
- Converts its children into a new Binomial Heap.

Objective: extract the minimum element.

Time Complexity: $O(\log n)$.

6. insert(_heap, key)

- Creates a new Binomial Tree with the key.
- Inserts it into the existing heap using insertATreeInHeap.

Time Complexity: $O(\log n)$.

7. getMin(_heap)

- Finds the node with the smallest value in the heap.
- Traverses all root nodes to find the minimum.

Time Complexity: $O(\log n)$.

8. extractMin(_heap)

- Finds and removes the minimum element in the heap.
- Merges the remaining heap with the heap formed by the children of the minimum element.
- Calls unionBinomialHeap and adjust to restore heap properties.

Objective: Extracts the smallest value in the heap.

Time Complexity: $O(\log n)$.

9. printTree(h) and printHeap(_heap)

- Prints the Binomial Tree and the entire heap.
- Recursively traverses and displays nodes.

Code Comparison with Haskell

<i>Feature</i>	<i>Python</i>	<i>Haskell</i>
Language Paradigm	Object-Oriented (Classes and Methods).	<i>Functional Programming (Pure Functions and Immutable Data).</i>
Implementation	<i>Uses classes (Node) and dynamic data structures (lists) for heap storage.</i>	<i>Defines recursive types for nodes and trees; relies on immutability and recursion.</i>
State Management	<i>Explicit tracking of parent, sibling, and child relationships through pointers.</i>	<i>Recursive pattern matching to traverse and manipulate data.</i>
Performance Profiling	<i>Profiling uses Python libraries like memory_profiler and time to track execution time and memory usage.</i>	<i>Profiling relies on GHC tools like Criterion for benchmarking; no explicit profiling in code.</i>
Debugging	<i>Python's syntax make debugging more intuitive.</i>	<i>Haskell's declarative style and lazy evaluation can make debugging challenging but concise.</i>



Conclusion

Purely functional data structures are ideal when immutability, safety, and concurrency are priorities.

However, they may come with performance trade-offs, particularly in memory

Where are we using FUNCTIONAL data structures nowadays?

- In environments that require high concurrency, such as multi-core processors or distributed systems (immutability)
- Blockchain and Cryptocurrencies
- Many NoSQL databases use immutable data structures to support features like distributed transactions, and versioned data.
- In real-time systems (e.g., embedded systems, robotics), where consistency, safety, and predictable behavior are critical

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