Repetition Codes

(n,1,n) Repetition Codes

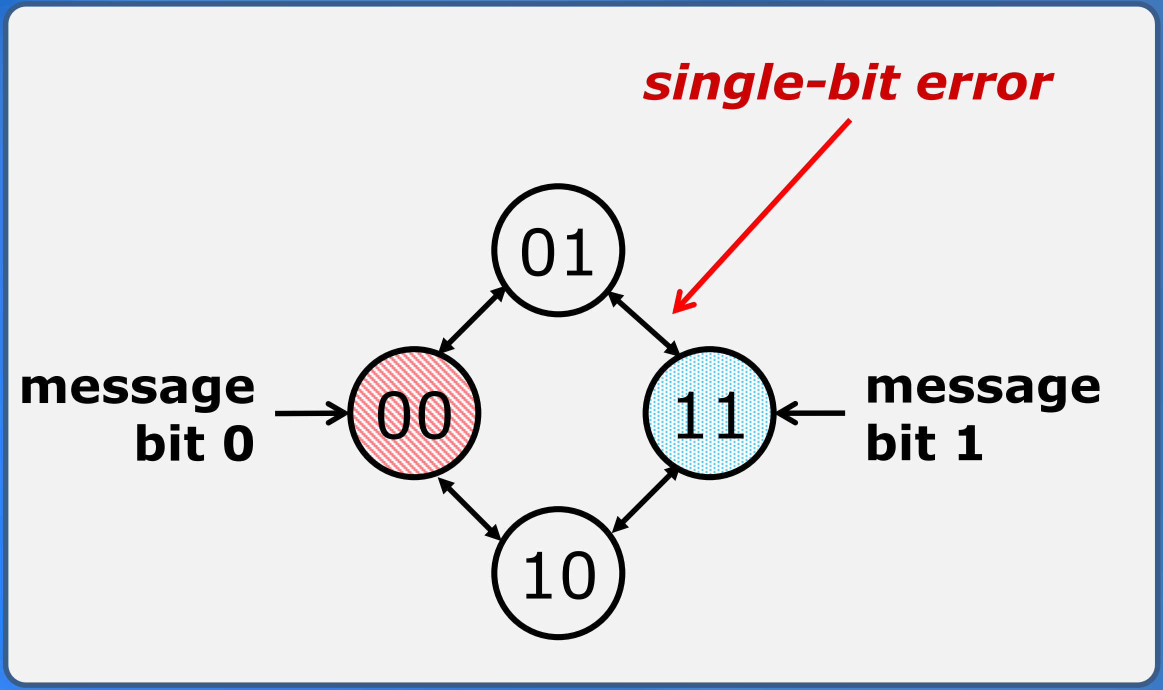
- Repetition codes are block codes, where
  - The message is split to 1-bit blocks.
  - Blocks are expanded to n bits by repeating the bit n times.
- Examples
  - (2,1,2) repetition code
    - message bit 0 → Codeword 00
    - message bit 1 → Codeword 11

    code rate = $\frac{1}{2}$

  - (3,1,3) repetition code
    - message bit 0 → Codeword 000
    - message bit 1 → Codeword 111

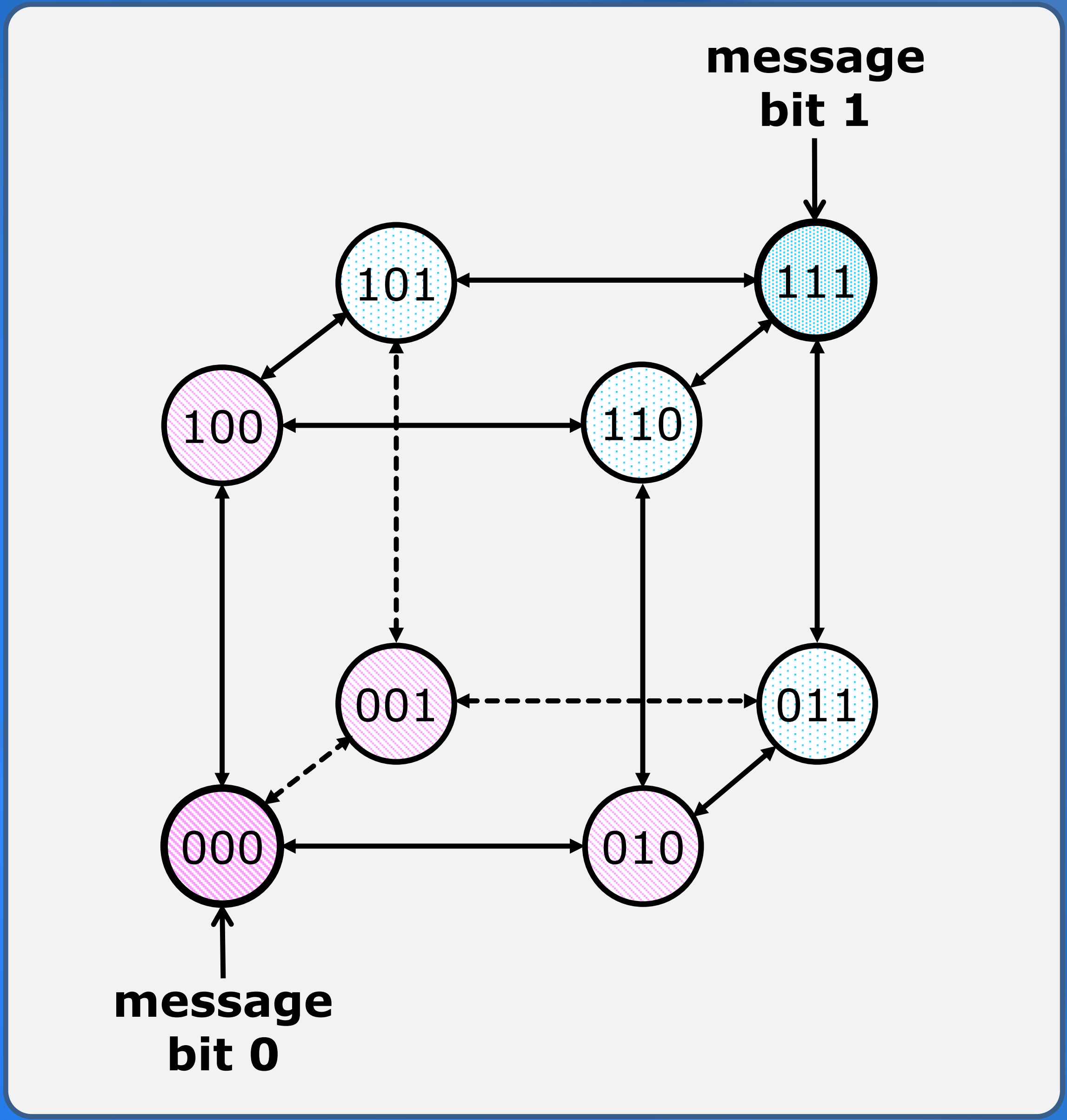
    code rate = $\frac{1}{3}$

The (2,1,2) Repetition Code



- **Each bit is repeated twice.**
  - **Each code word has an even number of "1" bits. We refer to this as "even parity".**
- **The Hamming distance between code words is d=2.**
- **This code can be used to detect errors in up to d-1=1 bit .**
  - **There is an error if the number of received "1" bits in the code word is odd.**

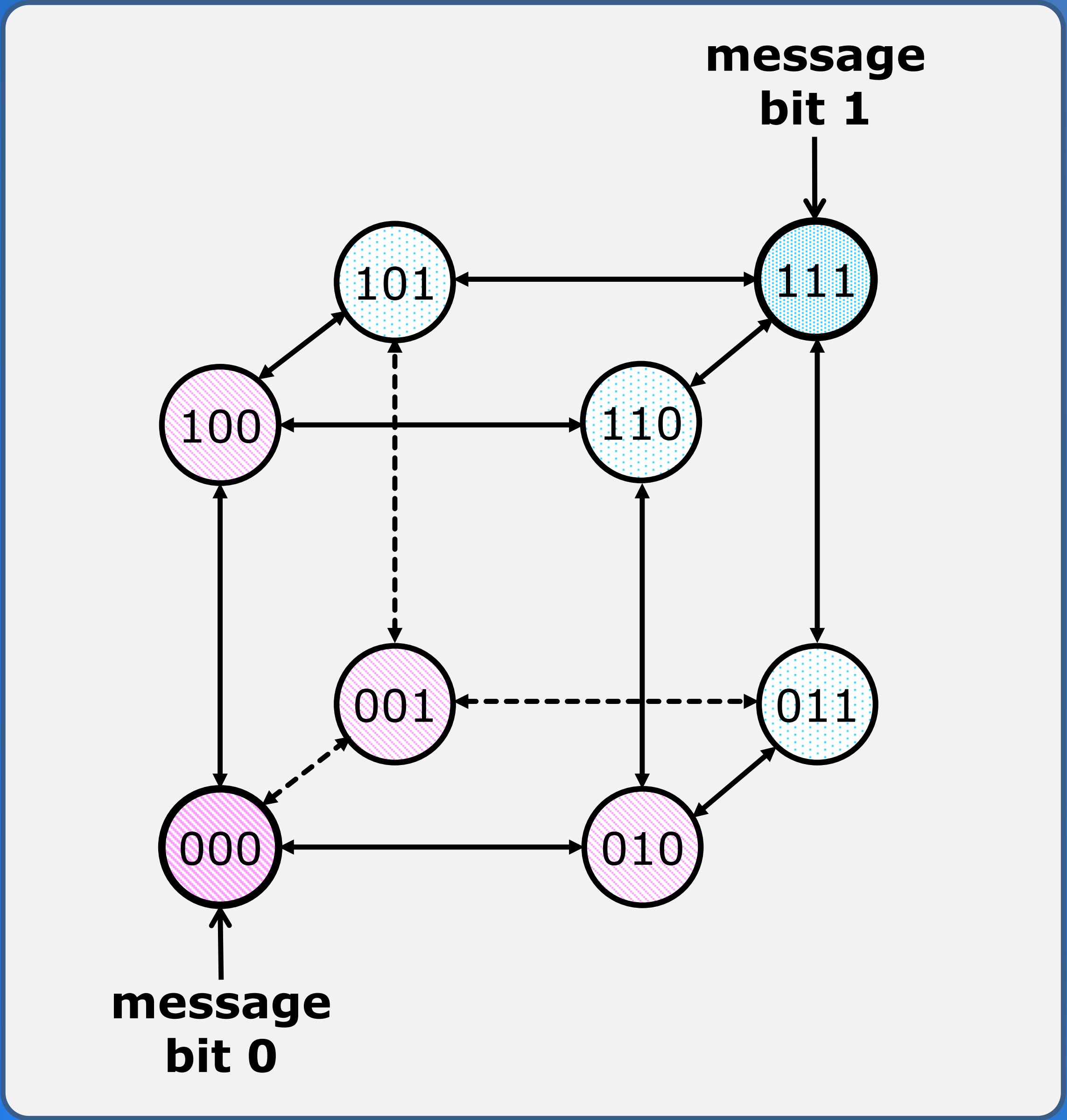
The (3,1,3) Repetition Code



- Each bit is repeated 3 times.
- The Hamming distance between code words is d=3.
- We can EITHER
  - Detect errors in up to d-1=2 bits

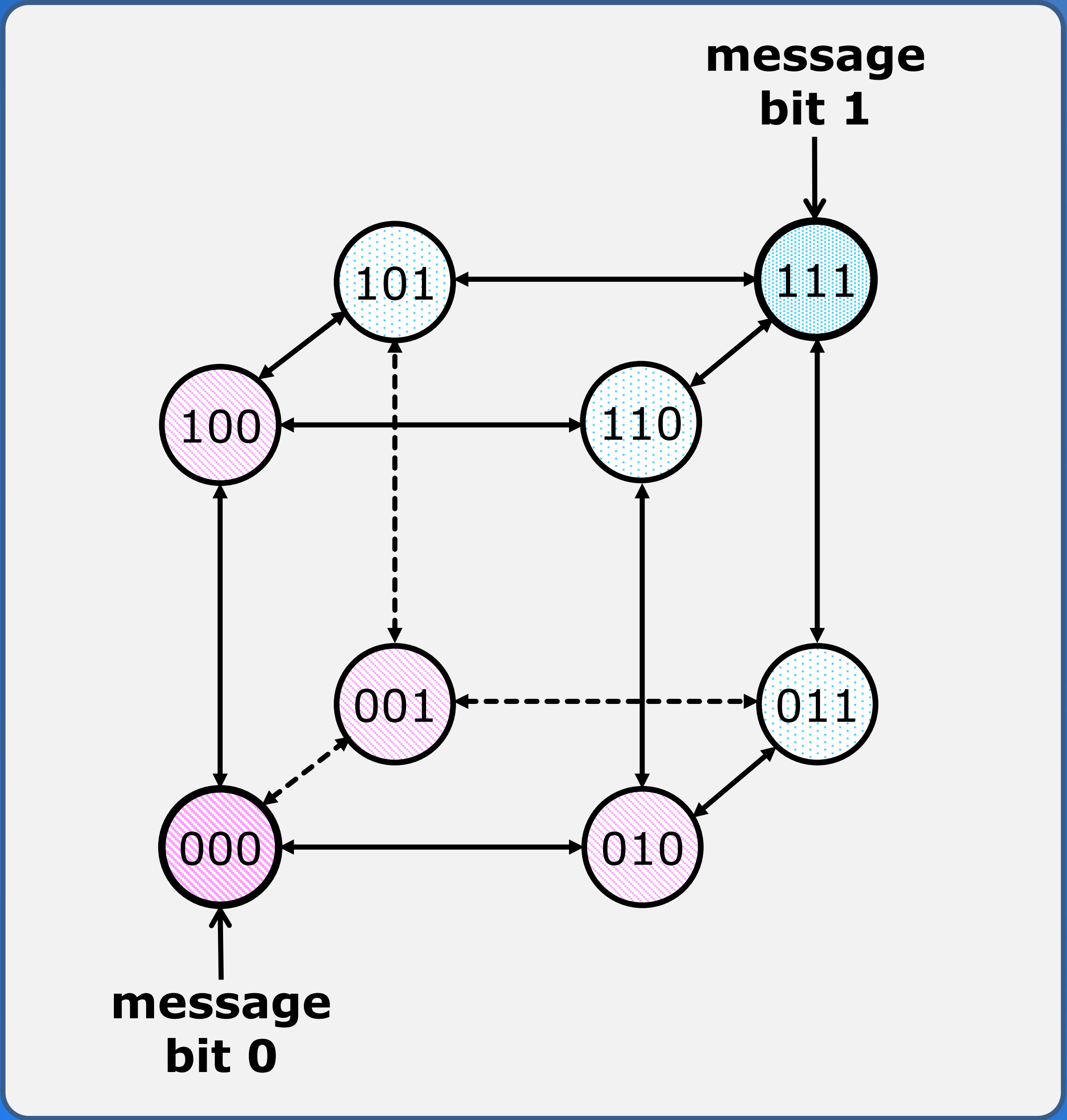
    OR
  - Detect and correct errors in up to $\frac{d-1}{2} = 1$ bit

Detecting Errors



- If we receive and observe a codeword with a mixture of 0's and 1's, we know that an error has occurred.
- If we receive 100, either
  - 000 was sent and a 1 bit error occurred.
  - 111 was sent and a 2 bit error occurred.

Correcting Errors



- If we assume that at most 1-bit error can occur, we can do error correction.
- If we receive 100, since at most 1-bit error occurred, 000 must have been sent.
- We can correct errors by seeing whether 0 or 1 has the most "votes".

The (4,1,4) Repetition Code

- We can EITHER
  - Detect errors in up to d-1=3 bits.

    OR
  - ~~Detect and correct errors in up to $\frac{d-1}{2}$ = 1.5 bits?~~

    Detect and correct 1 bit error, and detect 2 bit errors.

- For example
  - If we observe 1000, then
    - Either a 1 bit or a 3 bit error occurred.
    - If we correct, we assume the 3 bit error did not occur.
  - If we observe 1001, the codewords 0000 or 1111 are equidistant. We have no reliable way to decide which was transmitted.