# Derivation of Equalizer

# Deriving the Equalizer

- **According to Model 2:**

$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$$

- **Solving for x(n):**

$$x(n) = \frac{1}{(1-a) \cdot k}\left[y(n) - a \cdot y(n-1)\right]$$

**This is only true if the output of the channel is exactly described by this equation, however, there may be unmodeled effects, such as nonlinearity and noise or incorrect parameters.**

- **Thus, this is just an approximation to the input:**

$$\tilde{x}(n) = \frac{1}{(1-a) \cdot k}\left[y(n) - a \cdot y(n-1)\right]$$

# Interpretation in Terms of Changes

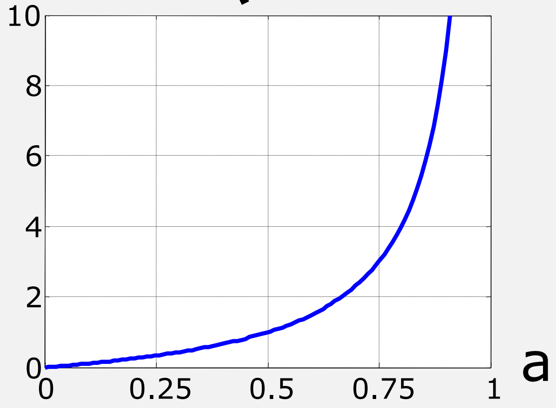
$$x(n) = \frac{1}{(1-a)\cdot k}\left[y(n) - a\cdot y(n-1)\right]$$

$$= \frac{1}{(1-a)\cdot k}\left[(1-a)y(n) + ay(n) - a\cdot y(n-1)\right]$$

$$= \frac{1}{k}\left[y(n) + \underbrace{\frac{a}{(1-a)}}\underbrace{\left(y(n) - y(n-1)\right)}_{\text{change in output}}\right]$$

current channel output

$$\frac{a}{1-a}$$




$$= \begin{cases} 0 & \text{if} \quad a = 0 \\ \infty & \text{if} \quad a = 1 \end{cases}$$

# Example

**Suppose we have a channel whose step response can be described by**

$$s(n) = \frac{1}{2}\left(1 - \left(\frac{2}{3}\right)^{n+1}\right)u(n)$$

**where u(n) is the unit step function.**
**What is the equation for the equalizer for this channel?**

## Solution:

**Step 1: Find the equivalent recursive model**

$$s(n) = k\left(1 - a^{n+1}\right)u(n) \qquad y(n) = a\cdot y(n-1) + (1-a)\cdot k\cdot x(n)$$

$$k \Downarrow \frac{1}{2} \qquad a \Downarrow \frac{2}{3}$$

$$y(n) = \frac{2}{3}y(n-1) + \frac{1}{6}x(n)$$

# Example

**Solution:**

**Step 2: Invert the recursive model**

$$y(n) = \frac{2}{3} y(n-1) + \frac{1}{6} x(n)$$

$$\Downarrow$$

$$x(n) = 6 \cdot y(n) - 4 \cdot y(n-1)$$