Examples

Example

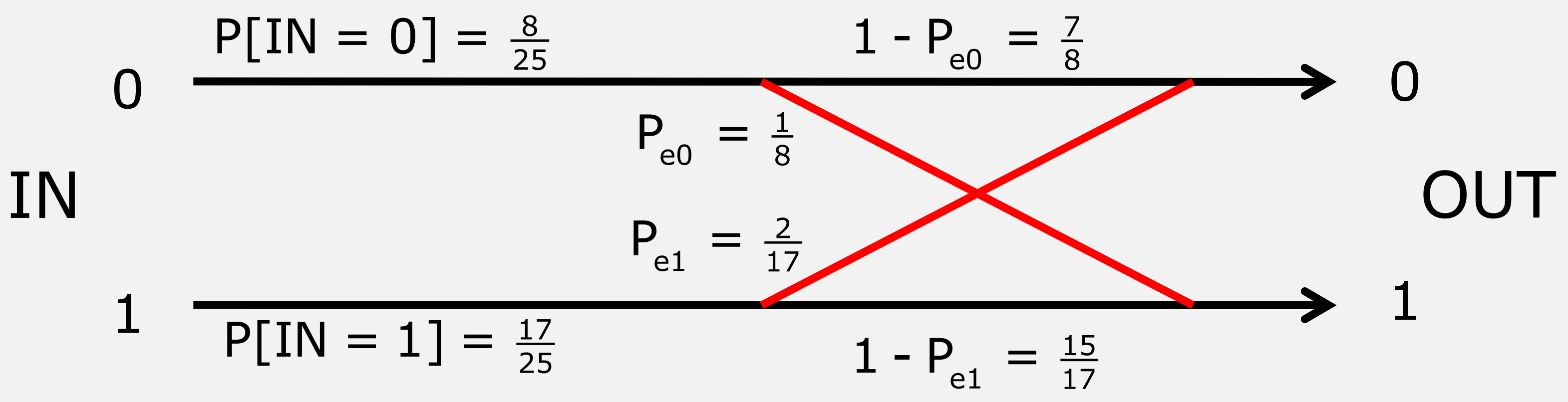
Input/Output Bit Streams

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
IN	1	1	0	1	1	0	0	1	1	1	0	1	1	0	1	0	0	1	1	1	1	0	1	1	1
OUT	1	1	0	1	1	0	0	1	1	1	1	1	1	0	1	0	0	1	1	0	1	0	0	1	1

By definition:

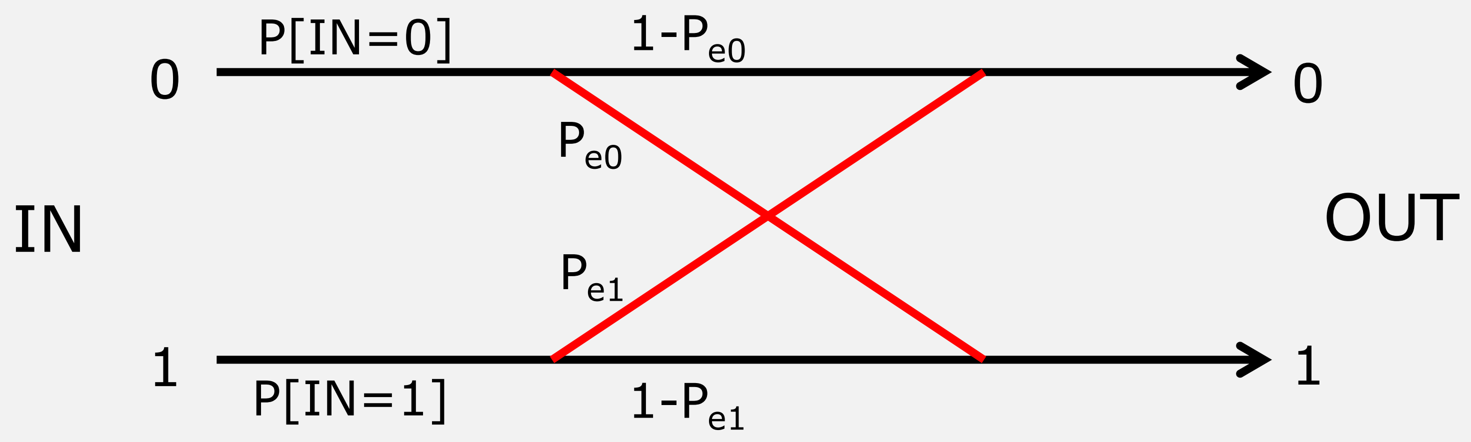
$$\text{BER} \approx \frac{\#\text{ of errors}}{\#\text{ of bit pairs}} = \frac{3}{25} = 12\%$$

Using our formula:



$$\textbf{BER} = P_{e0} \times [IN = 0] + P_{e1} \times [IN = 1] = \frac{1}{8} \times \frac{8}{25} + \frac{2}{17} \times \frac{17}{25} = \frac{3}{25}$$

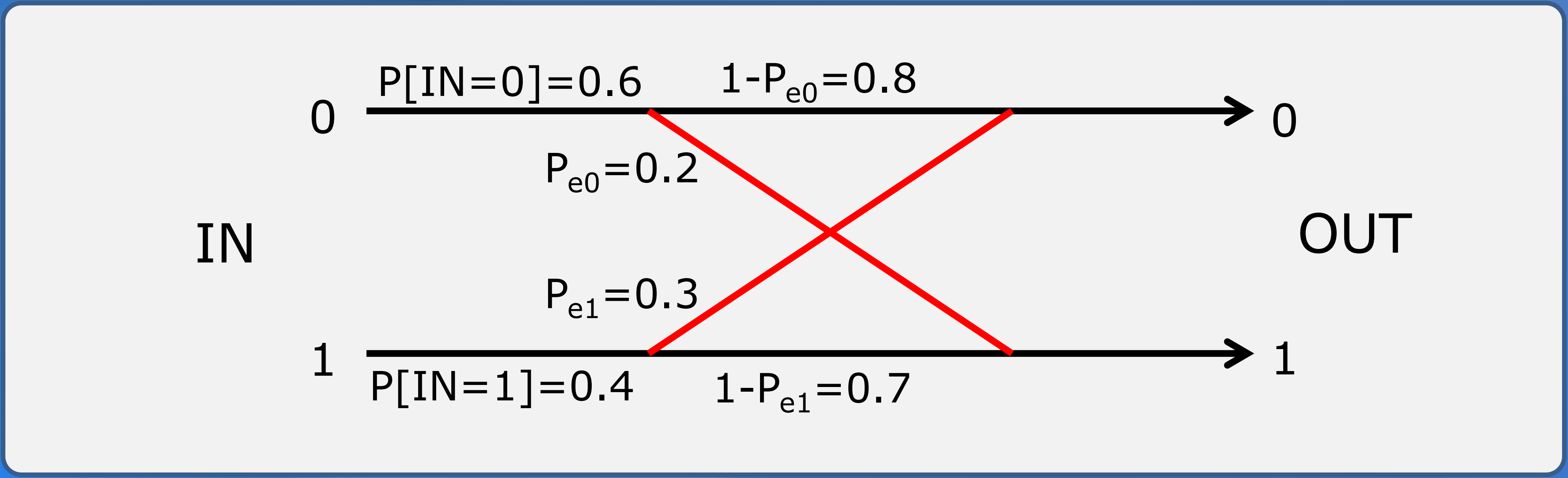
Intuition



$$BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1]$$

- **Since P[IN=0]+P[IN=1]=1,**
  - **The BER is a weighted average of P_{e0} and P_{e1}**
  - **The BER is between P_{e0} and P_{e1}**
  - **If IN=0 is more likely, the BER is closer to P_{e0}**
  - **If IN=1 is more likely, the BER is closer to P_{e1}**
  - **If IN=0 and 1 are equally likely, BER = ½(P_{e0} + P_{e1})**
  - **If $P_{e0} = P_{e1}$, BER = $P_{e0} = P_{e1}$.**

Example BER Calculation

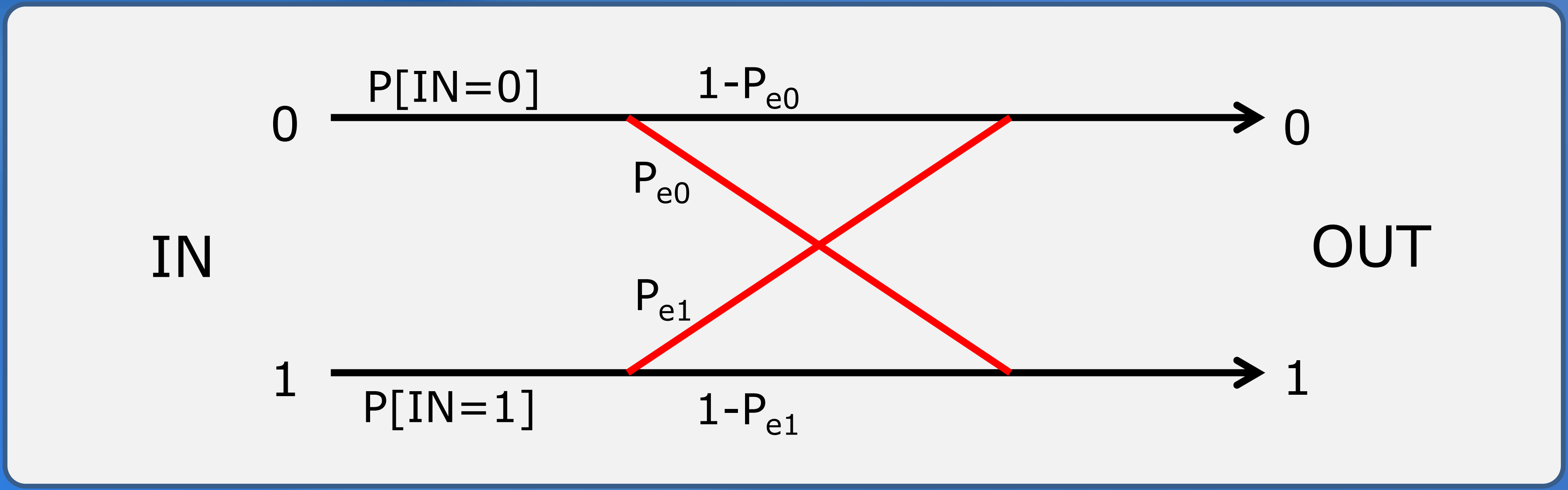


Question: What is the BER for the Binary Channel above?

Solution: $0.2<BER<0.3$

$$\begin{aligned} BER &= P_{e0}\cdot P[IN=0] + P_{e1}\cdot P[IN=1] \\ &= 0.2\times 0.6 + 0.3\times 0.4 \\ &= 0.12+0.12 \\ &= 0.24 \end{aligned}$$

What we need to know to predict BER



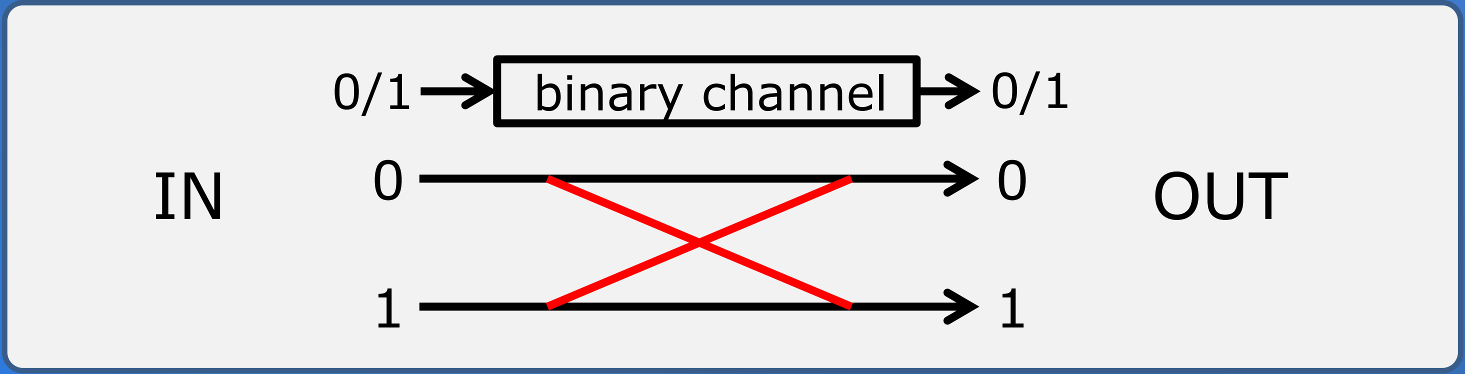
- **In order to predict the BER, we need to know**
  - **P[IN=0] (we can find P[IN=1] = 1 - P[IN=0])**
  - **P_{e0}**
  - **P_{e1}**
- **Usually, the transmitter determines P[IN=0]**
  - **e.g. P[IN=0] = P[IN=1] = 0.5**
- **P_{e0} and P_{e1} depend on**
  - **the transmit levels (r_{min},r_{max})**
  - **the "size" of the noise**

Summary

Noise is one of the critical and fundamental concepts in communications.

Without noise, there would be no difficulty in communication!

We started our analysis by considering only input/output bits using a simple binary channel model.



We use probability to get a formula for BER

$$BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1]$$

Usually, the transmitter controls P[IN=0] and P[IN=1]

- **e.g. P[IN=0] = P[IN=1] = 0.5**