Recursive Channel Model

Recursive Models

- Recursive = involving repeated application of a rule
- A recursive model for a discrete time waveform $x(n)$ has two parts
  - a formula that defines the nth sample in terms of the past samples, e.g.

$$x(n) = f(x(n-1))$$

  - an initial (starting) condition, e.g.

$$x(0) = 0$$

- Generating the waveform by recursion:
  - Given $x(0)$, find $x(1) = f(x(0))$.
  - Given $x(1)$, find $x(2) = f(x(1))$.
  - and so on…

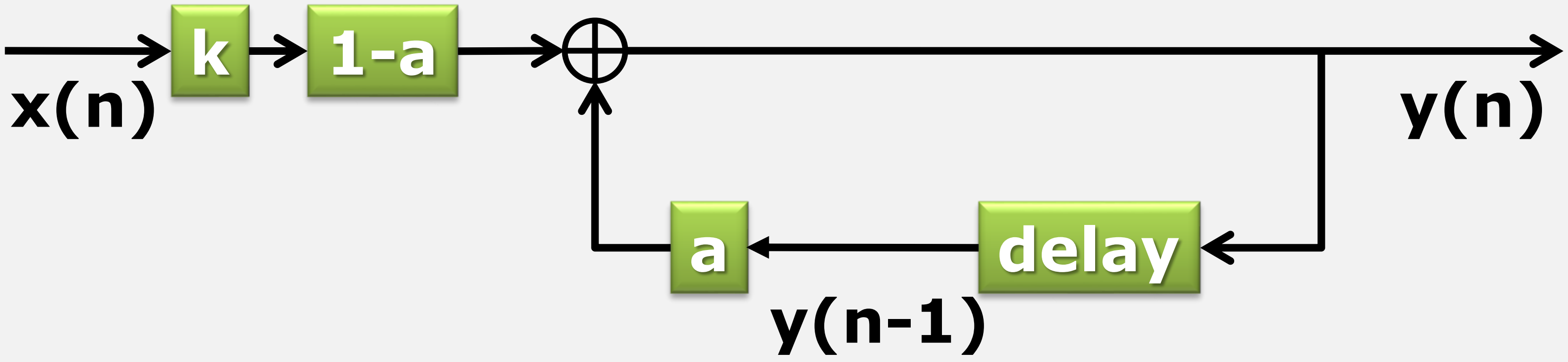
Examples

- Can you think of recursive models for the following sequences:
  - $x(n) = c$ (a constant) $x(0) = c$ $x(n) = x(n-1)$
  - $x(n) = n$ (a linear ramp) $x(0) = 0$ $x(n) = x(n-1) + 1$
  - $x(n) = 0.2n$ $x(0) = 0$ $x(n) = x(n-1) + 0.2$
  - $x(n) =$ an alternating bit stream (0 1 0 1 0 1…) $x(0) = 0$ $x(n) = 1- x(n-1)$

Recursive Model of IR Channel

- It turns out that the response of the IR channel y(n) to an input x(n) can be described by a recursive formula:

$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$$



- The parameter *a* lies between 0 and 1.
- The parameter *k* scales the input.
- This is also known as a “feedback” system, since the output feeds back as an input to the system to determine the next output.

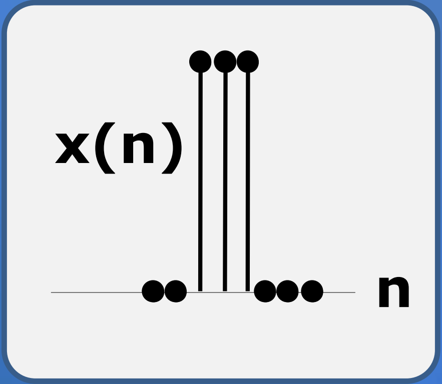
Example

- Given the channel model:

$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$$

- Assume that $a = \frac{1}{2}$ and $k = 1$, i.e.,

$$y(n) = \frac{1}{2}\, y(n-1) + \frac{1}{2}\, x(n)$$



- Find the output of the channel if the input is

n	0	1	2	3	4	5	6	7
x(n)	0	0	1	1	1	0	0	0
y(n)	0	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{16}$	$\frac{7}{32}$	$\frac{7}{64}$

Effect of the Parameter "a"

$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$$

- The parameter a determines the "memory" in the channel

- a = 0
  - no memory of the past
  - $y(n) = k \cdot x(n)$
  - the channel output is just the input multiplied by *k*

- a = 1
  - infinite memory of the past
  - $y(n) = y(n-1)$
  - the channel output is constant, ignores the channel input