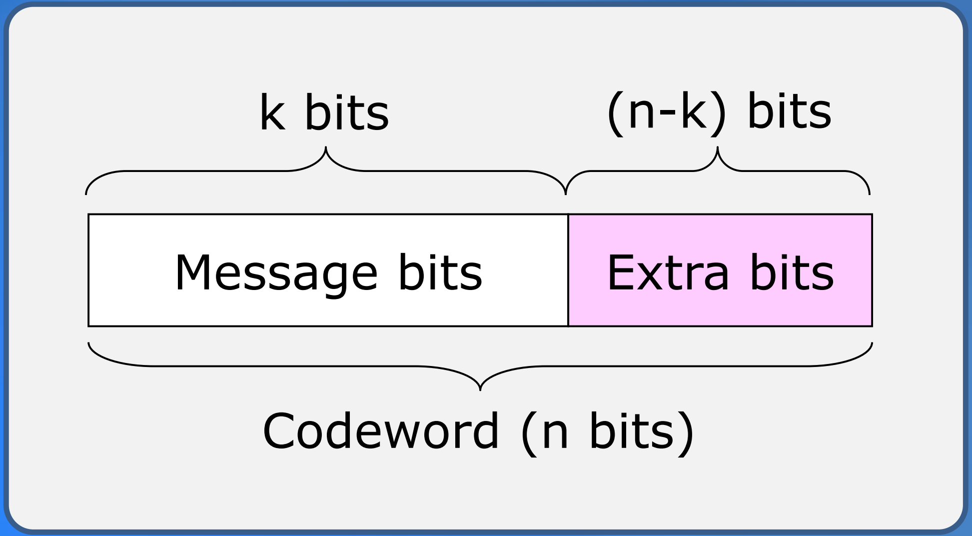
Block Codes

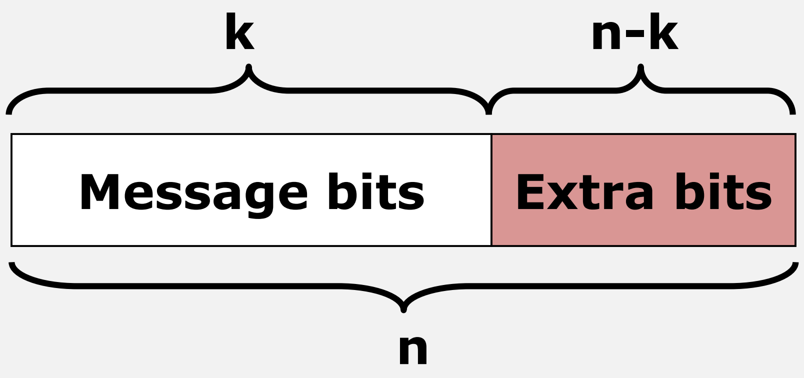
(n,k,d) Block Codes



- Split the message into k-bit blocks
- Create a codeword by adding (n-k) extra bits to each block.
  - The extra bits are computed based on the message bits.
  - Thus, they contain no new information.
- d = minimum Hamming Distance between codewords
- Sometimes we drop the d and indicate only (n,k)

Code Rate

- **Code rate**: the fraction of sent bits that contain useful information (i.e. the message).
- For the (n,k,d) block code



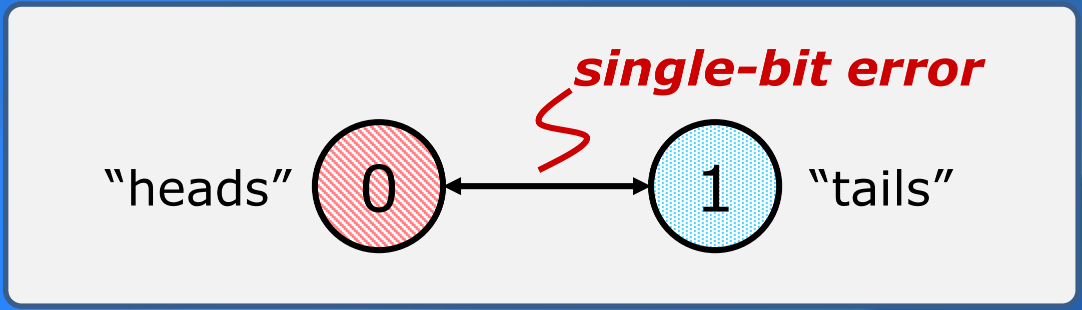
$$\text{code rate} = \frac{k}{n}$$

- Related terms
  - **Gross bit rate**: rate that all bits are sent $= \frac{F_s}{SPB}$
    - Also called the data signaling rate
  - **Net bit rate**: rate that useful bits are sent = code rate × gross bit rate

Hamming Distance

- The **Hamming Distance** between two codewords is the number of bit positions where the corresponding bits are different.
- For example
  - The Hamming distance between (00) and (10) is 1.
  - The Hamming distance between (00000000) and (11110011) is 6.

- The Hamming distance measures the number of bit errors it takes transform one codeword to another.
  - For example, if we use no coding, each bit is represented by one of two code words ("0" and "1").
  - Since the Hamming distance is 1, a single-bit error changes one code word the other.



Error Detection vs Correction

- Error detection
  - We can detect errors
  - But, we don’t know how to fix them
- Error correction
  - We can detect errors
  - And, we can correct them
- For a given code, the receiver can choose whether to use the code to detect errors or to correct them.

The Error Detection/Correction Capability

- The minimum Hamming distance determines the maximum number of bit errors the receiver can detect or correct.
- If the minimum Hamming distance is d, the receiver can either
  - Detect but not correct errors in at most d-1 bits of each codeword

    OR

  - Detect and correct errors in at most (d-1)/2 bits of each code word
- For example, if d = 3, the receiver can either
  - Detect 1 or 2 bit errors in each codeword.
  - Detect and correct 1 bit errors in each codeword.
    - If a 2 bit error does occur, it will be detected, but incorrectly corrected.