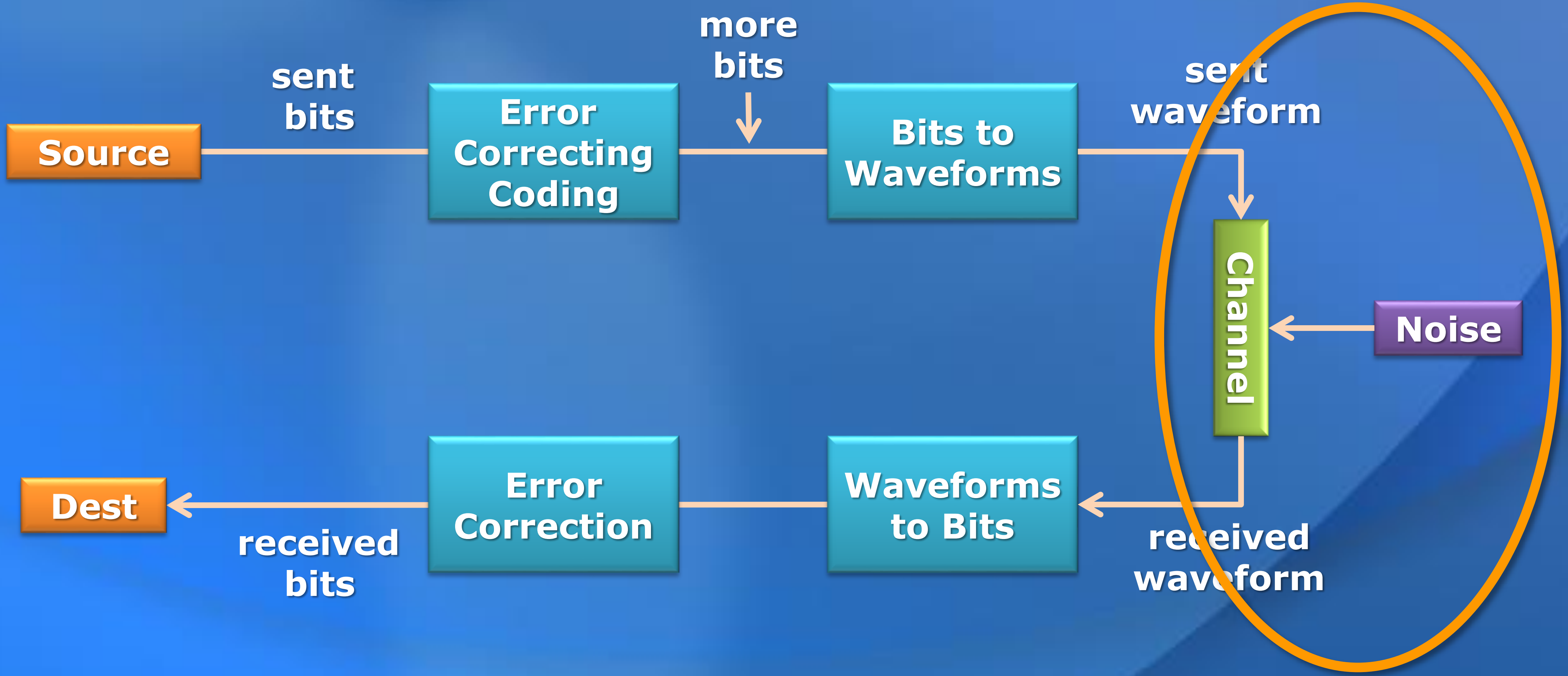
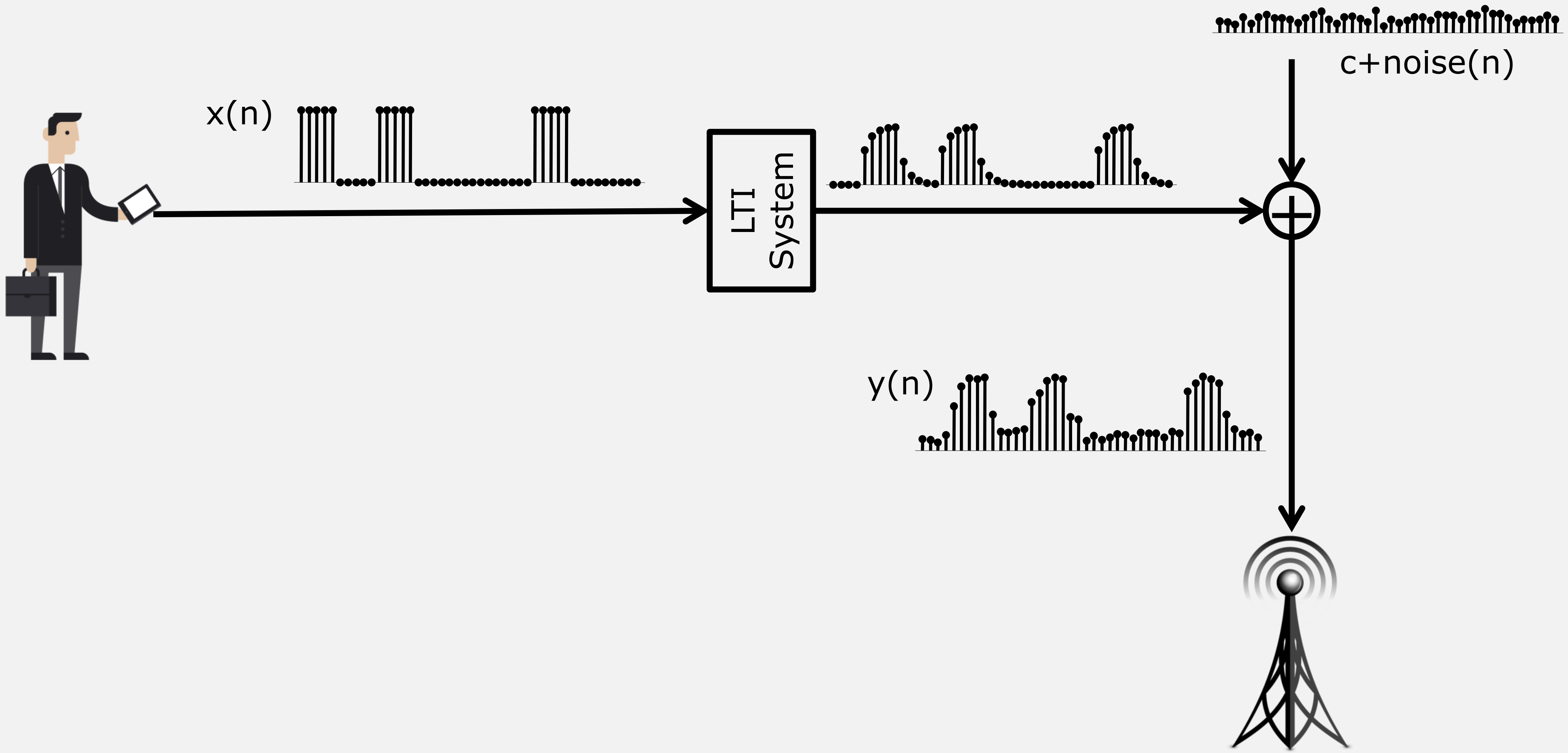
Equivalent Representations and Models

Communication System

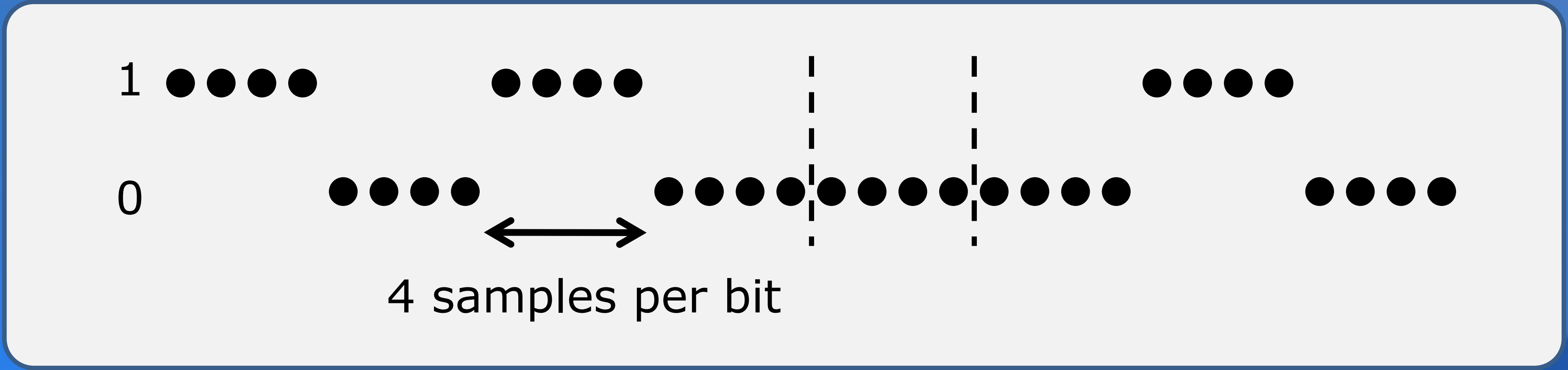


Model of the Channel



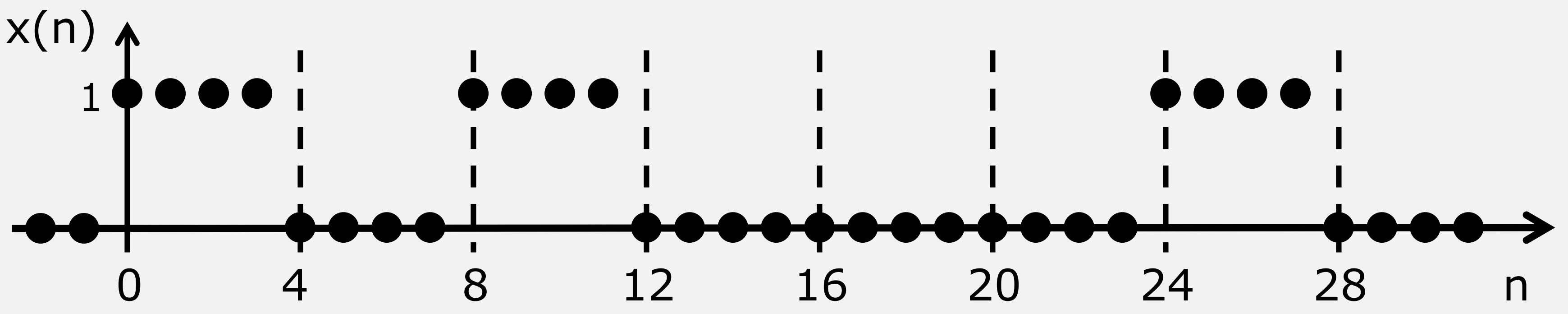
Describing the Input to a Channel

- We can describe the input to the channel as a discrete time signal or waveform:

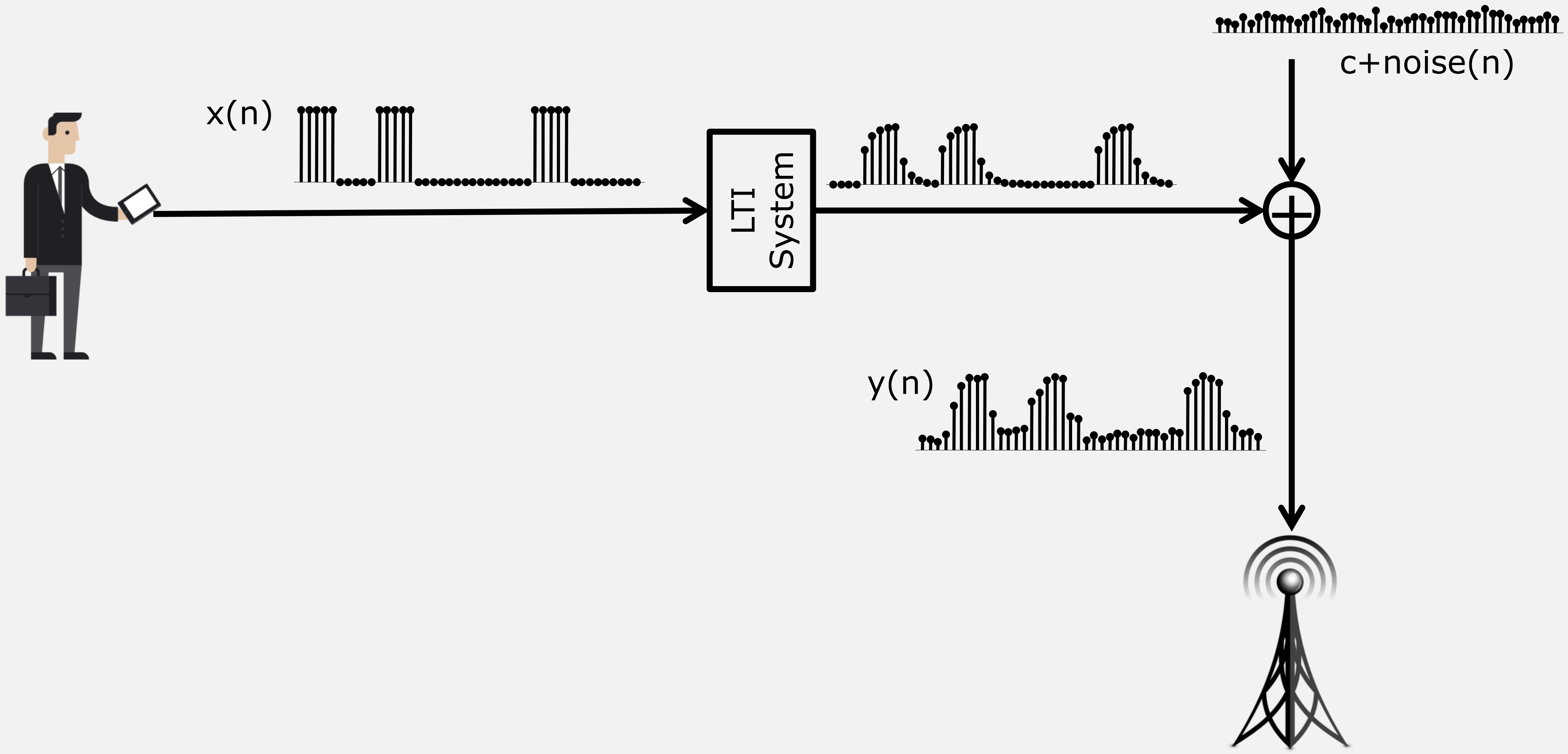


- We can describe a discrete time signal in many different ways, e.g.
  - Verbal
  - Graph
  - List of values for each sample n.
  - Sum of unit step functions

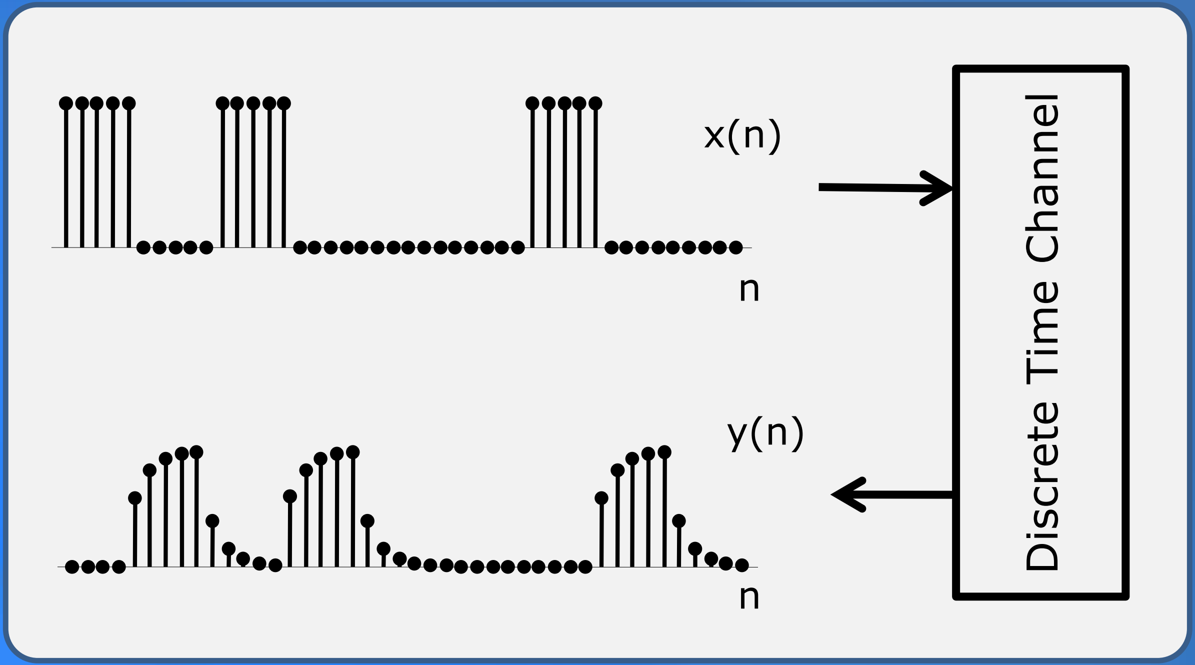
Equivalent Representations

Verbal	"Encoding of the bit sequence 1,0,1,0,0,0,1 at 4 samples per bit"
Graph	
List, table or vector of values	n = [0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 …] x(n) = [1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 0 0 …]
Sum of unit step functions	$x(n) = u(n) - u(n\text{-}4) + u(n\text{-}8) - u(n\text{-}12) + u(n\text{-}24) - u(n\text{-}28)$

Model of the Channel



Equivalent Models



- We described two equivalent models for the response of the channel due to the input
  - Model 1:
    - Channel is linear and time invariant
    - Channel has step response

      $s(n) = k(1-a^{n+1})u(n)$
  - Model 2:
    - Let $x(n)$ = channel input

      $y(n)$ = channel output

      $y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$

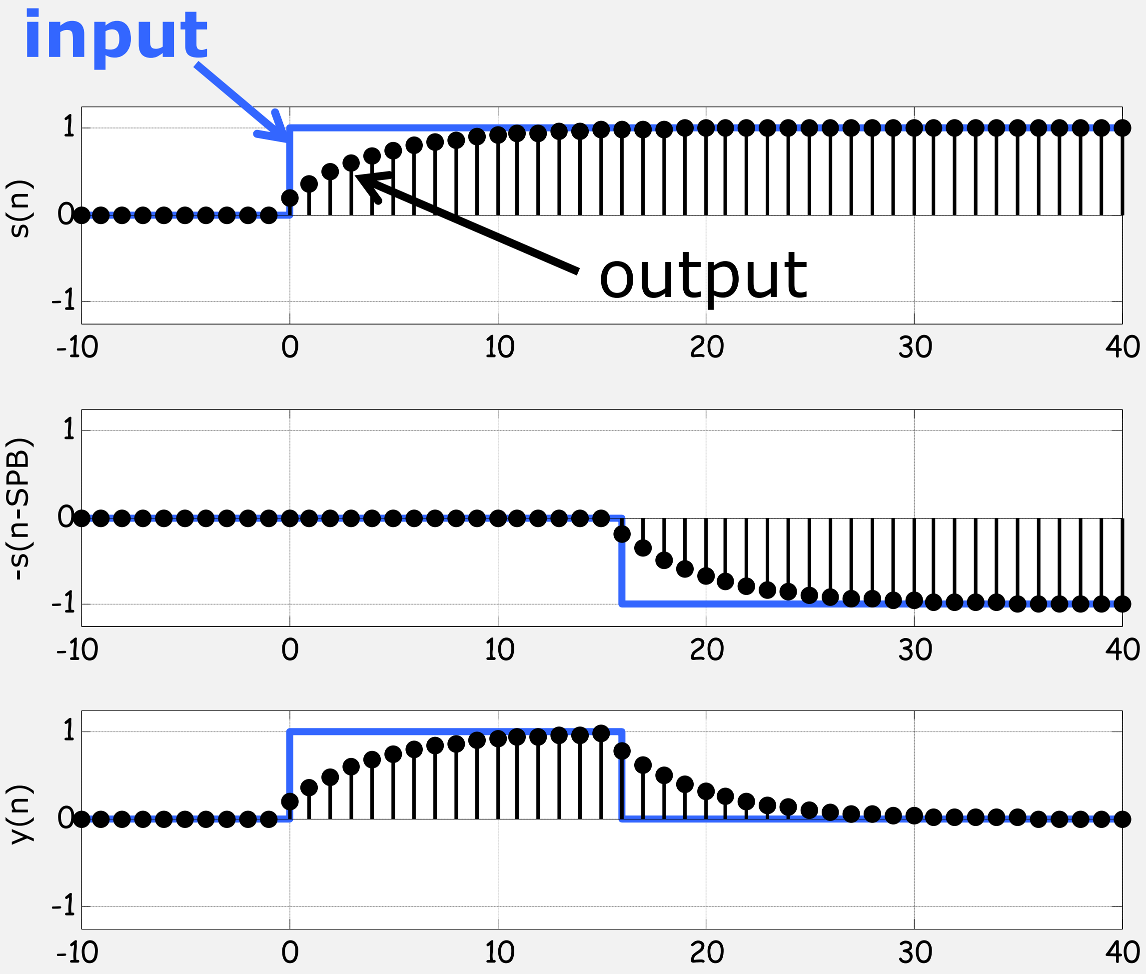
Example: Response to a pulse

unit step = u(n) → **LTI system** → s(n) = step response

$$x(n) = u(n) - u(n-16)$$

↓ channel ↓ channel ↓ channel

$$y(n) = s(n) - s(n-16)$$

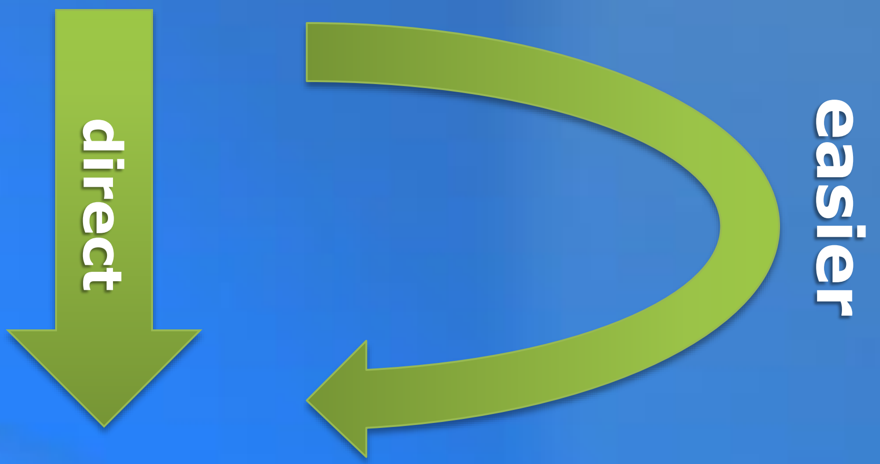


s(n) = Exponential approach, a = 0.8, k = 1

Key Idea

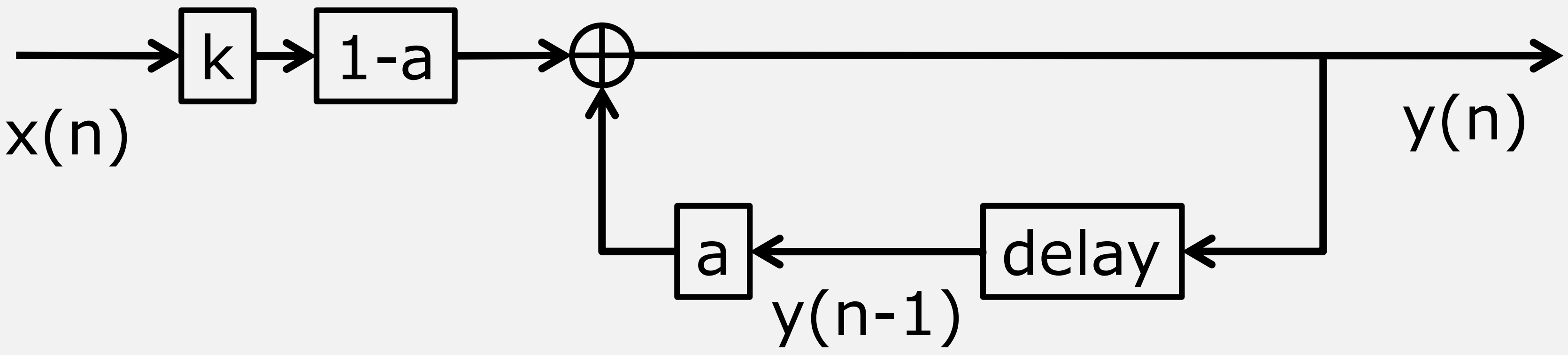
- Replace a direct path by a longer path with easier steps

$$x(n) = u(n) - u(n-16)$$



$$y(n) = s(n) - s(n-16)$$

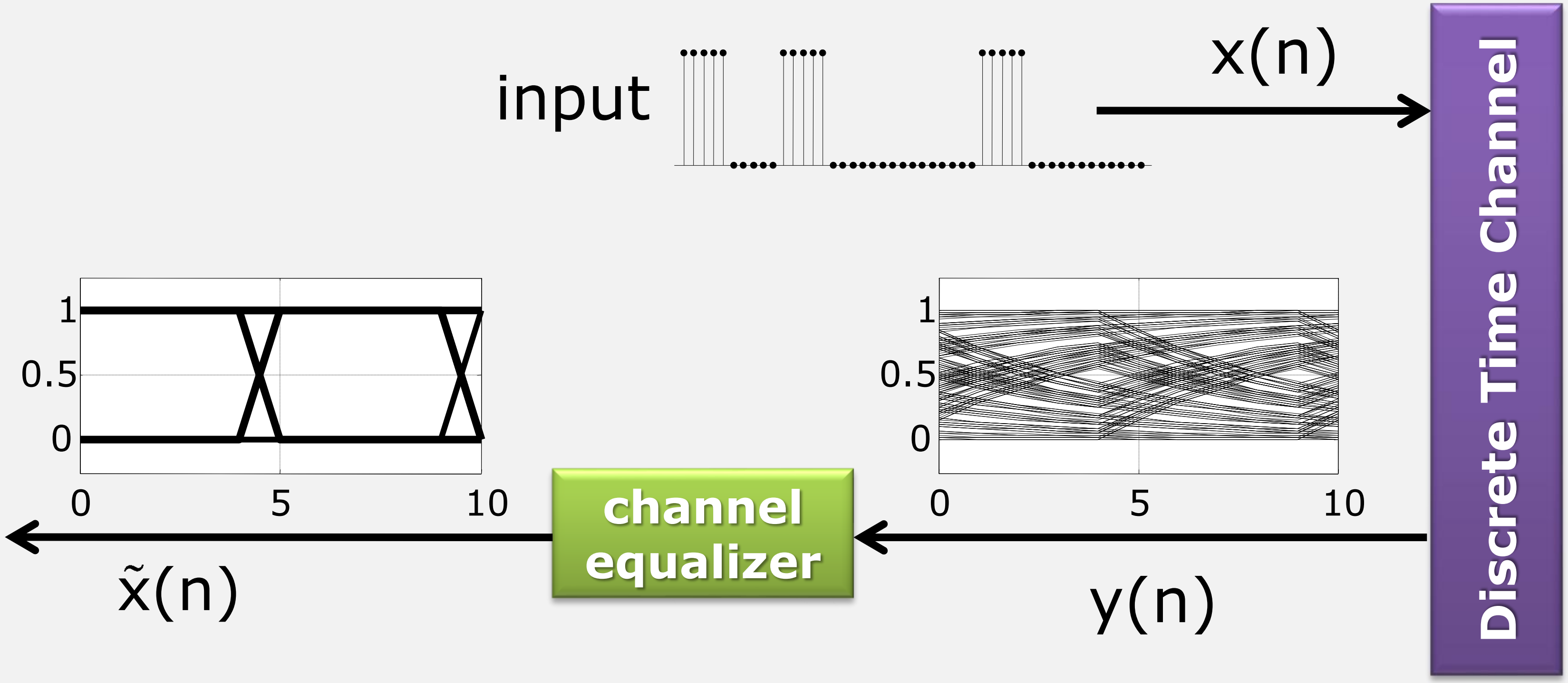
Recursive Model of IR Channel



$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$$

- $k > 0$ is a scaling parameter
- $0 \leq a \leq 1$ determines the "memory" in the channel
  - $a = 0 \rightarrow$ no memory of the past
  - $a = 1 \rightarrow$ infinite memory (never forgets, never changes)

Channel Equalization



Channel model: $y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n)$

invert!

Equalizer: $\tilde{x}(n) = \dfrac{1}{(1-a) \cdot k}\left[y(n) - a \cdot y(n-1)\right]$