An Expression for BER with Gaussian Noise

The Q-function

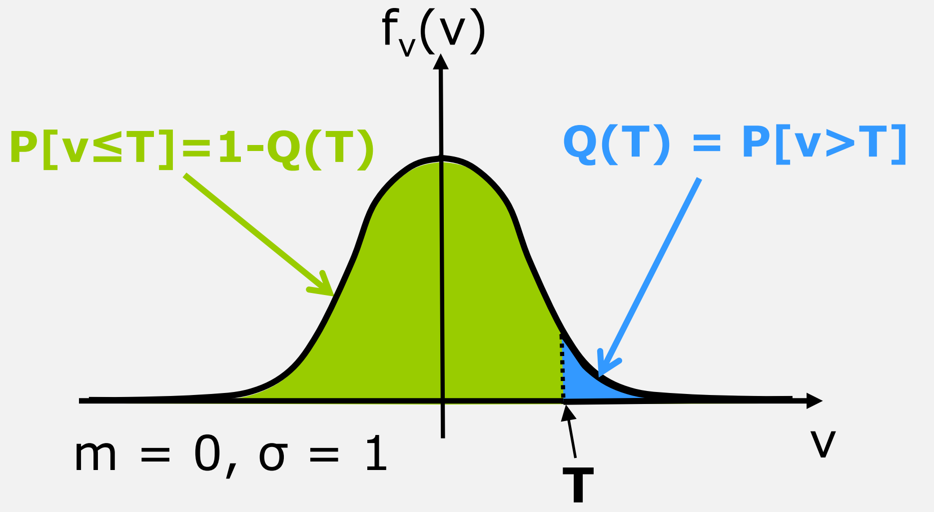
- Suppose that v is Gaussian with m = 0 and σ = 1.
- The probability that v is greater than a particular value T is given by the Q-function

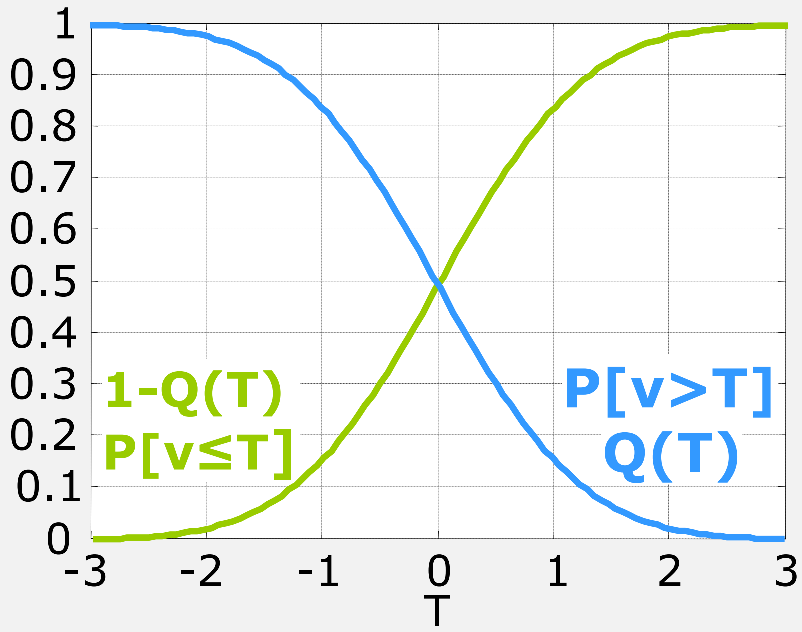
$$Q(T) = P[v > T]$$

- There is no closed-form expression for Q(T). Its value must be found numerically, e.g.
  - from tables, or
  - the MATLAB function `qfunc(T)`
- Properties

$$P[v \leq T] = 1 - Q(T)$$

$$Q(0) = \tfrac{1}{2}$$

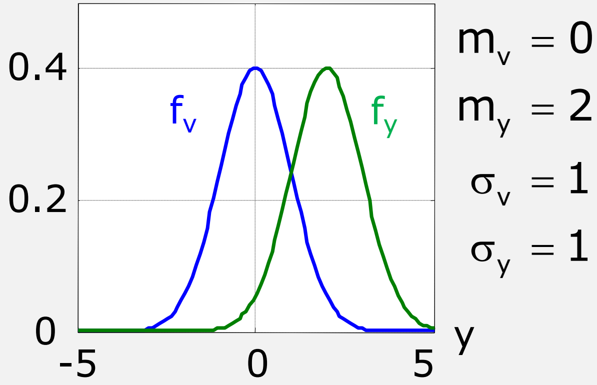


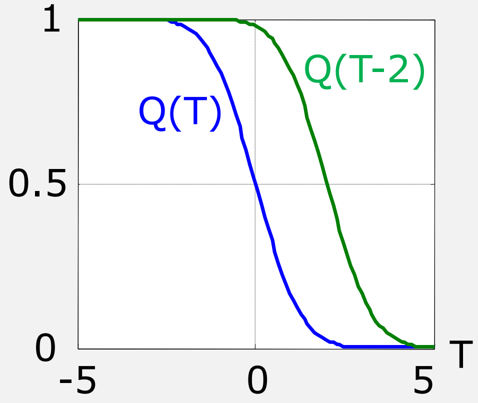


Probabilities for Other Gaussians

If y is Gaussian with $m_y \neq 0$ and $\sigma_y = 1$,

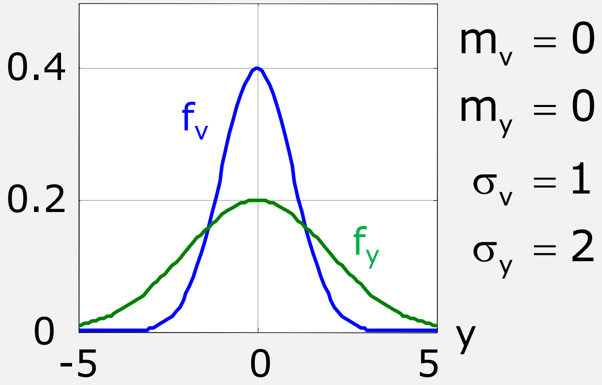
$$P[y > T] = Q\left(T - m_y\right)$$

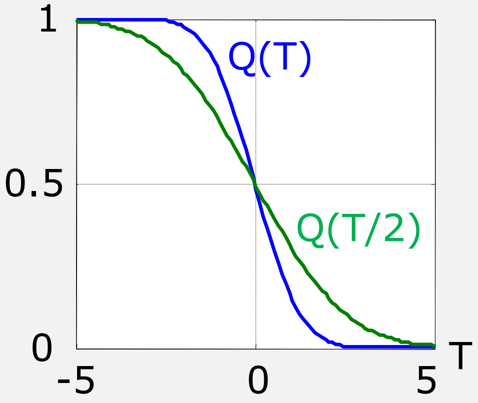




- **If y is Gaussian with $m_y = 0$ and $\sigma_y \neq 1$,**

$$P[y > T] = Q\left(\frac{T}{\sigma}\right)$$





- **In general, if y is Gaussian with $m_y \neq 0$ and $\sigma_y \neq 1$,**

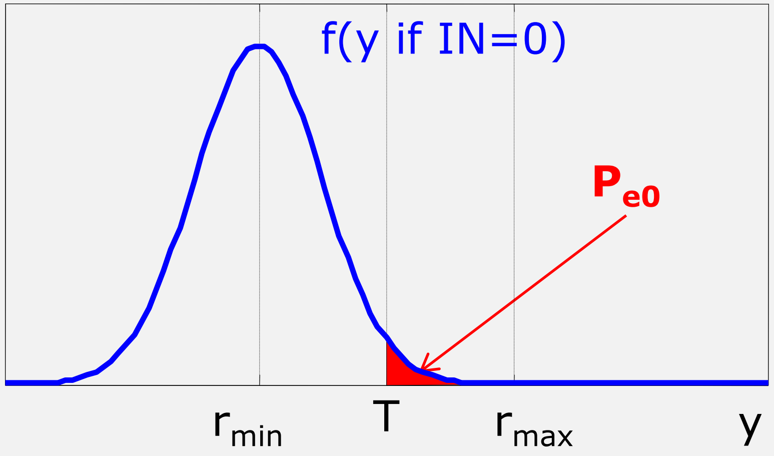
$$P[y > T] = Q\left(\frac{T - m_y}{\sigma_y}\right)$$

Expressions for P_{e0} and P_{e1}

- **If IN = 0, there is an error if**
  - **OUT = 1**
  - **The noise pushes y above T**

$$P_{e0} = P[y > T \text{ if } IN = 0]$$

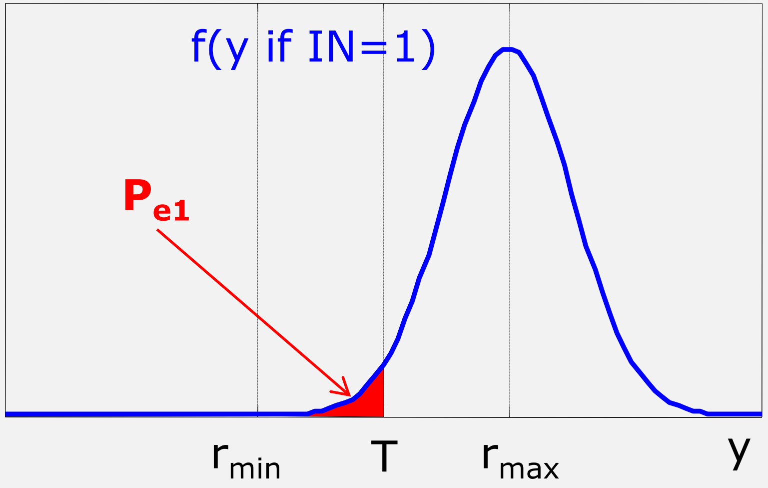
$$= Q\left(\frac{T - r_{min}}{\sigma}\right)$$



- **If IN = 1, there is an error if**
  - **OUT = 0**
  - **The noise pushes y below T**

$$P_{e1} = P[y < T \text{ if } IN = 1]$$

$$= 1 - Q\left(\frac{T - r_{max}}{\sigma}\right)$$



Predicting BER

If 0 and 1 input bits are equally likely

$$\mathrm{BER} = \tfrac{1}{2}P_{e0} + \tfrac{1}{2}P_{e1} = \tfrac{1}{2}Q\left(\frac{T-r_{min}}{\sigma}\right) + \tfrac{1}{2}\left[1 - Q\left(\frac{T-r_{max}}{\sigma}\right)\right]$$