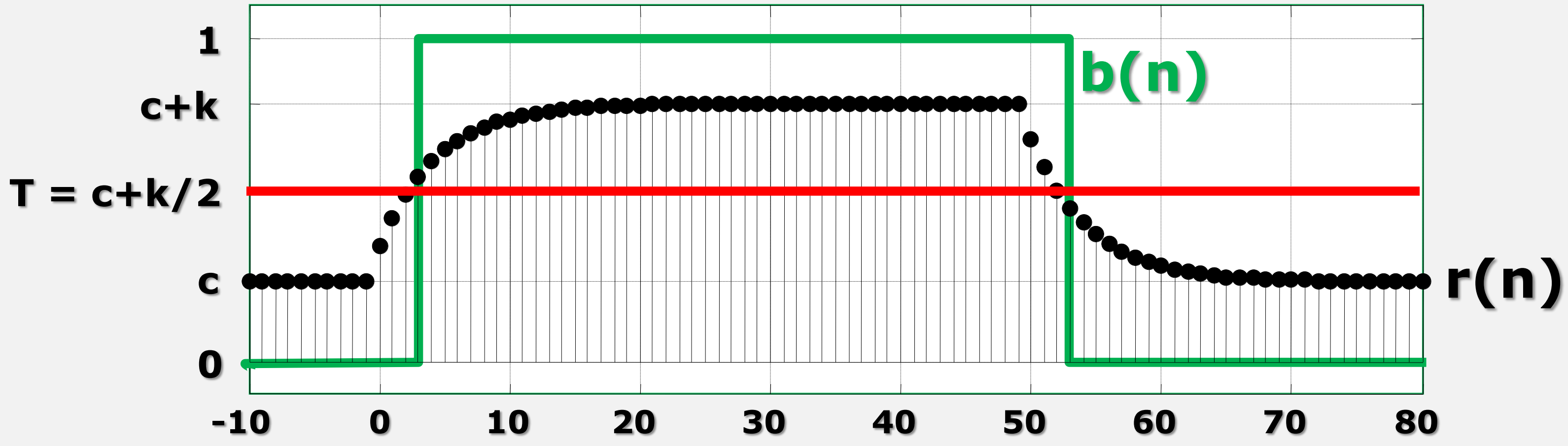
Thresholding

Thresholding

- In our system, for long SPBs at the receiver
  - 1 bits usually result in received values close to c+k
  - 0 bits usually result in received values close to c
- This suggests we can recover original bits by comparing received value with a threshold T
  - Intuitively, a good threshold is halfway between c and c+k
  - i.e. T = c+k/2

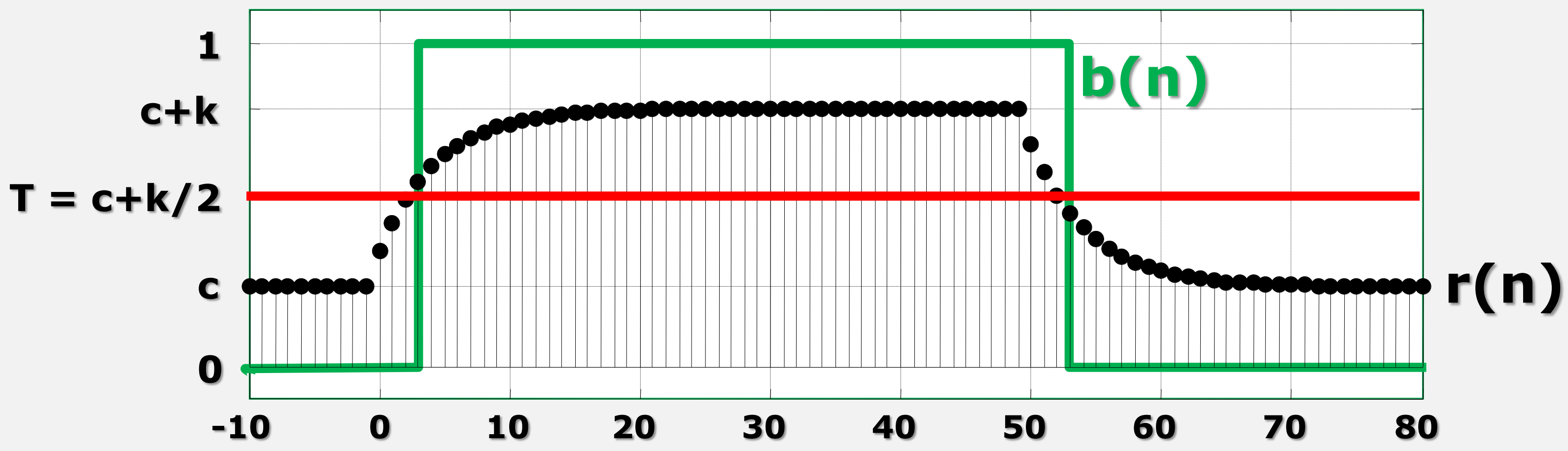


r(n): received signal at time n
b(n): thresholded signal

$$b(n) = \begin{cases} 1 & r(n) \geq T \\ 0 & r(n) < T \end{cases}$$

Training Sequence

- In order to choose a threshold, the receiver needs to know *c* and *k*.
- Unfortunately, these may change over time.
- To help the receiver estimate *c* and *k*, the transmitter sends a “training sequence”

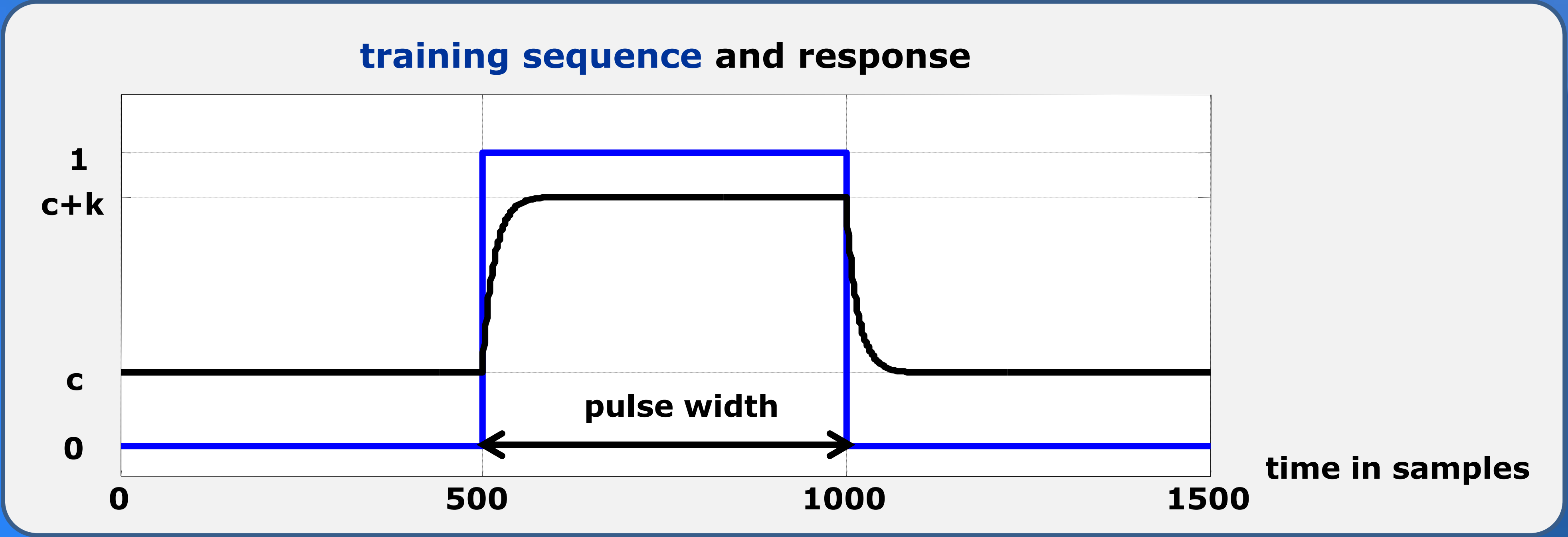


r(n): received signal at time n
b(n): thresholded signal

$$b(n) = \begin{cases} 1 & r(n) \geq T \\ 0 & r(n) < T \end{cases}$$

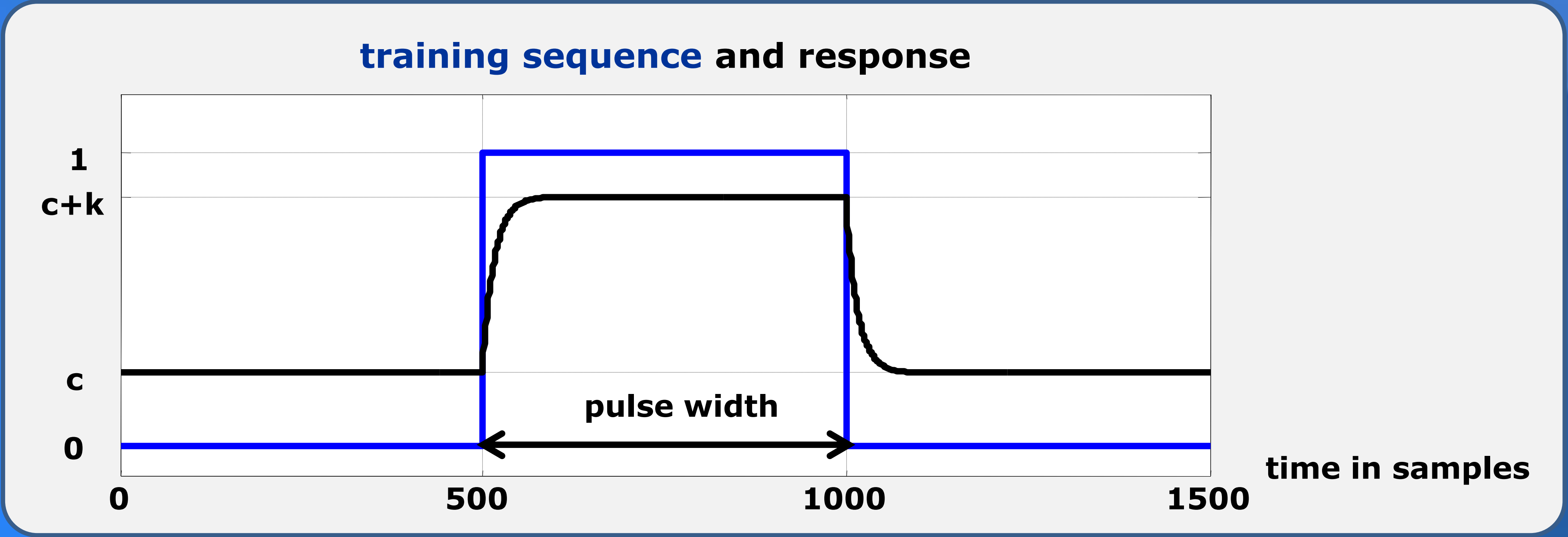
Our Training Sequence

- Assuming F_s = 1MHz, the training sequence consists of 500μs of 0, followed by 500 μs of 1, followed by 500μs of 0.
- Estimating channel parameters from the response
  - estimate of *c* = minimum value of the response
  - estimate of *k* = difference between minimum and maximum



Length of the training sequence

- Trade-off in the choice of the pulse width
  - Shorter pulse widths mean more time available to transmit data.
  - Longer pulse widths enable better estimates of channel parameters (*c, k*)

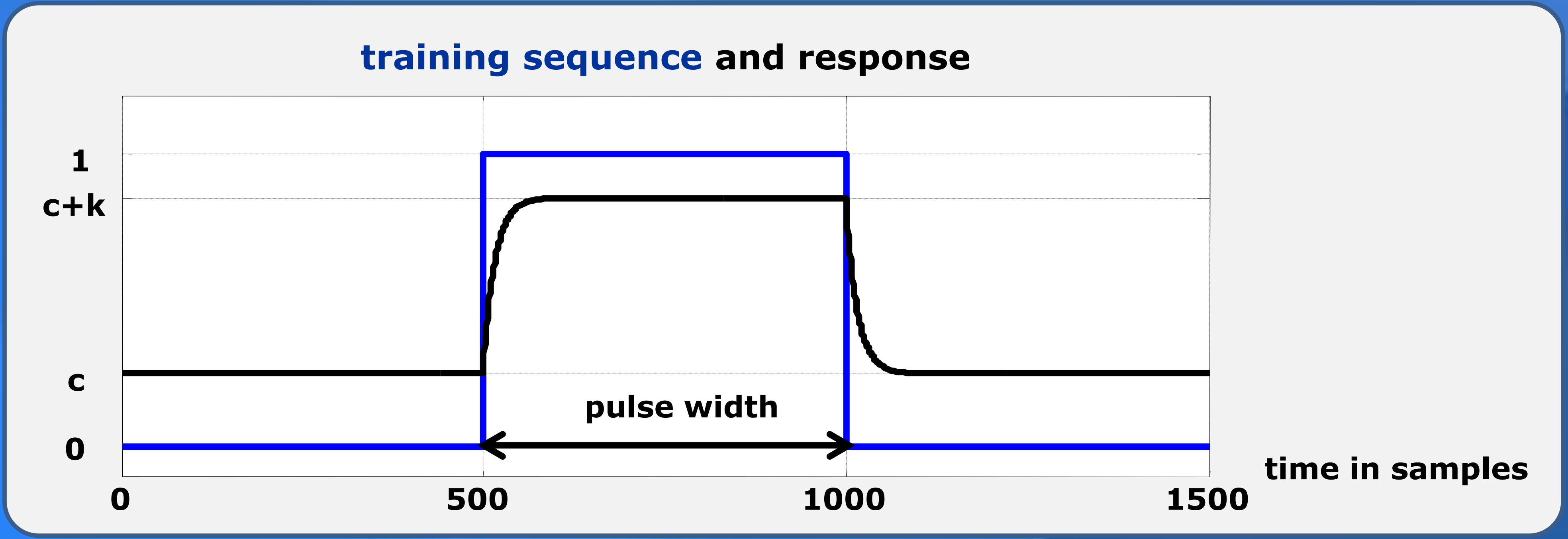


Length of the training sequence

- The choice of the pulse width is based on an assumption about the value of “a” in the step response.

$$s(n) = k(1 - a^{n+1})\, u(n)$$

- **Question:** If the value of “a” is larger (closer to 1), should the pulse width be made longer or shorter?

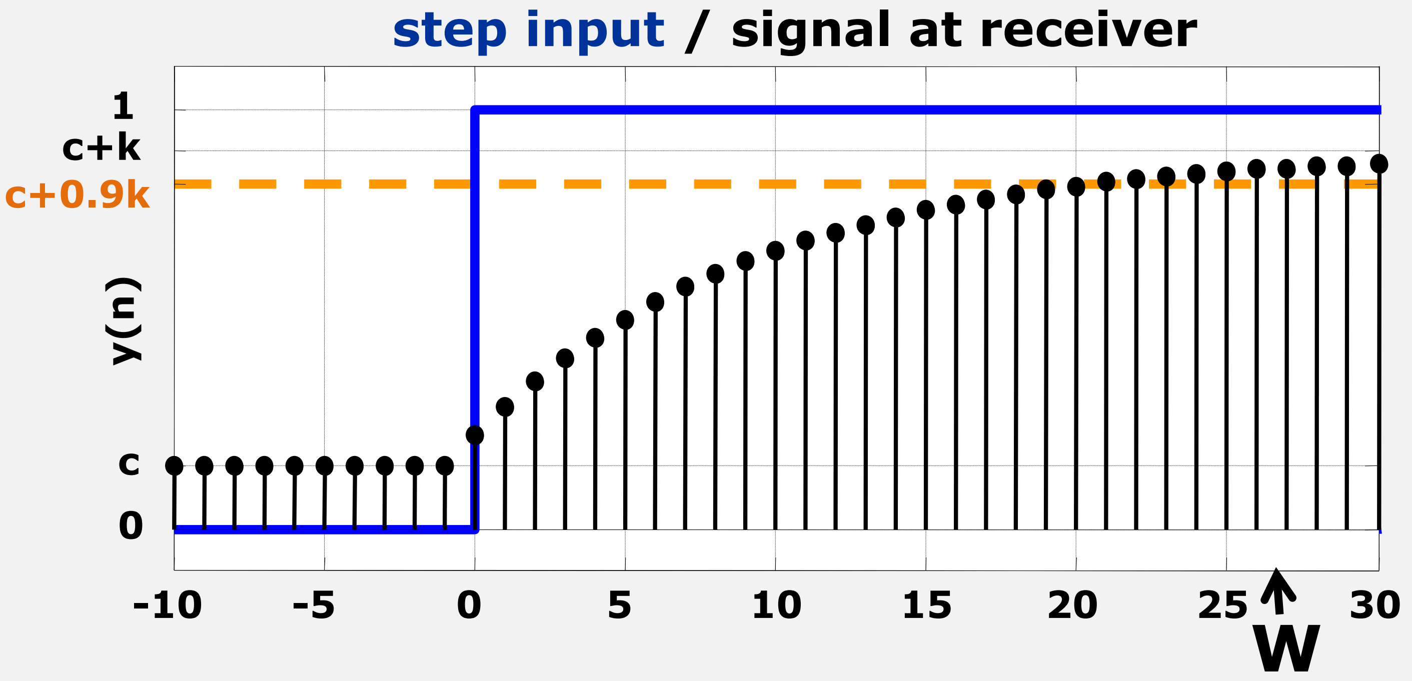


Example

- Consider a channel with a step response given by $s(n) = k(1 - a^{n+1})\, u(n)$
- During transmission, the environment adds a constant offset c, so the signal at the receiver is

$$y(n) = c + k(1 - a^{n+1})\, u(n)$$

- **Question:** How long should the pulse width *W* be so that the maximum value of the pulse response is larger than c+0.9k?



Example

Solution: Let the pulse width be W samples.

Since the maximum (max) occurs at the end of the pulse: $max = c + k(1 - a^{w+1})$

To ensure that:

$$max > c + 0.9k \;\Longrightarrow\; c + k(1 - a^{w+1}) > c + 0.9k$$

$$c + k\left(1 - a^{W+1}\right) > c + 0.9k$$

$$k\left(1 - a^{W+1}\right) > 0.9k$$

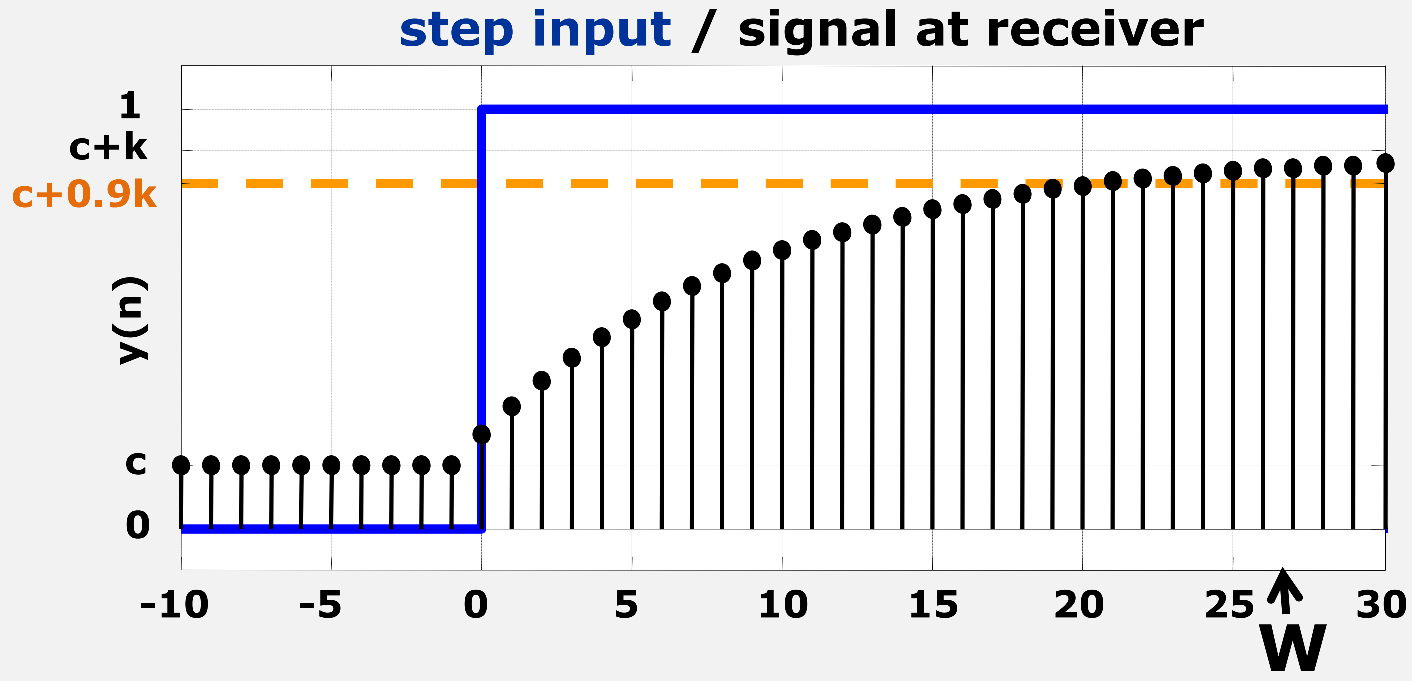
$$1 - a^{W+1} > 0.9$$

$$0.1 > a^{W+1}$$

$$\ln 0.1 > (W+1)\ln a$$

$$\frac{\ln 0.1}{\ln a} - 1 < W$$

$$0 < a < 1 \Rightarrow \ln a < 0$$



Example

Suppose we have a communication channel with c=0.15, k=0.75, and a=0.9. What is the minimum pulse width so that at the end of the pulse, the response is greater than c+0.9k = 0.825?

Solution:
By our prior analysis

$$W > \frac{\ln 0.1}{\ln a} - 1 \approx 20.85$$