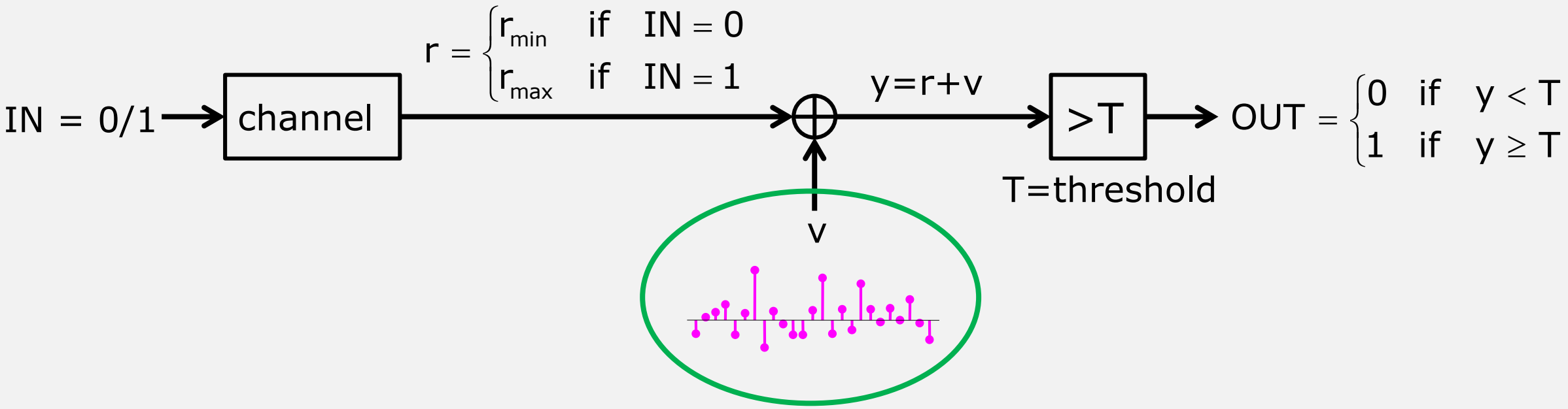
Gaussian Noise Model

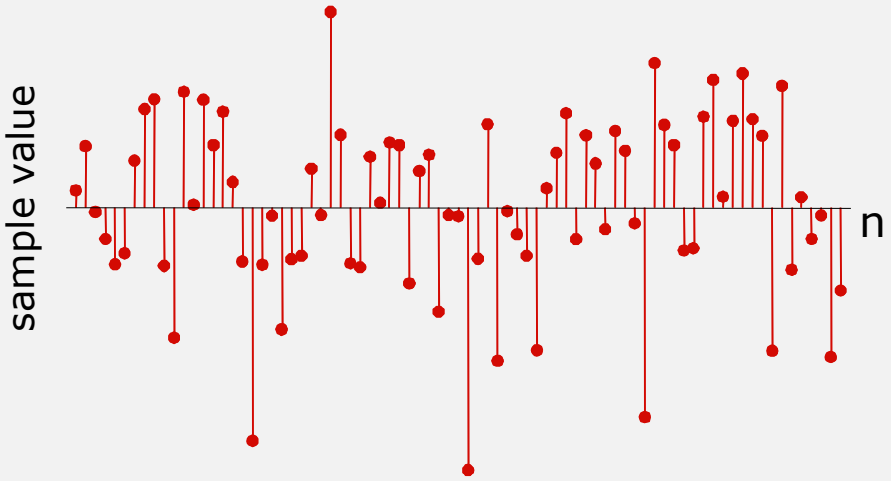
Inside the Binary Channel

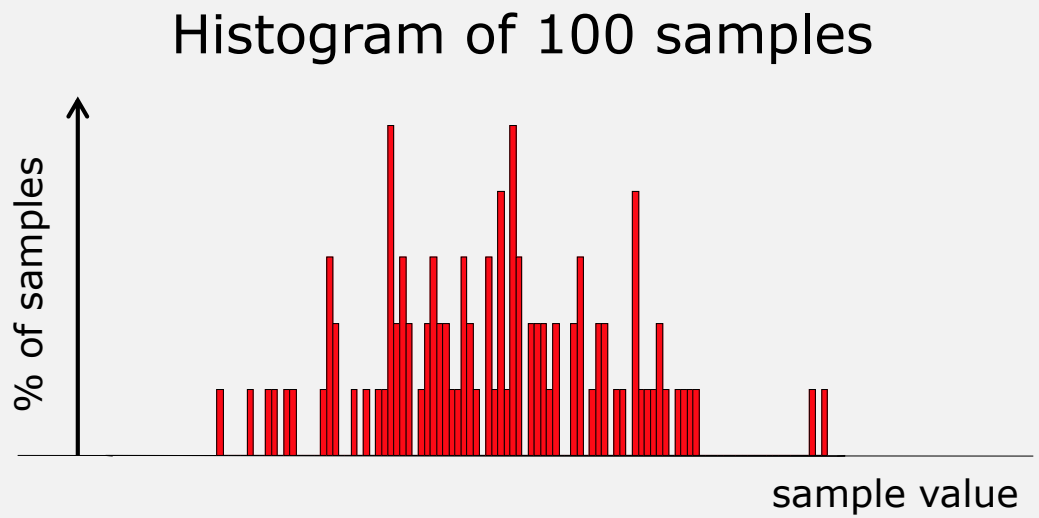


We call the noise, v, a random variable.

Statistics of the Noise

- **The value of each noise sample is random, but the statistics of a large number of samples is predictable.**





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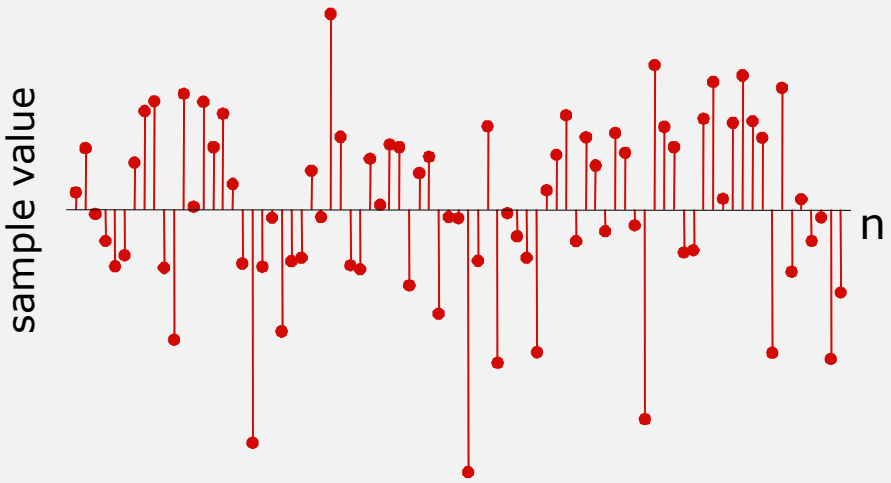

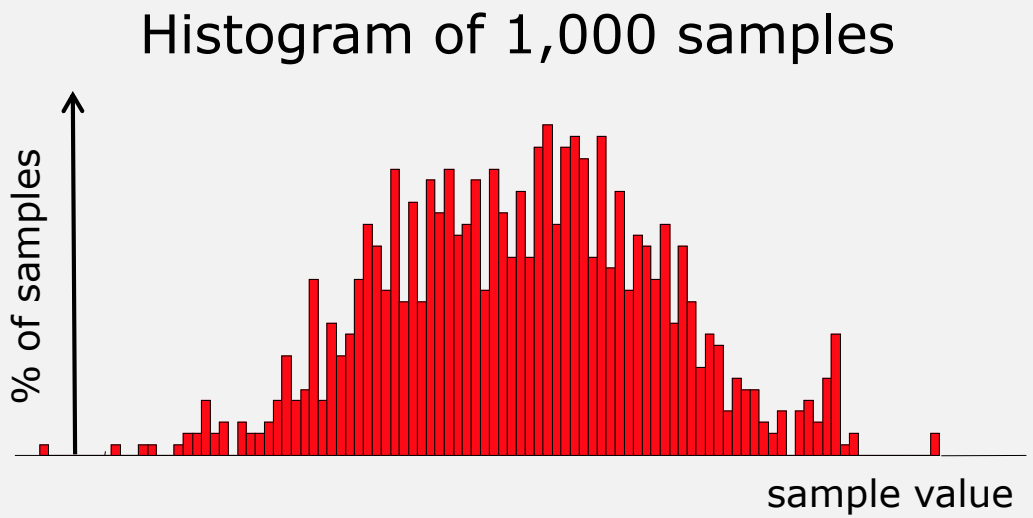

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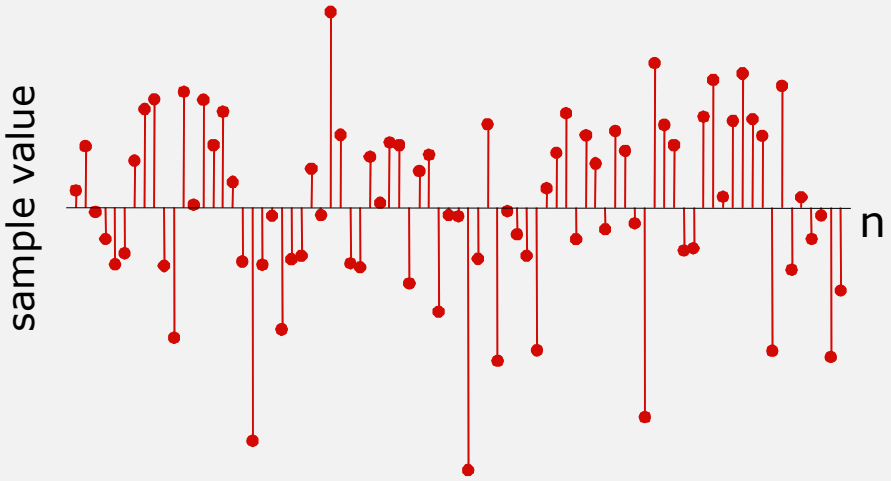

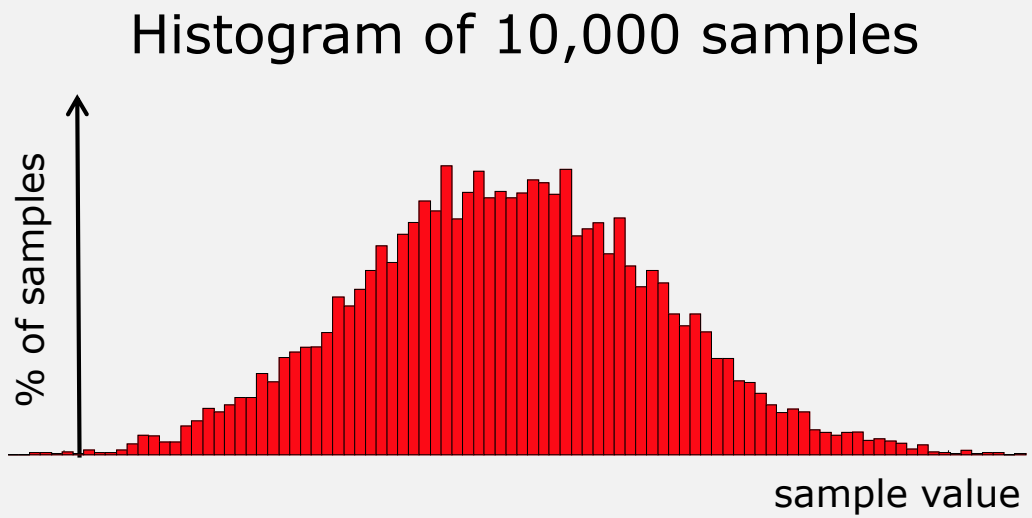

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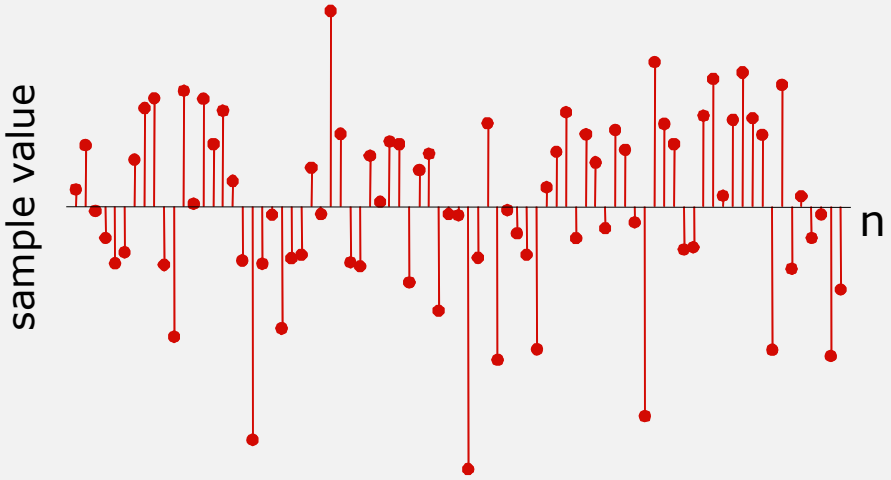
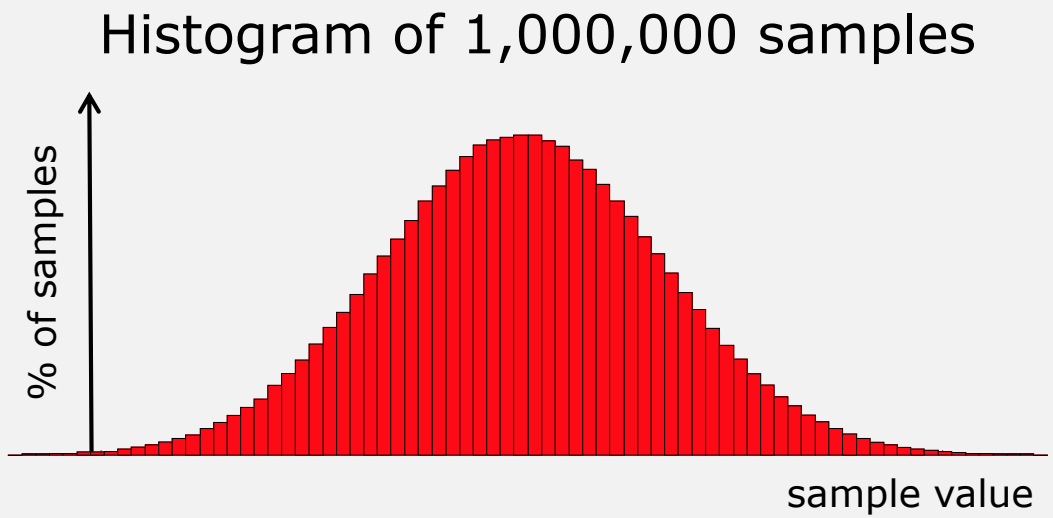
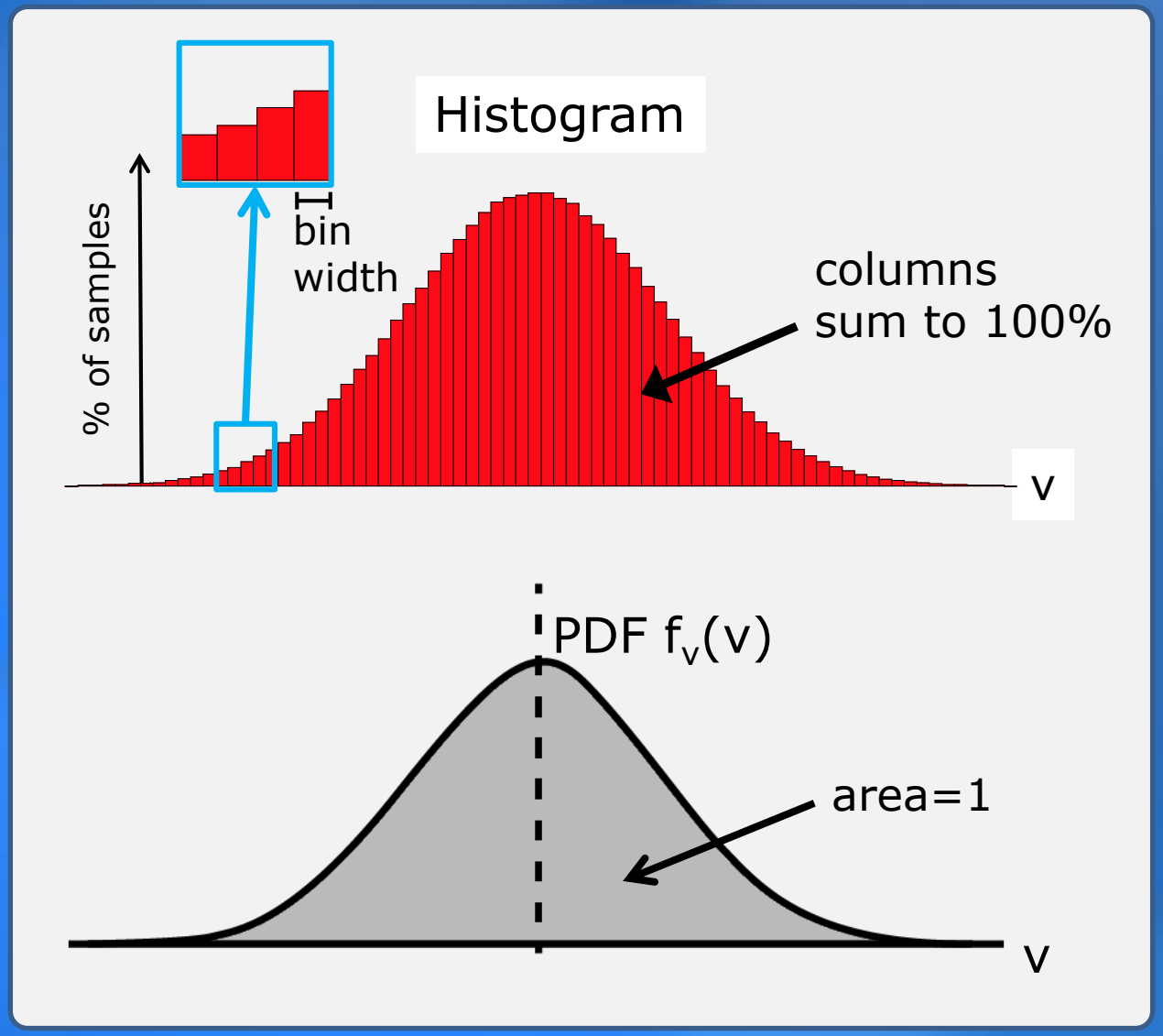




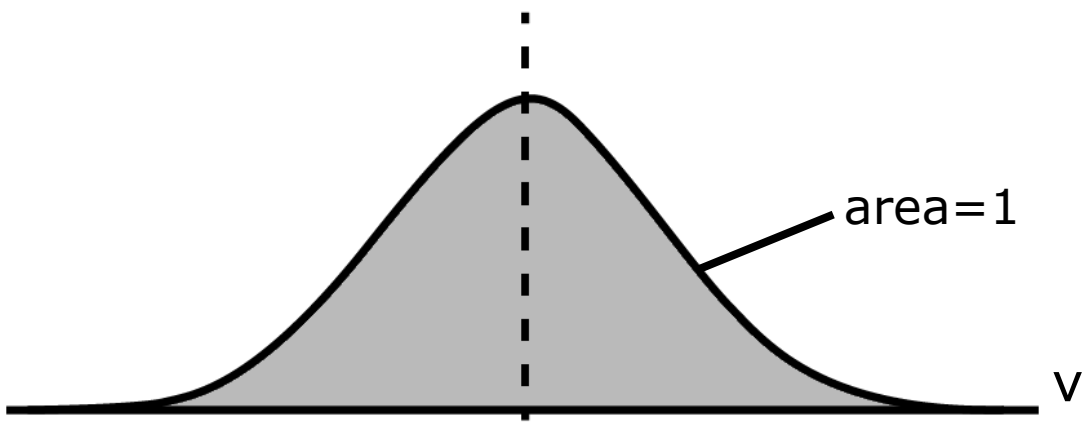
Probability Density Function



- The histogram is not totally smooth since we count the samples in bins of finite width.
- As the bins get smaller and smaller, the curve gets smoother and smoother.
- It approaches a function known as the probability density function (pdf), $f_v(v)$

Gaussian Density Function

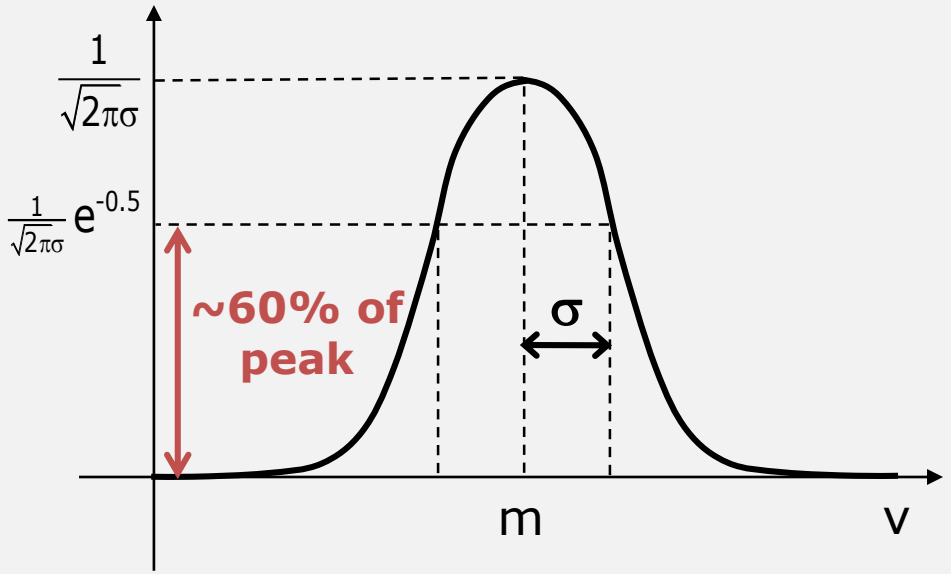
$$f_v(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(v-m)^2}{2\sigma^2}}$$



- The probability density function of many naturally occurring random quantities, such as noise, tends to have a bell-like shape, known as a Gaussian distribution.
- This very important result is called the Central-Limit Theorem.
- The Gaussian distribution is so common that it is also called the "normal" distribution.
- Applications:
  - Noise in communication systems
  - Particles in Brownian motion
  - Voltage across a resistor

Parameters Controlling the Shape

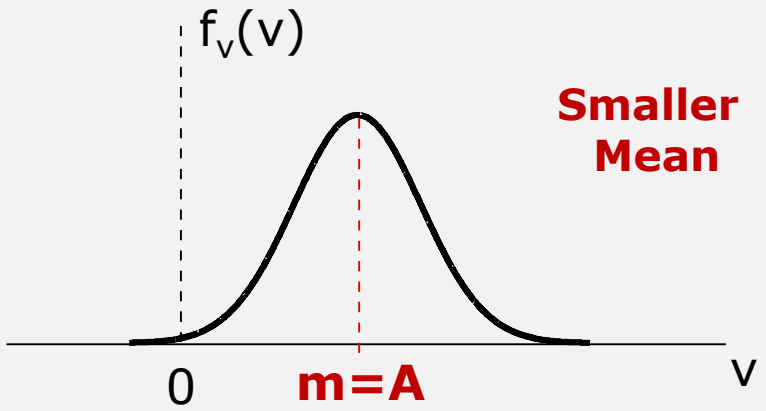
$$f_v(v) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(v-m)^2}{2\sigma^2}}$$

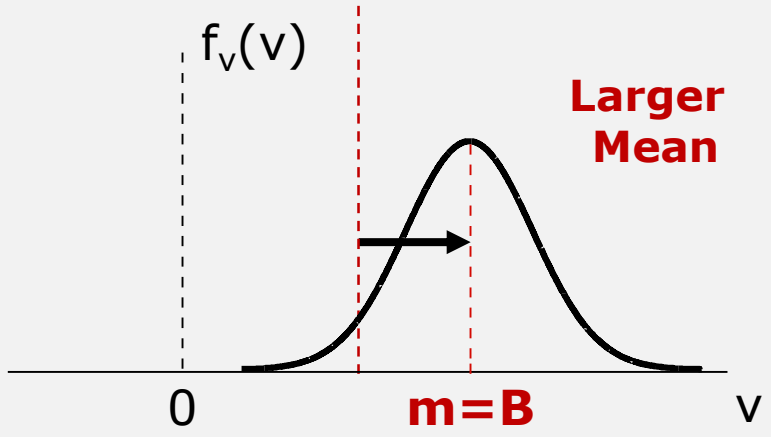


- The mean (m) is
  - Its average value over many samples
  - The center location of the pdf
- The standard deviation (σ) is
  - An indication the "spread" of the samples
  - A measure of the width of the pdf
- The variance (σ^2) is
  - The square of the standard deviation
  - The average power over many samples

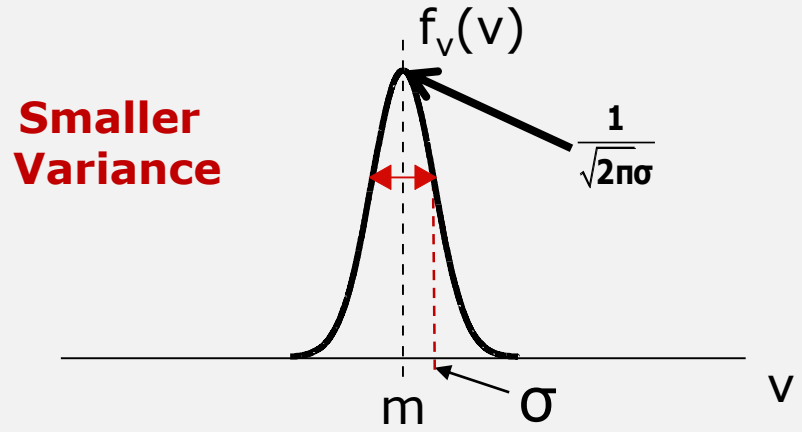
Changing the Mean and Variance

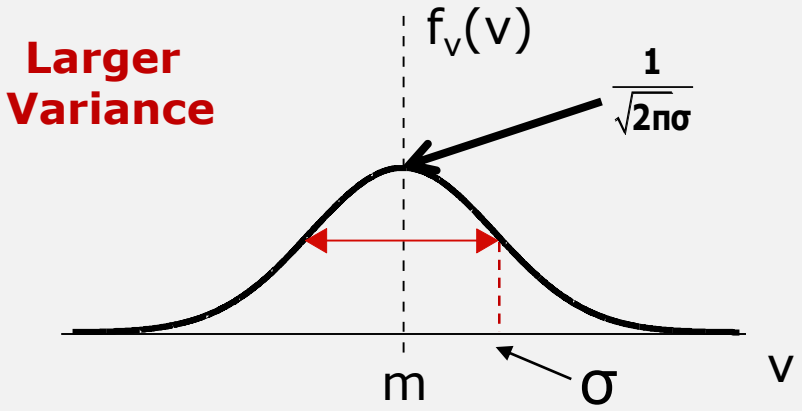
Changes in mean shift the *center of mass* of PDF





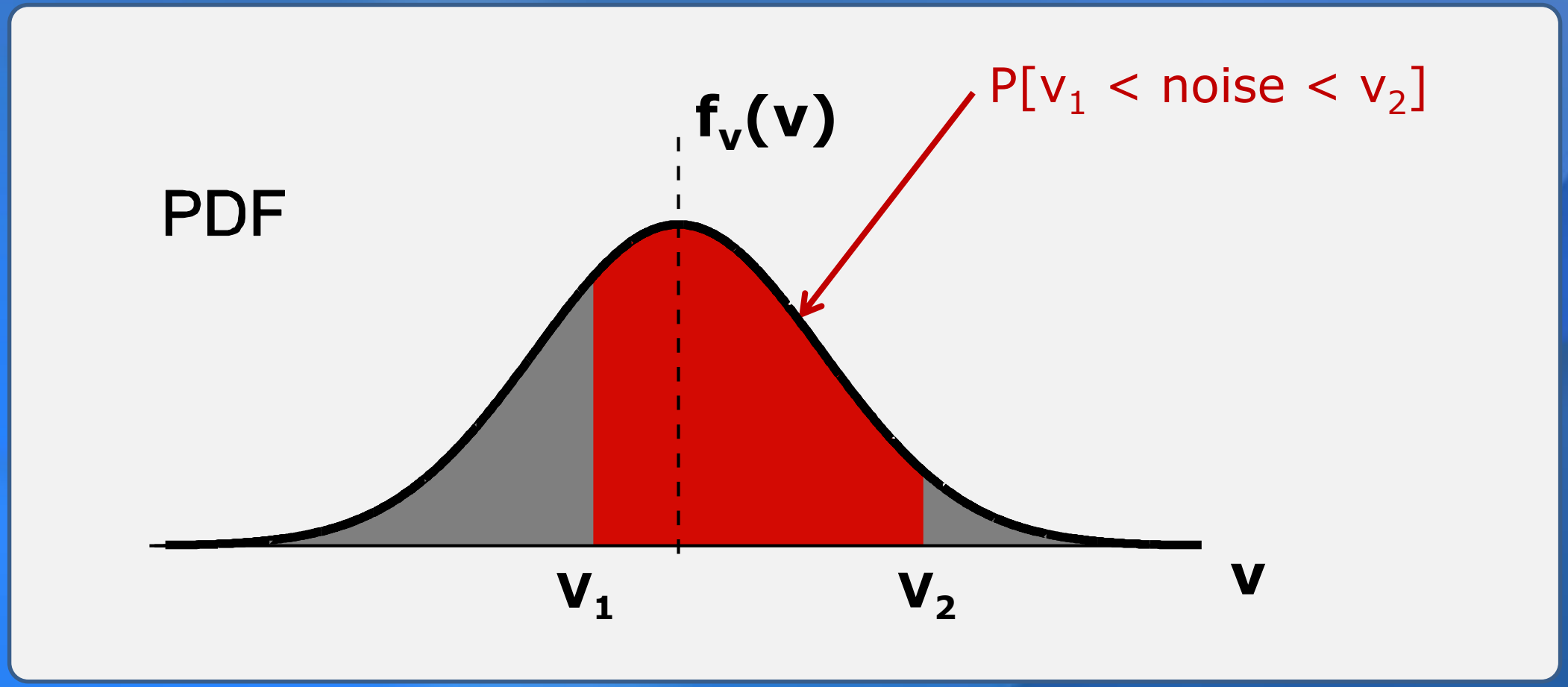
Changes in variance narrow or broaden the PDF



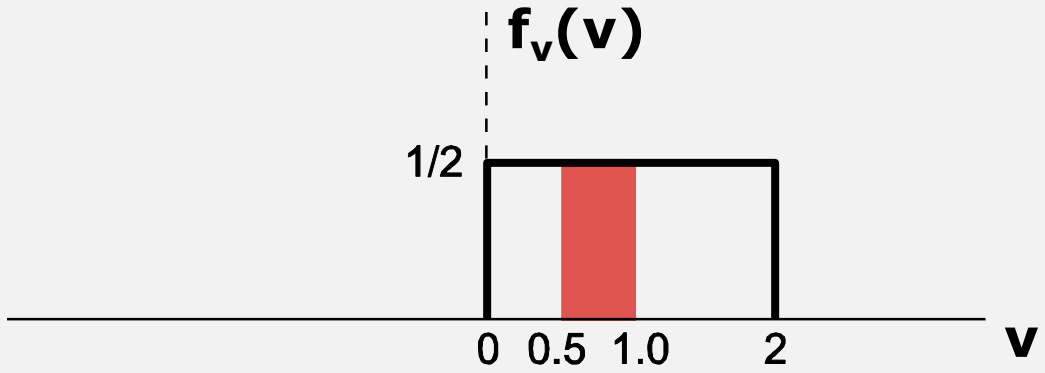


Calculating Probability by Integrating

- The probability that the noise v is between v_1 and v_2 is the area under the probability density function between v_1 and v_2



Example Probability Calculation



- **Verify that overall area is 1:**
  - **Since the curve defines a rectangle, the area is base × height:**

$$2 \times \frac{1}{2} = 1$$

- **Find the probability that v is between 0.5 and 1.0:**
  - **The area of the shaded region is** $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
  - **Thus,** $P[0.5 < v < 1.0] = \frac{1}{4}$