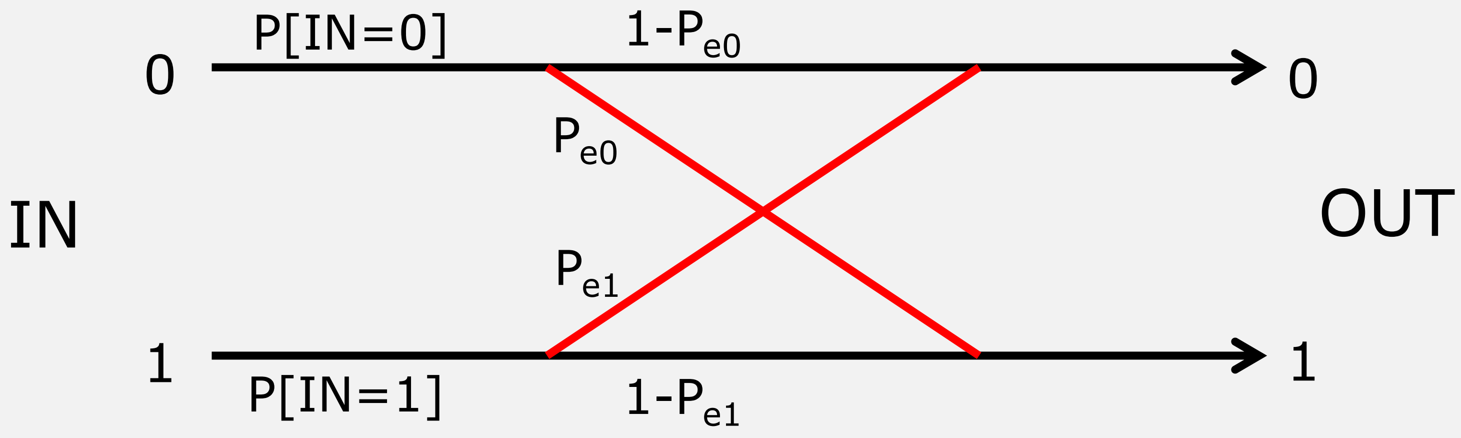
Average Power in Signals

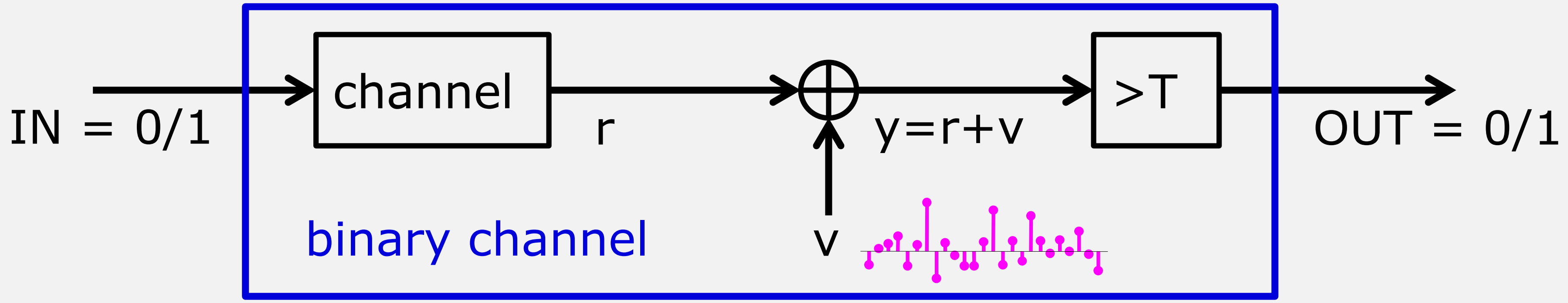
Binary Channel Model



$$\boxed{BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1]}$$

- **Usually, the transmitter determines P[IN=0/1]**
  - **e.g. P[IN=0] = P[IN=1] = 0.5**
- **P_{e0} and P_{e1} depend on**
  - **the transmit levels (r_{min},r_{max})**
  - **the power in the noise**
  - **the threshold**

Inside the Binary Channel



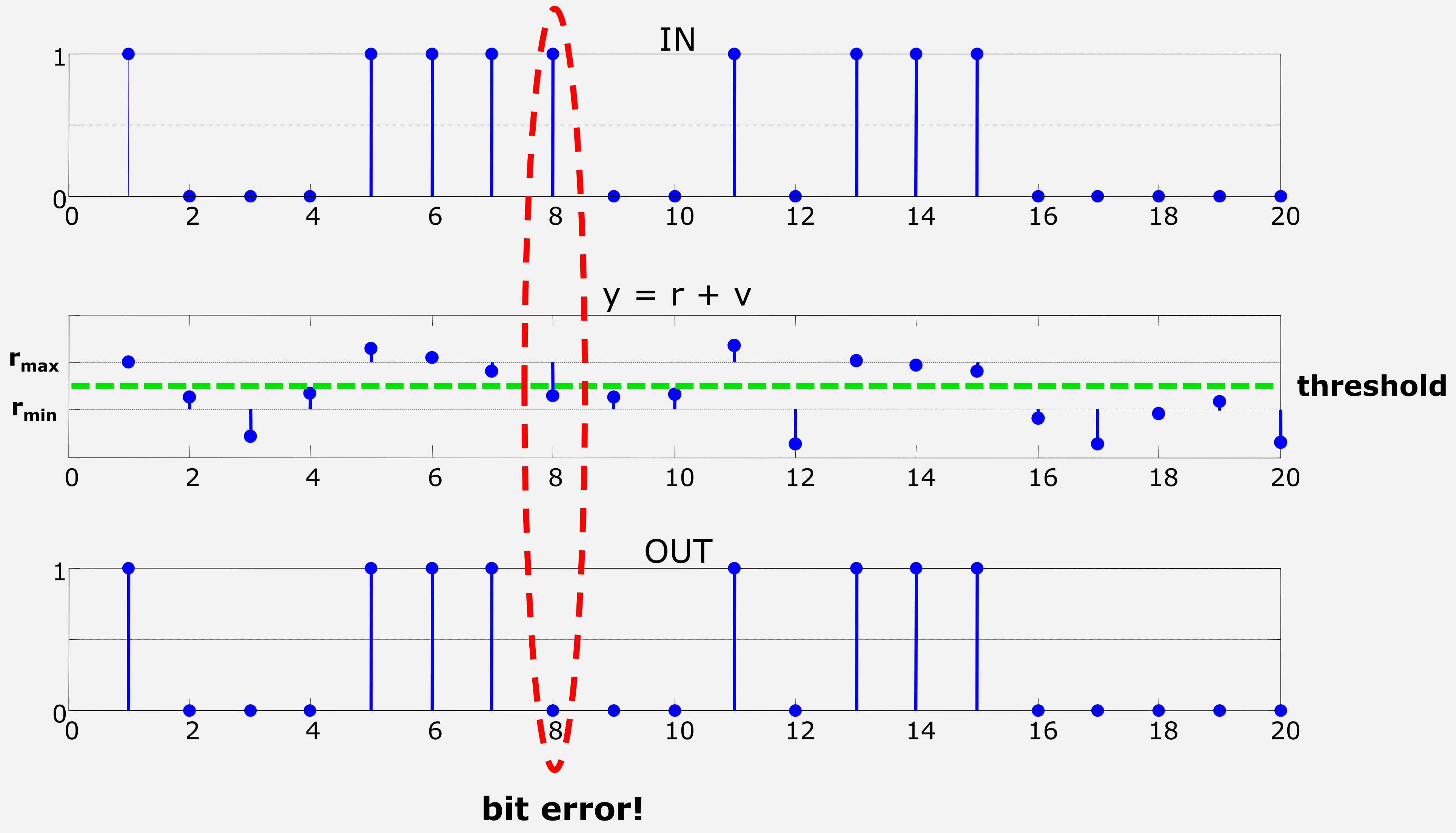
- **Under our simplifying assumptions, we can consider one bit at a time.**
- **The channel adds an offset r_{min} and scaling by r_{max}-r_{min}**

$$r = \begin{cases} r_{min} & \text{if} \quad IN = 0 \\ r_{max} & \text{if} \quad IN = 1 \end{cases}$$

- **The noise v is additive: y = r + v**
- **The output is obtained by thresholding y:**

$$OUT = \begin{cases} 0 & \text{if} \quad y < T \\ 1 & \text{if} \quad y \geq T \end{cases} \qquad T\text{=threshold}$$

Noise Leads to Bit Errors



Power Consumption

- Power is energy used per unit time:
  - $$\text{power} = \frac{\text{energy}}{\text{time}}$$
  - 1 Watt = Unit of Power
  - Lifting an apple (~100g) up by 1m in 1s requires ~1W

- Batteries contain a fixed amount of energy.
  - The higher the power consumption of the device they are powering, the faster this energy is used up.
  - $$\text{usable time} = \frac{\text{energy}}{\text{power consumption}}$$

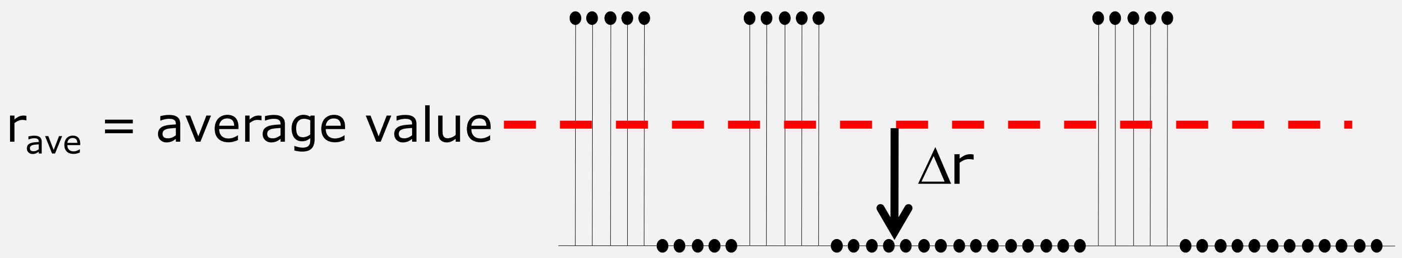
Power Consumption

- **Calculating the amount of energy in a battery**
  - **Batteries are typically rated at fixed voltage in volts (V) and a charge capacity in milliamp-hours (mAh)**
  - **Multiplying these together gives the total energy stored in the battery in milliwatt-hours (mWh)**
  - **For example, this mobile phone battery contains 3700mWh of energy**

- **Typical power consumption:**
  - **microwave oven 1000W**
  - **desktop computer 120W**
  - **notebook computer 40W**
  - **human brain 10W**
  - **mobile phone 1W**

Average Power in Signals

- **For communication, we usually have signals that vary around an average value**



- **For communication, we are interested in how much the signals differ from their average:** $\Delta r = r - r_{ave}$

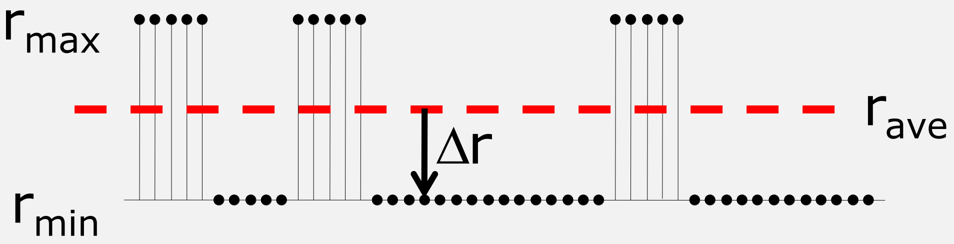
- **Since Δr can be both positive and negative, its average value over many samples is zero:**

$$\frac{1}{N}\sum_{n=1}^{N}\Delta r(n) = 0$$

- **The average power is the average squared value over many samples:**

$$P = \frac{1}{N}\sum_{n=1}^{N}\left(\Delta r(n)\right)^2$$

Average Power for Bit Signals



$$P_{signal} = \frac{1}{N}\sum_{n=1}^{N}\left(\Delta r\left(n\right)\right)^2$$

- **If 0 and 1's are equally likely,**

$$r_{ave} = \tfrac{1}{2} r_{min} + \tfrac{1}{2} r_{max}$$

- **If IN = 0,**

$$\Delta r = r_{min} - r_{ave} = r_{min} - \left(\tfrac{1}{2} r_{min} + \tfrac{1}{2} r_{max}\right) = \tfrac{1}{2}\left(r_{min} - r_{max}\right)$$

- **If IN = 1,**

$$\Delta r = r_{max} - r_{ave} = r_{max} - \left(\tfrac{1}{2} r_{min} + \tfrac{1}{2} r_{max}\right) = \tfrac{1}{2}\left(r_{max} - r_{min}\right)$$

- **The average power is**

$$P_{signal} = \frac{1}{2}\left[\tfrac{1}{2}\left(r_{max} - r_{min}\right)\right]^2 + \frac{1}{2}\left[\tfrac{1}{2}\left(r_{max} - r_{min}\right)\right]^2 = \frac{\left(r_{max} - r_{min}\right)^2}{4}$$