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Help

We have considered two models of our communication channel

- Model 1: The channel is a linear time invariant (LTI) system with an exponential step response

$$s(n) = k(1 - a^{n+1})u(n) \quad \text{where } 0 < a < 1$$

- Model 2: The channel has input  $x(n)$  and output  $y(n)$  determined by

$$y(n) = a \cdot y(n-1) + (1-a) \cdot k \cdot x(n) \quad \text{where } 0 < a < 1$$

In order to establish equivalence, it is sufficient to show that Model 2 is LTI and that Model 2 has an exponential step response. In the lecture video, we showed that Model 2 is LTI, but only gave an example showing that the response  $y(n)$  of Model 2 when the input  $x(n)$  is a unit step is the same as  $s(n)$  for a particular choice of  $a$  and  $k$  and limited number of values of  $n$ ,

In this document, we prove that the response  $y(n)$  of Model 2 when  $x(n) = u(n)$  is given by  $s(n)$  for all values of  $a \in (0, 1)$ ,  $k \in (-\infty, \infty)$  and  $n \in (-\infty, \infty)$ .

### **Proof:**

To start, we prove that  $y(n) = 0$  for all  $n < 0$ . Since  $u(n) = 0$  for all  $n < 0$ ,

$$y(n) = a \cdot y(n-1) \quad \text{for all } n < 0$$

Since  $a \neq 0$ , this is satisfied if and only if  $y(n) = 0$  for all  $n < 0$ . Since  $s(n) = 0$  for all  $n < 0$ , we have that  $y(n) = s(n)$  for all  $n < 0$ .

We then prove that  $y(n) = s(n) = k(1 - a^{n+1})$  for all  $n \geq 0$  using mathematical induction.

First note that since  $u(n) = 1$  for all  $n \geq 0$ ,  $y(n) = ay(n-1) + k(1-a)$  for all  $n \geq 0$ .

Base step: Since  $y(-1) = 0$ ,

$$\begin{aligned}
 y(0) &= ay(-1) + k(1 - a) \\
 &= k(1 - a) \\
 &= k(1 - a^{0+1}) \\
 &= s(0)
 \end{aligned}$$

Help

Inductive step: Assume that for some  $n \geq 0$ ,  $y(n) = s(n)$ , then

$$\begin{aligned}
 y(n+1) &= ay(n) + k(1 - a) \\
 &= ak(1 - a^{n+1}) + k(1 - a) \\
 &= k(a - a^{n+2} + 1 - a) \\
 &= k(1 - a^{n+2}) \\
 &= s(n+1)
 \end{aligned}$$

Thus, by mathematical induction,  $y(n) = s(n)$  for all  $n \geq 0$ .



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