Discrete-time sinusoids

Discrete-time cosines

Consider a discrete-time cosine waveform with N samples:

$$\cos(2\pi f n + \phi) \text{ for } n = 0, 1, \ldots (N-1)$$

In the following, we assume N is even.

If we have only N samples, it turns out that we only need to consider N/2+1 frequencies:

$$f_k = \frac{k}{N} \text{ for } k \in \left\{0, 1, \ldots \tfrac{N}{2}\right\} \longleftarrow$$

f_k is called a normalized frequency
It has units of cycles/sample

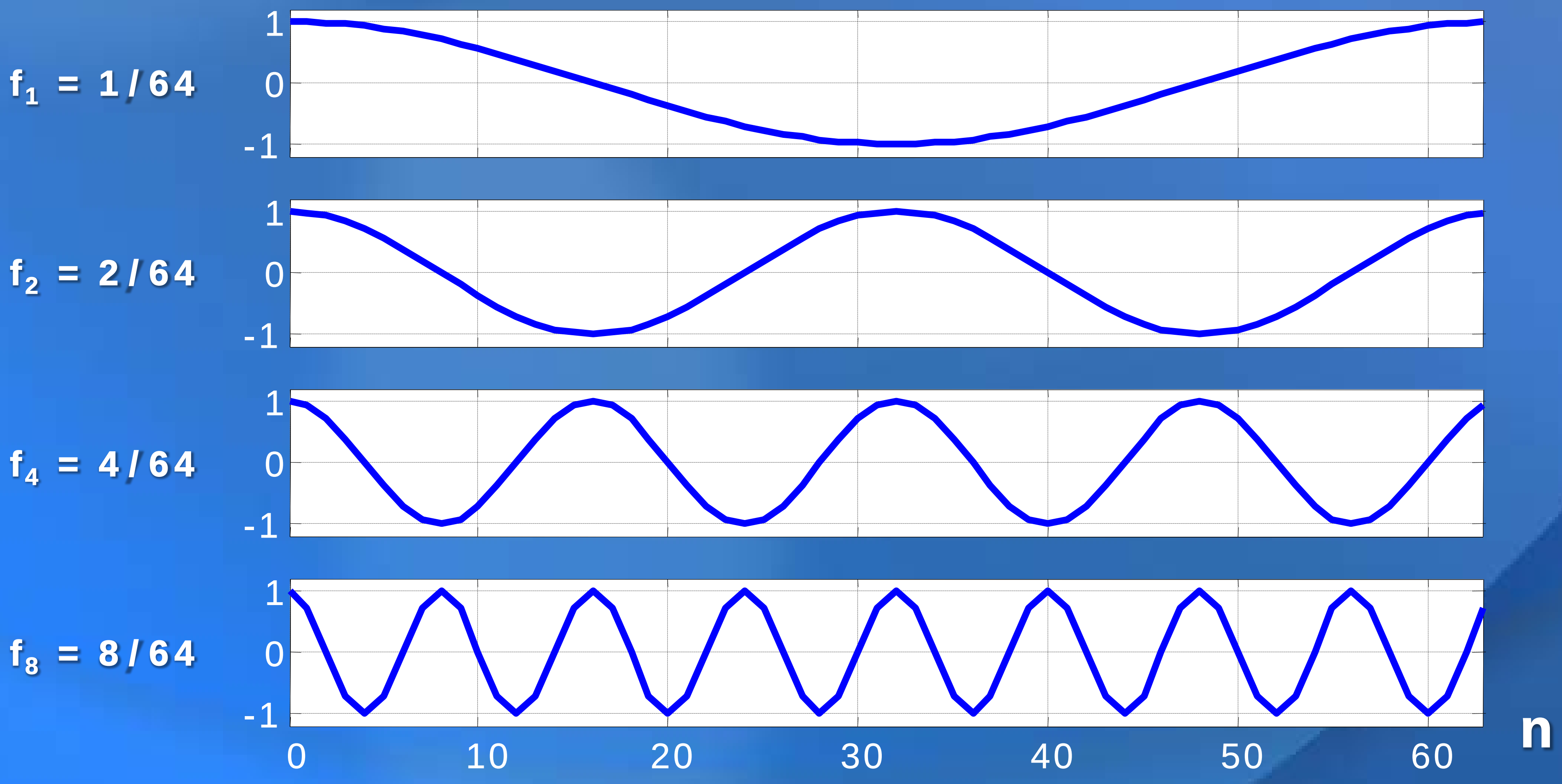
For all N, it is always true that $0 \leq f_k \leq 0.5$

k indicates how many times the cosine repeats in N samples

$$\cos(2\pi f_k n) = \cos(2\pi k \frac{n}{N})$$

The larger k, the higher the frequency.

Discrete-time cosines



Normalized frequency f

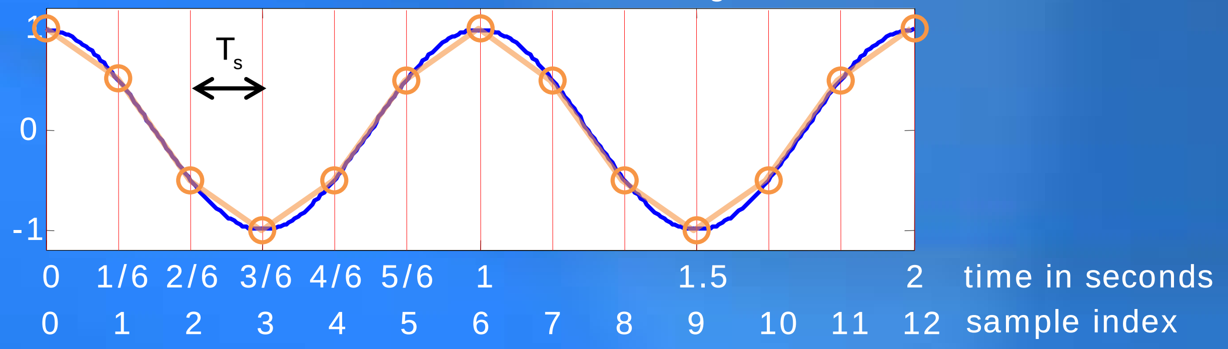
Sampling a continuous cosine $\cos(2\pi\tilde{f}t)$ **at frequency** F_s**,**

$$x(n) = \cos(2\pi\tilde{f}\frac{n}{F_s}) \Longleftarrow \text{Since sample period } T_s = \frac{1}{F_s},\ \text{sample } n \text{ at time } t = \frac{n}{F_s}$$

$$= \cos(2\pi\frac{\tilde{f}}{F_s}n)$$

$$= \cos(2\pi f n) \Rightarrow \boxed{f = \frac{\tilde{f}}{F_s}}$$

$\tilde{f} = 1\text{Hz}$ cosine sampled at $F_s = 6\text{Hz}$



Units of normalized frequency:

$$\frac{\tilde{f} \quad \text{cycles}/\text{sec}}{F_s \quad \text{samples}/\text{sec}} = f \quad \boxed{\text{cycles}/\text{samples}}$$