Entropy of a discrete random variable

Discrete Random Variable

A random variable X that can achieve one of K possible values x_k for $k=0,1,\ldots K-1$ each with probability p_k is said to be discrete.

Examples:

A random bit ($K = 2$)

Pick a number from 1 to 10 ($K = 10$)

Throw a 6 sided die ($K = 6$)

Entropy

The entropy of a discrete random variable that has *K* possible values each with probability p_k is

$$H = -\sum_{k=0}^{K-1} p_k \log_2(p_k)$$

Note that H is independent of the possible values. It depends only upon the probabilities.

$$\text{If } K = 2,\ H = -\sum_{k=0}^{1} p_k \log_2(p_k) = -p_0 \log_2(p_0) - p_1 \log_2(p_1)$$

Uniform Distribution

If all outcomes equally likely: $p_k = \frac{1}{K}$ for all k

The entropy is $H = -\sum_{k=0}^{K-1} p_k \log_2(p_k) = -\frac{1}{K}\sum_{k=0}^{K-1} \log_2\left(\frac{1}{K}\right) = \log_2 K$

Examples

- $K = 2$, $H = \log_2 2 = 1$ bit
- $K = 4$, $H = \log_2 4 = 2$ bits
- $K = 6$, $H = \log_2 6 \approx 2.6$ bits
- $K = 8$, $H = \log_2 8 = 3$ bit

Non-Uniform Distribution

k	x_k	p_k	$-\log_2(p_k)$
0	Blue	1 / 2	1 bit
1	Yellow	1 / 4	2 bits
2	Green	1 / 8	3 bits
3	Red	1 / 8	3 bits

Entropy: $H = -\sum_{k=0}^{K-1} p_k \log_2(p_k) = \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3 + \frac{1}{8}\cdot 3 = 1.75$

If all choices were equally likely, $H = \log_2(4) = 2$ bits