Complex Numbers

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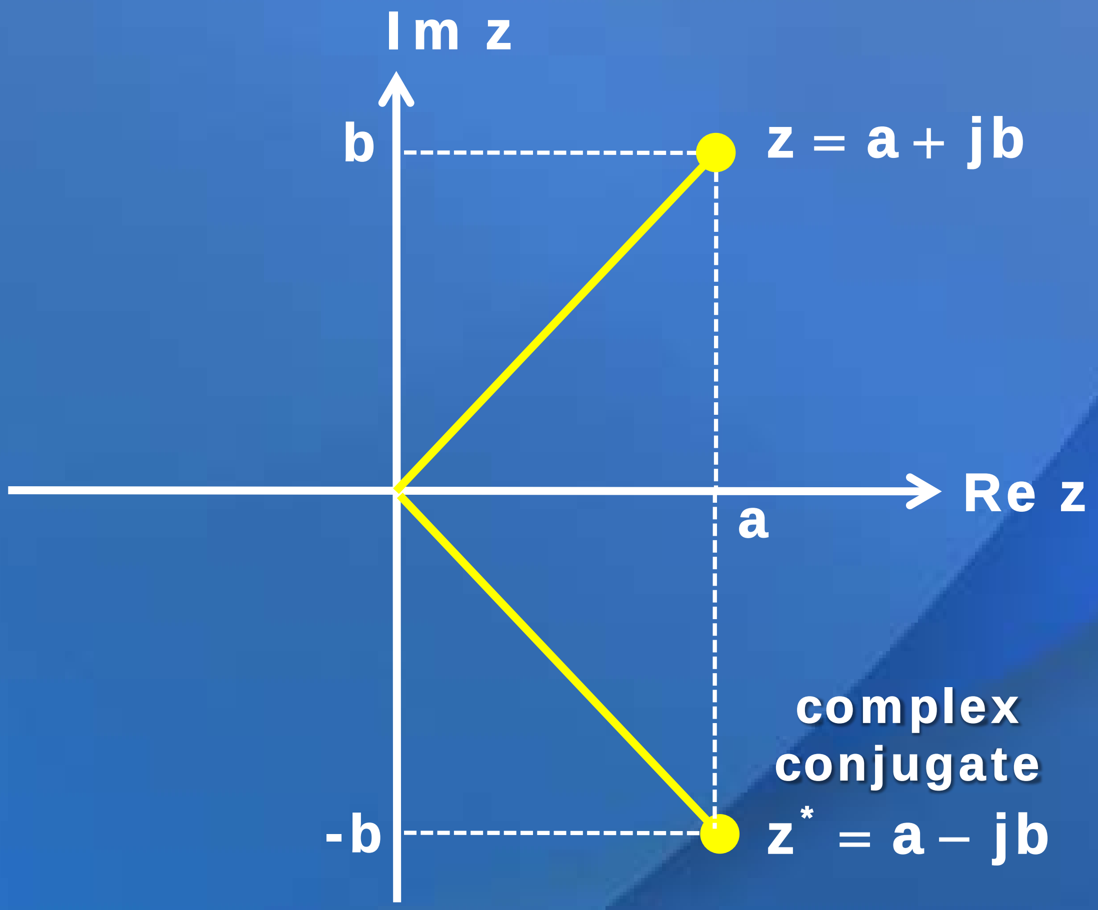
Define $j = \sqrt{-1}$

A complex number is given by

$$z = a + jb$$

where a and b are real

a = Re{z} (real part)
b = IM{z} (imaginary part)



complex plane

Properties

Let $z_1 = a_1 + jb_1$

$z_2 = a_2 + jb_2$

Addition

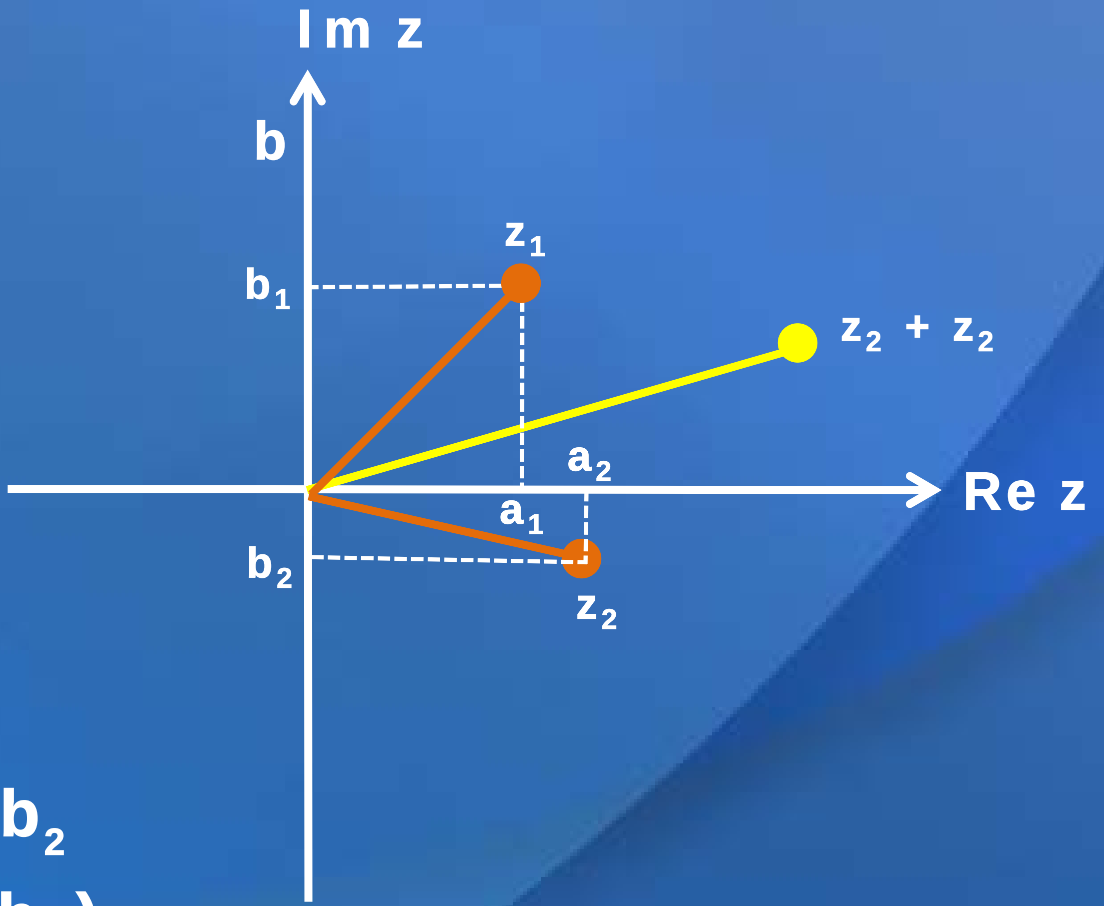
$$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2)$$

Multiplication

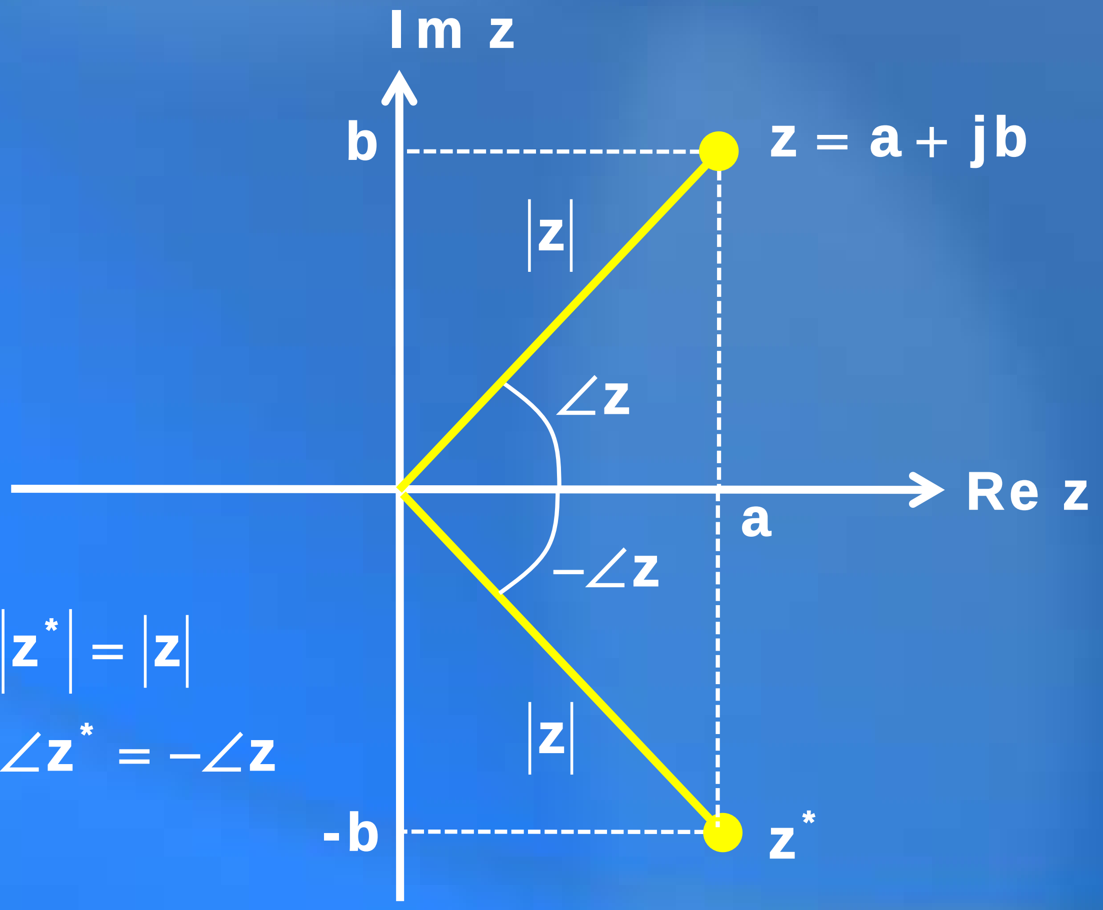
$$z_1 \cdot z_2 = (a_1 + jb_1)(a_2 + jb_2)$$

$$= a_1a_2 + ja_1b_2 + ja_2b_2 + j^2b_1b_2$$

$$= (a_1a_2 - b_1b_2) + j(a_1b_2 + a_2b_2)$$



Polar Representation



$$|z^*| = |z|$$

$$\angle z^* = -\angle z$$

By analogy with polar coordinates, we can define the magnitude and phase of a complex number.

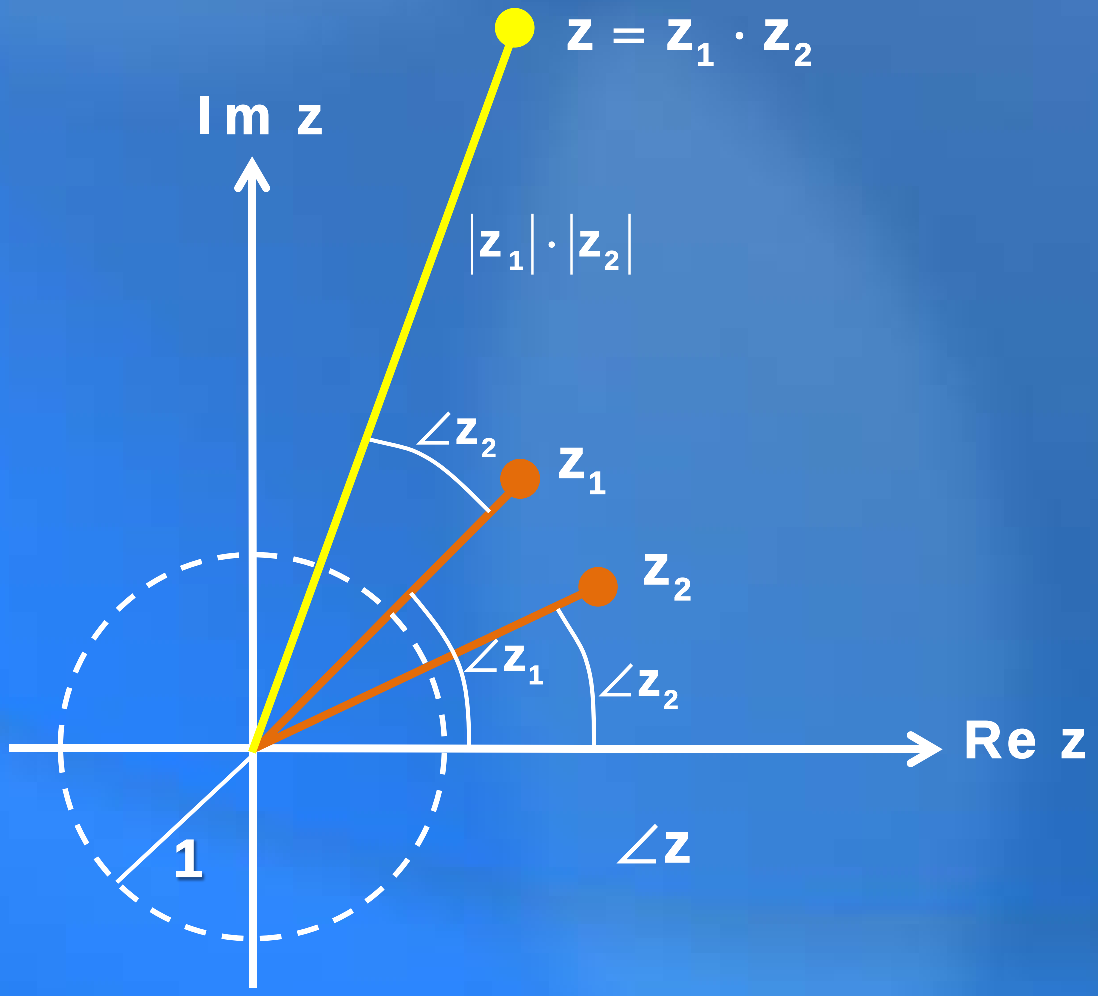
$$|z| = \sqrt{a^2 + b^2}$$

$$\angle z = \arctan(b\ /\ a)$$

$$a = |z|\cos(\angle z)$$

$$b = |z|\sin(\angle z)$$

Multiplication



Multiplication is much simpler using the polar representation:

If $z = z_1 \cdot z_2$,

$$|z| = |z_1| \cdot |z_2|$$

$$\angle z = \angle z_1 + \angle z_2$$