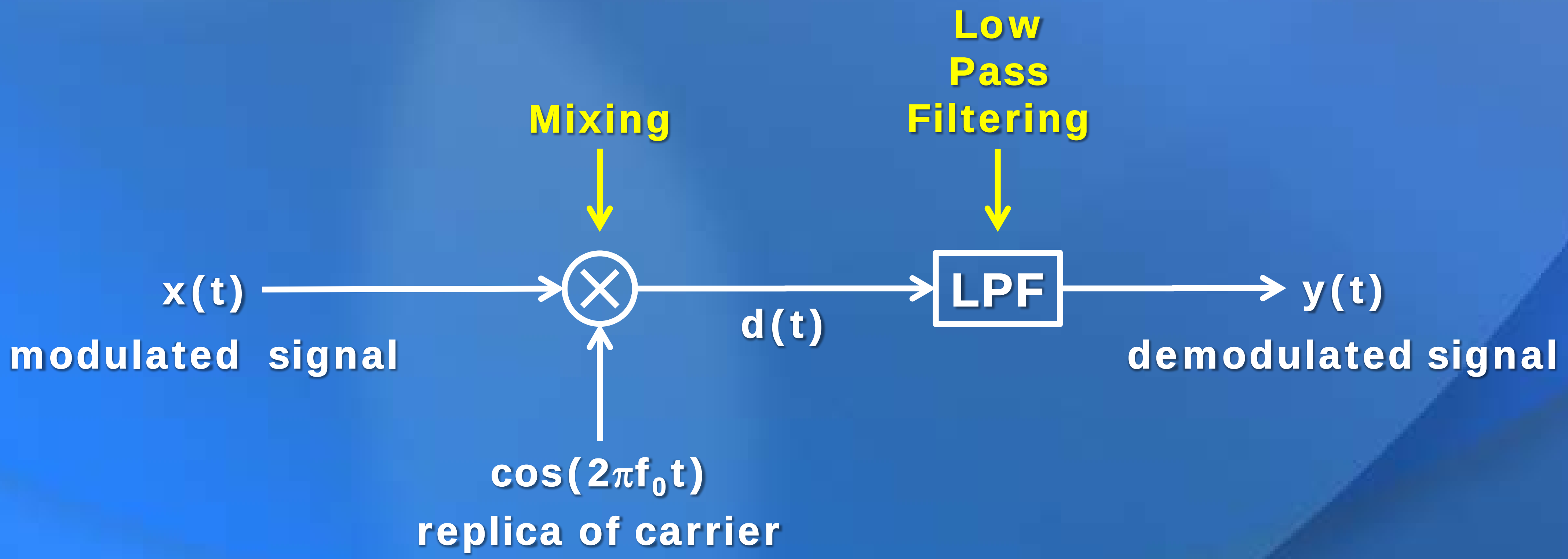
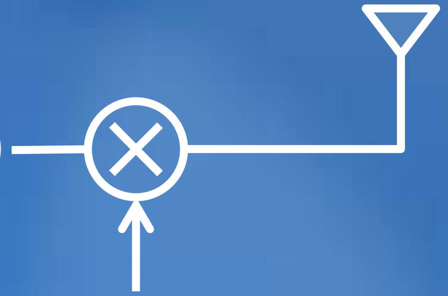
Analysis of Mixing using Cosines

Demodulation



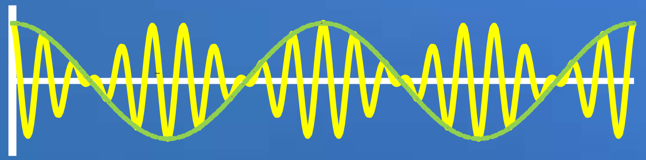
Modulation with Cosines

$m(t) = \cos(2\pi f_s t)$



$c(t) = \cos(2\pi f_0 t)$

$$x(t) = m(t) \times \cos(2\pi f_0 t)$$
$$= \tfrac{1}{2}\cos(2\pi(f_0 - f_s)t) + \tfrac{1}{2}\cos(2\pi(f_0 + f_s)t)$$



Analysis of Mixing

$$d(t) = x(t) \times \cos(2\pi f_0 t)$$

$$= \left[m(t) \times \cos(2\pi f_0 t) \right] \times \cos(2\pi f_0 t)$$

Analysis of Mixing

$$d(t) = \left[m(t) \times \cos(2\pi f_0 t)\right] \times \cos(2\pi f_0 t)$$

$$= m(t) \times \left[\cos(2\pi f_0 t) \times \cos(2\pi f_0 t)\right]$$

Analysis of Mixing

$$d(t) = m(t) \times \left[\cos(2\pi f_0 t) \times \cos(2\pi f_0 t)\right]$$

$$= m(t) \times \left[\tfrac{1}{2}\cos\left(2\pi(f_0 - f_0)t\right) + \tfrac{1}{2}\cos\left(2\pi(f_0 + f_0)t\right)\right]$$

Analysis of Mixing

$$
\begin{aligned}
d(t) &= m(t) \times \left[\tfrac{1}{2}\cos\left(2\pi(f_0 - f_0)t\right) + \tfrac{1}{2}\cos\left(2\pi(f_0 + f_0)t\right)\right] \\
&= m(t) \times \left[\tfrac{1}{2} + \tfrac{1}{2}\cos\left(2\pi(2f_0)t\right)\right]
\end{aligned}
$$

Analysis of Mixing

$$d(t) = m(t) \times \left[\tfrac{1}{2} + \tfrac{1}{2}\cos\left(2\pi(2f_0)t\right)\right]$$
$$= \tfrac{1}{2}m(t) + \tfrac{1}{2}m(t) \times \cos\left(2\pi(2f_0)t\right)$$

Analysis of Mixing

$$d(t) = \frac{1}{2}m(t) + \underbrace{\frac{1}{2}m(t)\times\cos\big(2\pi(2f_0)t\big)}_{\text{extra we need to remove}}$$

original message

Example

If $m(t) = \cos(2\pi f_s t)$, then

$$d(t) = \tfrac{1}{2}m(t) + \tfrac{1}{2}m(t) \times \cos\big(2\pi(2f_0)t\big)$$

$$= \underbrace{\tfrac{1}{2}\cos(2\pi f_s t)}_{\text{message}} + \underbrace{\tfrac{1}{4}\cos\big(2\pi(2f_0 - f_s)t\big) + \tfrac{1}{4}\cos\big(2\pi(2f_0 + f_s)t\big)}_{\text{extra}}$$