Discrete Fourier Transform

Discrete Fourier Series

The Fourier Series expansion expresses a signal as a sum of scaled and shifted cosines.

$$x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos(2\pi f_k n + \phi_k) \qquad f_k = \frac{k}{N}$$

Discrete Fourier Transform

Combining: $x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos\left(2\pi f_k n + \phi_k\right)$

$$\uparrow$$

$$\cos\left(2\pi f_k n + \phi_k\right) = \frac{1}{2}\left(e^{j\phi_k} e^{j2\pi f_k n} + e^{-j\phi_k} e^{-j2\pi f_k n}\right)$$

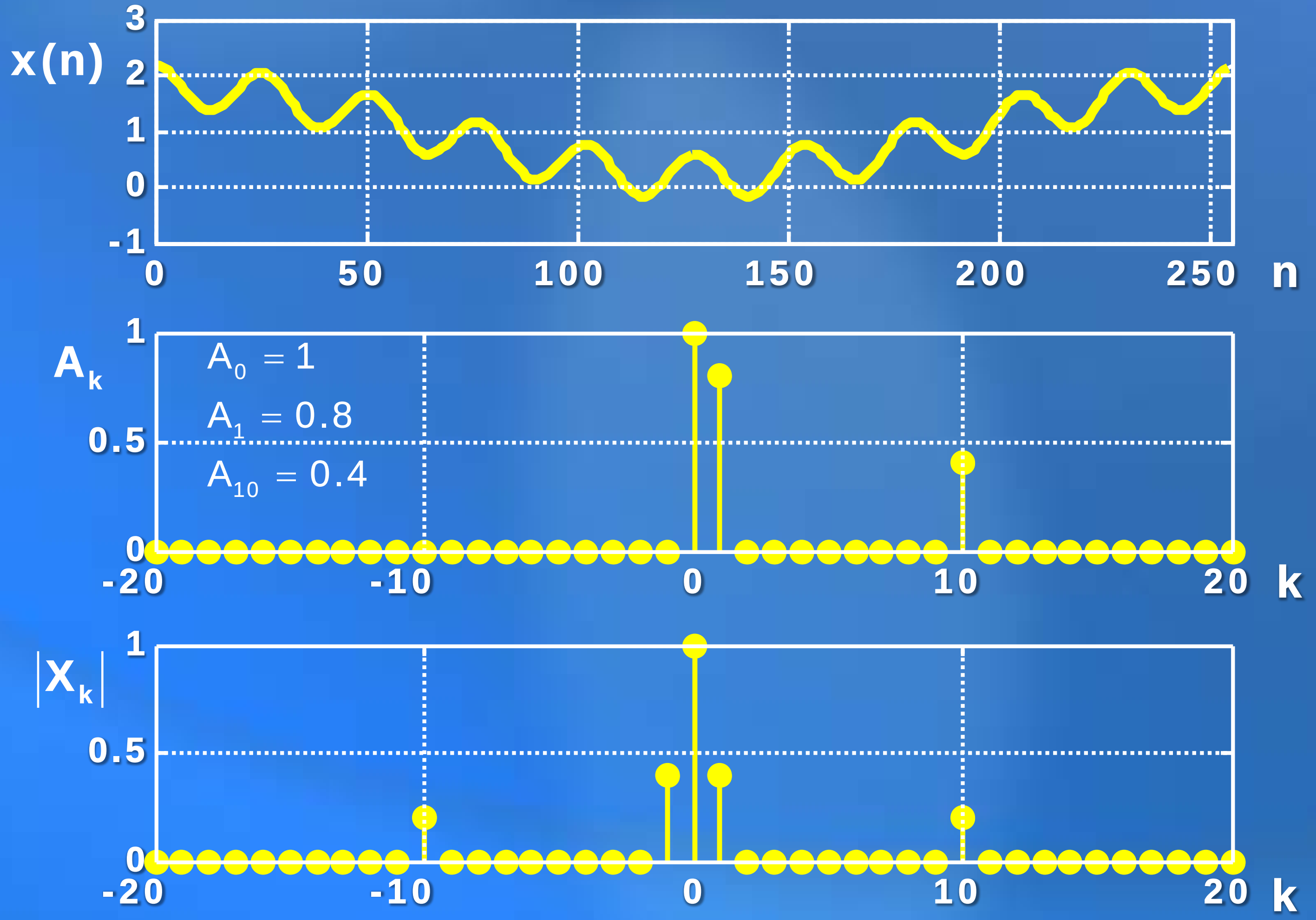
We obtain:

$$x(n) = A_0 + \sum_{k=1}^{N/2} \frac{A_k}{2} e^{j\phi_k} \cdot e^{j2\pi f_k n} + \sum_{k=1}^{N/2} \frac{A_k}{2} e^{-j\phi_k} \cdot e^{-j2\pi f_k n}$$

$$= \sum_{k=-N/2}^{N/2} X_k \cdot e^{j2\pi f_k n}$$

$$\boxed{X_k = \begin{cases} \frac{A_{-k}}{2} e^{-j\phi_{-k}} & k < 0 \\ A_0 & k = 0 \\ \frac{A_k}{2} e^{j\phi_k} & k > 0 \end{cases}}$$

DFT Amplitude



The DFT amplitude spectrum and discrete Fourier series amplitude spectra are related by

$$|X_k| = \begin{cases} \dfrac{A_{-k}}{2} & k < 0 \\ A_0 & k = 0 \\ \dfrac{A_k}{2} & k > 0 \end{cases}$$