Fourier Series

Fourier Series

Any sampled data waveform x(n) with N samples: $x(n) \text{ for } n = 0, 1, \ldots (N-1)$

can be expressed as the sum of N/2+1 cosine waves with frequencies

$$f_k = \frac{k}{N} \text{ for } k \in \left\{0, 1, \ldots \tfrac{N}{2}\right\}$$

using the Fourier Series: $x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos(2\pi f_k n + \phi_k)$

Different N sample waveforms have <u>different</u> values of A_k and ϕ_k, but the <u>same</u> values of f_k

Each term in the sum is called a <u>frequency component.</u>

Amplitude, Frequency, Phase

$$x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos(2\pi f_k n + \phi_k)$$

Amplitude A_k

- Tells us how large the cosine is.
- The frequency components with the largest amplitude are the “most important.”

Frequency f_k

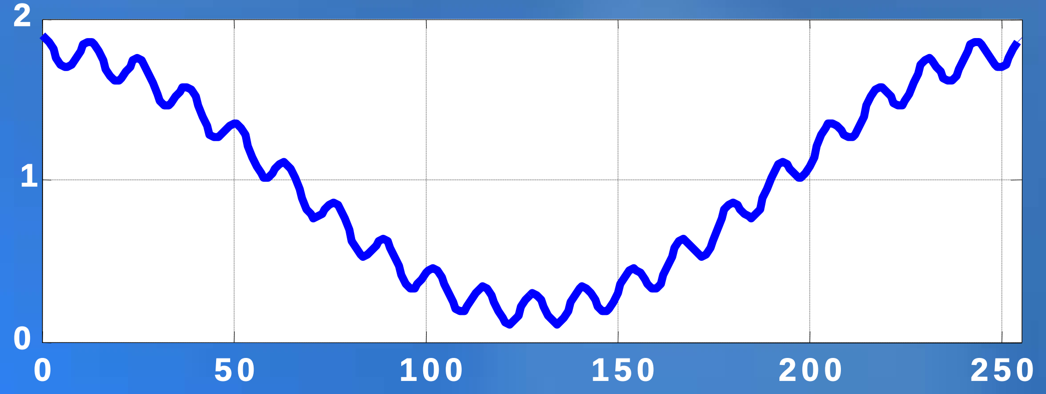
- Tells us how quickly the cosine changes
- small k = low frequency = slowly changing
- large k = high frequency = quickly changing

Phase ϕ_k

- Not as important, just shifts each cosine left or right.

Example

$$x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos(2\pi f_k n + \phi_k)$$

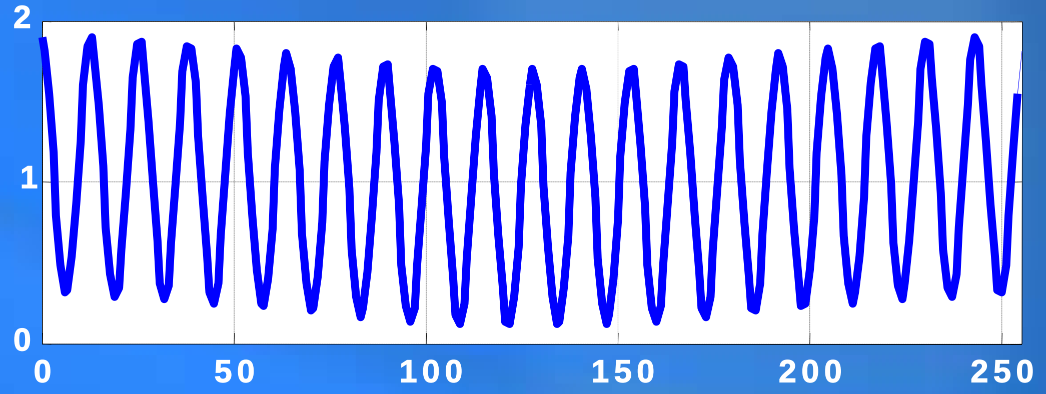


$A_0 = 1$

$A_1 = 0.8$

$A_{20} = 0.1$

large low frequency component
and
small high frequency component



$A_0 = 1$

$A_1 = 0.1$

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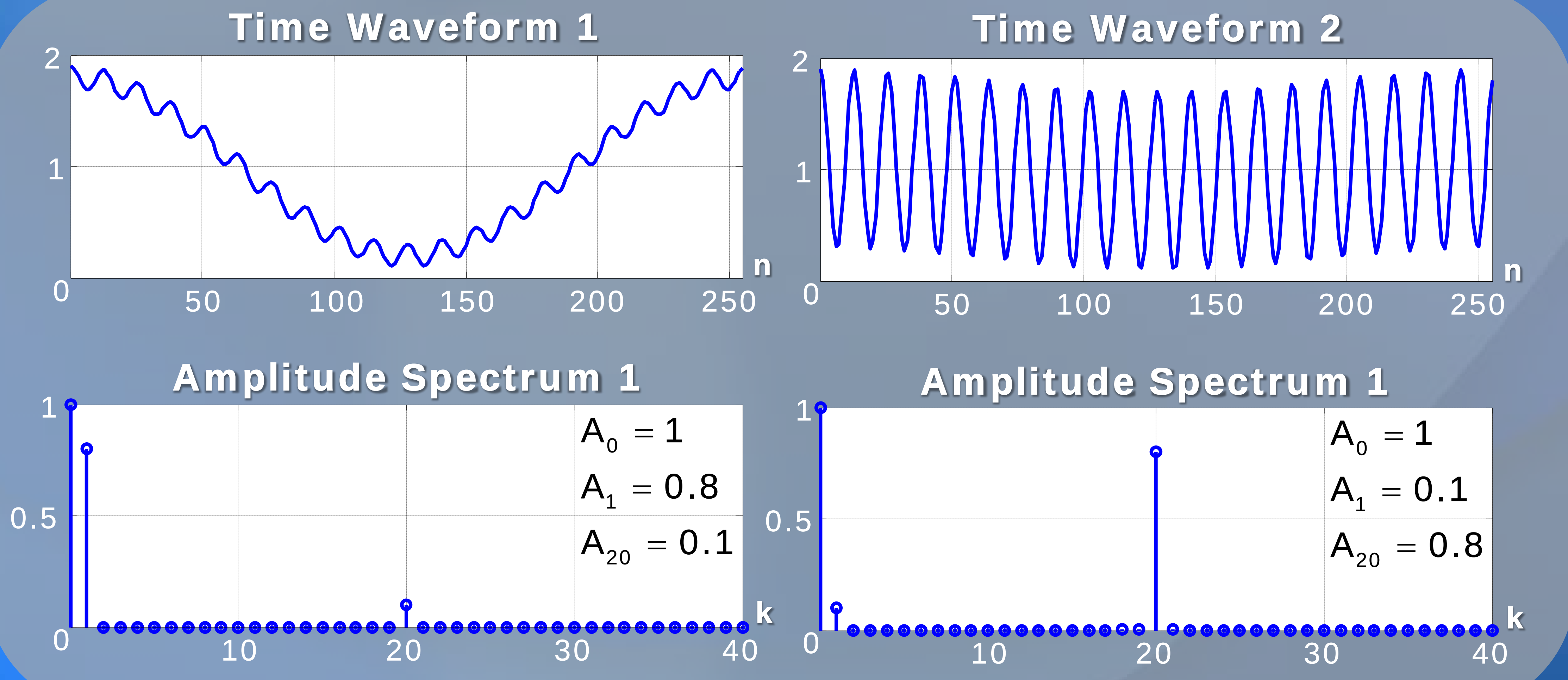
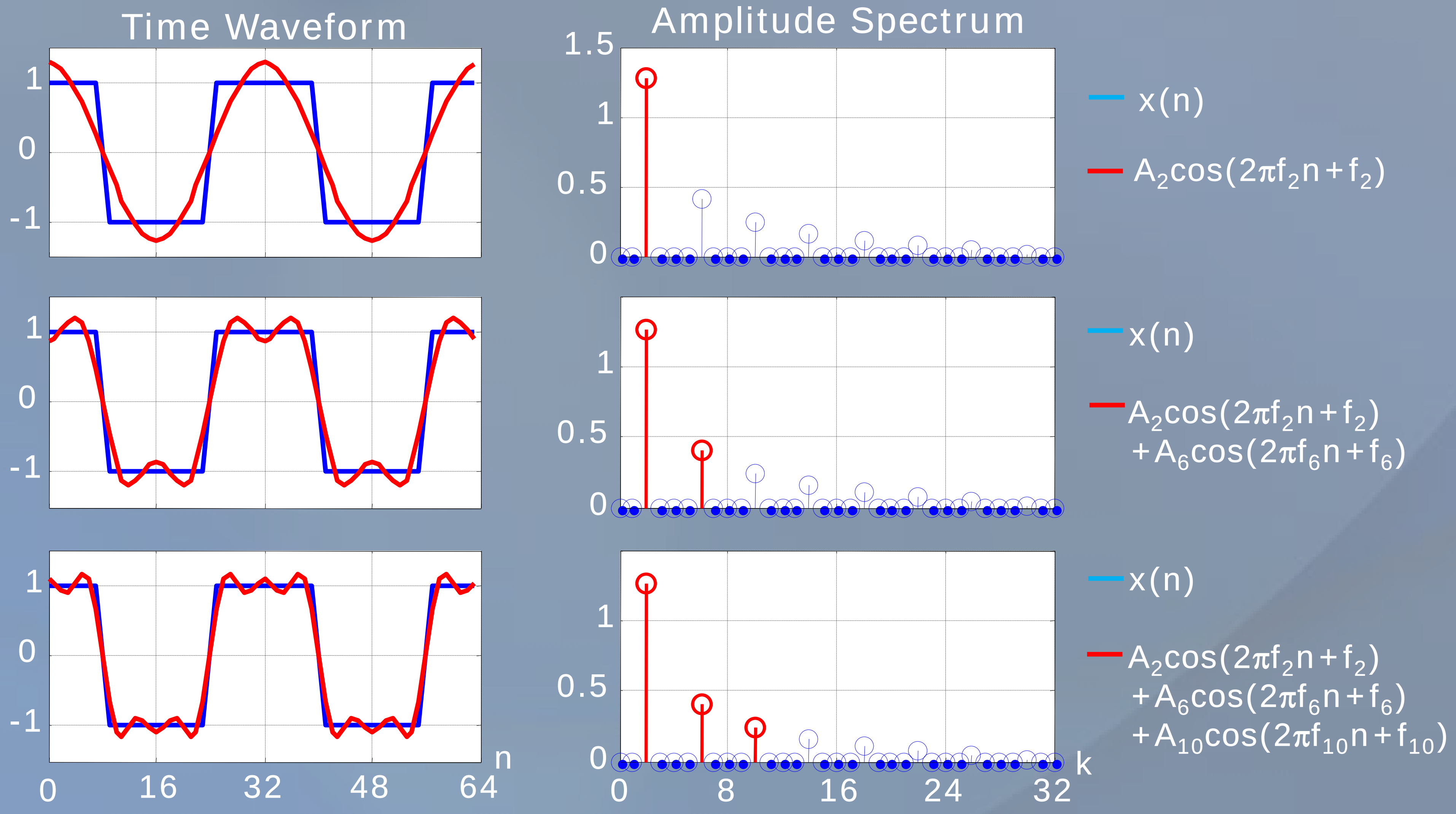
small low frequency component
and
large high frequency component

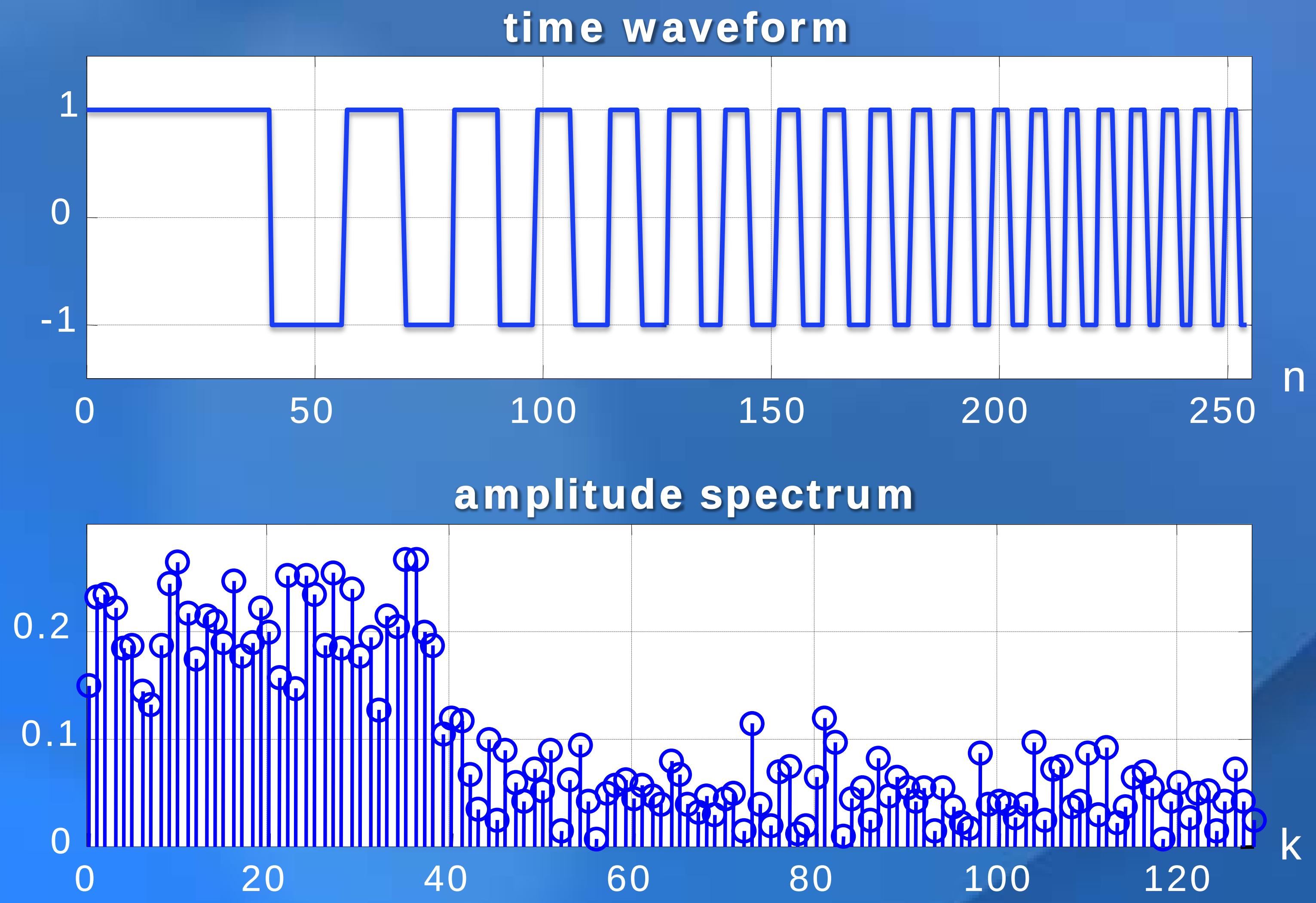
Amplitude Spectrum



Example: Square Wave



More Complex Example



Transforms

The Fourier Series is only one of many transforms:

- Fourier Transform
- Laplace Transform
- Z Transform

Transforms are merely a different way of expressing the same data. No information is lost or gained when taking a transform.

We use a transform because

- It gives us a different way of viewing or understanding the signal.
- Some operations on signals are easier to understand / analyze after taking the transform.