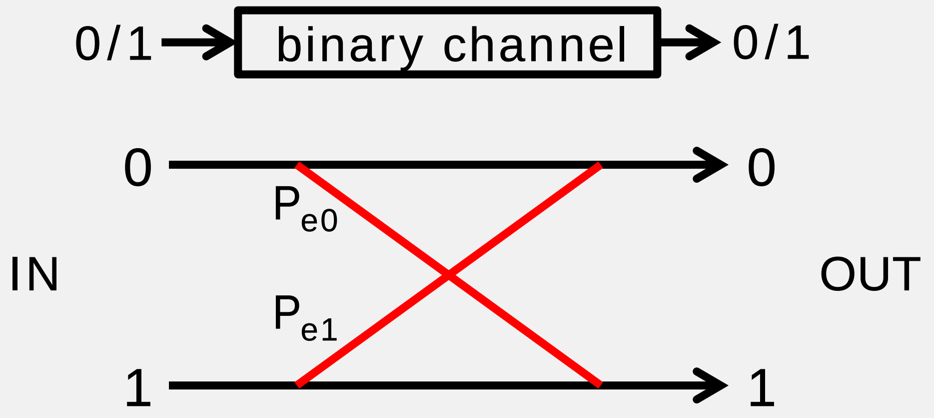
Entropy of a bit

Binary Channel Model

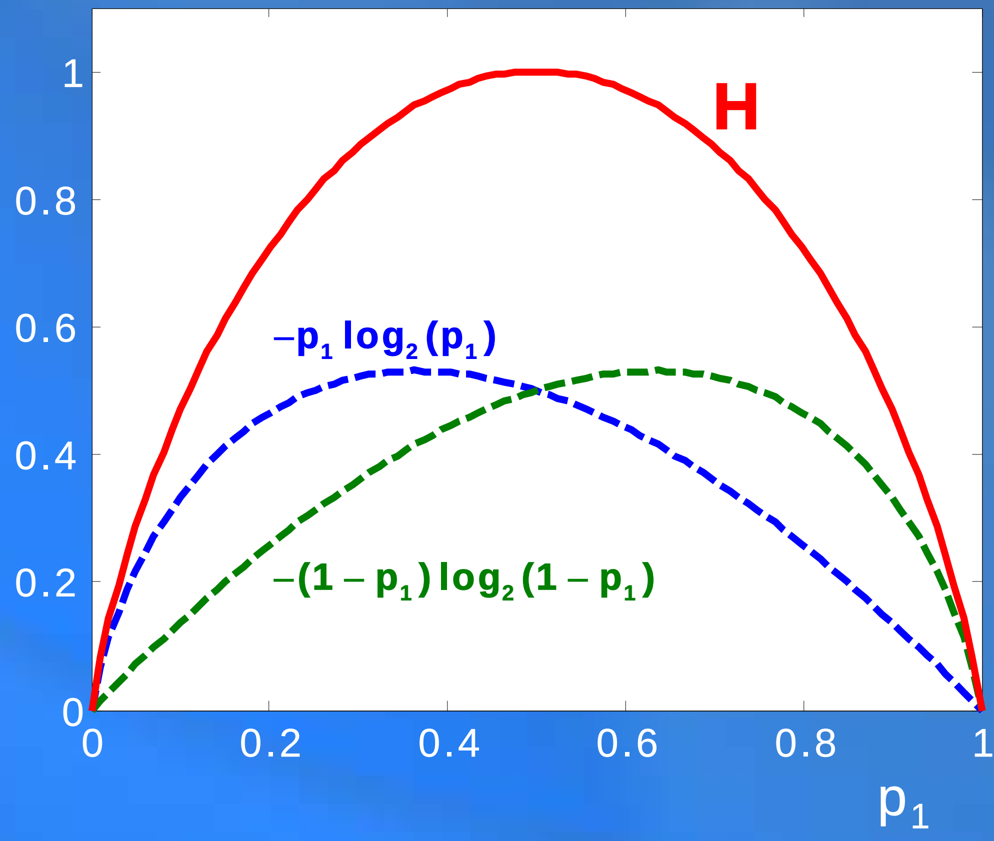


$$BER = P_{e0} \cdot P[IN=0] + P_{e1} \cdot P[IN=1]$$

Usually, the transmitter controls P[IN=0] and P[IN=1]

- **e.g. P[IN=0] = P[IN=1] = 0.5**

Entropy



Let p_1 = P[IN=1]

p_0 = P[IN=0]

Entropy:

$$H = -p_0 \log_2(p_0) - p_1 \log_2(p_1)$$

$$= -(1-p_1)\log_2(1-p_1) - p_1 \log_2(p_1)$$

(since $p_0 + p_1 = 1 \Rightarrow p_0 = 1 - p_1$)

Expected Value

Consider a random variable I that assumes one of two possible values:

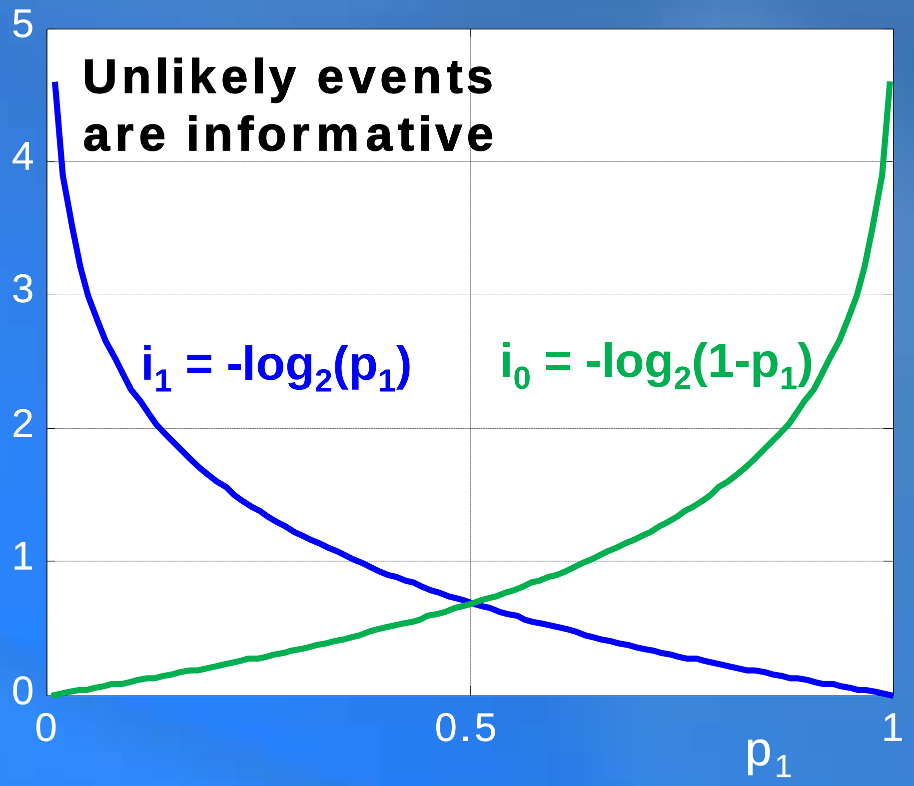
$I = i_0$ with probability p_0

$I = i_1$ with probability p_1

The expected value of X is $E[X] = p_0 \cdot i_0 + p_1 \cdot i_1$

Intuitively, the expected value is the long term average.

Entropy as Average Information



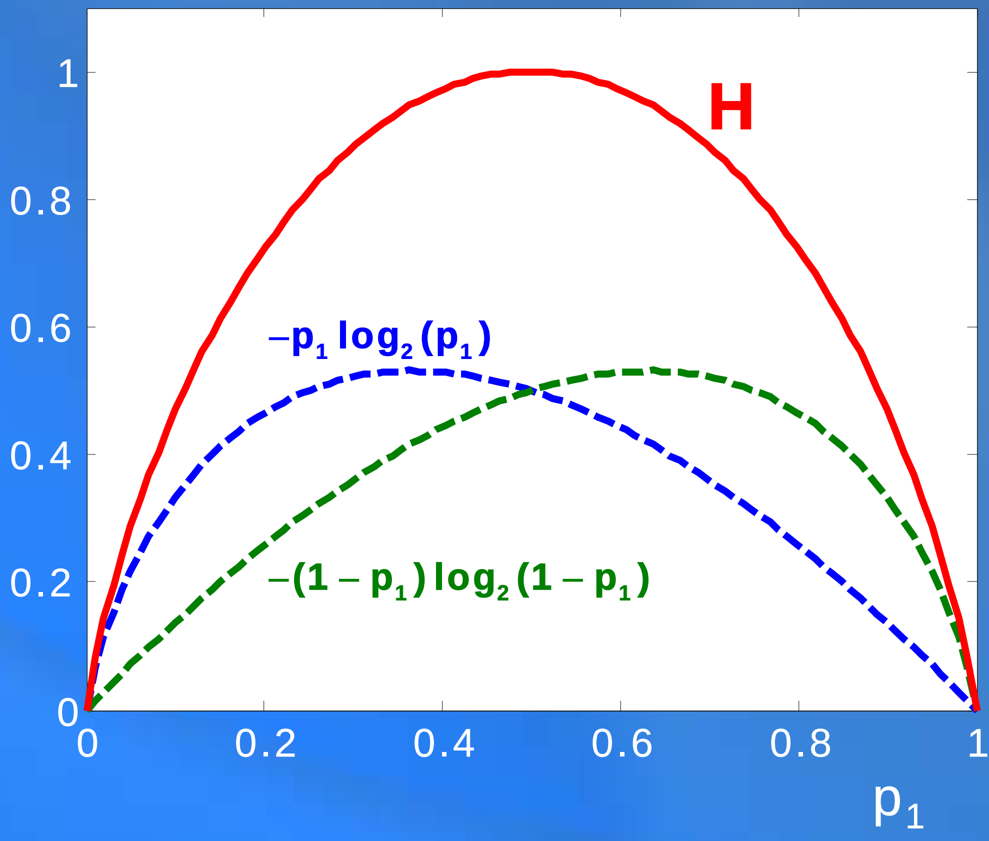
Define the "information" gained when observing a bit is 0 or 1 as

$$i_0 = -\log_2(p_0) = -\log_2(1-p_1)$$
$$i_1 = -\log_2(p_1)$$

The entropy of the bit is the average information

$$H = -p_0 \cdot \log_2(p_0) - p_1 \cdot \log_2(p_1)$$

Entropy



Entropy is a measure of the average information carried by a binary random variable.

$$H = -p_0 \log_2(p_0) - p_1 \log_2(p_1)$$
$$= -(1-p_1)\log_2(1-p_1) - p_1 \log_2(p_1)$$

The units of entropy are bits.