Aliasing

Sampling a Cosine

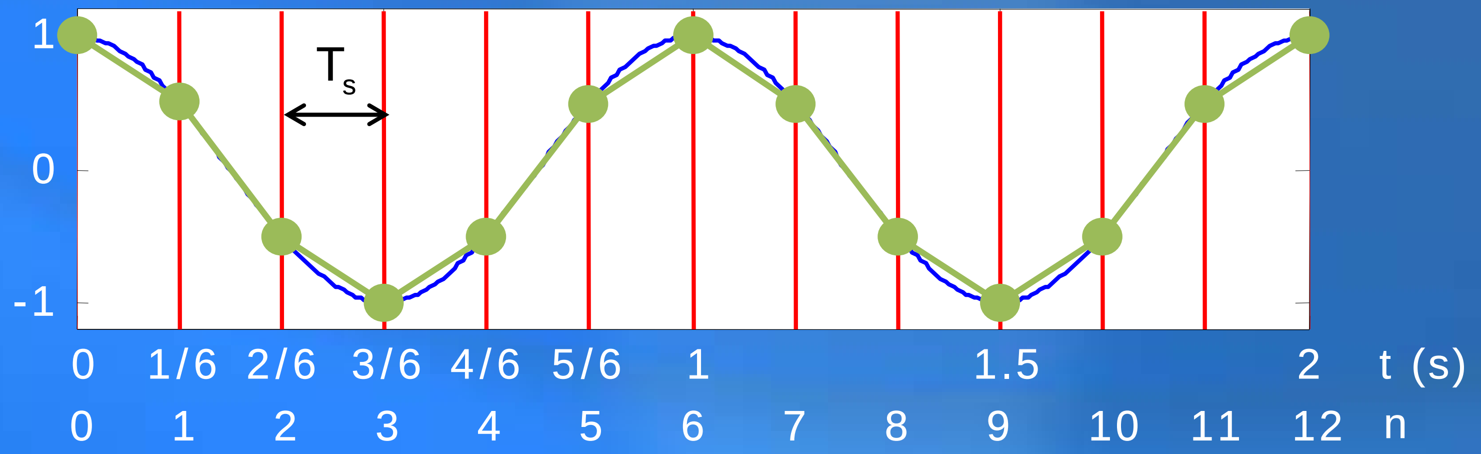
Consider a continuous time cosine: $\tilde{x}(t) = \cos(2\pi\tilde{f}t)$

Sampling this at F_s, we obtain samples

$$x(n) = \cos\left(2\pi\frac{\tilde{f}}{F_s}n\right)$$

$$= \cos(2\pi f n)$$

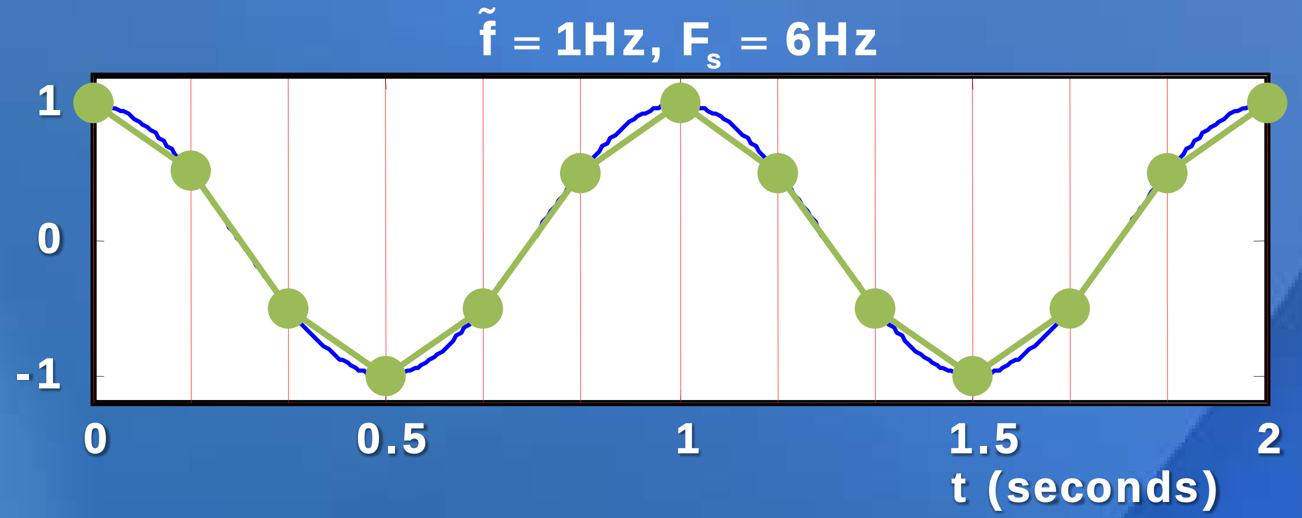
$\tilde{f} = 1\text{Hz},\ F_s = 6\text{Hz}$

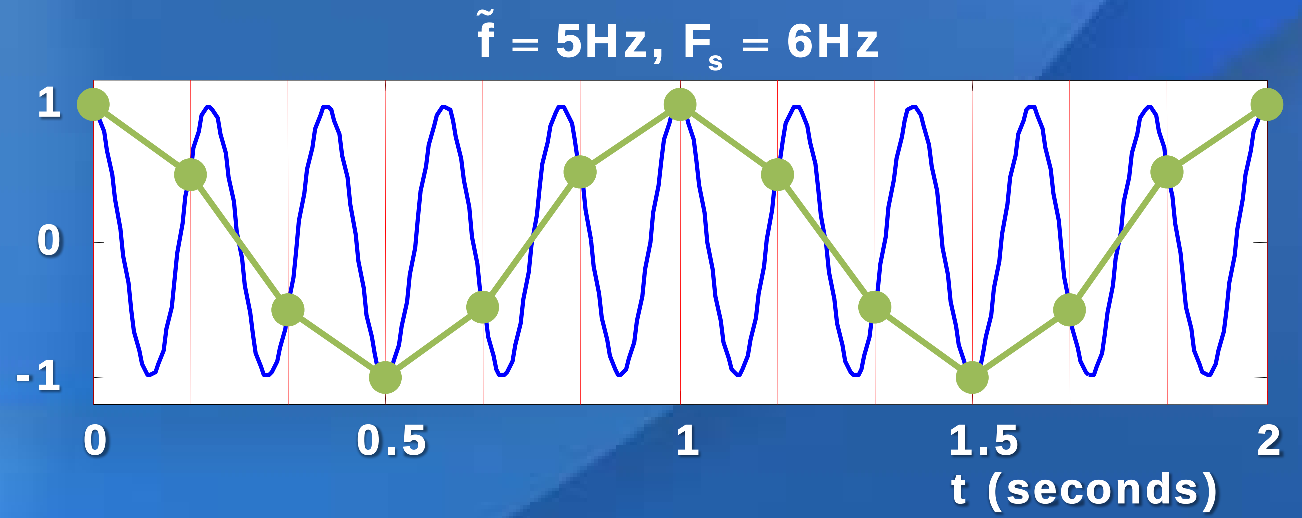


Normalized frequency: $f = \dfrac{\tilde{f}}{F_s}$

Aliasing

If the frequency of the cosine wave is too high in comparison with the sampling frequency, it appears to be (is <u>aliased</u> to) a lower frequency.





red lines = sample points
blue lines = original waveform
green circles = sample values

Nyquist Limit

Since $\cos(2\pi f n) = \cos\left(2\pi f n - 2\pi n\right)$

$$= \cos\left(2\pi\left(f-1\right)n\right)$$

$$= \cos\left(2\pi\left(1-f\right)n\right)$$

$$f = \frac{\tilde{f}}{F_s}$$

A discrete time cosine with f > 0.5 will be aliased to a cosine with a lower frequency, 1-f.

Equivalently, a continuous time cosine with $\tilde{f} > \underbrace{0.5 \cdot F_s}_{\text{Nyquist limit}}$ will be aliased.

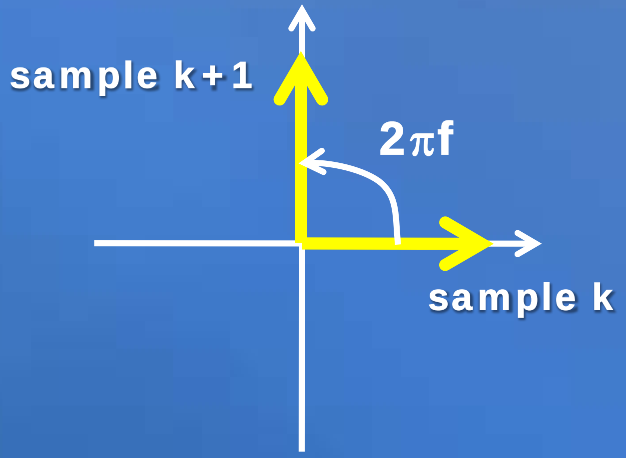
Sampling Complex Exponentials

Since $e^{j2\pi f(k+1)} = e^{j2\pi f} \cdot e^{j2\pi fk}$

between samples k and k+1, the complex exponential rotates by $2\pi f$.

Aliasing occurs when f > 0.5. The rotation is equivalent to a smaller negative rotation.

$f = 0.25$



$f > 0.75$

aliasing!