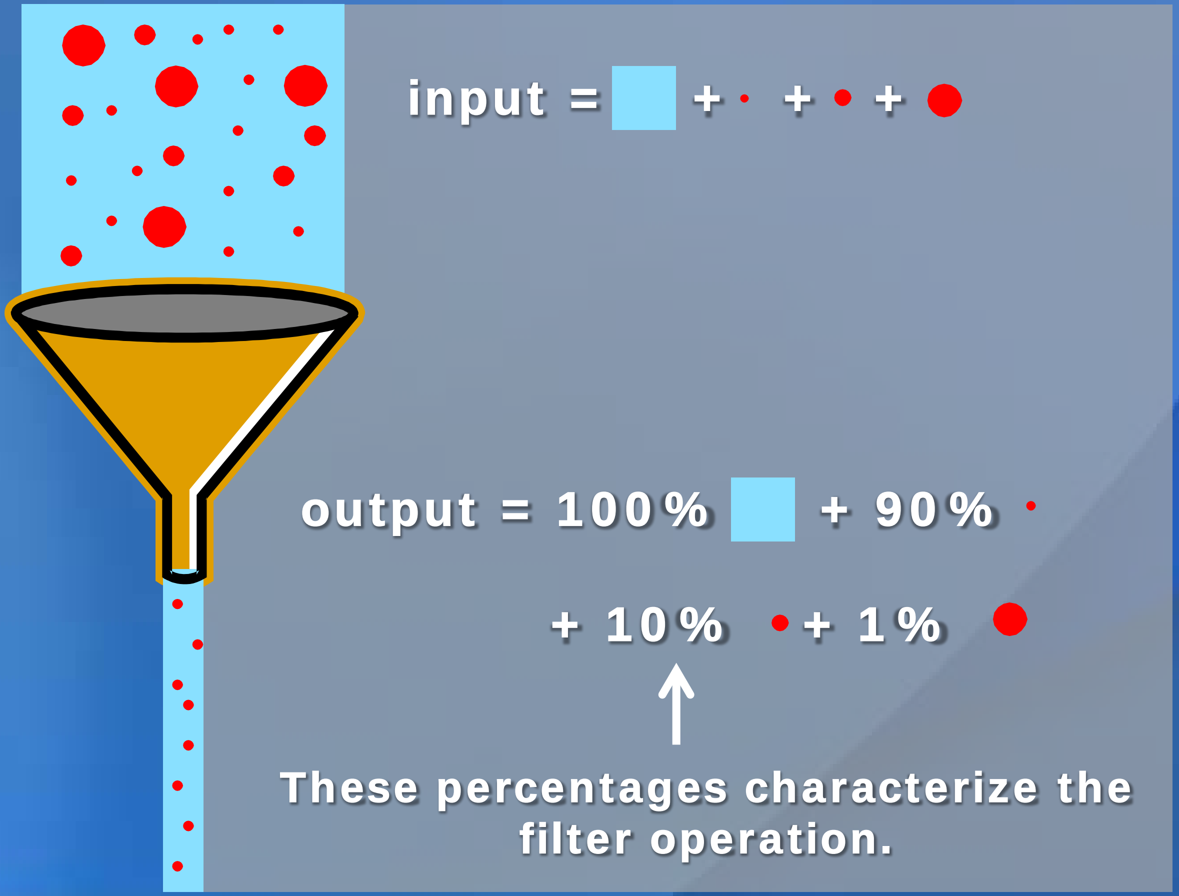
Channels as Filters

Motivation: Filters

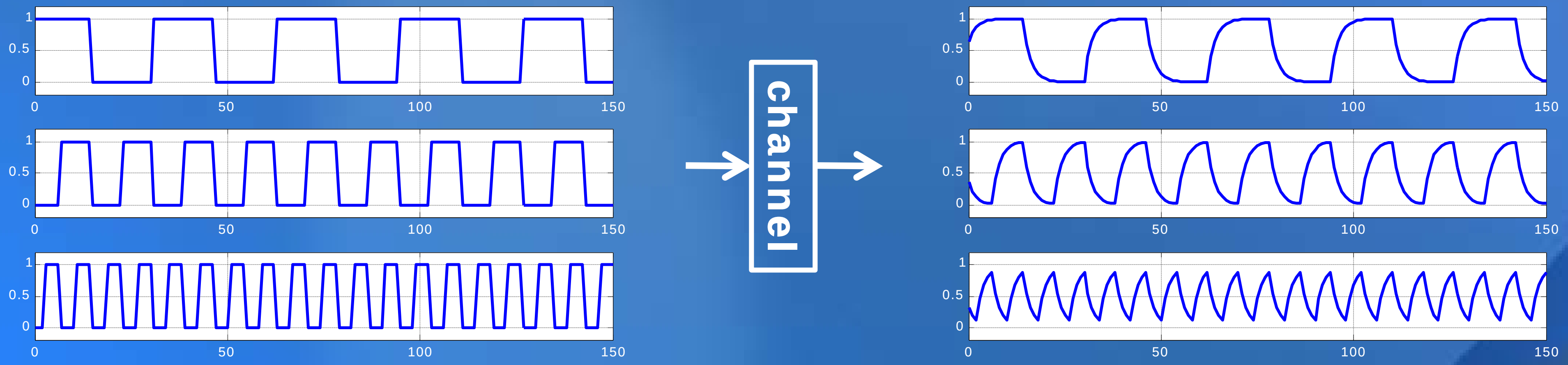
A filter is something that allows only certain parts (components) of the input to pass through to the output.

For example, a water filter might let all small molecules (e.g. water) through, but block most large particles.



Channels as Filters

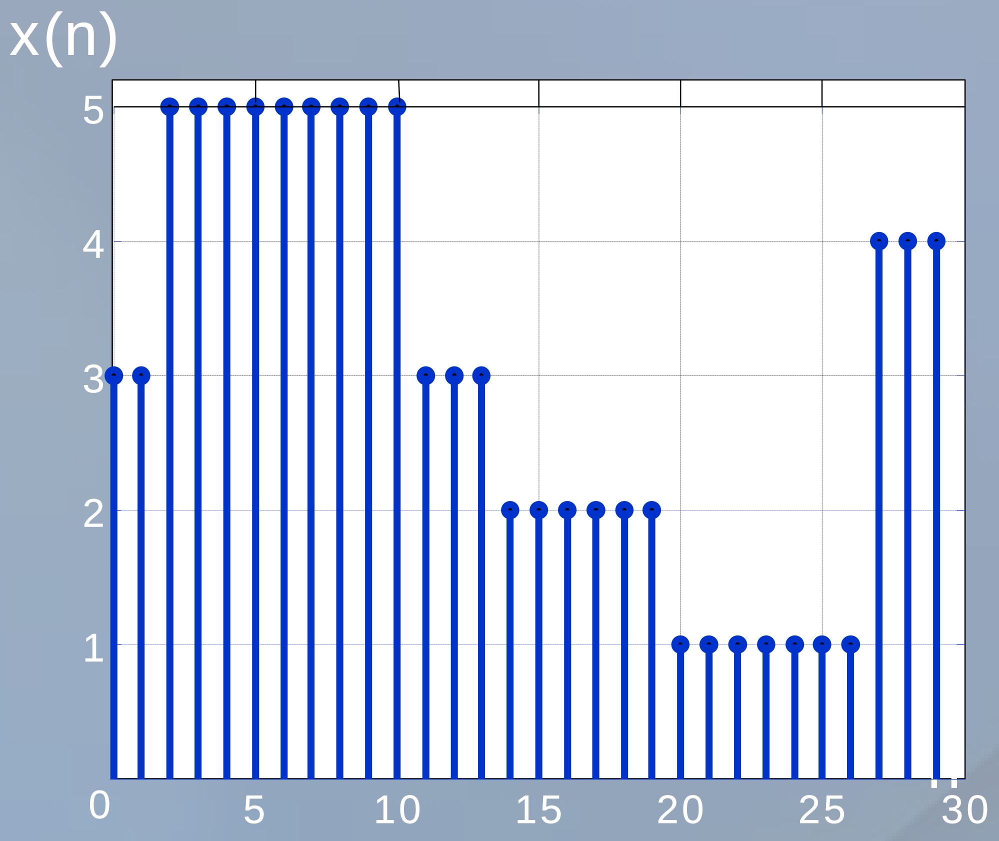
A communication channel can be viewed as a filter. Some parts of the signal (e.g. the long flat parts) pass through, but others (e.g. the fast changes) do not.



What are the “components” of the input signal?
How do we describe how much of each component is passed from input to output

Signal Components

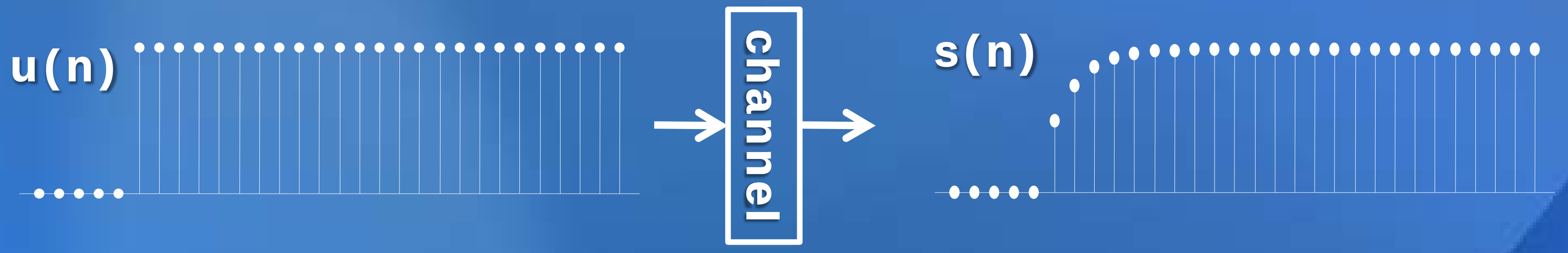
We can decompose any signal by expressing it as a sum of “simpler” signals, e.g. scaled and delayed unit steps.



$$
\begin{aligned}
x(n) = {} & 3 \cdot u(n) \\
& + 2 \cdot u(n-2) \\
& - 2 \cdot u(n-11) \\
& - u(n-14) \\
& - u(n-20) \\
& + 3 \cdot u(n-27)
\end{aligned}
$$

Problem with Unit Steps

If we treat unit steps as our basic components, the components of the output don’t “look like” the components of the input.



We wish to find a set of components such that

- Any signal can be expressed as the sum of scaled and delayed versions of these components
- The components still “look the same” after passing through the channel

Fourier Series

The Fourier Series expansion is an alternative decomposition of a signal as a sum of scaled and shifted cosines.

$$x(n) = A_0 + \sum_{k=1}^{N/2} A_k \cos(2\pi f_k n + \phi_k) \qquad f_k = \frac{k}{N}$$

$$= \sum_{k=0}^{N/2} A_k \cos(2\pi f_k n + \phi_k)$$