Quadrature amplitude modulation

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Quadrature amplitude modulation (**QAM**) is both an analog and a digital modulation scheme. It conveys two analog message signals, or two digital bit streams, by changing (*modulating*) the amplitudes of two carrier waves, using the amplitude-shift keying (ASK) digital modulation scheme or amplitude modulation (AM) analog modulation scheme. The two carrier waves, usually sinusoids, are out of phase with each other by 90° and are thus called quadrature carriers or quadrature components — hence the name of the scheme. The modulated waves are summed, and the final waveform is a combination of both phase-shift keying (PSK) and amplitude-shift keying (ASK), or (in the analog case) of phase modulation (PM) and amplitude modulation. In the digital QAM case, a finite number of at least two phases and at least two amplitudes are used. PSK modulators are often designed using the QAM principle, but are not considered as QAM since the amplitude of the modulated carrier signal is constant. QAM is used extensively as a modulation scheme for digital telecommunication systems. Arbitrarily high spectral efficiencies can be achieved with QAM by setting a suitable constellation size, limited only by the noise level and linearity of the communications channel.[1]

QAM is being used in optical fiber systems as bit rates increase; QAM16 and QAM64 can be optically emulated with a 3-path interferometer.[2]

Contents

Introduction

Like all modulation schemes, QAM conveys data by changing some aspect of a carrier signal, or the carrier wave, (usually a sinusoid) in response to a data signal. In the case of QAM, the amplitude of two waves, 90° out-of-phase with each other (in quadrature) are changed (*modulated* or *keyed*) to represent the data signal. Amplitude modulating two carriers in quadrature can be equivalently viewed as both amplitude modulating and phase modulating a single carrier.

Phase modulation (analog PM) and phase-shift keying (digital PSK) can be regarded as a special case of QAM, where the magnitude of the modulating signal is a constant, with only the phase varying. This can also be extended to frequency modulation (FM) and frequency-shift keying (FSK), for these can be regarded as a special case of phase modulation.

Analog QAM

When transmitting two signals by modulating them with QAM, the transmitted signal will be of the form:

$$\begin{aligned} s(t) &= \Re\left\{[I(t) + iQ(t)]\, e^{i2\pi f_0 t}\right\} \\ &= I(t)\cos(2\pi f_0 t) - Q(t)\sin(2\pi f_0 t) \end{aligned}$$

where $i^2 = -1$, $I(t)$, and $Q(t)$ are the modulating signals, f_0 is the carrier frequency and $\Re\{\}$ is the real part.

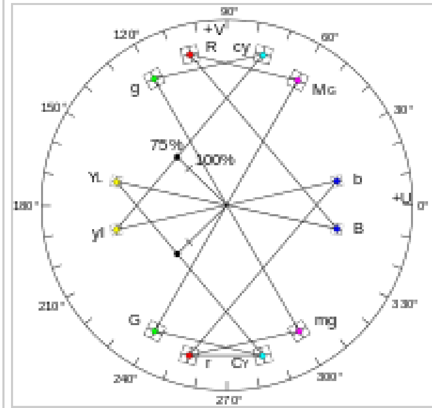
At the receiver, these two modulating signals can be demodulated using a coherent demodulator. Such a receiver multiplies the received signal separately with both a cosine and sine signal to produce the received estimates of $I(t)$ and $Q(t)$ respectively. Because of the orthogonality property of the carrier signals, it is possible to detect the modulating signals independently.

In the ideal case $I(t)$ is demodulated by multiplying the transmitted signal with a cosine signal:

$$\begin{aligned} r(t) &= s(t)\cos(2\pi f_0 t) \\ &= I(t)\cos(2\pi f_0 t)\cos(2\pi f_0 t) - Q(t)\sin(2\pi f_0 t)\cos(2\pi f_0 t) \end{aligned}$$

Using standard trigonometric identities, we can write it as:

$$\begin{aligned} r(t) &= \frac{1}{2}I(t)\left[1+\cos(4\pi f_0 t)\right] - \frac{1}{2}Q(t)\sin(4\pi f_0 t) \\ &= \frac{1}{2}I(t) + \frac{1}{2}[I(t)\cos(4\pi f_0 t) - Q(t)\sin(4\pi f_0 t)] \end{aligned}$$



Analog QAM: measured PAL colour bar signal on a vector analyser screen.

Low-pass filtering $r(t)$ removes the high frequency terms (containing $4\pi f_0 t$), leaving only the $I(t)$ term. This filtered signal is unaffected by $Q(t)$, showing that the in-phase component can be received independently of the quadrature component. Similarly, we may multiply $s(t)$ by a sine wave and then low-pass filter to extract $Q(t)$.

Analog QAM suffers from the same problem as Single-sideband modulation: the exact phase of the carrier is required for correct demodulation at the receiver. If the demodulating phase is even a little off, it results in crosstalk between the modulated signals. This issue of carrier synchronization at the receiver must be handled somehow in QAM systems. The coherent demodulator needs to be exactly in phase with the received signal, or otherwise the modulated signals cannot be independently received. This is achieved typically by transmitting a burst subcarrier or a Pilot signal.

Analog QAM is used in:

- NTSC and PAL analog Color television systems, where the I- and Q-signals carry the components of chroma (colour) information. The QAM carrier phase is recovered from a special Colorburst transmitted at the beginning of each scan line.
- C-QUAM ("Compatible QAM") is used in AM stereo radio to carry the stereo difference information.

Fourier analysis of QAM

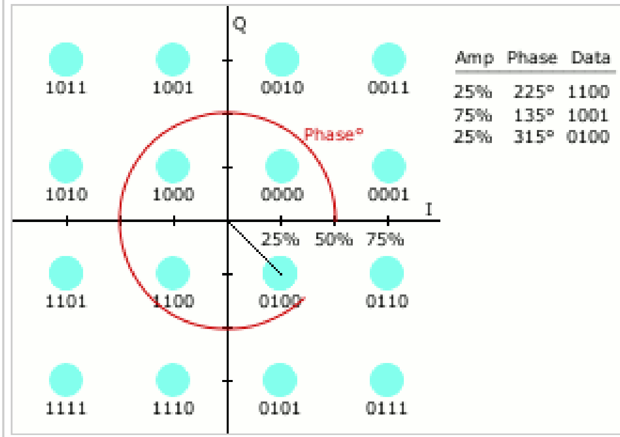
In the frequency domain, QAM has a similar spectral pattern to DSB-SC modulation. Using the properties of the Fourier transform, we find that:

$$S(f) = \frac{1}{2}\left[M_I(f - f_0) + M_I(f + f_0)\right] + \frac{i}{2}\left[M_Q(f - f_0) - M_Q(f + f_0)\right]$$

where $S(f)$, $M_I(f)$ and $M_Q(f)$ are the Fourier transforms (frequency-domain representations) of $s(t)$, $I(t)$ and $Q(t)$, respectively.

Quantized QAM

As in many digital modulation schemes, the constellation diagram is useful for QAM. In QAM, the constellation points are usually arranged in a square grid with equal vertical and horizontal spacing, although other configurations are possible (e.g. Cross-QAM). Since in digital telecommunications the data are usually binary, the number of points in the grid is usually a power of 2 (2, 4, 8, …). Since QAM is usually square, some of these are rare—the most common forms are 16-QAM, 64-QAM and 256-QAM. By moving to a higher-order constellation, it is possible to transmit more bits per symbol. However, if the mean energy of the constellation is to remain the same (by way of making a fair comparison), the points must be closer together and are thus more susceptible to noise and other corruption; this results in a higher bit error rate and so higher-order QAM can deliver more data less reliably than lower-order QAM, for constant mean constellation energy. Using higher-order QAM without increasing the bit error rate requires a higher signal-to-noise ratio (SNR) by increasing signal energy, reducing noise, or both.



Digital 16-QAM with example constellation points.

If data-rates beyond those offered by 8-PSK are required, it is more usual to move to QAM since it achieves a greater distance between adjacent points in the I-Q plane by distributing the points more evenly. The complicating factor is that the points are no longer all the same amplitude and so the demodulator must now correctly detect both phase and amplitude, rather than just phase.

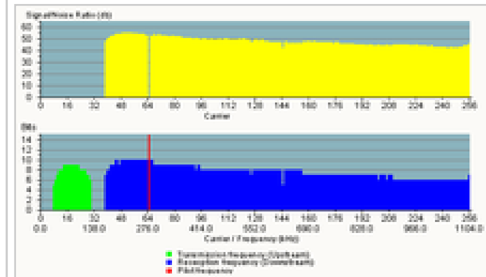
64-QAM and 256-QAM are often used in digital cable television and cable modem applications. In the United States, 64-QAM and 256-QAM are the mandated modulation schemes for digital cable (see QAM tuner) as standardised by the SCTE in the standard ANSI/SCTE 07 2013 (http://www.scte.org/FileDownload.aspx?A=3445). Note that many marketing people will refer to these as QAM-64 and QAM-256. In the UK, 64-QAM is used for digital terrestrial television (Freeview) whilst 256-QAM is used for Freeview-HD.

Communication systems designed to achieve very high levels of spectral efficiency usually employ very dense QAM constellations. For example, current Homeplug AV2 500-Mbit powerline Ethernet devices use 1024-QAM and 4096-QAM,[3] as well as future devices using ITU-T G.hn standard for networking over existing home wiring (coaxial cable, phone lines and power lines); 4096-QAM provides 12 bits/symbol. Another example is ADSL technology for copper twisted pairs, whose constellation size goes up to 32768-QAM (in ADSL terminology this is referred to as bit-loading, or bit per tone, 32768-QAM being equivalent to 15 bits per tone).[4]

Ultra-high capacity Microwave Backhaul Systems also use 1024-QAM.[5] With 1024-QAM, Adaptive Coding and Modulation (ACM), and XPIC, Vendors can obtain Gigabit capacity in a single 56 MHz channel.

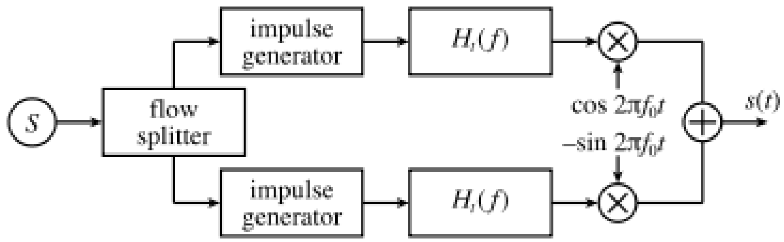
Ideal structure

Transmitter



bit-loading (bits per QAM constellation) on an ADSL line

The following picture shows the ideal structure of a QAM transmitter, with a carrier center frequency f_0 and the frequency response of the transmitter's filter H_t:



First the flow of bits to be transmitted is split into two equal parts: this process generates two independent signals to be transmitted. They are encoded separately just like they were in an amplitude-shift keying (ASK) modulator. Then one channel (the one "in phase") is multiplied by a cosine, while the other channel (in "quadrature") is multiplied by a sine. This way there is a phase of 90° between them. They are simply added one to the other and sent through the real channel.

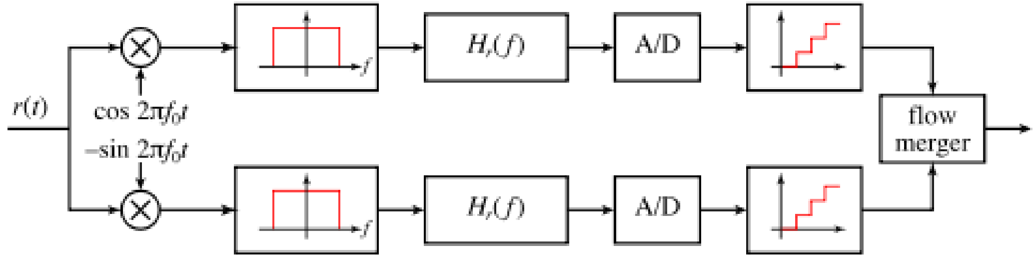
The sent signal can be expressed in the form:

$$s(t) = \sum_{n=-\infty}^{\infty} \left[v_c[n] \cdot h_t(t - nT_s) \cos(2\pi f_0 t) - v_s[n] \cdot h_t(t - nT_s) \sin(2\pi f_0 t) \right]$$

where $v_c[n]$ and $v_s[n]$ are the voltages applied in response to the n^{th} symbol to the cosine and sine waves respectively.

Receiver

The receiver simply performs the inverse operation of the transmitter. Its ideal structure is shown in the picture below with H_r the receive filter's frequency response :



Multiplying by a cosine (or a sine) and by a low-pass filter it is possible to extract the component in phase (or in quadrature). Then there is only an ASK demodulator and the two flows of data are merged back.

In practice, there is an unknown phase delay between the transmitter and receiver that must be compensated by *synchronization* of the receivers local oscillator; i.e., the sine and cosine functions in the above figure. In mobile applications, there will often be an offset in the relative *frequency* as well, due to the possible presence of a Doppler shift proportional to the relative velocity of the transmitter and receiver. Both the phase and frequency variations introduced by the channel must be compensated by properly tuning the sine and cosine components, which requires a *phase reference*, and is typically accomplished using a Phase-Locked Loop (PLL).

In any application, the low-pass filter and the receive H_r filter will be implemented as a single combined filter. Here they are shown as separate just to be clearer.

Quantized QAM performance

The following definitions are needed in determining error rates:

- M = Number of symbols in modulation constellation
- E_b = Energy-per-bit
- E_s = Energy-per-symbol = kE_b with *k* bits per symbol
- N_0 = Noise power spectral density (W/Hz)
- P_b = Probability of bit-error
- P_{bc} = Probability of bit-error per carrier
- P_s = Probability of symbol-error
- P_{sc} = Probability of symbol-error per carrier
- $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}t^2} dt,\ \ x\geq 0$

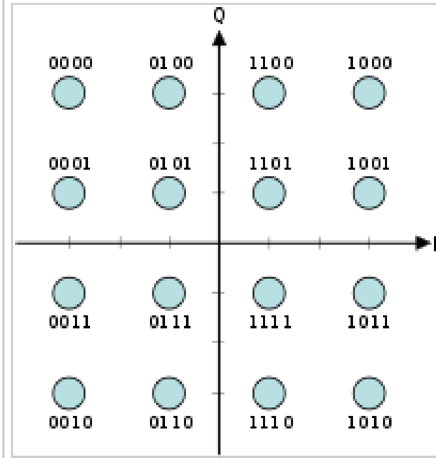
$Q(x)$ is related to the complementary Gaussian error function by: $Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{\sqrt{2}}x\right)$, which is the probability that *x* will be under the tail of the Gaussian PDF towards positive infinity.

The error rates quoted here are those in additive white Gaussian noise (AWGN).

Where coordinates for constellation points are given in this article, note that they represent a *non-normalised* constellation. That is, if a particular mean average energy were required (e.g. unit average energy), the constellation would need to be linearly scaled.

Rectangular QAM

Rectangular QAM constellations are, in general, sub-optimal in the sense that they do not maximally space the constellation points for a given energy. However, they have the considerable advantage that they may be easily transmitted as two pulse amplitude modulation (PAM) signals on quadrature carriers, and can be easily demodulated. The non-square constellations, dealt with below, achieve marginally better bit-error rate (BER) but are harder to modulate and demodulate.



Constellation diagram for rectangular 16-QAM.

The first rectangular QAM constellation usually encountered is 16-QAM, the constellation diagram for which is shown here. A Gray coded bit-assignment is also given. The reason that 16-QAM is usually the first is that a brief consideration reveals that 2-QAM and 4-QAM are in fact binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK), respectively. Also, the error-rate performance of 8-QAM is close to that of 16-QAM (only about 0.5 dB better), but its data rate is only three-quarters that of 16-QAM.

Expressions for the symbol-error rate of rectangular QAM are not hard to derive but yield rather unpleasant expressions. For an even number of bits per symbol, k, exact expressions are available. They are most easily expressed in a *per carrier* sense:

$$P_{sc} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_0}}\right)$$

so

$$P_s = 1 - (1 - P_{sc})^2$$

The bit-error rate depends on the bit to symbol mapping, but for $E_b/N_0 \gg 1$ and a Gray-coded assignment—so that we can assume each symbol error causes only one bit error—the bit-error rate is approximately

$$P_{bc} \approx \frac{P_{sc}}{\frac{1}{2}k} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right).$$

Since the carriers are independent, the overall bit error rate is the same as the per-carrier error rate, just like BPSK and QPSK.

$$P_b = P_{bc}$$

An exact and general closed-form expression of the Bit Error Rates (BER) for rectangular type of Quadrature Amplitude Modulation (QAM) over AWGN and slow, flat, Rician fading channels were derived analytically. Consider a (L×M)-QAM system with 2 · log2 L levels and 2 · log2M levels in the I-channel and Q-channel, respectively and a two-dimensional grey code mapping employed. It was shown[5] that the generalized expression for the conditional BER on SNR ρ over AWGN channel is

$$P_b(E|\rho) = \frac{1}{\log_2 (L \cdot M)} \left(\sum_{i=1}^{\log_2 L} P_b(E_i^L|\rho) + \sum_{i=1}^{\log_2 M} P_b(E_i^M|\rho) \right)$$

where

$$P_b(E_i^P|\rho) = \frac{2}{P} \sum_{j=0}^{(1-2^{-i})P-1} (-1)^{\left\lfloor \frac{j \cdot 2^{i-1}}{P} \right\rfloor} \cdot \left(2^{i-1} - \left\lfloor \frac{j \cdot 2^{i-1}}{P} + \frac{1}{2} \right\rfloor \right) \cdot \mathbb{Q} \left[(2j+1) \sqrt{\frac{6\rho}{(L^2 + M^2 - 2)}} \right].$$

Odd-*k* QAM

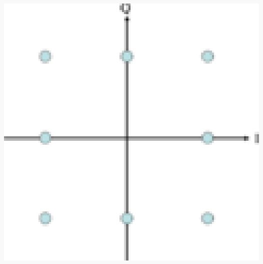
For odd k, such as 8-QAM ($k = 3$) it is harder to obtain symbol-error rates, but a tight upper bound is:

$$P_s \leq 4Q \left(\sqrt{\frac{3kE_b}{(M-1)N_0}} \right)$$

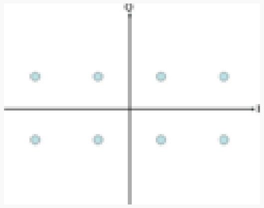
Two rectangular 8-QAM constellations are shown below without bit assignments. These both have the same minimum distance between symbol points, and thus the same symbol-error rate (to a first approximation).

The exact bit-error rate, P_b will depend on the bit-assignment.

Note that both of these constellations are seldom used in practice, as the non-rectangular version of 8-QAM is optimal.



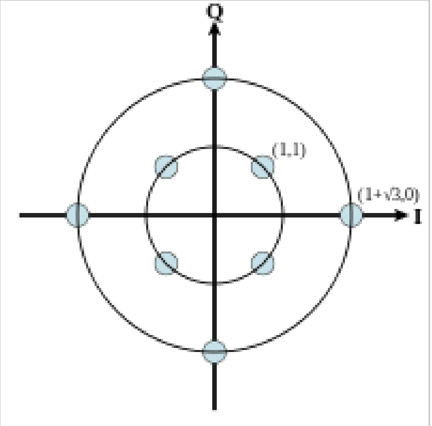
Constellation diagram for rectangular 8-QAM.



Alternative constellation diagram for rectangular 8-QAM.

Non-rectangular QAM

It is the nature of QAM that most orders of constellations can be constructed in many different ways and it is neither possible nor instructive to cover them all here. This article instead presents two, lower-order constellations.



Constellation diagram for circular 8-QAM.

Two diagrams of circular QAM constellation are shown, for 8-QAM and 16-QAM. The circular 8-QAM constellation is known to be the optimal 8-QAM constellation in the sense of requiring the least mean power for a given minimum Euclidean distance. The 16-QAM constellation is suboptimal although the optimal one may be constructed along the same lines as the 8-QAM constellation. The circular constellation highlights the relationship between QAM and PSK. Other orders of constellation may be constructed along similar (or very different) lines. It is consequently hard to establish expressions for the error rates of non-rectangular QAM since it necessarily depends on the constellation. Nevertheless, an obvious upper bound to the rate is related to the minimum Euclidean distance of the constellation (the shortest straight-line distance between two points):

$$P_s < (M-1)Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

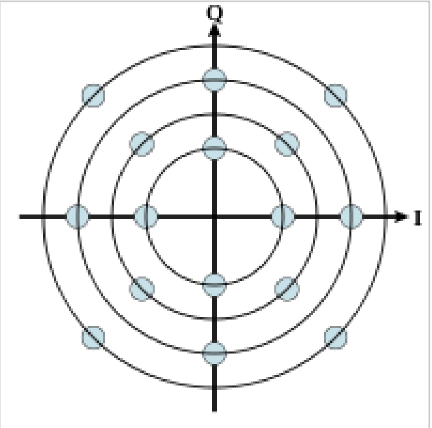
Again, the bit-error rate will depend on the assignment of bits to symbols.

Although, in general, there is a non-rectangular constellation that is optimal for a particular M, they are not often used since the rectangular QAMs are much easier to modulate and demodulate.

Hierarchical QAM

Hierarchical QAM is a form of Hierarchical modulation. For example, Hierarchical QAM is used in DVB, where the constellation points are grouped into a high-priority QPSK stream and a low-priority 16-QAM stream. The irregular distribution of constellation points improves the reception probability of the high-priority stream in low SNR conditions, at the expense of higher SNR requirements for the low-priority stream.[6]

Interference and noise



Constellation diagram for circular 16-QAM.

In moving to a higher order QAM constellation (higher data rate and mode) in hostile RF/microwave QAM application environments, such as in broadcasting or telecommunications, multipath interference typically increases. There is a spreading of the spots in the constellation, decreasing the separation between adjacent states, making it difficult for the receiver to decode the signal appropriately. In other words, there is reduced noise immunity. There are several test parameter measurements which help determine an optimal QAM mode for a specific operating environment. The following three are most significant:[7]

- Carrier/interference ratio
- Carrier-to-noise ratio
- Threshold-to-noise ratio

See also

- Amplitude and phase-shift keying or Asymmetric phase-shift keying (APSK)
- Carrierless Amplitude Phase Modulation (CAP)
- In-phase and quadrature components
- Modulation for other examples of modulation techniques
- Phase-shift keying
- QAM tuner for HDTV
- Random modulation

References

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2. Kylia products (http://kylia.com/QAM.html), dwdm mux demux, 90 degree optical hybrid, d(q) psk demodulatorssingle polarization
3. http://www.homeplug.org/media/filer_public/a1/46/a1464318-f5df-46c5-89dc-7243d8ccfcee/homeplug_av2_whitepaper_150907.pdf Homeplug_AV2 whitepaper

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7. Howard Friedenberg and Sunil Naik. "Hitless Space Diversity STL Enables IP+Audio in Narrow STL Bands" (PDF). *2005 National Association of Broadcasters Annual Convention*. Retrieved April 17, 2005.

5. Jonqyin (Russell) Sun "Linear diversity analysis for QAM in Rician fading channels", IEEE WOCC 2014

The notation used here has mainly (but not exclusively) been taken from

- *John G. Proakis*, "*Digital Communications, 3rd Edition*",

External links

Wikimedia Commons has media related to ***Quadrature amplitude modulation***.

- Interactive webdemo of QAM constellation with additive noise (http://webdemo.inue.uni-stuttgart.de/webdemos/02_lectures/uebertragungstechnik_1/qam_constellation_diagram_from_snr) Institute of Telecommunicatons, University of Stuttgart
- QAM bit error rate for AWGN channel – online experiment (http://www.etti.unibw.de/labalive/experiment/qam/)
- How imperfections affect QAM constellation (http://www.blondertongue.com/QAM-Transmodulator/QAM_defined.php)
- Microwave Phase Shifters (http://www.herley.com/index.cfm?act=app_notes&notes=iqv_phaseshift) Overview by Herley General Microwave
- Simulation of dual-polarization QPSK (DP-QPSK) for 100G optical transmission (http://www.vpiphotonics.com/Applications/TransmissionSystems/ModFormat_PolMuxQPSK.php)

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