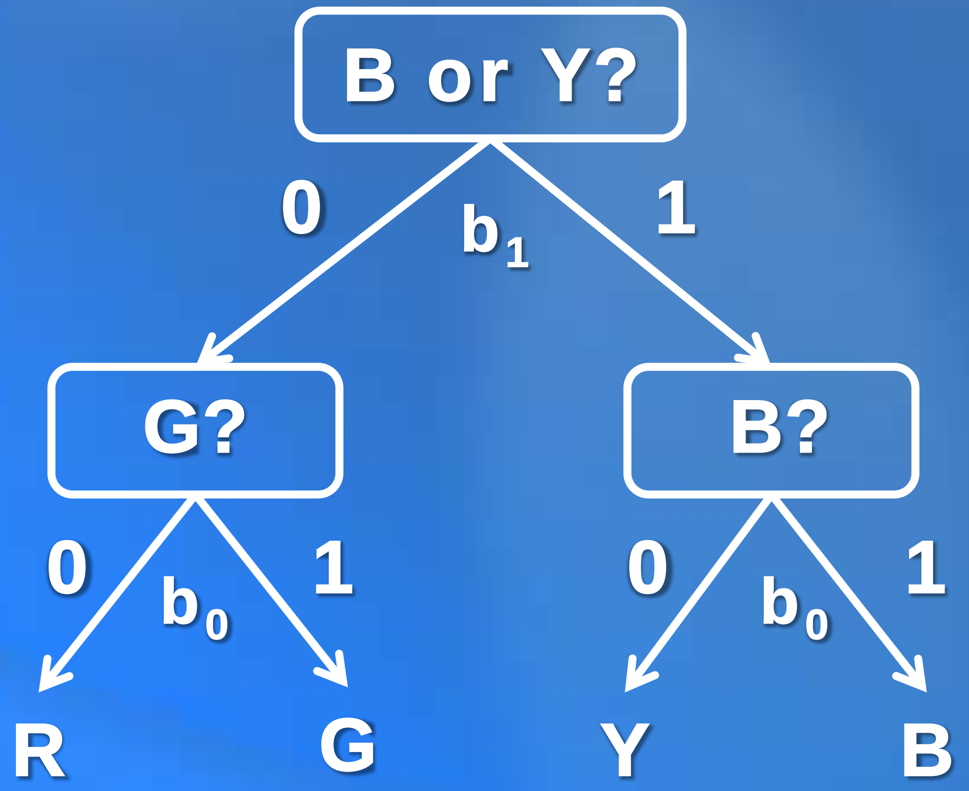
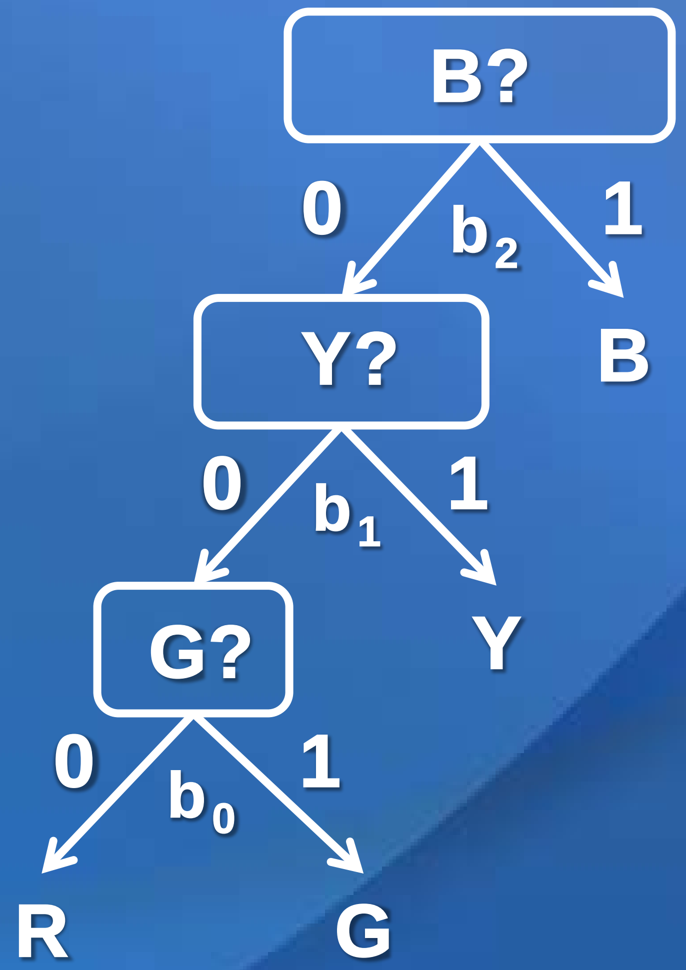
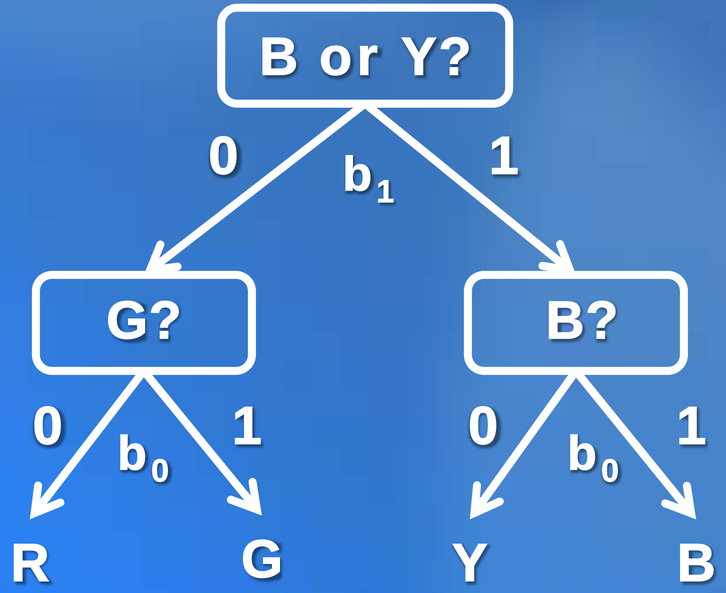
Average Code Length

Two Strategies

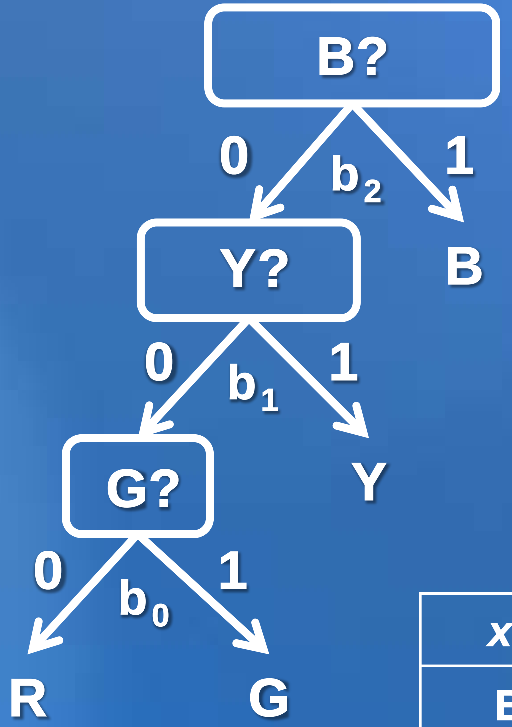




Encoding Tables



x_k	codeword_k	l_k
B	11	2
Y	10	2
G	01	2
R	10	2



l_k = length of codeword_k

x_k	codeword_k	l_k
B	1	1
Y	01	2
G	001	3
R	000	3

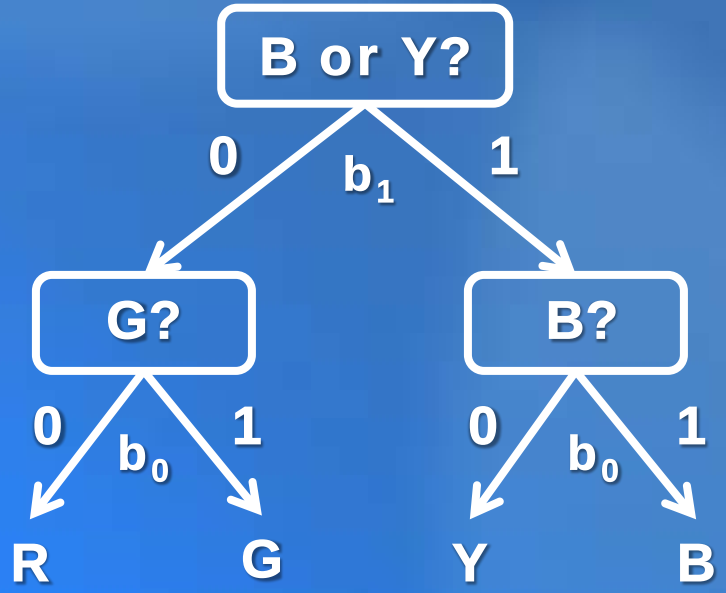
Average Code Length

The average length of a random code is the expected value of its length:

$$\bar{L} = \sum_{k=0}^{K-1} p_k \, l_k$$

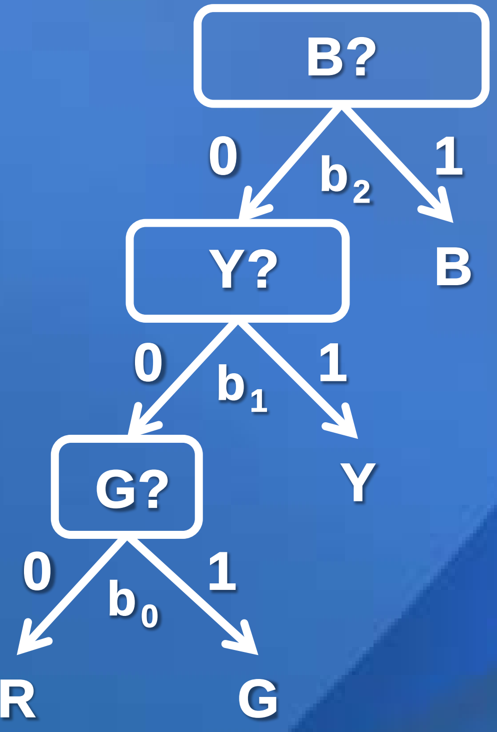
The entropy of the distribution p_k is a lower bound on the average code length!

Example



x_k	p_k
B	1 / 4
Y	1 / 4
G	1 / 4
R	1 / 4

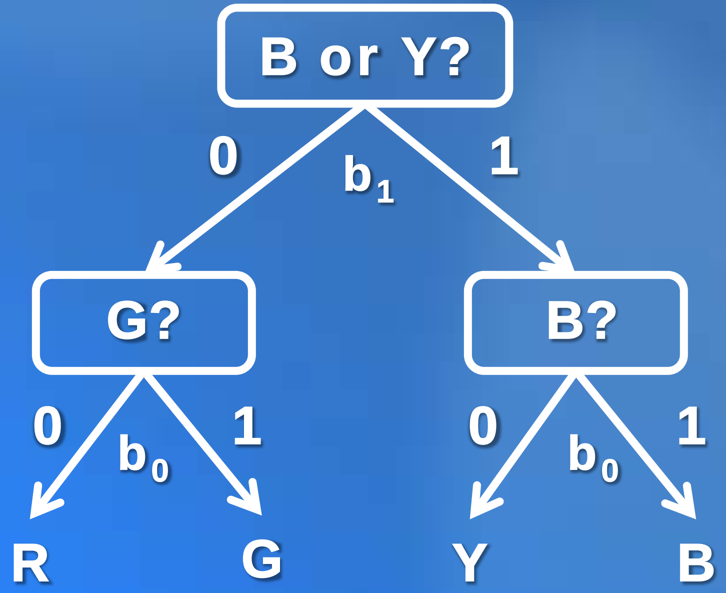
H = 2



$$\bar{L} = \tfrac{1}{4}\cdot 2 + \tfrac{1}{4}\cdot 2 + \tfrac{1}{4}\cdot 2 + \tfrac{1}{4}\cdot 2$$
$$= 2$$

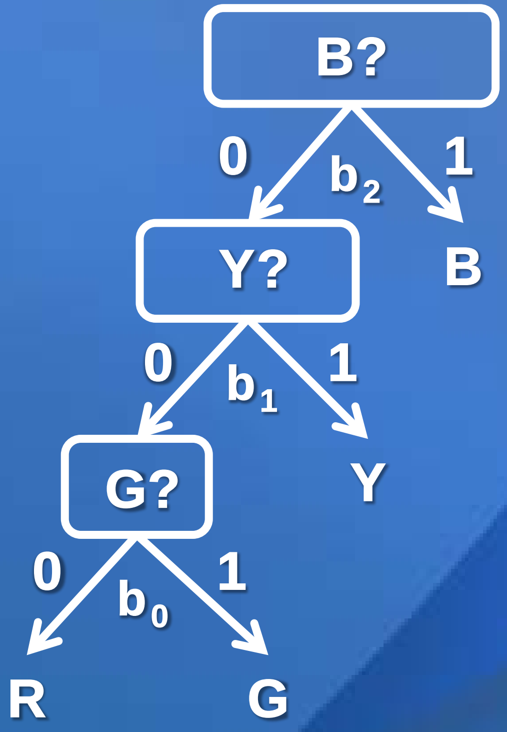
$$\bar{L} = \tfrac{1}{4}\cdot 1 + \tfrac{1}{4}\cdot 2 + \tfrac{1}{4}\cdot 3 + \tfrac{1}{4}\cdot 3$$
$$= 2.25$$

Example



x_k	p_k
B	1 / 2
Y	1 / 4
G	1 / 8
R	1 / 8

$$H = 1.75$$



$$\bar{L} = \tfrac{1}{2}\cdot 2 + \tfrac{1}{4}\cdot 2 + \tfrac{1}{8}\cdot 2 + \tfrac{1}{8}\cdot 2$$
$$= 2$$

$$\bar{L} = \tfrac{1}{2}\cdot 1 + \tfrac{1}{4}\cdot 2 + \tfrac{1}{8}\cdot 3 + \tfrac{1}{8}\cdot 3$$
$$= 1.75$$