Proving the Nyquist Theorem mathematically is beyond the scope of this course, but we can present a couple examples to support the basic idea of the Nyquist Theorem: we must sample at a rate faster than twice the rate at which signal itself is oscillating.

VIDEO 13.2. THE NYQUIST THEOREM ILLUSTRATED

Help

C13 Video 2 Two examples to Illustrate the Nyquist Theorem YouTube

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DR. JONATHAN VALVANO: Let's look at the Nyquist Theorem.

The Nyquist Theorem says that if a signal is oscillating at frequency F,

in order to capture it faithfully, we must

sample at a frequency that is strictly larger

than two times F. The essence of the Nyquist Theorem

is that if a signal is oscillating we must sample faster than it oscillates.

The mathematical proof for the Nyquist Theorem

is beyond the scope of this class.

But let me show you an example that illustrates the concept.

We have a circular running track, and we have

a person who is running around the track.

And our job is to count the number of

times they go around the track.

Let's say the fastest one could run is around the track in one minute.

So every 60 seconds, a fast person could get around the track.

I'm a lazy person, and I want to know, how often do I

have to look at my runner in order to tell how many times they've

gone around?

So for instance, if the runner is running

at a rate of one lap per one minute, and I look every 15 seconds,

I will see them here.

I will see them over here.

I will see them over here.

And I will see them over here.

And I can count because I've seen them before the finish line,

and I've seen them after the finish line.

And so I can count that I've seen it.

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However, if I count every two minutes, then I'm going to see them here.

They're going to go around the track twice,

and I'm going to see them again there.

And they're going to go around the track, and I'm going to see them there.

And so I will be unable to count how many times they've gone.

There's something special about 30 seconds.

If I count slower than every 30 seconds, I will see them.

So in other words, if I were to count at 40 seconds or 45 seconds,

I could potentially only seen them once per lap.

And if I only see them once per lap, I can't

tell whether they've going around once or they've not gone around at all

and not moving.

And so it turns out that this magic number, in order to solve my problem,

is, in order to tell how many times they've gone around the loop,

I have to look at least every 30 seconds, which is my Nyquist Theorem.

The oscillation of my runner has a

frequency of one lap per minute.

And I need to look at least twice per minute in order to tell.

Here's another example of the Nyquist Theorem.

We're stuck back on the island, and we want to get to the mainland.

But this time there's a boat.

And luckily for us, this boat takes a periodic trip

between the island, the mainland, and back to the island.

It takes about 12 hours for the boat to go

one trip, island to mainland and back again.

And while it's at the island, it'll sit on the dock.

And luckily for us, it'll sit at the dock for 12 hours.

So if we look at the signal, is the boat at the dock?

We will see that it oscillates.

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The boat is at the dock for 12 hours.

And the boat is not at the dock for about 12 hours.

At the dock, not at the dock, at the dock, not at the dock.

This is a periodic wave.

And we see that the signal oscillates at once

per day, which means, according to the Nyquist Theorem,

we must sample it at twice per day.

For example, if I go down to the dock every 11 hours,

I'm guaranteed to see the boat.

There's no time that the boat could come, stay at the dock

and leave that I wouldn't notice it.

So regardless of when I start looking, I will get onto the next boat.

On the other hand, if I sample the signal every 13 hours,

I am slower than the twice per day, and it's possible--

In this case here, we see that the boat came to the dock right

after I looked at it and left right before I looked at it again.

And so we see there's something magical about this 12 hours.

And looking at the boat every 12 hours is

equivalent to looking at it twice per day.

And so we see that if we sample faster than twice the frequency,

we will capture everything we need to know about that signal.

And that's the Nyquist Theorem.

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Example 1) There is a long distance race with runners circling around an oval track and it is your job to count the number of times a particular runner circles the track. Assume the fastest time a runner can make a lap is 60 seconds. You want to read a book and occasionally look up to see where your runner is. How often to you have to look at your runner to guarantee you properly count the laps? If you look at a period faster than every 30 seconds you will see the runner at least twice per lap and properly count the laps. If you look at a period slower than every 30 seconds, you may only see the runner once per lap and not know if the runner is going very fast or very slow. In this case, the runner oscillates at most 1 lap per minute and thus you must observe the runner at a rate faster than twice per minute.

Example 2) You live on an island and want to take the boat back to the mainland as soon as possible. There is a boat that arrives at the island once a day, waits at the dock for 12 hours and then it sets sail to the mainland. Because of weather conditions, the exact time of arrival is unknown, but the boat will always wait at the dock for 12 hours before it leaves. How often do you need to walk down to the dock to see if the boat is there? If the boat is at the dock, you get on the boat and take the next trip back to the mainland. If you walk down to the dock every 13 hours, it is possible to miss the boat. However, if you walk down to the dock every 12 hours or less, you'll never miss the boat. In this case, the boat frequency is once/day and you must sample it (go to the dock) two times/day.

A continuous waveform like Figure 13.1 is shown, V = 1.5 + 1*sin(2pi50t)+0.5*cos(2pi 200t). You may select the sampling rate and the precision (in bits) to see the signal captured. Notice that at sampling rates above 100 Hz you capture the essence of the 50Hz periodic wave, and above 400 Hz you capture the essence of both the 50 and 200 Hz waves. To "capture the essence" means the analog and digital signal go up and down at the same rate.

To view the interactive demonstrating the Nyquist Theorem, please visit:

http://users.ece.utexas.edu/~valvano/Volume1/E-Book/C13_DACSound.htm#ITool13.1 (http://users.ece.utexas.edu

/~valvano/Volume1/E-Book/C13_DACSound.htm#ITool13.1)