HUDM 5123 - Linear Models and Experimental Design
Lab 06 (There was no Lab 05) - Two-Way ANOVA

1 Introduction

The data for lab today are available through package **car**. The data set is called `Prestige`, and it deals with data from the Canadian Census from about 1970. Each observation (row) of the data represents an occupation. The variables (columns) we will be interested in are

1. `type`: Type of occupation. A factor with levels `bc`, Blue Collar; `prof`, Professional, Managerial, and Technical, `wc`, White Collar. Note that *blue collar* refers to workers whose job requires them to wear work clothes (e.g., manual labor); whereas, *white collar* typically refers to salaried employees whose job does not require them to wear work clothes (e.g., office work). Blue collar jobs are typically associated with lower pay and lower prestige than white collar jobs.

2. `incomeF`: Average income category (above or below the median income) in 1971.

3. `prestige`: The prestige rating for each occupation (higher value = more prestigious occupation). I don't know how the prestige rating was calculated and the documentation isn't helpful.

2 Data Set 1: Fake (Simulated) Data

For this lab we will use two data sets. The first is fake data that I simulated based on the Prestige data set to have no two-way interaction. The second is the real Prestige data set, which does have a two-way interaction. The research question of interest is whether the type of occupation (blue collar, white collar, professional) has an effect on prestige and, if so, if it is moderated by income category.

Table 1: Group sample sizes for `dat1`

		Type of Work			
		Blue-Collar	White-Collar	Professional	Total
	Low	12	12	12	36
Income	High	12	12	12	36
	Average	24	24	24	72

Note that the levels of the `type` factor are not in the right order, so we will reorder them.

```
levels(dat1$type)
[1] "bc"   "prof" "wc"
dat1$type <- factor(x = dat1$type,
                    levels = c("bc", "wc", "prof"))
dat2$type <- factor(x = dat2$type,
                    levels = c("bc", "wc", "prof"))
```

Table 2: Table of cell means for factors "Income" and "Type of Work"

		Type of Work			
		Blue-Collar	White-Collar	Professional	Average
	Low	26.9	39.6	67.3	44.6
Income	High	37.9	46.4	74.0	52.8
	Average	32.4	43.0	70.6	48.7

Before running regression, we need to tell R to us deviation coding instead of dummy coding. The default options for unordered factors and ordered factors, respectively, are dummy coding (`contr.treatment`) and polynomial coding (`contr.poly`). We need to change the default handling of unordered factors to deviation coding (`contr.sum`).

```
# Inspect settings
options("contrasts")
$contrasts
        unordered           ordered
"contr.treatment"      "contr.poly"

# Change dummy to deviation for unordered factors
options(contrasts = c("contr.sum", "contr.poly"))
```

We can check the contrast coding with the `contrasts()` function as follows. Note that the factors will be deviation-coded, as intended. The function `model.matrix()` may be used to examine the actual design matrix for `lm1`.

```
contrasts(x = dat1$type)
         [,1] [,2]
1 - bc      1    0
2 - wc      0    1
3 - prof   -1   -1
contrasts(x = dat1$incomeF)
         [,1]
1 - high    1
2 - low    -1
```

Now we will (a) run the full regression model and call it `lm1`, (b) load package **car** and use the `Anova()` function to extract the ANOVA table for the two-way design, and (c) pass `lm1` to **emmeans** to do follow-up testing and plot marginal means. The model `lm1`, fit with the code below, is specified as follows:

$$Y_i = \beta_0 + \beta_1 R1_i + \beta_2 C1_i + \beta_3 C2_i + \beta_4 R1_i C1_i + \beta_5 R1_i C2_i + \epsilon_i$$

```
lm1 <- lm(formula = prestige ~ incomeF*type, data = dat1)
summary(lm1)
```

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)     48.6889     1.2193  39.931  < 2e-16 ***
incomeF1         4.0889     1.2193   3.353  0.00133 **
type1          -16.2722     1.7244  -9.437 7.34e-14 ***
type2           -5.6806     1.7244  -3.294  0.00159 **
incomeF1:type1   1.4028     1.7244   0.813  0.41886
incomeF1:type2  -0.6972     1.7244  -0.404  0.68728
---

Residual standard error: 10.35 on 66 degrees of freedom
Multiple R-squared:  0.7387,Adjusted R-squared:  0.7189
F-statistic: 37.31 on 5 and 66 DF,  p-value: < 2.2e-16
```

Note, from the `model.matrix()` output below, that $R1$ corresponds with `incomeF1`, $C1$ with `type1`, and so on.

```
model.matrix(lm1)
   (Intercept) incomeF1 type1 type2 incomeF1:type1 incomeF1:type2
1            1       -1    -1    -1              1              1
2            1       -1    -1    -1              1              1
3            1       -1    -1    -1              1              1
4            1       -1    -1    -1              1              1
5            1       -1    -1    -1              1              1
6            1       -1    -1    -1              1              1
7            1       -1    -1    -1              1              1
8            1       -1    -1    -1              1              1
9            1       -1    -1    -1              1              1
10           1       -1    -1    -1              1              1
11           1       -1    -1    -1              1              1
12           1       -1    -1    -1              1              1
13           1       -1     0     1              0             -1
14           1       -1     0     1              0             -1
15           1       -1     0     1              0             -1
16           1       -1     0     1              0             -1
17           1       -1     0     1              0             -1
18           1       -1     0     1              0             -1
19           1       -1     0     1              0             -1
20           1       -1     0     1              0             -1
21           1       -1     0     1              0             -1
22           1       -1     0     1              0             -1
23           1       -1     0     1              0             -1
24           1       -1     0     1              0             -1
25           1       -1     1     0             -1              0
26           1       -1     1     0             -1              0
```

3 Two-Way ANOVA for Data Set 1

Load package **car** and use the `Anova()` function to produce the two-way ANOVA table for the full model fit in `lm1`.

```
library(car)
Anova(lm1, type = 3)
Anova Table (Type III tests)

Response: prestige
```

```
              Sum Sq Df   F value    Pr(>F)    
(Intercept)  170684  1 1594.4845 < 2.2e-16 ***
incomeF        1204  1   11.2453  0.001326 ** 
type          18695  2   87.3242 < 2.2e-16 ***
incomeF:type     71  2    0.3309  0.719469    
Residuals      7065 66                        
```

Note that there is no evidence of a significant interaction based on the ANOVA table. Next, we will use **emmeans** to create an interaction plot to visually assess the evidence for or against a two-way interaction between income level (low vs high) and type of occupation (bc, wc, prof). Visual examination of the plot confirms that there does not appear to be strong evidence of an interaction. Although the cell means for high vs low differ at the bc level more than at the other two, this appears to be withing the realm of sampling variability. Thus, since both the statistical and graphical evidence point toward no important interaction, we will proceed with analysis of main effects.

```
library(emmeans)
emm1 <- emmeans(object = lm1,
+                 specs = ~ incomeF*type)
emm1
 incomeF type emmean   SE df lower.CL upper.CL
 high    bc     37.9 2.99 66     31.9     43.9
 low     bc     26.9 2.99 66     21.0     32.9
 high    wc     46.4 2.99 66     40.4     52.4
 low     wc     39.6 2.99 66     33.7     45.6
 high    prof   74.0 2.99 66     68.1     80.0
 low     prof   67.3 2.99 66     61.3     73.2

Confidence level used: 0.95
p <- emmip(object = emm1,
      formula = incomeF ~ type,
      xlab = "Type of Profession",
      ylab = "Mean Prestige Score")
p$labels$colour <- "Income" # Change legend title
print(p)
```

3.1 Analysis of Main Effects

Having first examined the interaction and found it to be unimportant, we may examine the effects of one factor while averaging over the levels of the other, and vice versa; these are the main effects. According to the ANOVA table, there is a significant main effect for income level ($F(1,66) = 11.2$; $p = .001$) as well as for job type ($F(2,66) = 87.3$; $p < .0001$). The income factor only has two levels, so there is no need to follow up with pairwise tests. The type factor, however, has three levels. Assuming we planned to use Shaffer's planned post-omnibus modification, we would now move on to test the three pairwise main effect

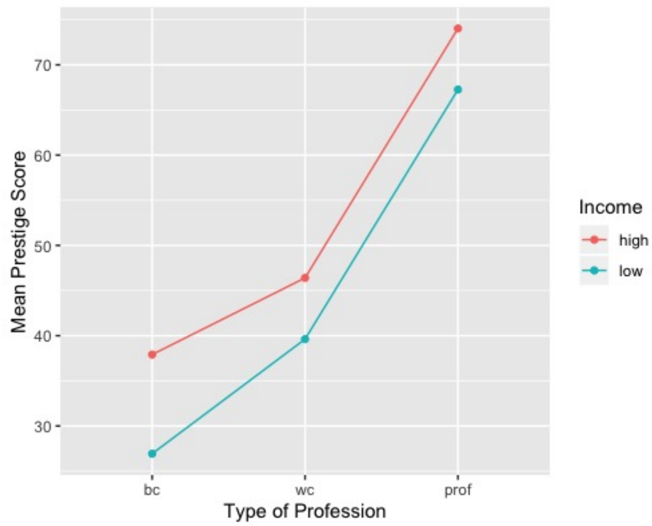

Figure 1: Interaction plot for `dat1`

comparisons, each at the full .05 level. Begin by fitting an **emmeans** model that averages over income level by using the `specs` argument to only condition on `type`. Note the warning that results are averaged across income.

```
emm3 <- emmeans(object = lm1,
                specs = ~ type)
emm3
 type emmean   SE df lower.CL upper.CL
 bc     32.4 2.11 66     28.2     36.6
 wc     43.0 2.11 66     38.8     47.2
 prof   70.6 2.11 66     66.4     74.9

Results are averaged over the levels of: incomeF
Confidence level used: 0.95
```

Next, test the pairwise main effects comparisons.

```
pairs(emm3, adjust = "none")
 contrast  estimate   SE df t.ratio p.value
 bc - wc      -10.6 2.99 66  -3.546 0.0007
 bc - prof    -38.2 2.99 66 -12.798 <.0001
 wc - prof    -27.6 2.99 66  -9.252 <.0001
```

Thus, we may conclude that professional jobs are considered more prestigious than both blue collar ($t(66) = -12.8$; $p < .0001$) and white collar ($t(66) = -9.3$; $p < .0001$) jobs, and which collar jobs are considered more prestigious than blue collar jobs ($t(66) = -3.6$; $p = .0007$). Furthermore, jobs that fetch higher income are considered more prestigious than those that fetch lower incomes ($t(66) = 3.4$; $p = .001$). Note that we got the t statistic value by taking the square root of the F value of 11.25 from the ANOVA table. Equivalently, though, we could have gotten it from **emmeans** as follows.

```
emm2 <- emmeans(object = lm1,
                specs = ~ incomeF)
emm2
 incomeF emmean   SE df lower.CL upper.CL
 high      52.8 1.72 66     49.3     56.2
 low       44.6 1.72 66     41.2     48.0

Results are averaged over the levels of: type
Confidence level used: 0.95

pairs(emm2, adjust = "none")
 contrast   estimate   SE df t.ratio p.value
 high - low     8.18 2.44 66 3.353   0.0013

Results are averaged over the levels of: type
```

4 Data Set 2: Real Prestige Data From Canadian Census

Begin by fitting the full linear model (same as above) with the second data set.

```
lm2 <- lm(formula = prestige ~ incomeF*type, data = dat2)
summary(lm2)

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      49.331      1.005  49.067  < 2e-16 ***
incomeF1          3.648      1.005   3.628 0.000469 ***
type1           -11.776      1.249  -9.427 3.66e-15 ***
type2            -6.092      1.470  -4.145 7.54e-05 ***
incomeF1:type1    3.790      1.249   3.034 0.003133 **
incomeF1:type2   -1.102      1.470  -0.750 0.455408
---

Residual standard error: 8.194 on 92 degrees of freedom
Multiple R-squared:  0.7821,Adjusted R-squared:  0.7703
F-statistic: 66.05 on 5 and 92 DF,  p-value: < 2.2e-16
```

Then pass the model output `lm1` to the `Anova()` function.

```
Anova(lm2, type = 3)
Anova Table (Type III tests)

Response: prestige
             Sum Sq Df   F value    Pr(>F)
(Intercept)  161631  1 2407.5431 < 2.2e-16 ***
incomeF         884  1   13.1625 0.0004687 ***
```

```
type          10568  2   78.7039 < 2.2e-16 ***
incomeF:type    630  2    4.6909 0.0114831 *
Residuals      6176 92
```

Here, the interaction is significant ($F(2, 92) = 4.7$; $p = .011$). Next, set up **emmeans** to create an interaction plot with `emmip()`.

```
emm4
 incomeF type emmean   SE df lower.CL upper.CL
 high    bc     45.0 2.05 92     40.9     49.1
 low     bc     30.1 1.55 92     27.0     33.2
 high    wc     45.8 3.10 92     39.6     51.9
 low     wc     40.7 2.05 92     36.6     44.8
 high    prof   68.2 1.61 92     65.0     71.3
 low     prof   66.2 3.66 92     59.0     73.5

emmip(object = emm4,
      formula = incomeF ~ type,
      xlab = "Type of Profession",
      ylab = "Mean Prestige Score")
```

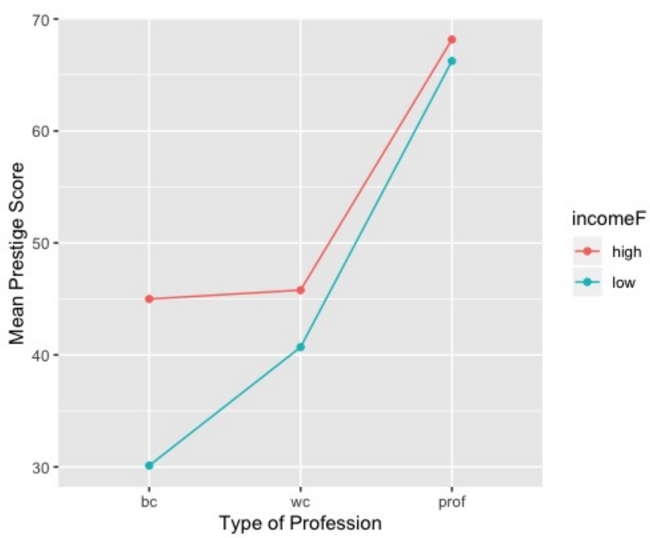

Figure 2: Interaction plot for `dat2`

4.1 Analysis of Simple Effects

Because there is a significant interaction that also appears visually important, we should not examine main effects or main effects contrasts, which average over the levels of the other factor. Instead, we will examine conditional (i.e., simple) effects of each factor, conditional on the levels of the other factor. There are two ways to do this in a two way design: we might examine the levels of income (high vs low) at each level of job type (bc, wc, and prof), or vise versa, we might compare bc, wc, and prof at both levels of income (high and low).

Here, we will do both. We will begin by examining job type conditional on the levels of income. The first step is to run the simple omnibus test to see if there is evidence of an overall effect of job type at either level of income.

```
joint_tests(lm2, by = "incomeF")
incomeF = high:
 model term df1 df2 F.ratio p.value
 type         2  92  47.783 <.0001

incomeF = low:
 model term df1 df2 F.ratio p.value
 type         2  92  43.317 <.0001
```

The null hypotheses for the two omnibus tests of the simple effects shown above are as follows:

$$\text{Simple Effect of Type} \mid \text{Income} = \text{Low:} \quad H_0 : \mu_{11} = \mu_{12} = \mu_{13}$$
$$\text{Simple Effect of Type} \mid \text{Income} = \text{High:} \quad H_0 : \mu_{21} = \mu_{22} = \mu_{23}$$

Thus, we conclude that job type has an effect on prestige for both the lower ($F(2,92) = 47.8$; $p < .0001$) and higher ($F(2,92) = 43.3$; $p < .0001$) income categories. Next we will follow up with simple pairwise comparisons. Again, if we had decided to use Shaffer's planned post-omnibus modification, we would test at the nominal .05 level. To do simple pairwise comparisons in **emmeans**, we first fit an **emmeans** object using the `|` character, which means "conditional on". Note that the output gives the conditional means of type at both levels of income.

```
emm5 <- emmeans(object = lm2,
                specs = ~ incomeF*type)
emm5
incomeF = high:
 type emmean   SE df lower.CL upper.CL
 bc     45.0 2.05 92     40.9     49.1
 wc     45.8 3.10 92     39.6     51.9
 prof   68.2 1.61 92     65.0     71.3

incomeF = low:
 type emmean   SE df lower.CL upper.CL
 bc     30.1 1.55 92     27.0     33.2
 wc     40.7 2.05 92     36.6     44.8
 prof   66.2 3.66 92     59.0     73.5

Confidence level used: 0.95
```

Next, carry-out pairwise comparisons of the levels of type, conditional on the levels of income. The null hypothesis for the prof vs wc comparison at income = low, for example, is

$$\text{Prof vs WC} \mid \text{Income} = \text{Low:} \quad H_0 : \mu_{11} = \mu_{12}.$$

```
pairs(emm5, adjust = "none")
incomeF = high:
 contrast  estimate   SE df t.ratio p.value
 bc - wc     -0.792 3.71 92 -0.213  0.8316
 bc - prof  -23.164 2.60 92 -8.897  <.0001
 wc - prof  -22.372 3.49 92 -6.412  <.0001

incomeF = low:
 contrast  estimate   SE df t.ratio p.value
 bc - wc    -10.576 2.57 92 -4.119  0.0001
 bc - prof  -36.122 3.98 92 -9.080  <.0001
 wc - prof  -25.546 4.20 92 -6.085  <.0001
```

Here, we find evidence that all types differ significantly at both levels of income with professional jobs having higher prestige than white collar jobs which have higher prestige than blue collar jobs with one exception. For those jobs that are in the upper 50th percentile of income, there is no discernible difference in prestige between blue collar and white collar jobs ($p = .83$). That is, a blue collar job that earns income in the top half of all jobs is indistinguishable from white collar jobs in terms of prestige. To take a closer look at which jobs are involved, here is a list of the job names that are both blue collar and in the higher income category:

```
dat2$X[which(dat2$type == "bc" & dat2$incomeF == "high")]
 [1] firefighters          policemen             funeral.directors
 [4] rotary.well.drillers tool.die.makers      machinists
 [7] sheet.metal.workers  welders              aircraft.workers
[10] aircraft.repairmen   electrical.linemen   electricians
[13] construction.foremen plumbers             train.engineers
[16] typesetters
```

And those that are both blue collar and in the lower income category:

```
dat2$X[which(dat2$type == "bc" & dat2$incomeF == "low")]
 [1] nursing.aides              service.station.attendant cooks
 [4] bartenders                 launderers                janitors
 [7] elevator.operators         farm.workers              bakers
[10] slaughterers.1             slaughterers.2            canners
[13] textile.weavers            textile.labourers         auto.workers
[16] electronic.workers         radio.tv.repairmen        sewing.mach.operators
[19] auto.repairmen             railway.sectionmen        carpenters
[22] masons                     house.painters            construction.labourers
[25] bus.drivers                taxi.drivers              longshoremen
[28] bookbinders
```

4.2 Conditioning on Type Instead of Income

First, test the simple omnibus tests of income level (low vs high) at each level of type (bc, wc, and prof). Note that the only place where the low vs high comparison is significant is when type = blue collar. Since this is a comparison that only has two groups, we need not follow up with pairwise comparisons. We may simply conclude that income level has an effect on prestige only for blue collar workers ($F(1, 92) = 33.6$; $p < .0001$).

```
joint_tests(object = lm2, by = "type")
type = bc:
 model term df1 df2 F.ratio p.value
 incomeF      1  92  33.561 <.0001

type = wc:
 model term df1 df2 F.ratio p.value
 incomeF      1  92   1.881 0.1736

type = prof:
 model term df1 df2 F.ratio p.value
 incomeF      1  92   0.230 0.6329
```

Again, though not necessary because the income factor only has two levels, we could also get the same result for the simple pairwise comparison by following up with a conditional pairwise test. Note that we get the same results, as expected.

```
emm6 <- emmeans(object = lm2,
                specs = ~ incomeF | type)
pairs(emm6, adjust = "none")
type = bc:
 contrast   estimate   SE df t.ratio p.value
 high - low    14.88 2.57 92 5.793   <.0001

type = wc:
 contrast   estimate   SE df t.ratio p.value
 high - low     5.09 3.71 92 1.371   0.1736

type = prof:
 contrast   estimate   SE df t.ratio p.value
 high - low     1.92 4.00 92 0.479   0.6329
```

5 Tasks for Lab 06

Both data sets for the lab are in a file called "Lab_06_Therapy.Rdata" in the "Code and Data" folder on Canvas. Suppose that a clinical psychologist is interested in comparing the relative effectiveness of three forms of psychotherapy for alleviating depression. Fifteen individuals are randomly assigned to each of three treatment groups: cognitive-behavioral (CBT), Rogerian, and assertiveness training. The Depression Scale of the MMPI serves as the dependent variable. Although the effect of psychotherapy is of primary interest, subjects are also classified according to the severity of their baseline depression as mild, moderate, or severe. I created two data sets that correspond with this scenario; one has a significant interaction and the other does not. Analyze them both down to the level of pairwise comparisons, making decisions about whether to compare the efficacy of treatments either averaged over levels of initial severity or not based on the presence or absence of a two-way interaction. Your analysis and write-up for each data set should include

1. Two-way ANOVA source tables using Type III sums of squares,
2. interaction plots with meaningful labels,
3. null hypotheses and results for conditional or overall main effects (as appropriate),
4. null hypotheses and results for pairwise comparisons,
5. a brief write-up, in paragraph form, summarizing the results.