L8 Slides Modified: 11, 64-65

L8 Roadmap Alighned with W8: Forecasting with DSGE Models

This roadmap keeps the **official L8 slides** as the raw material, but narrows the lecture to four functional blocks that map cleanly into Dynare work in W8.

Overview: four functional blocks

1. **From estimation to shocks**
2. **Unconditional forecasting (baseline)**
3. **Conditional forecasting (Dynare-relevant only)**
4. **Scenario analysis for W8** (baseline vs scenario; counterfactuals using estimated shocks)

Block 1. From estimation to shocks

Goal

Understand what "structural shocks" mean in this workflow, why we smooth, and where Dynare stores the objects used later for forecasting and scenarios.

Focus slides (core)

- Slide **2**: DSGE uses (story, policy experiments, forecasting).
- Slide **3**: Roadmap (historical decomposition; forecasts/scenarios).
- Slides **4-5**: Historical decompositions motivation.
- Slide **10**: Estimating shocks via Kalman filter; **use smoothing**.
- Slides **11-13**: "Simulation with some shocks" + Dynare `shock_decomposition` command.
- Slides **28-29**: After estimation: `oo_.SmoothedVariables`, `oo_.SmoothedShocks` (where the shocks live).

Dynare touchpoints to emphasize

- Estimation with smoothing produces smoothed states and shocks.
- Objects used downstream:
  - `oo_.SmoothedShocks.<shockname>`
  - `oo_.SmoothedVariables.<varname>`
  - later also `oo_.dr` (decision rules / solution object; used in simulations).

Block 2. Unconditional forecasting (baseline)

Goal

Understand what an "unconditional forecast" is operationally (forecast given information up to T, with future shocks not pinned down) and how to interpret forecast bands.

Focus slides (core)

- Slides **38-40**: Forecasting workflow and crisis-vs-baseline framing (high level).
- Slide **44**: Practical real-time forecasting problems.
- Slide **45**: Forecasting dataset timing (nowcast vs missing vs external forecasts).
- Slide **46**: What follows (unconditional vs conditional forecasts).
- Slide **48**: Sources of uncertainty (parameters; states; future shocks).
- Slides **49-51**: Bayesian intervals picture + "This is done in Dynare!" (interpretation and implementation message).

Block 3. Conditional forecasting (Dynare-relevant only)

Goal

(i) Explain what is being conditioned on, (ii) understand the "n conditioned paths need n shocks" logic, and (iii) run and interpret Dynare conditional forecasts and outputs.

Focus slides (core)

- Slides **59-63**: Why conditional forecasts; two methods; exogenous vs endogenous conditioning idea.
- Slides **64-65**: Conditioning exogenous variables (recover innovations consistent with a path).
- Slides **67-69**: Conditioning endogenous variables; key takeaway: conditioning n endogenous series requires n shocks.
- Slide **71: Dynare commands `conditional_forecast_paths` and `conditional_forecast` (how one does it).
- Slide **72**: Where Dynare stores conditional vs unconditional forecasts (`forecasts` structure).

Block 4. Scenario analysis for W8

Goal

Run baseline vs scenario exercises, and can explain what is held fixed across runs.

Baseline vs scenario narrative (communication)

Focus slides

- Slides **40-43** (PDF p.36-39): Baseline vs crisis scenario example (use as the “what scenario analysis looks like” anchor).
- Slide **45** (PDF p.41): Forecasting dataset timing (where scenarios plug in: external paths, nowcasts, missing observations).

Quick crosswalk table (blocks → slides)

Block	Core slide numbers	Optional / advanced slide numbers
(1) Estimation → shocks	2-5, 10-13, 28-29	8-9, 15, 93-96, 114-118
(2) Unconditional forecast	38-40, 44-46, 48-51	41-43, 55-58
(3) Conditional forecast	59-65, 67-69, 71-72	75-82
(4) Scenarios for W8	40-43, 45	17-22, 28-32

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Key point

After solving and estimating a linearized DSGE model, we can represent the solution in **state-space** form and use **smoothed structural shocks** to (i) replicate the data and (ii) compute **historical decompositions**.

Clean notation (consistent with the previous lectures and the rest of the course)

Let x_t be the **state vector** (predetermined states and latent states, including AR(1) shock *states*), and let ε_t be the vector of **structural innovations** ("shocks" in Dynare terminology). A first-order solution can be written as

$$x_t = Ax_{t-1} + B\varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \Omega).$$

Let y_t denote the vector of model variables of interest (i.e., observables). Then (no measurement errors):

$$y_t = Hx_t.$$

Important clarification

- x_t contains *state variables* (e.g., capital, lagged endogenous variables, and the AR(1) components of shocks).
- **The shocks are the innovations ε_t, not the state vector.**

What we do in practice

1. **Estimate the model** and run the **Kalman smoother** to obtain smoothed states and shocks:
    - $\{x_t\}_{t=1}^T$ (smoothed states)
    - $\{\varepsilon_t\}_{t=1}^T$ (smoothed structural innovations)
2. **Historical decomposition / "simulation with some shocks"**

    Using the estimated initial condition x_0 and the sequence of smoothed shocks $\{\varepsilon_t\}$, we simulate the model forward to recover the contribution of each shock to a variable (e.g., output growth, inflation, policy rate). Conceptually, for shock j:

    - set $\varepsilon_{t,j}$ equal to its smoothed estimate,
    - set all other shocks to zero,
    - simulate forward and record the implied path of y_t,
    - repeat for each j, and include an "initial conditions" component if desired.

Dynare implementation (what participants should remember)

- After `estimation(..., smoother);`, Dynare stores smoothed objects in:
    - `oo_.SmoothedShocks`
    - `oo_.SmoothedVariables`

- Historical decomposition plots can be produced with:

```
estimation(..., smoother);
shock_decomposition rnom_obs pic_obs y_obs;
```

Common pitfall (to avoid confusion)

Do **not** say "assume the states are shocks."

- States are *propagating variables*.
- Shocks are *innovations* ε_t that hit the transition equation and move the states.

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Practical note

W8 does **not** practice this exogenous-variable conditioning. We present this as a short, optional “how it works in principle” section.

Notation for the forecast horizon.

Let T be the **forecast origin** (last observed period).

- Let $h \geq 1$ denote the **forecast horizon** (number of future periods).
- Future dates are $T + 1, \ldots, T + h$.

(The original L8 slides sometimes use F or F^* for the horizon. To avoid confusion, this note uses h.)

What it means to condition on an exogenous path (state-space interpretation)

Setup

Let z_t denote an **exogenous process** in the DSGE model (e.g., external demand, commodity price, foreign interest rate). Suppose that, at time T, we are given a *target future path* for the next H periods:

$$\{z_{T+1}^*, z_{T+2}^*, \dots, z_{T+h}^*\}.$$

Write this compactly as $z_{T+1:T+h}^*$.

Link to DSGE state-space shocks

In the DSGE state-space representation, exogenous variables such as z_t evolve **only because of their associated structural shock(s)**. In particular, the transition equation contains future structural innovations $\{\varepsilon_t\}_{t=T+1:T+h}$.

Therefore:

> Conditioning on a path for an exogenous variable z_t is equivalent to choosing a sequence of future structural shocks $\{\varepsilon_t\}_{t=T+1:T+h}$ so that the state equation generates $z_{T+1:T+h}^*$.

Concretely, if the model implies a law of motion of the form

$$z_t = g(z_{t-1}; \theta) + \varepsilon_{z,t},$$

then the imposed path $z_{T+1:T+h}^*$ uniquely pins down the shock sequence $\{\varepsilon_{z,t}\}_{t=T+1:T+h}$ via

$$\varepsilon_{z,t} = z_t^* - g(z_{t-1}^*; \theta), \qquad t = T+1, \dots, T+h.$$

Once this sequence of future shocks is fixed, the rest of the model variables are obtained by simulating the DSGE solution forward exactly as in an unconditional forecast.

AR(1) special case (often used for exogenous processes)

If

$$z_t = (1-\rho)\bar{z} + \rho z_{t-1} + \varepsilon_{z,t},$$

then the implied innovations that rationalize the target path are

$$\varepsilon_{z,T+1} = z_{T+1}^* - (1-\rho)\bar{z} - \rho z_T,$$

$$\varepsilon_{z,T+h} = z_{T+h}^* - (1-\rho)\bar{z} - \rho z_{T+h-1}^*, \qquad h = 2, \dots, H.$$

Key idea

If we know the law of motion for z_t, we can back out the sequence of **innovations** (shocks) that makes the process follow the imposed path $z_{T+1:T+h}^*$.