INSTITUTE FOR
CAPACITY DEVELOPMENT

Workshop 7 – Bayesian Estimation of DSGE Models

JANUARY 19 – JANUARY 30, 2026

Tao Zha

Course on Monetary and Fiscal Policy Analysis with DSGE Models (OT26.08)

Outline

1. Estimation of the Baseline NK Model
   a) The model
   b) Data and transformations
   c) Prior distributions
   d) Simulation of the posterior distribution
      i. Obtaining an initial estimation of the mode
      ii. Metropolis-Hastings Algorithm
2. Estimation of the NK model with habits and persistence
   i. Model Comparison

Exercise 2: The Baseline New Keynesian Model

- Endogenous variables :

$$c_t = -\sigma\left(i_t - E_t\pi_{t+1}\right) + E_t c_{t+1} + \left(1-\rho_c\right)\sigma g_t$$

$$rw_t = \sigma_L l_t + \frac{1}{\sigma} c_t$$

$$y_t = a_t + l_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(rw_t - a_t)$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t + z_t$$

$$y_t = c_t$$

- Exogenous variables :

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$$

NKM Parameters

- Phillips Curve parameter:

$$\kappa = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$$

- Economic parameters:

$$\sigma, \sigma_L, \beta, \theta_p, \phi_\pi, \phi_y$$

- Shock parameters

$$\rho_a, \rho_g, \rho_z, \sigma_a, \sigma_g, \sigma_z .$$

Data

- We estimate the closed economy NKM with US data spanning 1984:1 to 2007:4. Why?
- 3 macroeconomic series: interest rate on three-month bills, core CPI inflation, output gap. We have put the data in Excel (`Data_USA.xlsx`) and Eviews (`Data_USA.wf1`) files
- 3 shocks to the model. No stochastic singularity problems.

Data

Remember there are two ways to extract the steady state values and trends from the data:

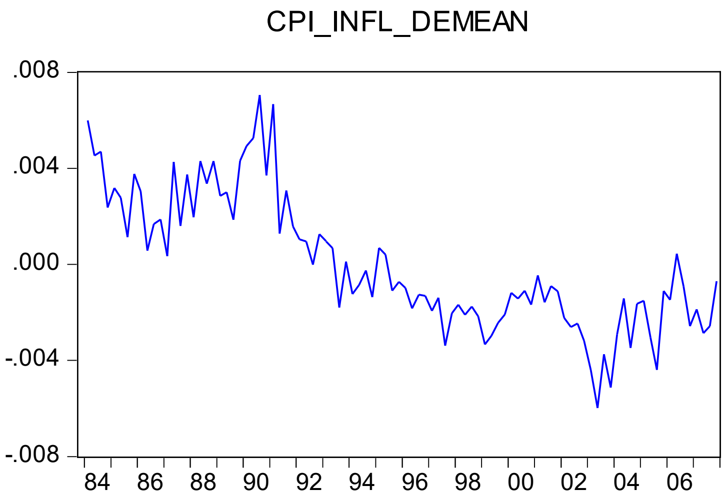
1. In a non-model consistent way: subtract the sample average of inflation and interest rate, detrend real GDP.
2. Model-consistent: estimate inflation target, the real growth rate, and the equilibrium interest rate.

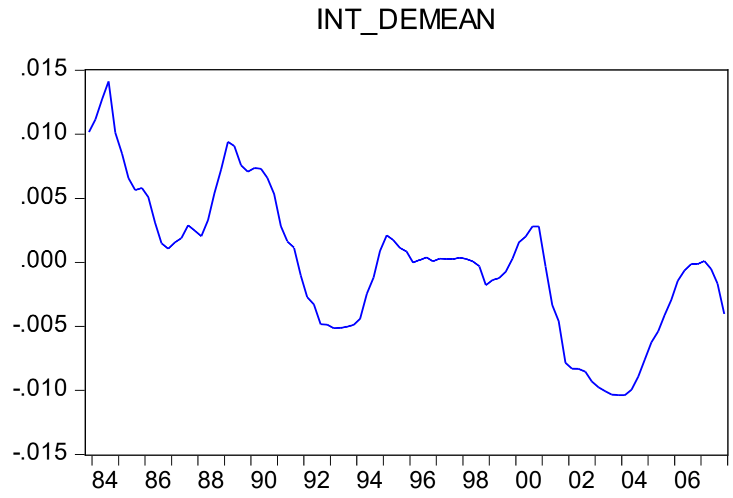
Data

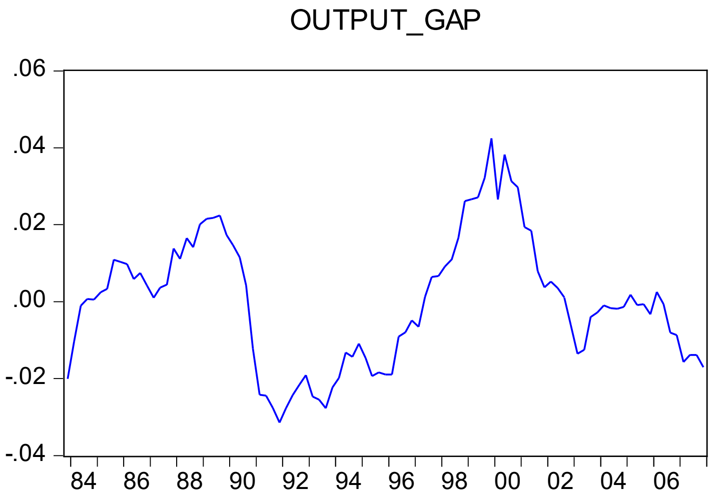
1. Not model-consistent way.
   a) We create the inflation series as $\ln(CPI_t/CPI_{t-1})$ and then subtract its sample mean.
   b) We divide the interest rate by 400 and subtract the sample mean.
   c) Extract a linear-quadratic trend from $\ln(RGDP_t)$.

The transformed data are saved in this Excel file (CSV format): `data_trans.csv`

Data: Transformed





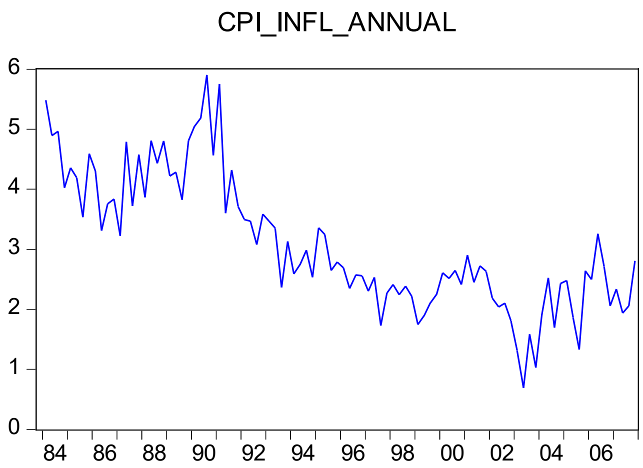


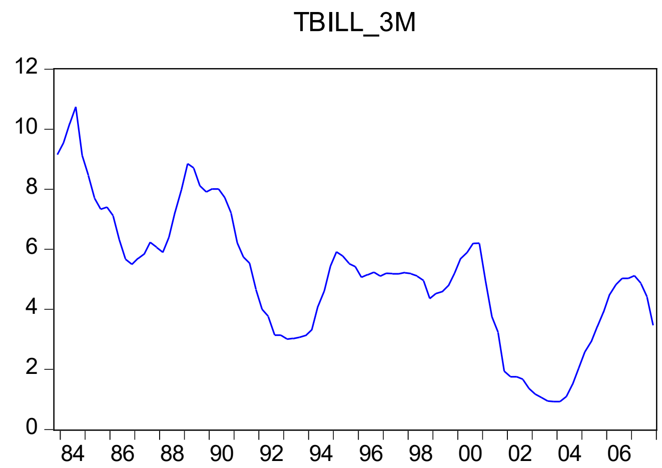
Data

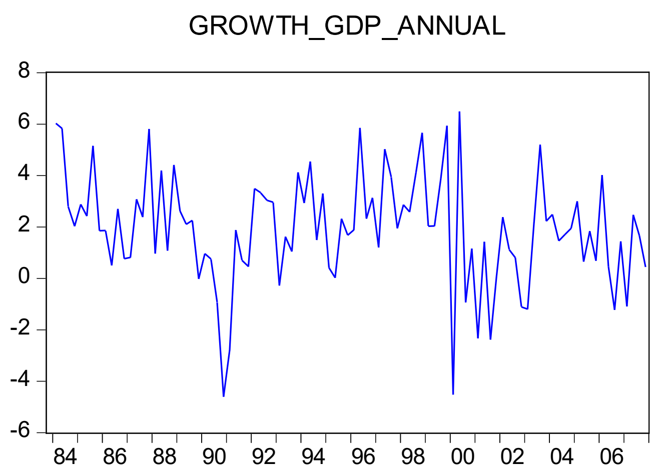
2. Consistent with the model.
   a) We create the annualized quarterly inflation series as $400*\ln(CPI_t/CPI_{t-1})$.
   b) The interest rate is the original series.
   c) We take the annualized quarterly real GDP growth rate as $400*\ln(GDP_t/GDP_{t-1})$.

The transformed data are saved in this Excel file (CSV format): `data_cons.csv`

Data: Model-Consistent







Moving to DYNARE

We will work with the file:

`nk_closedv1_est_trans.mod`

"_est" for estimation.

"_trans" for variables transformed outside the model.

Moving to DYNARE

Two preliminary issues:

1. Introduction of the measurement equations:

Line 19: `var c rnom pic lab rw y mc a g z rnom_obs pic_obs y_obs;`

Lines 52-54:

```
rnom_obs=rnom;
pic_obs=pic;
y_obs=y;
```

Evaluation of the Likelihood Function

Let's return to our closed economy NK model, which we want to estimate with data on the inflation rate, interest rate, and real GDP. In this case, the observable variables would be:

$$X_t^{obs} = \begin{bmatrix} \pi_t^{obs} \\ i_t^{obs} \\ y_t^{obs} \end{bmatrix} = \begin{bmatrix} \ln(P_t^{obs} / P_{t-1}^{obs}) - \overline{\pi} \\ (I_t^{obs} - \bar{I}) / 400 \\ \ln(GDP_t^{obs}) - \ln(GDP_t^{TREND}) \end{bmatrix}$$

IMF INSTITUTE FOR CAPACITY DEVELOPMENT

Evaluation of the Likelihood Function

And the measurement equation is (without measurement errors or a constant term)

$$X_t^{OBS} = HX_t \qquad (2') \qquad H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X_t = \left(c_t, l_t, y_t, rw_t, mc_t, \pi_t, i_t, g_t, a_t, z_t\right)'$$

Moving to DYNARE

Two preliminary issues:

2. In lines 24-33 we assigned initial values to the model's parameters (they do not affect the estimation, but they have to be initialized, otherwise DYNARE crashes).

The commands of the estimation start after line 66.

Bayesian Methods

- Objects of interest:
  - Prior distribution: $p(\theta)$
  - Likelihood function: $L(\mathbf{X}_T^{OBS} \mid \theta)$
  - Joint distribution: (of parameters and observable variables): $f(\mathbf{X}_T^{OBS}, \theta) = p(\theta)L(\mathbf{X}_T^{OBS} \mid \theta)$

Bayesian Methods

- Objects of interest:
  - Posterior distribution: $$p(\theta \mid \mathbf{X}_T^{\mathbf{OBS}}) = \frac{f(\mathbf{X}_T^{\mathbf{OBS}}, \theta)}{f(\mathbf{X}_T^{\mathbf{OBS}})}$$

    In other words, what we have learned about the model's parameters having observed the data.

  - Marginal likelihood:

$$f(\mathbf{X}_T^{\mathbf{OBS}}) = \int_{\theta \in \Theta} f(\mathbf{X}_T^{\mathbf{OBS}} \mid \theta) d\theta = \int_{\theta \in \Theta} p(\theta) L(\mathbf{X}_T^{\mathbf{OBS}} \mid \theta) d\theta$$

Choice of Priors in DYNARE

```
estimated_params;
beta_C,0.995,,,GAMMA_PDF,0.995,0.001;
sigma,1,,,GAMMA_PDF,1,0.5;
sigma_L,1,,,GAMMA_PDF,1,0.5;
theta_p,0.75,,,BETA_PDF,0.75,0.05;
phi_pic,1.5,,,GAMMA_PDF,1.5,.25;
phi_y,.125,,,GAMMA_PDF,.125,.05;
```

Choice of Priors in DYNARE

```
rho_a,0.9,,,BETA_PDF,0.8,0.1;
rho_g,0.8,,,BETA_PDF,0.8,0.1;
rho_z,0.7,,,BETA_PDF,0.8,0.1;
stderr e_a,0.01,,,GAMMA_PDF,0.01,0.005;
stderr e_g,0.01,,,GAMMA_PDF,0.01,0.005;
stderr e_z,0.01,,,GAMMA_PDF,0.01,0.005;
end;
```

Estimation

- In the following line 81 we are telling DYNARE what the observable variables are (they should have the same name in the `data_trans.m` file

```
varobs rnom_obs pic_obs y_obs;
```

- Lastly, the estimation command is (lines 83-90):

```
estimation(...);
```

Metropolis-Hastings

We start with an initial value ω_0

To obtain i=1:N draws, we select a new candidate:

$\omega_{new} = \omega_{i-1} + e$, where $e : N(0, c\Sigma)$,

$$R(\omega_{new}) = \frac{p(\omega_{new})}{p(\omega_{i-1})} \frac{L(\{\mathbf{X}_T^{\mathbf{OBS}}\} \mid \omega_{new})}{L(\{\mathbf{X}_T^{\mathbf{OBS}}\} \mid \omega_{i-1})}.$$

We follow the decision rule:

-If R>1, $\omega_i = \omega_{new}$.

-If R<1, P$(\omega_i = \omega_{new}) = R$.

Estimation

- `datafile='data_trans.csv',` data file
- `mh_replic=5000,` value of N
- `mh_nblocks=1,` MH chains
- `mh_jscale=0.3,` value of "c"
- `mh_drop=0.2,` initial fraction discarded
- `prior_trunc=1e-32,` truncate extreme values
- `nodiagnostic,` does not calculate convergence diagnostics (we haven't talked about this).
- `graph_format=none,` does not create PDF graphs.

Metropolis-Hastings: Practical tips

- To obtain a more efficient algorithm, DYNARE uses the following steps (An and Schorfheide, 2007):
  - It searches for an initial approximation to the mode of the distribution, using numerical methods to maximize $\omega_0 = \text{Arg Max} \log p(\omega \mid \mathbf{X}_{\mathbf{T}}^{\mathbf{OBS}})$
  - It calculates Σ^{-1} as the Hessian of log p() evaluated at ω_0.
  - "c" is then calibrated to ensure that the acceptance rate is around 35%.

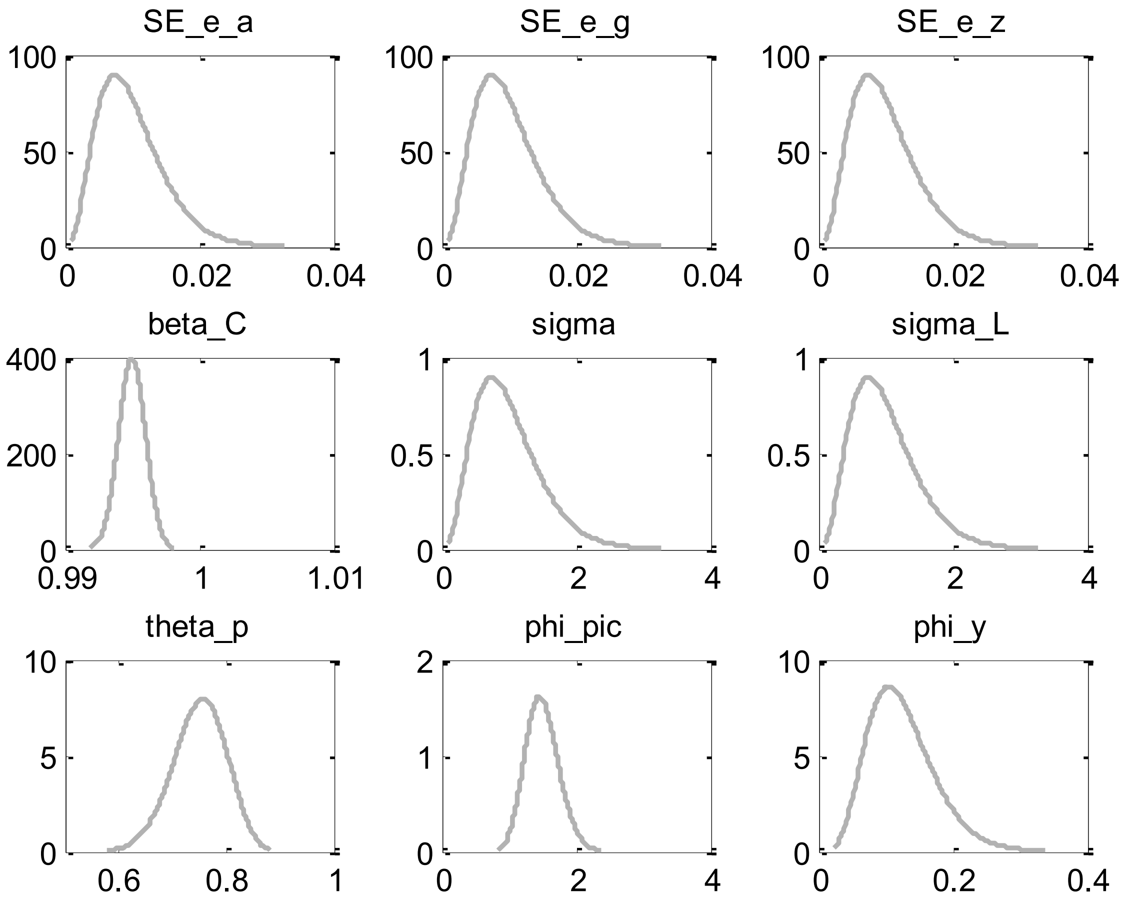
IMF INSTITUTE FOR CAPACITY DEVELOPMENT

Estimation

- Run `dynare nk_closedv1_est_trans`

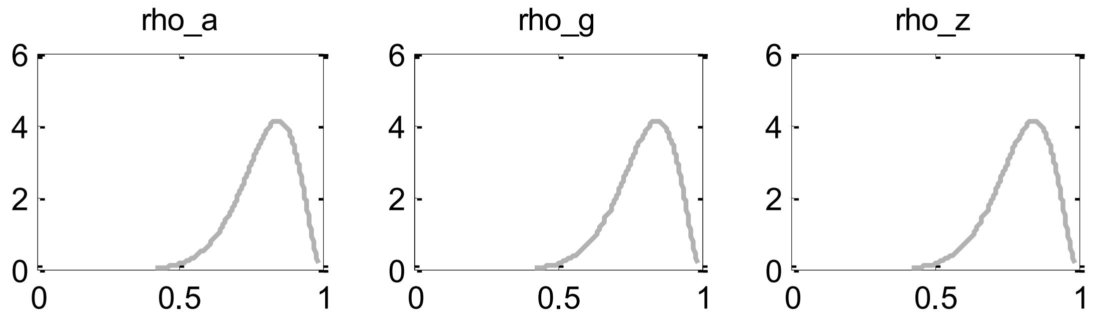
- DYNARE takes the initial value of the parameters vector and tries to obtain an approximation to the mode of the distribution (the standard deviation is based on the inverse Hessian evaluated at the mode).
- This is the initial maximization process seen at the start (it uses the `csminwel` algorithm written by Chris Sims)

Estimation: Prior Distributions



IMF INSTITUTE FOR CAPACITY DEVELOPMENT

Estimation: Prior Distributions



Estimation: First Approximation

RESULTS FROM POSTERIOR ESTIMATION

parameters

	prior mean	mode	s.d.	prior	pstdev
beta_C	0.995	0.9950	0.0010	gamm	0.0010
sigma	1.000	0.3802	0.1200	gamm	0.5000
sigma_L	1.000	0.9124	0.4778	gamm	0.5000
theta_p	0.750	0.7168	0.0373	beta	0.0500
phi_pic	1.500	2.1386	0.2010	gamm	0.2500
phi_y	0.125	0.0374	0.0152	gamm	0.0500
rho_a	0.800	0.9966	0.0019	beta	0.1000
rho_g	0.800	0.9467	0.0144	beta	0.1000
rho_z	0.800	0.6361	0.0503	beta	0.1000

standard deviation of shocks

	prior mean	mode	s.d.	prior	pstdev
e_a	0.010	0.0099	0.0025	gamm	0.0050
e_g	0.010	0.0138	0.0034	gamm	0.0050
e_z	0.010	0.0031	0.0003	gamm	0.0050

Log data density [Laplace approximation] is 1323.254249

Results: Posterior Distribution

ESTIMATION RESULTS

Log data density is 1322.451938.

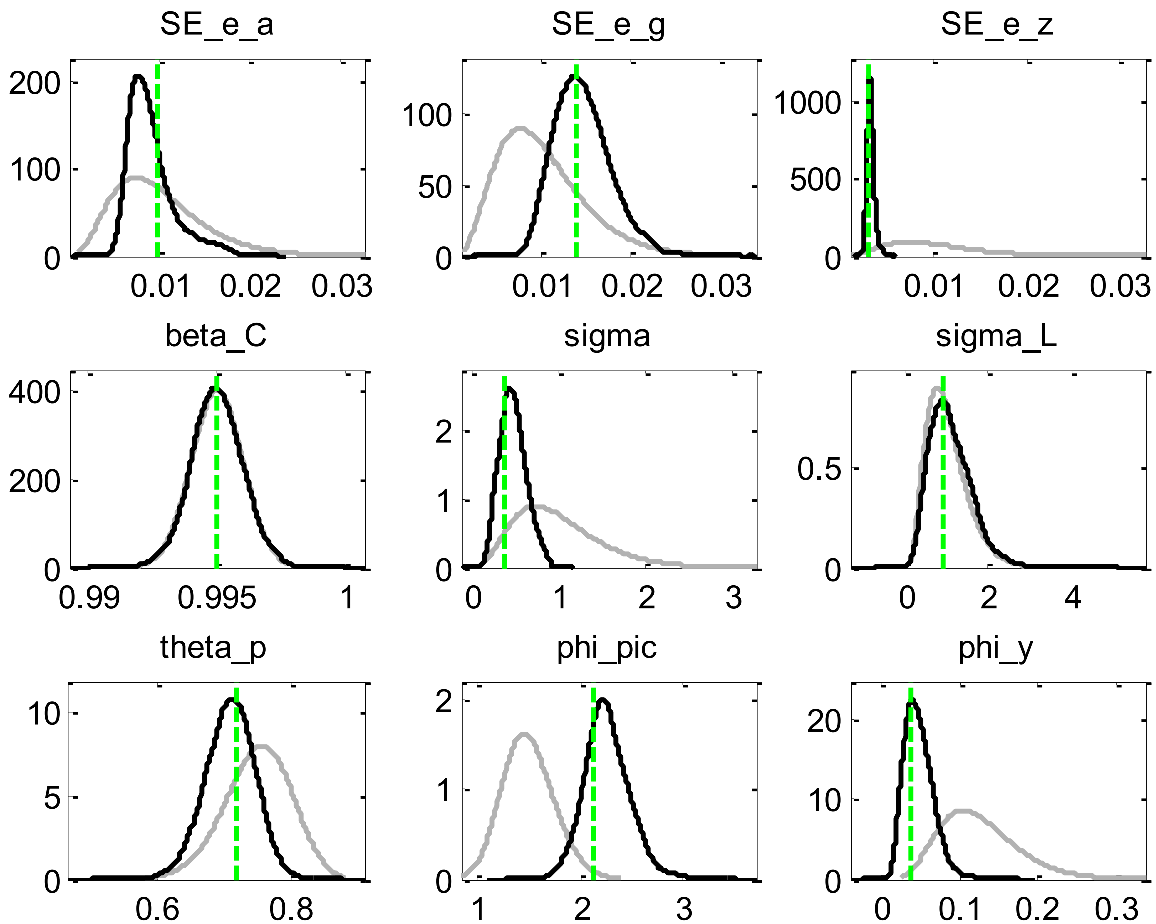
parameters

	prior mean	post. mean	90% HPD interval		prior	pstdev
beta_C	0.995	0.9950	0.9933	0.9966	gamm	0.0010
sigma	1.000	0.5614	0.3286	0.8547	gamm	0.5000
sigma_L	1.000	1.1611	0.4764	1.9944	gamm	0.5000
theta_p	0.750	0.7101	0.6561	0.7658	beta	0.0500
phi_pic	1.500	2.2785	1.9606	2.6619	gamm	0.2500
phi_y	0.125	0.0571	0.0199	0.0866	gamm	0.0500
rho_a	0.800	0.9929	0.9881	0.9986	beta	0.1000
rho_g	0.800	0.9395	0.9136	0.9657	beta	0.1000
rho_z	0.800	0.5977	0.5039	0.6822	beta	0.1000

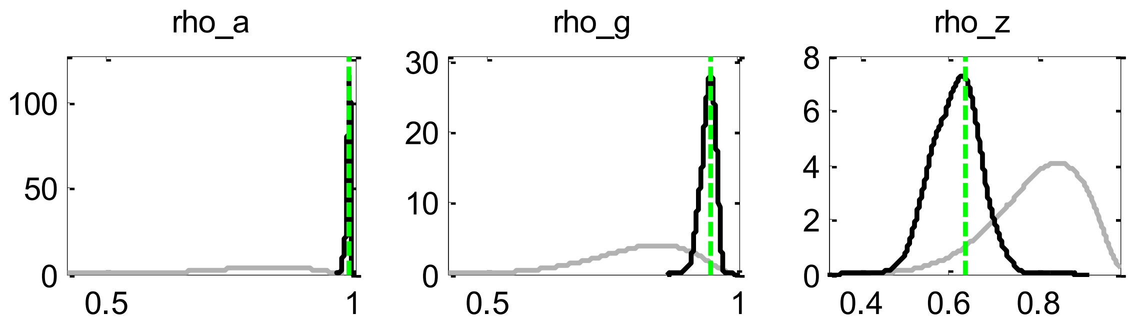
standard deviation of shocks

	prior mean	post. mean	90% HPD interval		prior	pstdev
e_a	0.010	0.0081	0.0055	0.0110	gamm	0.0050
e_g	0.010	0.0143	0.0089	0.0191	gamm	0.0050
e_z	0.010	0.0033	0.0027	0.0038	gamm	0.0050

Estimation: Posterior Distribution



Estimation: Posterior Distribution



Metropolis-Hastings: Practical tips

- Scale adjustment. The DYNARE results tell us that

```
Estimation::mcmc: Current acceptance
ratio per chain:
                  Chain  1: 52.32%
```

For this we want to modify `mh_jscale=0.5`. If it increases, the acceptance rate decreases.

To save time, we introduce the option

`mode_file= nk_closedv1_est_trans_mode`,

so we don't have to recalculate the mode.

Additional Exercises with NKM

2. Joint estimation with the parameters that determine the long-run behavior.
   - Save the file: `nk_closedv1_est_trans.mod` as `nk_closedv1_est_cons.mod`
   - What changes should be made in this file?
     - Define the steady state growth and inflation parameters, and assign them a prior
     - Modify the measurement equations
     - Change the name of the data file to `data_cons.csv` (the name of the observable variables does not change).
   - Estimate the model with and without endogenous persistence and compare the results.

Evaluation of the Likelihood Function

In this case the data transformation is minimal, in annualized terms:

$$X_t^{obs} = \begin{bmatrix} \pi_t^{obs} \\ i_t^{obs} \\ y_t^{obs} - y_{t-1}^{obs} \end{bmatrix} = \begin{bmatrix} 400\ln(P_t^{obs} / P_{t-1}^{obs}) \\ I_t^{obs} \\ 400\ln(PIB_t^{obs} / PIB_{t-1}^{obs}) \end{bmatrix}$$

and in the measurement equations:

$$X_t^{obs} = \begin{bmatrix} \pi_t^{obs} \\ i_t^{obs} \\ y_t^{obs} - y_{t-1}^{obs} \end{bmatrix} = \begin{bmatrix} \pi^* + 400\pi_t \\ \pi^* + \gamma - 400\ln(\beta) + 400 i_t \\ \gamma + 400(y_t - y_{t-1}) \end{bmatrix} = \Delta + HX_t$$

Evaluation of the Likelihood Function

Where:

$$\tilde{\Delta} = \begin{pmatrix} \pi^* \\ \gamma + \pi^* - 400\ln(\beta) \\ \gamma \end{pmatrix}$$

$$\tilde{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 400 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 400 & 0 & 0 & 0 & 0 \\ 0 & 0 & 400 & 0 & 0 & 0 & 0 & -400 & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{X}_t = \left(c_t, l_t, y_t, rw_t, mc_t, \pi_t, i_t, y_{t-1}, g_t, a_t, z_t\right)'$$

IMF INSTITUTE
FOR CAPACITY DEVELOPMENT

Additional Exercises with NKM

3. Estimate the model with habits in consumption habits and price indexation.

- To do so, use a priori distributions on the two parameters (h, chi_p) that are Beta with mean 0.5 and standard deviation 0.2.
- Simulate the posterior distribution with N= 25,000.
- Compare the models with and without inertia. What to do the values of the marginal likelihood of the four models suggest?

The decision rule for model comparison using marginal densities is: higher = better I.e. model A: -950, model B: -1000 ==> choose model A. That is, you want the marginal data density to be a high as possible