Lecture note

L3 - Labor Market Frictions in the NK Model

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1 Divine coincidence and basic NK-models

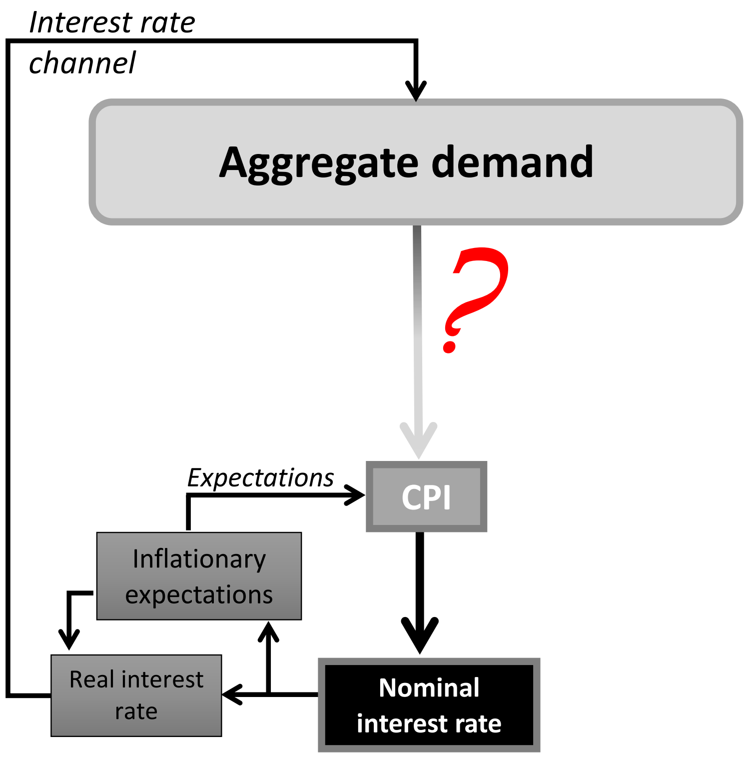
1.1 The beauty of 'Mickey-mouse' NK model

1. **The labor market in DSGE-models has a prominent role in the transmission mechanism**
   - Crucial role in the stabilization of the inflation and output gap
   - Real wage in the marginal cost proxies the aggregate demand
   - Basic New Keynesian model implies volatile wages

2. **But the real wage is rather like an acyclical variable**
   - Co-movement is weaker between the output gap and real wage
   - Inertia between the aggregate demand and inflation

3. **Common stabilization of output gap and inflation ("divine coincidence") less trivial**
   - Reacting too strongly to inflation, not always efficient in bringing back the economy to the natural level

The flowchart below summarizes the main channels and features of the New-Keynesian models in closed economies:



Question: in general, we want to understand the transmission mechanism between the real economy and inflation. More specifically, how sensitive the model to the labor market assumptions.

1.2 New-Keynesian models

1. Euler-equation/Dynamic IS-curve:

$$\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + \frac{1}{\sigma}\left(\xi_t^{\hat{C}} - E_t\xi_{t+1}^{\hat{C}}\right)$$

2. Labor supply:

$$\hat{w}_t = \varphi\hat{L}_t + \sigma\hat{C}_t$$

3. Labor demand:

$$\hat{L}_t = \frac{1}{1-\alpha}\left(\hat{Y}_t - \hat{A}_t\right)$$

4. Marginal cost:

$$\hat{mc}_t = \hat{w}_t + \frac{\alpha}{1-\alpha}\widehat{Y}_t - \frac{1}{1-\alpha}\hat{A}_t$$

where $\hat{A}_t$ is exogenously given

5. New-Keynesian Phillips-curve:

$$\pi_t = \kappa\hat{mc}_t + \beta E_t\pi_{t+1} + \xi_t^{\pi}$$

6. Goods market equilibrium:

$$\hat{Y}_t = \hat{C}_t$$

7. Taylor rule:

$$i_t = \phi_\pi\pi_t + \phi_x x_t + \xi_t^i$$

We can simplify the model and substitute out ...

1. ... consumption
2. ... labor demand from the labor supply curve

$$
\begin{aligned}
\hat{w}_t &= \varphi \frac{1}{1-\alpha}\left(\hat{Y}_t - \hat{A}_t\right) + \sigma \hat{Y}_t \\
&= \left(\frac{\varphi}{1-\alpha} + \sigma\right)\hat{Y}_t - \frac{\varphi}{1-\alpha}\hat{A}_t
\end{aligned}
$$

3. and plug the real wage into the marginal cost function:

$$
\begin{aligned}
\hat{mc}_t &= \left(\frac{\varphi}{1-\alpha} + \sigma\right)\hat{Y}_t - \frac{\varphi}{1-\alpha}\hat{A}_t + \frac{\alpha}{1-\alpha}\widehat{Y}_t - \frac{1}{1-\alpha}\hat{A}_t \\
&= \left(\frac{\varphi+\alpha}{1-\alpha} + \sigma\right)\hat{Y}_t - \frac{1+\varphi}{1-\alpha}\hat{A}_t
\end{aligned}
$$

The *natural level of the economy*: friction less case, no real and nominal rigidities, the nominal prices are flexible and immediately follow the marginal cost. In the current framework with one domestic consumption basket it implies $\hat{mc}_t^n = 0$. Based on the calculations above we can express the natural level of output:

$$
\begin{aligned}
0 = \quad \hat{mc}_t^n &= \left(\frac{\varphi+\alpha}{1-\alpha} + \sigma\right)\hat{Y}_t^n - \frac{1+\varphi}{1-\alpha}\hat{A}_t \\
\hat{Y}_t^n &= \left(\frac{\varphi+\alpha}{1-\alpha} + \sigma\right)^{-1}\frac{1+\varphi}{1-\alpha}\hat{A}_t
\end{aligned}
$$

The natural level of output is the function of the technology shock (RBC). The technology shock is exogenously given, and the current and future level of the natural output can be determined. The dynamic IS curve can be used to express the interest rate that is consistent with the natural output, it is called natural rate of interest then:

$$
\hat{Y}_t^n = E_t\hat{Y}_{t+1}^n - \frac{1}{\sigma}r_t^n
$$

We define the output gap (x) as difference between the total and potential GDP:

$$
x_t = \hat{Y}_t - \hat{Y}_t^n
$$

We can rewrite all equations as the deviation from the natural equilibrium.

The model so far:

1. Dynamic IS curve becomes:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) + \frac{1}{\sigma}\left(\xi_t^{\hat{C}} - E_t \xi_{t+1}^{\hat{C}}\right)$$

2. Marginal cost:

$$\hat{mc}_t = \left(\frac{\varphi + \alpha}{1 - \alpha} + \sigma\right) x_t$$

3. New-Keynesian Phillips-curve:

$$\pi_t = \kappa \hat{mc}_t + \beta E_t \pi_{t+1} + \xi_t^{\pi}$$

where $\kappa = (1 - \theta)(1 - \beta\theta)/\theta \cdot \frac{1}{1+\varepsilon\frac{\alpha}{1-\alpha}}$

4. Taylor rule (with reaction to the output gap):

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t + \xi_t^i$$

Role of the central bank: anchoring the nominal variables...

1. bringing back the inflation to the target (now to the zero steady-state)
2. closing the output gap or keeping the total output at the potential or natural level

Divine coincidence in basic NK model: the monetary policy has a convenient role ...

- If the inflation and its expectation are zero/on target, the output gap is closed too
- Inflation is good proxy for an overheated economy
- In order to close the output gap, the central bank has to keep the nominal interest rate elevated until the inflation does not get back to the target

What is the minimal reaction of the monetary policy?

- The real interest rate gap should be positive

$$i_t - E_t\pi_{t+1} - r_t^n > 0$$

- Do we find any condition that satisfies the positive real interest rate gap?

Stability condition under sticky prices

- Assume a very little deviation in the inflation $\pi_t \approx E_t\pi_{t+1}$ that implies for the New-Keynesian Phillips Curve:

$$\begin{aligned} \pi_t &\approx \kappa' x_t + \beta\pi_t \\ x_t &\approx \frac{1-\beta}{\kappa'}\pi_t \end{aligned}$$

  where $\kappa' = \kappa \cdot \left(\frac{\varphi+\alpha}{1-\alpha} + \sigma\right)$

- Substituting out the interest rate, expectation and output gap in the condition above:

$$\begin{aligned} \phi_\pi\pi_t + \phi_x x_t - E_t\pi_{t+1} &> 0 \\ (\phi_\pi - 1)\pi_t + \phi_x\frac{1-\beta}{\kappa'}\pi_t &> 0 \\ \kappa'(\phi_\pi - 1) + \phi_x(1-\beta) &> 0 \end{aligned}$$

- β close to 1, ϕ_x close to zero, then $\phi_\pi > 1$ should satisfy

2 Real wage rigidity

1. Basic NK model: the real wage adjustment immediately follows the volatility of output gap
2. Empirical fact: the wages are not procyclical and react with some delay to the real economic activity
   - It has impact on the monetary transmission mechanism and the dynamics of inflation

Modification of the model:

- Simplification: constant return to scale technology, $\alpha = 0$
- Modified labor supply curve:

$$\hat{w}_t = \rho^w \hat{w}_{t-1} + (1-\rho^w)\underbrace{\left(\varphi \hat{L}_t + \sigma \hat{C}_t\right)}_{'MRS'}$$

- Substituting out consumption and labor demand:

$$\hat{w}_t = \rho^w \hat{w}_{t-1} + (1-\rho^w)\left(\varphi \hat{Y}_t - \varphi \hat{A}_t + \sigma \hat{Y}_t\right)$$

- Real marginal cost by constant return to scale:

$$\begin{aligned} \hat{mc}_t &= \hat{w}_t - \hat{A}_t \\ \hat{w}_t &= \hat{mc}_t + \hat{A}_t \end{aligned}$$

- Instead of real wage, we can use marginal cost and productivity in the labor supply:

$$\begin{aligned} \hat{mc}_t + \hat{A}_t &= \rho^w\left(\hat{mc}_{t-1} + \hat{A}_{t-1}\right) + (1-\rho^w)\left((\varphi+\sigma)\hat{Y}_t - \varphi\hat{A}_t\right) \\ \hat{mc}_t &= \rho^w\left(\hat{mc}_{t-1} - \Delta\hat{A}_t\right) + (1-\rho^w)\left((\varphi+\sigma)\hat{Y}_t - (1+\varphi)\hat{A}_t\right) \end{aligned}$$

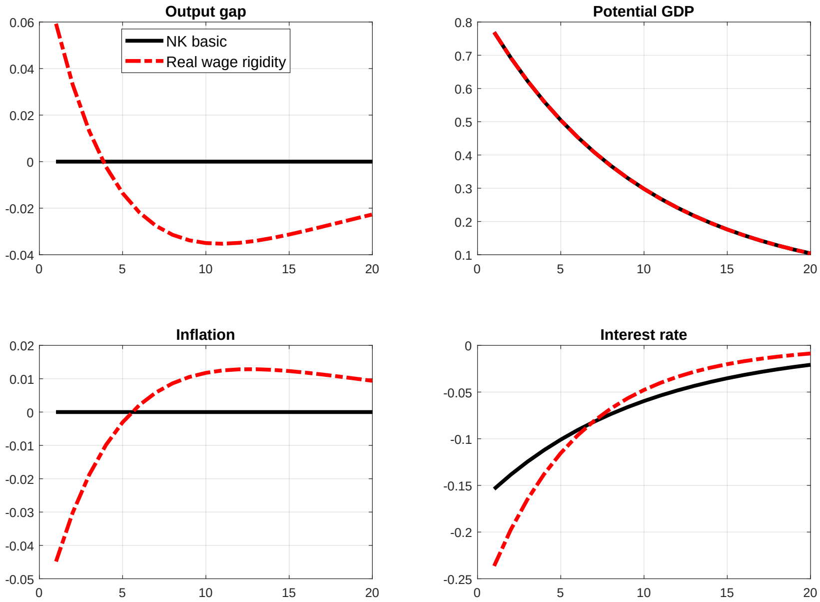
- Previously, we have derived the natural level of output, now we can express the output gap:

$$\hat{mc}_t = \rho^w\left(\hat{mc}_{t-1} - \Delta\hat{A}_t\right) + (1-\rho^w)(\varphi+\sigma)x_t$$

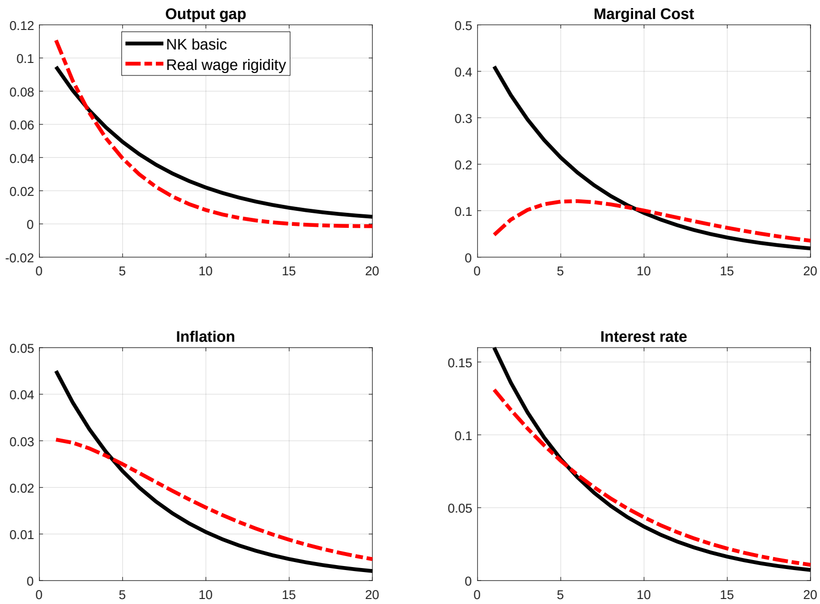
- In this setup the real marginal cost (and the inflation) is the function of the current and previous output gaps and productivity changes (the technology shock is non-neutral to the inflation and output gap):

$$\hat{mc}_t = \sum_{i=0}^{\infty} \rho^{wi}\left\{(1-\rho^w)(\varphi+\sigma)x_{t-i} - \rho^w\Delta\hat{A}_{t-i}\right\}$$

Productivity shock



Demand shock



Results

1. No ’divine coincidence’: behind the inflation and marginal cost, other factors (productivity) also play role, not the output gap only...
   - Strong reaction to the inflation only is not trivial if the central bank wants to close the output gap
   - Output gap stabilization could be more important
2. Inertia
   - The accommodation in marginal cost (and inflation) takes more time
   - Due to the elongated wage response, the history of the output gap does also matter ...
   - Too strong reaction to the inflation can generate long lasting negative output gap and negative inflation later

3 Nominal wage setting a la Calvo

In the previous step we have seen that the real wage rigidity modifies the transmission mechanism (and eliminates the divine coincidence). In this part we want to give a formal model that is consistent with the agents' optimization behavior, and similarly to the previous one generates real wage rigidity.

1. Monopolistic competition on the labor market, labor union collects the individual labor force ($L_t(j)$) and supplies it to the firms (L_t)

    - Households have market power
    - Compared to the perfect competition, this wage is adjusted by mark-up component

    $$L_t = \left[\int_0^1 L_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

    - Subject to the household's budget constraint and individual labor demand for household j's labor supply
    - Monopolistic competition: the households set such a wage that maximizes their own utilities

2. Nominal wage setting is not possible in every period (or costly)

    - Calvo: randomly given fraction of the households are able to set their wages in each period

    $$\sum_{n=0}^{\infty} \theta_w^n \beta^n U_{t+n}(j) \left\{ C_{t+n}(W_t^*(j)), L_{t+n}(W_t^*(j)) \right\} \longrightarrow \max_{W_t^*(j)}$$

    where θ_w of the probability of household j can not set a new wage
    - Rotemberg: nominal wage setting is coupled with additional adjustment cost

Optimization

Lagrangian:

$$\mathcal{L} = \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ \frac{C_{t+n}(j)^{1-\sigma}}{1-\sigma} - \Psi \frac{L_{t+n}(j)^{1+\varphi}}{1+\varphi} \right\} \longrightarrow \max_{W_t^*(j)}$$

subject to

$$\begin{aligned} P_{t+n}C_{t+n}(j) &= W_t^*(j)L_{t+n}(j) + others \\ L_{t+n}(j) &= \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w} L_{t+n} \end{aligned}$$

First-order condition:

$$\frac{\partial \mathcal{L}}{\partial W_t^*(j)} = \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ C_{t+n}(j)^{-\sigma} \frac{\partial C_{t+n}(j)}{\partial W_t^*(j)} - \Psi L_{t+n}(j)^{\varphi} \frac{\partial L_{t+n}(j)}{\partial W_t^*(j)} \right\} = 0$$

where

$$\begin{aligned} \frac{\partial C_{t+n}(j)}{\partial W_t^*(j)} &= \frac{1}{P_{t+n}} L_{t+n}(j) + \frac{1}{P_{t+n}} W_t^*(j) \frac{\partial L_{t+n}(j)}{\partial W_t^*(j)} \\ \frac{\partial L_{t+n}(j)}{\partial W_t^*(j)} &= -\varepsilon_w \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w - 1} \frac{1}{W_{t+n}} L_{t+n} \end{aligned}$$

First-order condition:

$$\begin{aligned} &\sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ C_{t+n}(j)^{-\sigma} \left(\frac{W_t^*(j)}{P_{t+n}} \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w} L_{t+n} - \varepsilon_w \frac{W_t^*(j)}{P_{t+n}} \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w} L_{t+n} \right) \right. \\ &\left. + \varepsilon_w \Psi \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w(1+\varphi)} L_{t+n}^{1+\varphi} \right\} = 0 \end{aligned}$$

Optimal wage:

$$W_t^*(j) = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\sum_{n=0}^{\infty} \theta_w^n \beta^n \Psi \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w(1+\varphi)} L_{t+n}^{1+\varphi}}{\sum_{n=0}^{\infty} \theta_w^n \beta^n C_{t+n}(j)^{-\sigma} \frac{1}{P_{t+n}} \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w} L_{t+n}}$$

Total wage index can be given from the composite term:

$$\begin{aligned} W_t &= \left[(1-\theta_w) W_t^*(j)^{1-\varepsilon_w} + \int_0^{\theta_w} W_t(j)^{1-\varepsilon_w} \right]^{\frac{1}{1-\varepsilon_w}} \\ &= \left[(1-\theta_w) W_t^*(j)^{1-\varepsilon_w} + \theta_w W_{t-1}^{1-\varepsilon_w} \right]^{\frac{1}{1-\varepsilon_w}} \end{aligned}$$

Log-linearization of nominal wage Phillips curve

One can rewrite the optimal wage equation in the following form:

$$\sum_{n=0}^{\infty} \theta_w^n \beta^n C_{t+n}(j)^{-\sigma} \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w} L_{t+n} \left\{ \frac{W_t^*(j)}{P_{t+n}} - \frac{\varepsilon_w}{\varepsilon_w - 1} \Psi \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w \varphi} L_{t+n}^{\varphi} C_{t+n}(j)^{\sigma} \right\} = 0$$

Outside of the big brackets we can take the steady-state of the variables and simplify with them

$$E_t \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ e^{\ln \frac{W_t^*(j)}{P_{t+n}}} - \frac{\varepsilon_w}{\varepsilon_w - 1} \Psi e^{-\varepsilon_w \varphi \ln \frac{W_t^*(j)}{W_{t+n}} + \varphi \ln L_{t+n} + \sigma \ln C_{t+n}(j)} \right\} = 0$$

Log-linearization of the terms within the brackets:

$$E_t \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ \frac{\widehat{W_t^*(j)}}{P_{t+n}} + \varepsilon_w \varphi \frac{\widehat{W_t^*(j)}}{W_{t+n}} - \varphi \widehat{L_{t+n}} - \sigma \widehat{C_{t+n}(j)} \right\} = 0$$

The log-linearized wage index is defined as:

$$\frac{\widehat{W_t^*(j)}}{P_{t+n}} = \ln \frac{W_t^*(j)}{P_{t+n}} - \ln \frac{W^*(j)}{P}$$

But it can be rewritten with the W_t:

$$\begin{aligned} \frac{\widehat{W_t^*(j)}}{P_{t+n}} &= \ln \frac{W_t^*(j)}{W_t} + \ln \frac{W_t}{P_{t+n}} - \ln \frac{W^*(j)}{W} - \ln \frac{W}{P} \\ &= \frac{\widehat{W_t^*(j)}}{W_t} + \frac{\widehat{W_t}}{P_{t+n}} \end{aligned}$$

Using them to rearrange the sum above:

$$E_t \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ \frac{\widehat{W_t^*(j)}}{W_t} + \frac{\widehat{W_t}}{P_{t+n}} + \varepsilon_w \varphi \frac{\widehat{W_t^*(j)}}{W_t} + \varepsilon_w \varphi \frac{\widehat{W_t}}{W_{t+n}} - \varphi \widehat{L_{t+n}} - \sigma \widehat{C_{t+n}(j)} \right\} = 0$$

We can express the optimal relative wage and rearrange the sum:

$$\frac{1 + \varepsilon_w \varphi}{1 - \theta_w \beta} \frac{\widehat{W_t^*(j)}}{W_t} = E_t \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ \varphi \widehat{L_{t+n}} + \sigma \widehat{C_{t+n}(j)} - \frac{\widehat{W_t}}{P_{t+n}} - \varepsilon_w \varphi \frac{\widehat{W_t}}{W_{t+n}} \right\}$$

Open the sum for $n = 0$:

$$\begin{aligned} \frac{1 + \varepsilon_w \varphi}{1 - \theta_w \beta} \frac{\widehat{W_t^*(j)}}{W_t} = \varphi \widehat{L_t} \quad &+ \quad \sigma \widehat{C_t(j)} - \frac{\widehat{W_t}}{P_t} \\ &+ \quad E_t \sum_{n=1}^{\infty} \theta_w^n \beta^n \left\{ \varphi \widehat{L_{t+n}} + \sigma \widehat{C_{t+n}(j)} - \frac{\widehat{W_t}}{P_{t+n}} - \varepsilon_w \varphi \frac{\widehat{W_t}}{W_{t+n}} \right\} \end{aligned}$$

Stepping ahead the equation from the previous step

$$\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}\frac{\widehat{W^*_{t+1}(j)}}{W_{t+1}} = E_t\sum_{n=1}^{\infty}\theta_w^{n-1}\beta^{n-1}\left\{\varphi\widehat{L_{t+n}}+\sigma\widehat{C_{t+n}(j)}-\frac{\widehat{W_{t+1}}}{P_{t+n}}-\varepsilon_w\varphi\frac{\widehat{W_{t+1}}}{W_{t+n}}\right\}$$

We can rewrite this equation in order to substitute out the sum:

$$\begin{aligned}\theta_w\beta\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}\frac{\widehat{W^*_{t+1}(j)}}{W_{t+1}} = E_t\sum_{n=1}^{\infty}\theta_w^{n}\beta^{n}\Bigg\{\varphi\widehat{L_{t+n}}+\sigma\widehat{C_{t+n}(j)} &- \frac{\widehat{W_t}}{P_{t+n}}-\frac{\widehat{W_{t+1}}}{W_t}- \\ &- \varepsilon_w\varphi\frac{\widehat{W_t}}{W_{t+n}}-\varepsilon_w\varphi\frac{\widehat{W_{t+1}}}{W_t}\Bigg\}\end{aligned}$$

And the sum is equal to:

$$\theta_w\beta\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}E_t\left(\frac{\widehat{W^*_{t+1}(j)}}{W_{t+1}}+\frac{\widehat{W_{t+1}}}{W_t}\right) = E_t\sum_{n=1}^{\infty}\theta_w^{n}\beta^{n}\left\{\varphi\widehat{L_{t+n}}+\sigma\widehat{C_{t+n}(j)}-\frac{\widehat{W_t}}{P_{t+n}}-\varepsilon_w\varphi\frac{\widehat{W_t}}{W_{t+n}}\right\}$$

Now we can simplify the optimal wage equation:

$$\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}\frac{\widehat{W^*_t(j)}}{W_t} = \varphi\widehat{L_t}+\sigma\widehat{C_t(j)}-\frac{\widehat{W_t}}{P_t}+\theta_w\beta\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}E_t\left(\frac{\widehat{W^*_{t+1}(j)}}{W_{t+1}}+\frac{\widehat{W_{t+1}}}{W_t}\right)$$

The nominal wage index can be written as:

$$\begin{aligned}W_t &= \left[(1-\theta_w)W^*_t(j)^{1-\varepsilon_w}+\theta_w W_{t-1}^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}} \\ 1 &= (1-\theta_w)\left(\frac{W^*_t(j)}{W_t}\right)^{1-\varepsilon_w}+\theta_w\left(\frac{W_{t-1}}{W_t}\right)^{1-\varepsilon_w}\end{aligned}$$

The log-linearized version of this:

$$\begin{aligned}0 &= (1-\theta_w)\frac{\widehat{W^*_t(j)}}{W_t}+\theta_w\frac{\widehat{W_{t-1}}}{W_t} \\ \frac{\widehat{W^*_t(j)}}{W_t} &= \frac{\theta_w}{1-\theta_w}\frac{\widehat{W_t}}{W_{t-1}}\end{aligned}$$

Substituting out the optimal wage index:

$$\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}\frac{\theta_w}{1-\theta_w}\frac{\widehat{W_t}}{W_{t-1}} = \varphi\widehat{L_t}+\sigma\widehat{C_t(j)}-\frac{\widehat{W_t}}{P_t}+\theta_w\beta\frac{1+\varepsilon_w\varphi}{1-\theta_w\beta}E_t\left(\frac{\theta_w}{1-\theta_w}\frac{\widehat{W_{t+1}}}{W_t}+\frac{\widehat{W_{t+1}}}{W_t}\right)$$

Rearranging and introducing wage inflation and real wage, we can get the New-Keynesian Phillips curve for the nominal wages:

$$\pi^w_t = \frac{(1-\theta_w)(1-\theta_w\beta)}{(1+\varepsilon_w\varphi)\theta_w}\left\{\varphi\widehat{L_t}+\sigma\widehat{C_t(j)}-\widehat{w_t}\right\}+\beta E_t\pi^w_{t+1}$$

Because all households follows the same optimization problem, the individual consumption can be rewritten to the aggregate one. And the the wage inflation:

$$\pi_t^w = \pi_t + \hat{w}_t - \hat{w}_{t-1}$$

HOMEWORK: Derive the nominal wage Phillips curve with nominal wage indexation (Hint: check my appendix to the basic NK-model!)

Model so far

- Dynamic IS curve:

$$\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + \frac{1}{\sigma}\left(\xi_t^{\hat{C}} - E_t\xi_{t+1}^{\hat{C}}\right)$$

- New-Keynesian Nominal Wage Phillips curve:

$$\pi_t^w = \kappa_w\left\{\varphi\hat{L}_t + \sigma\hat{C}_t - \hat{w}_t\right\} + \beta E_t\pi_{t+1}^w$$

- The wage inflation:

$$\pi_t^w = \pi_t + \hat{w}_t - \hat{w}_{t-1}$$

- Labor demand:

$$\hat{L}_t = \hat{Y}_t - \hat{A}_t$$

- Marginal cost (substituting out labor and consumption):

$$\hat{mc}_t = \hat{w}_t - \hat{A}_t$$

- New-Keynesian Phillips-curve:

$$\pi_t = \kappa\hat{mc}_t + \beta E_t\pi_{t+1} + \xi_t^\pi$$

- Goods market equilibrium:

$$\hat{Y}_t = \hat{C}_t$$

- Taylor rule:

$$i_t = \phi_\pi\pi_t + \phi_{\pi_w}\pi_t^w + \phi_x\pi_t^x + \xi_t^i$$

Further simplification

Based on the basic NK-model, in the flexible price equilibrium the marginal cost is equal to zero. Then the natural level of real wage is equal to the productivity. It implies that the real wage gap is the following:

$$\hat{mc}_t = x_t^w = \hat{w}_t - \hat{A}_t$$

We can use it and substituting out the consumption and the real wage in the newly derived New-Keynesian wage Phillips curve:

$$\pi_t^w = \kappa_w \left\{ \varphi \hat{L}_t + \sigma \hat{Y}_t - x_t^w - \hat{A}_t \right\} + \beta E_t \pi_{t+1}^w$$

Plugging back the labor demand curve:

$$\pi_t^w = \kappa_w \left\{ \varphi \hat{Y}_t - \varphi \hat{A}_t + \sigma \hat{Y}_t - x_t^w - \hat{A}_t \right\} + \beta E_t \pi_{t+1}^w$$

Based on the basic NK-model (and the constant return to scale technology), the natural level of output can be given as:

$$(\varphi + \sigma) \hat{Y}_t^n = (1 + \varphi) \hat{A}_t$$

It means that the nominal wage Phillips curve become:

$$\pi_t^w = \kappa_w \left\{ (\sigma + \varphi) x_t - x_t^w \right\} + \beta E_t \pi_{t+1}^w$$

And the wage inflation:

$$\pi_t^w = \pi_t + \hat{x}_t^w - \hat{x}_{t-1}^w + \hat{A}_t - \hat{A}_{t-1}$$

The model with gaps

- Dynamic IS curve:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t^n) + \frac{1}{\sigma}\left(\xi_t^{\hat{C}} - E_t\xi_{t+1}^{\hat{C}}\right)$$

- New-Keynesian Nominal Wage Phillips curve:

$$\pi_t^w = \kappa_w\Big\{(\sigma + \varphi)x_t - x_t^w\Big\} + \beta E_t\pi_{t+1}^w$$

- The wage inflation:

$$\pi_t^w = \pi_t + \hat{A}_t - \hat{A}_{t-1} + \hat{x}_t^w - \hat{x}_{t-1}^w$$

- New-Keynesian Phillips-curve:

$$\pi_t = \kappa x_t^w + \beta E_t\pi_{t+1} + \xi_t^\pi$$

- Monetary policy rule:

$$i_t = r_t^n + \phi_\pi\pi_t + \phi_{\pi_w}\pi_t^w + \phi_x x_t + \xi_t^i$$

Why does it matter? Is it similar to the version with real wage rigidity?

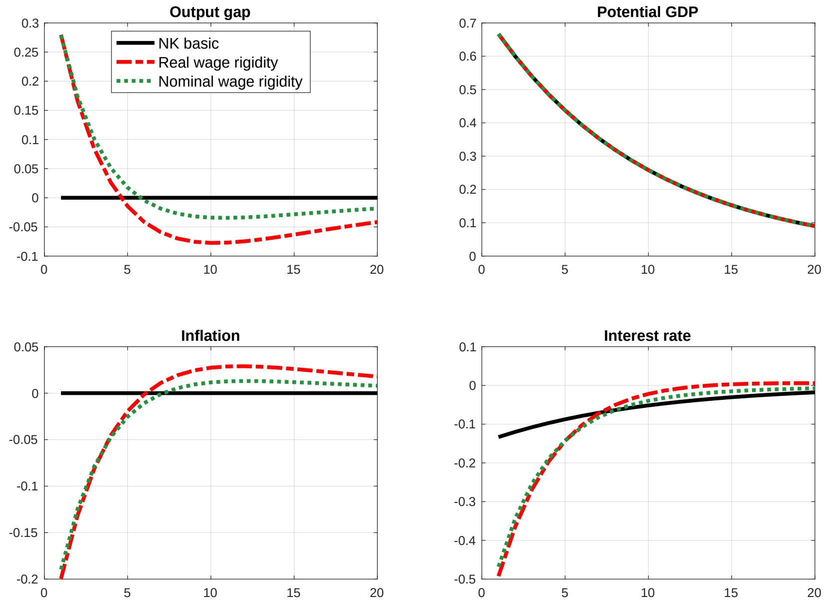
1. The marginal cost is equal to the real wage gap. The real wage gap is driven by the previous real wage gap, nominal wage inflation minus price inflation minus changes of the productivity:

$$\hat{x}_t^w = \hat{x}_{t-1}^w + \pi_t^w - \pi_t - \Delta\hat{A}_t$$

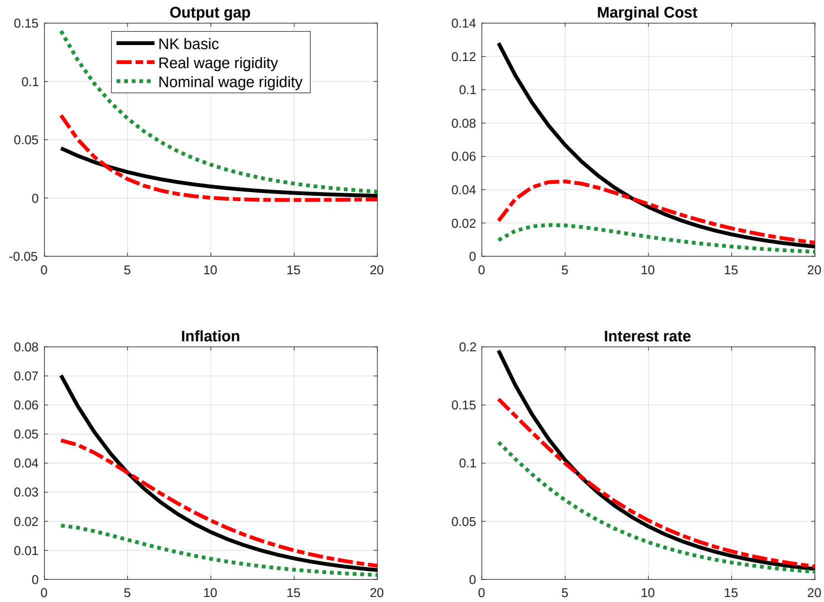
   The nominal wages and prices are sticky, then the real wage gap is sticky either. This is similar to the previous model. The sticky nominal wages slow down the accommodation of marginal cost and then the inflationary processes.

2. The technology shock affects the real wage gap (and later the output gap too), the 'divine coincidence' does not hold in this case.

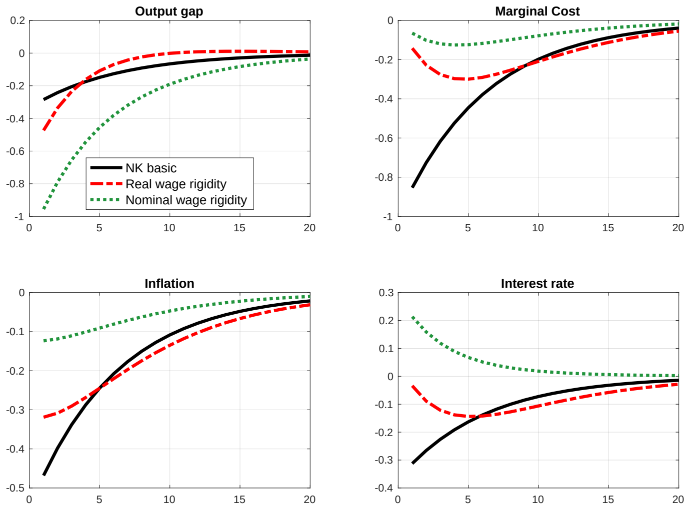
Productivity shock



Demand shock



Monetary policy shock



Last question: *what is the stability condition under sticky prices and wages?*

- The real interest rate gap should be positive

$$i_t - E_t\pi_{t+1} - r_t^n > 0$$

- Assume a very little deviation in the inflations and gaps:

$$\begin{aligned}
\pi_t &\approx \kappa x_t^w + \beta\pi_t \Rightarrow x_t^w \approx \frac{1-\beta}{\kappa}\pi_t \\
\pi_t^w &\approx \kappa_w\left\{(\sigma+\varphi)\hat{x}_t - \hat{x}_t^w\right\} + \beta\pi_t^w \Rightarrow x_t \approx \frac{1}{\sigma+\varphi}\left[\frac{1-\beta}{\kappa_w}\pi_t^w + x_t^w\right] = \frac{1-\beta}{\sigma+\varphi}\left[\frac{\pi_t^w}{\kappa_w} + \frac{\pi_t}{\kappa}\right] \\
\pi_t^w &\approx \pi_t
\end{aligned}$$

- The real interest rate gap can be rewritten with the interest rate rule:

$$\phi_\pi\pi_t + \phi_{\pi_w}\pi_t^w + \phi_x x_t - E_t\pi_{t+1} \quad > \quad 0$$

- Based on the calculations above we can substitute out everything as the function of price inflation:

$$\begin{aligned}
\phi_\pi\pi_t + \phi_{\pi_w}\pi_t + \phi_x\frac{1-\beta}{\sigma+\varphi}\left[\frac{\pi_t}{\kappa_w} + \frac{\pi_t}{\kappa}\right] - \pi_t \quad &> \quad 0 \\
\phi_\pi + \phi_{\pi_w} + \phi_x\frac{1-\beta}{\sigma+\varphi}\left[\frac{1}{\kappa_w} + \frac{1}{\kappa}\right] \quad &> \quad 1
\end{aligned}$$

- β close to 1, ϕ_x close to zero, then $\phi_\pi + \phi_{\pi_w} > 1$ should satisfy

4 Unemployment a la Blanchard-Galí (2010)

1. Walrasian labor market:
   - Households' 'leisure' decision is consistent with the utility maximization
   - Unemployment is voluntary
2. Model with unemployment and Nash-bargaining:
   - Labor and unemployment choices are function of the current and discounted pay-offs
   - Bargaining power determines the equilibrium wages and employment status

Firms

Firm i maximizes its profit and hires labor for production:

$$\mathcal{L} = \sum_{n=0}^{\infty} \theta^n \Delta_{t,t+n} \left\{ P_t^*(i) Y_{t+n}(i) - W_{t+n} L_{t+n}(i) - P_{t+n} hc_{t+n} H_{t+n}(i) \right\} \longrightarrow \max_{P_t^*(i)}$$

subject to

$$\begin{aligned}
Y_{t+n}(i) &= \left(\frac{P_t^*(i)}{P_{t+n}} \right)^{-\varepsilon} Y_{t+n} \\
L_{t+n}(i) &= (1 - pr^F) L_{t+n-1}(i) + H_{t+n}(i) \\
Y_{t+n}(i) &= A_{t+n} L_{t+n}(i)
\end{aligned}$$

where pr^F and hc are the economy wide firing probability and real hiring cost respectively, H is the hiring in each period. The real hiring cost assumed to be a function of hiring probability (pr^H):

$$\begin{aligned}
hc_t &= \kappa (pr_t^H)^{\alpha_{HC}} \\
pr_t^H &= \frac{H_t}{U_{t-1} + pr^F L_{t-1}}
\end{aligned}$$

where U is the unemployment rate.

Lagrangian:

$$\begin{aligned}
\mathcal{L} = \sum_{n=0}^{\infty} \theta^n \Delta_{t,t+n} \Bigg\{ \quad & P_t^*(i) \left(\frac{P_t^*(i)}{P_{t+n}} \right)^{-\varepsilon} Y_{t+n} - W_{t+n} L_{t+n}(i) - P_{t+n} hc_{t+n} H_{t+n}(i) \\
& + MC_{t+n} \left(A_{t+n} L_{t+n}(i) - \left(\frac{P_t^*(i)}{P_{t+n}} \right)^{-\varepsilon} Y_{t+n} \right) \\
& + \lambda_{t+n}^H \Big((1 - pr^F) L_{t+n-1}(i) + H_{t+n}(i) - L_{t+n}(i) \Big) \Bigg\}
\end{aligned}$$

First-order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_t^*(i)} &= \sum_{n=0}^{\infty} \theta^n \Delta_{t,t+n} Y_{t+n} \left\{ \left(\frac{P_t^*(i)}{P_{t+n}}\right)^{-\varepsilon} - \varepsilon \left(\frac{P_t^*(i)}{P_{t+n}}\right)^{-\varepsilon-1} \frac{P_t^*(i)}{P_{t+n}} + \varepsilon \left(\frac{P_t^*(i)}{P_{t+n}}\right)^{-\varepsilon-1} \frac{MC_{t+n}}{P_{t+n}} \right\} = 0 \\
&= \sum_{n=0}^{\infty} \theta^n \Delta_{t,t+n} \left(\frac{P_t^*(i)}{P_{t+n}}\right)^{-\varepsilon} Y_{t+n} \left\{ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} MC_{t+n} \right\} = 0 \\
\frac{\partial \mathcal{L}}{\partial L_t(i)} &= -W_t + MC_t A_t - \lambda_t^H + \theta \Delta_{t,t+1} (1 - pr^F) \lambda_{t+1}^H = 0 \\
\frac{\partial \mathcal{L}}{\partial H_t(i)} &= -P_t hc_t + \lambda_t^H = 0
\end{aligned}
$$

Optimal price:

$$
P_t^*(i) = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{n=0}^{\infty} \theta^n \Delta_{t,t+n} \left(\frac{P_t^*(i)}{P_{t+n}}\right)^{-\varepsilon} Y_{t+n} MC_{t+n}}{\sum_{n=0}^{\infty} \theta^n \Delta_{t,t+n} \left(\frac{P_t^*(i)}{P_{t+n}}\right)^{-\varepsilon} Y_{t+n}}
$$

Nominal marginal cost function:

$$
MC_t = \frac{W_t + P_t hc_t - \frac{\theta}{1+i_t} (1 - pr^F) P_{t+1} hc_{t+1}}{A_t}
$$

And the real marginal cost function:

$$
mc_t = \frac{w_t + hc_t - \theta \frac{1+\pi_{t+1}}{1+i_t} (1 - pr^F) hc_{t+1}}{A_t}
$$

Aggregate labor demand function:

$$
L_t = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t^*(i)}{P_t}\right)^{-\varepsilon} di
$$

Households

Households - beyond the intertemporal optimization - have to decide about their labor market status (being employed or unemployed), outcomes have different pay-offs.

1. Pay-off after being employed:

$$
V_t^W = \underbrace{w_t - \Psi C_t^{\sigma} L_t^{\varphi}}_{\text{Surplus}} + \beta \left\{ \left(\underbrace{1 - pr^F + pr^F pr_{t+1}^H}_{\text{Not fired and fired but hired}} \right) V_{t+1}^W + \underbrace{pr^F (1 - pr_{t+1}^H)}_{\text{Fired and not hired}} V_{t+1}^U \right\}
$$

2. Pay-off after being unemployed:

$$V_t^U = \underbrace{w_t^U}_{\text{Benefit}} + \beta\left\{\underbrace{(1 - pr_{t+1}^H)}_{\text{Not hired}} V_{t+1}^U + \underbrace{pr_{t+1}^H}_{\text{Hired}} V_{t+1}^W\right\}$$

Wage bargaining

Households and firms choose such a common wage, that maximizes their joint welfare function:

$$\max_{w_t} \underbrace{\left(V_t^W - V_t^U\right)^\nu}_{\text{Gain of emp.}} \underbrace{hc_t^{1-\nu}}_{\text{Cost of hiring}}$$

where ν expresses the bargaining power of the households.

First order condition (and optimal wage contract):

$$\nu\left(V_t^W - V_t^U\right)^{\nu-1} hc_t^{1-\nu}\frac{\partial V_t^W}{\partial w_t} + (1-\nu)\left(V_t^W - V_t^U\right)^\nu hc_t^{-\nu}\frac{\partial hc_t}{\partial w_t} = 0$$

Based on the marginal cost function and the pay-off after the employment, the partial derivatives are:

$$\frac{\partial V_t^W}{\partial w_t} = 1 \quad \text{and} \quad \frac{\partial hc_t}{\partial w_t} = -1$$

Substituting out the partial derivatives and some rearrangements:

$$\begin{aligned}
\nu\left(V_t^W - V_t^U\right)^{\nu-1} hc_t^{1-\nu} - (1-\nu)\left(V_t^W - V_t^U\right)^\nu hc_t^{-\nu} &= 0 \\
\nu\left(V_t^W - V_t^U\right)^{-1} hc_t - (1-\nu) &= 0 \\
V_t^W - V_t^U &= \frac{\nu}{1-\nu} hc_t
\end{aligned}$$

Implicit labor supply curve

We can express the difference of the two pay-offs

$$V_t^W - V_t^U = w_t - w_t^U - \Psi C_t^\sigma L_t^\varphi + \beta(1 - pr^F)(1 - pr_{t+1}^H)(V_{t+1}^W - V_{t+1}^U)$$

Plugging back the optimal wage contract:

$$\frac{\nu}{1-\nu} hc_t = w_t - w_t^U - \Psi C_t^\sigma L_t^\varphi + \beta(1 - pr^F)(1 - pr_{t+1}^H)\frac{\nu}{1-\nu} hc_{t+1}$$

Model equations

1. Dynamic IS-curve (after substituting out consumption)

$$Y_t^{-\sigma} = \beta E_t Y_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \frac{\xi_t^C}{\xi_{t+1}^C}$$

2. Nash wage equation (after substituting out consumption)

$$\frac{\nu}{1-\nu} hc_t \quad = \quad w_t - w_t^U - \Psi Y_t^{\sigma} L_t^{\varphi} + \beta(1-pr^F)(1-pr_{t+1}^H)\frac{\nu}{1-\nu} hc_{t+1}$$

3. Labor demand

$$L_t = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t^*(i)}{P_t}\right)^{-\varepsilon} di$$

4. Marginal cost function:

$$mc_t = \frac{1}{A_t}\left(w_t + hc_t - \theta \frac{1+\pi_{t+1}}{1+i_t}(1-pr^F)hc_{t+1}\right)$$

5. Unemployment

$$U_t = 1 - L_t$$

6. Hiring probability

$$pr_t^H \quad = \quad \frac{L_t - (1-pr^F)L_{t-1}}{U_{t-1} + pr^F L_{t-1}}$$

7. Hiring cost

$$hc_t \quad = \quad \kappa (pr_t^H)^{\alpha_{HC}}$$

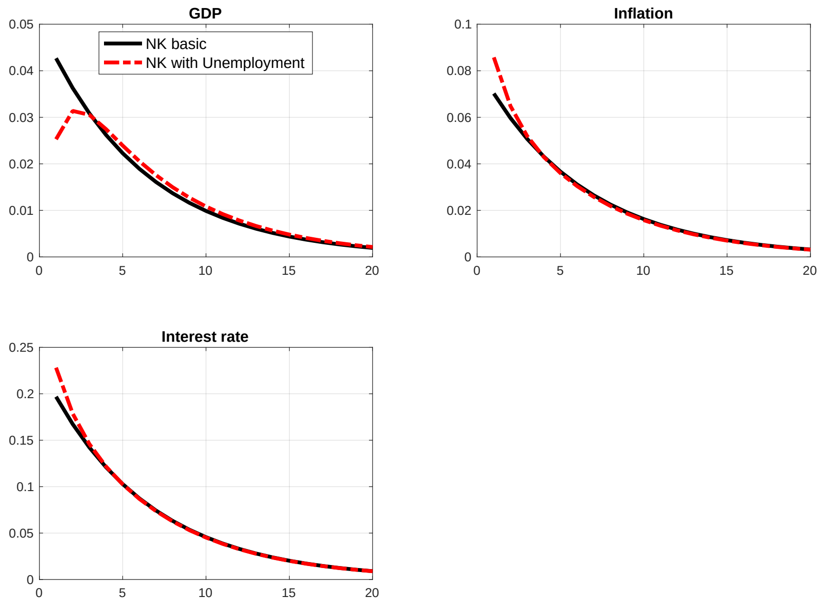
8. New-Keynesian Phillips curve:

$$\pi_t \quad = \quad \kappa \hat{mc}_t + \beta \pi_{t+1} + \xi_t^{\pi}$$

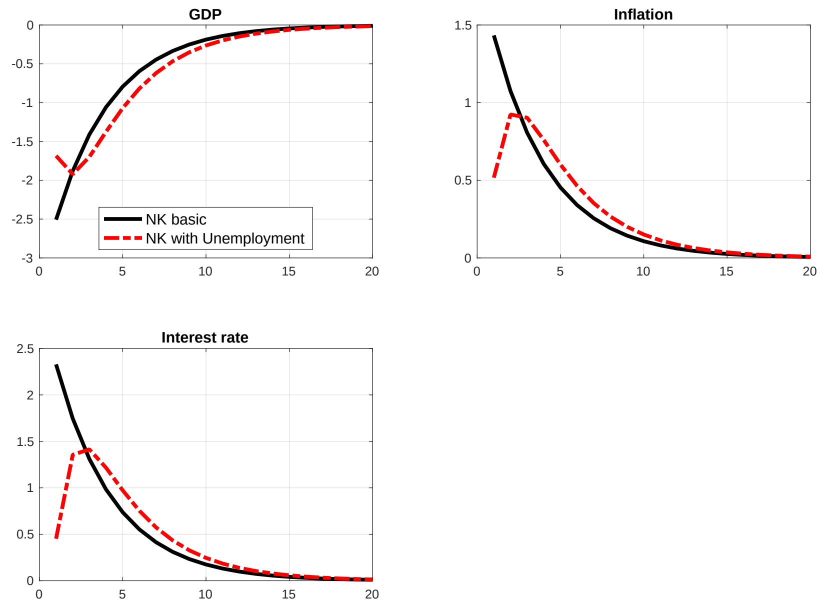
9. Taylor rule:

$$(1+i_t) \quad = \quad \frac{1}{\beta}(1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_x} e^{\xi_t^i}$$

Demand shock



Supply shock (cost-push shock)



What do we learn from this?

1. Hump-shaped reaction in aggregate demand
2. Cost-push shock: the hiring cost are lower (less tight labor market), partly offset the inflationary pressure
3. Potential next step: merge the *wage rigidity* (makes the marginal cost more persistent) and *unemployment* (makes the aggregate demand more hump-shaped)