INSTITUTE FOR CAPACITY DEVELOPMENT

Lecture 2 – The Basic New-Keynesian Model

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Course on DSGE Models, Rwanda

Learning Objectives

- Understand the key assumptions and features of the *basic* New-Keynesian (NK) model
- Analyze the transmission mechanism of shocks in the NK model
- Code, solve, and simulate the NK model in Dynare (W-2)

Outline

- The basic NK model
- Workshop: How to code, solve, and simulate the NK model in Dynare

Why a New-Keynesian (NK) Model?

- The RBC model is the backbone of DSGE models, but monetary policy does not play a role

- The foundations and implications of the RBC model are at odds with empirical evidence that
  - Nominal prices and wages are sticky
  - Monetary policy shocks are non-neutral

Evidence of Sticky Nominal Prices

Evidence: Frequency of price change

		Frequency of Price Changes (%)		Duration of Fixed Prices (months)	
Study	**Country**	**Average**	**Median**	**Average**	**Median**
Aucremanne and Dhyne (2004)	Belgium	16.9	13.3	5.4	7.0
Baharad and Eden (2006)	Israel	24.0	21.0	3.6	4.2
Baudry et al (2004)	France	18.9	14.9	4.8	6.2
Bils and Klenow (2004)	USA	26.1	20.9	3.3	4.3
Dhyne et al (2006)	Europe	15.1		6.1	
Gagnon (2009)	Mexico	30.4 - 36.6		2.2 - 2.8	
Klenow and Kryvstov (2008)	USA	29.3		2.9	
Medina et al (2007)	Chile	46.1	33.3	1.6	2.5
Nakamura and Steinsson (2008)	USA	21.1	8.7	4.2	11.0
Nunes (2006)	Brazil	40.3		1.9	

Non-neutrality of Monetary Policy Shocks

- What happens after a monetary policy shock?

- Christiano, Eichenbaum, and Evans (1999) find that in response to a contractionary monetary policy
  - The short-term interest rate (Fed Fund Rate) increases
  - Output and employment decline
  - Prices respond very slowly
  - Wages fall but little

Response to a Monetary Policy Shock

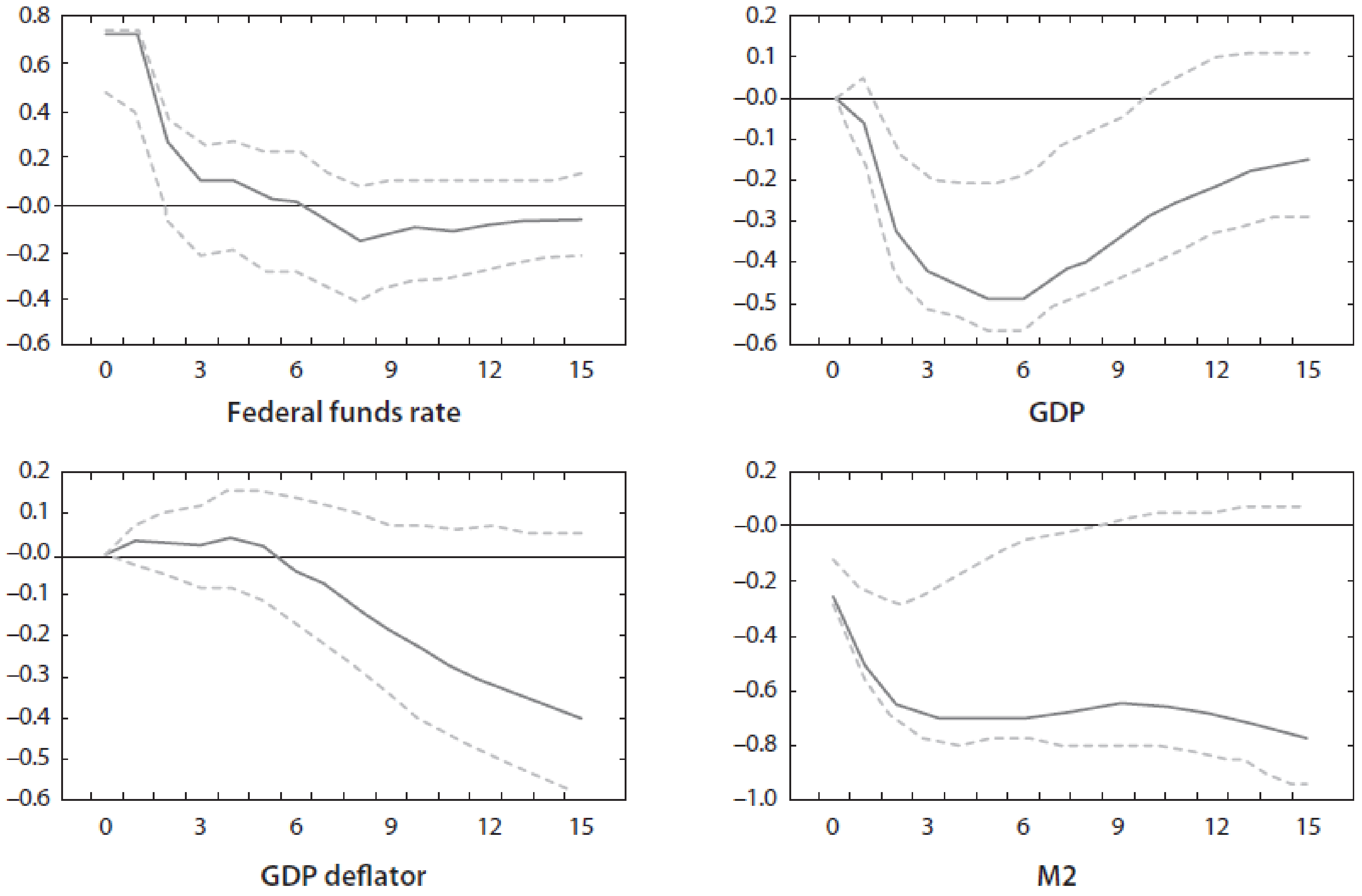


Figure 1.1. Estimated Dynamic Response to a Monetary Policy Shock
Source: Christiano, Eichenbaum, and Evans (1999).

Why a NK Model?

- To address the conflict between theoretical predictions (e.g., the classical monetary model) and empirical evidence

- The NK approach combines the DSGE structure of the RBC model with two key assumptions:
  - Monopolistic competition
  - Nominal price rigidities

- In the short run, output is demand-determined and nominal interest rates are not matched one-for-one with changes in expected inflation, leading to changes in real interest rate, which generates fluctuations in consumption and investment and therefore in output

The Basic NK Model in a Nutshell

1) The Aggregate Supply (AS) or New-Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t$$

2) The Aggregate Demand (AD) or Dynamic IS Equation (DISE)

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n)$$

with $r_t^n = \rho + (1-\rho_a)\sigma\psi_{ya}^n a_t + (1-\rho_z) z_t$

3) A Taylor rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

Three shocks: $s_t = v_t, a_t, z_t$ (to monetary policy, productivity, and preferences)

$$s_t = \rho_s s_{t-1} + \varepsilon_t^s, \quad \rho_s \in (0,1), \quad \varepsilon_t^s \sim N(0, \sigma_s^2)$$

Households' Preferences

HH's problem

$$\underset{C_t, C_t(j), B_t, N_t}{\text{Max}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t; Z_t) \qquad s.t.$$

- Aggregate consumption (over a continuum of goods on $j \in [0,1]$)

$$C_t = \left(\int_0^1 C_t\,(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Budget constraint

$$\int_0^1 P_t\,(j) C_t(j) dj + B_t - B_{t-1} \leq i_{t-1} B_{t-1} + W_t N_t + D_t$$

- The non-Ponzi game constraint

$$\lim_{T \to \infty} \mathbb{E}_t \left\{ \left[\beta^{T-t} \frac{\lambda_T}{\lambda_t} \right] B_T \right\} \geq 0$$

Households' Preferences

How to solve this problem? Two steps:

- Static problem: Solve for the optimal allocation of consumption $C_t(j)$
- Dynamic problem: Solve for the intertemporal consumption decisions C_t

Static Problem: Optimal Intra-temporal Consumption Allocation

- HH's consumption allocation problem: cost minimization problem

$$\underset{C_t(j)}{\text{Min}} \int_0^1 P_t\,(j) C_t(j) dj \qquad \text{s.t.} \quad C_t = \left(\int_0^1 C_t\,(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- First-order conditions (FOCs) lead to demand schedule

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t \qquad (1)$$

- The price index

$$P_t \equiv \left[\int_0^1 P_t\,(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$$

Dynamic Problem: Optimal Intertemporal Consumption

$$\underset{C_t, B_t, N_t}{\text{Max}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t; Z_t) \qquad s.t.$$

$$P_t C_t + B_t - B_{t-1} = i_{t-1} B_{t-1} + W_t N_t + D_t$$

FOCs:

- Intra-temporal condition

-

$$\underbrace{\frac{U'_{n,t}}{U'_{c,t}}}_{MRS} = \underbrace{\frac{W_t}{P_t}}_{\substack{Real \\ wage}}$$

In other words, the disutility from one hour of labor should be compensated by the utility we gain from the goods we can purchase with the salary earned from that hour of labor supply

$$U'_{n,t} = \frac{W_t}{P_t} U'_{c,t}$$

Dynamic Problem: Optimal Intertemporal Consumption

$$\underset{C_t, B_t, N_t}{\text{Max}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t; Z_t) \qquad s.t.$$

$$P_t C_t + B_t - B_{t-1} = i_{t-1} B_{t-1} + W_t N_t + D_t$$

FOCs:

- Intertemporal condition

$$U'_{c,t} = \beta \mathbb{E}_t \left\{ U'_{c,t+1} (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \Longrightarrow U'_{c,t} = \beta \mathbb{E}_t \left\{ U'_{c,t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \right\}$$

where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

In other words, at equilibrium, the marginal utility of today's consumption should be equal to the marginal utility we would gain from consuming tomorrow, if we choose to save today and consume later.

$$\frac{1}{P_t} U'_{c,t} = \beta \mathbb{E}_t \left\{ U'_{c,t+1} \frac{1 + i_t}{P_{t+1}} \right\}$$

First Order Conditions

- Utility function

$$U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t$$

with $z_t = \log(Z_t) = \rho_z z_{t-1} + \varepsilon_t^z, \rho_z \in (0,1), \varepsilon_t^z \sim N(0, \sigma_z^2)$

- Labor supply

$$\frac{N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}$$

- Euler equation

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \left(\frac{Z_{t+1}}{Z_t}\right) \right\}$$

First Order Conditions

- Log-linearizing the FOCs and rearranging yields

Labor supply

$$n_t = \frac{1}{\varphi}(w_t - p_t) - \frac{\sigma}{\varphi} c_t (2)$$

Euler Equation (Dynamic IS Curve)

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z) z_t (3)$$

or,

$$c_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}[(\mathbb{E}_t r_{t+k} - \rho) - (1-\rho_z)\rho_z^k z_t]$$

where $\rho \equiv -\log(\beta)$, $x_t \equiv \log(X_t)$, $i_t \cong \log(1+i_t)$, $\pi_t \cong \log(1+\pi_t)$, $z_t \equiv \log(Z_t) = \rho_z z_{t-1} + \varepsilon_t^z$

Firms' Technology and Pricing

- A continuum of firms that only use labor to produce differentiated goods indexed by $j \in [0,1]$

$$Y_t(j) = A_t N_t(j)^{1-\alpha} (4)$$

with $a_t = \log(A_t) = \rho_a a_{t-1} + \varepsilon_t^a, \rho_a \in (0,1), \varepsilon_t^a \sim N(0, \sigma_a^2)$

- Face an isoelastic demand as described by (1)
- Face price rigidity a la Calvo (1983)

Firms' Technology

- A continuum of firms that only use labor to produce differentiated goods indexed by $j \in [0,1]$

$$Y_t(j) = A_t N_t(j)^{1-\alpha} (4)$$

with $a_t = \log(A_t) = \rho_a a_{t-1} + \varepsilon_t^a, \rho_a \in (0,1), \varepsilon_t^a \sim N(0, \sigma_a^2)$

- Firms profit maximization problem for factor demand:

$$\min_{N_t(j)} \left[N_t(j) \frac{W_t}{P_t} + \lambda_t^F (Y_t(j) - A_t N_t(j)^{1-\alpha}) \right]$$

$$\frac{W_t}{P_t} = mc_t \frac{\partial Y_t(j)}{\partial N_t(j)}, \qquad mc_t \equiv \lambda_t^F$$

Real marginal cost can be defined as a real cost (wage) of one unit labor divided by the marginal product of the unit of labor

$$mc_t \equiv \frac{W_t/P_t}{MPN_t} = \frac{W_t/P_t}{(1-\alpha) A_t N_t^{-\alpha}}$$

Price setting

- Firms face an isoelastic demand as described by (1)

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t \quad (1)$$

Let's first assume that **prices are flexible**, firms can easily change prices every time, so they solve static profit maximization problem:

$$\max_{P_t(j)} [P_t(j) Y_t(j) - \boldsymbol{\psi}(Y_t(j))]$$

$P_t(j) = P_t^*$, which solves this optimization problem, taking demand function into consideration. Having in mind that mc_t is the first derivative of $\boldsymbol{\psi}$ deflated by P_t :

$$\frac{\partial}{\partial P_t^*}\left[P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t - \boldsymbol{\psi}\left(\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t\right)\right]^{-\varepsilon} = 0 \rightarrow \frac{P_t^*}{P_t} = \mathcal{M} mc_t\,, \qquad \mathcal{M} = \frac{\varepsilon}{\varepsilon - 1} \quad \text{is a price markup}$$

The higher the elasticity of substitution (ε) is, market is more competitive

$$\lim_{\varepsilon \to \infty} P_t^* = P_t mc_t$$

Price setting, when prices are sticky

- Face price rigidity a la Calvo (1983)
  - Only some firms change the price to P_t^*
  - The others keep the price at $P_{t-1}(j)$
  - Average duration of a price is $\frac{1}{1-\theta}$

$$P_t(j) = \begin{cases} P_{t-1}(j) & \text{with probability } \theta \\ P_t^* & \text{with probability } 1-\theta \end{cases}$$

- When the firms have a chance to update prices, they took into account that in the future, they might not be able to re-optimize it with the probability θ^k. Being Q the subjective discount factor of nominal cash flows, i.e., $Q_{t,t+k} \equiv \beta^k \frac{C_t P_t}{C_{t+k} P_{t+k}}$:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} [P_t(j) Y_{t+k}(j) - \boldsymbol{\psi}(Y_{t+k}(j))]$$

Considering the isoelastic demand schedule for every firm, the FOC with respect to optimal prices can be written as:

$$FOC\text{:}\ \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \{Q_{t,t+k} Y_{t+k}(j)(P_t(j) - \boldsymbol{\psi}_{Y,t+k}) \frac{\partial Y_{t+k}(j)}{\partial P_t(j)}\} = 0$$

Price setting, when prices are sticky

Re-arranging the first order condition, and because of the symmetry between firms:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t\left[Q_{t,t+k} Y_{t+k} \boldsymbol{\psi}_{Y,t+k}\right]}{\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t\left[Q_{t,t+k} Y_{t+k}\right]}$$

Replacing the demand schedule, expressing the optimal reset price in terms of real marginal costs, and log-linearizing around a zero-inflation steady state we arrive to the following expression:

$$\hat{p}_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\widehat{mc}_{t+k} + \hat{p}_{t+k})$$

Where μ is the steady-state markup, equal to $\log(\frac{\varepsilon}{\varepsilon-1})$, and the steady state real marginal cost is given by $\frac{\varepsilon-1}{\varepsilon}$.

According to this result, optimal prices reflects the full path of future marginal costs, given the impossibility of resetting every period. Or put another way:

$$\hat{p}_t^* = \mu + (1 - \beta\theta)(\widehat{mc}_t + \hat{p}_t) + \beta\theta \mathbb{E}_t \hat{p}_{t+1}^*$$

Aggregate Price

- Remember that not all the firms are able to update price optimally. θ portion of firms keep prices unchanged $P_t(j) = P_{t-1}(j)$. In a symmetric equilibrium $P_{t-1}(j) = P_t$.
- Recall the price index derived in HH's intra-temporal optimization:

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$$

- Let's transform variables into exponential functions, considering that the SS is characterized by a zero inflation: $P_t = Pe^{\hat{p}_t}$; $P_{t-1} = Pe^{\hat{p}_{t-1}}$; $P_t^* = Pe^{\widehat{p^*}_t}$. And let's use also that $e^x \sim 1 + x$. Replacing and simplifying:

$$p_t = \theta \hat{p}_{t-1} + +(1-\theta)\hat{p}_t^*$$

- And regrouping terms:

$$\pi_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$$

- Combining this to the optimal pricing equation derived above, we get NK Philips Curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{mc}_t \quad \rightarrow \quad \pi_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \, \widehat{mc}_{t+k}$$

The NKPC Under the Rotemberg (1982) Approach

A firm faces quadratic price adjustment costs and chooses price P_t^* to

$$\underset{P_t^*}{\mathrm{Max}}\, \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{t,t+1} \left[P_t^* Y_t - \Psi_t(Y_t) - \frac{\zeta}{2} \left(\frac{P_t^*}{P_{t-1}^*} - 1 \right)^2 P_t C_t \right]$$

subject to the demand schedule $Y_t = \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon} C_t$, where $\Psi_t(Y_t)$ is the cost function, and $Q_{t,t+1} \equiv \beta^t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}$ is the stochastic discount factor

The solution gives a similar NKPC to that under Calvo's approach

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \left(\frac{\varepsilon - 1}{\zeta} \right) \widehat{mc}_t$$

where ζ measures the degree of price stickiness

Equilibrium

- Good market clearing condition

$$Y_t(j) = C_t(j) \qquad\qquad Y_t \equiv \left(\int_0^1 Y_t\,(j)^{1-\frac{1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} = C_t$$

- Labor market clearing condition

$$N_t = \int_0^1 N_t\,(j) dj$$

- Given equations (1) and (4)

$$N_t = \int_0^1 \left(\frac{Y_t(j)}{A_t} \right)^{\frac{1}{1-\alpha}} dj = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} dj$$

- Log-linearize around SS (where $\int_0^1 \left(\frac{P_t(\mathrm{j})}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} dj = 0$ up to a 1st-order approximation)

$$y_t = a_t + (1-\alpha) n_t \tag{9}$$

Equilibrium

- Real marginal cost of production

$$MC_t \equiv \frac{W_t}{P_t MPN_t} = \frac{W_t}{P_t(1-\alpha)A_t N_t^{-\alpha}}$$

- Log-linearize it

$$mc_t = (w_t - p_t) - mpn_t$$

$$mc_t = (w_t - p_t) - (a_t - \alpha n_t) - \log(1-\alpha) \quad (10)$$

- Use equations (2), (9), and (10), and $c_t = y_t$, to derive

$$mc_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \quad (11)$$

Natural level of the economy

- Let's recall the marginal price setting behavior in frictionless economy:

$$\frac{P_t^*}{P_t} = \mathcal{M}mc_t$$

Considering that everyone can update price every period, $P_t = P_t^*$, therefore

$$\mathcal{M}mc_t = 1 \qquad \rightarrow \qquad \widehat{mc}_t = 0$$

Applying this to the eq(11), of the log-linearized marginal cost function, and solving for $y_t = y_t^n$:

$$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t^n - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) = 0$$

From the DISE, applying $y_t = y_t^n \; \forall t$, and solving for real interes rate $r_t = r_t^n$

$$y_t^n = \mathbb{E}_t\{y_{t+1}^n\} - \frac{1}{\sigma} (r_t^n - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t$$

The Dynamic IS Equation (DISE)

Now we can rewrite equilibrium conditions in terms of output gap.

- DISE:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

- Subtracting DISE for natural level of output

$$y_t - y_t^n = \mathbb{E}_t\{y_{t+1} - y_{t+1}^n\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho - \sigma\mathbb{E}_t\{y_{t+1}^n - y_t^n\}\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right) \qquad (14)$$

where $r_t^n = \rho + \sigma\psi_{ya}^n\mathbb{E}_t\{\Delta a_{t+1}\} \quad = \rho - (1-\rho_a)\sigma\psi_{ya}^n a_t + (1-\rho_z)z_t$ is the natural real interest rate and

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \qquad \rho_a \in (0,1), \qquad \varepsilon_t^a \sim N(0,\sigma_a^2)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \qquad \rho_z \in (0,1), \qquad \varepsilon_t^z \sim N(0,\sigma_z^2)$$

The Dynamic IS Equation (DISE)

- From the DISE (14) and $\lim_{T\to\infty} \mathbb{E}_t\{\tilde{y}_{t+T}\} = 0$, we obtain

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty} \mathbb{E}_t\{r_{t+k} - r_{t+k}^n\}$$

where $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ is the real interest rate

- The DISE determines the output gap given the expected paths for the natural and actual real interest rates

Monetary Policy

- The NKPC and DISE determine recursively inflation and the output gap, given expected paths for the (exogenous) natural rate and the actual real interest rate

- To close the model, we need one more equation to determine how the nominal interest rate i_t evolves over time, i.e., how monetary policy is conducted

- Real variables in the economy cannot be determined independently of monetary policy – monetary policy is non-neutral!

Monetary Policy: A Taylor Rule

- A Taylor rule, i.e., an interest rate feedback rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \tag{15}$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad \rho_v \in (0,1), \quad \varepsilon_t^v \sim N(0, \sigma_v^2)$$

- Policy parameters ϕ_π and ϕ_y are selected by monetary authority

The Basic Three-Equation NK Model

1) The Aggregate Supply (AS) or New-Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t$$

2) The Aggregate Demand (AD) or Dynamic IS Equation (DISE)

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right)$$

with $r_t^n = \rho - (1 - \rho_a)\sigma\psi_{ya}^n a_t + (1 - \rho_z) z_t$

3) A Taylor rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

Three shocks: $s_t = \{v_t, a_t, z_t\}$ (to monetary policy, productivity and preferences)

$$s_t = \rho_s s_{t-1} + \varepsilon_t^s, \quad \rho_s \in (0,1), \quad \varepsilon_t^s \sim N(0, \sigma_s^2)$$

Solving the NK Model

- Substitute the Taylor rule into the DISE, then we can write the basic NK model as

$$\begin{bmatrix} \mathbb{E}_t\{\tilde{y}_{t+1}\} \\ \mathbb{E}_t\{\pi_{t+1}\} \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} + \mathbf{B}_T(\hat{r}_t^n - v_t)$$

where $\mathbf{A}_T$ is 2×2 matrix and $\mathbf{B}_T$ is 2×1 matrix of exogenous parameters

- We solve the system numerically. More details in the W-2

- We can also append a demand for money $m_t - p_t = y_t - \eta i_t$

Solving the NK Model

- Given parameter values, for a unique equilibrium, $\mathbf{A}_T$ should have both eigenvalues outside the unit circle (explosive eigenvalues), since $\tilde{y}_t$ and π_t are jump variables —the Taylor principle is important here!
- For the rule in (15) then the condition that guarantees a unique equilibrium is

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

- What is the Taylor principle? Usually, it is expressed as $\phi_\pi > 1$ — i.e., respond strong enough to inflationary pressures
- Why? For simplicity, and to grasp the intuition, assume

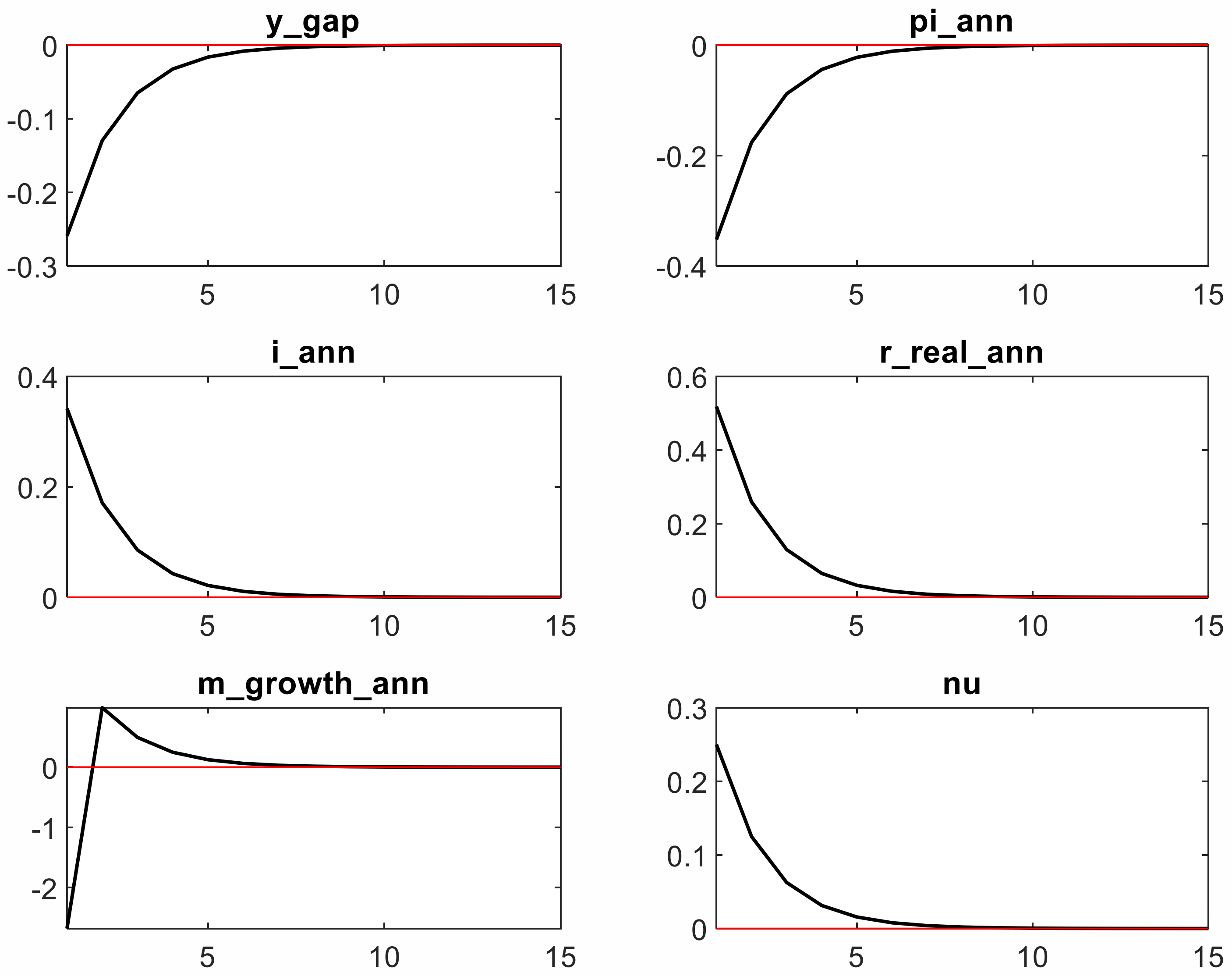
$$i_t = \rho + \phi_\pi \mathbb{E}_t\{\pi_{t+1}\}$$

Solving the NK Model

- Assume $i_t = \rho + \phi_\pi \mathbb{E}_t\{\pi_{t+1}\}$ with ϕ_π<1 then the Taylor rule induces macro instability
  - If expectations are such that $\mathbb{E}_t\{\pi_{t+1}\}\uparrow \Rightarrow$ CB, by Taylor rule, $i_t\uparrow \Rightarrow r_t\downarrow$, since ϕ_π<1 $\Rightarrow \tilde{y}_t\uparrow$ by DISE $\Rightarrow \pi_t \uparrow$ by NKPC, validating original expectations of $\mathbb{E}_t\{\pi_{t+1}\}\uparrow$

- Assume $i_t = \rho + \phi_\pi \mathbb{E}_t\{\pi_{t+1}\}$ with ϕ_π>1 then the Taylor rule induces macro stability
  - If expectations are such that $\mathbb{E}_t\{\pi_{t+1}\}\uparrow \Rightarrow$ CB, by Taylor rule, $i_t\uparrow \Rightarrow r_t \uparrow$, since ϕ_π>1 $\Rightarrow \tilde{y}_t\downarrow$ by DISE $\Rightarrow \pi_t \downarrow$ by NKPC, invalidating original expectations of $\mathbb{E}_t\{\pi_{t+1}\}\uparrow$

The Effects of a Monetary Policy Shock $\nu\uparrow$

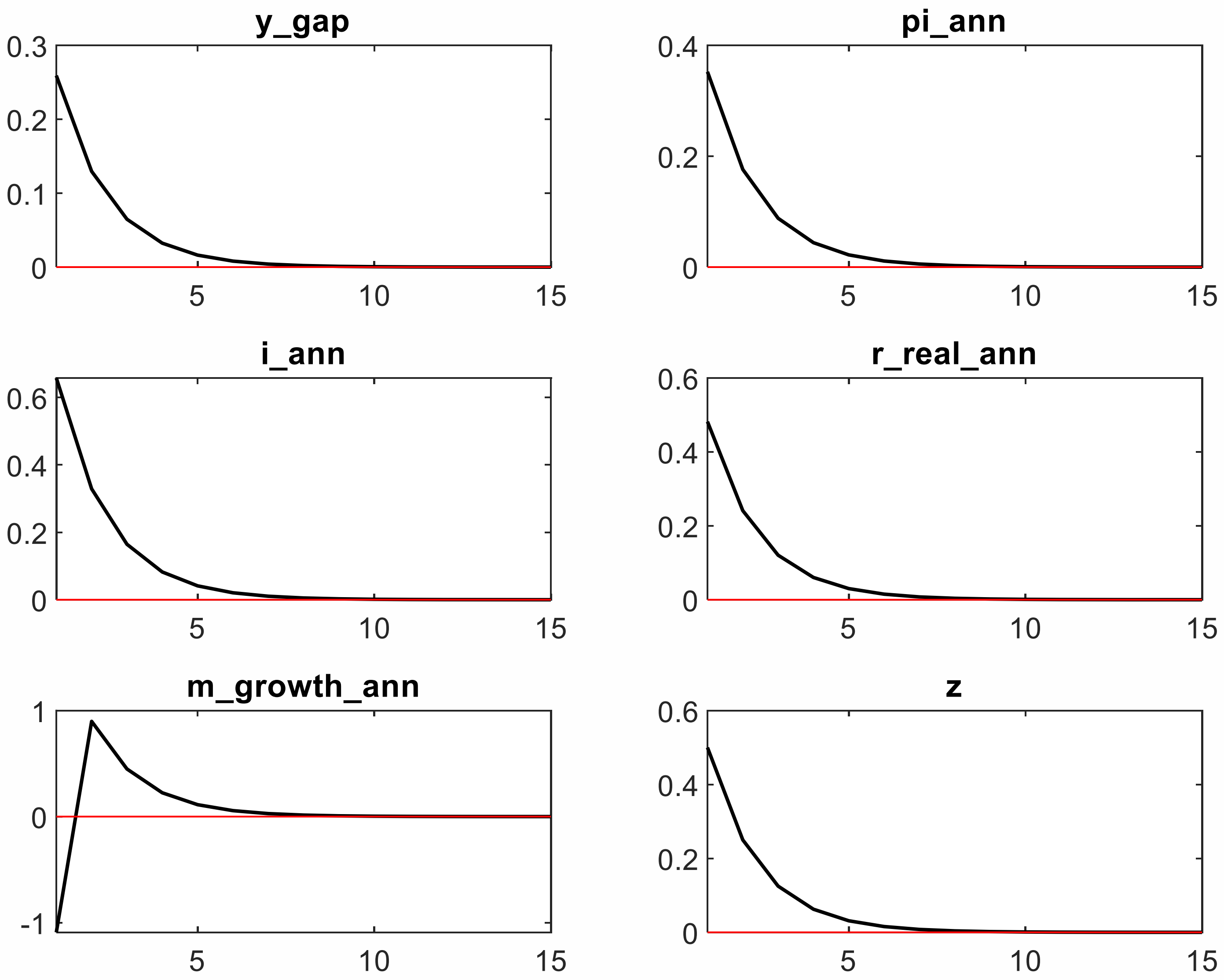


The Effects of a Monetary Policy Shock $v \uparrow$

Monetary tightening $v \uparrow$

- Output gap $\downarrow$, inflation $\downarrow$, nominal and real interest rates $\uparrow$ $(r_t = i_t - \mathbb{E}_t\{\pi_{t+1}\})$
- Transmission: $v \uparrow \xRightarrow{\text{Taylor Rule}} i_t \uparrow \xRightarrow{\text{DISE}} \tilde{y}_t \downarrow \xRightarrow{\text{NKPC}} \pi_t \downarrow$
- Qualitatively consistent with VAR evidence
- Matching the quantitative features of empirical IRFs requires enriching the basic NK model in several dimensions

The Effects of a Preference Shock $z \uparrow$

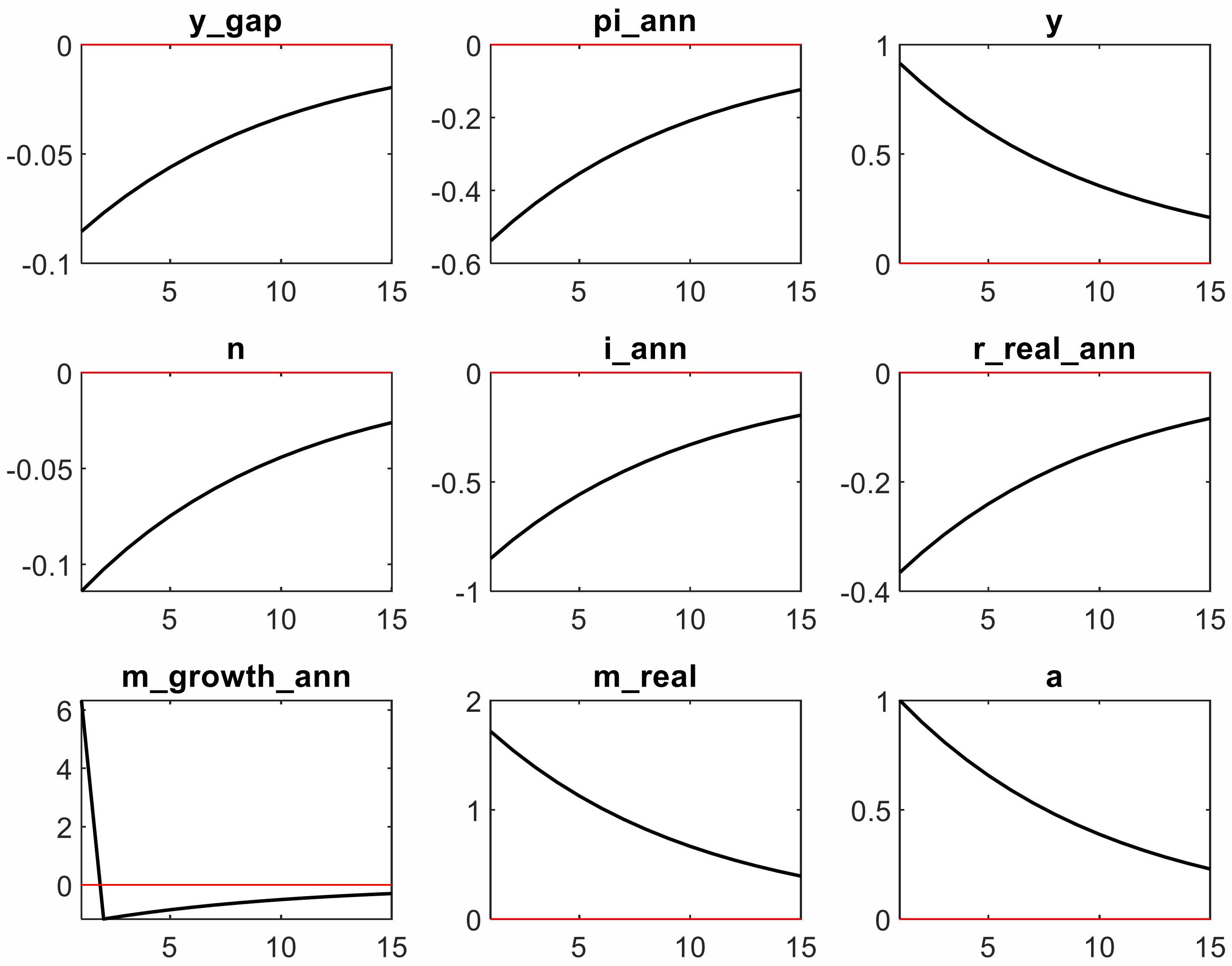


The Effects of a Preference Shock $z \uparrow$

A preference shock $z \uparrow$ implies an increase in the weight that households put in current relative to future utility, which induces an increase in consumption (aggregate demand)

- Output gap ↑, inflation ↑, nominal and real interest rates ↑
- Transmission: $z \uparrow \xrightarrow[\text{Via } r_t^n]{\text{DISE}} \tilde{y}_t \uparrow \xrightarrow{\text{NKPC}} \pi_t \uparrow \xrightarrow{\text{Taylor Rule}} i_t \uparrow$

The Effects of a Technology Shock $a \uparrow$



The Effects of a Technology Shock $a \uparrow$

Technology improvement $a\uparrow$

- Employment ↓, output gap ↓, inflation ↓, nominal and real interest rates ↓

- Transmission: $a\uparrow \Longrightarrow \begin{cases} y\uparrow \\ \tilde{y}\downarrow\ (y^n\uparrow > y\uparrow) \\ \mathrm{n}\downarrow \end{cases} \xRightarrow{\text{NKPC}} \pi\downarrow \xRightarrow{\text{Taylor Rule}} i\downarrow$

- Employment ↓ (consistent with SVAR evidence of Gali and Rabanal (2004), note that RBC will predict the opposite) but output ↑

How to Introduce "Supply" Shocks in the Basic NK Model?

- Literature makes assumptions on technology, preferences, and labor markets to derive

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\tilde{y}_t$$

- Deviations from this condition, e.g., by movements in nominal wages that push real wages away from their equilibrium values due to frictions in the wage contracting process, may give rise to cost push shocks "u_t"

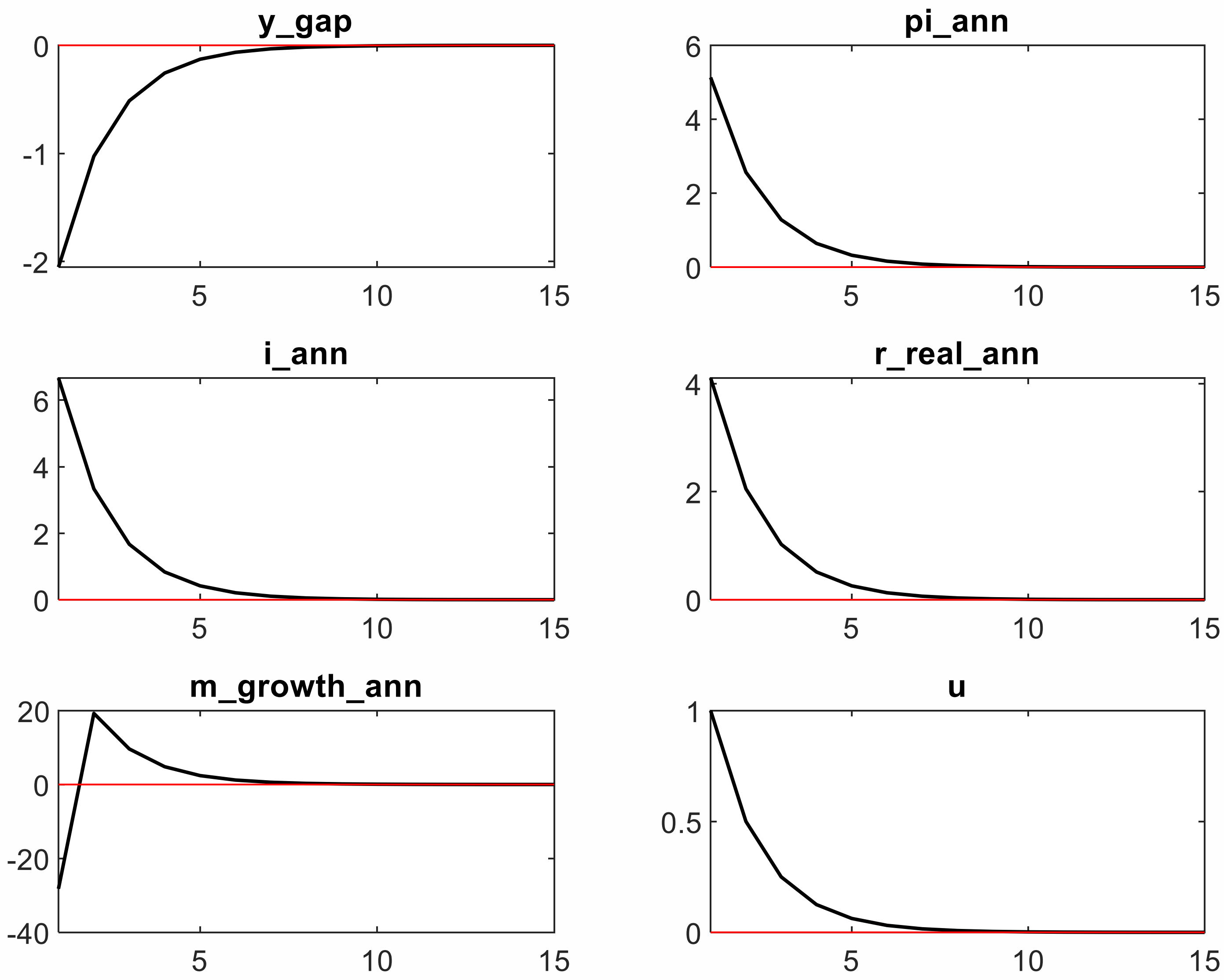
- Then, abstracting from micro-foundations

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t + u_t$$

with

$$u_t = \rho_u z_{t-1} + \varepsilon_t^u, \qquad \rho_u \in (0,1), \qquad \varepsilon_t^u \sim N(0, \sigma_u^2)$$

The Effects of a "Supply" Shock $u \uparrow$



The Effects of "Supply" Shock $u \uparrow$

Cost push shock $u \uparrow$

- Output gap $\downarrow$, inflation $\uparrow$, nominal and real interest rates $\uparrow$
- Transmission: $u \uparrow \xRightarrow{\text{NKPC}} \pi_t \uparrow \xRightarrow{\text{Taylor Rule}} i_t \uparrow \xRightarrow{\text{DISE}} \tilde{y}_t \downarrow$

Conclusions

- Recent empirical evidence shows that monetary shocks are non-neutral
- A simple NK model, as a synthesis of the RBC model and short-run nominal price rigidities, provides theoretical foundations for the need of monetary policy as it becomes non-neutral

Thank you!