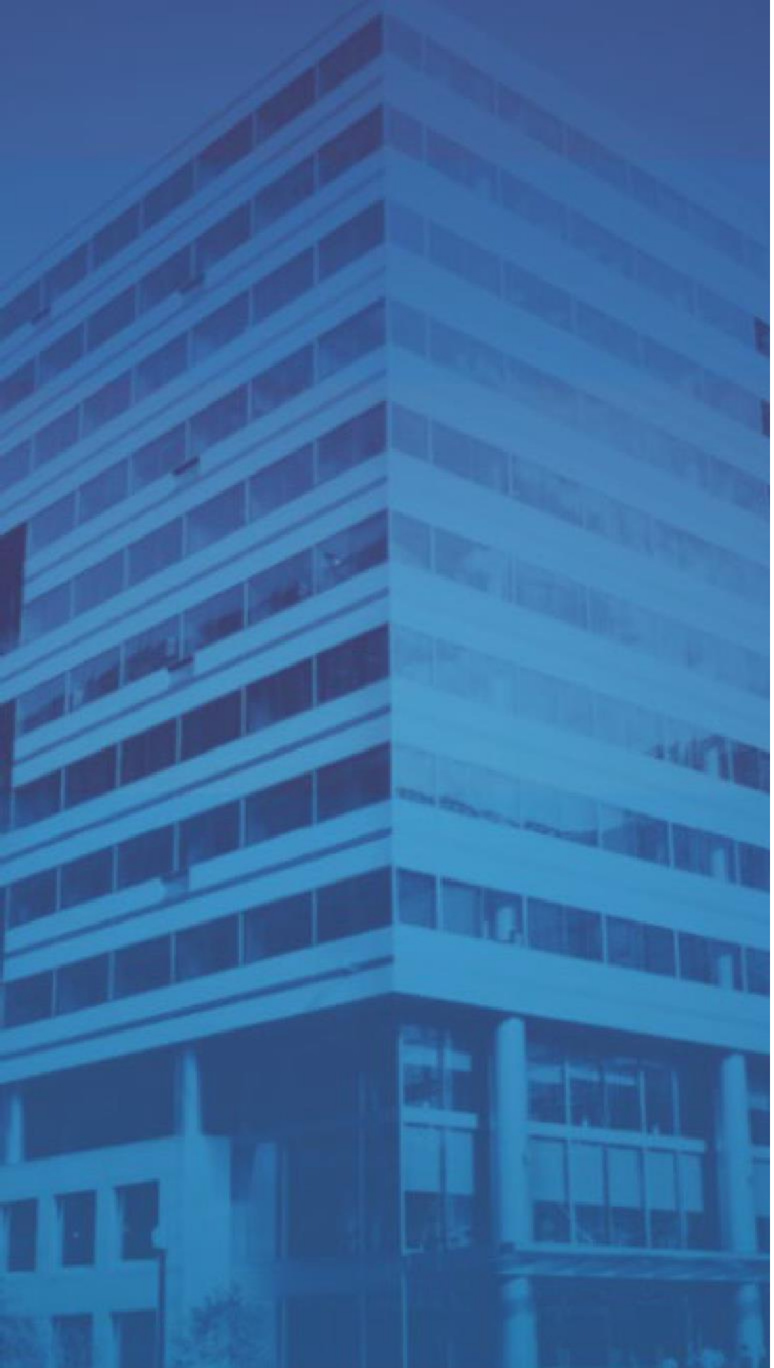

INSTITUTE FOR CAPACITY DEVELOPMENT

Lecture 6: A New Keynesian Model with Financial Frictions

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Course on Monetary and Fiscal Policy Analysis with DSGE Models (OT26.08)

Models without financial frictions

Households

- Consumption
- Investment
- Labor supply

Firms

- Demand for labor
- Demand for capital
- Production

Monetary Policy

- Taylor Rule
- Inflation Target
- Output Gap

Fiscal Policy

- Ricardian Consumers
  - Ricardian Equivalence
- Non-Ricardian Consumers

Households and Firms

- Nominal rigidities
  - Sticky prices
  - Sticky wages
  - Partial indexation to inflation
- Real frictions
  - Investment adjustment costs
  - Variable capital utilization

Shocks

- Monetary policy shocks
- Preference shocks
- Productivity shocks (temporary or permanent)
- Price and wage markup shocks
- Risk premium shocks

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Model without financial friction (I)

- **The typical DSGE model assumes investment adjustment cost. The capital producers maximize their income from capital investments:**

$$\max_{K_{t+n}, Inv_{t+n}} E_t \sum_{n=0}^{\infty} \Theta_{t,t+n} \left\{ R_{t+n}^K K_{t+n-1} - P_{t+n}^{Inv} Inv_{t+n} \right.$$
$$\left. + Q_{t+n} \left(Inv_{t+n} \left(1 - S\left(\frac{Inv_{t+n}}{Inv_{t+n-1}} \right) \right) + (1-\delta) K_{t+n-1} - K_{t+n} \right) \right\}$$

- where the discount factor can be given as $\Theta_{t,t+1} = 1/(1+i_t)$, and the second derivative of the $S(\cdot)$ in the steady-state is denoted by ζ_{Inv}

- Solution of the capital producers problem (after log-linearization)

(i) investment demand equation:

$$inv_t = \frac{1}{1+\beta} inv_{t-1} + \frac{\beta}{1+\beta} E_t inv_{t+1} + \frac{1}{\zeta_{Inv}(1+\beta)} (q_t - p_t^{Inv})$$

(ii) and no-arbitrage conditions in real terms:

$$q_t = \big(1 - \beta(1-\delta)\big) E_t r_{t+1}^K + \beta(1-\delta) E_t q_{t+1} - r_t$$

Model without financial friction (II)

- **In DSGE-models without financial frictions the expected return on capital assets are must be equal with the return on risk-free government bonds:**

$$E_t Ret^K_{t+1} \equiv E_t \frac{R^K_{t+1} + Q_{t+1}(1-\delta)}{Q_t} = 1 + i_t$$

- Modigliani-Miller neutrality theorem: it does not matter which agents are responsible for investment activity and capital accumulation. The financial intermediaries have no role.

- Empirical facts contradict to the theorem:

$$E_t Ret^K_{t+1} > 1 + i_t$$

there is a spread between the risk-free bonds and risky assets.

Models with financial frictions

Financial Markets

Former consensus:

- "price stability is sufficient to ensure macroeconomic stability"
- "the worsening of world financial markets is just a reflection of the worsening economy"

Financial Crisis

- Financial vulnerability
- Feedback loop from financial sector to real economy
- Interlinkages between domestic and international financial markets
- Greater interest in the concept of financial accelerator

Financial Sector

- Credit-market frictions
- Demand for credit
  - BGG(1999): financial accelerator
- Credit supply
  - Gertler and Kiyotaki (2014) and Gertler and Karadi (2011): bank runs equilibrium exists in periods of crisis, with significant contraction in banking and economic activities

Outline

1. Introduction to the financial accelerator
2. A log-linearized model for the closed economy with financial accelerator
3. A log-linearized model for the open economy with financial accelerator
4. A model of Unconventional Monetary Policy

1. Introduction to the financial accelerator

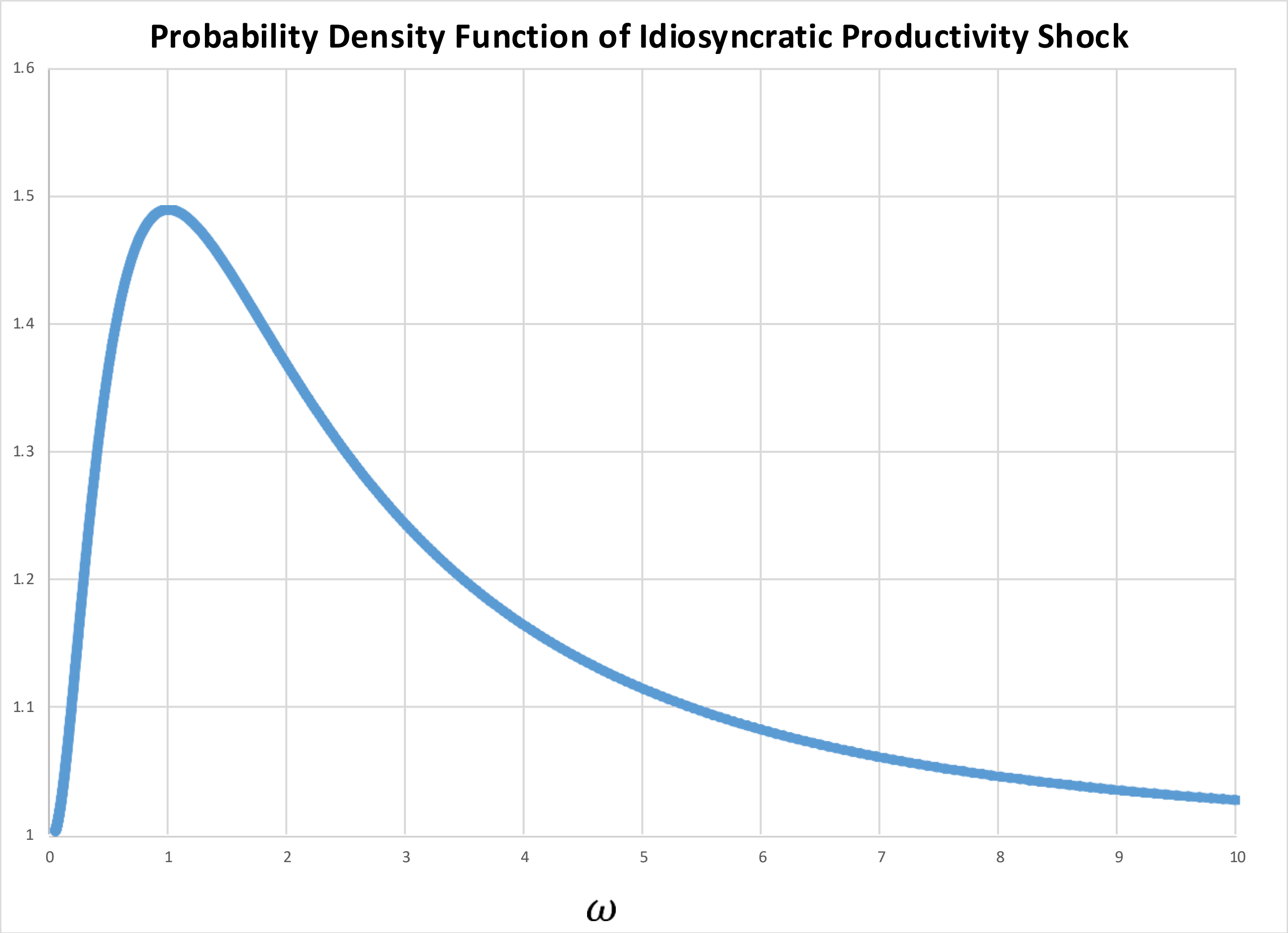
Origins of the financial constraint

- Heterogeneous agents: investors are different from lenders. Principal–agent problem in credit markets.
- Because of information asymmetries, lenders find it costly to verify the underlying financial position of the firm (agency costs).
- Thus, the financial contract between the investor and the lender will take this feature into account, incorporating a premium over the opportunity cost of internal funds to obtain compensation for the agency costs.

Bernanke, Gertler and Gilchrist (1998) (BGG)

- There is a continuum of entrepreneurs, each having net worth equal to $\boldsymbol{N_t}$
- Entrepreneurs combine $\boldsymbol{N_t}$ with new loans $\boldsymbol{Loan_t}$ to buy capital $\boldsymbol{Q_t K_t}$, where each unit of capital costs $\boldsymbol{Q_t}$ (Tobin's Q)
- A unit of capital has an average (gross) return of $\boldsymbol{Ret_t^K}$
- After investing in new capital, each entrepreneur receives an uncertain return equal to $\boldsymbol{\omega Ret_{t+1}^K Q_t K_t}$
- $\boldsymbol{\omega}$ is an idiosyncratic productivity shock with density function $\boldsymbol{f(\omega)}$, such that $\boldsymbol{E[\omega] = \int_0^\infty \omega f(\omega) d\omega = 1}$
- **Information asymmetry**: only the entrepreneur observes $\boldsymbol{\omega}$. The lender can observe $\boldsymbol{\omega}$ only after paying the monitoring cost $\boldsymbol{\mu}$

$f(\omega)$



Probability Density Function of Idiosyncratic Productivity Shock
1.6
1.5
1.4
1.3
1.2
1.1
1
0
1
2
3
4
5
6
7
8
9
10
ω

Banks (lenders)

The lender (bank) offers an **incentive-compatible contract** that ensures a particular return for itself such that bank profits are zero. Townsend (1979) showed that in this case a **non-contingent debt contract** becomes optimal

- Lender finances $\boldsymbol{Loan_t = Q_t K_t - N_t}$
- Entrepreneur commits to repay $\boldsymbol{(1 + i_t^e) Loan_t = (1 + i_t^e)(Q_t K_t - N_t)}$ before knowing the realization of the idiosyncratic shock $\boldsymbol{\omega}$
- If the shock is "good", entrepreneur returns $\boldsymbol{(1 + i_t^e) Loan_t}$
- If the shock is "bad", entrepreneur is unable to repay and defaults; the lender seizes whatever assets are left after paying the monitoring costs: $\boldsymbol{(1 - \mu)\omega Ret_{t+1}^K Q_t K_t}$.

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Entrepreneurs

Entrepreneur's decisions: **a value $\bar{\omega}$ exists**, such that

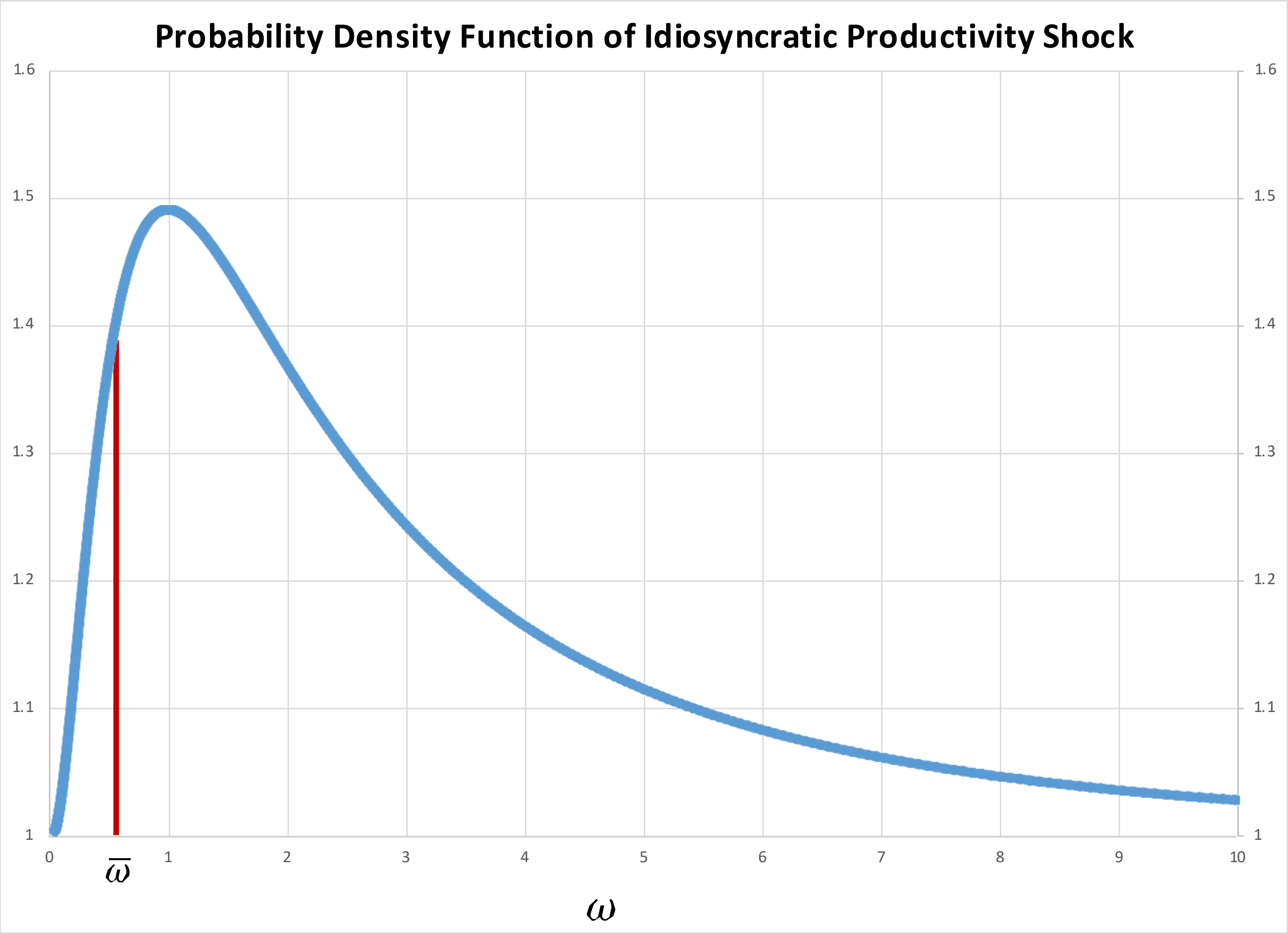
- If $\omega \geq \bar{\omega}$,the entrepreneur pays the agreed amount;
- If $\omega < \bar{\omega}$,the entrepreneur declares default

- Opportunity cost for the lender: safe investments at $(1 + i_t)$
- Thus, $(1 + i_t^e)/(1 + i_t) > 1$ will be a spread or premium on the risky debt
- Threshold $\bar{\omega}$ satisfies:

$$\bar{\omega} Ret_{t+1}^{K} Q_t K_t = (1 + i_t^e) Loan_t = (1 + i_t^e)(Q_t K_t - N_t)$$

- If Q_t, Ret_{t+1}^{K} and N_t are given, a debt contract that determines $(1 + i_t^e)$ and $(1 + i_t^e) Loan_t$ is equivalent to one that determines $\bar{\omega}$ is K_t
- **Thus, the contractual problem can be fully solved considering the demand for credit only!**

$f(\omega)$



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Banks

- **Lenders are perfectly competitive as they lend to "many" entrepreneurs and average out the returns from granted loans across both bankrupt and successful ones.**
- Lender's expected profits from loans:

$$\int_{\bar{\omega}}^{\infty} f(\omega)(1+i_t^e)Loan_t d\omega + (1-\mu)\int_0^{\bar{\omega}} \omega f(\omega)d\omega\, Ret_{t+1}^K Q_t K_t =$$

$$\int_{\bar{\omega}}^{\infty} f(\omega)\, \bar{\omega} Ret_{t+1}^K Q_t K_t d\omega + (1-\mu)\int_0^{\bar{\omega}} \omega f(\omega)d\omega\, Ret_{t+1}^K Q_t K_t =$$

$$Ret_{t+1}^K Q_t K_t \left[\bar{\omega}\int_{\bar{\omega}}^{\infty} f(\omega)\, d\omega + (1-\mu)\int_0^{\bar{\omega}} \omega f(\omega)d\omega\right] = Ret_{t+1}^K Q_t K_t[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]$$

- Perfect competition implies zero profits, so profits from loans should equal to opportunity cost of investing funds in safe assets:

$$Ret_{t+1}^K Q_t K_t[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] = (1+i_t)Loan_t = (1+i_t)(Q_t K_t - N_t)$$

- **Optimal contract maximizes entrepreneurs' expected profits given banks have zero profits**

Standard debt contract is optimal (1)

- Entrepreneur's expected profits, discounted by the interest rate:

$$\frac{1}{1+i_t}\left\{\int_{\bar{\omega}}^{\infty}\omega f(\omega)Ret_{t+1}^{K}Q_tK_t d\omega-\int_{\bar{\omega}}^{\infty}f(\omega)(1+i_t)Loan_t\,d\omega\right\}=$$

$$\frac{1}{1+i_t}\left\{Ret_{t+1}^{K}Q_tK_t\int_{\bar{\omega}}^{\infty}\omega f(\omega)d\omega-\bar{\omega}Ret_{t+1}^{K}Q_tK_t\int_{\bar{\omega}}^{\infty}f(\omega)d\omega\right\}=$$

$$\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t\left[\int_{\bar{\omega}}^{\infty}\omega f(\omega)d\omega-\bar{\omega}\int_{\bar{\omega}}^{\infty}f(\omega)d\omega\right]=[1-\Gamma(\bar{\omega})]\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t$$

- The optimal debt contract solves the following problem:

$$\max_{K_t,\bar{\omega}}[1-\Gamma(\bar{\omega})]\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t$$

$$\text{s.t. }\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]=Q_tK_t-N_t$$

$$\mathcal{L}=[1-\Gamma(\bar{\omega})]\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t+\lambda_t\left[\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]-(Q_tK_t-N_t)\right]$$

- The first order conditions (FOCs) are then:

$$\frac{\partial\mathcal{L}}{\partial K_t}=[1-\Gamma(\bar{\omega})]\frac{Ret_{t+1}^{K}}{1+i_t}Q_t+\lambda_t\left[\frac{Ret_{t+1}^{K}}{1+i_t}Q_t[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]-Q_t\right]=0$$

$$\frac{\partial\mathcal{L}}{\partial\bar{\omega}}=-\Gamma'(\bar{\omega})\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t+\lambda_t[\Gamma'(\bar{\omega})-\mu G'(\bar{\omega})]\frac{Ret_{t+1}^{K}}{1+i_t}Q_tK_t=0$$

Standard debt contract is optimal (2)

- After some rearranging, the two FOCs are:

$$\lambda_t = \boldsymbol{\Gamma'(\bar{\omega})/[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})]}$$

$$[\mathbf{1} - \boldsymbol{\Gamma}(\bar{\boldsymbol{\omega}})]\frac{\boldsymbol{Ret_{t+1}^K}}{\mathbf{1} + \boldsymbol{i_t}} + \frac{\boldsymbol{\Gamma'(\bar{\omega})}}{\boldsymbol{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})}}\left[\frac{\boldsymbol{Ret_{t+1}^K}}{\mathbf{1} + \boldsymbol{i_t}}[\boldsymbol{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})}] - \mathbf{1}\right] = 0$$

- The lenders' zero-profit condition:

$$\frac{\boldsymbol{Ret_{t+1}^K}}{\mathbf{1} + \boldsymbol{i_t}}\boldsymbol{Q_t K_t}[\boldsymbol{\Gamma(\bar{\omega}) - \mu G(\bar{\omega})}] = \boldsymbol{Q_t K_t - N_t}$$

- We can introduce a new notation for spread (sp_t) and leverage ratio (Lev_t)

$$\boldsymbol{sp_t} = \frac{\boldsymbol{Ret_{t+1}^K}}{\mathbf{1} + \boldsymbol{i_t}} \text{ and } \boldsymbol{Lev_t} = \frac{\boldsymbol{Q_t K_t}}{\boldsymbol{N_t}}$$

Standard debt contract is optimal (cont.)

- Further rearranging FOC with expressing the spread (demand side):

$$sp_t = \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \left[1 - \Gamma(\bar{\omega}) + \frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \right]^{-1}$$

- Using the leverage ratio in lenders' zero-profit condition gives (supply side):

$$sp_t = \frac{1}{[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]} \left(1 - \frac{1}{Lev_t} \right)$$

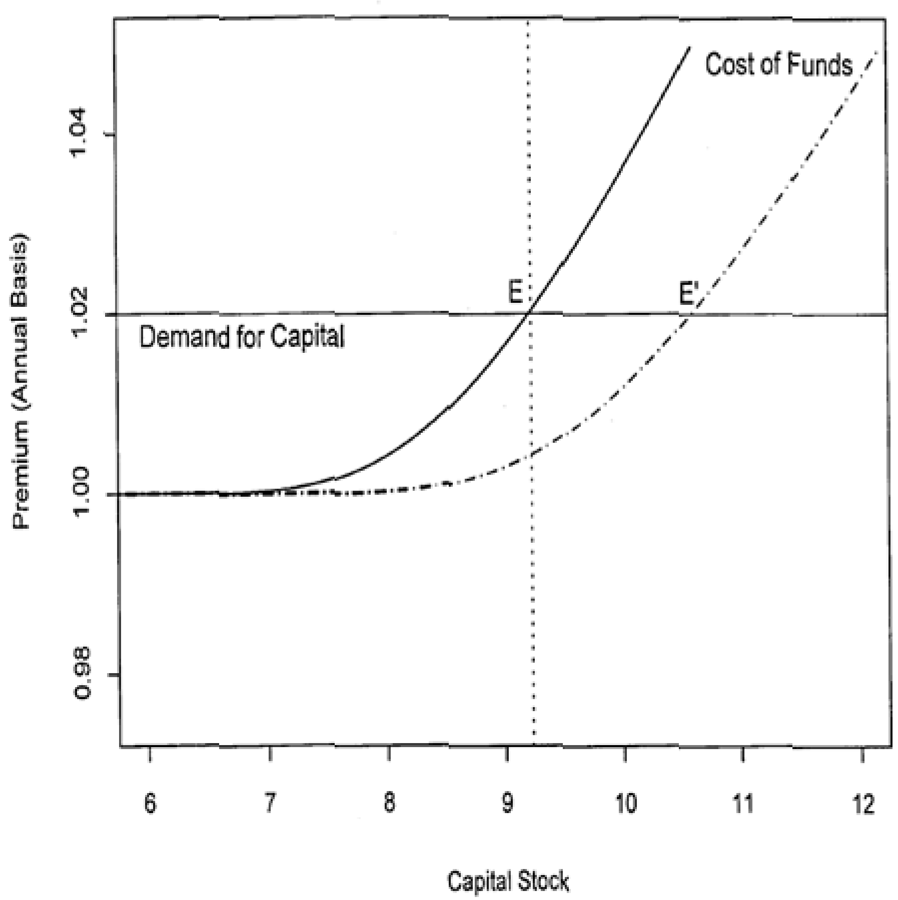
- Combining the two expressions, and noting that from demand side we have $\bar{\omega}$ depending on ***sp*** only, one finally obtains:

$$sp_t = \psi(Lev_t)$$

- Imposing certain conditions on the distribution of ω and given the value of μ, it follows that $\psi'(Lev_t) > 0$ → **risk premium increases with leverage!**

Entrepreneurs’ financial position will affects equilibrium level of investment and capital stock

Figure 1: The Effect of an Increase in Net Worth



For a capital demand which can be financed entirely with net worth, sp=0.

When external finance is necessary, the risk premium becomes upward sloping (expected default costs increase with higher debt to net worth, or riskiness).

An increase in net worth causes the external premium curve to shift to the right (decrease in premium for given demand of capital).

Source: Bernanke, Gertler, and Gilchrist (1998)

BGG introduce this risky debt contract in a DSGE model

- Defining *real* return on capital as $ret^K_{t+1} = Ret^K_{t+1}/(1 + E_t\pi_{t+1})$, spread is then given by:

$$sp_t = E_t\left[\frac{ret^K_{t+1}(1 + \pi_{t+1})}{1 + i_t}\right] = \psi\left(\frac{Q_t K_t}{N_t}\right) = \psi(Lev_t)$$

- The spread between the return on capital and the cost of funds depends on entrepreneur's financial position (leverage).
- Define $\boldsymbol{v} = \boldsymbol{\psi}'(\boldsymbol{Lev_t})\boldsymbol{Lev}/\boldsymbol{\psi}(\boldsymbol{Lev_t})$ as the elasticity of $\boldsymbol{sp}$ to leverage in steady state
- This will be critical in determining the magnitude of the financial accelerator mechanism.

BGG introduce this risky debt contract in a DSGE model

- To complete the description of the DSGE model, we need to provide an evolution of entrepreneurs' net worth.
- A fraction $1-\gamma$ of entrepreneurs dies each period, while another fraction γ survives into the following period.
- Entrepreneurs who die consume all their net worth.
- Entrepreneurs who die are replaced by a new cohort to keep population constant. Newly-born receive an initial resource transfer w^e (for simplicity this transfer is assumed to be received also by the entrepreneurs who survive).

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Net worth accumulation in BGG model

- The aggregate *net worth* is: (BGG.a3)

$$N_t = \gamma\left(1 - \Gamma(\overline{\omega}_t)\right) Ret_t^K Q_{t-1} K_{t-1} + W^e$$

- Substituting out the $\Gamma(\overline{\omega})$ term:

$$N_t = \gamma\left(Ret_t^K\left(1 - \mu G(\overline{\omega})\right) - (1 + i_{t-1})\right) Q_{t-1} K_{t-1} + \gamma(1 + i_{t-1}) N_{t-1} + W^e$$

- Which can be written as in real terms:

$$n_t = \gamma\left(ret_t^K\left(1 - \mu G(\overline{\omega})\right) - \frac{1 + i_{t-1}}{1 + \pi_t}\right) q_{t-1} K_{t-1} + \gamma \frac{1 + i_{t-1}}{1 + \pi_t} n_{t-1} + W^e$$

The effects of monetary policy are amplified by the financial accelerator

Monetary tightening

- Reduces consumption and investment
- Inflation and asset prices fall
- Real interest rate rises through the ***Fisher effect***
- Entrepreneurs' ***net worth*** falls and investment financing ***spread*** increases

Accelerator effect

Note: The contribution of financial accelerator is different for demand- vs. supply-type shocks!

An increase in public expenditure tends to "liquidate" investors' debt through higher inflation

Fiscal expansion

Multiplier effect

Accelerator Effect

- Increases aggregate demand
- Boosts inflation and asset prices
- High inflation increases investors' net worth (real value of the debt falls: ***Fisher effect***): low leverage
- This reduces financing spread
- As a result, there is lower crowding out of private investment
- Fernández-Villaverde (2010) uses this model and mechanism to argue that additional government spending could be more effective than reducing taxes for stimulating output in the short run.

2. A log-linearized (and simplified) model for the closed economy with financial accelerator

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BGG model simplified to facilitate intuition

- Aggregate demand (BGG2.1) :

$$y_t = \frac{C}{Y} c_t + \frac{Inv}{Y} inv_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c_t^e$$

- Euler equation for consumption (BGG2.2):

$$c_t = -\sigma \frac{1-h}{1+h}(i_t - E_t \pi_{t+1}) + \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t c_{t+1}$$

- Consumption of entrepreneurs (BGG2.3):
-

$$c_t^e = n_t$$

- Definition of risk spread of entrepreneurs (BGG2.4):

$$E_t[ret_{t+1}^K + \pi_{t+1} - i_t] = sp_t$$

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BGG model simplified to facilitate intuition

- Risk spread and leverage (BGG2.5) :

$$sp_t = \nu(q_t + k_t - n_t)$$

- Real return on capital (BGG2.6):

$$ret_t^K = (1-\varepsilon)(mc_t + y_t + k_{t-1}) + \varepsilon q_t - q_{t-1}, \text{ where } \varepsilon = (1-\delta)/ret^K$$

- Investment dynamics (BGG2.7):

$$q_t = \zeta_{Inv}(inv_t - inv_{t-1}) - \beta\zeta_{Inv}E_t(inv_{t+1} - inv_t)$$

- Aggregate supply / production function (BGG2.8):

$$y_t = a_t + \alpha k_{t-1} + (1-\alpha)l_t$$

BGG model simplified to facilitate intuition

- Labor supply (BGG2.9) :

$$rw_t = \sigma_L l_t + \frac{1}{\sigma}\frac{1}{1-h}c_t - \frac{1}{\sigma}\frac{h}{1-h}c_{t-1}$$

- Labor demand (BGG2.10):

$$rw_t = mc_t + y_t - l_t$$

- Phillips Curve (BGG2.11):

$$\pi_t = \frac{\chi_p}{1+\beta\chi_p}\pi_{t-1} + \frac{\beta}{1+\beta\chi_p}E_t\pi_{t+1} + \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p(1+\beta\chi_p)}mc_t$$

- Capital accumulation (BGG2.12):

$$k_t = \delta inv_t + (1-\delta)k_{t-1}$$

BGG model simplified to facilitate intuition

- Evolution of the entrepreneurs' net worth (BGG2.13):

$$n_t = \frac{K}{N} ret_t^K - \left(\frac{K}{N} - 1\right)(sp_{t-1} + i_{t-1} - \pi_t) + n_{t-1}$$

Fisher effect from inflation "surprises"

- Monetary policy rule (BGG2.14):

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\Phi_\pi \pi_t + z_t$$

BGG model simplified to facilitate intuition

- Exogenous process for government consumption (BGG2.15), productivity (BGG2.16) and monetary policy (BGG2.17) shocks:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$
$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$
$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$$

- Variables (17):

$$c_t,\, i_t\,,\pi_t,\, l_t,\, rw_t\,,\, y_t\,, mc_t\,,\, inv_t\,,\, k_t\,, n_t, c_t^e, ret_t^k, q_t, sp_t, g_t, a_t, z_t$$

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Calibration

- Quarterly frequency

- Discount factor, inflation and monetary policy rate in steady state:

$$\beta = 0.99, \pi = 0, 1 + i = \frac{1+\pi}{\beta} \approx 1.01\%, i = 1\%$$

- Household preferences: $\sigma = 1, \sigma_L = 1/3$

- Depreciation rate and capital adjustment costs:

$$\delta = 0.025, \frac{Inv}{K} = \delta$$

Calibration

- Financial contract:

$$\ln(\omega) \sim N\left(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2\right). \quad ret^K - r = 0.03 \text{ [annual]}, \int_0^{\bar{\omega}} f(\omega)\, d\omega = 0.0075,$$

$$\frac{K}{N} = 2 \text{ and } \gamma = 0.975 \rightarrow \sigma_\omega = 0.2741, \mu = 0.17 \rightarrow \nu = 0.0556, \frac{C^e}{Y} = 0.1058$$

- Price rigidities, monetary policy rule, and persistence of shocks:

$$\theta = 0.75, \rho_i = 0.90, \Phi_\pi = 1.5, \Phi_y = 0, \rho_g = 0.95, \rho_a = 0.99, \rho_z = 0$$

- Matlab codes BGG_ss.m and fun_bgg_ss.m are run in Dynare to obtain the rest of the parameters consistent with these SS assumptions (workshop)

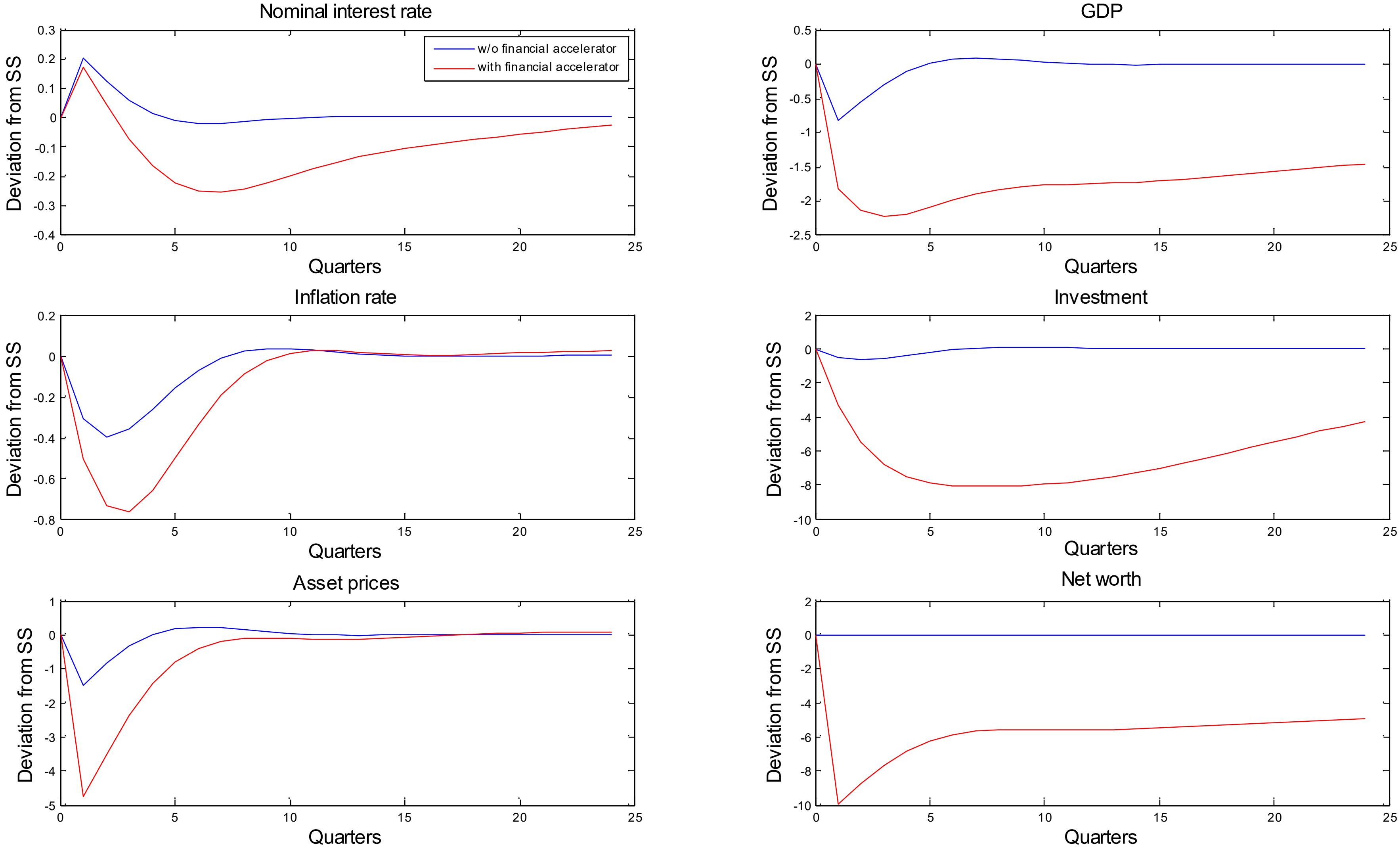
Comparison with the model without financial accelerator

- To eliminate the financial accelerator channel, we have the same model, except for:

$$sp_t = 0, n_t = 0$$

- Let's see the response of a monetary policy tightening with and without the financial accelerator…

Amplification of monetary policy with financial accelerator



Amplification of monetary policy with financial accelerator

- **Transmission mechanism in standard New Keynesian Model**

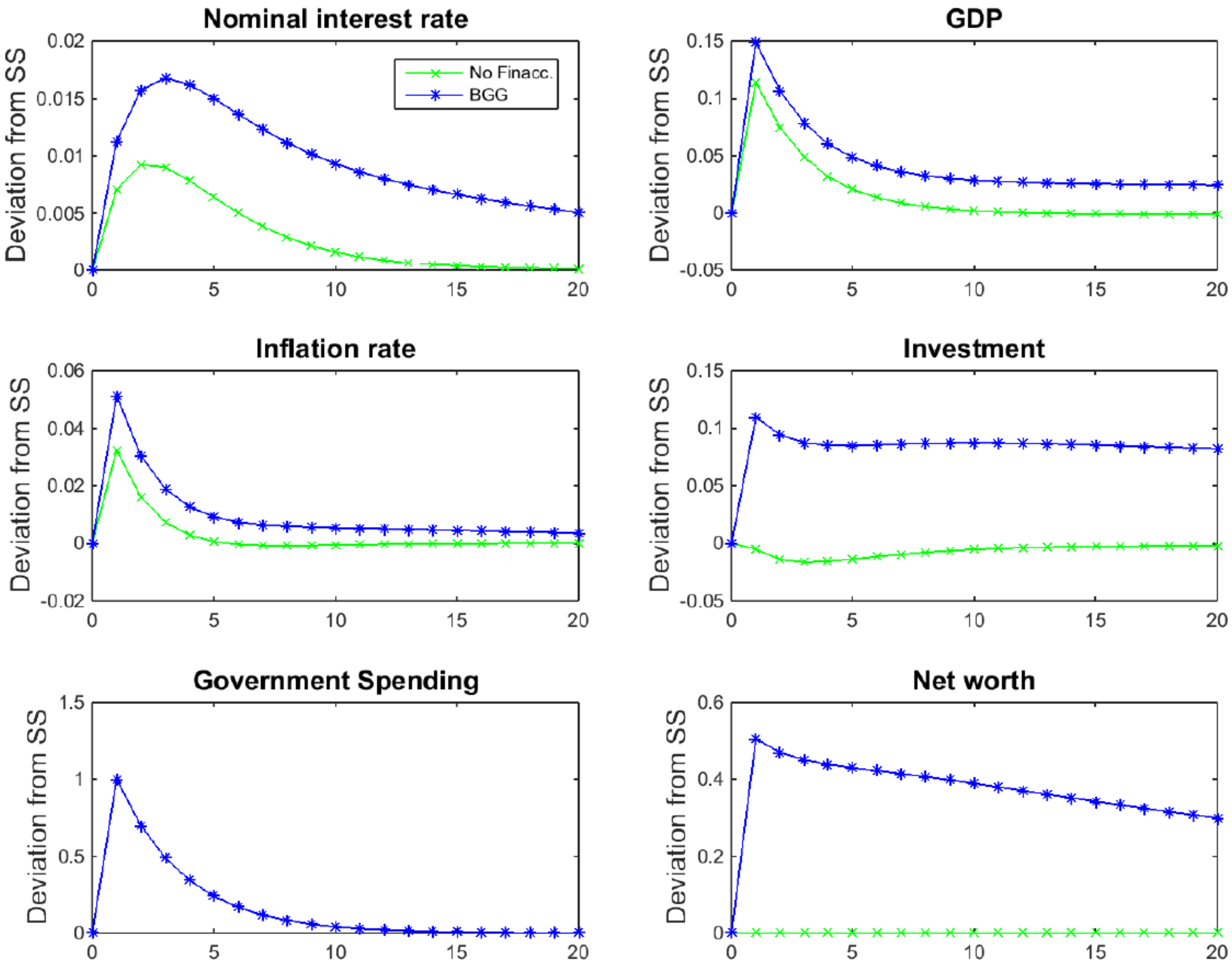
- Increase in policy rate
- Lower consumption, investment, and GDP
- Lower inflation
- Lower assets prices

Amplification of monetary policy with financial accelerator

- **Transmission mechanism with financial accelerator**

- Increase in policy rate
- Lower consumption, investment, and GDP
- Lower inflation
- Lower assets prices
- Lower net worth and higher risk premium
- Loans decline
- Decline investment and GDP

Government spending shock



Government spending shock

- **Transmission Mechanism in standard New Keynesian model**

- Increase in government spending
- Increase in GDP
- Higher inflation
- Higher assets prices

Government spending shock

- **Transmission mechanism with financial accelerator**

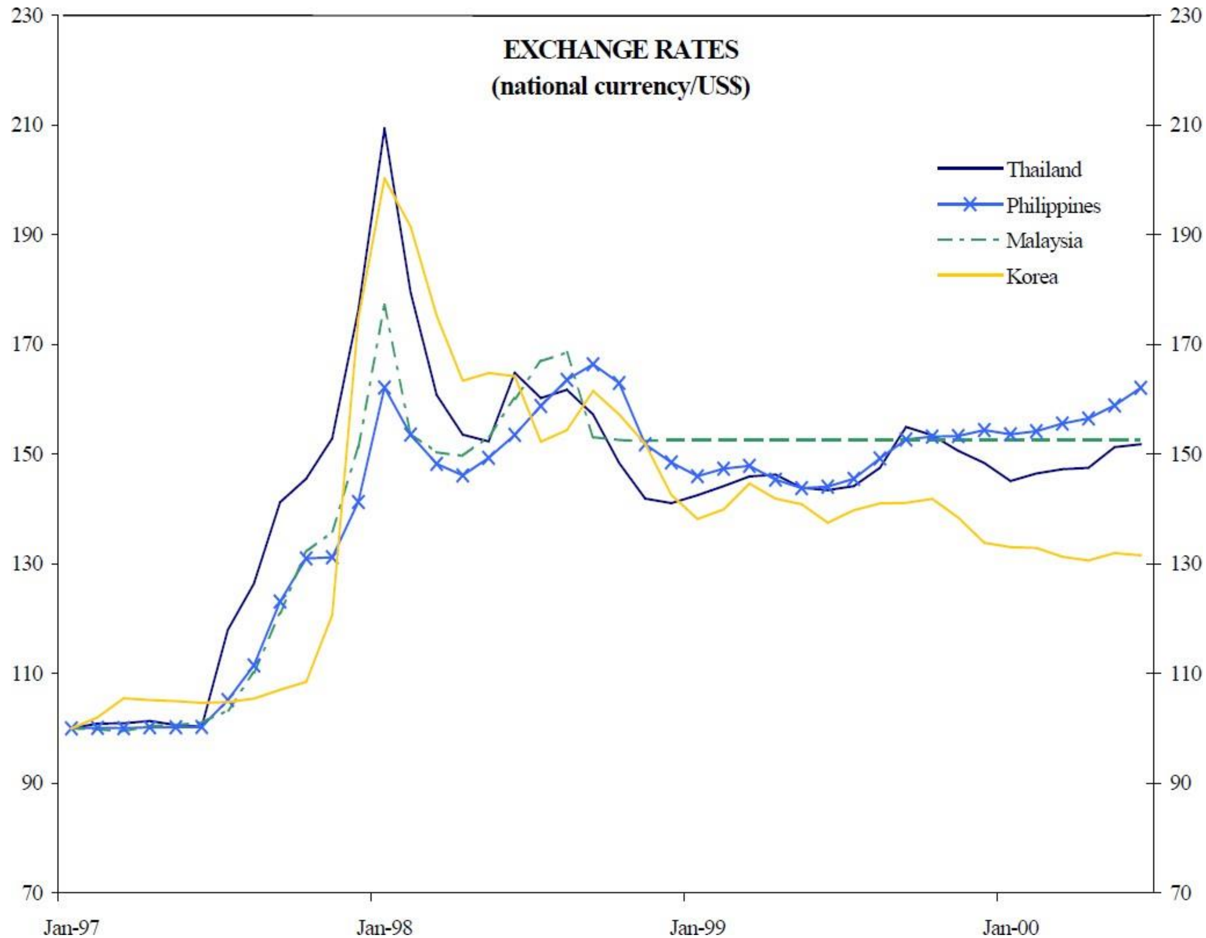
- Increase in government spending
- Increase in GDP
- Higher inflation
- Higher assets prices
- Increase in net worth and lower risk premium
- Loans increase
- Increase in investment and GDP

3. A log-linearized model for an open economy with financial accelerator

How to respond to capital outflows?

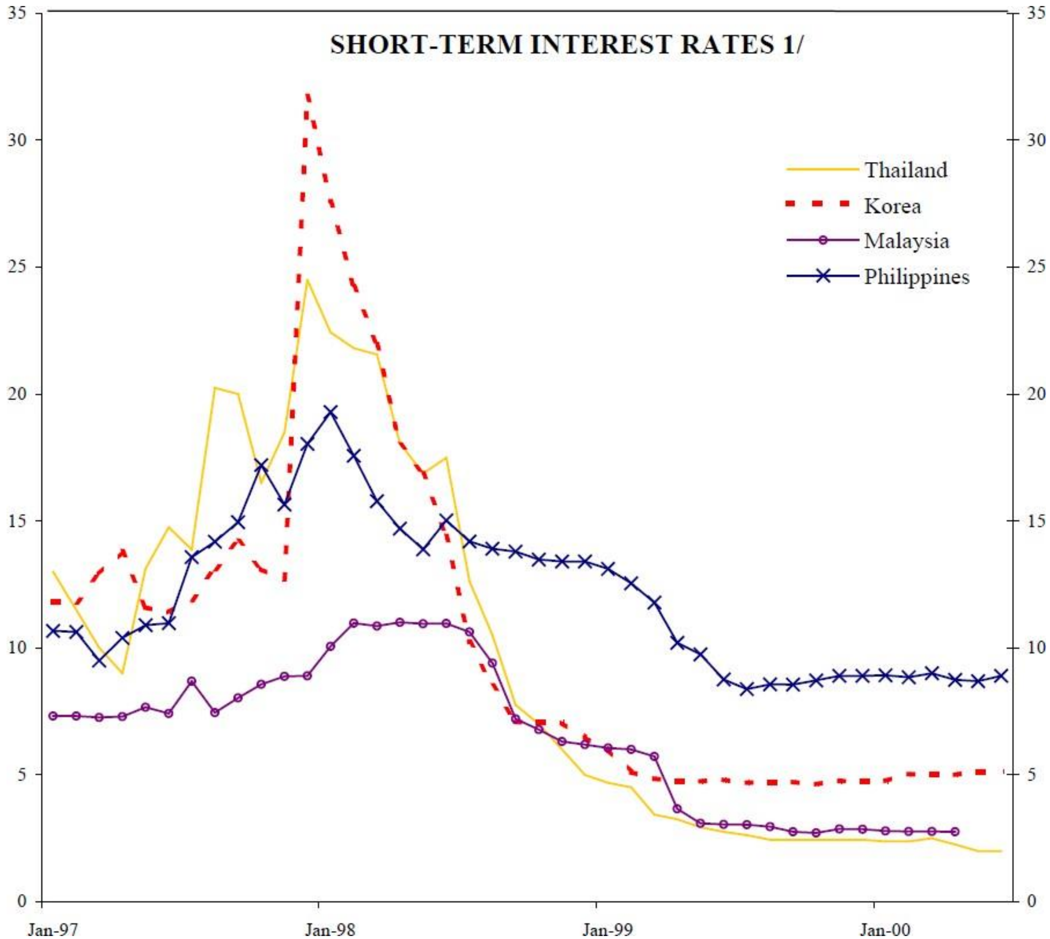
- Financial crises have given rise to a debate on how to respond to episodes of financial stress
- In emerging economies, financial stress is quite often the result of a sudden stop in capital flows
- One response is to cut the monetary policy (MP) interest rate to moderate output contraction
- But a monetary expansion could lead to exchange rate depreciation…
  - Currency mismatches in debt could make the economic contraction more severe
  - Central banks might need to defend the currency (“fear of floating”)
- … and inflation

Exchange rates during the Asian crisis



Source: Braggion, Christiano, and Roldos (2007)

What did the Asian countries do?



Source: Braggion, Christiano, and Roldos (2007)

BGG model extended to an open economy setting

The model is more complex, including new variables and equations

- GDP is not equal to domestic absorption (economy is open)
- There are goods produced locally and externally
- New relative prices
- Balance of payments identity
- We incorporate also: imperfect exchange rate pass-through, commodity exports and wage rigidities [**important for the final workshop**]
- Foreign variables
- Then, we can analyze how financial accelerator amplifies the effects of shocks (e.g., domestic or foreign interest rate increases)

GDP will include goods produced domestically and imported (I)

- We will denote by $\boldsymbol{H}$ the goods produced domestically, and by $\boldsymbol{F}$ the imported goods.
- Private consumption and investment are a composite of domestic ($\boldsymbol{H}$) and imported ($\boldsymbol{F}$) goods
  - $(1-\alpha_C)$: share of **domestic goods** in private consumption basket. Households and entrepreneurs have the same consumption basket.
  - $(1-\alpha_I)$: share of **domestic goods** in investment basket
  - Note: government consumption is only in domestic goods
  - Demand for domestic goods comes from **internal private demand** for $\boldsymbol{H}$ goods ($d_{H,t}$), government consumption (g_t) and foreign demand for $\boldsymbol{H}$ goods ($c^*_{H,t}$) (BGG4.1):

$$\frac{Y_H}{Y} y_{H,t} = \left((1-\alpha_C) \left(\frac{C}{Y} + \frac{C^e}{Y} \right) + (1-\alpha_I) \frac{I}{Y} \right) d_{H,t} + \frac{G}{Y} g_t + \frac{C_H^*}{Y} c^*_{H,t}$$

GDP will include goods produced domestically and imported (II)

- Internal private demand for $\boldsymbol{H}$ goods depends on the degree of substitution between $\boldsymbol{H}$ and $\boldsymbol{F}$ goods in the consumption and investment baskets
  - η_C: elasticity of substitution between H and F goods in consumption basket.
  - η_I: elasticity of substitution between H and F goods in investment basket.
  - Equation (BGG4.2):

$$\left((1-\alpha_C)\left(\frac{C}{Y}+\frac{C^e}{Y}\right)+(1-\alpha_I)\frac{I}{Y}\right)d_{H,t} =$$

$$(1-\alpha_C)\frac{C}{Y}c_t+(1-\alpha_C)\frac{C^e}{Y}c_t^e-(1-\alpha_C)\left(\frac{C}{Y}+\frac{C^e}{Y}\right)\eta_c\left(p_{H,t}-p_t\right)$$

$$+(1-\alpha_I)\frac{I}{Y}\left(inv_t-\eta_I\left(p_{H,t}-p_t\right)\right)$$

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GDP will include goods produced domestically and imported (III)

- Likewise, we will assume that foreign consumption of H goods depends on its relative price
  - η^*: price elasticity of foreign consumption for H goods
  - y_t^*: Foreign demand
  - LOOP holds for exported good $P_{H,t} = e_t P_{H,t}^*$
  - Equation (BGG4.3):

$$c_{H,t}^* = y_t^* - \eta^* \left(p_{H,t} - e_t - p_t^* \right)$$

GDP will include goods produced domestically and imported (IV)

- The complement of internal demand which is not spent on domestic goods is imported
  - Equation (BGG4.4) for the volume of imports:

$$\frac{M}{Y}m_t = \alpha_C \frac{C}{Y}c_t + \alpha_C \frac{C^e}{Y}c_t^e - \alpha_C\left(\frac{C}{Y} + \frac{C^e}{Y}\right)\eta_c(p_{F,t} - p_t) + \alpha_I \frac{I}{Y}\big(inv_t - \eta_I(p_{F,t} - p_t)\big)$$

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GDP will include goods produced domestically and imported (V)

- There is also a part of GDP linked to natural resources.
- Natural resources are commodities ($y_{CO,t}$) which are fully exported in the international markets at a given price ($p^*_{CO,t}$). For simplicity, it will be assumed that commodity production is exogenous.
  - Equation (BGG4.5) for the volume of exports:

$$\frac{X}{Y} x_t = \frac{C^*_H}{Y} c^*_{H,t} + \frac{Y_{CO}}{Y} y_{CO,t}$$

  - Equation (BGG4.6) for the export deflator:

$$\frac{X}{Y}(p_{X,t} - p_t) = \frac{C^*_H}{Y}(p_{H,t} - p_t) + \frac{Y_{CO}}{Y}(p^*_{CO,t} + e_t - p_t)$$

  - Note that: $\frac{X}{Y} = \frac{C^*_H}{Y} + \frac{Y_{CO}}{Y}$

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New relative prices (I)

- Nominal exchange rate: e_t
- Real exchange rate: $rer_t = e_t + p_t^* - p_t$
- Also: $p_{CO,t}^* + e_t - p_t = p_{CO,t}^* - p_t^* + rer_t$

  – Equation (BGG4.7) for the dynamics of the real exchange rate:

$$rer_t = rer_{t-1} + \Delta e_t + \pi_t^* - \pi_t$$

  – Nominal depreciation: $\Delta e_t = e_t - e_{t-1}$
  – CPI inflation: $\pi_t = p_t - p_{t-1}$
  – Foreign inflation: $\pi_t^* = p_t^* - p_{t-1}^*$

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New relative prices (II)

- Equation (BGG4.8) for the dynamics of relative price of $\boldsymbol{H}$ goods:

$$p_{H,t} - p_t = p_{H,t-1} - p_{t-1} + \pi_{H,t} - \pi_t$$

- Equation (BGG4.9) for interest rate parity (UIP):

$$i_t = i_t^* + E_t[\Delta e_{t+1}] + \zeta b_t^*$$

  - b_t^*: external debt
  - ζ: elasticity of risk premium (sovereign spread) to external debt

Balance of payments identity

- Changes in external debt are related to current account balance (trade balance plus net payments of factors from abroad). Equation (BGG4.10):

$$\frac{B^*}{Y}(rer_t + b_t^*) = \frac{B^*}{Y\beta}(i_{t-1}^* + (1+\zeta)b_{t-1}^* + rer_t - \pi_t^*) + \frac{M}{Y}(rer_t + m_t)$$
$$+\chi\frac{Y_{CO}}{Y}\left(p_{CO,t}^* - p_t^* + rer_t + y_{CO,t}\right) - \frac{X}{Y}\left(p_{X,t} - p_t + x_t\right)$$

  - χ : share of commodity good owned by foreigners. There is a fraction of the commodity good taken out from the country every period.

Imperfect pass-through from foreign inflation

- We will assume that importers that sell $\boldsymbol{F}$ goods in the domestic market face nominal rigidities in adjusting their prices
  - Marginal cost of imported goods: $e_t + p_t^*$
  - Marginal cost relative to sell price: $e_t + p_t^* - p_{F,t}$
  - Phillips curve for the price of $\boldsymbol{F}$ goods (BGG4.11):

$$\pi_{F,t} = \frac{\chi_F}{1+\beta\chi_F}\pi_{F,t-1} + \frac{\beta}{1+\beta\chi_F}E_t\pi_{F,t+1} + \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F(1+\beta\chi_F)}\underbrace{(e_t + p_t^* - p_t)}_{rer_t - (p_{F,t} - p_t)}$$

  - θ_F: Calvo parameter of nominal rigidities for $\boldsymbol{F}$ prices
  - χ_F: degree of indexation to past inflation
  - Evolution of relative price of $\boldsymbol{F}$ goods (BGG4.12):

$$p_{F,t} - p_t = p_{F,t-1} - p_{t-1} + \pi_{F,t} - \pi_t$$

Relative investment/consumption prices and real GDP

- The price of investment basket is a weighted average of ***H*** and ***F*** prices (BGG4.13):

$$p_{I,t} - p_t = (1 - \alpha_I)(p_{H,t} - p_t) + \alpha_I(p_{F,t} - p_t)$$

- Likewise, the price of consumption basket (CPI) is a weighted average of ***H*** and ***F*** prices (BGG4.14):

$$0 = (1 - \alpha_C)(p_{H,t} - p_t) + \alpha_C(p_{F,t} - p_t)$$

- Real GDP is the combination of production of ***H*** goods and commodities (BGG4.15):

$$y_t = \frac{Y_H}{Y} y_{H,t} + \frac{Y_{CO}}{Y} y_{CO,t}\ , \text{ note that } \quad 1 = \frac{Y_H}{Y} + \frac{Y_{CO}}{Y}$$

Consumption of households and entrepreneurs, and opportunity cost of capital

- Same equations as (BGG2.2), (BGG2.3), (BGG2.4)

  - Euler equation for households' consumption (BGG4.16):

$$c_t = -\sigma \frac{1-h}{1+h}(i_t - E_t\pi_{t+1}) + \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1}$$

  - Entrepreneurs' consumption (BGG4.17):

$$c_t^e = n_t$$

  - Spread between the return on capital and the opportunity cost of lent funds (BGG4.18):

$$E_t[ret_{t+1}^K + \pi_{t+1} - i_t] = sp_t$$

Spread, return on capital, investment demand, and supply of H goods

- Spread is a function of the financial position of entrepreneurs (BGG4.19, same as BGG2.5):

$$sp_t = \nu(q_t + k_t - n_t)$$

- Return on capital (BGG4.20, same as BGG2.6):

$$ret_t^K = (1-\varepsilon)(mc_t + y_t + k_{t-1}) + \varepsilon q_t - q_{t-1}, \text{ where } \varepsilon = (1-\delta)/ret^K$$

- Dynamics of investment (BGG4.21, same as BGG2.7):

$$inv_t = \frac{1}{1+\beta} inv_{t-1} + \frac{\beta}{1+\beta} E_t inv_{t+1} + \frac{1}{\zeta_{Inv}(1+\beta)} q_t$$

- Production function of ***H*** goods (BGG4.22, similar to BGG2.8):

$$y_{H,t} = a_t + \alpha k_{t-1} + (1-\alpha) l_t$$

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Labor market

- Labor supply is determined by the marginal rate of substitution between labor and consumption (BGG4.23):

$$mrs_t = \sigma_L l_t + \frac{1}{\sigma}\frac{1}{1-h}c_t - \frac{1}{\sigma}\frac{h}{1-h}c_{t-1}$$

- Labor demand (BGG4.24, same as BGG2.10):

$$rw_t = mc_t + y_t - l_t$$

Prices of H goods also face nominal rigidities

- Phillips curve for the price of ***H*** goods (BGG4.25, similar to BGG2.11):

$$\pi_{H,t} = \frac{\chi_p}{1+\beta\chi_H}\pi_{H,t-1} + \frac{\beta}{1+\beta\chi_H}E_t\pi_{H,t+1} + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H(1+\beta\chi_H)}\left(mc_t - (p_{H,t} - p_t)\right)$$

- Capital accumulation (BGG4.26, same as BGG2.12):

$$k_t = \delta inv_t + (1-\delta)k_{t-1}$$

Entrepreneurs’ net worth and monetary policy

- Entrepreneurs’ net worth (BGG4.27, same as BGG2.13):

$$n_t = \frac{K}{N} ret_t^K - \left(\frac{K}{N} - 1\right)(sp_{t-1} + i_{t-1} - \pi_t) + n_{t-1}$$

- Monetary policy rule (BGG4.28, modified from BGG2.14):

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left(\Phi_\pi \pi_t + \Phi_y y_t + \Phi_{\Delta e} \Delta e_t\right) + z_t$$

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We also include wage rigidities

- Phillips curve for nominal wages (BGG4.29):

$$\pi_t^w = \frac{\chi_w}{1+\beta\chi_w}\pi_{t-1}^w + \frac{\beta}{1+\beta\chi_w}E_t\pi_{t+1}^w + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\beta\chi_w)}(mrs_t - rw_t)$$

  - θ_w: Calvo parameter for nominal wage rigidities
  - χ_w: degree of indexation to past inflation

- Wage inflation (BGG4.30):

$$\pi_t^w = rw_t - rw_{t-1} + \pi_t$$

Same three internal shocks but adding commodity production as another exogenous variable

- Exogenous process for the government consumption (BGG4.31), productivity (BGG4.32) and monetary policy (BGG4.33) shocks:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$
$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$$
$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$$

- Commodity production follows an exogenous process given by (BGG4.34):

$$y_{co,t} = \rho_{y,co} y_{co,t-1} + \varepsilon_{y,co,t}$$

External conditions are not affected by the small open economy

- Exogenous process for foreign interest rate (BGG4.35), foreign inflation (BGG4.36), international commodity price (BGG4.37), and foreign demand (BGG4.38):

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_{i^*,t}$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t}$$

$$p_{CO,t}^* - p_t^* = \rho_{p,co^*}\left(p_{CO,t-1}^* - p_{t-1}^*\right) + \varepsilon_{p,CO^*,t}$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_{y^*,t}$$

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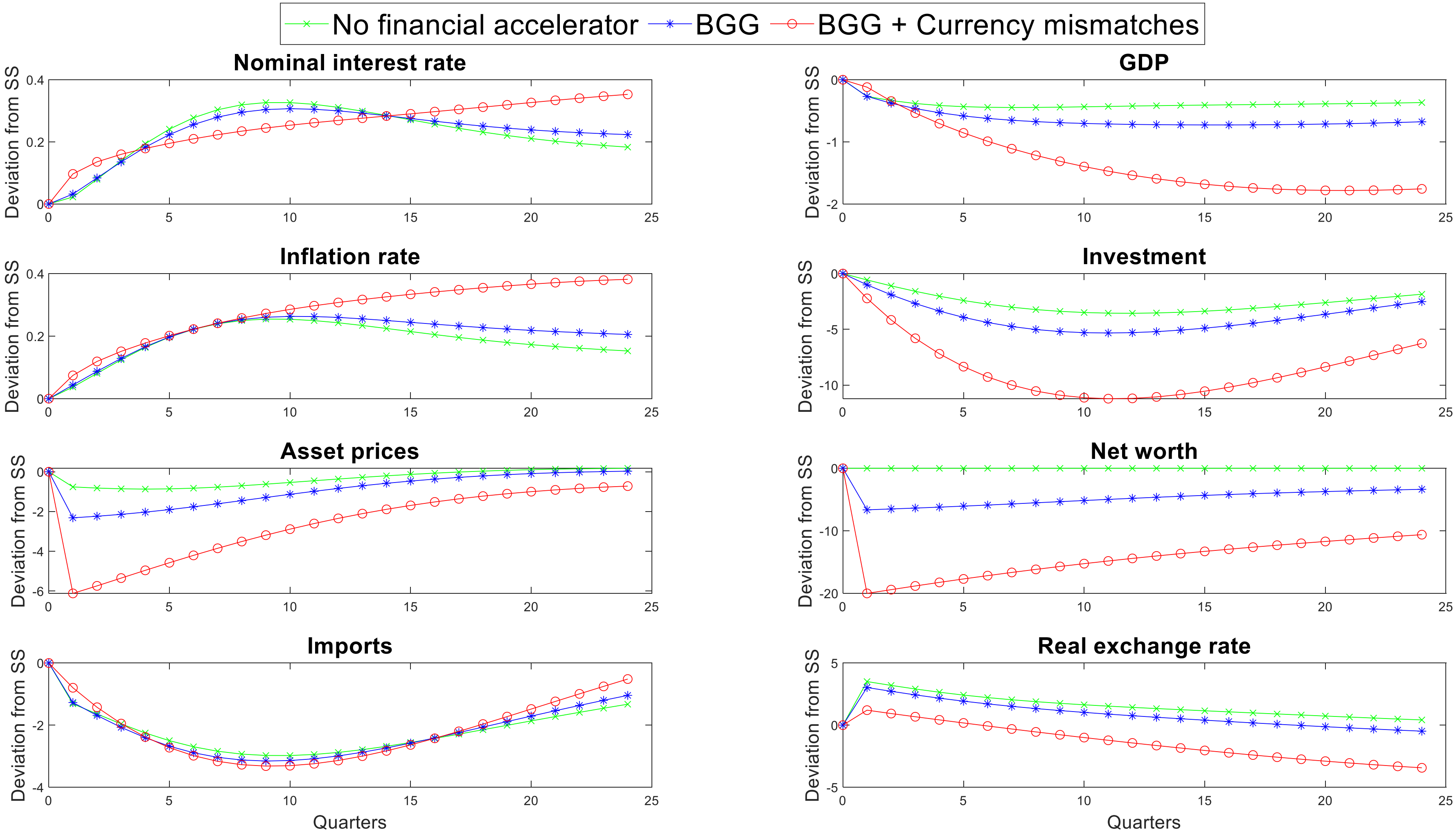
Financial Accelerator in a Small Open Economy Model: *i** shock



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4. A model for unconventional monetary policy

Gertler and Karadi (2011)

- At any moment in time the fraction $1 - f$ of household's member are workers and the fraction f are bankers.
- They switch randomly between the two occupations. A banker this period remains as a banker for the next period with probability ω^b.
- To avoid bankers reaching a point where they can fund all investments from their own capital, in every period $(1 - \omega^b)f$ bankers become workers.
- The same number of randomly selected workers become bankers, keeping the relative proportion of each type fixed.
- The household provides its new bankers with some start up funds

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Introducing financial intermediaries (bank)

- Bankers lend funds obtained from households to non-financial firms. Let $N_{j,t}$ be the amount of net worth that an intermediary *j* has at the end of period t ; $B_{j,t}$ are the deposits the intermediary obtains from households; $S_{j,t}$ is the quantity of financial claims on non-financial firms that the intermediary holds; and Q_t is the relative price of each claim. Balance sheet is:

$$Q_t S_{j,t} = B_{j,t} + N_{j,t}$$

- Household's deposits with the intermediary at time t pay the non-contingent gross return R_t. Loans from intermediaries obtain the stochastic return R_t^L
- The banker's equity capital evolves as the difference between earnings on loans and interest payments on deposits:

$$N_{j,t+1} = (R_t^L - R_t) Q_t S_{j,t} + R_t N_{j,t}$$

Financial intermediaries' optimization problem

- The banker's objective is to maximize expected terminal wealth:

$$\max V_{j,t} = E_t \sum_{s=0}^{\infty}(1-\omega^b)\left(\omega^b\right)^s \Theta_{t,t+s} N_{j,t+s+1}$$

$$\max V_{j,t} = E_t \sum_{s=0}^{\infty}(1-\omega^b)\left(\omega^b\right)^s \Theta_{t,t+s}\left((R^L_{t+s} - R_{t+s})Q_{t+s}S_{j,t+s} + R_{t+s}N_{j,t+s}\right)$$

where $\Theta_{t,t+s}$ is the stochastic discount-factor.

- A moral hazard/costly enforcement problem constrains the ability of bankers to obtain funds from households. That is, the intermediary can divert an exogenous fraction μ_t of total assets $Q_t S_{j,t}$
- Accordingly, for households to be willing to supply funds to the banker, the following incentive constraint must be satisfied

$$V_{j,t} \quad \geq \quad \mu_t Q_t S_{j,t}$$

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Value function can be rewritten as

- The value function for banks may be written as in recursive forumla

$$V_{j,t} = E_t \sum_{s=0}^{\infty} (1-\omega^b) \left(\omega^b\right)^s \Theta_{t,t+s} \left((R^L_{t+s} - R_{t+s}) Q_{t+s} S_{j,t+s} + R_{t+s} N_{j,t+s} \right)$$

$$V_{j,t} \quad = \quad \nu_t Q_t S_{j,t} + \eta_t N_{j,t}$$

- where ν_t is the expected discount value of expanding assets by a unit, holding net worth $N_{j,t}$ constant,

$$\nu_t \quad = E_t \left[(1-\omega^b) \Theta_{t,t+1} (r^L_{t+1} - r_{t+1}) + \Theta_{t,t+1} \omega^b \frac{Q_{t+1} S_{j,t+1}}{Q_t S_{j,t}} \nu_{t+1} \right]$$

- And η_t is the expected discounted value of having another unity of $N_{j,t}$, holding $S_{j,t}$ constant

$$\eta_t \quad = E_t \left[(1-\omega^b) + \Theta_{t,t+1} \omega^b \frac{N_{j,t+1}}{N_{j,t}} \eta_{t+1} \right]$$

Financial intermediary solution

- Combining the value function with the binding incentive compatible constraint:

$$V_{j,t} \quad = \quad \nu_t Q_t S_{j,t} + \eta_t N_{j,t} = \mu_t Q_t S_{j,t}$$

- When the incentive constraints is binding, the intermediary's assets are constrained by its equity capital
- The assets the banker can acquire will depend positively on its equity capital:

$$Q_t S_{j,t} = \frac{\eta_t}{\mu_t - \nu_t} N_{j,t}$$

$$Q_t S_{j,t} = \phi_t N_{j,t}$$

- where ϕ_t is the ratio of privately intermediated assets to equity, referred to as the leverage ratio

Concluding remarks

- Financial frictions **amplify the** business cycles. In open economy, they can magnify capital flows cycles.
- In general, financial frictions and currency mismatches do not modify the benefits of having a flexible exchange rate as a shock absorber over the medium term.
- However, some other frictions (e.g., dollarization) can justify a temporary defense of the exchange rate, as discussed in the workshop.

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