The Linerized Model

Government spending shock

Note that g_t is not wasteful government spending as in the standard literature. Rather, it is a utility-based demand shifter that behaves like fiscal demand in the IS equation, consistent with $y_t = c_t$.

Step 1: Start from the linearized Euler (IS) equation with fiscal demand

A common log-linearized Euler equation can be written as

$$c_t = E_t c_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + \sigma\, E_t \sum_{j=0}^{\infty} g_{t+j}.$$

The last term says: if government spending is expected to be higher in the future, it raises (private) demand today via the intertemporal condition.

Step 2: Impose the AR(1) process for government spending

Assume (as in the slides)

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}, \qquad |\rho_g| < 1.$$

Then

$$E_t g_{t+j} = \rho_g^j g_t.$$

Step 3: Sum the expected future path

Compute the geometric sum:

$$E_t \sum_{j=0}^{\infty} g_{t+j} = \sum_{j=0}^{\infty} \rho_g^j g_t = \frac{1}{1-\rho_g}\, g_t.$$

Step 4: Substitute back into the IS equation

Plugging this into Step 1 yields:

$$c_t = E_t c_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) + \frac{\sigma}{1-\rho_g}\, g_t .$$

So the coefficient is $\sigma/(1-\rho_g)$ (a division), not $\sigma(1-\rho_g)$.

Dynare form

```
c = c(+1) - sigma*(i - pi(+1)) + sigma/(1-rho_g)*g; or
c = -sigma*(rnom-pic(+1)) + c(+1)  + g*(1-rho_g)*sigma;
```

Dynare Error Diagnosis and Fix: `nk_closed_est.dyn`

Symptom

Running

```
dynare nk_closed_est.dyn
```

produces the error

> Initial value(s) of `phi_pic` are outside parameter bounds
> Error using `check_prior_bounds`

This occurs **before estimation starts**, even though the Blanchard–Kahn conditions are satisfied.

What did *not* go wrong

The message

> There are 2 eigenvalue(s) larger than 1 in modulus for 2 forward-looking variable(s)

> The rank condition is verified.

is **not an error**. It confirms determinacy and is unrelated to the crash.

What actually went wrong

The failure is caused by an interaction between:

- a very tight prior truncation
  `prior_trunc = 1e-32`
- and Dynare's internal check of admissible parameter bounds in `check_prior_bounds`.

Even when `mode_file` is commented out, Dynare still constructs admissible bounds from the prior distribution and truncation rule. With such an extreme truncation level, Dynare numerically rejects reasonable initial values for `phi_pic`.

Setting `prior_trunc = 1e-8` or `1e-10` is **still too tight** and leads to the same failure.

The fix

Set

```
prior_trunc = 0
```

This disables artificial truncation of the prior support and allows Dynare to accept the initial value for `phi_pic`. Estimation then proceeds normally.

Key takeaway

This was **not** a model indeterminacy problem and **not** a bad initial value problem. It was a **numerical prior-truncation issue**. Extremely small `prior_trunc` values can cause Dynare to reject valid parameter values during initialization.

Use `prior_trunc = 0` unless you have a very specific reason to enforce hard truncation.

Interpreting Shock-Conditioned DSGE Forecasts

(W8, Q4–Q5)

This note explains how to interpret the out-of-sample forecast figure generated by `nk_closed_est.dyn` for **Questions 4–5** in *W8 – Forecasting with DSGE Models*.

- Questions 4 and 5 use the same shock-conditioned forecast figure.
- Q4 concerns shock attribution.
- Q5 concerns how to interpret those conditional forecasts.
- They are conceptually one exercise viewed from descriptive versus interpretive perspectives.

What the colored lines represent

The figure plots **shock-conditioned forecasts**, not unconditional predictions.

Each color shows forecast paths when **only one structural shock is active**:

- **Blue**: technology shocks (a_t)
- **Green**: government / demand shocks (g_t)
- **Red**: monetary policy shocks (z_t)

Within each color:

- multiple lines reflect **posterior draws** of parameters and shocks,
- the spread represents **forecast uncertainty** conditional on that shock alone.

The **black line** is the realized data (or last observed history).

How to interpret these forecasts

These forecasts answer **counterfactual questions** of the form:

> *If only this shock were driving the economy, what paths would the model imply?*

They are **not** meant to be realistic forecasts taken literally.

Key points:

- Each shock acts **in isolation** (no cancellation from other shocks).
- Large movements or drifts are expected because offsetting forces are absent.
- Wide fan charts reflect **posterior uncertainty**, not model instability.

What we can and cannot conclude

Valid conclusions (what W8 Q4–Q5 want)

- Different shocks drive different variables:
  - policy shocks mainly affect interest rates and inflation,
  - technology shocks matter more for output,
  - government/demand shocks shift private demand.
- The signs and persistence of responses are economically coherent.
- The DSGE forecast is **structurally interpretable** via shock decomposition.

Invalid conclusions

- You **cannot** say that a shock “behaves badly” based on its isolated forecast.
- Individual colored paths are **not supposed to match** the realized data.
- The realized forecast arises from the **interaction and partial cancellation** of all shocks.

Summary

> The forecast fan charts indicate that conditional on monetary policy shocks, the posterior predictive distribution is substantially more dispersed than for technology or government shocks, implying that monetary policy shocks are relatively **weakly informative** for forecasting in this model.

W8 – Question 6

This note explains **what Question 6 is about**, how the code `nk_closed_est.dyn` implements it, and how the **tables and figures** are connected.

1. What Question 6 is asking conceptually

Question 6 is about estimation and historical decomposition, not forecasting.

Specifically, it asks you to:

1. Estimate the DSGE model using observed data.
2. Recover:

- smoothed structural shocks,
- smoothed constants (steady-state components),
- fitted paths of observables.

3. Use these objects to assess:
   - which shocks were important historically,
   - whether the model explains the data in a reasonable way.

So Q6 moves from *forecasting* (Q4–Q5) to **ex-post interpretation**.

2. How `nk_closed_est.dyn` answers Question 6

The code does three key things:

(i) Bayesian estimation

The block `dynare estimation(datafile=..., ...)` runs Bayesian estimation and produces posterior draws of: - parameters, - latent states, - structural shocks.

(ii) Smoothing (Kalman smoother)

Dynare then computes **smoothed objects**, i.e. estimates conditional on the *entire sample*:

- **Smoothed shocks**: best estimate of past e<u>a,t, e</u>g,t, e_z,t
- **Smoothed constants**: estimated steady-state components
- **Smoothed observables**: fitted paths of rnom, pi, y

This is why the figures are labeled *"Smoothed …"*.

(iii) Reporting summary statistics (the table in W8)

The table in Question 6 reports statistics such as: - posterior means, - posterior standard deviations, - shock variances or persistence.

These numbers come **directly from the estimation output** used to generate the plots.

3. Figures

Figure 8: Smoothed shocks

Panels labeled: - ea (technology), - eg (government/demand), - e_z (monetary policy)

show: - **black line**: posterior mean of the smoothed shock, - **green lines**: credible bands / posterior draws.

> These are the model-implied structural disturbances that best rationalize the observed data.

Figure 9: Smoothed constants

These plots show estimated **time-invariant components** (means / steady states).

Key point:

> Flat lines indicate stability; dispersion reflects posterior uncertainty (uncertainty bands).

Figures 11-12: Forecasted variables (mean / point)

They show how well the estimated model tracks: - nominal interest rate, - inflation, - output.

Figure 11:

- The black line is the conditional mean path $E_t x_{t+h}$ conditional on the posterior mean of the paramgers.
- The uncertainty intervals (green lines) for $E_t x_{t+h}$ are derived from the MCMC draws of parameters (no shocks).

Figure 12:

- The "point" forecast (black lines) conditions on fixed parameter and state estimates (median or mode).
- The green lines reflects future shock uncertainty, given the parameter values.

W8 – Question 7

Take output as an example.

- **Figure 12**: *Forecasted variables (point)* (unconditional forecast), and
- **Figure 15**: *Conditional forecast (Posterior Mean): y_obs* (forecast **conditional on an imposed path**).

1. Analyzing `nk_closed_est.dyn`

Figure 12 (unconditional "point" forecasts)

Generated by the `forecast=8` **option inside** `estimation(...)`:

```
estimation(...,
  nobs = 104,
  forecast=8,
  smoother
) rnom_obs pic_obs y_obs ;
```

This produces Dynare's standard forecast plots, including *"Forecasted variables (point)"* for `rnom_obs`, `pic_obs`, `y_obs`.

Interpretation: an **unconditional** forecast from the model at the forecast origin (end of sample), with no additional information imposed about future paths.

Figure 15 (conditional forecast)

Generated by the **three blocks after estimation**:

1) **Impose** future values of an observable (here `rnom_obs`) for specific periods:

```
conditional_forecast_paths;
  var rnom_obs;
  periods 1, 2;
  values 0.30, 0.807;
end;
```

2) Dynare computes a **conditional forecast** by adjusting a *controlled* shock (here monetary policy shock `e_z`):

```
conditional_forecast(parameter_set = posterior_mean,
                     controlled_varexo = (e_z),
                     replic = 3000);
```

3) Plot the results:

```
plot_conditional_forecast(periods = 10) rnom_obs pic_obs y_obs;
```

So **Figure 15 is a conditional forecast**: it answers

> “Given that `rnom_obs` must follow the specified path for periods 1–2, what does the model imply for `y_obs`?”

3. How to read Figure 15 (shaded region vs dashed lines)

In Figure 15,

- **Thick solid black line**: conditional mean path for `y_obs`
- **Thick dashed black line**: unconditional mean path for `y_obs` (benchmark)
- **Shaded grey area**: conditional predictive region (credible set) for `y_obs`
- **Dashed lines**: credible region for conditional forecasts.

Key Dynare lines (Q7)

A typical Q7 block has the structure:

```
conditional_forecast_paths;
  var rnom_obs;
  periods 1, 2;
  values 0.30, 0.807;
end;

conditional_forecast(parameter_set = posterior_mean,
                     controlled_varexo = (e_z),
                     replic = 3000);

plot_conditional_forecast(periods = 10) rnom_obs pic_obs y_obs;
```

Fixed (no draws)

- **Model parameters**: fixed at the posterior mean because of `parameter_set = posterior_mean`.
- **Initial state at the forecast origin**: fixed (consistent with the filtered/smoothed state under the chosen parameter set).
- **Imposed future path**: `rnom_obs(1)=0.30`, `rnom_obs(2)=0.807` is enforced.

Random (drawn)

- **Future shocks**: the distribution in Figure 15 is generated by drawing future innovations (after the forecast origin), with the number of simulations controlled by `replic = 3000`.

- **Controlled shock sequence**: `controlled_varexo = (e_z)` means Dynare uses the monetary policy shock to satisfy the imposed interest-rate path; the conditional distribution still reflects uncertainty from future shocks.