INSTITUTE FOR CAPACITY DEVELOPMENT

Lecture 3 – Labor Market Frictions in the NK Model

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Course on Monetary and Fiscal Policy Analysis with DSGE Models (OT26.08)

The beauty of ‘Mickey-mouse’ NK model

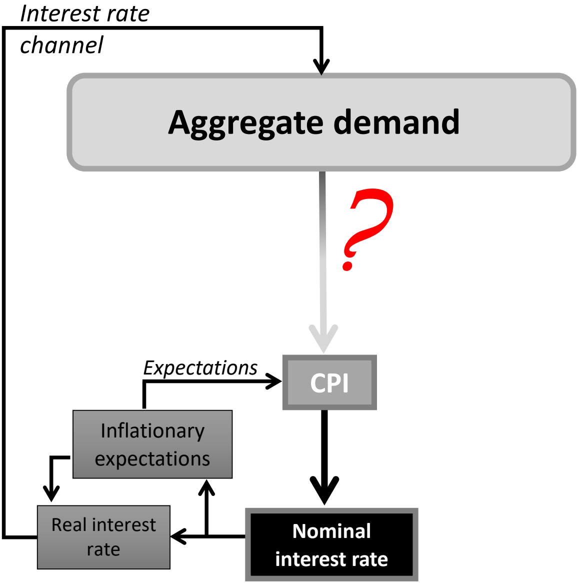
1. **The labor market in DSGE-models has a prominent role in the transmission mechanism**
   - Crucial role in the stabilization of the inflation and output gap
   - Real wage in the marginal cost proxies the aggregate demand
   - Basic New Keynesian model implies volatile wages
2. **But the real wage is rather like an acyclical variable**
   - Co-movement is weaker between the output gap and real wage
   - Inertia between the aggregate demand and inflation
3. **Common stabilization of output gap and inflation (“divine coincidence”) less trivial**
   - Reacting too strongly to inflation, not always efficient in bringing back the economy to the natural level

Content

1. Divine coincidence and basic NK-models
2. Real wage rigidity
3. Sticky wages a la Calvo
4. Unemployment in search models (Monacelli-Perotti-Trigari)

Divine coincidence and basic NK-models

New-Keynesian models in closed economy



New-Keynesian models: Households

1. Euler-equation/Dynamic IS-curve:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\sigma}(z_t - E_t z_{t+1})$$

2. Labor supply:

$$w_t - p_t = \varphi n_t + \sigma c_t$$

New-Keynesian models: Firms

3. Labor demand:

$$n_t \;=\; \frac{1}{1-\alpha}\left(y_t - a_t\right)$$

4. Marginal cost:

$$mc_t \;=\; w_t - p_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t$$

5. New-Keynesian Phillips-curve:

$$\pi_t \;=\; \lambda \hat{mc}_t + \beta E_t \pi_{t+1}$$

6. Goods market equilibrium:

$$y_t \;=\; c_t$$

where a_t is exogenously given

New-Keynesian models: Monetary policy

7. Taylor rule:

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

After some simplifications...

1. Dynamic IS-curve (substituting out consumption):

$$y_t \quad = \quad E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\sigma}(z_t - E_t z_{t+1})$$

2. Marginal cost (substituting out labor and consumption):

$$\hat{mc}_t \quad = \quad \left(\frac{\varphi + \alpha}{1-\alpha} + \sigma\right) y_t - \frac{1+\varphi}{1-\alpha} a_t$$

3. New-Keynesian Phillips-curve:

$$\pi_t \quad = \quad \lambda \hat{mc}_t + \beta E_t \pi_{t+1} + u_t$$

4. Taylor rule:

$$i_t \quad = \quad \rho + \phi_\pi \pi_t + v_t^i$$

The natural level of the economy: first best, flexible prices

1. The natural level of GDP:

$$y_t^n = \left(\frac{\varphi+\alpha}{1-\alpha}+\sigma\right)^{-1}\frac{1+\varphi}{1-\alpha}a_t$$

2. The 'natural rate of interest' from the Dynamic IS curve:

$$y_t^n = E_t y_{t+1}^n - \frac{1}{\sigma}r_t^n + \frac{1}{\sigma}\left(z_t - E_t z_{t+1}\right)$$

3. After some simplifications:

$$r_t^n = \rho - \sigma\left(\frac{\varphi+\alpha}{1-\alpha}+\sigma\right)^{-1}\frac{1+\varphi}{1-\alpha}(1-\rho^A)a_t + (1-\rho^z)z_t$$

Define the output gap (x) as difference between the total and potential GDP:

$$\tilde{y}_t = y_t - y_t^n$$

The model so far

1. Dynamic IS curve becomes:

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t^n)$$

2. Marginal cost:

$$\hat{mc}_t = \left(\frac{\varphi + \alpha}{1-\alpha} + \sigma\right)\tilde{y}_t$$

3. New-Keynesian Phillips-curve:

$$\pi_t = \lambda\hat{mc}_t + \beta E_t\pi_{t+1}$$

where $\lambda = (1-\theta)(1-\beta\theta)/\theta \cdot \frac{1}{1+\varepsilon\frac{\alpha}{1-\alpha}}$

4. Taylor rule:

$$i_t = \rho + \phi_\pi\pi_t + \phi_{\tilde{y}}\tilde{y}_t + v_t$$

Divine coincidence

Role of the central bank: anchoring the nominal variables...

1. bringing back the inflation to the target (now it is zero)
2. closing the output gap or keeping the total output at the potential or natural level

Divine coincidence in basic NK model: the monetary policy has a convenient role ...

- If the inflation and its expectation are zero/on target, the output gap is closed either
- Inflation is good proxy for an overheated economy
- In order to close the output gap, the central bank has to keep the nominal interest rate elevated until the inflation does not get back to the target

Divine coincidence does not hold if a cost-push shock hits the economy (Blanchard and Gali, 2007): Inflation increases, output gap drops:

$$\pi_t = \lambda \hat{mc}_t + \beta E_t \pi_{t+1} + u_t$$

Stability condition

- The real interest rate gap should be non-negative

$$i_t - E_t \pi_{t+1} - r_t^n > 0$$

- Enough if the monetary policy follows only the natural real interest rate?

- No! First, this reaction itself would not anchor the inflation expectations! Second, it is hard to observe the natural rate in real-time...

- We need a condition that satisfies the positive real interest rate gap? How should be design the policy rule (assuming no shocks to the natural rate) ?

Stability condition under sticky prices

- Assume a very little deviation in the inflation $\pi_t \approx E_t\pi_{t+1}$ that implies for the New-Keynesian Phillips Curve:

$$\begin{aligned} \pi_t &\approx \kappa\tilde{y}_t + \beta\pi_t \\ \tilde{y}_t &\approx \frac{1-\beta}{\kappa}\pi_t \end{aligned}$$

where $\kappa = \lambda \cdot \left(\frac{\varphi+\alpha}{1-\alpha} + \sigma\right)$

- Substituting out the interest rate, expectation and output gap in the condition above:

$$\begin{aligned} \phi_\pi\pi_t + \phi_{\tilde{y}}\tilde{y}_t - E_t\pi_{t+1} &> 0 \\ (\phi_\pi - 1)\pi_t + \phi_{\tilde{y}}\frac{1-\beta}{\kappa}\pi_t &> 0 \\ \kappa(\phi_\pi - 1) + \phi_x(1-\beta) &> 0 \end{aligned}$$

- β very close to 1 and $\phi_{\tilde{y}}$ close to zero, then $\phi_\pi > 1$ should satisfy

Real wage rigidity

Real wage rigidity

1. Basic NK model: the real wage adjustment immediately follows the volatility of output gap
2. Empirical fact: the wages are not procyclical and react with some delay to the real economic activity
   - It has impact on the monetary transmission mechanism and the dynamics of inflation

Rigid real wages

- Simplification: constant return to scale ($\alpha = 0$) and $\hat{mc}_t = \hat{w}_t - a_t$
- Modified labor supply curve (Blanchard and Gali, 2007):

$$\hat{w}_t = \rho^w \hat{w}_{t-1} + (1-\rho^w) \underbrace{(\varphi n_t + \sigma c_t)}_{'MRS'}$$

where real wage is $\hat{w}_t = w_t - p_t$.

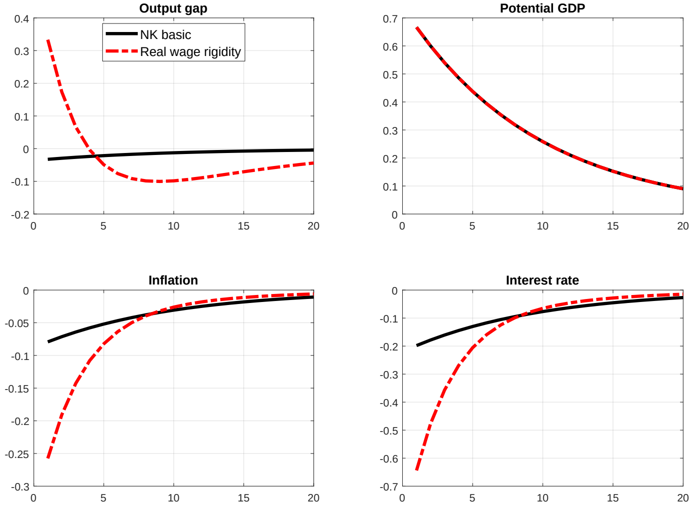
- Real marginal cost function becomes:

$$\begin{aligned}
\hat{mc}_t + a_t &= \rho^w (\hat{mc}_{t-1} + a_{t-1}) + (1-\rho^w) ((\varphi+\sigma) y_t - \varphi a_t) \\
\hat{mc}_t &= \rho^w (\hat{mc}_{t-1} - \Delta a_t) + (1-\rho^w) ((\varphi+\sigma) y_t - (1+\varphi) a_t) \\
\hat{mc}_t &= \rho^w (\hat{mc}_{t-1} - \Delta a_t) + (1-\rho^w)(\varphi+\sigma)\tilde{y}_t
\end{aligned}$$

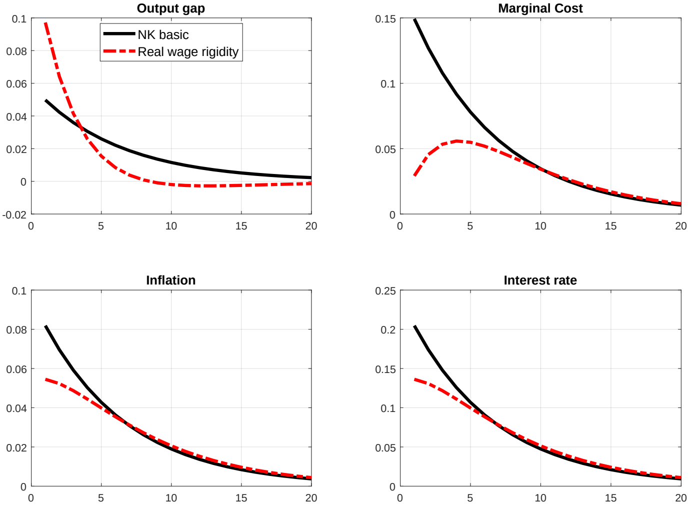
- The marginal cost (and then the inflation) is the function of the current and previous output gaps and changes in the productivity:

$$\hat{mc}_t = \sum_{i=0}^{\infty} \rho^{wi} \left\{ (1-\rho^w)(\varphi+\sigma)\tilde{y}_{t-i} - \rho^w \Delta \hat{A}_{t-i} \right\}$$

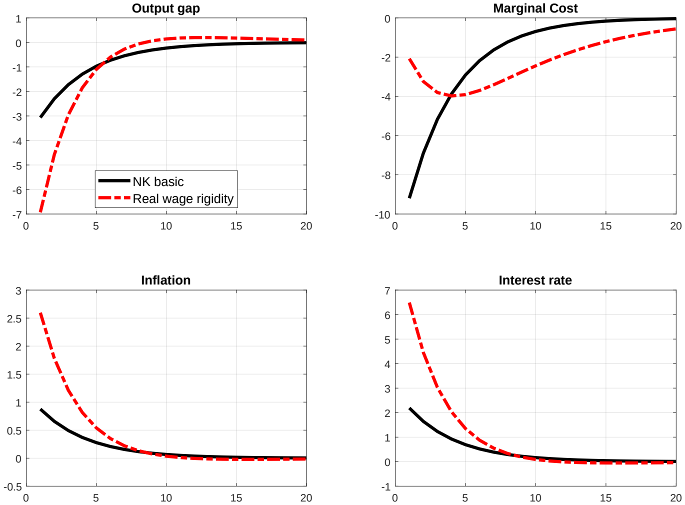
Productivity shock



Demand shock



Supply shock



Results

1. No 'divine coincidence': the marginal cost (sticky price level) is not disconnected from the productivity shock (and natural level), then the inflation is directly affected by the changes of productivity...
    - The direct link between the productivity and inflation enlarges the disinflationary pressure by positive productivity shock... in the original model the second round effect via the real natural rate affected the output gap and the inflation
    - Based on the mark-up shock, strong reaction to the inflation only is not trivial: generates huge real economic costs, especially, if the central bank wants to close the output gap

      Output gap stabilization should be more important
2. Inertia
    - Due to the lag reaction, the accommodation in marginal cost (and inflation) takes more time

      And cumulate the past output gaps and productivity changes
    - Then the strong reaction to the inflation may generate long lasting negative output gap and negative inflation later

Nominal wage stickiness

Nominal wage setting a la Calvo

Monopolistic competition on the labor market, labor union collects the individual labor force ($N_t(j)$) and supplies it to the firms (N_t):

$$N_t = \left[\int_0^1 N_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

- Households in the labor union have market power
- Monopolistic competition: the households set such a wage that maximizes their own utilities
- Compared to the perfect competition, their wage is adjusted by mark-up component
- The optimization is subject to the household's budget constraint and individual labor demand for household j's labor force

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t$$

Nominal wage setting a la Calvo

Nominal wage setting is not possible in every period (or costly)

- Calvo: $1-\theta_w$ share of households are able to set the new wage in the period t

$$\sum_{n=0}^{\infty} \theta_w^n \beta^n U_{t+n}(j) \left\{ C_{t+n}(W_t^*(j)), N_{t+n}(W_t^*(j)) \right\} \longrightarrow \max_{W_t^*(j)}$$

  where θ_w of the probability of household j can not set a new wage
- In Rotemberg-case the nominal wage setting is coupled with additional adjustment cost

Optimization (1)

Lagrangian:

$$\mathcal{L} = \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ \frac{C_{t+n}(j)^{1-\sigma}}{1-\sigma} - \Psi \frac{N_{t+n}(j)^{1+\varphi}}{1+\varphi} \right\} \longrightarrow \max_{W_t^*(j)}$$

subject to

$$\begin{aligned} P_{t+n}C_{t+n}(j) &= W_t^*(j)N_{t+n}(j) + others \\ N_{t+n}(j) &= \left(\frac{W_t^*(j)}{W_{t+n}}\right)^{-\varepsilon_w} N_{t+n} \end{aligned}$$

Optimization (2)

First-order condition:

$$\frac{\partial \mathcal{L}}{\partial W_t^*(j)} = \sum_{n=0}^{\infty} \theta_w^n \beta^n \left\{ C_{t+n}(j)^{-\sigma} \frac{\partial C_{t+n}(j)}{\partial W_t^*(j)} - \Psi N_{t+n}(j)^{\varphi} \frac{\partial N_{t+n}(j)}{\partial W_t^*(j)} \right\} = 0$$

where

$$\begin{aligned}
\frac{\partial C_{t+n}(j)}{\partial W_t^*(j)} &= \frac{1}{P_{t+n}} N_{t+n}(j) + \frac{1}{P_{t+n}} W_t^*(j) \frac{\partial N_{t+n}(j)}{\partial W_t^*(j)} \\
\frac{\partial N_{t+n}(j)}{\partial W_t^*(j)} &= -\varepsilon_w \left(\frac{W_t^*(j)}{W_{t+n}} \right)^{-\varepsilon_w - 1} \frac{1}{W_{t+n}} N_{t+n}
\end{aligned}$$

Optimization (3)

First-order condition:

$$\sum_{n=0}^{\infty} \theta_w^n \beta^n \Bigg\{ C_{t+n}(j)^{-\sigma} \left(\frac{W_t^*(j)}{P_{t+n}} \left(\frac{W_t^*(j)}{W_{t+n}} \right)^{-\varepsilon_w} N_{t+n} - \right.$$

$$\left. -\varepsilon_w \frac{W_t^*(j)}{P_{t+n}} \left(\frac{W_t^*(j)}{W_{t+n}} \right)^{-\varepsilon_w} N_{t+n} \right)$$

$$+\varepsilon_w \Psi \left(\frac{W_t^*(j)}{W_{t+n}} \right)^{-\varepsilon_w(1+\varphi)} N_{t+n}^{1+\varphi} \Bigg\} = 0$$

Optimal wage:

$$W_t^*(j) = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\sum_{n=0}^{\infty} \theta_w^n \beta^n \Psi \left(\frac{W_t^*(j)}{W_{t+n}} \right)^{-\varepsilon_w(1+\varphi)} N_{t+n}^{1+\varphi}}{\sum_{n=0}^{\infty} \theta_w^n \beta^n C_{t+n}(j)^{-\sigma} \frac{1}{P_{t+n}} \left(\frac{W_t^*(j)}{W_{t+n}} \right)^{-\varepsilon_w} N_{t+n}}$$

Optimization (4)

Total wage index can be given from the composite term:

$$
\begin{aligned}
W_t &= \left[(1-\theta_w)W_t^*(j)^{1-\varepsilon_w} + \int_0^{\theta_w} W_t(j)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}} \\
&= \left[(1-\theta_w)W_t^*(j)^{1-\varepsilon_w} + \theta_w W_{t-1}^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}
\end{aligned}
$$

Log-linearized equations

- After long derivations (see Erceg, Henderson, and Levin, JME, 2000)
- New-Keynesian Nominal Wage Phillips curve:

$$\pi_t^w = \frac{(1-\theta_w)(1-\beta\theta_w)}{(1+\varepsilon_w\varphi)\theta_w}\left\{\sigma c_t + \varphi n_t - \hat{w}_t\right\} + \beta E_t \pi_{t+1}^w$$

  where the wage inflation:

$$\pi_t^w = \pi_t + \hat{w}_t - \hat{w}_{t-1}$$

- Under sticky wages and sticky prices, the real wage should be also sticky (real wage rigidity)

Model with sticky prices and wages (1)

- Dynamic IS curve:

$$y_t \quad = \quad E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\sigma}\left(1 - \rho^z\right) z_t$$

- New-Keynesian Nominal Wage Phillips curve:

$$\pi_t^w = \kappa_w \Big\{ (\sigma + \varphi) y_t - \varphi a_t - \hat{\omega}_t \Big\} + \beta E_t \pi_{t+1}^w$$

- The wage inflation:

$$\pi_t^w = \pi_t + \hat{\omega}_t - \hat{\omega}_{t-1}$$

Model with sticky prices and wages (2)

- Marginal cost (substituting out labor and consumption):

$$\hat{mc}_t = \hat{w}_t - a_t$$

- New-Keynesian Phillips-curve:

$$\pi_t = \kappa \hat{mc}_t + \beta E_t \pi_{t+1} + u_t$$

- Taylor rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_{\pi_w} \pi_t^w + \phi_{\tilde{y}} \tilde{y}_t + v_t$$

With gaps

- Dynamic IS curve:

$$\tilde{y}_t \quad = \quad E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n) + \frac{1}{\sigma}(1 - \rho^z) z_t$$

- New-Keynesian Nominal Wage Phillips curve:

$$\pi_t^w = \kappa_w \Big\{ (\sigma + \varphi) \tilde{y}_t - \tilde{\omega}_t \Big\} + \beta E_t \pi_{t+1}^w$$

- The wage inflation:

$$\pi_t^w = \pi_t + a_t - a_{t-1} + \tilde{\omega}_t - \tilde{\omega}_{t-1}$$

- New-Keynesian Phillips-curve:

$$\pi_t \quad = \quad \kappa \tilde{\omega}_t + \beta E_t \pi_{t+1} + u_t$$

- Monetary policy rule:

$$i_t \quad = \quad \rho + \phi_\pi \pi_t + \phi_{\pi_w} \pi_t^w + \phi_{\tilde{y}} \tilde{y}_t + v_t$$

Stability condition under sticky prices and wages

- The real interest rate gap should be positive

$$i_t - E_t\pi_{t+1} - r_t^n > 0$$

- Assume a very little deviation in the inflations and gaps:

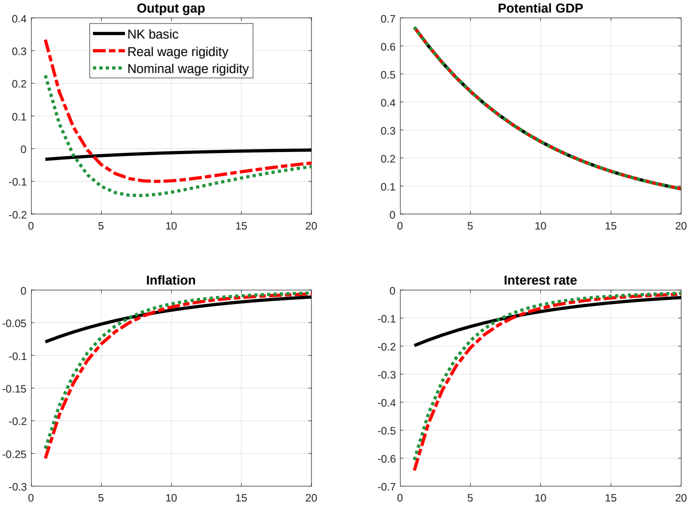
$$\begin{aligned}
\pi_t &\approx \kappa\tilde{w}_t + \beta\pi_t \Rightarrow \tilde{w}_t \approx \frac{1-\beta}{\kappa}\pi_t \\
\pi_t^w &\approx \kappa_w\Big\{(\sigma+\varphi)\tilde{y}_t - \tilde{w}_t\Big\} + \beta\pi_t^w \Rightarrow \tilde{y}_t \approx \frac{1}{\sigma+\varphi}\left[\frac{1-\beta}{\kappa_w}\pi_t^w + \tilde{w}_t\right] \\
\pi_t^w &\approx \pi_t
\end{aligned}$$

- Substituting out everything from the interest rate condition above:

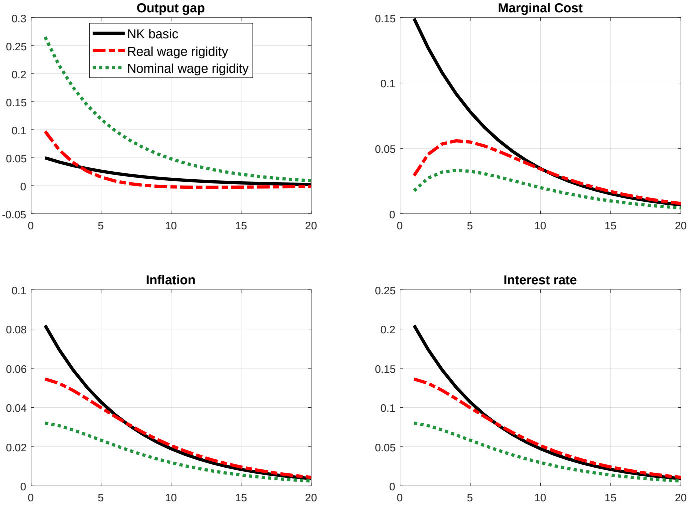
$$\begin{aligned}
\phi_\pi\pi_t + \phi_{\pi_w}\pi_t^w + \phi_{\tilde{y}}\tilde{y}_t - E_t\pi_{t+1} &> 0 \\
\phi_\pi\pi_t + \phi_{\pi_w}\pi_t + \phi_{\tilde{y}}\frac{1}{\sigma+\varphi}\left[\frac{1-\beta}{\kappa_w}\pi_t + \frac{1-\beta}{\kappa}\pi_t\right] - \pi_t &> 0 \\
\phi_\pi + \phi_{\pi_w} + \phi_{\tilde{y}}\frac{1-\beta}{\sigma+\varphi}\left[\frac{1}{\kappa_w} + \frac{1}{\kappa}\right] &> 1
\end{aligned}$$

- β close to 1, ϕ_x close to zero, then $\phi_\pi + \phi_{\pi_w} > 1$ should satisfy

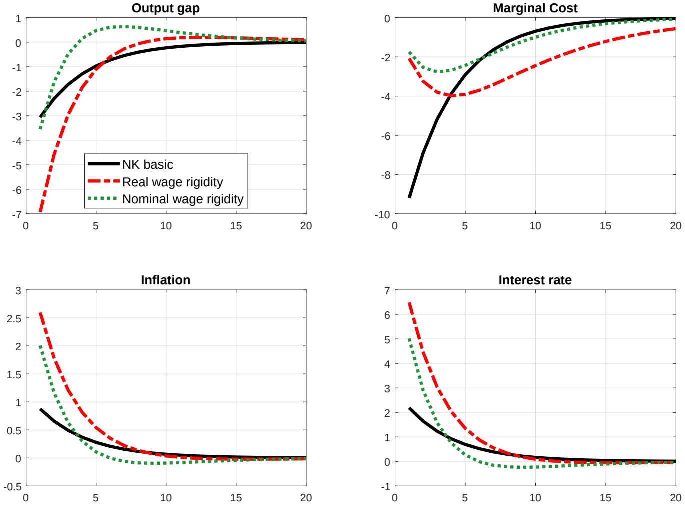
Productivity shock



Demand shock



Supply shock

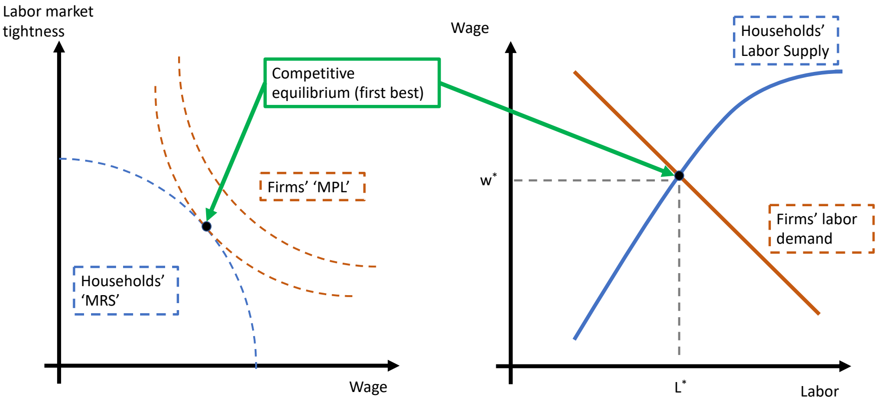


Unemployment in DSGE-models

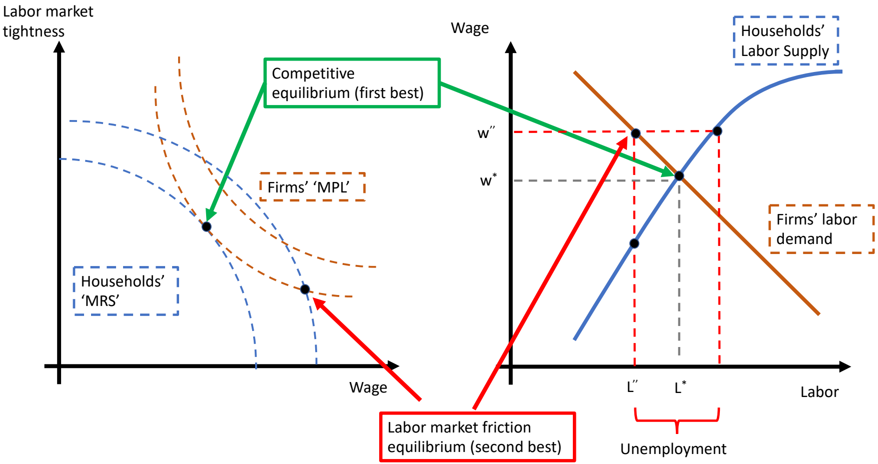
Unemployment a la Monacelli-Perotti-Trigari (2010)

1. Walrasian labor market (implicit assumptions so far):
   - Households' ‘leisure’ decision is consistent with the utility maximization
   - Unemployment is voluntary
2. Model with unemployment and Nash-bargaining:
   - Employment status follow a 'life-cycle': hiring, separation, dismissal, unemployment and re-hiring
   - Unemployment is not voluntary, results of labor market frictions and different interest of stakeholders
   - Bargaining power determines the equilibrium (suboptimal) wages and (un)employment status

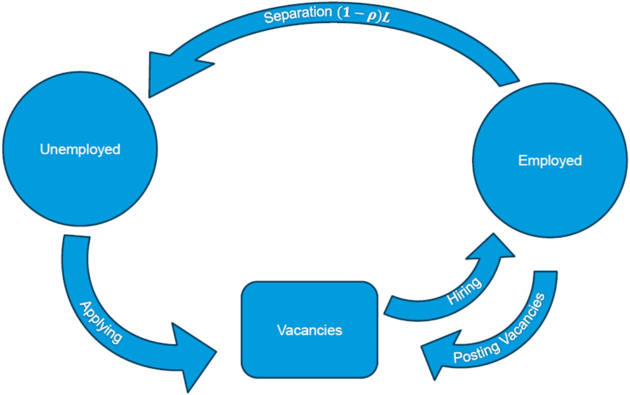
Competitive equilibrium



Non-Walrasian labor market equilibrium



Labor search



Labor market (1)

- Each firm employs L_t workers in the current period, but also hires new employees for the vacancies V_t
- The available or unemployed labor force is noted by U_t, and the population is is normalized to unity

$$U_t = 1 - L_{t-1}$$

where n_{t-1} denotes that the unemployment accounts the end of last period employment.

- The tightness of the labor market (vacancies vs unemployed population) is defined as

$$\omega_t = \frac{V_t}{U_t}$$

- The 'matches' quantifies potential link between the vacancy and unemployment (available free labor). Assume a Cobb-Douglas function:

$$M_t = \gamma_m U_t^{\gamma} V_t^{1-\gamma}$$

Labor market (2)

- Probability of firm finds new employee (based on the matching function):

$$q_t = \frac{M_t}{V_t} = \gamma_m U_t^{\gamma} V_t^{-\gamma} = \gamma_m \omega_t^{-\gamma}$$

  What does it mean? The probability of filling vacancy is lower if the tightness is higher (every firms look for new employees and it is **hard** to find a 'match')
- Probability of unemployed worker finds a job:

$$p_t = \frac{M_t}{U_t} = \gamma_m U_t^{\gamma-1} V_t^{1-\gamma} = \gamma_m \omega_t^{1-\gamma}$$

  What does it mean? The probability of hiring is higher if the tightness is higher (every firms look for new employees or the unemployment is low and it is **easy** to find a 'match')
- These probabilities are given for the firms and households.
- Introducing ρ, it denotes the probability if the worker 'survives' with the firm for the next period, $1-\rho$ is the probability of the separation

Firms (1)

- The firms i demand labor for its production but in every period $1-\rho$ of the labor force is separated or dismissed and the firm should search (and 'match') additional labor-force to fill the potential vacancy:

$$L_t(i) = \rho L_{t-1}(i) + M_t(i) = \rho L_{t-1}(i) + q_t V_t(i)$$

- Firm maximizes its profit and hires labor for production (with Rotemberg pricing):

$$\begin{aligned}
\mathcal{L} &= \sum_{n=0}^{\infty} \Delta_{t,t+n} \Bigg\{ P_{t+n}(i) Y_{t+n}(i) - W_{t+n} L_{t+n}(i) - P_{t+n}\kappa V_{t+n}(i) - \\
&- P_{t+n} Y_{t+n} R\left(\frac{P_{t+n}(i)}{P_{t+n-1}(i)}\right) \Bigg\} \longrightarrow \max_{P_t(i), L_t(i), V_t(i)}
\end{aligned}$$

subject to

$$\begin{aligned}
Y_{t+n}(i) &= \left(\frac{P_{t+n}(i)}{P_{t+n}}\right)^{-\varepsilon} Y_{t+n} \\
L_{t+n}(i) &= \rho L_{t+n-1}(i) + q_{t+n} V_{t+n}(i) \\
Y_{t+n}(i) &= A_{t+n} L_{t+n}(i)
\end{aligned}$$

where κ express the cost of keeping unfilled vacancy. $R(\cdot)$ Rotemberg cost

First-order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_t(i)} &= \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t - \varepsilon \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1} \frac{P_t(i)}{P_t} Y_t - P_t Y_t R' \left(\frac{P_t(i)}{P_{t-1}(i)}\right) \frac{1}{P_{t-1}(i)} + \\
&+ \varepsilon \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon-1} \frac{MC_t}{P_t} Y_t + \Delta_{t,t+1} P_{t+1} Y_{t+1} R' \left(\frac{P_{t+1}(i)}{P_t(i)}\right) \frac{P_{t+1}(i)}{P_t(i)^2} = 0 \\
\frac{\partial \mathcal{L}}{\partial L_t(i)} &= -W_t + MC_t A_t - \lambda_t^H + \Delta_{t,t+1} \rho \lambda_{t+1}^H = 0 \\
\frac{\partial \mathcal{L}}{\partial V_t(i)} &= -P_t \kappa + \lambda_t^H q_t = 0
\end{aligned}$$

Firms (3)

Optimal price:

$$\frac{\varepsilon}{\varepsilon-1}mc_t+\frac{1}{1+r_t}\frac{\phi_P}{\varepsilon-1}\frac{Y_{t+1}}{Y_t}\pi_{t+1}\left(1+\pi_{t+1}\right)=1+\frac{\phi_P}{\varepsilon-1}\pi_t\left(1+\pi_t\right)$$

Value of having additional labor force:

$$\lambda_t^H=\frac{P_t\kappa}{q_t}$$

Labor demand (MPL_t):

$$MPL_t \quad : \quad \frac{\kappa}{q_t}=mc_tA_t-w_t+\frac{1}{1+r_t}\rho\frac{\kappa}{q_{t+1}}$$

The production function

$$Y_t=A_tL_t$$

Households (1)

The households maxmize their lifetime utility:

$$\mathcal{L} = \sum_{n=0}^{\infty} \beta^n \left\{ \frac{C_{t+n}^{1-\sigma}}{1-\sigma} - \Psi \frac{L_{t+n}^{1+\varphi}}{1+\varphi} \right\} \longrightarrow \max_{C_t, L_t}$$

And the employment of the households are determined by

$$L_t = \rho L_{t-1} + p_t (1 - L_{t-1})$$

And their usual budget constraint is:

$$P_t C_t + B_t \quad = \quad W_t L_t + (1 + i_{t-1}) B_{t-1}$$

Households (2)

Lagrangian:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{n=0}^{\infty} \beta^n \left\{ \frac{C_{t+n}^{1-\sigma}}{1-\sigma} - \Psi \frac{L_{t+n}^{1+\varphi}}{1+\varphi} \right. \\
& + \lambda_{t+n} \left(W_{t+n} L_{t+n} + (1 + i_{t+n-1}) B_{t+n-1} - P_{t+n} C_{t+n} - B_{t+n} \right) \\
& + \left. \mu_{t+n} \left(\rho L_{t+n-1} + p_{t+n} (1 - L_{t+n-1}) - L_{t+n} \right) \right\}
\end{aligned}
$$

First-order condition:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= C_t^{-\sigma} - \lambda_t P_t = 0 \\
\frac{\partial \mathcal{L}}{\partial L_t} &= -\Psi L_t^{\varphi} + \lambda_t W_t - \mu_t + \beta \mu_{t+1} (\rho - p_{t+1}) = 0 \\
\frac{\partial \mathcal{L}}{\partial B_t} &= -\lambda_t + \beta \lambda_{t+1} (1 + i_t) = 0
\end{aligned}
$$

Households (3)

Optimal behavioral equations:

$$\begin{aligned} \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1+i_t}{1+\pi_{t+1}} &= 1 \\ \mu_t &= C_t^{-\sigma} w_t - \Psi L_t^{\varphi} + \beta \mu_{t+1}(\rho - p_{t+1}) \end{aligned}$$

The last equation is very similar to the labor supply from the original model:

$$\mu_t = C_t^{-\sigma} \left(w_t - \Psi \frac{L_t^{\varphi}}{C_t^{-\sigma}} \right) + \beta \mu_{t+1}(\rho - p_{t+1})$$

where the $\Psi \frac{L_t^{\varphi}}{C_t^{-\sigma}}$ is the intertemporal 'MRS' condition:

$$\mu_t = C_t^{-\sigma} \left(w_t - MRS_t \right) + \beta \mu_{t+1}(\rho - p_{t+1})$$

But the μ interpretation is more: it shows the households benefit of the current and expected employment.

Nash bargaining (1)

Households and firms choose such a common wage, that maximizes their joint welfare function:

$$\mu_t^{\eta} MPL_t^{1-\eta} \longrightarrow \max_{w_t}$$

η denotes the bargaining power in the decision (the closer to one the stronger households' power).

First order condition (and optimal wage contract):

$$\eta \mu_t^{\eta-1} MPL_t^{1-\eta} \frac{\partial \mu_t}{\partial w_t} + (1-\eta)\mu_t^{\eta} MPL_t^{-\eta} \frac{\partial MPL_t}{\partial w_t} = 0$$

Based on the previously derived decision function:

$$\frac{\partial MPL_t}{\partial w_t} = -1 \text{ and } \frac{\partial \mu_t}{\partial w_t} = C_t^{-\sigma}$$

If we rearrange, we get the equilibrium condition

$$(1-\eta)\frac{\mu_t}{C_t^{-\sigma}} \quad = \quad \eta MPL_t$$

Nash bargaining (2)

In the second best equilibrium:

- The firms may decide to hire relatively less (compared to the frictionless case) labor force and pay less wage (thereby improves its profit),
- The households offer less labor force as they have to evaluate their decision based on the future benefit of employment or the expected labor

The Nash-bargaining also tell us how to divide the surplus among the agents:

$$S_t = MPL_t + \frac{\mu_t}{C_t^{-\sigma}}$$

Based on the equilibrium condition:

$$\begin{aligned} S_t &= MPL_t + \frac{\eta}{1-\eta} MPL_t \\ (1-\eta) S_t &= MPL_t \end{aligned}$$

Nash bargaining (3)

Combining the surplus with the labor demand and supply functions:

$$\begin{aligned}(1-\eta)S_t &= mc_t A_t - w_t + \frac{1}{1+r_t}\rho(1-\eta)S_{t+1} \\ \eta S_t &= w_t - MRS_t + \eta S_{t+1}\frac{1}{1+r_t}(\rho - p_{t+1})\end{aligned}$$

We can express the surplus if we add these two equations:

$$S_t = mc_t A_t - MRS_t + \frac{S_{t+1}}{1+r_t}(\rho - \eta p_{t+1})$$

And we can substitute out the S with $(1-\eta)S_t = MPL_t = \frac{\kappa}{q_t}$ to express the equilibrium condition in terms of the firm value:

$$\frac{\kappa}{q_t} = (1-\eta)(mc_t A_t - MRS_t) + \frac{1}{1+r_t}(\rho - \eta p_{t+1})\frac{\kappa}{q_{t+1}}$$

Model equations

1. Euler equation:

$$\beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1+i_t}{1+\pi_{t+1}} \quad = \quad 1$$

2. Labor market equilibrium:

$$\frac{\kappa}{q_t} \quad = \quad (1-\eta)(mc_t A_t - MRS_t) + \frac{1}{1+r_t}(\rho - \eta p_{t+1})\frac{\kappa}{q_{t+1}}$$

3. MRS condition

$$MRS_t = \Psi \frac{L_t^{\varphi}}{C_t^{-\sigma}}$$

4. Labor demand:

$$\frac{\kappa}{q_t} = mc_t A_t - w_t + \frac{1}{1+r_t}\rho\frac{\kappa}{q_{t+1}}$$

Model equations

5. Production function:

$$Y_t = A_t L_t$$

6. Non-linear New-Keynesian Phillips-curve:

$$\frac{\varepsilon}{\varepsilon-1} mc_t + \frac{1}{1+r_t} \frac{\phi_P}{\varepsilon-1} \frac{Y_{t+1}}{Y_t} \pi_{t+1} \left(1+\pi_{t+1}\right) = 1 + \frac{\phi_P}{\varepsilon-1} \pi_t \left(1+\pi\right.$$

7. Flow of employment:

$$L_t = \rho L_{t-1} + q_t V_t$$

8. Unemployment

$$U_t = 1 - L_{t-1}$$

9. The tightness of the labor market:

$$\omega_t = \frac{V_t}{U_t}$$

Model equations

10. Probability of firm finds new employee

$$q_t = \gamma_m \omega_t^{-\gamma}$$

11. Probability of unemployed worker finds a job:

$$p_t = \gamma_m \omega_t^{1-\gamma}$$

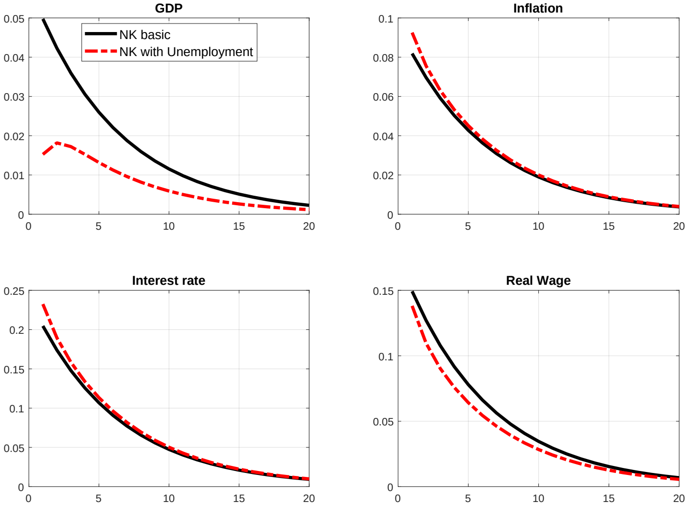
12. Taylor-rule:

$$(1 + i_t) = \frac{1}{\beta}(1 + \pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_x} e^{\xi_t^i}$$

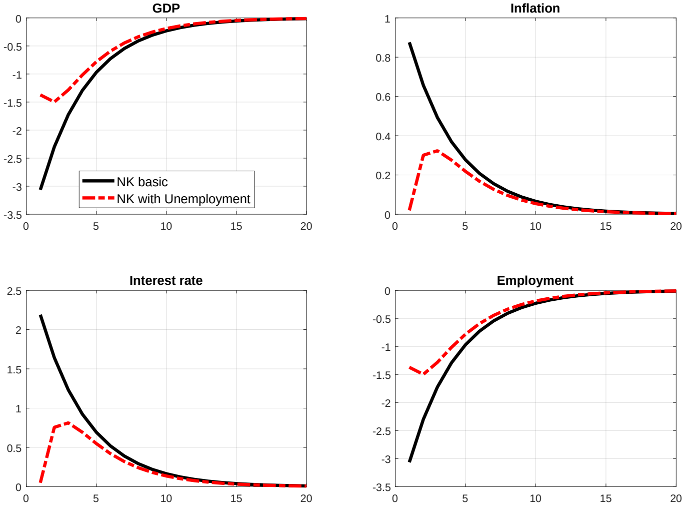
13. Good market equilibrium:

$$Y_t = C_t$$

Demand shock



Supply shock (cost-push shock)



What do we learn from this?

1. Hump-shaped reaction in aggregate demand (due to the inertia in employment)
2. Cost-push shock: the hiring cost are lower (less tight labor market), partly offset the inflationary pressure
3. Potential next step: merge the *wage rigidity* (makes the marginal cost more persistent) and *unemployment* (makes the aggregate demand more hump-shaped)

Thank you for your attention!