L5 - Derivation of Open Economy Model

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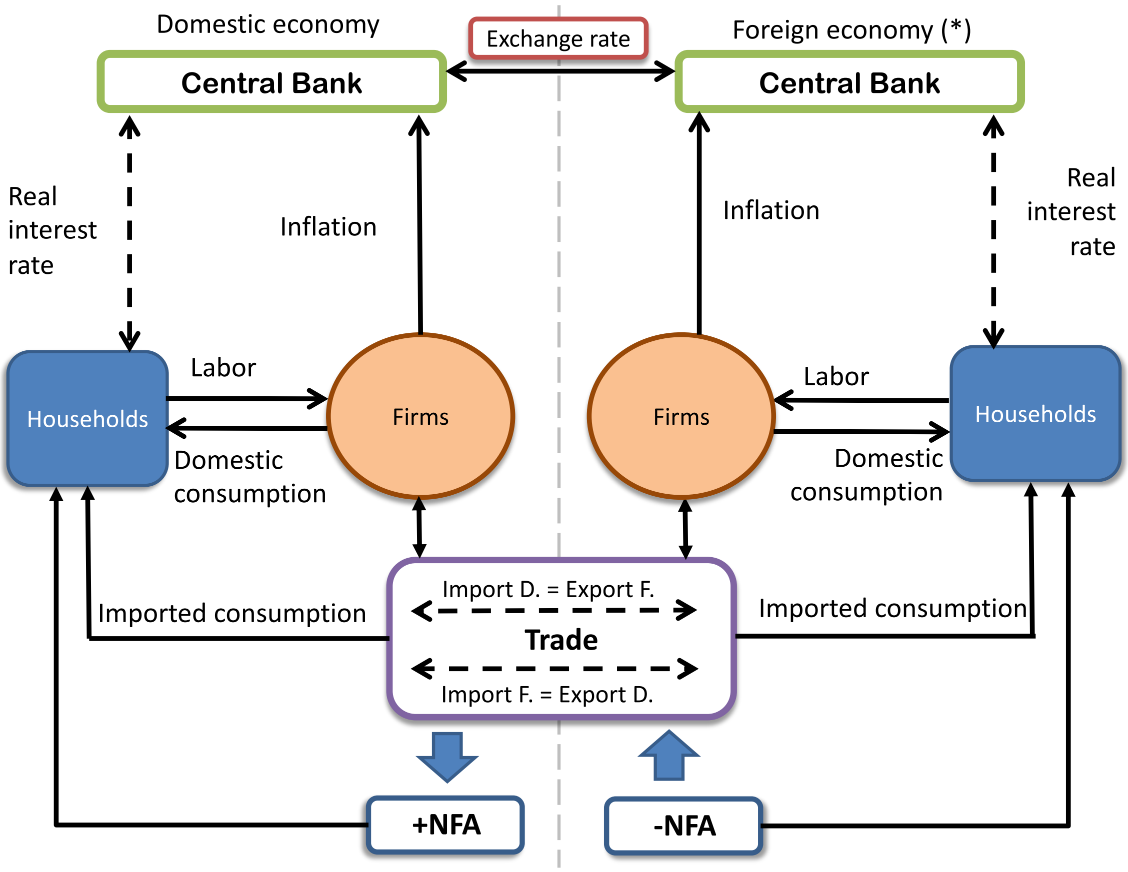
1 Derivation of the open economy version

The agents' behavior in the two country is the same: the households maximize their utilities; firms set their nominal prices; and the central banks set the interest rate by reacting to increasing inflationary pressure. If the domestic variable x is adjusted by *, it means that x^* is the same macroeconomic variable but belongs to the foreign economy.

The following flowchart summarizes the main agents, variables and channels of the model. As we mentioned above initially we assume 2 symmetric countries. We derive all equations from the perspective of the domestic economy.



1.1 Households' optimization

1. Utility maximization over the aggregate variables:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \exp\{g_t\} \left\{ \frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \zeta_L \frac{L_t^{1+\sigma_L}}{1+\sigma_L} \right\}$$

subject to

$$T_t + W_t L_t + R_{t-1} B_{t-1} - R^*_{t-1} \psi_{t-1} e_t B^*_{t-1} = B_t - e_t B^*_t + P_t C_t$$

where e is the nominal exchange rate, B and B^*_F denote domestic and foreign bonds (net foreign assets) respectively, R and R^* assign the domestic and foreign gross interest rate respectively, P is the nominal price level of consumption basket, C is the consumption basket of the representative households, W and L denote the nominal wage and labor supply respectively (as a simplification we do not assume any nominal or real wage rigidity), ψ is the risk premium, σ is the intertemporal elasticity of substitution, σ_L inverse of Frisch elasticity, T net lump-sum transfer or profit from the firms.

2. Optimal (cost-minimizing or utility-maximizing) combination of domestic and for-

eign goods

$$C_t = \left[(1-\alpha_C)^{\frac{1}{\eta_C}} {C_{H,t}}^{1-\frac{1}{\eta_C}} + (\alpha_C)^{\frac{1}{\eta_C}} {C_{F,t}}^{1-\frac{1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}}$$

where C_H and C_F are the domestic and foreign goods, α_C reflects their preference toward domestic goods, η_C denotes the elasticity of substitution between foreign and home good.

The risk premium is determined by the foreign investors:

$$\psi_t = (1 + \psi_B B^*_{F,t})e^{\varphi_t}$$

And ϕ is an exogenous AR(1) shock process.

Optimization problem (1): aggregate variables

Lagrangian of the utility maximization:

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \exp\{g_t\} \left\{ \frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \zeta_L \frac{L_t^{1+\sigma_L}}{1+\sigma_L} \right\} \\ &+ \lambda_t \left(T_t + W_t L_t + R_{t-1}B_{t-1} - R^*_{t-1}\psi_{t-1}e_t B^*_{t-1} - B_t + e_t B^*_t - P_t C_t \right) \end{aligned}$$

First-order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t} &= \beta^t \exp\{g_t\} C_t^{-1/\sigma} - \lambda_t P_t = 0 \\ \frac{\partial \mathcal{L}}{\partial L_t} &= -\beta^t \exp\{g_t\} \zeta_L L_t^{\sigma_L} + \lambda_t W_t = 0 \\ \frac{\partial \mathcal{L}}{\partial B_t} &= -\lambda_t + E_t \lambda_{t+1} R_t = 0 \\ \frac{\partial \mathcal{L}}{\partial B^*_t} &= \lambda_t e_t - E_t \lambda_{t+1} e_{t+1} R^*_t \psi_t = 0 \end{aligned}$$

Optimization problem (2): the composition of the consumption basket

Lagrangian of the cost minimaztion:

$$\begin{aligned}\mathcal{L} &= P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \\ &+ P_t\left(C_t - \left[(1-\alpha_C)^{\frac{1}{\eta_C}} C_{H,t}{}^{1-\frac{1}{\eta_C}} + (\alpha_C)^{\frac{1}{\eta_C}} C_{F,t}{}^{1-\frac{1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C - 1}}\right)\end{aligned}$$

where P_H is the domestic producer price index, P_F is the foreign price is the function of the nominal exchange rate and foreign producers price index $P_{F,t} = e_t P^*_{H,t}$.

First-order conditions:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial C_{H,t}} &= P_{H,t} - P_t C_t^{\frac{1}{\eta_C}} (1-\alpha_C)^{\frac{1}{\eta_C}} C_{H,t}{}^{-\frac{1}{\eta_C}} = 0 \\ \frac{\partial \mathcal{L}}{\partial C_{F,t}} &= P_{F,t} - P_t C_t^{\frac{1}{\eta_C}} (\alpha_C)^{\frac{1}{\eta_C}} C_{F,t}{}^{-\frac{1}{\eta_C}} = 0\end{aligned}$$

Households' behavior can be given by the following equations:

Euler equation (Dynamic IS curve):

$$\beta E_t \frac{\exp\{g_{t+1}\}}{\exp\{g_t\}} \frac{(C_{t+1})^{-1/\sigma}}{(C_t)^{-1/\sigma}} R_t \frac{P_t}{P_{t+1}} = 1$$

UIP condition:

$$E_t R_t^* \psi_t \frac{e_{t+1}}{e_t} = R_t$$

Labor supply curve:

$$\zeta_L L_t^{\sigma_L} C_t^{1/\sigma} = \frac{W_t}{P_t}$$

Demand for domestic good:

$$C_{H,t} = (1-\alpha_C)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta_C} C_t$$

Demand for foreign good:

$$C_{F,t} = \alpha_C \left(\frac{P_{F,t}}{P_t}\right)^{-\eta_C} C_t$$

We can plug back the demand for home and foreign goods to the CES function

$$
\begin{aligned}
C_t &= \left[(1-\alpha_C)^{\frac{1}{\eta_C}} {C_{H,t}}^{1-\frac{1}{\eta_C}} + (\alpha_C)^{\frac{1}{\eta_C}} {C_{F,t}}^{1-\frac{1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}} \\
C_t^{1-\frac{1}{\eta_C}} &= (1-\alpha_C)^{\frac{1}{\eta_C}} \left((1-\alpha_C)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta_C} C_t\right)^{1-\frac{1}{\eta_C}} \\
&+ (\alpha_C)^{\frac{1}{\eta_C}} \left(\alpha_C\left(\frac{P_{F,t}}{P_t}\right)^{-\eta_C} C_t\right)^{1-\frac{1}{\eta_C}} \\
1 &= (1-\alpha_C)^{\frac{1}{\eta_C}} \left((1-\alpha_C)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta_C}\right)^{1-\frac{1}{\eta_C}} \\
&+ (\alpha_C)^{\frac{1}{\eta_C}} \left(\alpha_C\left(\frac{P_{F,t}}{P_t}\right)^{-\eta_C}\right)^{1-\frac{1}{\eta_C}} \\
1 &= (1-\alpha_C)\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta_C} + (\alpha_C)\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta_C} \\
P_t^{1-\eta_C} &= (1-\alpha_C)P_{H,t}^{1-\eta_C} + (\alpha_C)P_{F,t}^{1-\eta_C}
\end{aligned}
$$

Plugging back the consumptions into the CES function, the consumer price index is:

$$
P_t = \left[(1-\alpha_C)P_{H,t}^{1-\eta_C} + (\alpha_C)P_{F,t}^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}
$$

The households' equations are the following (after the log-linearization):

$$
\begin{aligned}
g_{t+1} - g_t - \frac{1}{\sigma}(c_{t+1} - c_t) + r_t - E_t\pi_{t+1} &= 0 \\
r_t^* + \psi_t + e_{t+1} - e_t &= r_t \\
\sigma_L l_t + \frac{1}{\sigma} c_t &= w_t \\
c_{H,t} &= -\eta_C(p_{H,t} - p_t) + c_t \\
c_{F,t} &= -\eta_C(p_{F,t} - p_t) + c_t \\
p_t &= (1-\alpha_C)p_{H,t} + \alpha_C p_{F,t}
\end{aligned}
$$

And the risk premium is given:

$$
\psi_t = \psi_B b_t^* + \varphi_t
$$

1.2 Firms' optimization

Domestic final good is demanded by the domestic and foreign households (export)

The production of the domestic final good is a composite of intermediate individual goods, we apply a standard Dixit-Stiglitz aggregator function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

The domestic final good producer's demand can be derived from the cost-minimization problem:

$$Y_t(i) = \left(\frac{P_t^H(i)}{P_t^H} \right)^{-\varepsilon} Y_t$$

The intermediate firm i has a market power and able to set the profit-maximizing nominal price level, however not every firm can set their prices in every period. Only a random $1-\theta$ fraction is able to set the nominal price (Calvo, 1983).

The profit-maximization problem is the following:

$$\mathcal{L} = \sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left\{ P_t^{H,opt}(i) Y_{t+s}(i) - W_{t+s} L_{t+s}(i) \right\} \longrightarrow \max_{P_t^{H,opt}(i), L_{t+s}(i)}$$

subject to

$$\begin{aligned} Y_{t+s}(i) &= \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} \\ Y_{t+s}(i) &= A_{t+s} L_{t+s}(i) \end{aligned}$$

Plugging back all constraints:

$$\begin{aligned} \mathcal{L} &= \sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left\{ P_t^{H,opt}(i) \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} - W_{t+s} L_{t+s}(i) \right. \\ &+ \left. MC_{t+s} \left(A_{t+s} L_{t+s}(i) - \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} \right) \right\} \end{aligned}$$

First-order conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial P_t^{H,opt}(i)} &= \sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left\{ \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} - \right. \\ & -\varepsilon P_t^{H,opt}(i) \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon-1} Y_{t+s} \frac{1}{P_{t+s}^H} + \\ & \left. +\varepsilon MC_{t+s} \left(\frac{1}{P_{t+s}^H} \right)^{-\varepsilon-1} \frac{1}{P_{t+s}^H} Y_{t+s} \right\} = 0 \\ \frac{\partial \mathcal{L}}{\partial L_t(i)} &= -W_t + MC_t A_t = 0 \end{aligned}$$

Multiplying by $P_t^{H,opt}(i)$, after some rearrangements and simplifications:

$$\sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} \left\{ P_t^{H,opt}(i) - \frac{\varepsilon}{\varepsilon - 1} MC_{t+s} \right\} = 0$$

Optimal price can be given as the function of the mark-up and present value of the future flow of marginal costs:

$$P_t^{H,opt}(i) = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left(1/P_{t+s}^H \right)^{-\varepsilon} Y_{t+s} MC_{t+s}}{\sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left(1/P_{t+s}^H \right)^{-\varepsilon} Y_{t+s}}$$

where the nominal marginal cost function:

$$MC_t = \frac{W_t}{A_t}$$

Based on the Calvo-pricing the price index of domestically produced final good is the following:

$$P_t^H = \left[\theta \left(P_{t-1}^H \right)^{1-\varepsilon} + (1-\theta) P_t^{H,opt}(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

Producers' price inflation:

$$1 + \pi_t^H = \frac{p_t^H}{p_{t-1}^H} (1 + \pi_t)$$

Log-linearization of the New-Keynesian Phillips-curve:

Dividing the equation by P_t^H:

$$E_t \sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} \left(\frac{P_t^{H,opt}(i)}{P_t^H} - \frac{\varepsilon}{\varepsilon - 1} \frac{MC_{t+s}}{P_t^H} \right) = 0$$

In the model the numerae is the consumer price index P, then we can express the real marginal cost as $mc_t = MC_t / P_t$. It means that

$$E_t \sum_{s=0}^{\infty} \theta^s \Delta_{t,t+s} \left(\frac{P_t^{H,opt}(i)}{P_{t+s}^H} \right)^{-\varepsilon} Y_{t+s} \left(\frac{P_t^{H,opt}(i)}{P_t^H} - \frac{\varepsilon}{\varepsilon - 1} mc_{t+s} \frac{P_{t+s}}{P_t^H} \right) = 0$$

Log-linearization of each term in the bracket (outside terms are simplified to the steady-state values after the log-linearization):

$$E_t \sum_{s=0}^{\infty} \theta^n \beta^n \left(p_t^{H,opt}(i) - p_t^H - mc_{t+s} - (p_{t+s} - p_t^H) \right) = 0$$

We can rearrange terms independent from the sum:

$$\frac{1}{1 - \theta\beta} (p_t^{H,opt}(i) - p_t^H) = E_t \sum_{s=0}^{\infty} \theta^n \beta^n \left(mc_{t+s} - (p_t^H - p_{t+s}) \right)$$

Open the sum for $s = 0$:

$$\frac{1}{1-\theta\beta}(p_t^{H,opt}(i) - p_t^H) = \hat{mc}_t - (p_t^H - p_t) + E_t \sum_{s=1}^{\infty} \theta^n \beta^n \left(mc_{t+s} - (p_t^H - p_{t+s}) \right)$$

Stepping the previous equation one period ahead:

$$\frac{1}{1-\theta\beta}(p_{t+1}^{H,opt}(i) - p_{t+1}^H) = E_t \sum_{s=1}^{\infty} \theta^{n-1} \beta^{n-1} \left(mc_{t+s} - (p_{t+1}^H - p_{t+s}) \right)$$

We can involve P_t^H and rearrange it

$$\frac{1}{1-\theta\beta}(p_{t+1}^{H,opt}(i) - p_{t+1}^H) = E_t \sum_{s=1}^{\infty} \theta^{n-1} \beta^{n-1} \left(mc_{t+s} - (p_t^H - p_{t+s}) - (p_{t+1}^H - p_t) \right)$$

And now we can express the bracket starting from $s = 1$:

$$\frac{\theta\beta}{1-\theta\beta} \left(p_{t+1}^{H,opt}(i) - p_{t+1}^H + (p_{t+1}^H - p_t^H) \right) = E_t \sum_{s=1}^{\infty} \theta^n \beta^n \left(mc_{t+s} - (p_t^H - p_{t+s}) \right)$$

Plugging it back, we do not have sum any more

$$\frac{1}{1-\theta\beta}(p_t^{H,opt}(i) - p_t^H) = mc_t - (p_t^H - p_t) + E_t \frac{\theta\beta}{1-\theta\beta} \left(p_{t+1}^{H,opt}(i) - p_{t+1}^H + (p_{t+1}^H - p_t^H) \right)$$

The log-linearization of the price index can be given as:

$$\begin{aligned} P_t^H &= \left[\theta \left(P_{t-1}^H \right)^{1-\varepsilon} + (1-\theta) \left(P_t^{H,opt}(i) \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \\ p_t^H &= \theta p_{t-1}^H + (1-\theta) p_t^{H,opt}(i) \end{aligned}$$

where $p_t^H - p_{t-1}^H = \pi_t^H$.

We can introduce the definition of the inflation and rewrite the equation:

$$p_t^{H,opt}(i) - p_t^H = \frac{\theta}{1-\theta} \pi_t^H$$

Plug it back to the recursive equation above where we also introduce inflation and the relative price definition $rp_t^H = p_t^H - p_t$:

$$\frac{1}{1-\theta\beta}\frac{\theta}{1-\theta}\pi_t^H = mc_t - rp_t^H + E_t\frac{\theta\beta}{1-\theta\beta}\left(\frac{\theta}{1-\theta}\pi_{t+1}^H + \pi_{t+1}^H\right)$$

After some rearrangements

$$\pi_t^H = \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(mc_t - rp_t^H\right) + E_t\beta\pi_{t+1}^H$$

The behavior of the domestic firms can be given by the following equations

New-Keynesian Phillips curve:

$$\pi_t^H = \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(mc_t - rp_t^H\right) + \beta E_t\pi_{t+1}^H$$

Marginal cost function:

$$MC_t = \frac{W_t}{A_t}$$

in log-linearized, real version:

$$mc_t = w_t - a_t - p_t$$

Producers' price inflation:

$$1 + \pi_t^H = \frac{p_t^H}{p_{t-1}^H}(1+\pi_t)$$

in log-linearized version:

$$\pi_t^H = p_t^H - p_{t-1}^H + \pi_t$$

where

$$\pi_t = p_t - p_{t-1}$$

Labor demand function:

$$L_t = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t^H(i)}{P_t^H}\right)^{-\varepsilon} di$$

in log-linearized version:

$$l_t = y_t - a_t$$

1.3 International relative prices

Once we have open economy model, the relative price dynamic becomes a basic issue. It is common that in DSGE models we define real exchange rate (RER) variables. Below we can show the RER describe the same relative price development.

Domestic consumer price index:

$$P_t = \left[(1-\alpha_C)P_{H,t}^{1-\eta_C} + \alpha_C\left(e_t P_{H,t}^*\right)^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$$

The relative price of the domestic good can be expressed as the function of terms-of-trade:

$$1 = \frac{1}{p_t^H} = \left[(1-\alpha_C)\left(\frac{P_t^H}{P_t}\right)^{1-\eta_C} + \alpha_C\left(\frac{e_t P_{H,t}^*}{P_t}\right)^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$$
$$1 = \left[(1-\alpha_C)\left(p_t^H\right)^{1-\eta_C} + \alpha_C\left(\frac{e_t P_{H,t}^*}{P_t}\right)^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$$

Foreign consumer price index:

$$P_t^* = \left[(1-\alpha_C^*)P_{H,t}^{*\,1-\eta_C} + \alpha_C^*\left(e_t^{-1}P_{H,t}\right)^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$$

If the domestic economy is a small open economy, it does not really affect the foreign price level and consumption basket, then the $\alpha_C^* \approx 0$:

$$P_t^* = P_{H,t}^*$$

Real Exchange Rate (from the perspective of domestic economy): defined as the ratio of the ratio of foreign and domestic consumer price index, the foreign is expressed in domestic currency:

$$RER_t = \frac{e_t P_t^*}{P_t}$$

With small open economy assumption, the real exchange rate can be expressed as

$$RER_t = \frac{e_t P_{H,t}^*}{P_t}$$

Then the price equation can be rewritten as

$$1 \quad = \quad (1-\alpha_C)\left(P_t^H/P_t\right)^{1-\eta_C} + \alpha_C RER_t^{1-\eta_C}$$

In log-linear form it can be written as:

$$\begin{aligned} 0 &= (1-\alpha_C)\left(p_t^H - p_t\right) + \alpha_C rer_t \\ p_t^H - p_t &= -\alpha_C/(1-\alpha^C) rer_t \end{aligned}$$

1.4 Monetary policy

The central bank in both countries follows inflation targeting

It sets the interest rate based on the Taylor-rule:

$$R_t = (1+\pi_t)^{\phi_\pi} (Y_t/Y)^{\phi_y} e^{z_t}$$

Taking the log-linearized version:

$$r_t = \phi_\pi \pi_t + \phi_y y_t + z_t$$

where ϕ_π is the reaction to inflation, ϕ_y denotes the central bank's reaction to the cyclical position.

1.5 Budget constraints, market clearing conditions and good market equilibrium

By the end of the derivations, we have to show that all budget constraints, market clearing conditions give back the good market equilibrium or GDP-identity.

Among the same representative agents (without government and banking sector) the net domestic savings should be zero $B_t = 0$. Then the households budget constraint should be:

$$-e_t B_t^* + P_t C_t = T_t + W_t L_t - e_t R_{t-1}^* \psi_{t-1} B_{t-1}^*$$

The households are owner of the domestic firms, the aggregate profit can be given as:

$$T_t = P_{H,t} Y_t - W_t L_t$$

Once we plug it back to the budget constraint, we can rearrange it:

$$\underbrace{P_{H,t}Y_t}_{\text{GDP}} = \underbrace{P_t C_t}_{\text{Dom. Demand}} \underbrace{- e_t B_t^* + e_t R_{t-1}^* \psi_{t-1} B_{t-1}^*}_{\text{Trade Balance}}$$

In the followings we have to show that the trade balance is equal to the net-export. To do it, first we can use the market clearing condition of the domestic firms::

$$Y_t = C_{H,t} + C_{H,t}^*$$

where the domestic production should be equal with the sum of domestic and foreign demand for domestically produced good.

$$Y_t = C_{H,t} + \alpha_C \left(\frac{P_{H,t}}{e_t P_t^*} \right)^{-\eta_C^*} C_t^*$$

We also know that the domestic households' consumption basket is the sum of domestically produced good and imported good:

$$P_t C_t = P_{H,t} C_{H,t} + e_t P_{H,t}^* C_{F,t}$$

Substituting out the nominal consumption and domestic production:

$$P_{H,t} C_{H,t} + P_{H,t} C_{F,t}^* = P_{H,t} C_{H,t} + e_t P_{H,t}^* C_{F,t} - e_t B_t^* + e_t R_{t-1}^* \psi_{t-1} B_{t-1}^*$$

If we rearrange we can get the definition for the trade balance:

$$\underbrace{P_{H,t} C_{H,t}^*}_{\text{Export}} - \underbrace{e_t P_{H,t}^* C_{F,t}}_{\text{Import}} = TB_t = -e_t B_t^* + e_t R_{t-1}^* \psi_{t-1} B_{F,t-1}^*$$

- In real terms the trade balance is the following:

$$TB_t = \frac{P_{H,t}}{P_t} C_{H,t}^* - RER_t C_{F,t}$$

1.6 Complete market assumption

Assuming that free flow of capital and goods between the two economies, symmetric equilibrium, and there is no risk premium ($\psi_t = 1$ for all t). This implies the Euler-equations and UIP conditions:

Domestic Euler equation (Dynamic IS curve):

$$\beta E_t \frac{\exp\{g_{t+1}\}}{\exp\{g_t\}} \frac{(C_{t+1})^{-1/\sigma}}{(C_t)^{-1/\sigma}} R_t \frac{P_t}{P_{t+1}} = 1$$

Foreign Euler equation (Dynamic IS curve):

$$\beta E_t \frac{\exp\{g^*_{t+1}\}}{\exp\{g^*_t\}} \frac{(C^*_{t+1})^{-1/\sigma}}{(C^*_t)^{-1/\sigma}} R^*_t \frac{P^*_t}{P_{t+1}*} = 1$$

UIP condition:

$$E_t R^*_t \frac{e_{t+1}}{e_t} = R_t$$

Combining the two Euler equations:

$$\beta E_t \frac{\exp\{g_{t+1}\}}{\exp\{g_t\}} \frac{(C_{t+1})^{-1/\sigma}}{(C_t)^{-1/\sigma}} R_t \frac{P_t}{P_{t+1}} = \beta E_t \frac{\exp\{g^*_{t+1}\}}{\exp\{g^*_t\}} \frac{(C^*_{t+1})^{-1/\sigma}}{(C^*_t)^{-1/\sigma}} R^*_t \frac{P^*_t}{P_{t+1}*}$$

And substituting out R_t from UIP conditions:

$$\beta E_t \frac{\exp\{g_{t+1}\}}{\exp\{g_t\}} \frac{(C_{t+1})^{-1/\sigma}}{(C_t)^{-1/\sigma}} R^*_t \frac{e_{t+1}}{e_t} \frac{P_t}{P_{t+1}} = \beta E_t \frac{\exp\{g^*_{t+1}\}}{\exp\{g^*_t\}} \frac{(C^*_{t+1})^{-1/\sigma}}{(C^*_t)^{-1/\sigma}} R^*_t \frac{P^*_t}{P_{t+1}*}$$

Simplifying and rearraning it

$$E_t \frac{\exp\{g_{t+1}\}}{\exp\{g_t\}} \frac{(C_{t+1})^{-1/\sigma}}{(C_t)^{-1/\sigma}} \frac{e_{t+1} P^*_{t+1}}{P_{t+1}} = E_t \frac{\exp\{g^*_{t+1}\}}{\exp\{g^*_t\}} \frac{(C^*_{t+1})^{-1/\sigma}}{(C^*_t)^{-1/\sigma}} \frac{e_t P^*_t}{P_t}$$

Moving all $t+1$ variables to the LHS, and assuming no preference shock in foreign economy:

$$E_t \exp\{g_{t+1}\} \frac{(C_{t+1})^{-1/\sigma}}{(C^*_{t+1})^{-1/\sigma}} RER_{t+1} = \frac{(C_t)^{-1/\sigma}}{(C^*_t)^{-1/\sigma}} RER_t \exp\{g_t\}$$

The last equation also implies that in the steady-state it is also true, that the

$$\frac{(C_t)^{-1/\sigma}}{(C^*_t)^{-1/\sigma}} RER_t \exp\{g_t\} = ... = \frac{(C_0)^{-1/\sigma}}{(C^*_0)^{-1/\sigma}} RER_0 \exp\{g_0\} = const$$

If we take the log-linear approximation of the equation above, we get:

$$-\frac{1}{\sigma}(c_t - c^*_t) + rer_t + g_t = 0$$

2 Model equations under incomplete markets

The model has the following log-linear equations:

The households' equations:

$$
\begin{aligned}
g_{t+1} - g_t - \frac{1}{\sigma}\left(c_{t+1} - c_t\right) + r_t - E_t\pi_{t+1} &= 0 \\
r_t^* + \psi_B b_t^{F,*} + \varphi_t + e_{t+1} - e_t &= r_t \\
\sigma_L l_t + \frac{1}{\sigma} c_t &= rw_t \\
c_{H,t} &= -\eta_C rp_{H,t} + c_t \\
c_{F,t} &= -\eta_C rer_t + c_t
\end{aligned}
$$

The firms' equations:

$$
\begin{aligned}
\pi_t^H &= \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(mc_t - rp_t^H\right) + \beta E_t \pi_{t+1}^H \\
mc_t &= rw_t - a_t \\
\pi_t^H &= rp_t^H - rp_{t-1}^H + \pi_t \\
l_t &= y_t - a_t
\end{aligned}
$$

Good market equilibrium:

$$
\begin{aligned}
y_t &= (1-\alpha_C)c_{H,t} + \alpha_C\left(c_t^* - \eta_C\left(rp_{H,t} - rer_t\right)\right) \\
tb_t &= (rp_{H,t} + c_{H,t}^*) - (rer_t + c_{F,t}) \\
b_t^* &= \frac{1}{\beta} b_{t-1}^* - tb_t
\end{aligned}
$$

The central bank:

$$
r_t = \phi_\pi \pi_t + \phi_y y_t + z_t
$$

Foreign asset market equilibrium:

$$
\begin{aligned}
\pi_t &= \pi_{H,t} + \frac{\alpha_C}{1-\alpha_C}\Delta rer_t \\
\Delta rer_t &= \Delta e_t + \pi_t^* - \pi_t
\end{aligned}
$$

Foreign demand:

$$
c_{H,t}^* = -\eta_C^*(rp_{H,t} - rer_t) + c_t^*
$$

The relative prices are expressed in terms of domestic CPI(p). If the economy operates under complete market condition, the risk-premium and UIP equations disappear. And the exchange rate will be determined through the following

$$
-\frac{1}{\sigma}\left(c_t - c_t^*\right) + rer_t + g_t = 0
$$