INSTITUTE FOR CAPACITY DEVELOPMENT

L-8: Forecasting with DSGE Models and Policy Applications

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Course on Monetary and Fiscal Policy Analysis with DSGE Models (OT26.08)

Introduction

- DSGE models can be used for at least three purposes:
  - **Telling a story:** Which shocks can explain the current situation?
  - **Policy experiments:** What are the consequences in terms of welfare of switching from an inflation targeting system to a fixed exchange rate system?
  - **Forecasting and generating counterfactual scenarios**

Introduction

1. Historical Decomposition
   – Estimating shocks using the Kalman filter.
   – Analysis of alternative and counterfactual policies.

2. Forecasts and Scenarios
   – Unconditional and conditional forecasting.
   – Conditional forecasts using the Kalman filter.

Historical decompositions

In DGSE models, structural shocks determine economic cycles.

The type of shocks reflect the type of economy being modeled. For example,

- In a closed economy: the most common shocks are related to technology, preferences, financing, the fiscal sector, and monetary policy.
- In an open economy, other shocks are also important, such as: foreign interest rate shocks, commodity price shocks, global economic activity shocks.

Historical decompositions

- An important consequence of DSGE modeling is that a variable can be decomposed into structural shocks
- This allows us to answer questions like:
  - How a different shocks (PTF, demand, monetary) contributed to explain the observed variables (growth, inflation, wages).
  - How would the economy have fared if a different monetary or fiscal policy had been followed.
- It plays a major role in forecasting exercises.

Historical decompositions

- Shocks are decomposed based on the solution of the model. The VAR solution is represented as:

$$X_t = \mathbf{P} X_{t-1} + \mathbf{Q} \varepsilon_t$$

- Backward iteration produces:

$$X_t = \mathbf{P}^t X_0 + \sum_{i=0}^{t-1} \mathbf{P}^i \mathbf{Q} \varepsilon_{t-i}$$

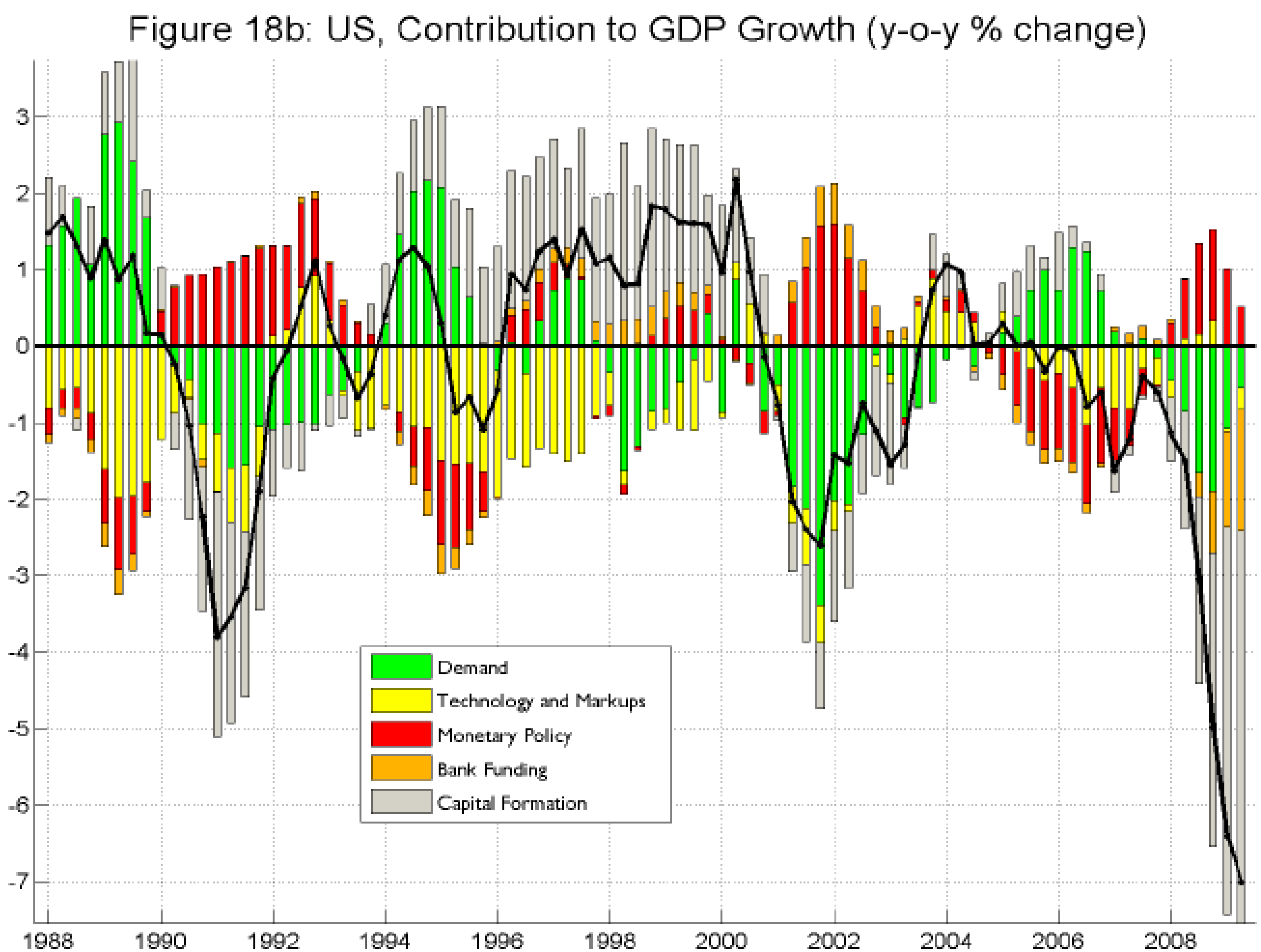
where the value of a variable today is a function of the shocks received up to period *t* and the initial conditions.

Historical decompositions

Some examples:

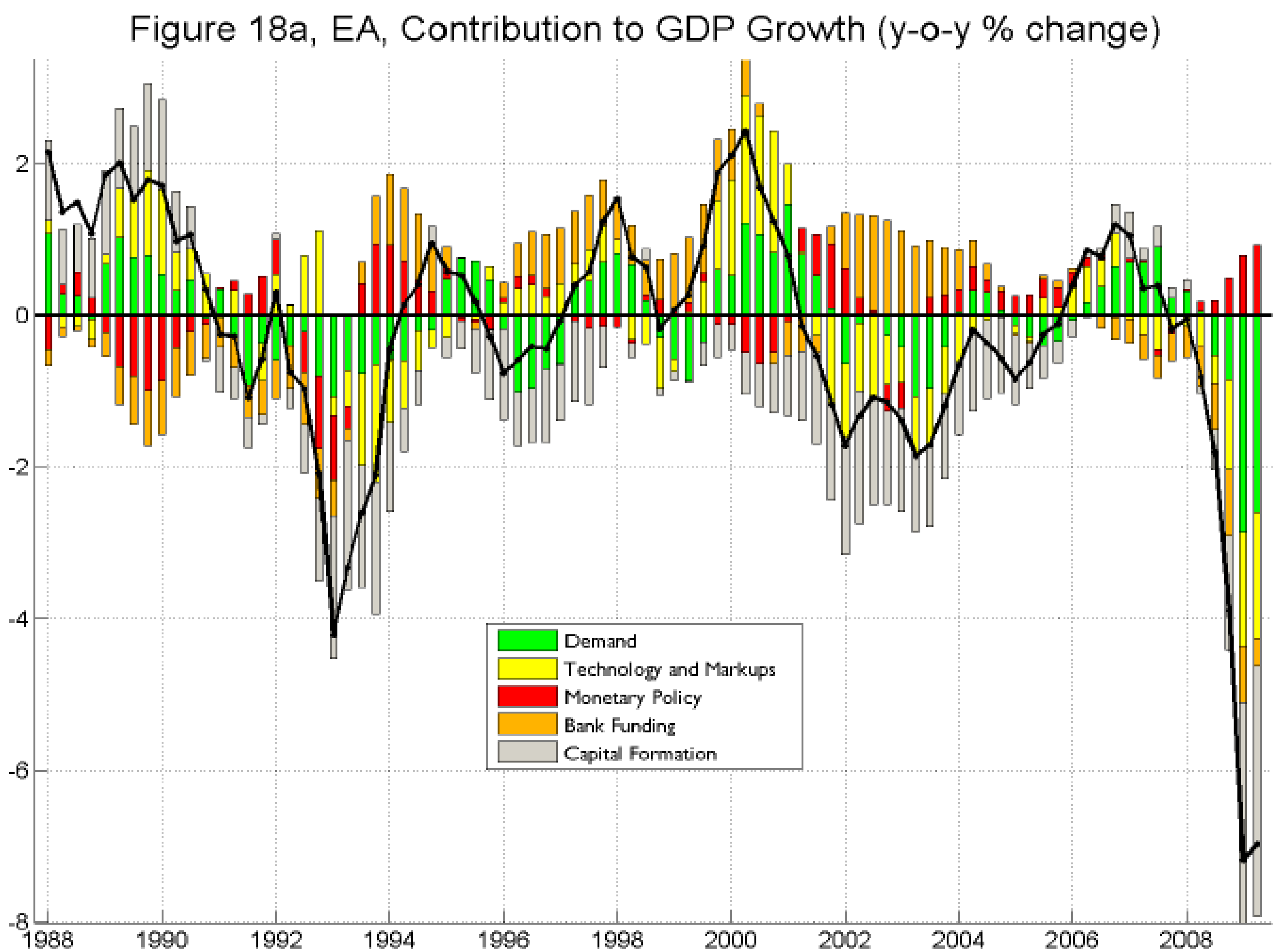
- Christiano, Motto, and Rostagno (2010): the effect of turbulence in the banking sector (bank funding) and in risk and technology in the investment sector (capital formation) on the reasons for the 2007-2009 recession in the USA and the euro zone.

Historical decompositions



Source: Christiano, Motto, and Rostagno (ECB WP 1192)

Historical decompositions



Source: Christiano, Motto, and Rostagno (ECB WP 1192)

Estimating structural shocks

- Using the Kalman filter, it is possible to estimate a series of unobservable shocks that are consistent with the model and the data.

- Some considerations:
  - When estimating the shocks it is better to use the smoothing algorithm. The reason is that, by applying this algorithm, shocks are estimated using all the available information.
  - See appendix for details.

Possible Exercise 1: Simulation with some of the shocks

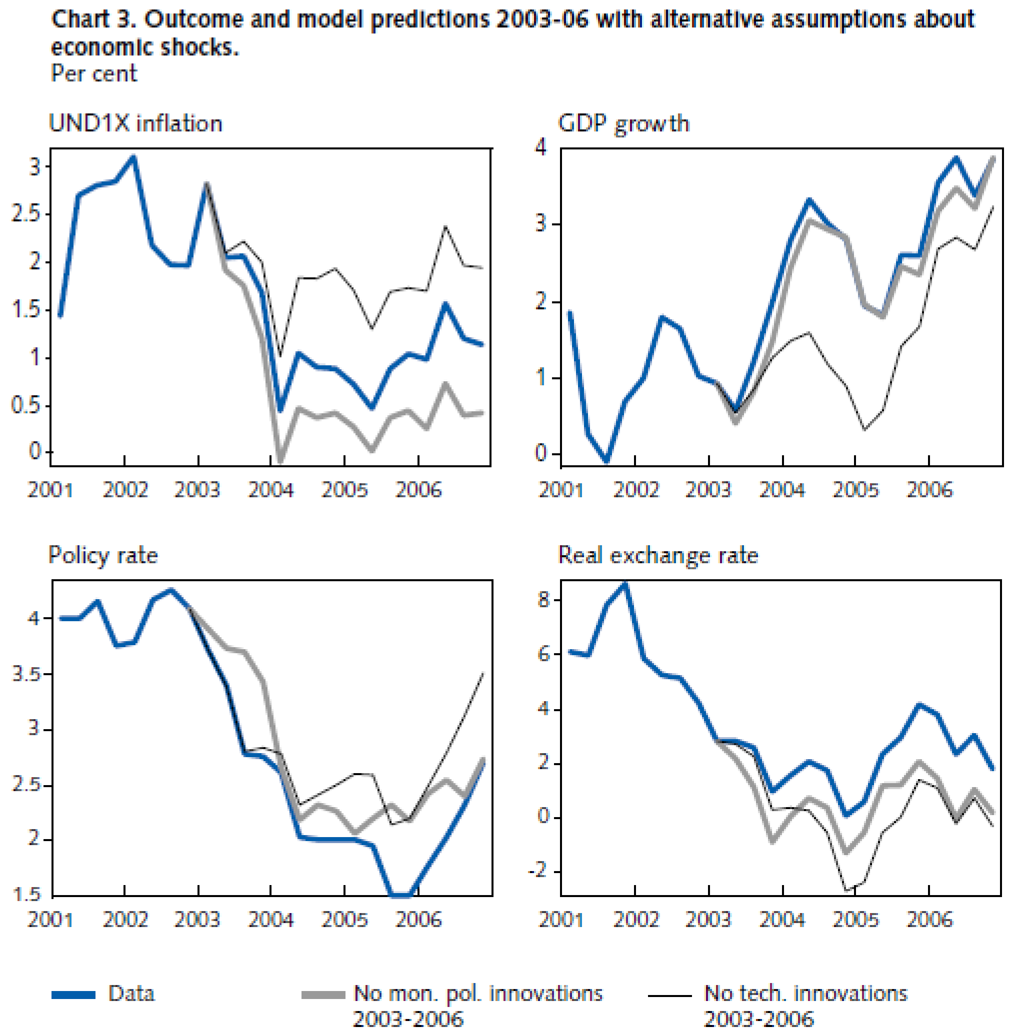
- Remember that the solution of a DSGE model can be written as follows:

$$s_t = \mathbf{A_1} s_{t-1} + \mathbf{A}_2 \varepsilon_t \qquad (1)$$

$$f_t = \mathbf{A}_3 s_t \qquad (2)$$

  where “s” are the state variables and “f” are the rest of the endogenous variables. Assuming that “s” are the model shocks, it is therefore possible to estimate the sequence of shocks “s” and innovations “ε”.
- We can determine the contribution of each shock to economic fluctuations through model simulation using estimated shock values.

Simulation with some of the shocks



Simulation with some of the shocks

- This is easily implemented in DYNARE.

```
estimation();
shock_decomposition rnom_obs pic_obs y_obs;
```

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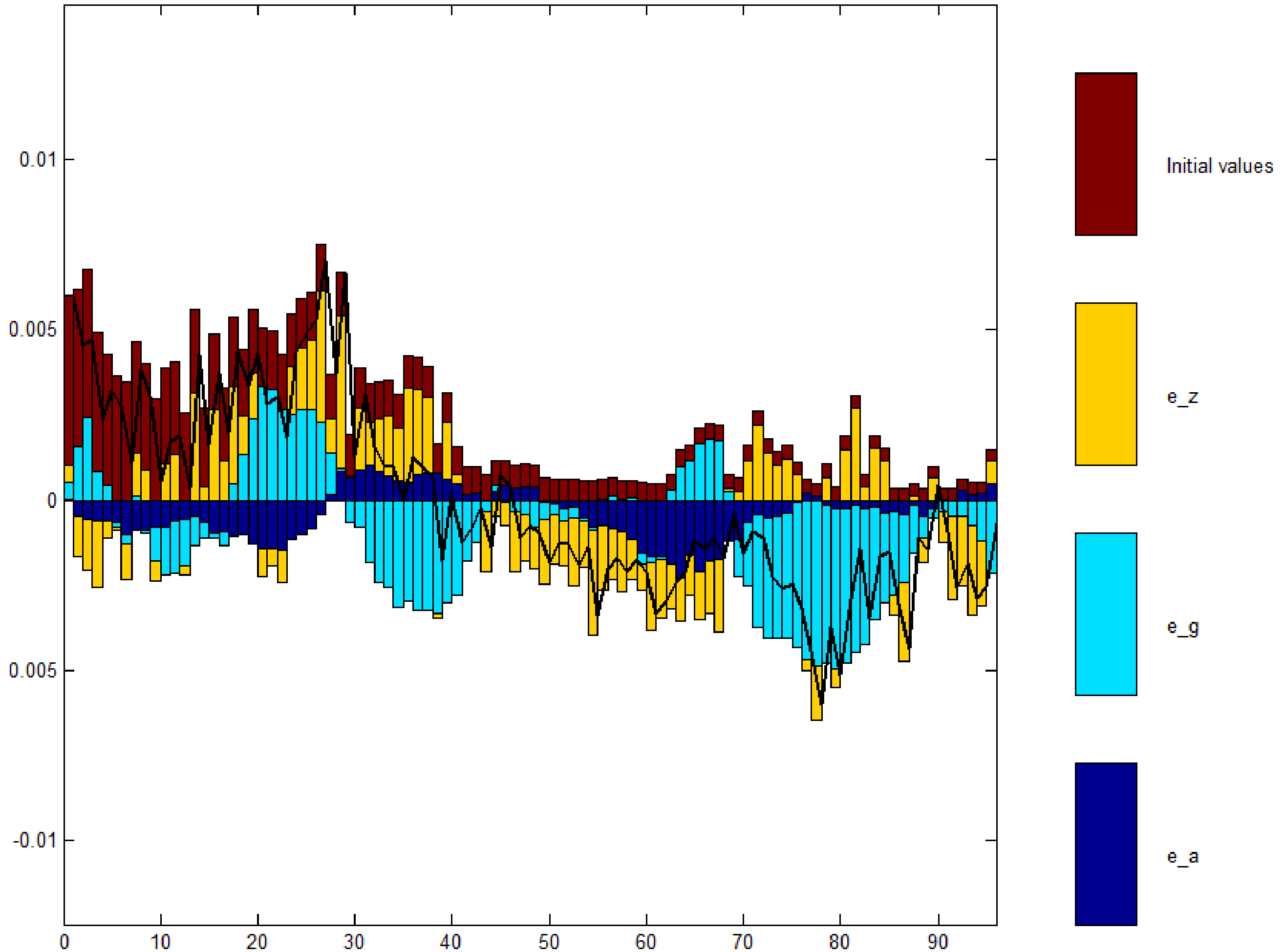
Simulation of one Shock at a Time: Inflation



Counterfactual policy analysis

- Using a realization of the shocks and the model, one can answer the following question:

  What would have happened if …?

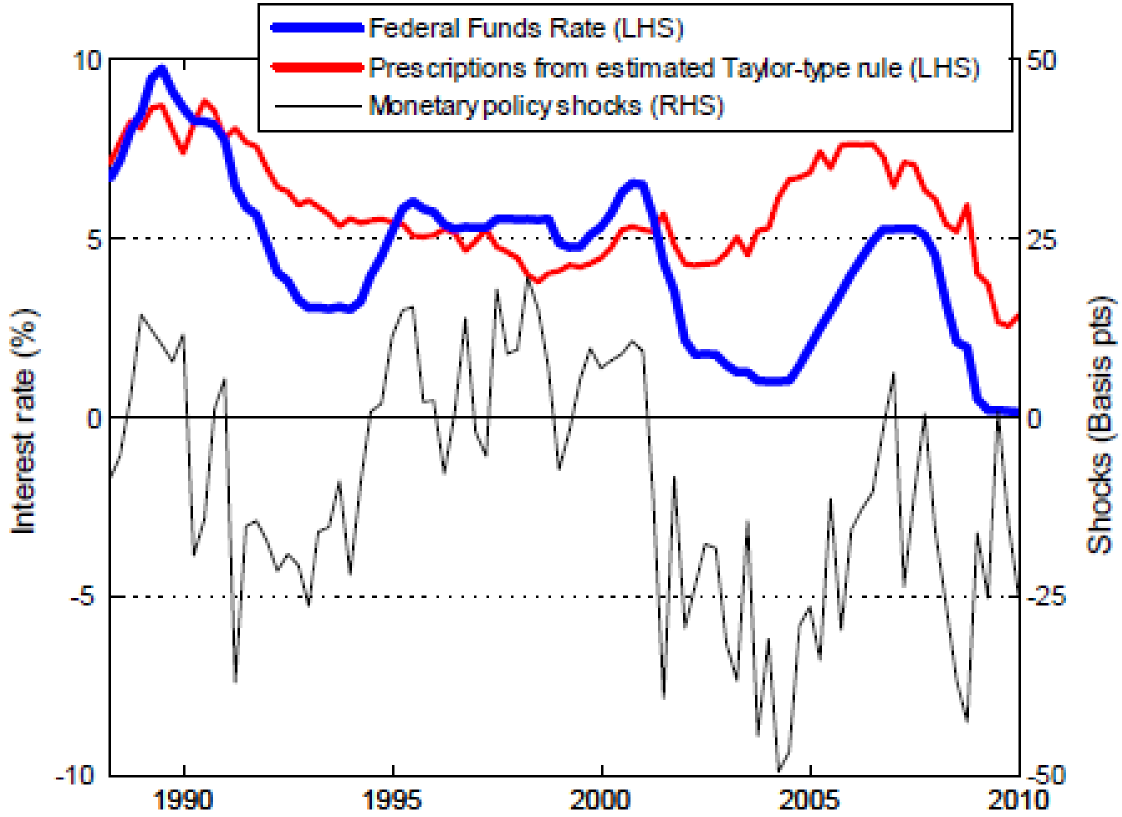
  - the monetary policy rule had been different;
  - the value of the model parameters had been different (slope of the Phillips curve, intertemporal elasticity of substitution, etc.), either as a result of private sector behavior changes or other policies (fiscal, banking regulation, competition, etc.).

Counterfactual policy analysis

Some examples:

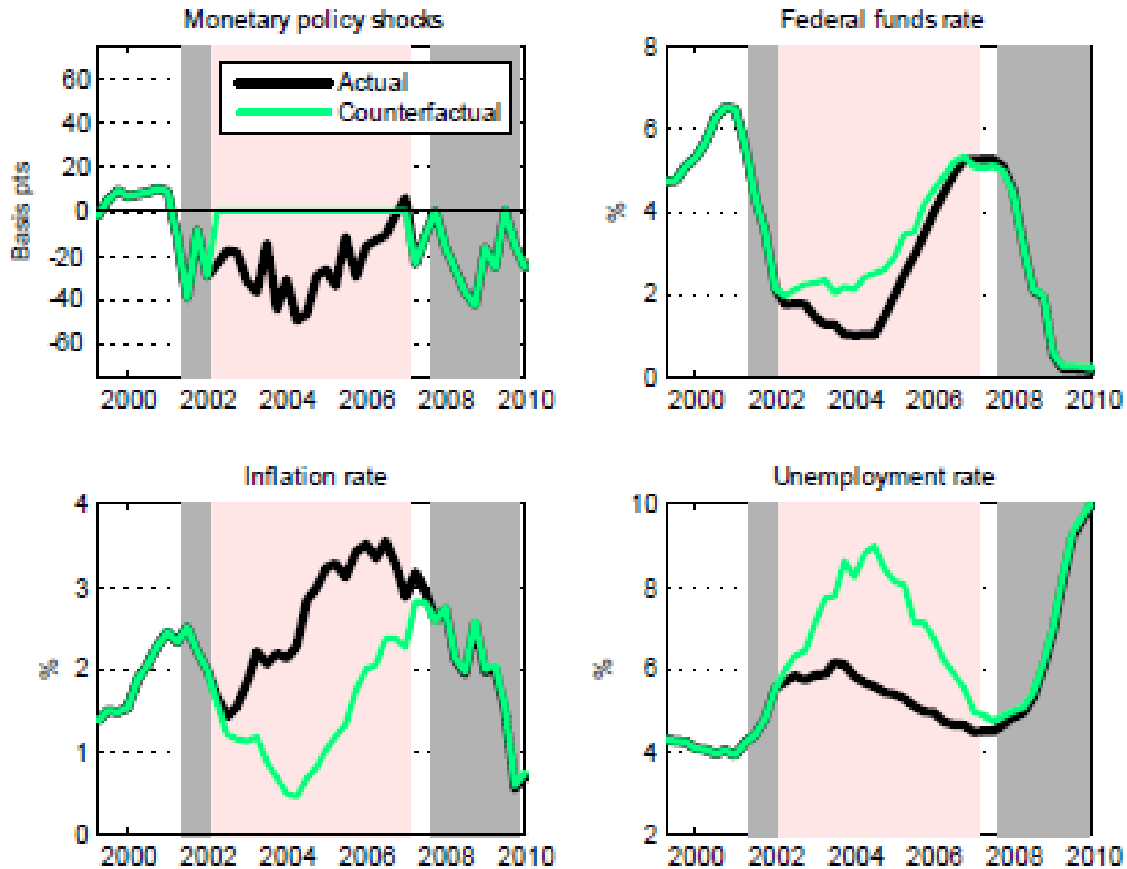
- Groshenny (2010): What would have happened if the Fed had not deviated from the Taylor rule during the period 2002-2006?

Counterfactual Policy Analysis



Source: Groshenny (2010)

Counterfactual Policy Analysis



Source: Groshenny (2010)

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Counterfactual experiments

- Assuming that we have a series of shocks estimated from the original model (a benchmark model)

$$\{\varepsilon_t\}_{t=1}^{T}$$

- We can evaluate the effect of changing an equation or a parameter of the model as follows:
  - Solve the alternative model
  - and simulate it using the estimated shocks from the benchmark model.

- The solution of the alternative model is:

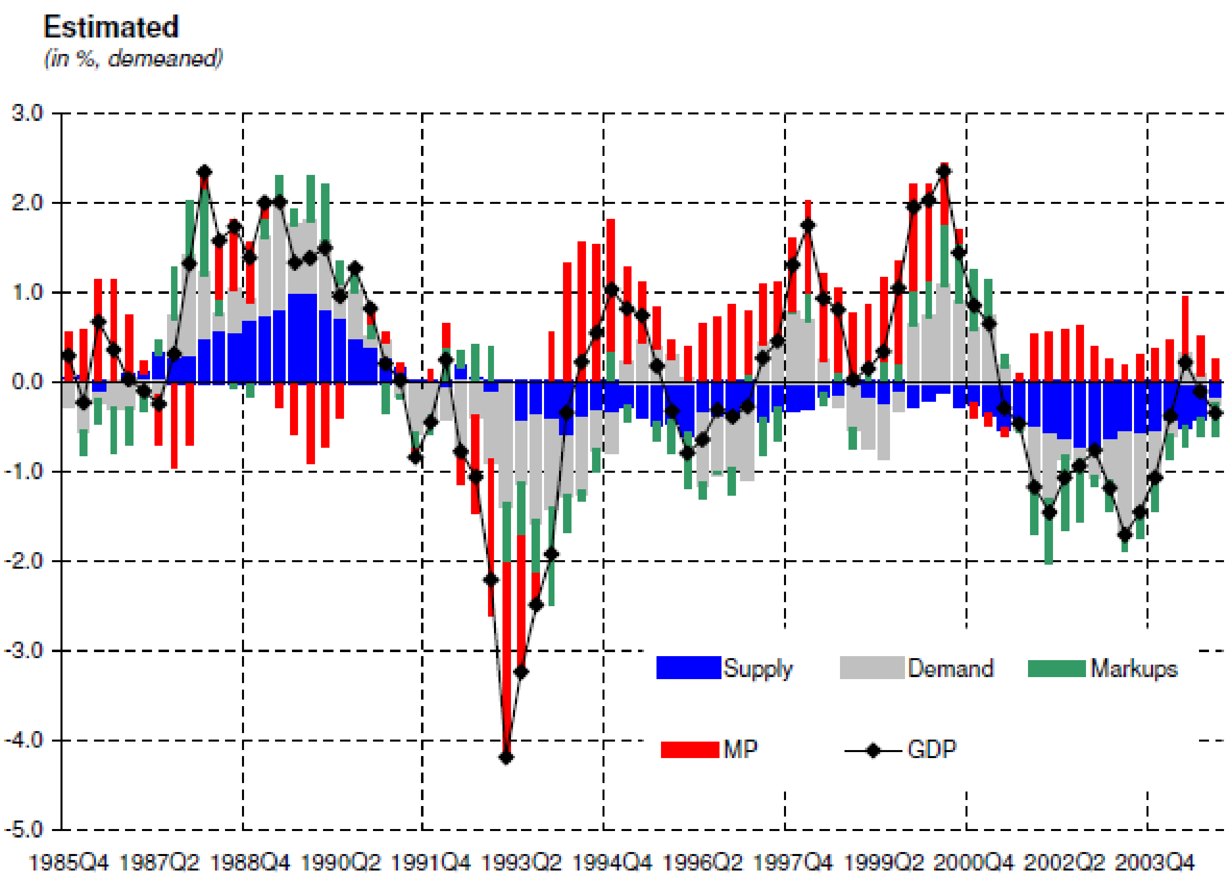
$$X_t = \tilde{P} X_{t-1} + \tilde{Q} \varepsilon_t$$
$$X_t^{obs} = H X_t$$

Counterfactual policy analysis

- Doing this, we are generating a hypothetical series that we can be compared with the original.

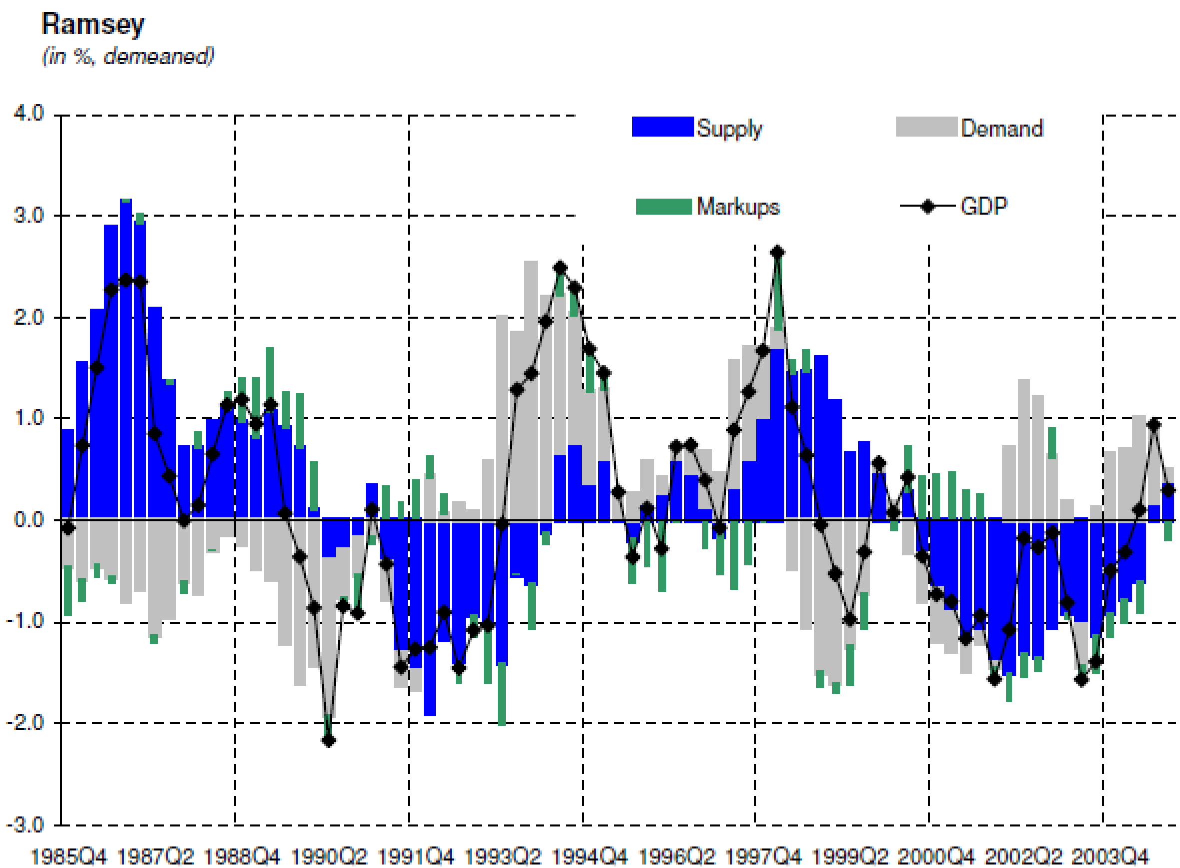
- Example: Adjemian, Darracq-Pariès, and Moyen (2007, ECB WP 803) compare the ECB's estimated monetary policy with optimal monetary policy.

Counterfactual Policy Analysis



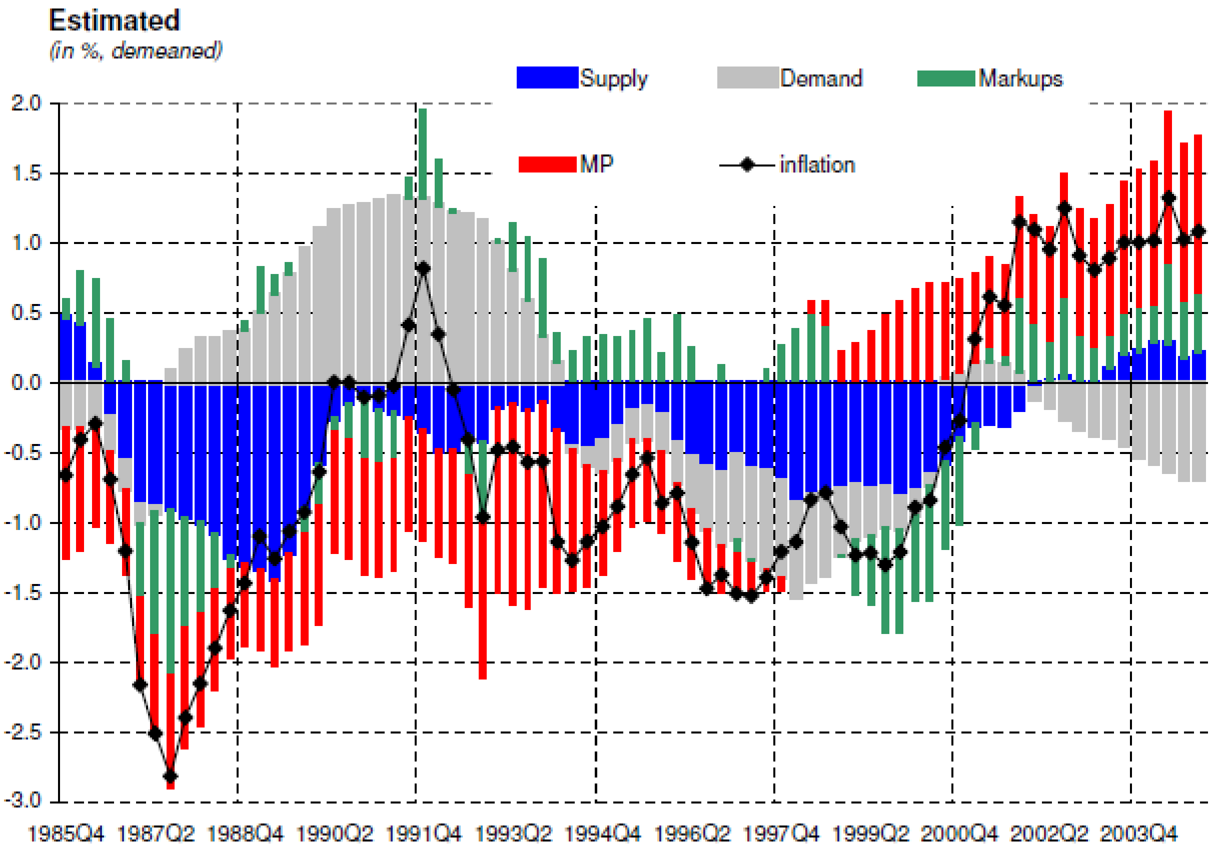
GDP growth under estimated monetary policy rules.
Source: Adjemian et al. (2007), ECB WP 803.

Counterfactual Policy Analysis



GDP growth under optimal monetary policy rules.
Source: Adjemian et al. (2007), ECB WP 803.

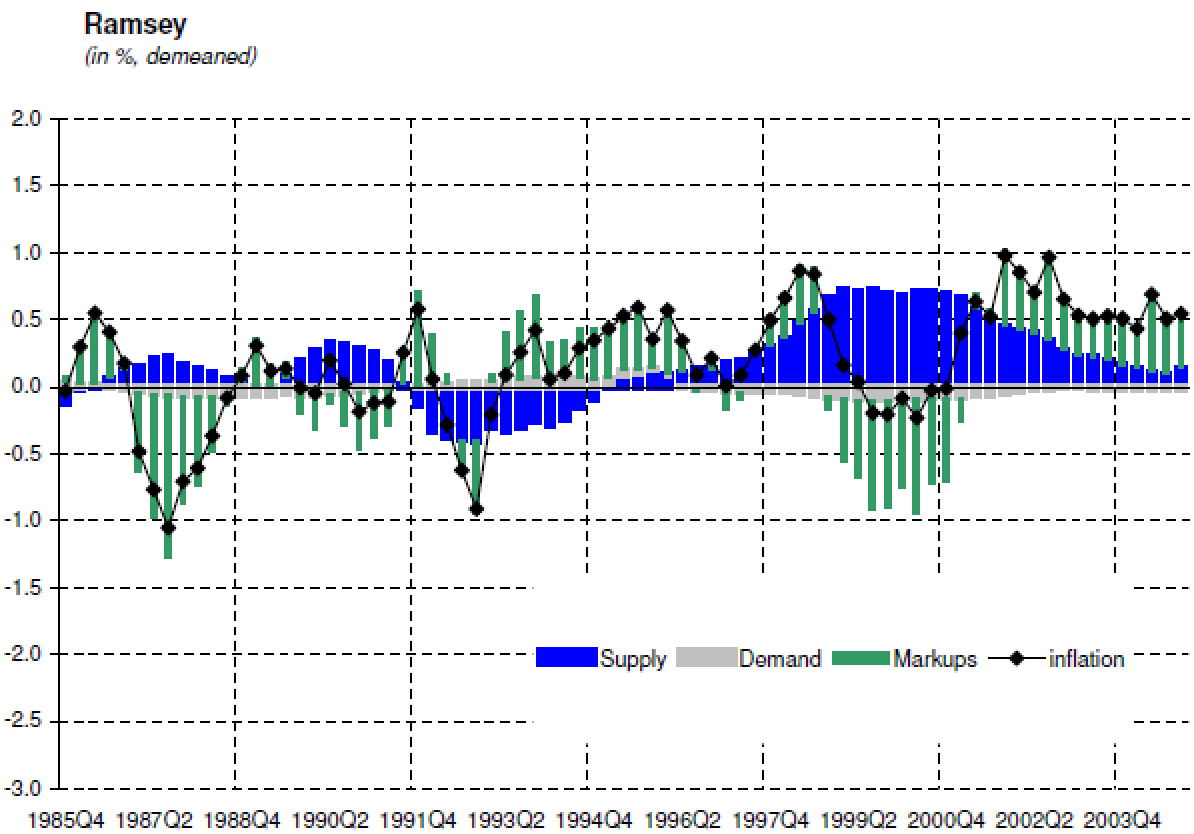
Counterfactual Policy Analysis



Inflation under estimated monetary policy rules.

Source: Adjemian et al. (2007), ECB WP 803.

Counterfactual Policy Analysis



Inflation under optimal monetary policy rules.
Source: Adjemian et al. (2007), ECB WP 803.

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Counterfactual policy analysis

Tab. 4 - SELECTED SECOND ORDER MOMENTS

	Estimated	Ramsey	Ramsey IR
Std. dev.			
Output	5.26	7.26	7.25
Consumption	6.28	7.61	7.59
Investment	12.27	17.42	17.44
Wage Inflation	1.11	0.29	0.32
Inflation	0.97	0.27	0.27
Interest Rate	0.91	3.13	0.74

Source: Adjemian et al. (2007), ECB WP 803.

Counterfactual policy analysis

- How can we do this in DYNARE? First, estimate the model and use smoother command in the estimation.

```
estimation(datos_trans,mh_replic=25000,mode_file=nk_
closedv1_est_trans_mode,mh_nblocks=1,mh_jscale=0.5,m
h_drop=0.2,prior_trunc=1e-32, smoother,
nodiagnostic, graph_format = none);
```

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Counterfactual policy analysis

- After estimating the model, the structure oo_. has the following variables:
  - `oo_.SmoothedVariables`
  - `oo_.SmoothedShocks`
- The variable `oo_.SmoothedShocks` contains the sequence of estimated shocks.
- Dynare will automatically save the structure oo_ in the results file.

Counterfactual policy analysis in Dynare

```
% Simulate the model with all shocks starting form an initial
   condition

% Initial condition in the first period

y0=[oo_.SmoothedVariables.c(1);oo_.SmoothedVariables.rnom(1);
    oo_.SmoothedVariables.pic(1);oo_.SmoothedVariables.lab(1);
    oo_.SmoothedVariables.rw(1);oo_.SmoothedVariables.y(1);
    oo_.SmoothedVariables.mc(1);oo_.SmoothedVariables.a(1);
    oo_.SmoothedVariables.g(1);oo_.SmoothedVariables.z(1);

   oo_.SmoothedVariables.rnom_obs(1);oo_.SmoothedVariables.pic_obs(1
   );
oo_.SmoothedVariables.y_obs(1)];

% Shocks from the second period

ex_todos=[oo_.SmoothedShocks.e_a(2:T),oo_.SmoothedShocks.e_g(2:T),
    oo_.SmoothedShocks.e_z(2:T)];
```

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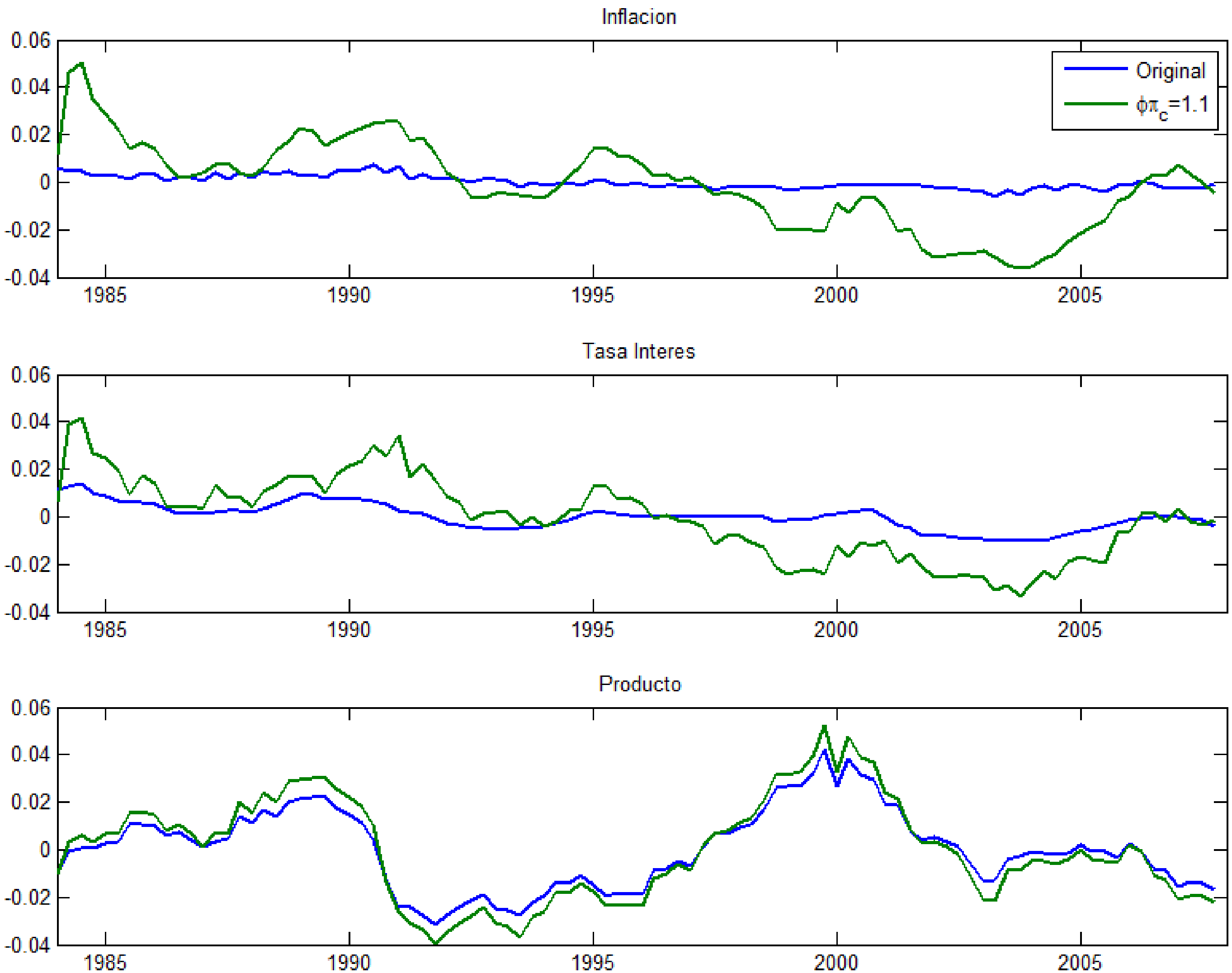
Counterfactual policy analysis

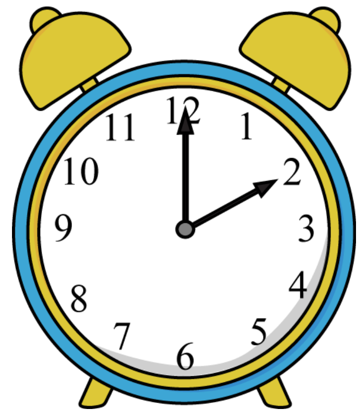
```
% Alternative model
% Different monetary policy rule
% Load the results from the alternative model
load('nk_closedv1_cal_trans_phi11_results.mat');

% Simulate the alternative model with the shocks from
  the benchmark model
  dr=oo_.dr;          % Solution of the model in DYNARE
  iorder=1;           % Order of simulation
  y_pic11=simult_(y0,dr,ex_todos,iorder);
```

Counterfactual Policy Analysis



Coffee Break!



10 minutes

Forecasting with DSGE Models

DSGE forecasting?

Not really?

Introduction to forecasting

- A typical forecasting exercise in a central bank includes the following steps:
  1. Estimation of shocks affecting the economy.
  2. Generate an unconditional forecast taking into account the estimated shock decomposition.
  3. Perform a scenario analysis and risk assessment: How would the baseline forecast change if a particular shock impacts the economy during the forecasting horizon?

Some advantages of DSGE forecasting

The forecast can be interpreted because

1. It is a function of structural shocks; and

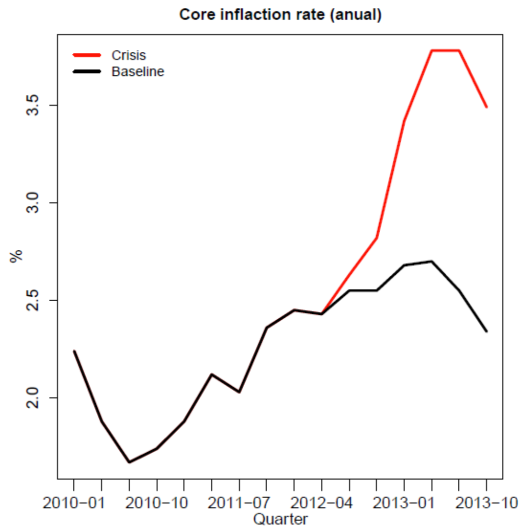
2. It is based on an economic model

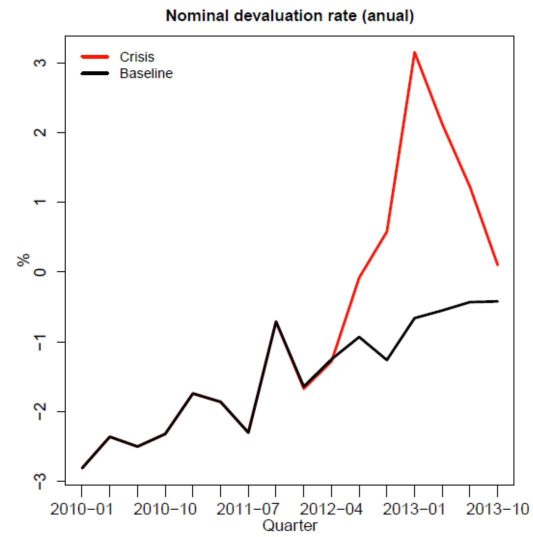
Let us examine a DSGE model for Colombia called PATACON or **P**olicy **A**nalysis **T**ool **A**pplied to **C**olombian **N**eeds

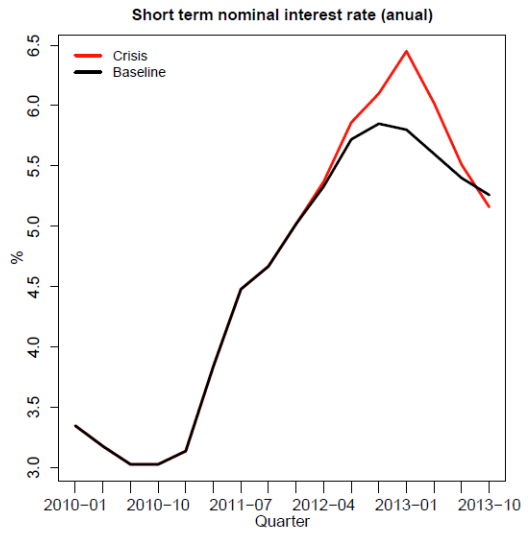
Example of a forecasting exercise

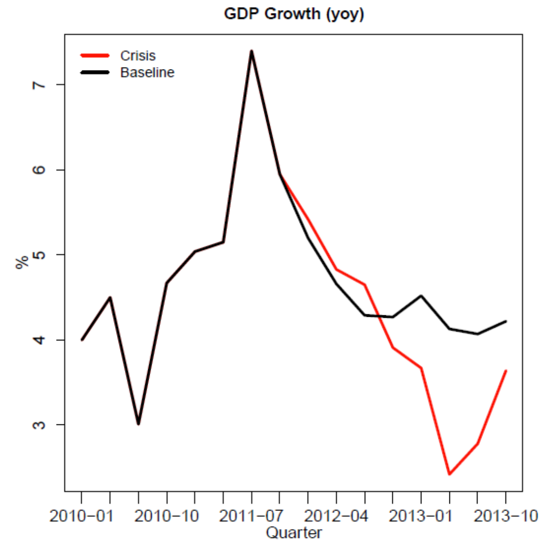
- A baseline forecast and a crisis forecast are generated

- The crisis scenario includes simultaneous shocks, namely:
  - Increase in the external interest rate
  - Decrease in external output
  - Deterioration in consumer confidence

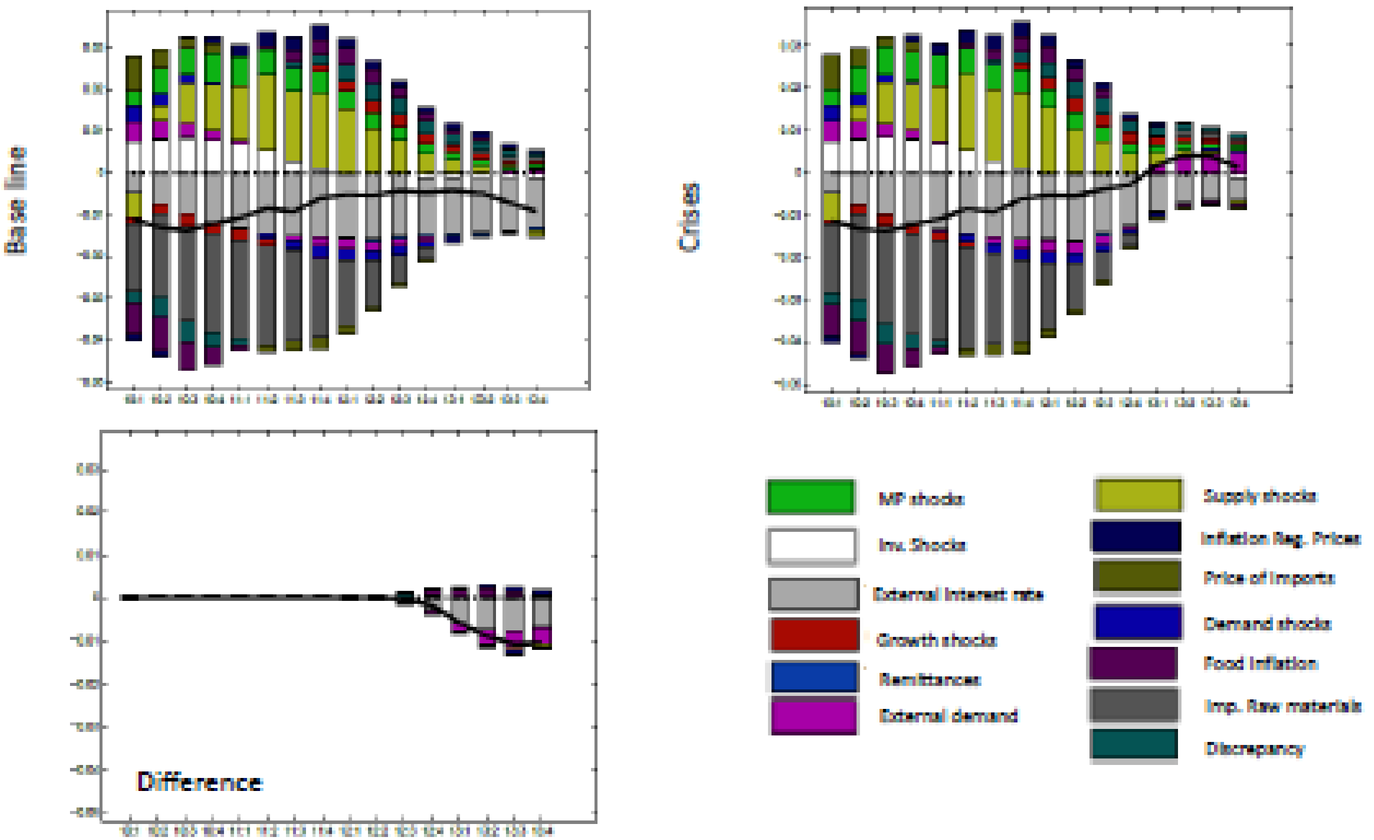

Core inflaction rate (anual)
Crisis
Baseline
%
2.0
2.5
3.0
3.5
2010−01
2010−10
2011−07
2012−04
2013−01
2013−10
Quarter


Nominal devaluation rate (anual)
Crisis
Baseline
%
−3
−2
−1
0
1
2
3
2010−01
2010−10
2011−07
2012−04
2013−01
2013−10
Quarter


Short term nominal interest rate (anual)
Crisis
Baseline
%
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
2010−01
2010−10
2011−07
2012−04
2013−01
2013−10
Quarter

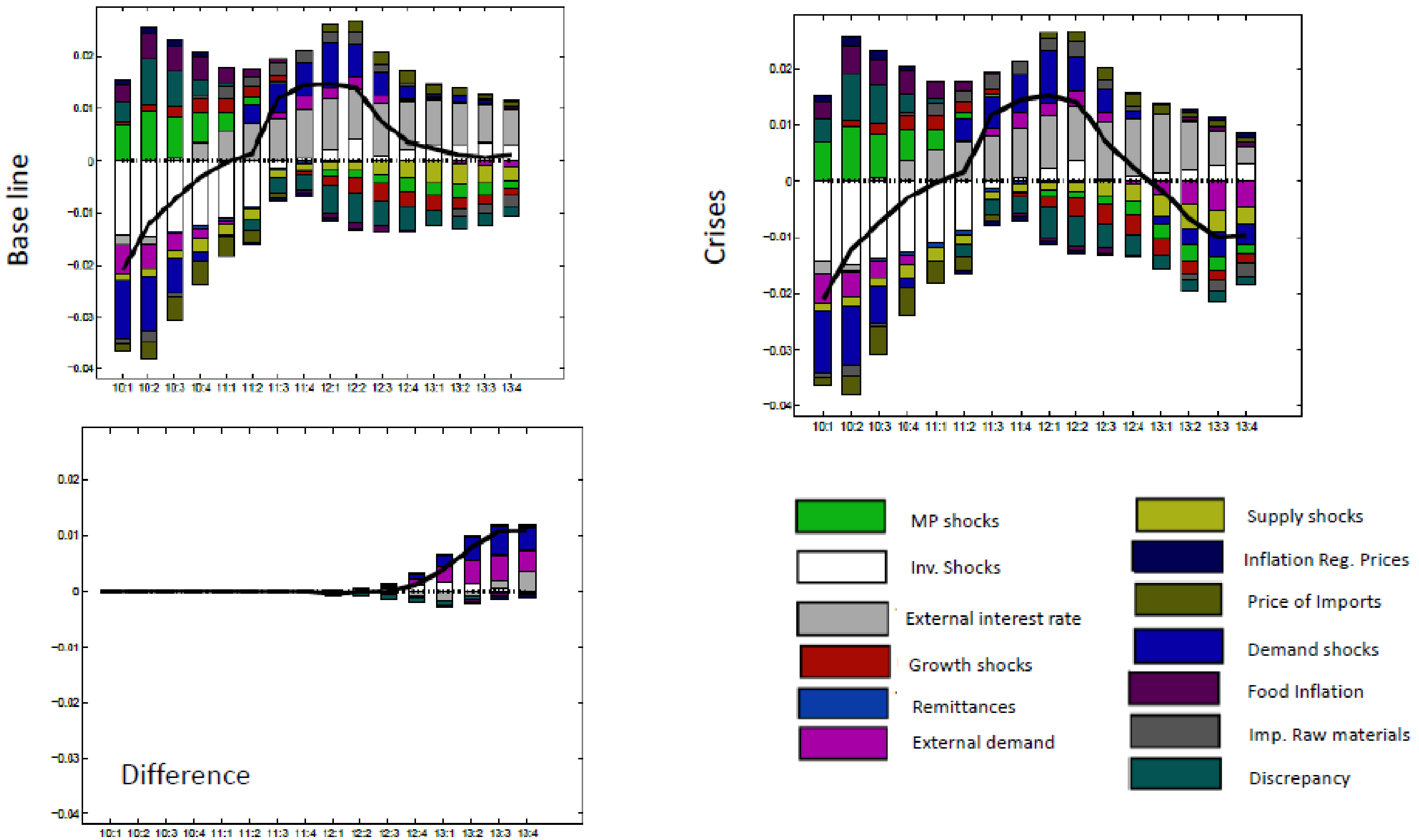

GDP Growth (yoy)
Crisis
Baseline
%
3
4
5
6
7
2010−01
2010−10
2011−07
2012−04
2013−01
2013−10
Quarter

Baseline Inflation Forecast



Forecast from Q4 2011 with two scenarios

Output Growth Forecast



Forecast from Q4 2011 with two scenarios

Practical problems with real-time forecasting

1. GDP data are available with a time lag and are revised after their initial release.

   We can add a measurement error to the GDP equation in the model to capture this data revision problem.

2. The DSGE models are usually estimated using quarterly data. However, there are monthly data for some variables that we would like to include in the forecast.

   Examples of available monthly information: interest rates, exchange rates, inflation rates, etc.

Forecasting Datasets

		Estimation sample				Forecasting Quarter	Forecast horizon			end
		T=1			t=T	t=T+1	t=T+2		t=T+F*	t=t+F
		Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	
Interest rates	…						NA	NA	NA	…
Inflation rate	…						Staff	NA	NA	…
GDP growth	…					Nowcast	NA	NA	NA	…
External interest rates	…						IMF forecast			…

Available data

What follows

- **Unconditional forecast**
  - Economic model -> Solution -> Kalman filter
  - We will discuss the goodness of the forecast

- **Conditional forecast**
  - How do we incorporate extra information into the forecasting exercise?
  - How do we generate forecasting scenarios?
  - What is the effect of the conditioning information on the goodness of the forecast?

DSGE model forecasts using the same structure as the Kalman filter

- To recap, the notation used for the Kalman filter is:

$$X_t = PX_{t-1} + Q\varepsilon_t \text{ (1')}$$
$$X_t^{obs} = HX_t \text{ (2')}$$
$$\varepsilon_t \sim iidN(0, \Omega)$$

- By applying the Kalman filter from t =1 to T, we estimate:

$$X_{T|T}, X_{T|T}^{obs} = X_T^{obs}$$

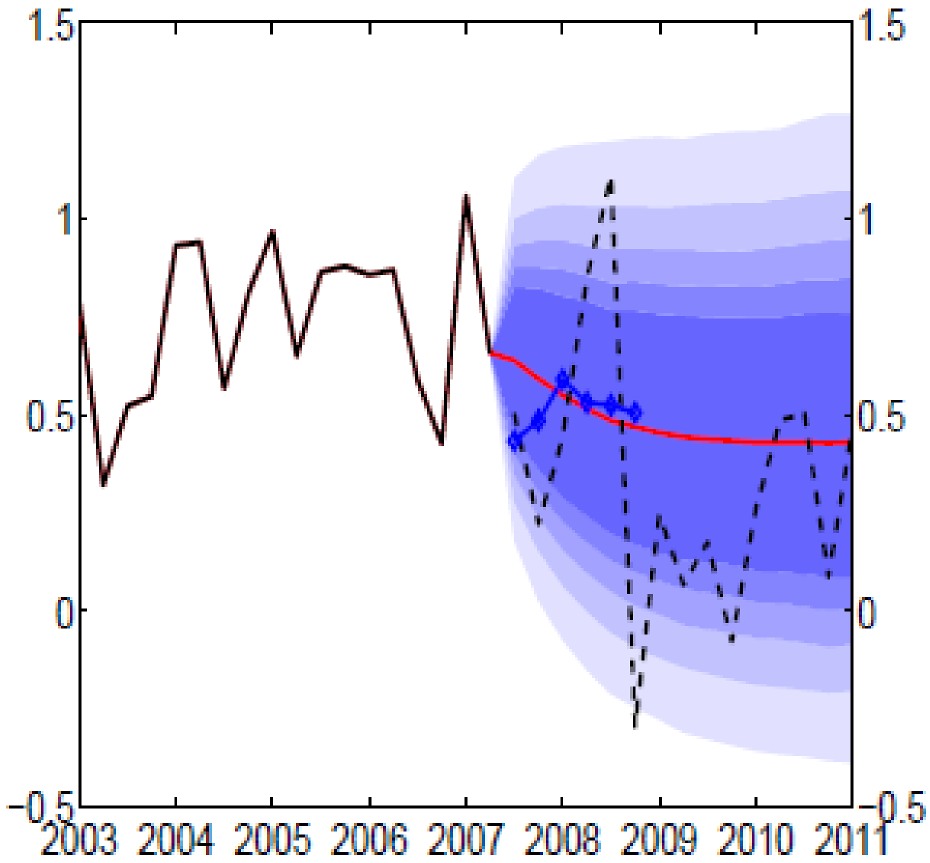
- Unconditional forecasts from T+1 up until T+k are obtained recursively:

$$X_{T+i|T} = \mathbf{P}X_{T+i-1|T},\ i = 1, \ldots, k$$
$$X_{T+i|T}^{obs} = \mathbf{H}X_{T+i|T},\ i = 1, \ldots, k$$

Sources of uncertainty in the forecast

- The sources of uncertainty in the forecast are the following:
  - Estimation of the parameters,
  - Estimation of the states X_T^T
  - Future shocks that can affect the economy during the forecasting horizon.
- Bayesian forecasting methods take into account these sources of uncertainty. This is another advantage of working with Bayesian estimation.

Example of Bayesian Confidence Intervals



Inflation forecast
Red line

See Appendix

How can we estimate this confidence interval?

CALM DOWN, NO NEED TO PANIC!

This is done in Dynare!

Interpretation of the forecast

- The structure of the model can help explain forecasts based on the economic shocks derived from the model.

- The forecast can be written in terms of the shocks:

$$X_{T+1|T} = \mathbf{P}^{T} X_{1|T} + \sum_{j=0}^{T-1} \mathbf{P}^{j+1} \mathbf{Q} \varepsilon_{T-j|T}$$

$$X_{T+i|T} = \mathbf{P}^{T-1+i} X_{1|T} + \sum_{j=0}^{T-1} \mathbf{P}^{j+i} \mathbf{Q} \varepsilon_{T-j|T}$$

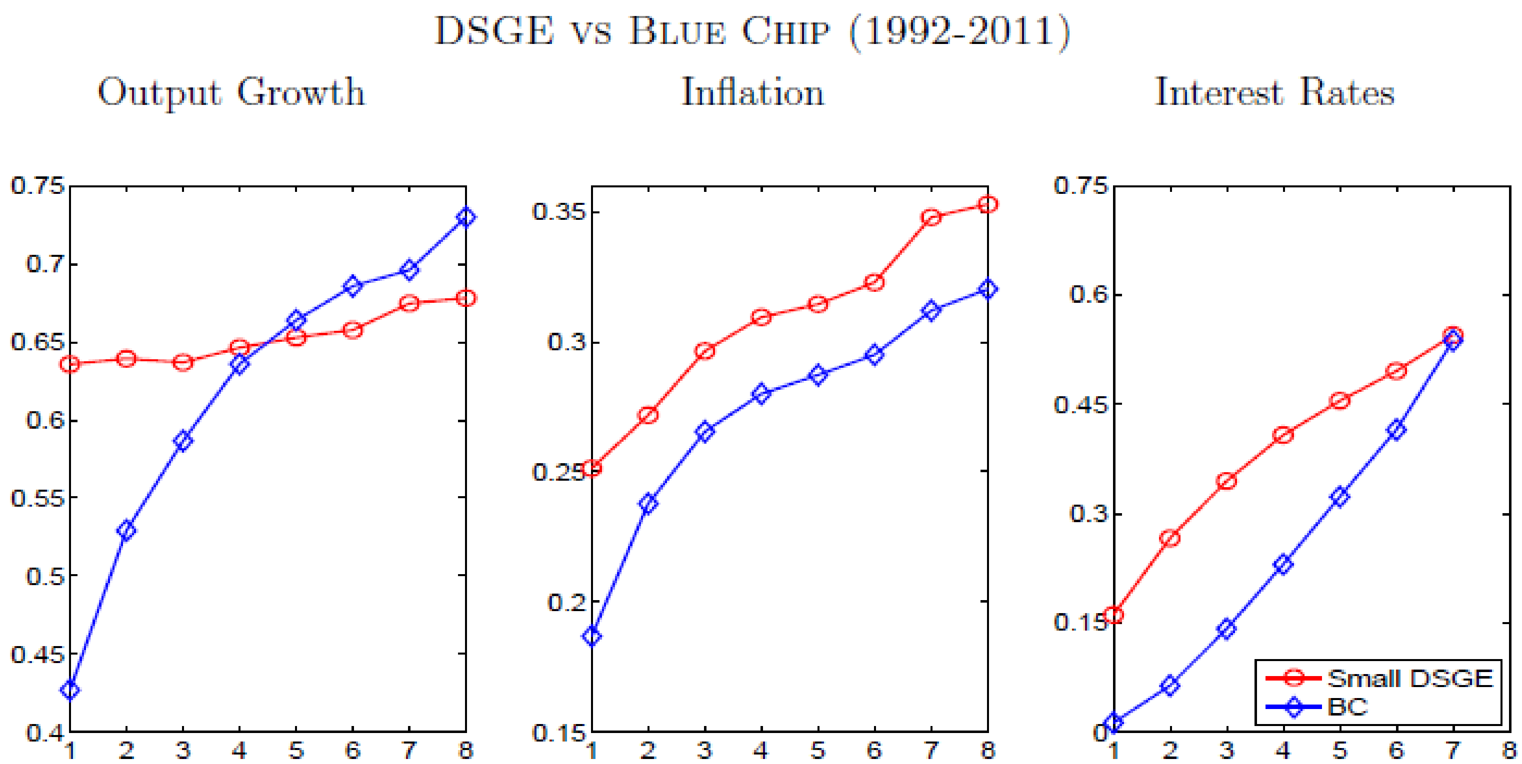
- In this way, the forecasts can be explained in terms of the shocks impacting the economy in the model.

Goodness of a DSGE forecast

Del Negro and Shorfheide (2012) compare the accuracy of DSGE model forecasts with Blue Chip consensus forecasts and Greenbook forecasts.

- **The Blue Chip Survey** of economic indicators provides forecasts for the end of the current year and for the following year. These forecasts are from a panel of experts in economic forecasting. Del Negro and Shorfheide use the consensus forecasts for comparison purposes.
- **Greenbook** uses the forecast by the Federal Reserve in advance of each meeting of the Federal Open Market Committee.
- **DSGE forecasts:**
  1. Small DSGE model: This is a model of a closed economy with three equations. The model is estimated using three observable variables.
  2. Smets and Wouters DSGE (2007) model: This model has several nominal and real rigidities. The model is estimated using seven observable variables.

Goodness of a DSGE forecast



Small RMSE DSGE model forecasts vs Blue Chip forecasts
Source: Del Negro and Shorfheide (2012)

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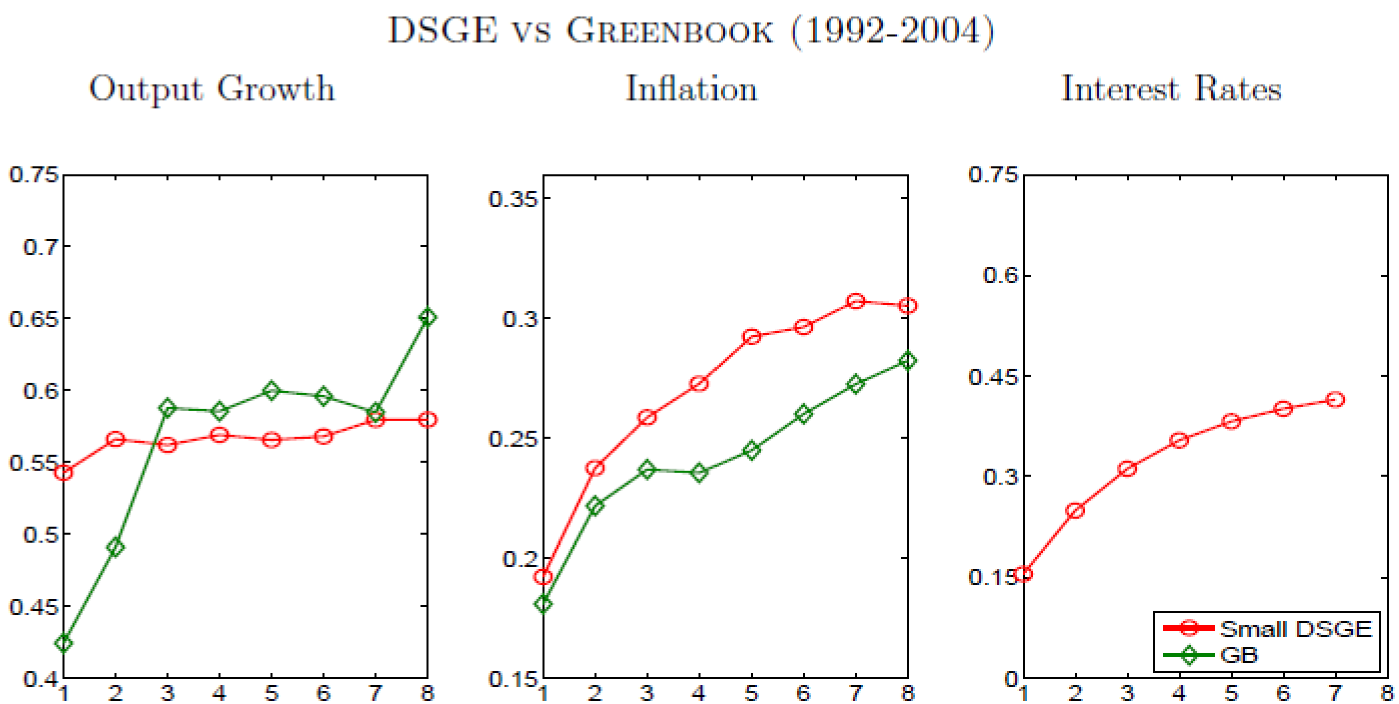
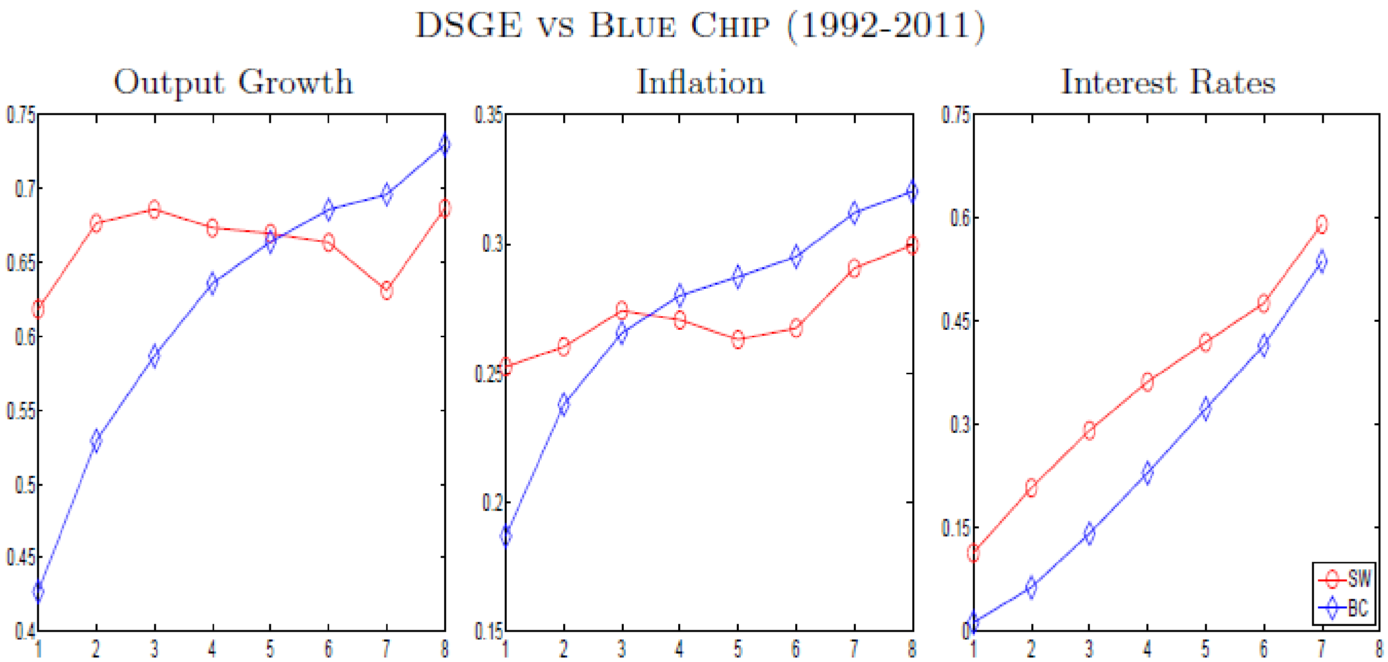
Goodness of a DSGE forecast



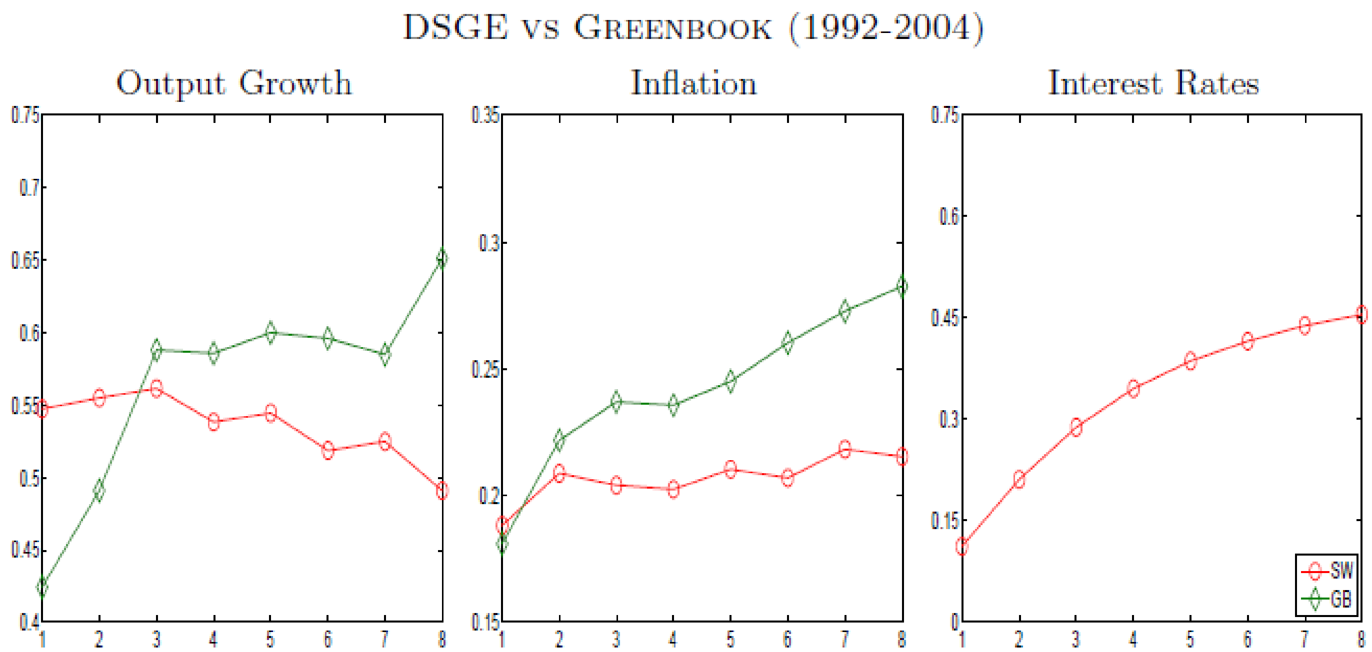
Simple RMSE DSGE forecasts vs Greenbook forecasts
Source: Del Negro and Shorfheide (2012)

Goodness of a DSGE forecast



RMSE Smets and Wouters Model forecasts vs Blue Chip consensus forecasts
Source: Del Negro and Shorfheide (2012)

Goodness of a DSGE forecast



RMSE Smets and Wouters (2007) model forecasts vs Greenbook forecasts
Source: Del Negro and Shorfheide (2012)

Conditional forecast

Conditional forecasts use “extra” information on the future in forecasting.

Advantages:

- Provided that the information is reliable, conditional forecasts can improve the forecasting capacity of our model.

- They are useful for policy simulation.

Conditional forecast
When would we want to use a conditional forecast?

There is a natural need to include conditional information in a forecast in circumstances where:

- **Some variables are observed with greater frequency**

  For example, interest and inflation rates are observed on a monthly basis while DGSE models are usually quarterly.

- **When there are available forecasts of some variables that are difficult to forecast using our DSGE model.**

  Typical examples are external variables, global GDP, and the external interest rate. Domestic inflation in the short term (one quarter) is another example.

Two conditioning methods
First method

With this method, the conditioning information is used to create alternative scenarios for the forecasting exercise.

What effect might an increase in the external interest rate have on the economy?

How would the economy behave if the short-term interest rate followed a certain path?

Two conditioning methods
Second

With this method, the conditioning information is used to add information to the forecasting exercise.

- GDP data are very commonly observed with a time lag. However, there are “nowcast” indicators available at the time of the forecast.

- We may also want to include the short-term forecasts from other models (or forecasts from other departments in the central bank) in our DSGE forecasting.

Two conditioning methods
Details of the first method

With the first method, the conditioning information is used to create alternative scenarios for the forecasting exercise.

1. Exogenous variables are conditioned:
   external interest rate, external demand, etc.

2. Endogenous variables are conditioned:
   paths for GDP, consumption, inflation, etc.

Details of the first method
Exogenous variables

- Let us assume that we have the path for the exogenous variable *R* from *T+1* up until *T+ F* *.
  It will be called
  $$R_{T+1:T+F^*}$$
- With a given law of movement for the exogenous process, it is possible to find the sequence of innovations consistent with this path.

Details of the first method
Exogenous variables

Assuming that the law of movement is

$$R_t = (1-\rho)\overline{R} + \rho R_{t-1} + \varepsilon_t$$

Then the sequence of innovations can be found through

$$\widehat{\epsilon}_{T+1} = (1-\rho)\bar{R} + \rho R_T - R^*_{T+1}$$

$$\widehat{\epsilon}_{T+2} = (1-\rho)\bar{R} + \rho R_{T+1} - R^*_{T+2}$$

$$\widehat{\epsilon}_{T+F^*} = (1-\rho)\bar{R} + \rho R_{T+F^*-1} - R^*_{T+F^*}$$

Details of the first method
Exogenous variables

- The forecasts for the other variables of the model can be recovered by means of a model simulation using the vector of shocks, consisting of the $\widehat{\epsilon}_{T+1:T+F^*}$ sequence:

- After solving for $\widehat{\epsilon}_{T+1}$ we can generate:

$$X_{T+1|T} = PX_T^T + Q\hat{\epsilon}_{T+1}$$

$$X_{T+2|T} = PX_{T+1|T} + Q\hat{\epsilon}_{T+2}$$

$$X_{T+F^*|T} = PX_{T+F^*-1|T} + Q\hat{\epsilon}_{T+F^*}$$

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Details of the first method
Endogenous variables

- Conditioning is more complicated with endogenous variables.
  - Example: In this case, forecasts of a particular value for the inflation rate, the GDP growth rate, or the short-term nominal interest rate are conditioned.

- To condition the model forecast with information on an endogenous variable we need to select one or more structural shocks and a sequence of innovations that are consistent with the conditioning information.
  - A simple example is conditioning a short-term interest rate path. In this case, we can use the monetary policy shock to reach a particular conditioned value. But this is not the only possibility!

Details of the first method
Endogenous variables

In calculating structural shocks, we need to bear two things in mind:

1. There are other shocks affecting endogenous variables simultaneously;

2. Any innovation has contemporaneous effects on all the endogenous variables.

A detailed explanation is given in the Appendix.

Details of the first method
Endogenous variables

To recap:

If the forecast is conditioned for v endogenous variables, it is necessary to find a sequence of v structural shocks that would be consistent with the conditioning information!

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In practice

"Oh! **So now**,

who will protect me?“

Dynare also does this!

Conditional forecasts in Dynare

- We make these forecasts in Dynare with the commands conditional_forecast_paths and conditional_forecast

```
conditional_forecast_paths;
var fedrate;
periods 1;
values -0.4375;
end;
```

- Conditioned variable
- Periods of conditioned forecasting (1: T+1; 2: T+2; etc)
- Values for those periods

Value of the parameters for the forecast

Shocks used to obtain the value of the conditioned variables in the forecasting periods stated above (1 in this case)

Forecasting periods

```
conditional_forecast(parameter_set = posterior_mode,controlled_varexo=(e_fedrate),periods=17);
```

Where are the conditional forecasts stored in Dynare?

- The conditional forecasts are stored in a Matlab structure called 'forecasts':
  - `forecasts.cond.Mean.y`: mean **conditional** forecast for the variable 'y'
  - `forecasts.cond.ci.y`: has the **confidence intervals** of the **conditional** forecast for the variable 'y'
  - `forecasts.uncond.Mean.y`: mean **unconditional** forecast for the variable 'y'
  - `forecasts.uncond.ci.y`: has the **confidence intervals** of the **unconditional** forecast for the variable 'y'

An exercise:
A forecasting exercise with changing parameters

- Alternative forecasting scenarios can be constructed using any parameter set.

- Assuming that the new model leads to the following solution:

$$X_t = PX_{t-1} + Q\epsilon_t$$

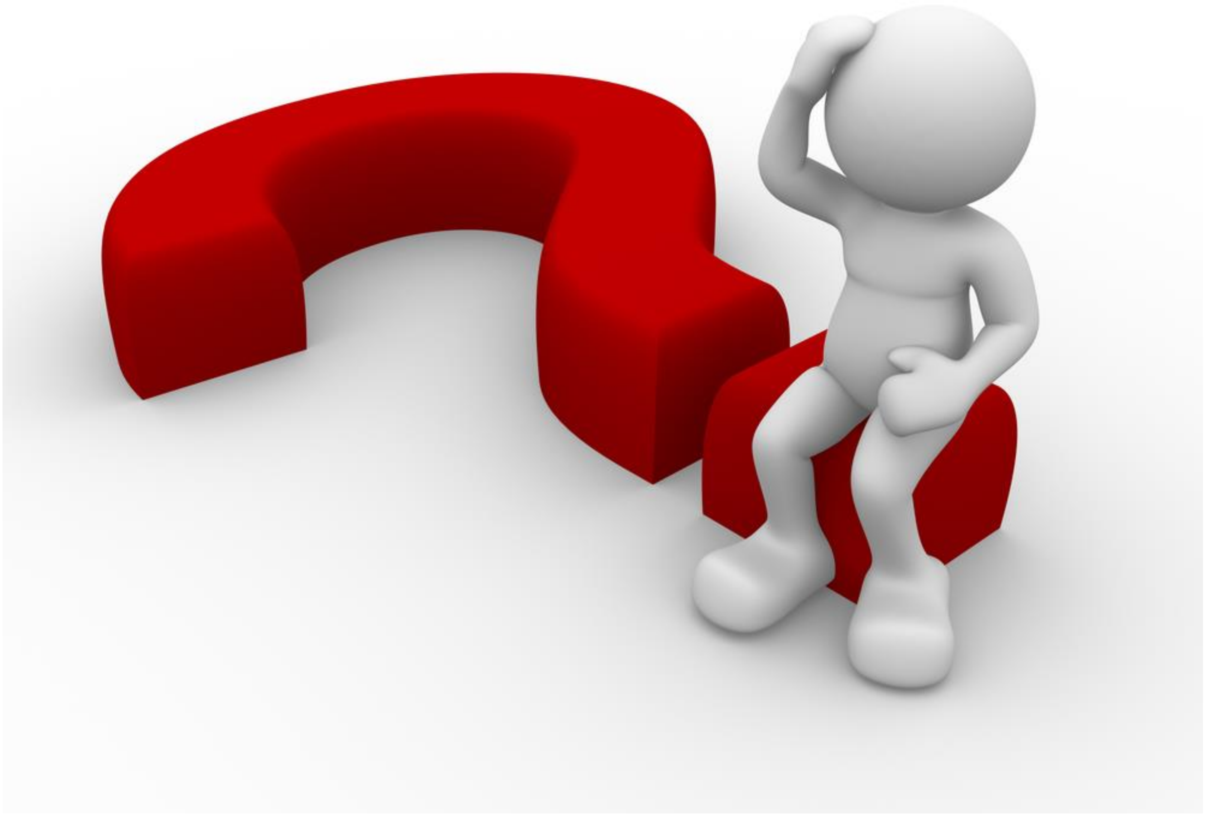
- The alternative forecasts will be:

$X(a)_{T|T}$ estimated with the original model

$$X(a)_{T+i|T} = PX(a)_{T+i-1|T}, i = 1, \ldots, k$$

$$X(a)_{T+i|T} = P^{new} X(a)_{T+i-1|T, i=k+1, \ldots. T^*}$$

How would you do this in Dynare?



Second conditioning model

Below is a description of a simple procedure for incorporating “nowcasts” and forecasts from other models in our dataset.

- With this method, there is no difference between endogenous and exogenous variables.
- The conditioning information is treated with uncertainty, unlike the previous method, in which the conditioning assumptions are deterministic.
- The Kalman filter will estimate the sequences of shocks that are consistent with our conditioning assumptions. So we do not have total control over which structural shocks are subject to adjustment.
  - Instead Mr Kalman, we will do it!

Incorporate nowcasts and forecasts from other models in the dataset

		Estimation sample				Forecasting Quarter	Forecast horizon			end
		T=1			t=T	t=T+1	t=T+2		t=T+F*	t=t+F
		Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	
Interest rates	...						NA	NA	NA	...
Inflation rate	...						Staff	NA	NA	...
GDP growth	...					Nowcast	NA	NA	NA	...
External interest rates	...						IMF forecast			...

Available data

The general idea
Incorporate **Nowcast** information

Nowcasts provide information on an observable variable that is unknown at the time of the forecast. One example is GDP growth.

Nowcasts are a noisy signal of the observable variable.

$$y^{NC}_{T+1} = \underbrace{y_{T+1}}_{\text{Obervable variable}} + \underbrace{\sigma \nu_{t+1}}_{Noise}$$

The general idea
Incorporate ***Forecasting*** information

- A forecast is a realization of the predictive density of any model.

- Thus a forecast can be interpreted as the expected value of an observable variable

$$y_{T+1} = \underbrace{y^F_{T+1}}_{Forecast\ from\ other\ model} + \underbrace{\sigma v_{t+1}}_{Noise}$$

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How can this be implemented in practice?

- To incorporate nowcasts and forecasts from other models in our dataset, all we need is the Kalman filter with matrices that change over time.
- The additional information is then added by means of the state-space representation,

$$X_t^{obs} = \mathbf{H}_t X_t + \mathbf{R}_t v_t$$ Measurement equation

$$X_t = \mathbf{P} X_{t-1} + \mathbf{Q} \varepsilon_t$$ Measurement equation

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How can this be implemented in practice?

- The matrices $\boldsymbol{H}_t$ and $\boldsymbol{R}_t$ change from nowcasts to forecasts.

		Estimation sample				Forecasting Quarter	Forecast horizon			end
		T=1			t=T	t=T+1	t=T+2		t=T+F*	t=t+F
		Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	
Interest rates	...						NA	NA	NA	...
Inflation rate	...						Staff	NA	NA	...
GDP growth	...					Nowcast	NA	NA	NA	...
External interest rates	...						IMF forecast			...

Available data

Estimation sample	Forecasting quarter	Forecast horizon
$H_t=H$; $R_t=R_1=0$	$H_t=H$; $R_t=R_2$	$H_t=H_t$; $R_t=R_3$

How can this be implemented in practice?

- With the **nowcast,** $\boldsymbol{H_t}$ remains constant up until *T+1* but there is an error in measurement after *T+1.*

  See the example of GDP growth.

- **With the forecast,** $\boldsymbol{H_{jt}} = 0$ when no data is available (NA) and $\boldsymbol{H_{jt}} = 1$ with $\boldsymbol{R_{jt}} = \sigma > 0$ when information is available.

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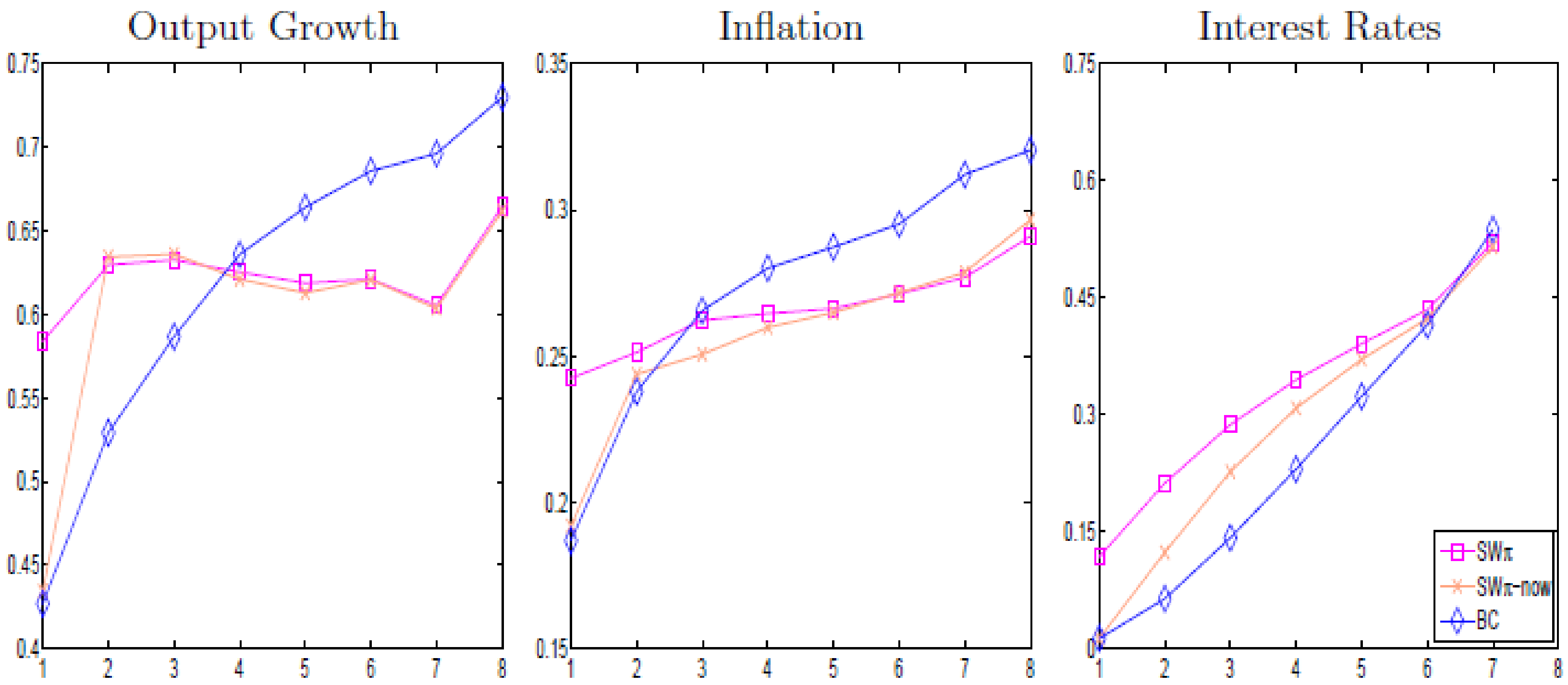
How is this done in Dynare?

Dynare does not do this directly but we can modify the dataset.

What do we gain from conditioning?

RMSE Smets Wouters with nowcast vs Bluechip



Source: Del Negro Schorfheide (2012)

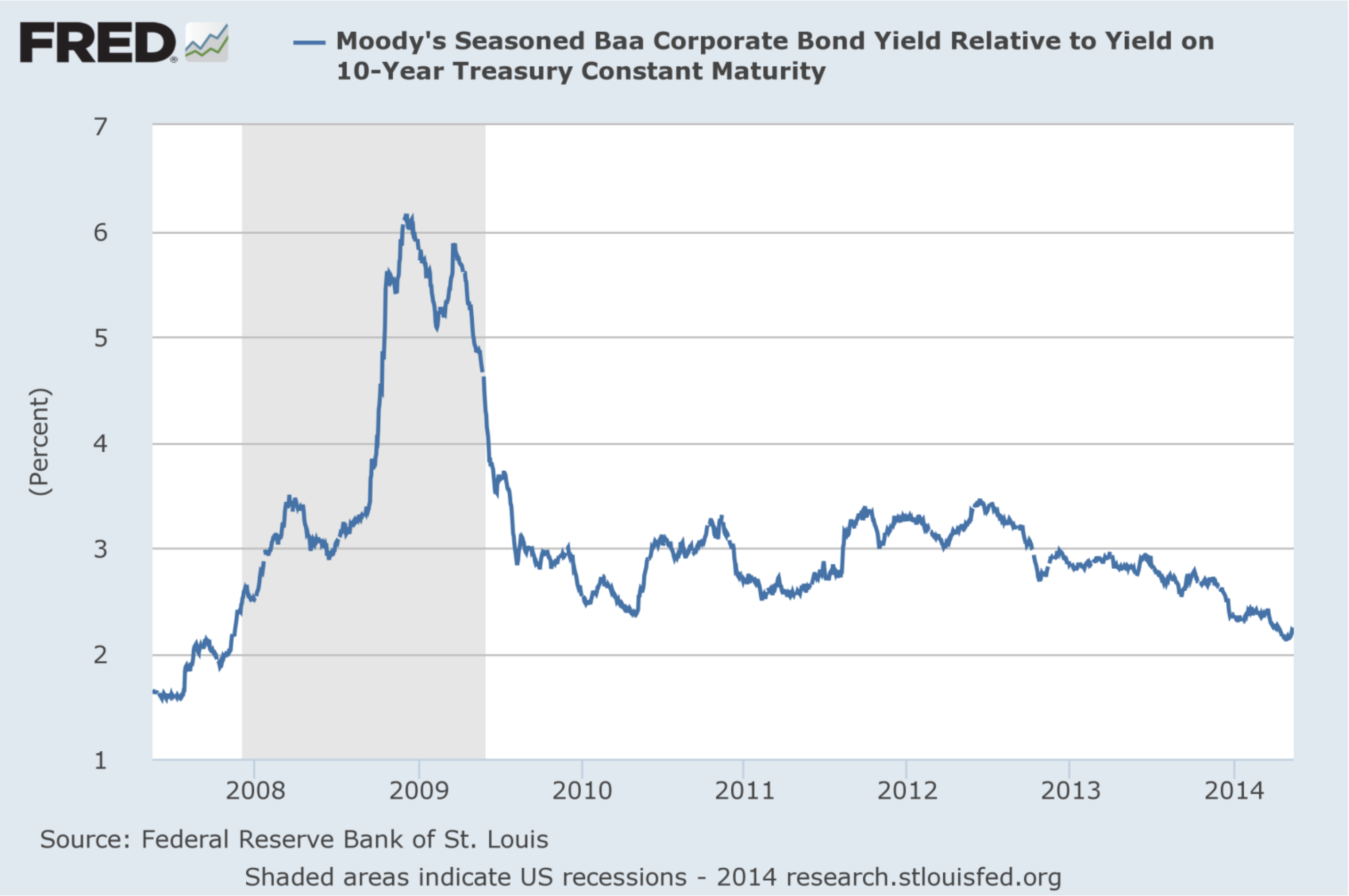
What do we gain from conditioning?

- Del Negro and Shorfheide (2012) modify the SW model and include financial frictions with the BGG model.

- They also add as an observable variable the spread between the yield on BAA corporate bonds and 10-year treasuries. This variable captures risk pricing on the financial markets.

- This variable is also a good candidate for the application of nowcasting techniques because it is observable on a daily basis.

BAA Corporate Bond Spread relative to 10-Year Treasuries



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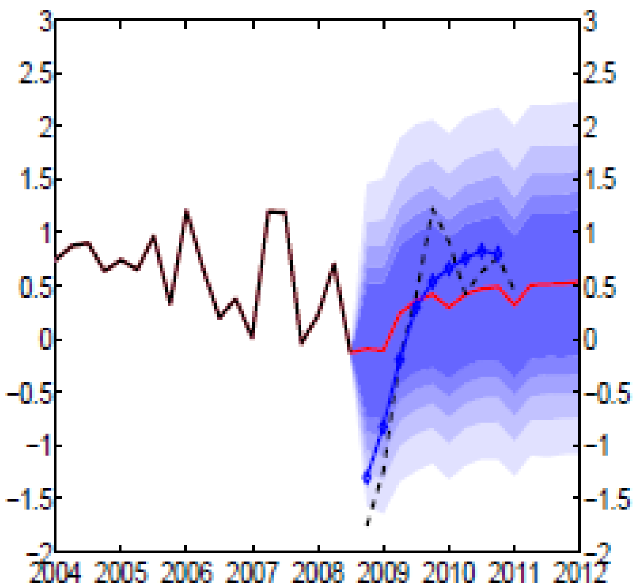
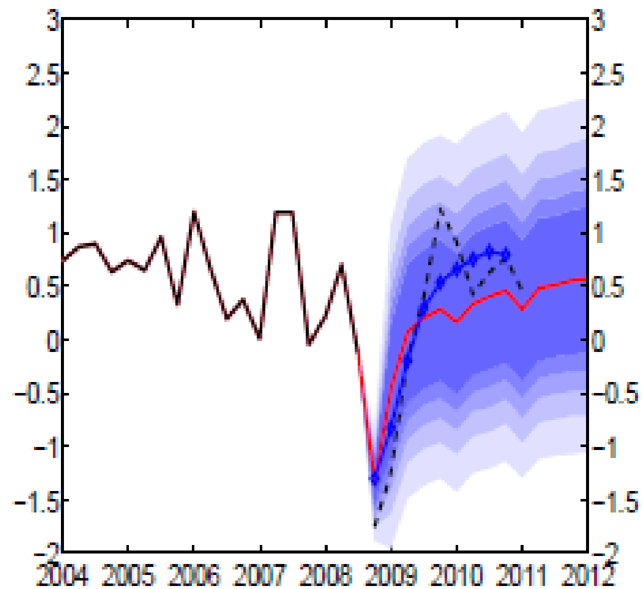
Goodness of Conditioned Forecasting

Forecasting output growth using the SW model with the financial accelerator



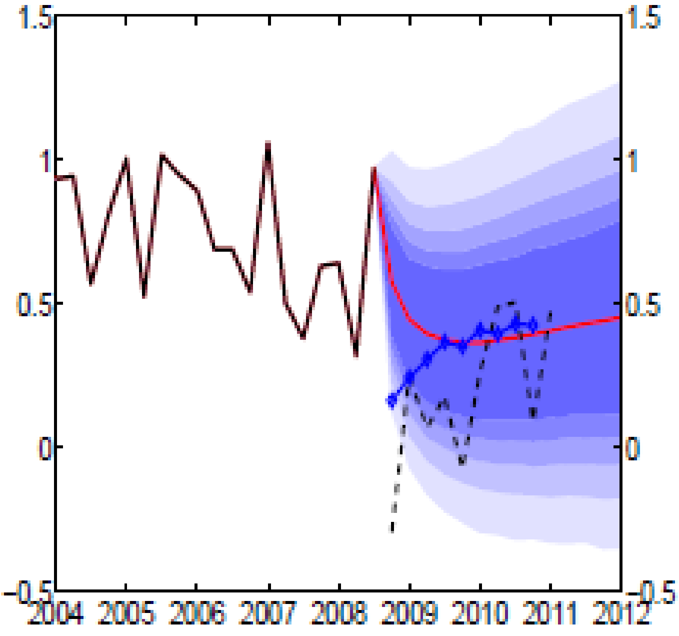
Unconditioned



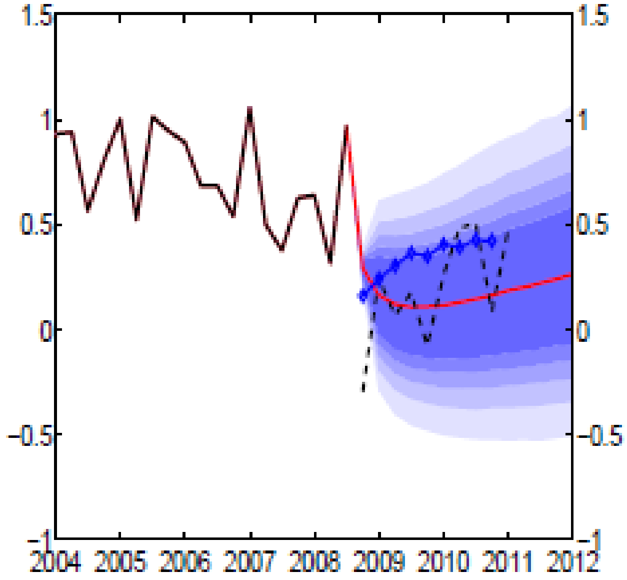
Conditioned

Goodness of Conditioned Forecasting

Forecasting inflation using the SW model
with the financial accelerator



Unconditioned



Conditioned

Conclusions

- When forecasting with a DSGE the following is needed:
  An estimate of the state vector of the economy. This is estimated using the Kalman filter and smoothed.

- Based on the solution, many interesting policy analysis exercises can be conducted:
  - unconditioned and conditioned forecasts
  - counterfactual exercises

- Working with missing data and heterogeneous datasets is relatively easy if the Kalman filter is used.

Conclusion and Discussion
DSGE models and other models

- What is forecasting capacity?

- Why do we use a structural macroeconomic model?

- How does the DSGE model compare with other macroeconomic and econometric models?

Conclusion and discussion
DSGE models and other models

Thanks

Model Solution and State Space Representation

Model solution

- Before the model can be used for forecasting it has to be solved and estimated /calibrated.
- The solution of a DSGE model is the nonlinear policy function

$$s_t = \Phi\left(s_{t-1}, \varepsilon_t; \theta\right)$$

  that gives the equilibrium law of motion for the state variables.
- Where
  - s_t are the state variables in the model. Examples are: the capital stock, debt, structural shocks, etc.
  - θ: is a vector of structural parameters
  - ε_t : is a vector of structural innovations

Solution of the model

- The solution to the nonlinear model is difficult to find and consequently, we usually work with a (log) linear approximation given by

$$s_t = A_1(\theta)s_{t-1} + A_2(\theta)\varepsilon_t$$

- Once we have the state vector we recover the remaining endogenous variables through the following equation

$$f_t = A_3(\theta)s_t$$

that relates the vector of states with a vector that contains both observable and non observable variables.

State-Space representation

- The solution to the model can be manipulated to get the state-space representation of the model.
- The transition equation is given by

$$x_t = P(\theta)x_{t-1} + Q(\theta)\varepsilon_t \qquad P(\theta) = \begin{bmatrix} 0 & A_3(\theta)A_1(\theta) \\ 0 & A_2(\theta) \end{bmatrix}$$

$$x_t = \begin{pmatrix} f_t \\ s_t \end{pmatrix} \qquad Q(\theta) = \begin{bmatrix} A_3(\theta)A_2(\theta) \\ A_2(\theta) \end{bmatrix}$$

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State-Space representation (cont ...)

- And the measurement equation

$$x_t^{obs} = Hx_t + u_t$$

- ***H*** is a matrix with zeros and ones that selects the observable variables out of the x_t vector.

Bayesian Forecasting
Sources of Uncertainty

Bayesian Forecasting

- Forecast uncertainty is a function of:

1. Parameter uncertainty that comes from the estimation of θ,
2. Uncertainty in the estimation of x_t^T
3. and uncertainty about future shocks that may affect the economy.

Bayesian forecasting takes into account these three sources of uncertainty.

Bayesian Forecasting

- Bayesian inference starts with a prior distribution *$p(\theta)$* and the likelihood function $L\left(x_{1:T}^{obs} \mid \theta\right)$
- As explained in previous lectures these two distributions (sources of information) are combined through the Bayesian theorem into the posterior distribution

$$p\left(\theta \mid x_{1:T}^{obs}\right) = \frac{p(\theta) L\left(x_{1:T}^{obs} \mid \theta\right)}{p\left(x_{1:T}^{obs}\right)}$$

where $p\left(x_{1:T}^{obs}\right) = \int p(\theta) L\left(x_{1:T}^{obs} \mid \theta\right) d\theta$

Bayesian Forecasting

- In the forecasting exercise the object of interest is not the posterior density but the posterior predictive density given by:

$$p\left(x_{T+1:T+F} \mid x_{1:T}^{obs}\right)$$

that gives the conditional probability $x_{T+1:T+F}$ given the information up to time *T*.

From posterior distribution to the predictive density

- The predictive density can be seen as an integral of the conditional distribution $p\left(x_{T+1:T+F} \mid x_{1:T}^{obs};\theta\right)$ with respect to the posterior density.

$$p\left(x_{T+1:T+F} \mid x_{1:T}^{obs}\right) = \int p\left(x_{T+1:T+F} \mid x_{1:T}^{obs};\theta\right) p\left(\theta \mid x_{1:T}^{obs}\right) d\theta$$

- This representation suggests that one can sample the posterior predictive density as follows

Posterior simulator for the predictive density

1. Draw θ^j from the posterior distribution $p\left(\theta \mid x_{1:T}^{obs}\right)$

2. and draw $x_{T+1:T+F}$ from $p\left(x_{T+1:T+F} \mid x_{1:T}^{obs}; \theta^j\right)$

- Based on these draws we can compute summary statistics for our forecast, such as: the mean, the variance, confidence intervals, etc.

Posterior simulator for the predictive density
An algorithm description (I)

1. Draw θ^j from the posterior distribution $p\left(\theta \mid x_{1:T}^{obs}\right)$ (MCMC output)
2. To get draws from $p\left(x_{T+1:T+F} \mid x_{1:T}^{obs};\theta^j\right)$ given θ^j. Use the Kalman filter to compute the mean and variance of $p\left(x_T^T \mid x_{1:T}^{obs};\theta^j\right)$
3. Generate a draw from

$$x_T^T \; : \; p\left(x_T^T \mid x_{1:T}^{obs};\theta^j\right) = N\left(x_T^T, V_T^T\right)$$

Posterior simulator for the predictive density
An algorithm description (II)

4. Draw a sequence of structural shocks $\varepsilon_{T+1:T+F}$ from $N\left(0,\Sigma_{\varepsilon}\right)$
5. Iterate

$$x_{T+1|T}^{(j)} = P\left(\theta^{j}\right)x_{T}^{T(j)} + Q\left(\theta^{j}\right)\varepsilon_{T+1}^{j}$$

$$x_{T+2|T}^{(j)} = P\left(\theta^{j}\right)x_{T+1}^{T(j)} + Q\left(\theta^{j}\right)\varepsilon_{T+2}^{j}$$

$$x_{T+F|T}^{(j)} = P\left(\theta^{j}\right)x_{T+F-1}^{T(j)} + Q\left(\theta^{j}\right)\varepsilon_{T+F}^{j}$$

Posterior simulator for the predictive density An algorithm description (III)

At this step you have a sequence $x^{(j)}_{T+1:T+F|T}$ that includes all variables in the model. If you only need the observable variables use the measurement equation

$$x^{obs(j)}_{T+s} = \Delta + Hx^{(j)}_{T+s|T}$$

6. Finally, repeat steps 1 through 5 *mc* times to get a sample of size *mc* from the predictive density.

Appendix 2
Conditioning with the Endogenous Variable

How to compute the sequence of structural shocks that is consistent with a given value of the endogenous variable?

- If you want to condition the forecast with v endogenous variables then you need v structural shocks.
- To understand the procedure it is easier to write the state space representation of the model as:

$$
\begin{aligned}
x_{T+1} &= \Delta + A_3(\theta) s_{T+1} \\
&= \Delta + A_3(\theta) A_1(\theta) s_{T+1} + A_3(\theta) A_2(\theta) \varepsilon_{T+1} \\
&= \Delta + T(\theta) s_{T+1} + R(\theta) \varepsilon_{T+1}
\end{aligned}
$$

How to compute the sequence of structural shocks that is consistent with a given value of the endogenous variable?

- Define the selection matrix M_ν $(\nu \; x \; \text{p})$ through

$$\bar{y}_{T+1} = M_\nu X_{T+1}$$

where $\bar{y}_{T+1}$ is a *($\nu \times$ 1)* vector that contains the conditioning information. *p* is the number of control variables in the model.

- Define the (k x 1) vector $\tilde{\varepsilon}_{T+1}$ as

$$\tilde{\varepsilon}_{jT+1} = 0 \quad \text{if } j \text{ is a conditioning shock}$$

How to compute the sequence of structural shocks that is consistent with a given value of the endogenous variable?

- The innovations to the selected structural shocks are given by

$$\bar{u}_{T+1} = \underbrace{\left(M_v R(\theta) M_u\right)^{-1}}_{\text{Contemporaneous effect}} \left[M_v \bar{y}_{T+1} - M_v \Delta - M_v T(\theta) s_{T+1} - \underbrace{M_v T(\theta)\tilde{\varepsilon}_{T+1}}_{\text{Shocks to other variables}} \right]$$

where M_u $(k \; x \; v)$ matrix that selects the subset of structural innovations that are used for conditioning.

- Finally,

$$\bar{y}_{T+1} = \Delta + T(\theta) s_{T+1} + R(\theta)\left(\tilde{\varepsilon}_{T+1} + M_u \bar{u}_{T+1}\right)$$

is the constrained vector of endogenous variables.

Appendix
Algorithm Description for Nowcasting and Forecasting

Algorithm description for nowcasting

1. Draw θ^j from the posterior distribution $p\left(\theta | x_{1:T}^{obs}\right)$
2. In period *T+1*

$$\underbrace{X_{T+1}^{\text{obs,nc}}}_{\text{Nowcast}} = \underbrace{H_{T+1} X_{T+1}}_{\text{Observable variable}} + \underbrace{\sigma v_{T+1}}_{\text{Noise}}$$

   is the measurement equation. Use the Kalman Filter updating to compute $p\left(x_{1:T}^{T+1} \mid x_{1:T}^{obs}, x_{1:T}^{obs,nc}; \theta^j\right)$ and generate a draw x_{T+1}^{j} from this distribution.

3. Given x_{T+1}^{j} and a sequence of structural shocks $\varepsilon_{T+1:T+F}$ drawn from $n\left(0, \Sigma_{\varepsilon}\right)$ iterate

$$x_{T+s}^{(j)} = P\left(\theta^j\right) x_{T+s-1} + Q\left(\theta^j\right) \varepsilon_{T+s}^{j} \qquad s = 1 \ldots F$$

4. Repeat 1 to 3 *mc* times to get *mc* samples from the posterior predictive density.

Algorithm description for forecasting

1. Draw θ^j from the posterior distribution $p\left(\theta | x_{1:T}^{obs}\right)$
2. Draw $x_{T+1}^{f(j)} = x_{T+1}^{f} + \sigma v_{T+1}$ from $v_{T+1} \sim n\left(0,1\right)$ which is the forecasting distribution of the alternative model.
3. In period *T+1*

$$\underbrace{X_{T+1}^{f(j)}}_{Forecast} = \underbrace{H_{T+1} X_{T+1}}_{\text{Observable variable}} + \underbrace{\sigma v_{T+1}}_{\text{Noise}}$$

is the measurement equation. Use the Kalman Filter updating to compute $p\left(x_{1:T}^{T+1} \mid x_{1:T}^{obs}, x_{1:T+1}^{f(j)}; \theta^j\right)$ and generate a draw x_{T+1}^{j} from this distribution.

4. Given x_{T+1}^{j} and a sequence of structural shocks $\varepsilon_{T+1:T+F}$ drawn from $n\left(0, \Sigma_\varepsilon\right)$ iterate

$$x_{T+s}^{(j)} = P\left(\theta^j\right) x_{T+s-1}^{j} + Q\left(\theta^j\right) \varepsilon_{T+s}^{j} \qquad s = 1 \ldots F$$

5. Repeat 1 to 4 *mc* times to get *mc* samples from the posterior predictive density.

Appendix
Kalman Filter Smoothed

Use of the Kalman filter

- We will use a simple notation in which we have manipulated the data so that they are stationary with a zero mean.

$$X_t = PX_{t-1} + Q\varepsilon_t \text{ (1')}$$
$$X_t^{obs} = HX_t \text{ (2')}$$
$$\varepsilon_t \sim iidN(0, \Omega)$$

- The idea is to use the observed data to draw the shocks and the other unobservable variables of the model (all contained in X).

Kalman filter: algorithm

- We start with the initial values $X_{0|0}$ y $V_{0|0}$, where:

$$V_{t|t-1} = E[(X_t - X_{t|t-1})(X_t - X_{t|t-1})'],$$
$$V_{t|t} = E[(X_t - X_{t|t})(X_t - X_{t|t})'].$$

- We use the model to draw:

$$X_{1|0} = \mathbf{P} X_{0|0},$$
$$X_{1|0}^{obs} = \mathbf{H} X_{1|0}$$
$$V_{1|0} = \mathbf{P} V_{0|0} \mathbf{P}' + \mathbf{Q \Omega Q}'.$$

- We draw the first observation and update the estimate of the state variable.

$$X_{1|1} = X_{1|0} + V_{1|0} \mathbf{H}'(\mathbf{H} V_{1|0} \mathbf{H}')^{-1} (X_1^{obs} - X_{1|0}^{obs})$$
$$V_{1|1} = V_{1|0} - V_{1|0} \mathbf{H}'(\mathbf{H} V_{1|0} \mathbf{H}')^{-1} \mathbf{H} V_{1|0}$$

Kalman filter: algorithm

- The same algorithm is repeated from t=2,…,T

$$X_{t|t-1} = \mathbf{P}X_{t-1|t-1},$$
$$X_{t|t-1}^{obs} = \mathbf{H}X_{t|t-1},$$
$$V_{t|t-1} = \mathbf{P}V_{t-1|t-1}\mathbf{P}' + \mathbf{Q\Omega Q}'$$

$$X_{t|t} = X_{t|t-1} + V_{t|t-1}\mathbf{H}'(\mathbf{H}V_{t|t-1}\mathbf{H}')^{-1}(X_t^{obs} - X_{t|t-1}),$$
$$V_{t|t} = V_{t|t-1} - V_{t|t-1}\mathbf{H}'(\mathbf{H}V_{t|t-1}\mathbf{H}')^{-1}\mathbf{H}V_{t|t-1}.$$

Kalman filter: smoothed

- We finally draw a sequence of the following variables:

  from t=1,…,T. $X_{t|t-1}, X_{t|t}, V_{t|t-1}, V_{t|t}$.

- To improve the estimation of state variables, we may want to use information from the entire sample.
- The process for obtaining $X_{t|T}$ y $V_{t|T}$ from t=1,…,T is called smoothing.

Kalman filter: smoothed

- As Hamilton (1994) points out, starting with $X_{T|T}$ and $V_{T|T}$ and applying standard formulas for orthogonal forecasts and the law of iterated expectations, we can obtain smoothed estimates of the states through the following iterations

$$X_{t|T} = X_{t|t} + J_t(X_{t+1|T} - X_{t+1|t})$$

$$J_t = V_{t|t}\mathbf{P'}(V_{t+1|t})^{-1}$$

$$V_{t|T} = V_{t|t} + J_t(V_{t+1|T} - V_{t+1|T})J_t^{'}.$$