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STRUCTURAL PATH ANALYSIS AND MULTIPLIER DECOMPOSITION WITHIN A SOCIAL ACCOUNTING MATRIX FRAMEWORK

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The main purpose of this paper is to apply structural path analysis to a Social Accounting Matrix (SAM) framework. Because the SAM is a comprehensive – essentially general equilibrium – data system, the whole network through which influence is transmitted can be identified and specified through structural path analysis. The latter provides an alternative and much more detailed way to decompose multipliers as compared with the traditional treatment of Stone (1978) and Pyatt and Round (1979).

This paper consists of five sections. The first one reviews the SAM framework as a basis for multiplier analysis and multiplier decomposition. In particular, the additive decomposition in terms of transfer, open-loop and closed-loop effects is succinctly presented. Section II applies this conventional decomposition to a SAM of South-Korea to illustrate with eleven specific cases the effects of an exogenous injection on the endogenous accounts of the SAM, i.e. the incomes, of the factors, household groups and production activities.

Section III is devoted to the presentation of the elements of structural analysis and, more particularly, the transmission of economic influence within a structure. Finally, Section IV applies structural path analysis to the South-Korean SAM and compares and contrasts the multiplier decomposition which it yields, with the alternative decomposition discussed in Section II. The comparison is the more significant in that the two decomposition methods are applied to the same eleven selected cases spanning a variety of sectors (i.e. poles) of origin (for the injection) and sectors (poles) of destination.

The empirical analysis in Section IV suggests that structural path analysis applied to a SAM is a potentially operationally useful technique within which a whole series of policy issues can be addressed. The final section is devoted to a brief summary and conclusions.

I. THE SOCIAL ACCOUNTING MATRIX, MULTIPLIER ANALYSIS AND DECOMPOSITION

The Social Accounting Matrix (SAM) has become used increasingly in the last years as a general equilibrium data system linking, among other accounts, production activities, factors of production and institutions (companies and households). As such, it captures the circular interdependence characteristic of any economic system among (*a*) production, (*b*) the factorial income distribution (i.e. the distribution of value added generated by each production activity to the

Table 1

Simplified Schematic Social Accounting Matrix

				Expenditures				
				Endogenous accounts			Exog.	
				Factors	Households	Production activities	Sum of other accounts	Totals
				1	2	3	4	5
Receipts	Endogenous accounts	Factors	1	o	o	$\mathbf{T}_{13}$	$\mathbf{x}_1$	$\mathbf{y}_1$
		Households	2	$\mathbf{T}_{21}$	$\mathbf{T}_{22}$	o	$\mathbf{x}_2$	$\mathbf{y}_2$
		Production activities	3	o	$\mathbf{T}_{32}$	$\mathbf{T}_{33}$	$\mathbf{x}_3$	$\mathbf{y}_3$
	Exog.	Sum of other accounts	4	$\mathbf{l}'_1$	$\mathbf{l}'_2$	$\mathbf{l}'_3$	$\mathbf{t}$	$\mathbf{y}_x$
	Totals		5	$\mathbf{y}'_1$	$\mathbf{y}'_2$	$\mathbf{y}'_3$	$\mathbf{y}_x$	

various factors), and (*c*) the income distribution among institutions and, particularly, among different socio-economic household groups.[1]

Under certain assumptions, such as excess capacity (i.e. availability of unused resources) and fixed prices, the SAM can be used as the basis for simple modelling. More specifically, the effects of exogenous injections on the whole economic system can be explored by multiplier analysis which requires partitioning the SAM into endogenous and exogenous accounts. Typically the former include (i) factors; (ii) institutions (companies and households); and, (iii) production activities; while the exogenous accounts consist of (iv) government; (v) capital; and (vi) rest of the world.

Table 1 shows this partition and the transformations (matrices) involving the three endogenous accounts. These matrices are, respectively, $\mathbf{T}_{13}$ which allocates the value added generated by the various production activities into income accruing to the factors of production; $\mathbf{T}_{33}$ which gives the intermediate input requirements (i.e. the input-output transactions matrix). $\mathbf{T}_{21}$ maps the factorial income distribution into the household income distribution (where households are distinguished according to socio-economic characteristics); $\mathbf{T}_{22}$ captures the income transfers within and among household groups; and finally $\mathbf{T}_{32}$ reflects the expenditure pattern of the various institutions (mainly the household groups) for

[1] For a discussion of the structure of the SAM and its potential use in policy analysis as a data system or as a basis for modelling, see Pyatt and Thorbecke (1976).

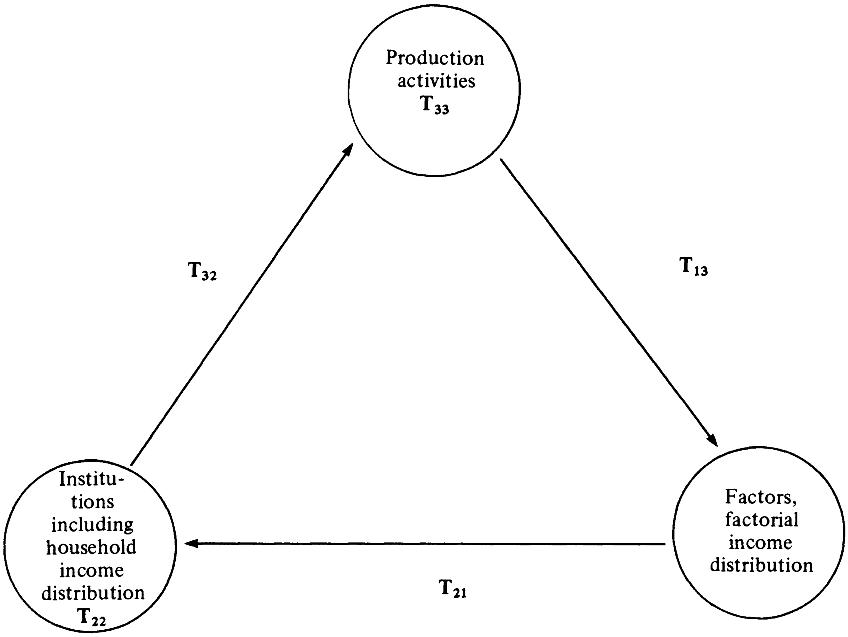

Fig. 1. Simplified interrelationship among principal SAM accounts (production activities, factors and institutions). $\mathbf{T}_{ij}$ stands for the corresponding matrix in the simplified SAM which appears on Table 1. Thus, for example, $\mathbf{T}_{13}$ refers to the matrix which appears at the intersection of row 1 (account 1), i.e. 'factors' and column 3 (account 3), i.e. 'production activities.'

the different commodities (production activities) which they consume. Fig. 1 shows this same triangular interdependence graphically using the same notation as in Table 1.

In Table 2 the row totals for incomes received by endogenous accounts are given by (the column vector) $\mathbf{y}_n$, which consists of two parts arising from, respectively, (i) expenditures by the endogenous accounts recorded as $\mathbf{T}_{nn}$ and summed up as column vector $\mathbf{n}$; and (ii) expenditures by the exogenous accounts recorded as $\mathbf{T}_{nx}$ and summed up as $\mathbf{x}$.[1] The latter part is referred to as injections. We have

$$\mathbf{y}_n = \mathbf{n} + \mathbf{x}. \tag{1}$$

Analogously for the incomes received by the exogenous accounts $\mathbf{y}_x$[2] (see Table 2)

$$\mathbf{y}_x = \mathbf{l} + \mathbf{t}. \tag{2}$$

The elements of the endogenous transaction matrix $\mathbf{T}_{nn}$ in Table 2 can be

[1] This section follows closely the notation given in Svejnar and Thorbecke (1983). See also for a similar treatment Pyatt and Round (1979), which uses, however, a somewhat different notation.

[2] It is to be noted that because Table 2 is a SAM, its corresponding row and column totals are equal – column totals for endogenous accounts are given by (the row vector) $\mathbf{y}'_n$, while those for exogenous accounts are given by $\mathbf{y}'_x$.

Table 2

Schematic Representation of Endogenous and Exogenous Accounts in a SAM

		Expenditures				
		Endogenous	Sum	Exogenous	Sum	Totals
Receipts	Endogenous	$\mathbf{T}_{nn}$	$\mathbf{n}$	Injections $\mathbf{T}_{nx}$	$\mathbf{x}$	$\mathbf{y}_n$
	Exogenous	Leakages $\mathbf{T}_{xn}$	$\mathbf{l}$	Residual balances $\mathbf{T}_{xx}$	$\mathbf{t}$	$\mathbf{y}_x$
Totals		$\mathbf{y}'_n$		$\mathbf{y}'_x$		

expressed as ratios of their corresponding column sums, i.e. as average expenditure propensities,[1]

$$\mathbf{T}_{nn} = \mathbf{A}_n \hat{\mathbf{y}}_n, \tag{3}$$

where $\hat{\mathbf{y}}_n$ is a diagonal matrix whose elements are y_i, $i = 1, \ldots, n$. Similarly

$$\mathbf{T}_{xn} = \mathbf{A}_1 \hat{\mathbf{y}}_n. \tag{4}$$

By introducing the matrices $\mathbf{A}_n$ and $\mathbf{A}_1$, $\mathbf{n}$ and $\mathbf{l}$ can now be expressed as

$$\mathbf{n} = \mathbf{A}_n \mathbf{y}_n, \tag{5}$$

and

$$\mathbf{l} = \mathbf{A}_1 \mathbf{y}_n. \tag{6}$$

Combining (1) and (5) gives the multiplier matrix $\mathbf{M}_a$,

$$\mathbf{y}_n = \mathbf{A}_n \mathbf{y}_n + \mathbf{x} = (\mathbf{I} - \mathbf{A}_n)^{-1} \mathbf{x} = \mathbf{M}_a \mathbf{x}. \tag{7}$$

Equation (7) yields endogenous incomes ($\mathbf{y}_n$) by multiplying injections ($\mathbf{x}$) by a multiplier matrix $\mathbf{M}_a$.[2] This matrix has been referred to as the accounting multiplier matrix because it explains the results observed in a SAM and not the process by which they are generated.[3]

As described previously, and as a comparison of Tables 1 and 2 shows, $\mathbf{T}_{nn}$ is partitioned. Corresponding to this partition the matrix of average expenditure propensities is as follows,

$$\mathbf{A}_n = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{bmatrix}. \tag{8}$$

At first glance the system specified in equations (7) and (8) appears analogous to the open Leontief model. In fact, the basic difference is that the SAM is closed

[1] Hence, columns of $\mathbf{A}_n$ in equation (3) show expenditures as proportions of total income (i.e. $\mathbf{y}'_n$ in Table 2) and not as absolute amounts as in $\mathbf{T}_{nn}$.

[1] More precisely, this equation yields the income levels of factors ($\mathbf{y}_1$), households ($\mathbf{y}_2$) and production activities ($\mathbf{y}_3$) which are endogenously determined as functions of the exogenous injections ($\mathbf{x}$).

[3] See Pyatt and Round (1979) for the distinction between accounting multiplier matrix and fixed-price multiplier matrix which is discussed subsequently.

with respect to the determination of the factorial and household income distribution and the consumption behaviour of households. In a SAM system, combining equations (7) and (8) and solving for the production activities vector ($\mathbf{y}_3$) yields

$$\mathbf{y}_3 = \mathbf{A}_{33}\mathbf{y}_3 + (\mathbf{A}_{32}\mathbf{y}_2 + \mathbf{x}_3) = (\mathbf{I} - \mathbf{A}_{33})^{-1}(\mathbf{A}_{32}\mathbf{y}_2 + \mathbf{x}_3). \tag{9}$$

This formulation generalises the Leontief model by including as one of the elements of final demand the effects of income distribution ($\mathbf{y}_2$) on household consumption (through $\mathbf{A}_{32}$ which reflects the consumption pattern of each group of households).[1]

One limitation of $\mathbf{M}_a$ as derived in equations (7) and (8) is that it implies unitary income elasticities (the prevailing average expenditure propensities in $\mathbf{A}_n$ are assumed to apply to any incremental injection). A more realistic alternative is to specify a matrix of marginal expenditure propensities ($\mathbf{C}_n$ below) corresponding to the observed income and expenditure elasticities of the different agents, under the assumption that prices remain fixed when income is altered. Expressing equation (1) in terms of changes in injections, one obtains

$$d\mathbf{y}_n = d\mathbf{n} + d\mathbf{x} \tag{10}$$

$$= \mathbf{C}_n d\mathbf{y}_n + d\mathbf{x} = (\mathbf{I} - \mathbf{C}_n)^{-1} d\mathbf{x} = \mathbf{M}_c d\mathbf{x}. \tag{11}$$

$\mathbf{M}_c$ has been coined the fixed-price multiplier matrix and its advantage is that it allows any non-negative income and expenditure elasticities to be reflected in $\mathbf{M}_c$.[2]

A rearrangement of the well-known multiplicative decomposition converts the matrix of accounting multipliers ($\mathbf{M}_a$) into four additive components, (i) the initial injection ($\mathbf{I}$); (ii) the net contribution of the transfer multiplier effects ($\mathbf{T}$); (iii) the net contribution of open-loop or cross multiplier effects ($\mathbf{O}$) and (iv) the net contribution of circular closed-loop effects ($\mathbf{C}$);[3]

$$\mathbf{M}_a = \mathbf{I} + \underbrace{(\mathbf{M}_{a1} - \mathbf{I})}_{\mathbf{T}} + \underbrace{(\mathbf{M}_{a2} - \mathbf{I})\mathbf{M}_{a1}}_{\mathbf{O}} + \underbrace{(\mathbf{M}_{a3} - \mathbf{I})\mathbf{M}_{a2}\mathbf{M}_{a1}}_{\mathbf{C}}.$$

$$= \mathbf{I} + \mathbf{T} + \mathbf{O} + \mathbf{C} \tag{12}$$

The transfer effects capture the multiplier effects resulting from direct transfers within endogenous accounts (in our particular case among institutions and households ($\mathbf{A}_{22}$) and the interindustry transfers ($\mathbf{A}_{33}$)). The open-loop effects capture the interactions among and between the three endogenous accounts, while the closed-loop effects ensure that the circular flow of income is completed among endogenous accounts, i.e. from production activities to factors to institutions and then back to activities in the form of consumption demand following the triangular pattern presented in Fig. 1.

[1] In contrast, the open Leontief model can be expressed as follows using the same notation $\mathbf{y}_3 = (\mathbf{I} - \mathbf{A}_{33})^{-1}\mathbf{f}$, where $\mathbf{A}_{33}$ is the input-output coefficient matrix and $\mathbf{f}$ is exogenous final demand. It is obvious that (9) contains more information and a higher degree of endogeneity since it captures the effects of income distribution on consumption which the Leontief formulation does not.

[2] Given the average expenditure propensities from the initial (base year) SAM table and a knowledge of the respective income elasticities, marginal expenditure propensities can be directly derived.

[3] For a detailed discussion and derivation of multiplier decomposition, see Pyatt *et al.* (1977); Pyatt and Round (1979); and Stone (1978).

It will be shown shortly that the above multiplier decomposition reveals only to a very limited extent how influence is transmitted within a structure. Because of the partitioning into three endogenous accounts, it can only decompose the effects of injections into total effects within and between accounts. As such it cannot identify the network of paths along which influence is carried among and between production activities, factors and households – which is the contribution of structural path analysis as shown in Sections III and IV.

II. MULTIPLIER DECOMPOSITION APPLIED TO SOUTH-KOREAN SAM

Before turning to structural path analysis the above multiplier analysis and decomposition is applied by way of illustration to a SAM which was built for South-Korea (1968).[1] The **A** matrix of this SAM appears in a truncated form in Table 3.[2] It can be seen that the factor account is broken down into 15 categories, i.e. six different labour skills, two types of self-employed, capital, five types of farmers and government workers. The classification of households is essentially similar to that of factors.[3] Production activities were divided into 29 activities on the basis of product-cum-technology characteristics. The other accounts (i.e. the exogenous ones) appearing in the SAM in Table 3 are government, capital and rest of the world.[4] The South-Korean SAM is meant only to capture in an approximate way the structure of the economy of South-Korea in 1968. It is used here only for demonstrational and illustrative purposes.

In Table 4, eleven cases are selected to illustrate the effects of an injection in one sector on another via the respective accounting multipliers. Table 4, furthermore, gives the decomposition of the multipliers into transfer effects, open loop effects and closed loop effects. These eleven cases are discussed very briefly in this section since they are analysed, in detail, on the basis of a different type of decomposition (i.e. structural path analysis) in Section IV. In fact, the purpose of this section is to illustrate the traditional multiplier decomposition as a background against which the alternative structural path analysis decomposition can be presented in Section IV.

The first two cases (I and II) in Table 4 involve the effects of an injection in one production activity on another. The initial injection could consist of government expenditure or export demand. Thus, for example, it can be asked what the consequences would be of an injection of 100 units (won) of exogenous demand (say, export demand) for mining products on 'other agriculture' (non-cereal)

[1] This SAM was built by Thorbecke based on the Adelman and Robinson (1978) data set as part of an NSF project dealing with the macroeconomic effects of the choice of technology. For detailed discussion of this SAM and, in particular, the attempt to distinguish activities according to product-cum-technology characteristics, see part 4 of Svejnar-Thorbecke (1983).

[2] To save space, Table 3 includes all the rows of the SAM but only selected columns, i.e. 3 factors, 3 household groups, 4 production activities and the exogenous accounts. The complete **A** matrix can be found in Svejnar and Thorbecke (1983). The corresponding accounting multipliers' Table is available upon request from the authors.

[3] It should be noted that the only institutions are households. Companies do not appear explicitly in the South-Korean SAM, instead the factors' account receives the capital value added and distributes to households directly.

[4] It can readily be ascertained that the **A** matrix (average expenditure propensities) in Table 3 has the same partition as Table 1 and equation (8).

Partial SAM Table for South Korea, 1968 – Partial Matrices of Av

						Endogenous		
						1 Factors of production		
						Engineers	Technicians	Farm size 1
						1	2	11
Receipts	1	Factors of production		Engineers	(1)			
				Technicians	(2)			
				Skilled workers	(3)			
				Apprentices	(4)			
				Unskilled workers	(5)			
				White-collar workers	(6)			
				Self-employed in manufacturing	(7)			
				Self-employed in services	(8)			
				Capital	(9)			
				Agricultural labourers	(10)			
				Farm size 1	(11)			
				Farm size 2	(12)			
				Farm size 3	(13)			
				Farm size 4	(14)			
				Government workers	(15)			
	2	Households	Wage earners	Engineers	(16)	0·529	0·002	0·0
				Technicians	(17)	0·0	0·827	0·0
				Skilled workers	(18)	0·0	0·0	0·0
				Apprentices	(19)	0·0	0·0	0·0
				Unskilled workers	(20)	0·0	0·0	0·0
				White-collar workers	(21)	0·0	0·104	0·0
			Self-employed	In manufacturing	(22)	0·0	0·0	0·0
				In services	(23)	0·0	0·0	0·0
			Agriculture	Capitalist	(24)	0·079	0·008	0·0
				Agricultural labourers	(25)	0·0	0·0	0·0
				Farm size 1	(26)	0·0	0·0	1·000
				Farm size 2	(27)	0·0	0·0	0·0
				Farm size 3	(28)	0·0	0·0	0·0
				Farm size 4	(29)	0·0	0·0	0·0
				Government workers	(30)	0·392	0·059	0·0
	3	Production activities		Cereals	(31)			
				Other agriculture	(32)			
				Fishing	(33)			
				Processed foods (L)	(34)			
				Processed foods (S+M+Self)	(35)			
				Mining	(36)			
				Textiles (L)	(37)			
				Textiles (S+M+self)	(38)			
				Finished textile products	(39)			
				Lumber and furniture	(40)			
				Chemical products (L)	(41)			
				Chemical products (S+M+self)	(42)			
				Energy (L+M)	(43)			
				Energy (S+self)	(44)			
				Cement, non-metallic mineral products	(45)			
				Metal products (L+M)	(46)			
				Metal products (S+self)	(47)			
				Machinery	(48)			
				Transport equipment	(49)			
				Beverages & tobacco (L)	(50)			
				Beverages & tobacco (S+M+self)	(51)			
				Other consumer products	(52)			
				Construction	(53)			
				Real estate	(54)			
				Transportation & communication	(55)			

Table 3

ble for South Korea, 1968 – Partial Matrices of Average Expenditure Propensities for Endogenous Accounts ($\mathbf{A}_n$) *an*

		Endogenous			Expenditures			Propensites				
		1 Factors of production			2 Households			3 Production activities				
		Engineers	Technicians	Farm size 1	Skilled workers	Apprentices	Unskilled workers	Other agriculture	Fishing	Processed foods (L)	Processed foods (S+M+self)	Government
		1	2	11	18	19	20	32	33	34	35	6
	(1)							0·0	0·011	0·006	0·004	
	(2)							0·0	0·015	0·006	0·006	
	(3)							0·0	0·020	0·031	0·027	
	(4)							0·0	0·004	0·003	0·007	
	(5)							0·0	0·182	0·016	0·030	
	(6)							0·0	0·050	0·014	0·029	
uring	(7)							0·0	0·041	0·0	0·010	
	(8)							0·0	0·0	0·0	0·0	
	(9)							0·291	0·303	0·121	0·084	
	(10)							0·038	0·0	0·0	0·0	
	(11)							0·099	0·0	0·0	0·0	
	(12)							0·091	0·0	0·0	0·0	
	(13)							0·096	0·0	0·0	0·0	
	(14)							0·043	0·0	0·0	0·0	
	(15)							0·0	0·0	0·0	0·0	
	(16)	0·529	0·002	0·0								
	(17)	0·0	0·827	0·0								
	(18)	0·0	0·0	0·0								
	(19)	0·0	0·0	0·0								
	(20)	0·0	0·0	0·0								
	(21)	0·0	0·104	0·0								
	(22)	0·0	0·0	0·0								
	(23)	0·0	0·0	0·0								
	(24)	0·079	0·008	0·0								
	(25)	0·0	0·0	0·0								
	(26)	0·0	0·0	1·000								
	(27)	0·0	0·0	0·0								
	(28)	0·0	0·0	0·0								
	(29)	0·0	0·0	0·0								
	(30)	0·392	0·059	0·0								
	(31)				0·161	0·022	0·197	0·102	0·0	0·008	0·008	708·
	(32)				0·124	0·144	0·122	0·141	0·016	0·236	0·236	466·
	(33)				0·023	0·023	0·018	0·0	0·0	0·066	0·066	71·0
	(34)				0·050	0·059	0·046	0·003	0·005	0·065	0·065	308·
lf)	(35)				0·043	0·051	0·040	0·002	0·004	0·056	0·056	266·
	(36)				0·0	0·0	0·0	0·001	0·004	0·006	0·006	198·
	(37)				0·002	0·002	0·001	0·0	0·001	0·0	0·0	6·93
	(38)				0·001	0·001	0·001	0·0	0·0	0·0	0·0	3·87
	(39)				0·065	0·075	0·078	0·003	0·034	0·004	0·004	170·
	(40)				0·001	0·002	0·001	0·002	0·007	0·002	0·002	546·
	(41)				0·014	0·017	0·014	0·015	0·002	0·017	0·017	1186
-self)	(42)				0·004	0·005	0·004	0·003	0·0	0·004	0·004	271·
	(43)				0·021	0·023	0·022	0·001	0·036	0·016	0·016	2737
	(44)				0·003	0·004	0·003	0·0	0·006	0·003	0·003	427·
ral products	(45)				0·002	0·002	0·002	0·0	0·0	0·002	0·002	131·
	(46)				0·001	0·002	0·001	0·008	0·003	0·004	0·004	121·
	(47)				0·0	0·001	0·0	0·003	0·001	0·001	0·001	41·6
	(48)				0·004	0·012	0·005	0·0	0·0	0·003	0·003	931·
	(49)				0·003	0·010	0·003	0·0	0·006	0·0	0·0	807·
	(50)				0·041	0·050	0·046	0·001	0·001	0·004	0·004	1856
M+self)	(51)				0·016	0·020	0·018	0·0	0·0	0·002	0·002	742·
	(52)				0·018	0·020	0·017	0·005	0·002	0·015	0·015	1261
	(53)				0·0	0·0	0·0	0·0	0·0	0·002	0·002	7929
	(54)				0·054	0·061	0·047	0·0	0·0	0·0	0·0	0·0
ication	(55)				0·067	0·084	0·055	0·006	0·007	0·014	0·014	4732

*re Propensities for Endogenous Accounts ($\mathbf{A}_n$) and Exogenous Accounts ($\mathbf{A}_1$)**

s	Propensites				Exogenous expenditures				
lds	3 Production activities				4	5	6		
Unskilled workers	Other agriculture	Fishing	Processed foods (L)	Processed foods (S+M+self)	Government expenditure	Capital account	Rest of the World	Sub-total: 4+5+6	Total income
20	32	33	34	35	61	62	63	64	65
	0·0	0·011	0·006	0·004					19639·7
	0·0	0·015	0·006	0·006					23774·2
	0·0	0·020	0·031	0·027					86006·9
	0·0	0·004	0·003	0·007					12666·4
	0·0	0·182	0·016	0·030					104686·0
	0·0	0·050	0·014	0·029					142743·0
	0·0	0·041	0·0	0·010					7395·87
	0·0	0·0	0·0	0·0					78371·9
	0·291	0·303	0·121	0·084					558745·0
	0·038	0·0	0·0	0·0					29132·3
	0·099	0·0	0·0	0·0					60709·2
	0·091	0·0	0·0	0·0					61179·1
	0·096	0·0	0·0	0·0					66680·4
	0·043	0·0	0·0	0·0					38986·9
	0·0	0·0	0·0	0·0					81720·0
							528·512	528·512	12320·0
							1074·93	1074·93	24948·8
							4386·12	4386·12	101306·0
							42·5918	42·5918	980·189
							4180·12	4180·12	96085·8
							7070·00	7070·00	163433·0
							1363·91	1363·91	31426·7
							8932·56	8932.56	206186·0
							0·687500	0·687500	122527·0
							1408·96	1408·96	31648·1
							4284·56	4284·56	96241·2
							5210·00	5210·00	117026·0
							6599·81	6599·81	147746·0
							2956·84	2956·84	66195·0
							4337·19	4337·19	100391·0
0·197	0·102	0·0	0·008	0·008	708·166	−11724·9	112·000	−10904·7	285056·0
0·122	0·141	0·016	0·236	0·236	466·054	771·884	3054·00	4291·94	295880·0
0·018	0·0	0·0	0·066	0·066	71·0000	44·8770	4425·00	4540·87	45409·1
0·046	0·003	0·005	0·065	0·065	308·284	425·145	6493·86	7227·29	91979·5
0·040	0·002	0·004	0·056	0·056	266·615	367·681	5616·13	6250·43	79544·1
0·0	0·001	0·004	0·006	0·006	198·000	1579·67	8583·00	10360·7	45027·0
0·001	0·0	0·001	0·0	0·0	6·93577	10826·1	11274·8	22107·8	85835·0
0·001	0·0	0·0	0·0	0·0	3·87316	6045·65	6296·21	12345·7	47934·0
0·078	0·003	0·034	0·004	0·004	170·036	1640·50	37136·0	38946·5	114861·0
0·001	0·002	0·007	0·002	0·002	546·857	1874·58	19319·0	21740·4	53206·5
0·014	0·015	0·002	0·017	0·017	1186·97	945·357	675·363	2807·69	55635·2
0·004	0·003	0·0	0·004	0·004	271·779	216·458	154·637	642·874	12738·4
0·022	0·001	0·036	0·016	0·016	2737·75	511·489	2650·95	5900·19	93449·0
0·003	0·0	0·006	0·003	0·003	427·608	79·8893	414·051	921·548	14595·2
0·002	0·0	0·0	0·002	0·002	131·390	−5476·98	1861·00	−3484·59	45461·5
0·001	0·008	0·003	0·004	0·004	121·150	558·663	2718·39	3398·21	57704·8
0·0	0·003	0·001	0·001	0·001	41·6968	192·278	935·606	1169·58	19860·4
0·005	0·0	0·0	0·003	0·003	931·883	15425·2	5287·00	21644·1	61398·4
0·003	0·0	0·006	0·0	0·0	807·623	31374·7	405·000	32587·4	64241·2
0·046	0·001	0·001	0·004	0·004	1856·51	3770·56	1883·66	7510·73	71587·6
0·018	0·0	0·0	0·002	0·002	742·480	1507·97	753·338	3003·79	28630·9
0·017	0·005	0·002	0·015	0·015	12616·3	2736·76	43789·0	59142·0	172127·0
0·0	0·0	0·0	0·002	0·002	7929·00	213775	8042·00	229746·0	254597·0
0·047	0·0	0·0	0·0	0·0	0·0	5508·89	0·0	5508·89	59983·9
0·055	0·006	0·007	0·014	0·014	4733·89	599·357	20822·0	35155·2	165290·0

	1	Factors of production		Technicians	(2)			
				Skilled workers	(3)			
				Apprentices	(4)			
				Unskilled workers	(5)			
				White-collar workers	(6)			
				Self-employed in manufacturing	(7)			
				Self-employed in services	(8)			
				Capital	(9)			
				Agricultural labourers	(10)			
				Farm size 1	(11)			
				Farm size 2	(12)			
				Farm size 3	(13)			
				Farm size 4	(14)			
				Government workers	(15)			
Receipts	2	Households	Wage earners	Engineers	(16)	0·529	0·002	0·0
				Technicians	(17)	0·0	0·827	0·0
				Skilled workers	(18)	0·0	0·0	0·0
				Apprentices	(19)	0·0	0·0	0·0
				Unskilled workers	(20)	0·0	0·0	0·0
				White-collar workers	(21)	0·0	0·104	0·0
			Self-employed	In manufacturing	(22)	0·0	0·0	0·0
				In services	(23)	0·0	0·0	0·0
			Agriculture	Capitalist	(24)	0·079	0·008	0·0
				Agricultural labourers	(25)	0·0	0·0	0·0
				Farm size 1	(26)	0·0	0·0	1·000
				Farm size 2	(27)	0·0	0·0	0·0
				Farm size 3	(28)	0·0	0·0	0·0
				Farm size 4	(29)	0·0	0·0	0·0
				Government workers	(30)	0·392	0·059	0·0
	3	Production activities		Cereals	(31)			
				Other agriculture	(32)			
				Fishing	(33)			
				Processed foods (L)	(34)			
				Processed foods (S+M+Self)	(35)			
				Mining	(36)			
				Textiles (L)	(37)			
				Textiles (S+M+self)	(38)			
				Finished textile products	(39)			
				Lumber and furniture	(40)			
				Chemical products (L)	(41)			
				Chemical products (S+M+self)	(42)			
				Energy (L+M)	(43)			
				Energy (S+self)	(44)			
				Cement, non-metallic mineral products	(45)			
				Metal products (L+M)	(46)			
				Metal products (S+self)	(47)			
				Machinery	(48)			
				Transport equipment	(49)			
				Beverages & tobacco (L)	(50)			
				Beverages & tobacco (S+M+self)	(51)			
				Other consumer products	(52)			
				Construction	(53)			
				Real estate	(54)			
				Transportation & communication	(55)			
				Trade and banking (S+M+L)	(56)			
				Trade and banking (self)	(57)			
				Education	(58)			
				Medical, personal & other services	(59)			
				Sub-total: 1+2+3	(60)	1·000	1·000	1·000
	4			Government income	(61)	0·0	0·0	0·0
	5			Capital account	(62)	0·0	0·0	0·0
	6			Rest of the World	(63)	0·0	0·0	0·0
				Sub-total: 4+5+6	(64)	0·0	0·0	0·0
				Total expenditures	(65)	19640	23774	60709

* For reasons of space, only some of the columns are shown in the above truncated Table. contained in rows 61–63, columns 1–59. Row 65 gives total expenditures in millions of won fo appear in columns 61–64 and are also expressed in millions of won.

	(2)							0·0	0·015	0·006	0·006	
	(3)							0·0	0·020	0·031	0·027	
	(4)							0·0	0·004	0·003	0·007	
	(5)							0·0	0·182	0·016	0·030	
	(6)							0·0	0·050	0·014	0·029	
uring	(7)							0·0	0·041	0·0	0·010	
	(8)							0·0	0·0	0·0	0·0	
	(9)							0·291	0·303	0·121	0·084	
	(10)							0·038	0·0	0·0	0·0	
	(11)							0·099	0·0	0·0	0·0	
	(12)							0·091	0·0	0·0	0·0	
	(13)							0·096	0·0	0·0	0·0	
	(14)							0·043	0·0	0·0	0·0	
	(15)							0·0	0·0	0·0	0·0	
	(16)	0·529	0·002	0·0								
	(17)	0·0	0·827	0·0								
	(18)	0·0	0·0	0·0								
	(19)	0·0	0·0	0·0								
	(20)	0·0	0·0	0·0								
	(21)	0·0	0·104	0·0								
	(22)	0·0	0·0	0·0								
	(23)	0·0	0·0	0·0								
	(24)	0·079	0·008	0·0								
	(25)	0·0	0·0	0·0								
	(26)	0·0	0·0	1·000								
	(27)	0·0	0·0	0·0								
	(28)	0·0	0·0	0·0								
	(29)	0·0	0·0	0·0								
	(30)	0·392	0·059	0·0								
	(31)				0·161	0·022	0·197	0·102	0·0	0·008	0·008	708·
	(32)				0·124	0·144	0·122	0·141	0·016	0·236	0·236	466·
	(33)				0·023	0·023	0·018	0·0	0·0	0·066	0·066	71·0
	(34)				0·050	0·059	0·046	0·003	0·005	0·065	0·065	308·
lf)	(35)				0·043	0·051	0·040	0·002	0·004	0·056	0·056	266·
	(36)				0·0	0·0	0·0	0·001	0·004	0·006	0·006	198·
	(37)				0·002	0·002	0·001	0·0	0·001	0·0	0·0	6·93
	(38)				0·001	0·001	0·001	0·0	0·0	0·0	0·0	3·87
	(39)				0·065	0·075	0·078	0·003	0·034	0·004	0·004	170·
	(40)				0·001	0·002	0·001	0·002	0·007	0·002	0·002	546·
	(41)				0·014	0·017	0·014	0·015	0·002	0·017	0·017	1186
- self)	(42)				0·004	0·005	0·004	0·003	0·0	0·004	0·004	271·
	(43)				0·021	0·023	0·022	0·001	0·036	0·016	0·016	2737
	(44)				0·003	0·004	0·003	0·0	0·006	0·003	0·003	427·
ral products	(45)				0·002	0·002	0·002	0·0	0·0	0·002	0·002	131·
	(46)				0·001	0·002	0·001	0·008	0·003	0·004	0·004	121·
	(47)				0·0	0·001	0·0	0·003	0·001	0·001	0·001	41·6
	(48)				0·004	0·012	0·005	0·0	0·0	0·003	0·003	931·
	(49)				0·003	0·010	0·003	0·0	0·006	0·0	0·0	807·
	(50)				0·041	0·050	0·046	0·001	0·001	0·004	0·004	1856
I+self)	(51)				0·016	0·020	0·018	0·0	0·0	0·002	0·002	742·
	(52)				0·018	0·020	0·017	0·005	0·002	0·015	0·015	1261
	(53)				0·0	0·0	0·0	0·0	0·0	0·002	0·002	7929
	(54)				0·054	0·061	0·047	0·0	0·0	0·0	0·0	0·0
ication	(55)				0·067	0·084	0·055	0·006	0·007	0·014	0·014	4733
+L)	(56)				0·048	0·052	0·050	0·009	0·017	0·025	0·025	1477
	(57)				0·062	0·067	0·064	0·012	0·021	0·033	0·033	1911
	(58)				0·017	0·024	0·003	0·0	0·0	0·0	0·0	3286
services	(59)				0·061	0·075	0·053	0·001	0·002	0·012	0·012	9469
	(60)	1·000	1·000	1·000	0·905	0·906	0·919	0·975	0·803	0·797	0·797	1682
	(61)	0·0	0·0	0·0	0·041	0·039	0·042	0·001	0·004	0·037	0·037	0·0
	(62)	0·0	0·0	0·0	0·034	0·034	0·018	0·001	0·075	0·009	0·009	1362
	(63)	0·0	0·0	0·0	0·020	0·021	0·021	0·022	0·117	0·157	0·157	3850
	(64)	0·0	0·0	0·0	0·095	0·094	0·081	0·025	0·197	0·203	0·203	1400
	(65)	19640	23774	60709	10131	980·2	96086	29588	45409	91980	79544	3083

f the columns are shown in the above truncated Table. In the original Table (see Svejnar and Thorbecke (1983)) $\mathbf{A}_n$ is contained i
9. Row 65 gives total expenditures in millions of won for each class and corresponds to column 65 which gives corresponding total
expressed in millions of won.

	0·0	0·015	0·006	0·006					23774·2
	0·0	0·020	0·031	0·027					86006·9
	0·0	0·004	0·003	0·007					12666·4
	0·0	0·182	0·016	0·030					104686·0
	0·0	0·050	0·014	0·029					142743·0
	0·0	0·041	0·0	0·010					7395·87
	0·0	0·0	0·0	0·0					78371·9
	0·291	0·303	0·121	0·084					558745·0
	0·038	0·0	0·0	0·0					29132·3
	0·099	0·0	0·0	0·0					60709·2
	0·091	0·0	0·0	0·0					61179·1
	0·096	0·0	0·0	0·0					66680·4
	0·043	0·0	0·0	0·0					38986·9
	0·0	0·0	0·0	0·0					81720·0
							528·512	528·512	12320·0
							1074·93	1074·93	24948·8
							4386·12	4386·12	101306·0
							42·5918	42·5918	980·189
							4180·12	4180·12	96085·8
							7070·00	7070·00	163433·0
							1363·91	1363·91	31426·7
							8932·56	8932.56	206186·0
							0·687500	0·687500	122527·0
							1408·96	1408·96	31648·1
							4284·56	4284·56	96241·2
							5210·00	5210·00	117026·0
							6599·81	6599·81	147746·0
							2956·84	2956·84	66195·0
							4337·19	4337·19	100391·0
0·197	0·102	0·0	0·008	0·008	708·166	−11724·9	112·000	−10904·7	285056·0
0·122	0·141	0·016	0·236	0·236	466·054	771·884	3054·00	4291·94	295880·0
0·018	0·0	0·0	0·066	0·066	71·0000	44·8770	4425·00	4540·87	45409·1
0·046	0·003	0·005	0·065	0·065	308·284	425·145	6493·86	7227·29	91979·5
0·040	0·002	0·004	0·056	0·056	266·615	367·681	5616·13	6250·43	79544·1
0·0	0·001	0·004	0·006	0·006	198·000	1579·67	8583·00	10360·7	45027·0
0·001	0·0	0·001	0·0	0·0	6·93577	10826·1	11274·8	22107·8	85835·0
0·001	0·0	0·0	0·0	0·0	3·87316	6045·65	6296·21	12345·7	47934·0
0·078	0·003	0·034	0·004	0·004	170·036	1640·50	37136·0	38946·5	114861·0
0·001	0·002	0·007	0·002	0·002	546·857	1874·58	19319·0	21740·4	53206·5
0·014	0·015	0·002	0·017	0·017	1186·97	945·357	675·363	2807·69	55635·2
0·004	0·003	0·0	0·004	0·004	271·779	216·458	154·637	642·874	12738·4
0·022	0·001	0·036	0·016	0·016	2737·75	511·489	2650·95	5900·19	93449·0
0·003	0·0	0·006	0·003	0·003	427·608	79·8893	414·051	921·548	14595·2
0·002	0·0	0·0	0·002	0·002	131·390	−5476·98	1861·00	−3484·59	45461·5
0·001	0·008	0·003	0·004	0·004	121·150	558·663	2718·39	3398·21	57704·8
0·0	0·003	0·001	0·001	0·001	41·6968	192·278	935·606	1169·58	19860·4
0·005	0·0	0·0	0·003	0·003	931·883	15425·2	5287·00	21644·1	61398·4
0·003	0·0	0·006	0·0	0·0	807·623	31374·7	405·000	32587·4	64241·2
0·046	0·001	0·001	0·004	0·004	1856·51	3770·56	1883·66	7510·73	71587·6
0·018	0·0	0·0	0·002	0·002	742·480	1507·97	753·338	3003·79	28630·9
0·017	0·005	0·002	0·015	0·015	12616·3	2736·76	43789·0	59142·0	172127·0
0·0	0·0	0·0	0·002	0·002	7929·00	213775	8042·00	229746·0	254597·0
0·047	0·0	0·0	0·0	0·0	0·0	5508·89	0·0	5508·89	59983·9
0·055	0·006	0·007	0·014	0·014	4733·89	599·357	29822·0	35155·2	165290·0
0·050	0·009	0·017	0·025	0·025	1477·16	18554·2	4656·40	24687·8	147053·0
0·064	0·012	0·021	0·033	0·033	1911·84	24014·1	6026·60	31952·5	190323·0
0·003	0·0	0·0	0·0	0·0	32864·1	25·9289	0·0	32890·0	53768·1
0·053	0·001	0·002	0·012	0·012	94696·1	−15269·1	8577·00	88003·9	240803·0
0·919	0·975	0·803	0·797	0·797	168231·0	310902·0	273339·0	752472·0	0·564488E+07
0·042	0·001	0·004	0·037	0·037	0·0	0·0	72300·0	72300·0	308301·0
0·018	0·001	0·075	0·009	0·009	136220·0	0·0	108030·0	244250·0	432992·0
0·021	0·022	0·117	0·157	0·157	385000·0	122090·0	0·0	125940·0	433669·0
0·081	0·025	0·197	0·203	0·203	140070·0	122090·0	180330·0	442490·0	0·119496E+07
96086	29588	45409	91980	79544	308301·0	432992·0	453669·0	0·1194960	0·6839840E7

ble (see Svejnar and Thorbecke (1983)) $\mathbf{A}_n$ is contained in rows 1–59, columns 1–59; $\mathbf{A}_1$ is
orresponds to column 65 which gives corresponding total incomes. Exogenous expenditures

1. Direct Influence

The direct influence of i on j transmitted through an elementary path is the change in income (or production) of j induced by a unitary change in i, the income (or the production) of all other poles except those along the selected elementary path remaining constant. The direct influence can be measured, respectively, along an arc or an elementary path as follows,

(*a*) Case of direct influence of i on j along arc (i,j)

$$I^D_{(i\to j)} = a_{ji}, \tag{13}$$

where a_{ji} is the (j,i)th element of the matrix of average expenditure propensities $\mathbf{A}_n$.[1] Matrix $\mathbf{A}_n$ can therefore be called the *matrix of direct influences* – it being understood that the direct influence is measured along arc (i,j).

(*b*) Case of direct influence along an elementary path $(i,...,j)$. The 'multiplication rule' applied to the influence graph shows that direct influence transmitted from a pole i to a pole j along a given elementary path is equal to the product of the intensities of the arcs constituting the path (Lantner, 1974, p. 53). Thus,

$$I^D_{(i\,...\,j)} = a_{jn} \dots a_{mi}. \tag{14}$$

For example, Fig. 2 below represents a given elementary path, $p = (l,x,y,j)$[2]

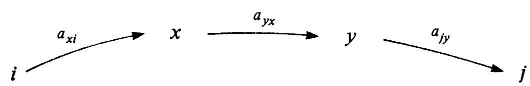

Fig. 2. Elementary path.

and
$$I^D_{(i\to j)_p} = I^D_{(i,x,y,j)} = a_{xi}\,a_{yx}\,a_{jy}. \tag{15}$$

2. Total Influence

In most structures, there exists a multitude of interactions among poles. In particular, poles along any elementary path are likely to be linked to other poles and other paths forming circuits which amplify in a complex way, the direct influence of that same elementary path. To capture these indirect effects Lantner (1974) introduced the concept of total influence.

Given an elementary path $p = (i,...,j)$ with origin i and destination j, the total influence is the influence transmitted from i to j along the elementary path p including all indirect effects within the structure imputable to that path. Thus, total influence cumulates, for a given elementary path p, the direct influence transmitted along the latter and the indirect effects induced by the circuits adjacent to that same path (i.e. these circuits which have one or more poles in common with path p). Fig. 3 reproduces the same elementary path $p = (i,x,y,j)$

[1] Indeed, according to the definition of average expenditure propensity: $t_{ji} = a_{ji}y_i$, where t_{ji} is the (j,i)th element of the transactions matrix of the SAM and y_i is the ith element of the row vector of column sums (representing the gross outputs of production activities, incomes of factors, and incomes of institutions respectively) from which it follows that $y_j = a_{ji}y_i = a_{ji}$ when the output or income of pole i increases by one unit $(y_i = 1)$.

[2] As will be seen subsequently, a multitude of different elementary paths may go from i to j.

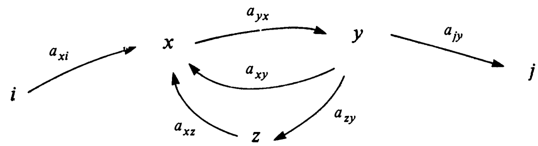

Fig. 3. Elementary path including adjacent circuits.

appearing in Fig. 2 and in addition incorporates explicitly all circuits adjacent to it.

It can readily be seen that between poles i and y the direct influence is $a_{xi}a_{yx}$ which is then transmitted back from y to x via the two loops yielding an effect $(a_{xi}a_{xy})\,(a_{xy}+a_{zy}a_{xz})$ which in turn has to be transmitted back from x to y. This process yields a series of dampened impulses between x and y

$$a_{xi}a_{yx}1\{1+a_{yx}(a_{xy}+a_{zy}a_{xz})+[a_{yx}(a_{xy}+a_{zy}a_{xz})]^2+\dots\}$$
$$=a_{xi}a_{yx}[1-a_{yx}(a_{xy}+a_{zy}a_{xz})]^{-1}. \quad (16)$$

To complete the transmission of influence along the above elementary path p the above effects have to travel along the last arc (y,j) so that the above effects have to be multiplied by a_{jy} to obtain the total influence along this path,

$$I^T_{(i\to j)_p}=a_{xi}a_{yx}a_{jy}[1-a_{yx}(a_{xy}+a_{zy}a_{xz})]^{-1}. \quad (17)$$

It can readily be seen that the first term on the right-hand side represents the previously defined direct influence, $I^D_{(i\to j)_p}$, the second term is the *path multiplier* $\mathbf{M}_p$, i.e.

$$I^T_{(i\to j)_p}=I^D_{(i\to j)_p}\mathbf{M}_p. \quad (18)$$

$\mathbf{M}_p$ captures the extent to which the direct influence along path p is amplified through the effects of adjacent feedback circuits. The measure of $\mathbf{M}_p$ is developed more formally in the Appendix. In general, within a structure, the path multiplier $\mathbf{M}_p$, of any elementary path p is equal to the ratio of two determinants $\Delta p/\Delta$ where Δ is the determinant $|\mathbf{I}-\mathbf{A}_n|$ of the structure represented by the SAM and Δ_p is the determinant of the structure excluding the poles constituting path p.

3. *Global Influence*

Global influence, in contrast with direct and total influences, does not refer to topology, namely, the specific paths followed in the transmission of influence. Global influence from pole i to pole j simply measures the total effects on income or output of pole j consequent to an injection of one unit of output or income in pole i.

The global influence is captured by the reduced form of the SAM model derived previously

$$\mathbf{y}_n=[\mathbf{I}-\mathbf{A}_n]^{-1}\mathbf{x}=\mathbf{M}_a\mathbf{x}. \quad (19)=(7)$$

Let $m_{a_{ji}}$ be the (j,i)th element of the matrix of accounting multipliers $\mathbf{M}_a$ then, as was seen previously, it captures the full effects of an exogenous injection x_i on the endogenous variable y_j. Hence

$$I^G_{(i\to j)}=m_{a_{ji}}, \quad (20)$$

and matrix $\mathbf{M}_a=[\mathbf{I}-\mathbf{A}_n]^{-1}$ can be called the *matrix of global influences*.

It is important to understand clearly the distinction between global influence and direct influence. The latter is linked to a particular elementary path which is entirely isolated from the rest of the structure (i.e. assuming *ceteris paribus*). It captures what could be called the immediate effect of an impulse following this particular path. Global influence, in contrast, differs from direct influence for two fundamental reasons:

(*a*) It captures the direct influence transmitted by *all* elementary paths linking (spanning) the two poles under consideration. Indeed, given two poles i and j, the effects of an injection affecting the output or income of i on the output or income of j manifest themselves through the intermediary of all paths with origin i and destination j. According to the 'additive rule' applied to the influence graph, the direct influence, transmitted by pole i to pole j along different elementary paths with the same origin and destination, is equal to the sum of the direct influences transmitted along each elementary path (Lantner (1974), p. 53).

(*b*) In addition, these paths are not considered in isolation but as an integral part of the structure from which they were separated to calculate the direct influence. Hence, global influence cumulates all induced and feedback effects resulting from the existence of circuits in the graph and is, as shown by Lantner (1974), pp. 246–7) and Gazon (1976, pp. 130–5) equal to the sum of the total influences of all elementary paths spanning pole i and pole j (see eq. 22).

An example should clarify this point. Fig. 4 reproduces the elementary path and adjacent circuits explored in Fig. 3 and adds two other elementary paths with the same origin i and destination j, i.e. (i,s,j) and (i,v,j).

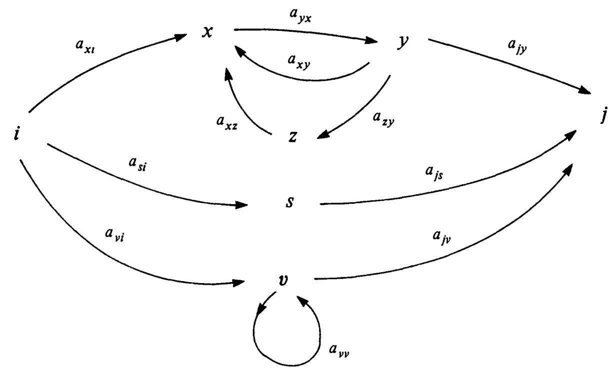

Fig. 4. Network of elementary paths and adjacent circuits linking poles i and j.

In the above example, it is clear that path (i,s,j) is an elementary path without any adjacent circuit while path (i,v,j) contains one loop centred on v. For simplicity, we can refer to these last two paths as 2 and 3, respectively – the initial path being referred to as 1.

$$\begin{aligned} I^G_{(i\to j)} = m_{aji} &= I^T_{(i,x,y,j)} + I^T_{(i,s,j)} + I^T_{(i,v,j)} \\ &= I^T_{(i\to j)_1} + I^T_{(i\to j)_2} + I^T_{(i\to j)_3} \\ &= I^D_{(i\to j)_1}\mathbf{M}_1 + a_{si}a_{js} + (a_{vi}a_{jv})(1-a_{vv})^{-1} \\ &= I^D_{(i\to j)_1}\mathbf{M}_1 + I^D_{(i\to j)_2} + I^D_{(i\to j)_3}\mathbf{M}_3. \end{aligned} \tag{21}$$

Note that in the case of the second path, the multiplier is one since the path has no adjacent circuits. Thus, in general, the global influence linking any two poles of a structure can be decomposed into a series of total influences transmitted along each and all elementary paths spanning i and j, i.e.

$$I^G_{(i\to j)} = m_{a_{ji}} = \sum_{p=1}^{n} I^T_{(i\to j)_p} = \sum_{p=n}^{n} I^D_{(i\to j)_p}\mathbf{M}_p, \tag{22}$$

where p stands for elementary paths $1, 2, k, \ldots, n$. This decomposition is more formally derived in the appendix where the relevant determinantal expansions are identified and related to path analysis interpretation.

IV. STRUCTURAL PATH ANALYSIS APPLIED TO SOUTH-KOREAN SAM

In order to illustrate the usefulness and the types of questions which path analysis can answer, it is applied in the present section to the South-Korean SAM structure presented in Section II. Given the questionable nature of some of the estimates which went into the SAM, this application should be considered as a demonstration exercise of the types of results which can be obtained with this approach.

It can be seen from Table 3 which gives the (truncated) matrix of average propensities ($\mathbf{A}_n$) that the endogenous part of the matrix consists of 59 poles (15 classes of factors, 15 classes of households and 29 production activities). There exists in such a structure a multitude of elementary paths.[1] One way of limiting the scope of the analysis is to study only those paths the length (i.e. number of arcs) of which does not exceed three. The more arcs a path contains, the weaker will be the direct and total influences transmitted along it.[2]

Even by limiting the scope of investigation in this fashion, this leaves many elementary paths which are not explicitly studied. Clearly the choice of paths to be explored depends on the questions raised. In what follows, we shall attempt to give a few selected examples organized according to (*a*) the SAM account in which the pole of origin is located and the SAM account containing the pole of destination; and (*b*) the type of question path analysis is supposed to elucidate.

Before actually embarking on the empirical analysis, it should be noted that the selected pole of origin (and its injection) within a SAM structure can be in any of the three endogenous accounts, production activities, factors, and institutions. However, the triangular interrelationship of the endogenous structure of the SAM means that an elementary path must always travel in the triangular direction as shown on Fig. 1. For example, if the injection occurs in a given production activity, all elementary paths originating with that activity would

[1] For example, in the structure represented by the 1966 French input–output table disaggregated in only six sectors, Lantner (1974, p. 257), has identified 844 elementary paths.

[2] One example suffices to illustrate this point: assume a path of length 4 (i.e. four consecutive arcs) and the intensity of the influence between any two poles equal to 0·5, then the direct influence would be equal to $(0{\cdot}5)^4$ or only 0·0625. Experimenting with the SAM South-Korea data set revealed that it is extremely rare to find a path of length four or longer transporting more than one half of a percent of the global influence transmitted from the pole of origin to the pole of destination. In any case, should such a path be presumed important, it could easily be identified by the computer.

affect, first, other production activities (through the induced demand for intermediate inputs represented by the I–O matrix $\mathbf{A}_{33}$) and factor demand (through the distribution of the value added among factors, i.e. matrix $\mathbf{A}_{13}$) before the influence is transmitted to institutions (in particular, the households) through matrix $\mathbf{A}_{21}$. Next in this sequence, transfers among institutions would be captured through $\mathbf{A}_{22}$ before the final link back to production activities (reflecting the consumption pattern of institutions, i.e. $\mathbf{A}_{32}$) can take place. Thus, there is an immutable ordering which is predetermined by the structure of the SAM. No elementary path can have arcs linking production activities directly to institutions (the $\mathbf{A}_{23}$ matrix is empty) or linking the latter directly to factors ($\mathbf{A}_{12}$ is, likewise, empty).

The examples which were used below have, in common, that the injection in each case takes place in one of the production activities except for the last one which originates with households. In principle, any other pole of origin – among factors or institutions – could equally as well have been selected. In order to provide a good basis for comparing the two types of multiplier decomposition structural path analysis is applied to the same eleven cases which were explored previously in Section II (see Table 4). These eleven different cases are analysed in Table 5. Each case (i) takes a given pole of origin (i) and destination (j) and measures the corresponding global influence; (ii) identifies the more important elementary paths spanning these two poles and measures their direct and total influences, respectively; and (iii) gives the proportion of the global influence between i and j transmitted through each specific path p.

These eleven cases can be, furthermore, broken down according to whether the pole of destination is a production activity, a factor or a household. Hence, these cases can be distinguished according to whether influence is transmitted (1) from production activity to production activity (Cases I and II); (2) from production activity to factors (Cases III–VIII); (3) from production activity to households (Cases IX and X); (4) from households to production activities (Case XI); and (5) through path multipliers.

1. Influence of Production Activities on Other Production Activities

It should be noted at the outset that the present structural path analysis applied to a SAM does not yield the same results as applied to only the input-output matrix. In a SAM-type framework, a production activity can influence another one through the intermediate effects on factors and institutions (households) which are considered exogenous in the input-output framework.[1]

Case I in Table 5 explores the path analysis from an injection into the construction sector to its effects on mining. From the matrix of accounting multipliers, the global influence can be obtained – i.e. an injection of 1,000 Won into the construction industry yields an increase of 68 Won in the output of mining products (see column 3). The path analysis which is undertaken shows that only 25·1 % (Column 8) of this additional production is caused directly by the demand for mining inputs by the construction sector through the elementary path (in this case, an arc) linking the two sectors without any intermediate poles.

[1] This was pointed out in Section I (see equation (9)) and footnote 1 on p. 115.

The other elementary paths shown under Case I reveal that a significant part of the global influence of construction on mining is exercised indirectly through the demand for, respectively, 'cement and non-metallic products' (26·3 % of the global influence), 'metal products (L + M)' (3·1 %) and 'energy' (3·0 %).[1]

The above type of analysis is potentially useful to the policymakers in the sense that it informs them of the principal axes along which a given injection (impulse) transmits itself to the rest of the economic structure. In particular, path analysis identifies the poles which play an important role in transmitting influence. In the same way as some materials are better conductors of electricity than others, certain poles are better transmitters of influence than others. In this sense structural path analysis might help a government identify potential bottlenecks (i.e. poles which do not relay influence well) which might occur in a public expenditure programme and vice-versa.

Case II illustrates the fact that a not insignificant part of the influence transmitted from one production activity to another may go through a path which includes factors and households groups. Indeed, as can be seen from Table 5, about 10 % (7·2 + 2·5 %) of the global influence exercised by an increase in mining output on 'other agriculture' follows two elementary paths which combine the whole triangular cycle from production (e.g. mining) to factorial income (skilled workers, and unskilled workers, respectively) to household income (skilled workers and unskilled workers, respectively) and back to production (in this case, the pole of destination, 'other agriculture').

One type of issue which is potentially interesting to the policymakers and which can be elucidated through path analysis is the extent of the linkages prevailing between formal and informal activities either directly or indirectly.

2. *The Influence of Production Activities on Factors of Production*

Here again, the matrix of accounting multipliers (global influences) yields the global effects of an exogenous expenditure on production activity i on income of factor j. This increase in income can be interpreted as a rise in the employment of factor j but as such does not identify in which sector the additional employment is to occur. Structural path analysis permits one to answer this question. More specifically, the sectoral breakdown of employment can be obtained within the following context.

(*a*) Decomposition of a single accounting multiplier: For example, in Case III, the question can be raised in which sectors the additional employment of skilled workers will occur consequent to increased exports of finished textile products of 1,000 Won. It can be seen from Table 5 that the income of skilled workers increases by 182 won with the bulk of the additional employment occurring in the same finished textile products sector (59·2 %) and to a much lesser extent in large textiles enterprises (12 %) and small and medium textiles (6·8 %). Case IV, in contrast, provides an example where the indirect effects on factor employment are larger than the direct ones. Thus, the proportion of global influence from

[1] In the eleven cases explored in Table 5, only those elementary paths transporting at least 2·5 % of the global influence are shown explicitly. This means that in a number of cases, a multitude of paths not explicitly mentioned here because each transmits only a small part of the global influence between the poles of origin and destination, carry together a substantial share of the global influence.

Table 5

Structural Path Analysis: Global Influence, Direct Influence and Total Influence for Selected Paths, South Korea SAM

(0)	(1) Path origin (i) →	(2) Path destination (j)‡	(3) Global influence $I^G_{(i\to j)} = m_{aji}$	(4) Elementary paths $(i\to j)_p$‡	(5) Direct influence $I^D_{(i\to j)p}$ ×	(6) Path multiplier M_p =	(7) Total influence $I^T_{(i\to j)p}$	(8)† $\frac{I^T_{(i\to j)p}}{I^G_{(i\to j)}}$ (in %)
I	Construction (Co)	Mining (Mi)	0·068	Co-Mi	0·016	1·071	0·017	25·1
				Co-Cement+Nonmet Min Prod-Mi	0·015	1·186	0·018	26·3
				Co-Metal Prods (L+M)-Mi	0·002	1·318	0·002	3·1
				Co-Energy (L+M)-Mi	0·002	1·199	0·002	3·0
II	Mining (Mi)	Other Agriculture (OA)	0·453	Mi-OA	0·058	1·646	0·096	21·2
				Mi-Skilled Workers-Skilled Workers*-OA	0·019	1·754	0·033	7·2
				Mi-Unskilled Workers-Unskilled Workers*-OA	0·006	1·796	0·011	2·5
III	Finished Textile Products (FTP)	Skilled Workers (SW)	0·182	FTP-SW	0·094	1·153	0·108	59·2
				FTP-Textiles (L)-SW	0·016	1·355	0·022	12·0
				FTP-Textiles (M+S+Self)-SW	0·009	1·266	0·012	6·8
IV	Processed Foods (Self+S+M) (PFS)	Capital (C)	0·748	PFS-Other Agriculture-C	0·069	2·331	0·160	21·3
				PFS-C	0·084	1·778	0·150	20·0
				PFS-Fishing-C	0·020	1·806	0·036	4·8
				PFS-Trade and Banking (self)-C	0·015	1·900	0·028	3·7
V	Processed Foods (L) (PFL)	Unskilled Workers (UW)	0·117	PFL-Fishing-UW	0·019	1·315	0·025	21·4
				PFL-UW	0·015	1·281	0·019	16·2
				PFL-Trade+Bank (Self)-UW	0·003	1·420	0·004	3·3
				PFL-Transport+Commun-UW	0·003	1·398	0·004	3·2
VI	Construction (Co)	Engineers (E)	0·058	Co-E	0·047	1·033	0·049	83·8
		Technicians (T)	0·053	Co-T	0·034	1·041	0·035	67·5
				Co-Metal Prods (L+M)-T	0·001	1·285	0·001	2·6
		Skilled Workers (SW)	0·132	Co-SW	0·045	1·101	0·050	37·5
				Co-Cement+Nonmet Min Prod-SW	0·008	1·221	0·010	7·8
				Co-Lumber+Furniture-SW	0·003	1·185	0·004	2·9
				Co-Machinery-SW	0·002	1·216	0·003	2·5
		Unskilled Workers (UW)	0·138	Co-UW	0·030	1·146	0·034	24·9
				Co-Transport+Commun-UW	0·004	1·253	0·007	5·0
				Co-Cement+Nonmet Min Prod-UW	0·003	1·270	0·004	3·0
		White-Collar Workers (WCW)	0·210	Co-WCW	0·088	1·155	0·102	48·5
				Co-Trade+Banking (Self)-WCW	0·007	1·261	0·009	4·1
				Co-Cement+Nonmet Min Prod-WCW	0·005	1·278	0·006	2·7
		Self-employed in Services (SES)	0·083	Co-Trade+Bank (Self)-SES	0·006	1·246	0·011	13·8
				Co-SES	0·004	1·114	0·004	5·0

VII Energy (L+M) (EL)	Unskilled Workers (UW)	0·101	EL-Transport+Commun-UW	0·016	1·385	0·022	22·2
			EL-Mining-UW	0·012	1·304	0·015	14·6
Energy (S+Self) (ES)	Unskilled Workers (UW)	0·110	EL-UW	0·009	1·285	0·011	10·8
			ES-Transport+Commun-UW	0·016	1·257	0·020	18·7
			ES-Mining-UW	0·011	1·189	0·013	12·3
			ES-UW	0·017	1·147	0·020	18·4
VIII Mining (Mi)	Skilled Workers (SW)	0·246	Mi-SW	0·172	1·121	0·193	78·4
Energy (L+M) (EL)	Skilled Workers (SW)	0·095	EL-Mining-SW	0·028	1·253	0·035	36·9
Energy (S+Self) (ES)	Skilled Workers (SW)	0·094	ES-Mining-SW	0·028	1·141	0·032	34·1
Cement, Nonmetallic Mining Prod (CNM)	Skilled Workers (SW)	0·155	CNM-Mining-SW	0·020	1·155	0·023	14·9
Metal Products (L+M) (MPL)	Skilled Workers (SW)	0·113	MPL-Mining-SW	0·005	1·378	0·007	6·0
Metal Products (S+Self) (MPS)	Skilled Workers (SW)	0·115	MPS-Mining-SW	0·005	1·209	0·006	5·2
IX Other Agriculture (OA)	Farmers (1) F. (1)*	0·265	OA-Farmers (1)-F. (1)*	0·101	1·721	0·173	65·3
	Farmers (2) F. (2)*	0·292	OA-Farmers (2)-F. (2)*	0·091	1·756	0·159	54·5
	Farmers (3) F. (3)*	0·349	OA-Farmers (3)-F. (3)*	0·096	1·778	0·171	49·0
	Farmers (4) F. (4)*	0·156	OA-Farmers (4)-F. (4)*	0·044	1·678	0·073	46·6
X Processed Foods (Self+S+M) PFS	Unskilled Workers (UW)*	0·119	PFS-Unskilled Workers-UW*	0·022	1·291	0·029	25·0
			PFS-Fishing-Unskilled Workers-UW*	0·009	1·327	0·012	10·0
			PFS-Apprentices-UW*	0·002	1·269	0·003	2·6
			PFS-Transport+Commun-Unskilled Workers-UW*	0.002	1·410	0·003	2·5
			PFS-Trade+Bank (Self)-Unskilled Workers-UW*	0·002	1·436	0·003	2·5
Processed Foods (L) (PFL)	Unskilled Workers (UW)*	0·106	PFL-Unskilled Workers-UW*	0·011	1·316	0·015	14·5
			PFL-Fishing-Unskilled Workers-UW*	0·009	1·352	0·012	11·5
			PFL-Transport-Commun-Unskilled Workers-UW*	0·002	1·437	0·003	2·7
			PFL-Trade+Bank (Self)-Unskilled Workers-UW*	0·002	1·462	0·003	2·7
XI Unskilled Workers (UW)*	Cereals (CE)	0·603	UW*-CE	0·197	1·280	0·252	41·8
Apprentices (AO)*	Cereals (CE)	0·398	AP*-CE	0·022	1·258	0·028	7·0
			AP*-Chemical Products (L)-CE	0·009	1·398	0·013	3·2
			AP*-Other Consumer Products-CE	0·007	1·413	0·010	2·6

† Percentages in this column were calculated by the computer directly on the basis of data not rounded off. Consequently, the figures in this column may differ slightly from the percentage ratio of column 7 and column 3.

‡ Since in the South Korean SAM, the classification of factors is the same as that of households, those poles referring to given socio-economic household groups – rather than the corresponding factor group – are indicated by way of an asterisk (e.g. 'Unskilled Workers*') means the household group headed by an unskilled worker and 'Unskilled Workers' (with no asterisk) denotes the corresponding factorial group.

* See footnote ‡.

processed foods to capital transmitted indirectly via the demand of the former for agricultural input (which comes under other agriculture) amounted to 21·3 % and was greater than the direct demand for capital which amounted to only 20 %.

Still another example of greater indirect income and employment effects on a factor (in this case, on unskilled workers) than direct effects is shown in Case V. The substantial amount of fish required by the large processed foods enterprises for canning purposes leads indirectly to more employment of unskilled workers (i.e. fishermen) than the direct demand of that sector for unskilled workers. As can be seen from Table 5, 21·4 % of the global influence is transmitted through the elementary path, processed foods (L)-fishing-unskilled workers compared to only 16·2 % through the direct path PFS-UW.

(*b*) Decomposition of accounting multipliers for a given column: This amounts to comparing, first, the global influence, $m_{a_{ji}}$, varying j over different factorial skill groups for a given i (in this case construction) and, secondly, decomposing $m_{a_{ji}}$ for given j's (specific skill groups) to determine in which productive activities the additional employment occurs.

If, for example, the construction industry is stimulated by the government in which sectors will the different types of employment occur? Case VI attempts to answer this question by decomposing the global influence on each one of six different labour skill groups to check not only the total employment effects of each of these groups but, more importantly, in which sectors the additional employment is to take place. It can be seen that, for most skill groups, it is mainly the construction sector itself which benefits from the additional employment which is created. The only exception applies to the 'self employed in services', for which the indirect employment generated via the sector trade and banking was greater than the direct employment within the construction industry *per se*.

(*c*) Decomposition of accounting multipliers along a row: This type of analysis is particularly appropriate within the context of exploring the macro economic consequences of the choice of alternative technologies by given industries and sectors.[1]

Thus, for example, one can ask in which sectors and to which extent additional jobs shall be created for unskilled workers depending on whether a more capital intensive technology is selected in producing energy (i.e. increasing the output of the large scale energy sector) or a less capital intensive technique (i.e. increasing the output of the smaller scale energy sector). This exploration is undertaken under Case VII where it can be seen that the income accruing to unskilled workers would be slightly higher if the increased production occurred in the smaller enterprises as opposed to the larger ones (110 units as against 101 for an initial injection of 1,000). In both cases, the additional employment of unskilled workers takes place mainly in three sectors: Transport and Communication; Mining, and Energy. On the other hand, an examination of Case VII reveals also that more jobs for unskilled workers would be created in the smaller scale and more labour intensive energy sector directly than in the larger scale one (20 as opposed to 11 as indicated in column 7 of Table 5).

[1] It amounts to identifying $m_{a_{ji}}$ and scrutinising the effects of alternative i's (production activities-cum-technology) on a given j (a factor).

(*d*) Instead of finding out in which sector additional employment is created as a result of exogenous injections in production activities, one can ask the reverse question, namely, which sectors should be stimulated to increase the employment of given skill groups in given sectors? For instance, if the objective were the creation of additional jobs for skilled workers in mining (see Case VIII), it can be seen that a few indirect elementary paths originating respectively in the energy (all sizes) cement, non metallic mining products and metal products (all sizes) sectors would induce the creation of jobs for skilled workers in the mining sector. Even though the direct path from mining to skilled workers yields more employment of skilled workers than each of the other individual indirect paths above, it is operationally useful for the policymaker to know that other paths not originating in the mining sector do provide important sources of additional employment in mining.

3. The Influence of Production Activities on Socio-economic Household Groups

The present subsection can be organised in essentially the same way as the preceding one according to the four types of questions and corresponding examples just discussed, substituting as poles of destination the incomes of socio-economic groups as opposed to factor income (and employment) as in the preceding section. Thus, to illustrate the decomposition of accounting multipliers of the same column, one can ask how the incomes of different classes of farmers as households would be affected by an increase in the production of the sector 'other agriculture' (agricultural production excluding cereals). In particular, one may be interested in the income effects on the group of small farmers. This is illustrated in Case IX which shows that it is the medium sized farmers (2 and 3) which will benefit more strongly from such an increase in production. (See column 3 of Table 5). However, it is interesting to note that the smaller farms transmit more total influence directly from other agriculture than any of the other three sized farms as can be seen from column 7 of Table 5. Likewise, an examination of column 8 reveals that the smaller the size farm, the greater the proportion of the global influence transmitted directly from an increased production in other agriculture to that farmers' group income.[1]

Our penultimate case X, illustrates this possibility by exploring the effects of an increase in production of processed foods on the income accruing to the household category headed by unskilled workers. Again, if one looks at the effects of alternative technologies on the incomes accruing to unskilled workers as

[1] It should be noted that in the previous example, the increased income of each group of farmers as households corresponds exactly to the increased factorial income. In other words, the incomes of these farmers as a household group comes entirely from the income received by the corresponding factorial group. Such a perfect mapping between factor income and household income is atypical. For most socio-economic groups, the total household income comes from different factorial sources generated in different production activities even though if the household groups are defined according to the occupation or skill level of the head of the household, the bulk of the household income in any given group is likely to come from the corresponding occupation or skill level. This means that the matrix $\mathbf{A}_{21}$ which maps the factorial income distribution into a household income distribution tends to have larger diagonal elements (in some cases equal to 1) than off-diagonal elements. This can be verified by looking at the truncated $\mathbf{A}_{21}$ matrix in Table 3. For example, 53% of the factorial income received by engineers goes to the household group headed by engineers ($a_{16,1}$ in Table 3) compared to 100% for farm size I income ($a_{26,11} = 1{\cdot}0$).

households, one can see that with the smaller scale technology (Self + S + M), the global influence would result from greater employment of unskilled workers as factors in respectively, processed foods (25 %), fishing (10 %), transport and communication (2·5 %) and trade and banking (2·5 %). Interestingly the direct employment and income effects on unskilled workers of an increase in the production of processed foods (in both smaller and larger firms) are considerably smaller than the indirect ones – amounting to, respectively, 25 and 14·5 % of the increase in income of that socioeconomic group (see column 8 of Table 5).[1] The principal difference between these effects and the alternative effects which would result from having chosen the more capital intensive and larger scale technology (L) is the much smaller direct contribution made by the unskilled workers employed in the processed food sector in the large enterprises as opposed to the small ones (the total influence in column 7 of Table 5 is equal to 0·015 in the former case and 0·029 in the latter).

4. *The Influence of Socio-economic Household Groups on Production Activities*

In this final example (Case XI), we illustrate the impact of exogenous changes in income of certain household categories on production activities. These changes could, for example, represent government subsidies or transfers to certain socio-economic groups. In Case XI, the effects of an increase in the incomes of two household groups 'apprentices' and 'unskilled workers' – say as a result of transfers – on cereals production are estimated. It can be seen that the global influence on the latter group is greater by about 50 % if the subsidies are directed towards 'unskilled workers' as compared to 'apprentices'. In turn, path analysis reveals that the bulk of the global influence consists of the direct demand for cereals by unskilled workers' households. In contrast the corresponding direct demand by 'apprentices' is about 9 times smaller (0·022 versus 0·197). Evidently, as can be seen from Table 5, about 90 % of the impact of higher incomes of apprentices on the production of cereals occurs indirectly via other sectors which themselves require cereals.

5. *Path Multipliers*

So far in the analysis of the eleven cases above, the meaning of path multipliers appearing in column 6 of Table 5 has not been discussed. As was pointed out previously, the multipliers $\mathbf{M}_p$ measure the degree of amplification conferred to these paths by adjacent circuits. In general, the size of these multipliers varies as a function of the length of a path. This is, of course, logical since the more poles a path contains, the higher the probability of adjacent circuits including one or more poles of this path being present. Going down column 6 of Table 5 it can be seen that with the exception of Cases II, IV and IX which are discussed subsequently, the path multipliers reach a level of up to 1·285, 1·420 and 1·462, respectively, depending on whether the paths are of length 1, 2 or 3.

[1] It should be recalled, in this connection, that only the indirect paths carrying at least 2·5 % of the global influence are included in Table 5. Examination of column 8 of Table 5 in these two specific cases reveals that the great bulk of the indirect influence must have been transmitted along many paths which individually, accounted for less than 2·5 % of the global influence.

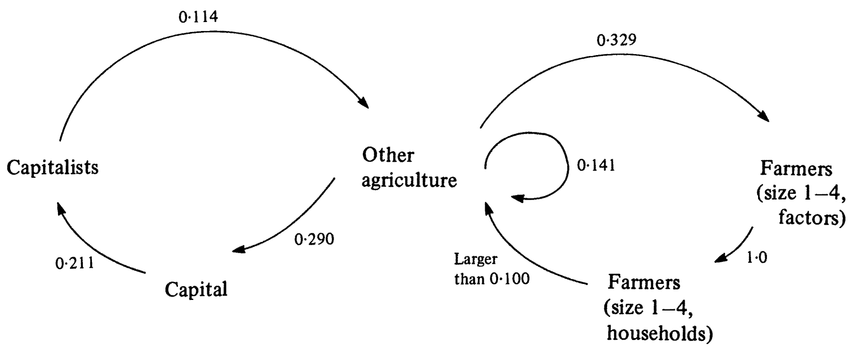

Fig. 5

In the other three cases, II, IV and IX, the multipliers are particularly high; they are all above 1·645 reflecting the amplifying action of powerful circuits. All these paths have in common that they embrace at least one of the poles forming the circuits represented above within which all of the arcs have a relatively high intensity (see Fig. 5).

An alternative way of presenting the path multiplier is to calculate its inverse, that is, the ratio of direct influence to total influence $1/\mathbf{M}_p = I^D_{(i\to j)_p}/I^T_{(i\to j)_p}$. This ratio shows the proportion of the total influence transmitted along an elementary path which is accounted for by the immediate effects, namely the direct influence. This parameter can be quite relevant in a policy context by indicating the extent to which an initial injection into a given pole will generate rapidly or only after a long period of time any increase in the production or the income of other poles in the economic structure.[1]

Thus, for example, an examination of Case IV reveals that the path multiplier of the elementary path from processed foods to other agriculture to capital is equal to 2·331, whereas it only equals 1·778 in the direct path from processed foods to capital. In other words, there might be a slight trade-off between the direct influence along these two paths – an injection of 1,000 units into processed foods yielding an increase of 69 units in the income accruing to capital through the first path as opposed to 84 units through the second path – and a greater total influence through the former as opposed to the latter necessitating presumably a longer period of time to be fully felt.

In the cases and corresponding paths which have been examined here, the relative importance of the immediate effects (i.e. the ratio of direct to total influence) is generally quite high. Nevertheless, it should not be overlooked that

[1] Strictly speaking, the above statement is not correct since the whole structural analysis abstracts from time. The various influences and effects occasioned by an exogenous injection are assumed to be instantaneous (including the multipliers). In reality, however, the transmission of economic influence from one pole to others takes time. In particular, it is reasonable to assume that the time required for the transmission of influence along a given elementary path would vary in function of the number and lengths of adjacent circuits. It is also reasonable to assume that the larger the number of poles contained in an elementary path or an adjacent circuit, the longer it will take for the influence to be transmitted from the pole of origin to the pole of destination. Consequently, the existence of relatively long and powerful circuits and correspondingly high path multipliers would seem to imply that the transmission of influence would tend to be slower than in the converse case of low path multipliers and a high ratio of direct to total influence.

many other paths carry together a considerable part of the economic influence. Since these paths can be very long, they can often have a high multiplier in the range of 3–4 – implying that the proportion of immediate effects to total effects could be in the range of 25–33 %.

V. SUMMARY AND CONCLUSIONS

The SAM framework represents an important addition to and generalisation of the input-output model since it captures the circular interdependence characteristic of any economic system among (*a*) production activities (*b*) the factorial income distribution (*c*) the income distribution among institutions (particularly among different socio-economic household groups), which, in turn, determines the expenditure pattern of institutions.

The global (direct and indirect) effects of injections from exogenous variables on the endogenous variables are captured, under certain conditions, by the accounting (or, alternatively, fixed price) multipliers provided by the reduced form of the SAM-model. These multipliers do not clarify the 'black box', i.e. the structural and behavioural mechanisms responsible for the final (reduced form) solution. The decomposition of accounting (and fixed price) multipliers proposed by Pyatt and Round (1979) constituted a first attempt to grasp the complexity of the network of relations among endogenous variables without, however, providing information of concrete usefulness to policymakers.

In contrast, the application to the SAM framework of the structural analysis of Lantner (1974) and Gazon (1976), founded on the concept of economic influence and its transmission among the agents (poles) of the structure under consideration, reveals much more explicitly and clearly the endogenous interaction process. In particular, this method of structural path analysis shows, respectively, how influence is diffused from a given pole, through which specific paths it is transmitted and the extent to which it is amplified by the circuits adjacent to these paths. The decomposition of accounting multipliers (or global influences) into total influences carried along the respective elementary paths spanning two given poles allows the decision-maker to capture in a distinctive and isolated way the reaction mechanisms of different economic agents within the complex network of structural relations. It would appear that this type of structural decomposition could help the policymaker and analyst break down the various channels through which influence is transmitted in a disaggregated macroeconomic system and thereby contribute to the quality of policy decisions.

Several developing countries have by now built SAM's – perhaps the most ambitious being the one just completed by the Central Statistical Bureau of Indonesia. As the underlying quality of these SAM's improves – in particular, by relying on a classification and disaggregation scheme reflecting well the prevailing production structure and socio-economic behaviour – their operational usefulness when combined with structural path analysis should be significantly enhanced.

Furthermore, there is some evidence that researchers in developed countries are becoming interested in using SAM-type data sets to explore, among others, the general equilibrium effects of changes in key policy instruments such as taxes (see e.g. Shoven-Whalley (1973; 1976)). Structural path analysis could likewise, provide a potentially important complementary tool in identifying the whole network through which the effects of policy measures are transmitted in an economy. Finally, path analysis could help specify more dynamic and eventually price endogenous models – by providing a better understanding of how influence travels within a structure.

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Appendix

Decomposition of Global Influence into Total Influences

Global influence linking any two poles of a structure can be decomposed into a series of total influences transmitted along each and all elementary paths spanning i and j, i.e.

$$I^G_{(i\to j)}(m_{a_{ji}}) = \sum_{p=1}^{n} I^T_{(i\to j)_p} = \sum_{p=1}^{n} I^D_{(i\to j)_p}\mathbf{M}_p. \qquad \text{(i)} = (22)$$

The above theorem was demonstrated by inductive methods (Lantner, 1974) and deductive methods (Gazon, 1976) but it can also be established by more conventional determinantal expansions as shown in Crama *et al.* (1983). Applying this demonstration to the particular case of the structure represented by the network in Fig. 4, it will be shown that the global influence of i on j is, in fact, equal to the sum of the total influences of the three elementary paths spanning poles i and j. Let Δ be the determinant of matrix $(\mathbf{I}-\mathbf{A}_n)$. In our specific case

$$\mathbf{I}-\mathbf{A}_n = \begin{array}{c} \\ i \\ x \\ y \\ z \\ s \\ v \\ j \end{array} \begin{array}{c} \begin{array}{ccccccc} i & x & y & z & s & v & j \end{array} \\ \begin{bmatrix} 1 & & & & & & \\ -a_{xi} & 1 & -a_{xy} & -a_{xz} & & & \\ & -a_{yx} & 1 & & & & \\ & & -a_{zy} & 1 & & & \\ -a_{si} & & & & 1 & & \\ -a_{vi} & & & & & 1-a_{vv} & \\ & & -a_{jy} & & -a_{js} & -a_{jv} & 1 \end{bmatrix} \end{array}. \qquad \text{(ii)}$$

From the expression of an element of a matrix in terms of the original matrix, global influence

$$I^G_{(i\to j)} = \frac{\Delta_{ij}}{\Delta}, \qquad \text{(iii)}$$

where Δ_{ij} is the ijth cofactor of Δ.

Expanding Δ_{ij} by minors according to elements of its first column, one obtains:

$$\Delta_{ij} = (-a_{xi}) \begin{vmatrix} -a_{xy} & 1 & 0 & 0 & 0 \\ 0 & -a_{zy} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1-a_{vv} \\ 0 & -a_{jy} & 0 & -a_{js} & -a_{jv} \end{vmatrix} \qquad i \xrightarrow{a_{xi}} x$$

$$-(-a_{si}) \begin{vmatrix} 1 & -a_{xy} & -a_{xz} & 0 & 0 \\ -a_{yx} & 1 & 0 & 0 & 0 \\ 0 & -a_{zy} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-a_{vv} \\ 0 & -a_{jy} & 0 & -a_{js} & -a_{jv} \end{vmatrix} \qquad i \xrightarrow{a_{si}} s$$

$$+(-a_{vi}) \begin{vmatrix} 1 & -a_{xy} & -a_{xz} & 0 & 0 \\ -a_{yx} & 1 & 0 & 0 & 0 \\ 0 & -a_{zy} & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -a_{jy} & 0 & -a_{js} & -a_{jv} \end{vmatrix} \qquad i \xrightarrow{a_{vi}} v \qquad \text{(iv)}$$

(The arcs corresponding to the graph in Fig. 4 are shown next to their respective minors.)

The first minor above can be further expanded by suppressing column x, the second by suppressing column s and the third one by suppressing column v.

$$\Delta_{ij} = (-a_{xi})(-a_{yx}) \begin{vmatrix} -a_{zy} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-a_{vv} \\ -a_{jy} & 0 & -a_{js} & -a_{jv} \end{vmatrix} \qquad i \xrightarrow{a_{xi}} x \xrightarrow{a_{yx}} y$$

$$+(-a_{si})(-a_{js}) \begin{vmatrix} 1 & -a_{xy} & -a_{xz} & 0 \\ -a_{yx} & 1 & 0 & 0 \\ 0 & -a_{zy} & 1 & 0 \\ 0 & 0 & 0 & 1-a_{vv} \end{vmatrix} \qquad i \xrightarrow{a_{si}} s \xrightarrow{a_{js}} j$$

$$+(-a_{vi})(-a_{jv}) \begin{vmatrix} 1 & -a_{xy} & -a_{xz} & 0 \\ -a_{yx} & 1 & 0 & 0 \\ 0 & -a_{zy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad i \xrightarrow{a_{vi}} v \xrightarrow{a_{jv}} j \qquad (\mathrm{v}a)$$

Hence, it can be verified that pole j has been reached with the last two determinants above (see the paths next to these determinants and in Fig. 4). The first determinant above can be further expanded by suppressing column y which yields

$$(-a_{xi})(-a_{yx})(-a_{zy})\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1-a_{vv} \\ 0 & -a_{js} & -a_{jv} \end{vmatrix}$$

$$+(-a_{xi})(-a_{yx})(+a_{jy})\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-a_{vv} \end{vmatrix} \qquad (\mathrm{v}\,b)$$

Since the first term in (v b) collapses, it follows that

$$\Delta_{ij} = a_{xi}a_{yx}a_{jy}\Delta_1 + a_{si}a_{js}\Delta_2 + a_{vi}a_{jv}\Delta_3 \qquad (\mathrm{vi})$$

where Δ_1, Δ_2 and Δ_3 are the determinants of the sub-structures excluding, respectively, the poles composing the three elementary paths $1 = (i,x,y,j)$, $2 = (i,s,j)$, and $3 = (i,v,j)$ (see Fig. 4)

Dividing (vi) by Δ and substituting equations (15) and (21) in the text, one obtains

$$\frac{\Delta_{ij}}{\Delta} = I^D_{(i\to j)_1}\mathbf{M}_1 + I^D_{(i\to j)_2}\mathbf{M}_2 + I^D_{(i\to j)_3}\mathbf{M}_3 \quad (\text{with } \mathbf{M}_2 = 1). \qquad (\mathrm{vii})$$

From (vii) and (iii), it follows that global influence can be decomposed into the sum of total influences.

Incidentally, it can be shown calculating the determinants of Δ_1 and Δ that

$$\mathbf{M}_1 = \frac{\Delta_1}{\Delta} = \frac{(1-a_{vv})}{(1-a_{vv})[1-a_{yx}(a_{xy}-a_{zy}a_{xz})]}$$
$$= [1-a_{yx}(a_{xy}-a_{zy}a_{xz})]^{-1}, \qquad (\mathrm{viii})$$

and simiilarly,

$$\mathbf{M}_2 = \frac{\Delta_2}{\Delta} = 1 \qquad (\mathrm{ix})$$

$$\mathbf{M}_3 = \frac{\Delta_3}{\Delta} = (1-a_{vv})^{-1}. \qquad (\mathrm{x})$$

The right-hand side expressions correspond to the values derived for these three path multipliers, respectively, in equations (17) and (21) of the text.

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