

$$1 a) \left(\frac{\phi(x+a) - \phi(x)}{a} \right)^2$$

$$= \frac{1}{a^2} \left[\phi^2(x+a) + \phi^2(x) - 2\phi(x)\phi(x+a) \right]$$

$$= \frac{1}{a^2} \left[\phi^2(x) + \phi^2(x) - 2\phi(x)\phi(x+a) \right]$$

$$= \frac{1}{a^2} \left[2\phi^2(x) - \phi(x)\phi(x+a) - \phi(x-a)\phi(x) \right]$$

$$= -\frac{\phi(x)}{a^2} \left[\phi(x+a)\phi(x) - 2\phi(x) + \phi(x)\phi(x-a) \right]$$

1b)

Expand $\phi(x \pm a)$

$$\phi(x \pm a) = \phi(x) \pm a \phi'(x)$$

$$+ \frac{1}{2} a^2 \phi''(x)$$

$$\pm \frac{1}{3!} a^3 \phi''' + \frac{1}{4!} a^4 \phi^{(4)} + \dots$$

\Rightarrow

$$S_L^2 = \sum a \frac{1}{2} \cancel{\frac{\phi(x)}{a^2}} \text{ from } x+a$$

$$\times \left[\frac{1}{2} \cancel{a^2} \phi''(x) \text{ from } x-a \right. \\ \left. + \frac{1}{2} \cancel{a^2} \phi''(x) + \mathcal{O}(a^4) \right]$$

a^3 terms will cancel out.

$$= - \sum a \frac{1}{2} \phi'' \phi + \mathcal{O}(a^2)$$