Lattice Field Theory - Exercise Session 25.1-01.2

January 26, 2024

This exercise session will be centered around autocorrelation time and behaviour of Ising 2D with different algorithms. You will need the Metropolis-Hastings version of the Ising 2D code used in the previous exercise. Feel free to use your own, or the uploaded Python solution of exercise 1 (on github). For this exercise session, we will operate with no external field $\bar{h} = 0$ and $\bar{J} = 1$.

1. Let us now implement the Cluster Wolff algorihm and plot the magnetization. Choose a warm up of 1000 steps. Add 1000 computation steps. For this algorithm, begin by,

   (a) 1. Choose site x,y at random

   (b) Precompute probability of adding site to cluster $P_{add} = 1 - exp(-2J/T)$

   (c) Initialize two lists of coordinates, one named Cluster another Pocket. One will contain all coordinates belonging to a cluster of spins, the other of immediate neighbours to which one should jump to, in order to grow the cluster. Something like Cluster = [[x,y]] should do the trick.

   (d) Now this is the hard part, and for this reason I've elected to provide some pseudocode:

```
while Pocket is not empty:
    Pocket_new = []
    for i,j coordinate pairs in Pocket:
        # find all the neighbours,
        #remember to wrap around boundaries
        ip1 = (i+1) % lattice_size
        im1 = (i-1+lattice_size) %lattice_size
        jp1 = (j+1) % lattice_size
        jm1 = (j-1+lattice_size) %lattice_size
        nbr = [[ip1,j],
               [im1,j],
               [i,jp1],
               [i,jm1]]
        #Now we go over all neighbours of current spin
```

```
        for l in nbr:
            if spin[l] == spin[current coordinate in Pocket]
                if l not in Cluster:
                    if random number < P_add:
                        Add spin[l] to Pocket_new
                        Add spin[l] to Cluster
    # Remember to reset Pocket,
    # we need to keep growing the cluster
    # so we must jump to new spins
    Pocket = Pocket_new
```

(e) Flip all spins in cluster - done!

Be very careful to get all of the indents right! After all of this a single update sweep of Cluster Wolff is coded. Show me what the magnetization $|M|$ looks like in the range $T \in [1, 5]$.

2. Let us implement the autocorrelation function of the magnetization observable. Reminder the autocorrelation function is given by,

$$C(t) = \frac{\frac{1}{N-t}\sum_{i=1}^{N-t}\Bigg[X_i X_{i+t}\Bigg] - \langle X\rangle_1 \langle X\rangle_2}{\langle X^2\rangle - \langle X\rangle^2} \tag{1}$$

where,

$$\langle X\rangle_1 = \frac{1}{N-t}\sum_{i=1}^{N-t} X_i, \tag{2}$$

$$\langle X\rangle_2 = \frac{1}{N-t}\sum_{i=t}^{N} X_i, \tag{3}$$

Plot the autocorrelation function normalized with the first step, $C(t)/C(0)$ as a function of steps for three different temperatures, $T \in [1, 5]$. Trick: compute C(t) with $t \in [0, t_{max}]$, where $t_{max} = 70$. This is necessary to also remove the decorrelated noise at large t. Has the advantageof being computationally easier. How does this compare with Metropolis, is the exponential fall-off faster or slower?