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Abstract Financial Market Risks

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Chapter 1

Introduction

1.1 Introduction

Financial assets grow a lot faster than GDP nowadays so it's interesting to study them.

From the viewpoint of firms, financial markets offer great sources of:

- Funding
- Return on excess cash or reserves
- Diversification of risks

There are different types of assets to invest in:

- **Common stocks**: Represent an ownership interest in a given company.
- **Bonds**: Form of debt financing issued by companies and governments. Typically lower return than stocks, but also lower volatility and a less of a chance of loss.
- **Mutual Funds and ETFs**: Collections of stocks and bonds that give investors easy access to a diversified portfolio of assets.
- **Real Estate**: Rental properties and investment real estate.
- **Commodities**: Physical products like oil, gold, silver, corn, soybeans or even livestock.
- **Derivatives**: Financial instruments / agreements between two people that have values determined by the prices of something else (underlying). For instance swaps, futures or options.
- **Foreign exchange market**: Decentralized over-the-counter financial market for the trading of currencies.

We have that assets = liabilities + equities where the assets are what the business has or owns (equipment, supplies, cash, accounts receivable), liabilities what the business owes outsiders (bank loans, accounts payable, issued bonds) and equities ownership interest in a corporation (investment & business profits).

Definition 1: Over-the-counter

Over-the-counter (OTC) or off-exchange trading is done directly between two parties, without the supervision of an exchange. It is contrasted with exchange trading, which occurs via exchanges. A stock exchange has the benefit of facilitating liquidity, providing transparency, and maintaining the current market price. In an OTC trade, the price is not necessarily publicly disclosed.

1.2 Future and Present Values

One of the most basic principle of finance is that a dollar today is worth more than a dollar tomorrow. If \$100 are invested at a rate r, after time t their value will be: $\$100 * (1+r)^t$ We can also ask what the present value (PV) of a future payment is. If you receive C_t dollars at the end of year t, this will be:

$$\text{Present value} = \text{PV} = \frac{C_t}{(1+r)^t}$$

The rate r is called the discount rate and the present value is the discounted value of the cash flow, C_t. If we have multiple cash flows, this is simply:

$$\text{PV} = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_T}{(1+r)^T}$$

If we assume continuous compounding (compound interest is calculated and reinvested into an account's balance over a theoretically infinite number of periods), we have:

$$\text{PV} = \frac{C_t}{e^{rt}}$$

Net present value (NPV) is the present value minus the required investment:

$$\text{NPV} = \text{PV} - \text{Investment}$$

Another second basic financial principle is that a safe dollar is worth more than a risky dollar. For instance, if you think a project is as risky as investment in the stock market and stocks are expected to provide a 12% return, then 12% is the opportunity cost of capital for the project (what you are giving up by investing in the office building and not in equally risky securities). Future payments of the project are discounted with a factor of 12%.

Definition 2: Opportunity Cost of Capital

The opportunity cost of capital is the incremental return on investment that a business foregoes when it elects to use funds for an internal project, rather than investing cash in a marketable security. Thus, if the projected return on the internal project is less than the expected rate of return on a marketable security, one would not invest in the internal project, assuming that this is the only basis for the decision. The opportunity cost of capital is the difference between the returns on the two projects.

Example 1: Real and Nominal Interest Rates

We usually are quoted nominal interest rates r_n. To get the real interest rate r_r for a given expected inflation rate f, we can simply calculate:

$$r_r = \frac{(1 + r_n)}{(1 + f)} - 1$$

Chapter 2

Risk

2.1 Risk Premium

Investors expect to be paid more to take more risk. This is true for fixed income, but also for equities. But there, the value of the risk premium is more difficult to estimate, owing to uncertainty of future cash-flows.

Definition 3: Risk Premium

The risk premium is some measure (or expectation) of excess return over the risk free rate, which is often the interest rate on Treasury bills.

The (market) risk premium can't be estimated exactly and it varies from country to country.

If we assume the dividend $\$d$ grows at rate $g\%$ and future dividends are discounted at the risk premium $r\%$, the share price $\$P$ should be:

$$P = d + d\frac{1+g}{1+r} + d\left(\frac{1+g}{1+r}\right)^2 + d\left(\frac{1+g}{1+r}\right)^3 + \dots$$

Which is approximately $P = \frac{d}{(r-g)}$. Therefore, if investors require less return to hold this risky asset (i.e. they demand a lower risk premium), the price must rise.

2.2 Measuring Portfolio Risk

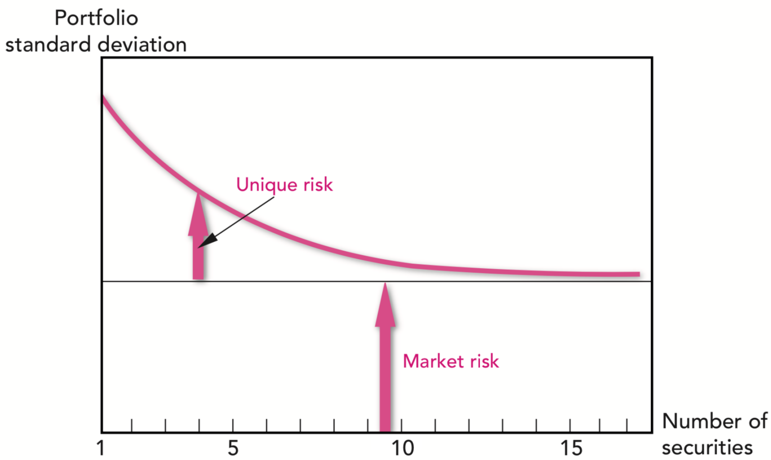
The variance of the market return is the expected squared deviation from the expected return:

$$\text{Variance}(\tilde{r}_m) = \mathbb{E}((\tilde{r}_m - r_m)^2)$$

Standard deviation is the square root of the variance. Value at risk (VaR) at confidence level $1-\alpha$ is the $(1-\alpha)$-quantile of the loss function. Conditional value at risk (CVaR) is the expected shortfall, i.e. the expectation value conditioned only on values greater than VaR.

Diversification reduces variability and therefore risk. The risk that potentially can be eliminated by diversification is called unique risk (unsystematic risk, residual risk, diversifiable risk). There is also some risk that can't be avoided regardless of diversification which is called market risk (systematic risk / undiversifiable risk). We generally want to have uncorrelated / negatively correlated assets, where correlation is defined as:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X) * Var(Y)}}$$



2.3 Calculating Portfolio Risk

The expected portfolio return is easy to compute, it's simply the weighted average of the expected returns on the individual stocks. For the portfolio variance, we have:

$$\text{Portfolio variance} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

Where σ_{ij} ($\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$) is the covariance between asset i and j ($\sigma_{ii} = \sigma_i^2$) and x_i is the fraction invested in asset i.

Example 2: Portfolio calculation

Suppose that 65% of your portfolio is invested in Coca-Cola and the remainder in Reebok. Coca-Cola has an expected return of 10 percent, Reebok 20 percent. The expected portfolio return is therefore $(0.65 * 10) + (0.35 * 20) = 13.5\%$ The portfolio variance is $x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12}\sigma_1\sigma_2)$, where $x_1 = 0.65$ and $x_2 = 0.35$.

Suppose we are dealing with portfolios in which equal investments are made in each of N stocks. Then:

$$\begin{aligned}\text{Portfolio variance } &= N\left(\frac{1}{N}\right)^2 * \text{average variance}\\ &+(N^2 - N)\left(\frac{1}{N}\right)^2 * \text{average covariance}\\ = \frac{1}{N} * \text{average variance} &+ \left(1 - \frac{1}{N}\right) * \text{average covariance}\end{aligned}$$

This converges to the average covariance, therefore the market risk is the average covariance.

2.4 Beta

If you want to know the contribution of an individual security to the risk of a well diversified portfolio, it is no good thinking about how risky that security is if held in isolation — you need to measure its market risk, and that boils down to measuring how sensitive it is to market movements. This sensitivity is called beta (β). Stocks with betas greater than 1.0 tend to amplify the overall movements of the market. Stocks with betas between 0 and 1.0 tend to move in the same direction as the market, but not as far.

For instance, a well-diversified portfolio with a beta of 1.5 will amplify every market move by 50 percent and end up with 150 percent of the market's risk.

Definition 4: Beta

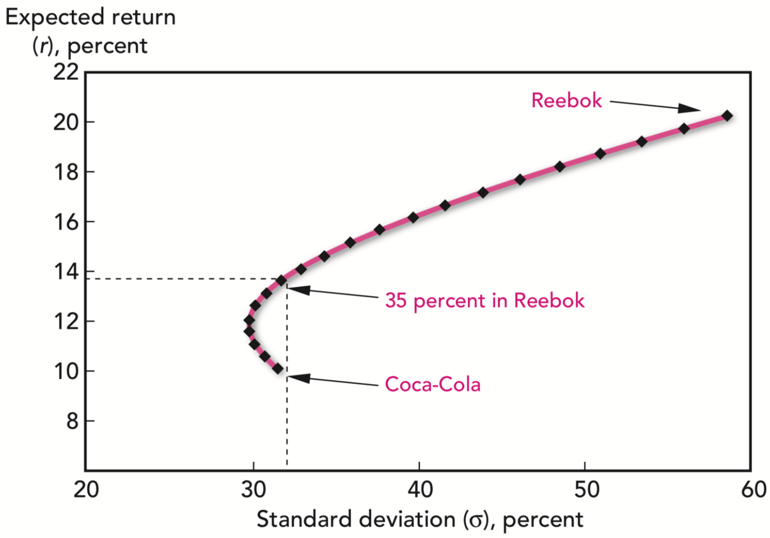
The beta of stock i is defined as

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

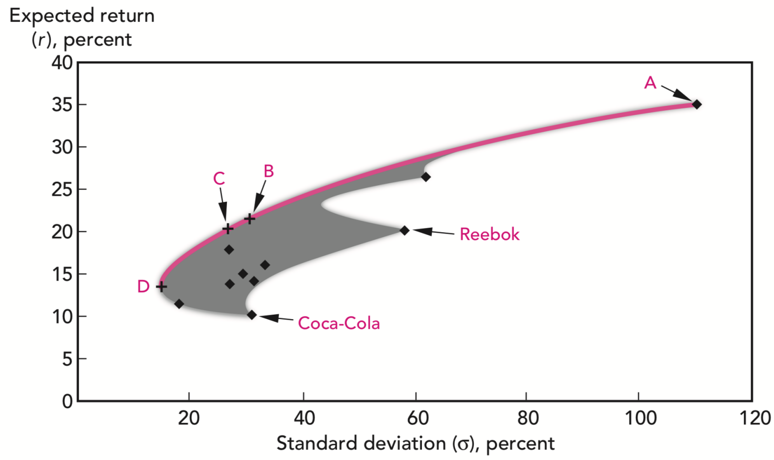
where σ_{im} is the covariance between stock i's return and the market return σ_m^2 and is the variance of the market return.

2.5 Markowitz Portfolio Optimization

Different combinations of two stocks can result in different return / risk profiles:

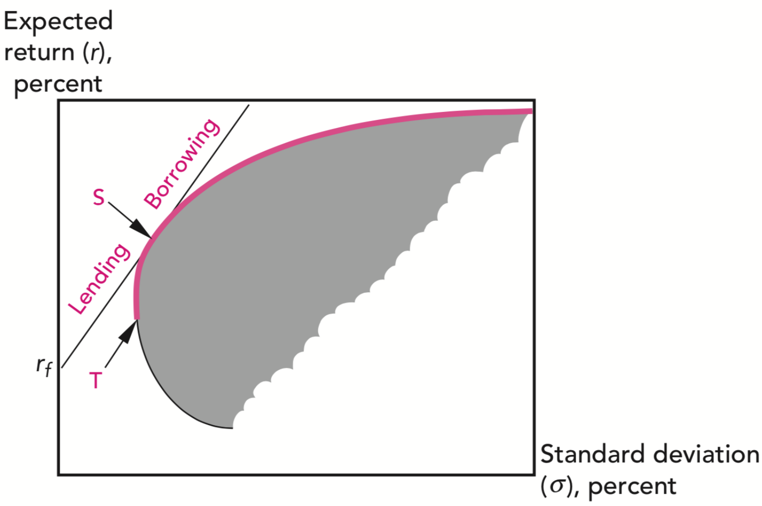


This can also be done for more than two portfolios:



Any combination of risk / return in the shaded area can be obtained by combination of portfolios. But you want to end up with the portfolios along the heavy solid line, which were called efficient portfolios by Markowitz. The problem can be solved using Lagrange multipliers.

Lending or borrowing at the risk free rate (r_f) allows us to exist outside the efficient frontier:



Example 3: Tangent Portfolio

Suppose that portfolio S has an expected return of 15 percent and a standard deviation of 16 percent. The risk free rate (r_f) is 5% and treasury bills are assumed to be risk-free (i.e. zero standard deviation).

If half is invested in S, the expected return is $r = \frac{1}{2} * 15\% + \frac{1}{2} * 5\% = 10\%$ and the standard deviation is $\frac{1}{2} * \sigma_S = 8\%$.
If you borrow an amount equal to your initial wealth, the expected return is $r = (2 * r_s) - 1 * r_f$ and the standard deviation $\sigma = 2 * \sigma_S = 32\%$.

To find the line for the tangent portfolio, you start at r_f on the vertical axis and draw the steepest line to the curved heavy line of efficient portfolios.

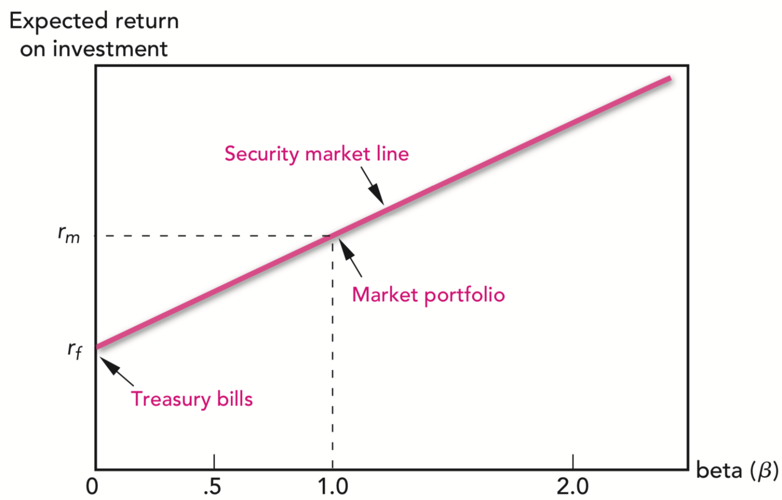
The formula for the return (the tangent) is then $r_P = \omega_1 r_f + \omega_2 r_S$ and $\sigma_P = \omega_2 \sigma_S$. We can rewrite this (note that $\omega_2 = \frac{\sigma_P}{\sigma_S}$) as: $r_P = r_f + S_S \sigma_P$ where S_S is the Sharpe ratio:

$$S_S = \frac{r_S - r_f}{\sigma_S}$$

The ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk.

2.6 Capital Asset Pricing Model

The capital asset pricing model states that the expected risk premium on each investment is proportional to its beta. This means that each investment should lie on the sloping security market line connecting Treasury Bills and the market portfolio:



The expected risk premium on an investment with a beta of 0.5 is therefore half the expected risk premium on the market or in other words:

$$\text{Expected risk premium on stock} = \beta * \text{Expected risk premium on market}$$
$$r - r_f = \beta(r_m - r_f)$$

The idea behind the CAPM is that the market is mean-variance efficient: Everybody has the same information and assessments. Then, each investor should hold the same portfolio as everybody else (namely the market portfolio). If everybody holds the market portfolio and beta measures each security's contribution to the market portfolio risk, then the risk premium demanded by investors should be proportional to beta. If a stock would be below the security market line, it wouldn't be sensible to buy it (higher return with same beta would be possible with the market portfolio / Treasury bills). Therefore, the price has to fall until the expected return matches the security market line. Since no stocks are below the line and the average is on the line, there are also no stocks above the line.

CAPM (and APT) can be used to calculate the discount rate of a firm for (the future cash flows of) an investment.

2.7 Arbitrage Pricing Theory

The arbitrage pricing theory (APT) assumes that each stock's return depends partly on pervasive macroeconomic influences or "factors" and partly on noise - events that are unique to that company. The return is then:

$$\text{Return} = a + b_1(r_{factor1}) + b_2(r_{factor2}) + \ldots + \text{noise}$$

Where the b_i's are the sensitivities of a stock to a certain factor. Diversification does eliminate unique risk, the expected risk premium is therefore only affected by macroeconomic risk:

$$\begin{aligned}\text{Expected risk premium} &= r - r_f \\ &= b_1(r_{factor1} - r_f) + b_2(r_{factor2} - r_f) + \ldots\end{aligned}$$

I.e. every factor has an associated expected risk premium and the overall premium is the weighted sum.

There are many possible factors, e.g. yield spread, interest rate, exchange rate, GDP, inflation, momentum, size, value / growth and many more.

2.7.1 Fama-French Three-Factor Model

Fama and French suggested the factors market (like in the CAPM), company size and the book-to-market ratio (ratio of the book value to the market cap), i.e.:

$$r - r_f = b_{market}(r_{market}) + b_{size}(r_{size}) + b_{book-to-market}(r_{book-to-market})$$

Chapter 3

Market Efficiency

3.1 Investment vs. Financing

Questions companies face are:

- Should the firm reinvest most of its earnings in the business, or distribute the cash to shareholders?
- Should the firm borrow short-term or long-term?
- Should it borrow by issuing a normal long-term bond or a convertible bond?

When the firm makes capital investment decisions, it does not assume that it is facing perfect competitive markets (because of few competitors, geographic area, patents, expertise, etc...) which gives the opportunity for superior profits (projects with positive NPV). In financial markets, there is strong competition with all other corporations and the state, local and federal government so the chance for projects with positive NPV is much smaller.

3.2 Efficient Market Hypothesis

In 1953, a British statistician observed that stock and commodity prices behave like a random walk (the price changes are independent of one another). The main idea behind this is: If past price changes could be used to predict future price changes, investors could make easy profits. But in competitive markets easy profits don't last. As investors try to take advantage of the information in past prices, prices adjust immediately until the superior profits from studying past price movements disappear. As a result, all the information in past prices will be reflected in today's stock price, not tomorrow's. Patterns in prices will no longer exist and price changes in one period will be independent of changes in the next. But why stop there? If markets are competitive, shouldn't today's stock price reflect all the information that is available to investors?

Definition 5: Efficient Market Theory

There are three forms of the efficient market theory:

- **Weak Form Efficiency**: Market prices reflect all historical price information.
- **Semi-Strong Form Efficiency**: Market prices reflect all publicly available information.
- **Strong Form Efficiency**: Market prices reflect all information, both public and private.

The more efficient the market, the more random the sequence of price changes generated by such a market. The most efficient market of all is one in which price changes are completely random and unpredictable.

There is evidence for the weak form efficiency as there's very little correlation between movements of different time periods.

To analyze the semi-strong form of the efficient-market hypothesis, researchers have measured how rapidly security prices respond to different items of news, such as earnings or dividend announcements, news of a takeover, or macroeconomic information. For this, the abnormal stock return was measured, which is defined as:

$$\begin{aligned}\text{Abnormal stock return} &= \text{actual stock return} - \text{expected stock return} \\ &= \tilde{r} - (\alpha + \beta \tilde{r}_m)\end{aligned}$$

They found out that the response is almost immediate.

To test the strong form, researchers have looked for portfolios of professional analysts and mutual funds which can outperform the market and found out that there are almost none (over a long period of time).

But there's also evidence against the EMH. Some examples are:

- Royal-Dutch and Shell merged with a fixed ratio of fundamental values, the ratio of prices is still different from this value and varies over time.
- The Earnings Announcement Puzzle: Firms with very good earnings continue to outperform those with very bad earnings for a long time after the announcement.

- The New-Issue Puzzle: Early gains after a new stock issuance / IPO often turn into losses in the long turn (compared to similar-sized stocks).

The EMH states that prices are just reflecting news (exogenous dynamics), where there's also the theory for the "reflexivity" of markets (endogenous dynamics). The theory states that markets are subjected to internal feedback loops and prices do influence the fundamentals, which in turn change the expectations, thus influencing prices.

3.2.1 Behavioral Finance

Some Anomalies can be explained with behavioral finance (a study of investor market behavior that derives from psychological principles of decision making, to explain why people buy or sell the stocks they do). For instance, there's the principle of loss aversion which states that people tend to avoid those actions that may result in loss. The pain of a loss seems to depend on whether it comes on the heels of earlier losses. In contrast, investors may be more prepared to run the risk of a stock market dip after they have experienced a period of substantial gains. Furthermore, there may be wrong beliefs about probabilities (assuming what happened in the past is representative of the future) or the bias of overconfidence (most investors thinking that they are better-than-average stock pickers).

3.2.2 Bubbles

Some people attribute the Dot-Com bubble to "irrational exuberance". As the bull market developed, it generated optimism about the future and stimulated demand for shares.

The present value can also be very sensitive to changes in expectation. For instance, if we assume an expected dividend of 18.7 with an expected growth rate of 10 percent a year and a risk premium of 11.7%, we get (if we simply discount future dividend payments) a present value of $\frac{d}{r-g} = \frac{18.7}{.117-.10} = 1100$. If now the dividend growth is assumed to be 9.5% per year, we get $\frac{18.7}{.117-.095} = 850$, i.e. a 23% decline in present value because of a 0.5% decline in expected dividend growth.

Bubbles result from positive feedbacks and there are many mechanisms for positive feedbacks in the stock market. They can be divided into technical and rational mechanisms (algorithmic trading, trend following strategies, etc...) and behavioral mechanisms (like imitation).

Chapter 4

Options

There are multiple usages for derivatives:

- To hedge risks
- To speculate
- To lock in an arbitrage profit
- To change the nature of a liability
- To change the nature of an investment without incurring the costs of selling one portfolio and buying another

The following terminology is used:

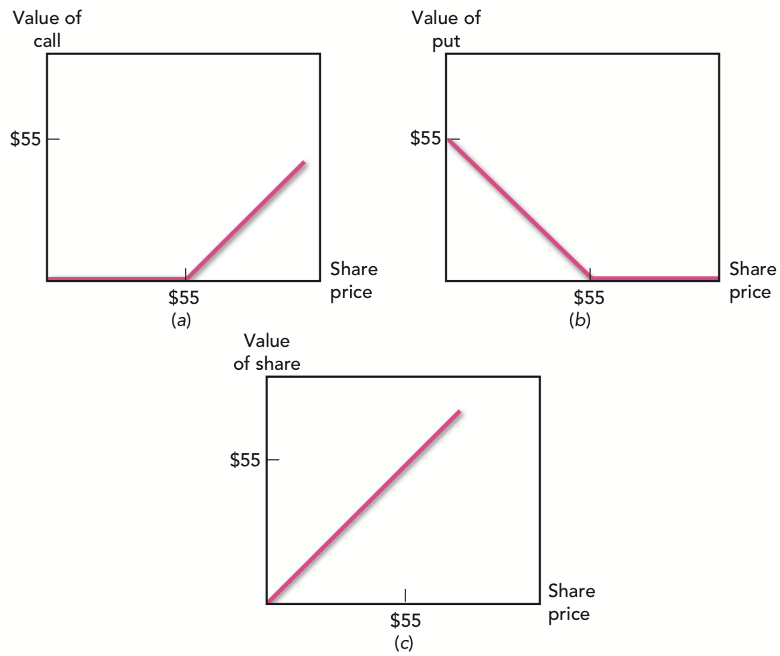
- **Option Premium**: The price paid for the option per share.
- **Intrinsic Value**: Difference between the strike / exercise price and the stock price.
- **Time Premium**: Value of an option above the intrinsic value.

4.1 Calls, Puts and Shares

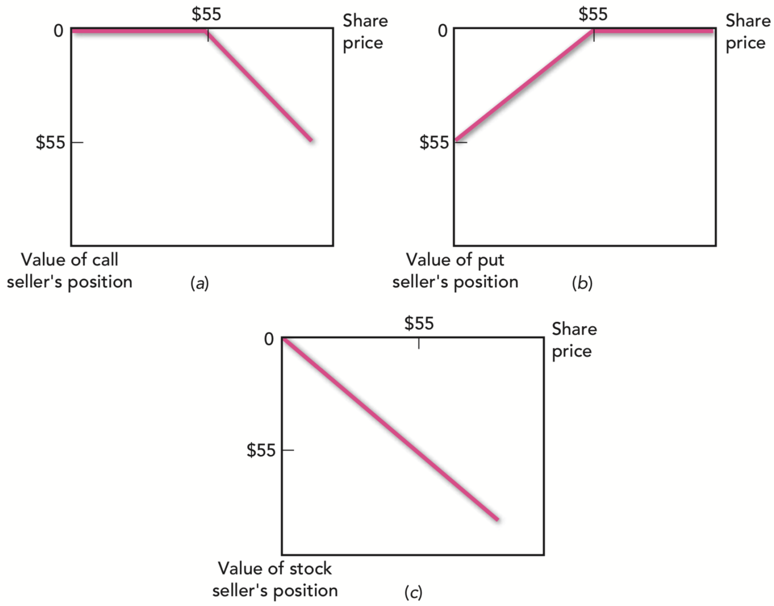
A call option gives its owner the right to buy stock at a specified exercise / strike price on or before a specified exercise date. If the option can be exercised only on one particular day, it is conventionally known as a European call; in other cases, the option can be exercised on or at any time before that day, and it is then known as an American call (mnemonic: American $\rightarrow$ more flexible). In general, the value of a call option goes down as the exercise price goes up.

A put option gives you the right to sell the share for the exercise price. The value of a put increases when the exercise price is raised.

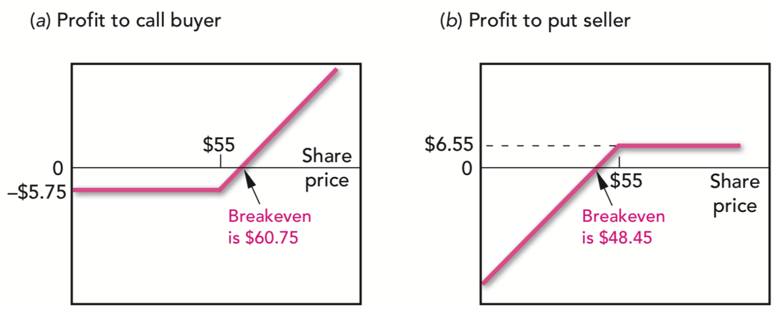
Extending the maturity date makes both puts and calls more valuable. Here is the payoff diagram of calls and puts:



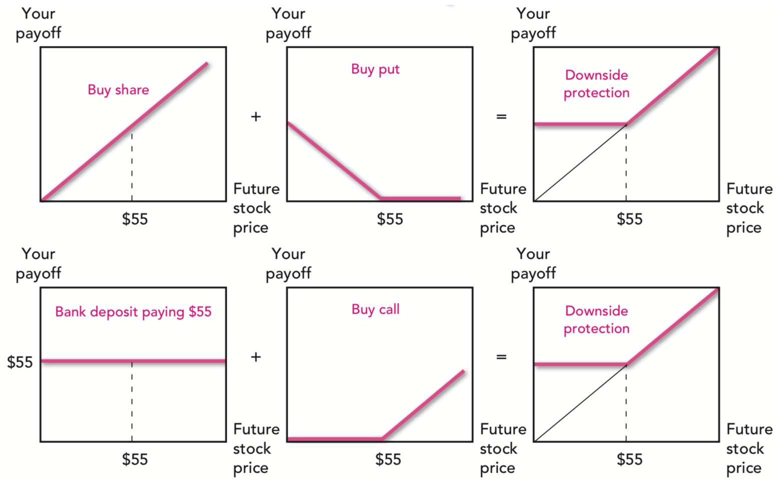
If you sell / write a call, you promise to deliver shares if asked to do so by the call buyer. If the call is exercised, the seller loses the difference between the share price and the exercise price received from the buyer. If you sell / write a put, you promise to pay the exercise price for the shares if the buyer should request it. The payoff diagram for the seller looks like this (where the third option is selling short, i.e. selling a stock without owning it):



The profit diagrams include the initial cost and are therefore a shifted version of the payoff diagrams:



The following picture shows that the same payoff profile can be achieved by buying a share and a put ("protective put") or by depositing the (present value of the) exercise price and buying a call:



Definition 6: Call-Put Parity

Because both illustrated strategies provide identical payoffs, the following fundamental relationship for European options holds:

$$\text{value of call} + \text{PV of exercise price} = \text{value of put} + \text{share price}$$

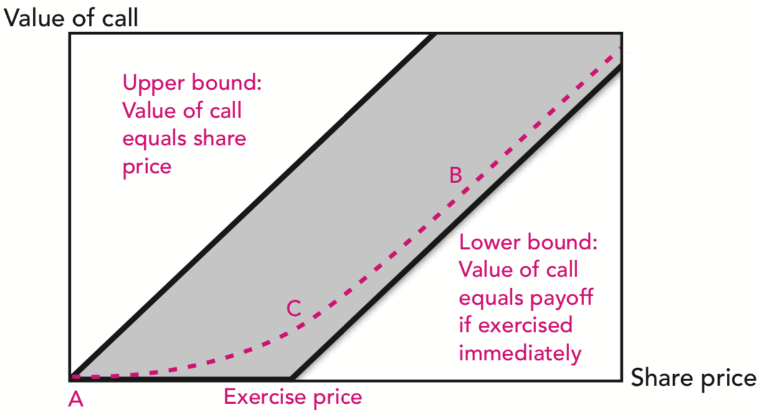
If a stock pays dividends before the final exercise date, the investor who buys the call misses out on this dividend, therefore the relationship becomes:

$$\begin{gathered}\text{value of call} + \text{PV of exercise price} = \\ \text{value of put} + \text{share price} - \text{PV of dividend}\end{gathered}$$

Any set of contingent payoffs—that is, payoffs which depend on the value of some other asset — can be constructed with a mixture of simple options on that asset.

4.2 Option values

The lower-bound for the option value is always the intrinsic value. The upper-bound is always the share price, an option can't be more valuable than the underlying share (because the stock gives a higher ultimate payoff then the option).



When the difference between share and strike price becomes large, the option value approaches the intrinsic value (because then it's basically a sure thing that the option will be exercised). When the share is worthless, the option is as well. When the share is around the exercise price, there's still a possibility that the option will have an intrinsic value at expiration, so the value is larger than 0. If the underlying stock is more volatile, the option is worth more. The relationship between increases in components and the call option price are summarized here:

1. If there is an *increase* in:	The change in the call option price is:
Stock price (P)	Positive
Exercise price (EX)	Negative
Interest rate (r_f)	Positive*
Time to expiration (t)	Positive
Volatility of stock price (σ)	Positive*

2. Other properties:
   a. *Upper bound.* The option price is always less than the stock price.
   b. *Lower bound.* The option price never falls below the payoff to immediate exercise (P − EX or zero, whichever is larger).
   c. If the stock is worthless, the option is worthless.
   d. As the stock price becomes very large, the option price approaches the stock price less the present value of the exercise price.

Note that the first three points are inverted for put options, whereas the last two are the same.

A higher interest rate leads to a higher call price because acquiring stock by call options is buying on credit (exercise price has to be paid at maturity), so the delay in payment is more valuable if interest rates are high.

4.3 Valuing Options

Valuing options can't be done with discounted cash flow analysis because the risk of the option (and therefore the expected rate of return) is constantly changing with the price. The trick is to set up an option equivalent by combining common stock investment and borrowing. The net cost of buying the option equivalent must equal the value of the option.

Example 4: Replicating Portfolio

Imagine a six-month call option on a stock with exercise price \$55 that is currently at the money. The risk free rate is 4% per year, i.e. 2% for 6 months. The stock can only do two things over the six months: It will fall to \$41.25 (-25%) or rise to \$73.33 (+33%). The possible payoffs for the call option are \$0 (for \$41.25) or \$18.33 (\$73.33 - \$55).
If we buy .5714 shares and borrow \$23.11 from the bank, the payoff is \$0 ($.5714 * \$41.25 - \$23.11 * 1.02$) or \$18.33 ($.5714 * \$73.33 - \$23.11 * 1.02$), i.e. the same.
We get the number of shares required to replicate one call (hedge ratio / option delta) as:

$$\text{Option delta} = \frac{\text{spread of possible option prices}}{\text{spread of possible share prices}} = \frac{18.33 - 0}{73.33 - 41.25}$$

The amount needed from the bank is the present value of the difference in the payments, in this example the present value of $.5714 * \$41.25 = \$23.57$, i.e. $\$23.57/1.02$.
The value of the call is then the difference between the value of the shares and the bank loan, i.e. $(55 * .5714) - \$23.11 = \$8.32$

Example 5: Risk-Neutral Valuation

Consider the same example. Another way to come to the call price is to pretend that investors are indifferent about risk[a]. Then, the expected return must be equal to the risk-free rate of interest, i.e. 2%. So we have

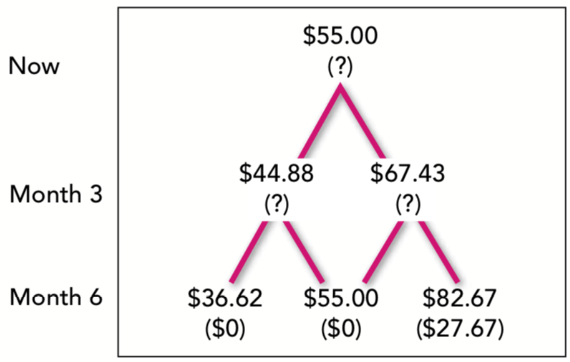
$$\text{Expected return} = p_r * 33\% + (1 - p_r) * (-25\%) = 2\%$$

Where p_r is the (risk-neutral) probability of rise that has nothing to do with the physical probability. We get $p_r = .463$ and can calculate the expected value of the call option as $p_r * \$18.33 + (1 - p_r) * \$0 = \$8.49$. Discounting by the interest rate gives \$8.32.

[a] This method doesn't actually assume that investors are risk neutral. But it can be shown that valuation using risk neutral probabilities is equivalent to replication / no-arbitrage.

4.3.1 Binomial Method

The binomial method starts by reducing the possible changes in the next period's stock price to an "up" (where the value is $(1+u)S$ and a "down" (where the value is $(1+d)S$) move. If we have the rise / fall amount per period, we can easily fill in the binomial tree with the possible stock prices and the call prices (in parentheses) for the last period, which is simply the intrinsic value:



To calculate the call values in month 3, we can use the following formula:

$$C = \frac{1}{1+r_f}\left(\frac{r_f - d}{u-d}C_u + \frac{u - r_f}{u-d}C_f\right)$$

Where $\pi_u = \frac{r_f - d}{u-d}$, $\pi_d = \frac{u-r_f}{u-d}$ are the risk-neutral probabilities. The same formula can be applied afterwards to calculate the present value.

How do we pick values for the up and down changes per period? If we have σ, the standard deviation of stock returns per year and h, the time period as fraction of a year, this can be done by:

$$1 + u = e^{\sigma\sqrt{h}}$$
$$1 + d = \frac{1}{1+u}$$

4.3.2 Black-Scholes Formula

If you let h go to 0 in the binomial model, you arrive at a lognormal distribution and arrive at the Black-Scholes formula for the value of a call option.

Definition 7: Black-Scholes Formula

$$C = (N(d_1) * S) - (N(d_2) * PV(K))$$

$$d_1 = \frac{\ln(\frac{S}{PV(K)}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Where:

- S: Price of underlying share
- t: Time to maturity (in % of year, i.e. years to maturity)
- K: Strike price
- r: Discount rate / risk free rate
- $PV(K) = K * e^{-rt}$

Instead of calculating the option price for a given σ, the Black-Scholes formula can be used to derive the implied volatility of a stock for a given option price. As the option (for the same expiration date) becomes increasingly in-the-money or out-of-the-money, volatility increases, which is called "volatility smile".

When a stock pays dividend, the price of the stock should be reduced by the present value of the dividends paid before the option's maturity.

Definition 8: The Greeks

- Delta (Δ): A measure of an option's sensitivity to changes in the price of the underlying asset. $\Delta = \frac{\partial C}{\partial S}$
- Gamma (Γ): A measure of delta's sensitivity to changes in the price of the underlying asset. $\Gamma = \frac{\partial^2 C}{\partial S^2}$
- Vega: A measure of an option's sensitivity to changes in the volatility of the underlying asset. $\mathcal{V} = \frac{\partial C}{\partial \sigma}$

- Theta (Θ): A measure of an option's sensitivity to time decay. $\Theta = \frac{\partial C}{\partial t}$
- Rho (ρ): A measure of an option's sensitivity to changes in the risk free interest rate. $\rho = \frac{\partial C}{\partial r}$

4.3.3 Dilution

To value a warrant, we have to calculate the numbers of warrants issued per share outstanding q. The warrant value is then:

$$\frac{1}{1+q} \times \text{ Value of call on alternative firm}$$

4.4 Real Options

Discounted cash flow doesn't incorporate real options attached to the project that sophisticated managers can take advantage of. A real option is the right, but not the obligation, to undertake some business decision, typically the option to make a capital investment. For companies fraught with uncertainty, the stock price is the sum of discounted cash flow value, representing the existing businesses, plus real options value. Real options capture the value of uncertain growth opportunities.

The real option framework can be used for capital budgeting (improves on discounted cash flow), structuring business decisions, aligning management's value creation decisions with the market and is the appropriate method for valuing IP and new technology (software, IT expenditures, etc...). There are four types of real options:

1. The option to expand and make follow-up investments if the immediate investment project succeeds.
2. The option to wait (and learn) before investing.
3. The option to shrink or abandon a project.
4. The option to vary the mix of output or the firm's production methods.

Standard finance theory says that the greater the uncertainty, the higher the appropriate discount rate and therefore the lower the present value. For real options, the greater the uncertainty surrounding the development of the company, the more valuable these options may be. Besides high uncertainty, real options are also more valuable when managerial flexibility is high.

4.4.1 The Option To Expand

Imagine a company wants to build a computer (Mark I). The project has a negative NPV, but it gives the option to build another computer (Mark II) in three years. The investment required for the Mark II is \$900 million and forecasted cash inflows are \$807 million. We assume a discount rate of 20% and a standard deviation of 35% on the future value of the Mark II cash flows (because the future value is highly uncertain). The present value of the forecasted cash flows is $\frac{807}{(1.2)^3} = \$467$ million.

We therefore have a three-year call option on an asset worth \$467 million with a \$900 million exercise price. We can use the Black Scholes formula to calculate the value of this call with $\sigma = 0.35$.

4.4.2 Timing Option

Even if a project has a positive NPV, it can be good to wait and see how the market develops. Imagine you have the option to build a factory immediately for \$180 million (with a present value of \$200 million, i.e. NPV = \$20 million) or wait a year. In the first year, the cash flow is either \$16 million (in which case the project value is \$160 million) or \$25 million (in which case the project value is \$250 million). If demand is high, the total return is $\frac{25+250}{200} - 1 = .375$, if demand is low $\frac{16+160}{200} - 1 = -.12$. In a risk-neutral world, the expected return would be equal to the interest rate (which is assumed to be 5%). We therefore have

$$\text{Expected return} = p_h * 37.5\% + (1 - p_h) * (-12\%) = 5\%$$

Where p_h is the (risk-neutral) probability of high demand. The binomial model can now be applied to calculate the value of the option. With probability p_h, the option value will be \$70 million (250 - 180) and with probability $(1 - p_h)$, the option will be worthless. The option value is then the expected (in a risk-neutral world) discounted return, i.e.

$$\frac{(p_h * 70) + ((1 - p_h) * 0)}{1.05} = \$22.9 \text{ million}$$

4.4.3 The Abandonment Option

When bad news arrives, it is useful to have the option to bail out and and recover the value of the project's plant, equipment and other assets. Imagine a project with initial costs of \$10 million for equipment and \$2 million for roads and site preparation. The equipment costs \$700'000 per year to operate. The project would generate revenues of \$1.7 million per year, cash flow would therefore be \$1.0 million per year. Annual price growth of 9% is assumed, the risk-adjusted rate is 9% and the risk-free rate 6%. It's possible to construct a binomial tree for the cash flow and the present value (if u and d is calculated / given). To do so, the risk-neutral probabilities are calculated and the present value of continuing is calculated. If it's lower than abandonment (in that year), this value is entered:

Year	0	1	2	3	4	5	6	7	8	9	10
Forecasted revenues Present value	 $17.00	1.85	2.02	2.20	2.40	2.62	2.85	3.11	3.39	3.69	4.02
Fixed costs Present value	 $5.15	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
NPV	-0.15										6.18 3.49
										5.28 8.61	
									4.50 12.22		4.50 3.49
								3.82 14.62		3.82 7.15	
							3.23 16.07		3.23 9.68		3.23 3.49
						2.72 16.75		2.72 11.31		2.72 6.05	
					2.27 16.86		2.27 12.23		2.27 7.77		2.27 3.49
				1.89 16.51		1.89 12.58		1.89 8.81		1.89 5.21	
			1.55 15.83		1.55 12.51		1.55 9.33		1.55 6.32		1.55 3.49
		1.26 14.91		1.26 12.10		1.26 9.43		1.26 6.92		1.26 4.58	
Cash flow Present value	 13.84		1.00 11.47		1.00 9.23		1.00 7.14		1.00 5.22		1.00 3.49
		0.78 10.73		0.78 8.83		0.78 7.09		0.78 5.50		0.78 4.11	
			0.59 8.49		0.59 6.95		0.59 5.60		0.59 4.43		0.59 3.49
				0.42 7.29		0.42 5.90		0.42 4.78		0.42 3.87	
					0.27 6.56		0.27 5.31		0.27 4.30		0.27 3.49
						0.15 5.90		0.15 4.78		0.15 3.87	
							0.03 5.31		0.03 4.30		0.03 3.49
								-0.06 4.78		-0.06 3.87	
									-0.14 4.30		-0.14 3.49
										-0.22 3.87	
											-0.28 3.49
Salvage value (years 1–10)		9.00	8.10	7.29	6.56	5.90	5.31	4.78	4.30	3.87	3.49

E.g. in the top entry of year 9, we calculate:

$$\mathrm{PV} = \frac{(p_u * (6.18 + 3.49)) + ((1 - p_u) * (4.5 + 3.49))}{1.06} = 8.61$$

With the option, the project has therefore a NPV of (13.84 - 12 = $1.84 million). The abandonment option has therefore increased NPV by $1.99 million and is worth that much.

Chapter 5

Bonds and Interest Rates

5.1 Bonds

Bonds are long-term debt. The face value of the bond is known as principal, on which a coupon at a coupon rate can be paid. The present value of a bond is simply the sum of the discounted cash flows. For instance, a 5-year Treasury bond with a 7 percent coupon has a present value of (where a similar security has a return of 4.8%):

$$\text{PV} = \frac{70}{1.048} + \frac{70}{(1.048)^2} + \frac{70}{(1.048)^3} + \frac{70}{(1.048)^4} + \frac{1070}{(1.048)^5} = 1'095.78$$

Bond prices are usually expressed as a percentage of the face value, i.e. a \$1'000 dollar with a PV of \$1'095.78 is worth 109.578%.

You can also ask the question: If the price of the bond is \$1'095.78, what return do investors expect? Then, the equation becomes:

$$1'095.78 = \frac{70}{1+r} + \frac{70}{(1+r)^2} + \frac{70}{(1+r)^3} + \frac{70}{(1+r)^4} + \frac{1070}{(1+r)^5}$$

The rate r is often called the bond's yield to maturity.

Definition 9: Duration, Sensitivity, Volatility

The duration is often used to describe the average time to each payment. If the bond has a total value V, it is defined as:

$$\text{Duration} = \frac{1 * PV(C_1)}{V} + \frac{2 * PV(C_2)}{V} + \frac{3 * PV(C_3)}{V} + \ldots = d \text{ years}$$

The sensitivity measures how the price of a bond evolves when the

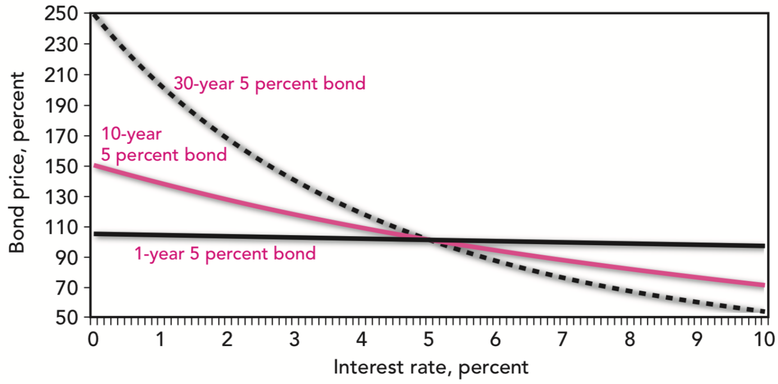
YTM changes. It is defined as:

$$\text{Sensitivity} = -\frac{1}{1+r}\sum_{i=1}^{N}\frac{i * PV(C_i)}{V} = -\frac{\text{Duration}}{1+r}$$

Volatility is defined as:

$$\text{Volatility} = -\text{ Sensitivity}$$

When interest rates fall, bond prices rise (because the discounting rate falls) and vice-versa. The effect is greatest when the cash flows on the bond last for many years:



5.1.1 Zero-Coupon Bond / Discount Bond

The simplest kind of bond is the zero-coupon bond. We call $B(t,s)$ the price, at time t, of a bond maturing at time $s = t + T \geq t$, with maturity value \$1. It is the present value (at t) of 1\$ received at s. The yield-to-maturity is simply:

$$R(t,T) = \frac{1}{T}(\ln(\$1) - \ln B(t,t+T)) = -\frac{1}{T}\ln B(t,t+T)$$

5.2 Interest Rates

According to Fisher, the real interest rate is the price which equates the supply and demand for capital. The supply depends on people's willingness to save and the demand depends on the opportunities for productive investment. Real interest rate change only slowly, but according to Fisher the nominal interest rate changes with the expected inflation rate.

If we have a loan that pays \$1 at time 1 and time 2, the present value is:

$$\mathrm{PV} = \frac{1}{1+r_1} + \frac{1}{(1+r_2)^2}$$

r_1 (that is fixed today) is called today's one-period spot rate and r_2 today's two-period spot rate. If we use the same discounting rate for all payments (as in section 5.1), we get the yield to maturity r.

5.2.1 Forward Rate

If we have a fixed one-year spot rate r_1 and two-year spot rate r_2, a dollar that is invested for one year gives $\$(1+r_1)$ whereas one for two years returns $\$(1+r_2)^2$. We can calculate the additional rate that is earned by keeping the money invested for two years, it is called the forward interest rate or f_2 with:

$$(1+r_2)^2 = (1+r_1) * (1+f_2)$$

In other words, one can think of the two-year investment as earning the one-year spot rate for the first year and the extra return, or forward rate, for the second year.

More generally, we have:

$$(1+r_n)^n = (1+r_1)(1+f_2)(1+f_3)\ldots(1+f_n)$$

Example 6

Given a 2 year treasury with YTM=8.995%, a 3 year treasury with YTM=9.660%, what is the 3rd year forward rate?
We have $r_2 = 0.08995$, $r_3 = 0.09660$ and want to find out f_3. We write

$$(1+r_3)^3 = (1+r_2)^2(1+f_3)$$

and derive $1+f_3 = \frac{(1+r_3)^3}{(1+r_2)^2}$, which gives $f_3 = 0.110$.

Example 7

Two years from now, you intend to begin a project that will last for 5 years. What discount rate should be used when evaluating the project? The 2 year spot rate is 5%, the 7 year spot rate 7.05%.
Writing

$$(1+r_7)^7 = (1+r_2)^2(1+f_{2,5})^5$$

and solving for $f_{2,5}$ gives $f_{2,5} = 7.88\%$.

Example 8

Given a 8% 2 year bond with a YTM of 9.43% and a 10% 2 year bond with a YTM of 9.43%, what is the forward rate?
We first value the two bonds and get for the 8% bond a value of 975\$ and for the 10% a value of 1'010\$. We can now define a system of equation with two unknowns and two equations:

$$975\$ = \frac{80\$}{1+f_1} + \frac{1'080\$}{(1+f_1)(1+f_2)}$$
$$1'010\$ = \frac{100\$}{1+f_1} + \frac{1'100\$}{(1+f_1)(1+f_2)}$$

The expectations theory of term structure states that the forward rate is simply the expected spot rate for the time period (e.g. f_2 is the expected spot rate of interest at year 1 on a loan maturing at the end of year 2).

5.2.2 Term structure

Generally, long rates of interest are higher than short rates which is called normal yield curve. In this scenario, investors expect the economy to grow in the future in association with a risk of rising inflation. When the yield curve is inverted, long-term investors think the economy will slow or even decline in the future and then will settle for lower yields. When the yield curve is flat or humped, there is large uncertainty about the future economic situation.

According to the expectation theory, the only reason for an upward-sloping term structure is that investors expect short-term interest rates to rise and the only reason for a declining term structure is that investors expect short-term interest rates to fall.

The liquidity-preference theory of the term structure states that if investors incur extra risk from holding long-term bonds, they will demand the compensation of a higher expected return. For this reason, the forward rate must be higher than the expected spot rate. This difference is usually called liquidity premium. If the liquidity-preference theory is right, the term structure should be upward-sloping more often than not. Of course, if future spot rates are expected to fall, the term structure could be downward-sloping and still reward investors for lending long. Inflation uncertainty may help to explain why long-term bonds provide a liquidity premium. If inflation creates additional risks for long-term lenders, borrowers must offer some incentive if they want investors to lend long.

Cash flow should be discounted using the term structure information, i.e. with the appropriate spot / forward rate.

There has been a historical inverse relationship between rates of unemployment and the inflation rate which is illustrated in the Phillips Curve.