Project 10 — Implied Volatility Surface (SVI)

Smile parametrization, calibration, and arbitrage-aware checks

Quant Finance Portfolio

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Abstract

This report supports **Project 10**, an implied-volatility surface construction workflow based on **SVI (Stochastic Volatility Inspired)** parametrization. Starting from option prices or quotes, the notebook computes implied volatilities, fits **SVI smiles** per maturity, and assembles a smooth **volatility surface**. The emphasis is on recruiter-relevant quality: clear definitions (moneyness, total variance), stable calibration, and **no-static-arbitrage** sanity checks (smile shape, convexity, calendar monotonicity).

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1 What you build in Project 10

An implied-volatility surface is the essential object behind modern option pricing, risk, and calibration. This project implements an end-to-end workflow:

- Ingest an option chain (strikes, maturities, prices or IV quotes) and normalize conventions.
- Convert prices to **implied volatilities** via inversion of Black–Scholes.
- Re-express strikes in **log-moneyness** and work with **total variance**.
- Fit an **SVI** smile for each maturity (fast, stable parametric fit).
- Assemble a smooth **IV surface** across strikes and maturities.
- Validate the result using **arbitrage-aware checks** and visual diagnostics.
- Provide interactive plots (smile per expiry, surface, parameter stability).

Recruiter takeaway. This project demonstrates a practical volatility-surface pipeline: data normalization, numerical implied-vol inversion, parametric smile calibration, and discipline around static-arbitrage sanity checks.

2 Prerequisites (math and option conventions)

2.1 Black–Scholes pricing and implied volatility

In Black–Scholes, a European option price is a function of volatility:

$$C = \mathrm{BS}(S_0, K, r, q, T, \sigma).$$

The **implied volatility** IV is the value of σ that reproduces the observed market price:

$$\mathrm{BS}(\cdot, \mathrm{IV}) = C_{\mathrm{mkt}}.$$

Numerically, IV is obtained by root-finding (bisection / Newton), with robust bounds and convergence checks.

2.2 Moneyness and log-moneyness

A standard coordinate for smiles is **log-moneyness**:

$$k = \log\left(\frac{K}{F}\right), \qquad F = S_0\, e^{(r-q)T}.$$

Using k makes cross-maturity comparisons more stable and helps interpret symmetry/asymmetry of smiles.

2.3 Total variance

SVI is typically expressed in terms of **total variance**:

$$w(k, T) = \sigma^2(k, T)\, T,$$

where $\sigma(k, T)$ is the implied volatility at log-moneyness k and maturity T. Working with w often improves numerical behavior across maturities.

2.4 Static arbitrage in option surfaces (high level)

A surface should avoid obvious arbitrage:

- **Butterfly arbitrage (in strike):** option prices should be convex in strike.
- **Calendar arbitrage (in maturity):** option prices should be non-decreasing in maturity (for fixed strike under consistent discounting).

This project does not claim a full proof of arbitrage-freeness, but implements **practical checks** used as quality gates in production pipelines.

3 SVI parametrization

3.1 Raw SVI formula

For a fixed maturity T, raw SVI models the total variance smile $w(k)$ as:

$$w(k) = a + b\left(\rho(k-m) + \sqrt{(k-m)^2 + \sigma^2}\right),$$

with parameters:

- a controls the overall level (vertical shift),
- $b > 0$ scales the slope/curvature,
- $\rho \in (-1, 1)$ controls skew/asymmetry,
- m shifts the smile horizontally,
- $\sigma > 0$ controls the smoothness near the minimum.

3.2 Interpretation

SVI is a flexible yet compact shape that captures:

- a convex "smile" profile (curvature),
- skew (equity-like negative skew is common),
- stable extrapolation in the wings compared to naive polynomials.

4 Calibration workflow

4.1 From option quotes to (k, w) points

For each maturity T:

1. compute forward price F and log-moneyness $k = \log(K/F)$,
2. compute implied volatility $\sigma_{\mathrm{mkt}}(k, T)$ from option prices,
3. compute total variance $w_{\mathrm{mkt}}(k, T) = \sigma_{\mathrm{mkt}}^2 T$.

4.2 Objective function

A typical fit minimizes squared error in total variance:

$$\min_{\theta} \sum_{j} \left(w_{\mathrm{SVI}}(k_j; \theta) - w_{\mathrm{mkt}}(k_j)\right)^2,$$

with $\theta = (a, b, \rho, m, \sigma)$ and constraints ($b > 0$, $\sigma > 0$, $\rho \in (-1, 1)$). Some implementations fit implied vol directly; fitting w is common and often numerically smoother.

4.3 Initialization and constraints

Calibration quality depends on:

- reasonable initial guesses (level/skew/curvature from data),
- parameter bounds to avoid degenerate shapes,
- robustness to sparse strikes or noisy quotes.

4.4 Surface assembly across maturities

Once each maturity has a calibrated smile, the surface is built by:

- interpolating $w(k, T)$ or parameters across T,
- evaluating on a grid (k, T),
- converting back to implied vol: $\sigma(k, T) = \sqrt{w(k, T)/T}$.

5 Arbitrage-aware sanity checks

This section summarizes quality gates commonly used in practice.

5.1 Strike direction checks (smile shape)

- total variance $w(k)$ should be non-negative,
- the smile should be reasonably convex and smooth,
- wing behavior should not explode unrealistically.

5.2 Calendar direction checks (maturity consistency)

A practical check is monotonicity of total variance:

$$w(k, T_2) \geq w(k, T_1) \quad \text{for } T_2 > T_1$$

(at fixed k), which helps reduce calendar arbitrage risk.

5.3 Repricing consistency

A strong diagnostic is to reprice options using the calibrated surface:

$$C_{\text{model}}(K, T) = \text{BS}(\cdot, \sigma_{\text{SVI}}(k, T)),$$

and compare C_{model} to market prices/IVs.

6 Implementation notes (what the notebook is doing)

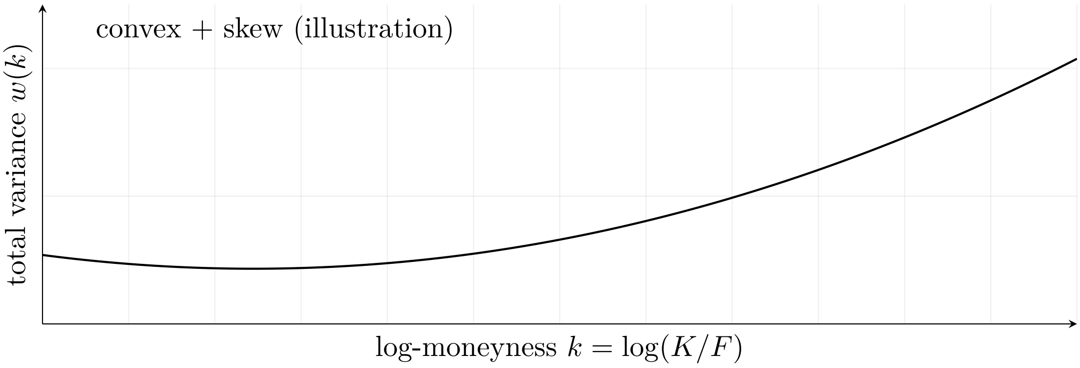
- Load option quotes and clean the dataset (remove obvious outliers, enforce bid/ask logic if available).
- Compute forwards, log-moneyness k, and implied volatilities by inverting Black–Scholes.
- Fit raw SVI per maturity (constrained optimization).
- Build a grid in (k, T), evaluate $w(k, T)$, convert to IV and plot the surface.
- Run checks: non-negativity, smoothness, monotonicity across maturities, and repricing residuals.
- Export interactive plots for the GitHub repo (smiles, surfaces, parameter evolution).

7 Sanity checks you should always do

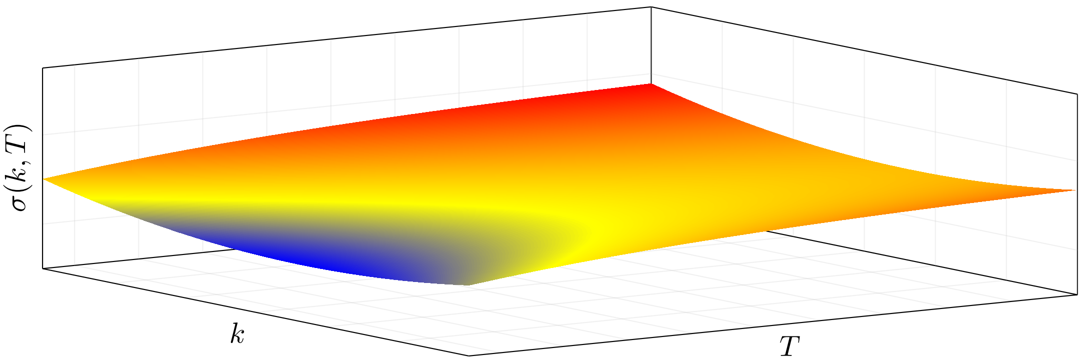
- Implied vols should be positive and within plausible ranges for the underlying.
- SVI parameters should satisfy constraints ($b > 0$, $\sigma > 0$, $\rho \in (-1, 1)$).
- The smile should be smooth; local oscillations often indicate overfitting or noisy data.
- Total variance should not decrease with maturity at fixed k (basic calendar check).
- Repricing errors should be small and not systematically biased (e.g. always underpricing OTM puts).

8 Overleaf plots (conceptual, fast to compile)

8.1 SVI smile shape in total variance (stylized)



8.2 Implied-vol surface intuition (stylized)



9 Interview pitch

This project builds an implied-volatility surface using SVI. Starting from option quotes, I compute implied volatilities via Black–Scholes inversion, transform strikes into log-moneyness, and fit SVI total-variance smiles per maturity with constrained optimization. I then assemble a smooth surface across maturities and validate it with practical arbitrage-aware checks (non-negativity, maturity monotonicity, smooth smile shape) and repricing residual diagnostics, with interactive plots exported for the repository.