Project 07 — Yield Curve Bootstrapping & Swap Pricing

Zero-coupon curve construction, forward rates, par swaps

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Abstract

This report supports **Project 07**, an interest-rate foundation project. It builds a **zero-coupon discount curve** (bootstrapping from market instruments), derives **spot and forward rates**, and prices **fixed-for-floating interest-rate swaps** from first principles using discount factors. The focus is on consistent day-count / compounding conventions, clean curve construction, and trader-facing checks (par-instrument repricing, monotonicity, and sensitivity).

Contents

1 What you build in Project 07

The notebook implements the standard IR curve workflow:

- Parse an input market sheet (deposits / short rates, FRAs or futures, swaps) and normalize conventions.
- Bootstrap **discount factors** $\mathrm{DF}(t)$ on pillar dates.
- Convert discount factors into **zero rates** and **instantaneous/period forward rates**.
- Price a **par swap** and recover the fixed rate from the curve.
- Provide interactive visualization (curve shape, forward curve, sensitivity to bumps).

Recruiter takeaway. This project demonstrates practical fixed-income skills: curve construction, instrument repricing, and correct discounting logic for swaps.

2 Prerequisites (math and fixed-income conventions)

2.1 Time, year fractions, and day-count

All IR pricing reduces cashflows to present value using a year fraction τ computed from a day-count convention (e.g. ACT/360, ACT/365F, 30/360). A payment at date T corresponds to a time $t = \tau(0, T)$ in year units.

2.2 Discount factors and present value

A discount factor $\mathrm{DF}(t)$ maps \$1 received at time t to today's value:

$$\mathrm{PV} = \mathrm{DF}(t) \times \mathrm{Cashflow}(t).$$

The discount curve is the object that makes **market instruments reprice**.

2.3 Compounding and zero rates

A continuously compounded zero rate $z(t)$ is defined by

$$\mathrm{DF}(t) = e^{-z(t)t}.$$

Under annual compounding, $\mathrm{DF}(t) = (1 + z(t))^{-t}$. The project keeps conventions explicit.

2.4 Forwards

A simple forward rate between t_1 and t_2 (with year fraction $\Delta = t_2 - t_1$) is derived from discount factors:

$$F(t_1, t_2) = \frac{1}{\Delta}\left(\frac{\mathrm{DF}(t_1)}{\mathrm{DF}(t_2)} - 1\right).$$

Forward rates are the natural pricing inputs for floating cashflows.

3 Curve construction: bootstrapping discount factors

3.1 Bootstrapping principle

Bootstrapping solves for discount factors sequentially on increasing maturities (pillar dates). For each quoted instrument, write its no-arbitrage PV equation in terms of $\mathrm{DF}(\cdot)$, then solve for the unknown pillar DF.

3.2 Example 1: deposit (single cashflow)

For a deposit from 0 to T with year fraction τ, the payoff is $(1 + R\tau)$ at T. No-arbitrage implies:

$$1 = \mathrm{DF}(T)\,(1 + R\tau) \quad \Rightarrow \quad \mathrm{DF}(T) = \frac{1}{1 + R\tau}.$$

3.3 Example 2: par swap (multiple cashflows)

A fixed-for-floating swap with fixed leg payments at dates $T_1, \ldots, T_n$ (year fractions τ_i) has fixed leg PV:

$$\mathrm{PV}_{\text{fixed}} = K \sum_{i=1}^{n} \tau_i \, \mathrm{DF}(T_i),$$

where K is the fixed rate.

For the floating leg (in a single-curve simplified setting), a standard result is:

$$\mathrm{PV}_{\text{float}} = 1 - \mathrm{DF}(T_n).$$

At par, PVs are equal, hence the par swap rate is

$$K_{\text{par}} = \frac{1 - \mathrm{DF}(T_n)}{\sum_{i=1}^{n} \tau_i \, \mathrm{DF}(T_i)}.$$

Bootstrapping uses this equation to solve for the next unknown discount factor once earlier DFs are known.

Practical note. Modern multi-curve setups separate discounting (OIS) from forwarding (IBOR). This project focuses on the core logic; the framework is structured so multi-curve can be added.

4 From discount factors to rates

4.1 Zero rates

With continuous compounding:

$$z(t) = -\frac{1}{t} \log(\mathrm{DF}(t)).$$

With simple compounding:

$$z(t) = \frac{1}{t} \left(\frac{1}{\mathrm{DF}(t)} - 1 \right).$$

4.2 Forward curve

Given $\mathrm{DF}(t)$ on a grid, compute forwards $F(t_i, t_{i+1})$ using the DF ratio formula. The forward curve provides a market-implied view of future short rates.

5 Swap pricing and sensitivity

5.1 Pricing a generic swap

Given a fixed rate K:

$$\mathrm{PV}_{\text{swap}} = \mathrm{PV}_{\text{float}} - \mathrm{PV}_{\text{fixed}} = (1 - \mathrm{DF}(T_n)) - K \sum_{i=1}^{n} \tau_i \, \mathrm{DF}(T_i).$$

A receiver swap has the opposite sign convention.

5.2 Key sensitivities

A basic sensitivity measure is **DV01** (PV change for a 1bp parallel curve bump). Numerically: bump the curve by 1bp, rebuild discount factors, reprice, and compute ΔPV.

6 Implementation notes (what the notebook is doing)

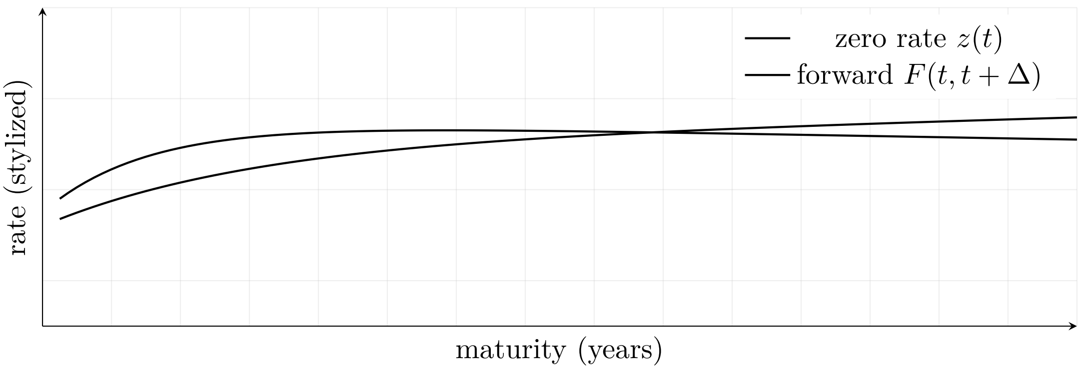
- Read market quotes, build a sorted list of pillar maturities.
- Bootstrap $\mathrm{DF}(T_i)$ sequentially from short to long maturities.
- Convert to zero rates and forward rates and plot them.
- Reprice par instruments to confirm the curve is correct (errors close to zero).
- Provide interactive charts (curve, forwards, swap PV vs rate, bump sensitivity).

7 Sanity checks you should always do

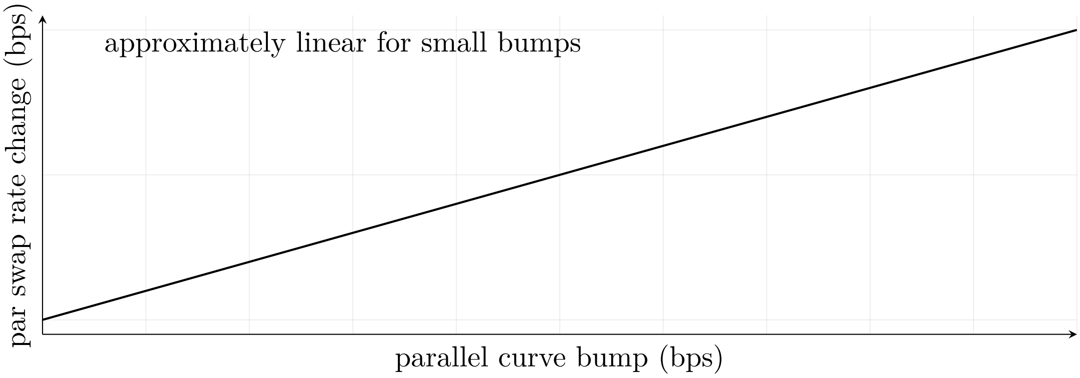
- **Repricing:** deposits and par swaps should reprice to their market quotes (tiny residuals).
- **DF bounds:** $\mathrm{DF}(0) = 1$ and typically $0 < \mathrm{DF}(t) \leq 1$ for positive rates.
- **Monotonicity:** DFs should be non-increasing with maturity in normal positive-rate environments.
- **Arbitrage:** forwards should not imply obvious negative discount factors or inconsistent rates.
- **Convention discipline:** day-count and compounding assumptions must be consistent throughout.

8 Overleaf plots (conceptual, fast to compile)

8.1 Typical shapes: zero curve and forward curve (stylized)



8.2 Par swap rate sensitivity to the curve level (stylized)



9 Interview pitch

I implemented a yield-curve bootstrapping engine: from market quotes I constructed discount factors on pillar dates, derived zero and forward curves, and priced fixed-for-floating swaps by discounting cashflows. I verified the curve by repricing par instruments, and I added sensitivity analysis (DV01-style bumps) and interactive plots to visualize curve shape and forwards.

The natural next step is a multi-curve setup: OIS for discounting and IBOR for forwarding, plus convexity adjustments for futures. The architecture is designed to accommodate that.