

# Probability Distributions

## Random Variables

Random variables are functions with numerical outcomes that occur with some level of uncertainty. For example, rolling a 6-sided die could be considered a random variable with possible outcomes  $\{1, 2, 3, 4, 5, 6\}$ .

## Discrete and Continuous Random Variables

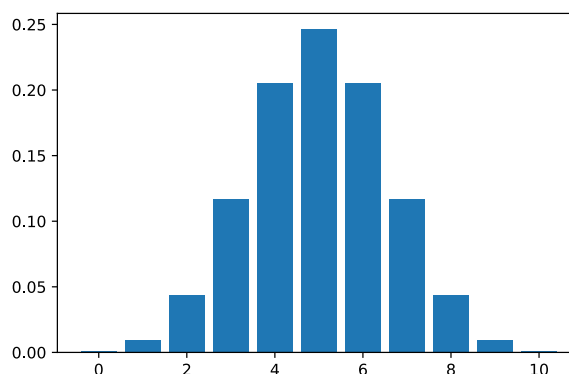
Discrete random variables have countable values, such as the outcome of a 6-sided die roll.

Continuous random variables have an uncountable amount of possible values and are typically measurements, such as the height of a randomly chosen person or the temperature on a randomly chosen day.

## Probability Mass Functions

A probability mass function (PMF) defines the probability that a discrete random variable is equal to an exact value.

In the provided graph, the height of each bar represents the probability of observing a particular number of heads (the numbers on the x-axis) in 10 fair coin flips.



## Probability Mass Functions in Python

The `binom.pmf()` method from the `scipy.stats` module can be used to calculate the probability of observing a specific value in a random experiment.

For example, the provided code calculates the probability of observing exactly 4 heads from 10 fair coin flips.

```
import scipy.stats as stats

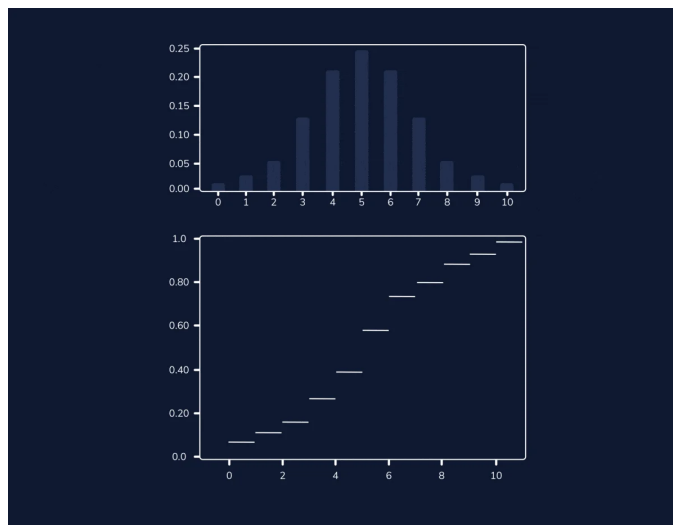
print(stats.binom.pmf(4, 10, 0.5))

# Output:
# 0.20507812500000022
```

## Cumulative Distribution Function

A cumulative distribution function (CDF) for a random variable is defined as the probability that the random variable is less than or equal to a specific value.

In the provided GIF, we can see that as  $x$  increases, the height of the CDF is equal to the total height of equal or smaller values from the PMF.



## Calculating Probability Using the CDF

The `binom.cdf()` method from the `scipy.stats` module can be used to calculate the probability of observing a specific value or less using the cumulative density function.

The given code calculates the probability of observing 4 or fewer heads from 10 fair coin flips.

```
import scipy.stats as stats

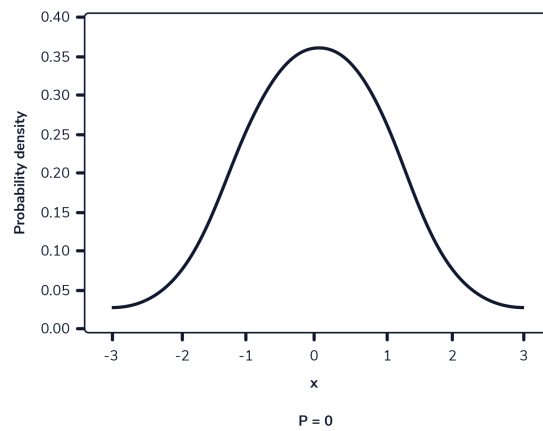
print(stats.binom.cdf(4, 10, 0.5))

# Output:
# 0.3769531250000001
```

## Probability Density Functions

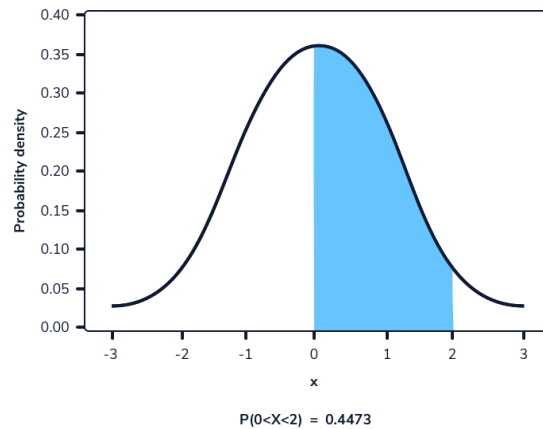
For a continuous random variable, the probability density function (PDF) is defined such that the area underneath the PDF curve in a given range is equal to the probability of the random variable equalling a value in that range.

The provided gif shows how we can visualize the area under the curve between two values.



## Probability Density Function at a Single Point

The probability that a continuous random variable equals any exact value is zero. This is because the area underneath the PDF for a single point is zero. In the provided gif, as the endpoints on the x-axis get closer together, the area under the curve decreases. When we try to take the area of a single point, we get 0.



## Parameters of Probability Distributions

Probability distributions have parameters that control the exact shape of the distribution. For example, the binomial probability distribution describes a random variable that represents the number of successes in a number of trials ( $n$ ) with some fixed probability of success in each trial ( $p$ ). The parameters of the binomial distribution are therefore  $n$  and  $p$ . For example, the number of heads observed in 10 flips of a fair coin follows a binomial distribution with  $n=10$  and  $p=0.5$ .

## The Poisson Distribution

The Poisson distribution is a probability distribution that represents the number of times an event occurs in a fixed time and/or space interval and is defined by parameter  $\lambda$  (lambda). Examples of events that can be described by the Poisson distribution include the number of bikes crossing an intersection in a specific hour and the number of meteors seen in a minute of a meteor shower.

## Expected Value

The *expected value* of a probability distribution is the weighted (by probability) average of all possible outcomes. For different random variables, we can generally derive a formula for the expected value based on the parameters.

For example, the expected value of the binomial distribution is  $n \cdot p$ .

The expected value of the Poisson distribution is the parameter  $\lambda$  (lambda).

Mathematically:

$$X \sim \text{Binomial}(n, p), E(X) =$$

$$Y \sim \text{Poisson}(\lambda), E(Y) =$$

## Variance of a Probability Distribution

The *variance* of a probability distribution measures the spread of possible values. Similarly to expected value, we can generally write an equation for the variance of a particular distribution as a function of the parameters. For example:

$$X \sim \text{Binomial}(n, p), \text{Var}(X)$$

$$Y \sim \text{Poisson}(\lambda), \text{Var}(Y)$$

## Sum of Expected Values

For two random variables,  $X$  and  $Y$ , the expected value of the sum of  $X$  and  $Y$  is equal to the sum of the expected values.

Mathematically:

$$E(X + Y) = E(X) + E(Y)$$

## Adding a Constant to an Expected Value

If we add a constant  $c$  to a random variable  $X$ , the expected value of  $X + c$  is equal to the original expected value of  $X$  plus  $c$ .

Mathematically:

$$E(X + c) = E(X) + c$$

## Multiplying an Expectation by a Constant

If we multiply a random variable  $X$  by a constant  $c$ , the expected value of  $c \cdot X$  equals the original expected value of  $X$  times  $c$ .

Mathematically:

$$E(c \times X) = c \times E(X)$$



## Adding a Constant to Variance

If we add a constant  $c$  to a random variable  $X$ , the variance of the random variable will not change.  
Mathematically:

$$\text{Var}(X + c) = \text{Var}(X)$$

## Multiplying Variance by a Constant

If we multiply a random variable  $X$  by a constant  $c$ , the variance of  $c \cdot X$  equals the original expected value of  $X$  times  $c$  squared.  
Mathematically:

$$\text{Var}(c \times X) = c^2 \times \text{Var}(X)$$

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