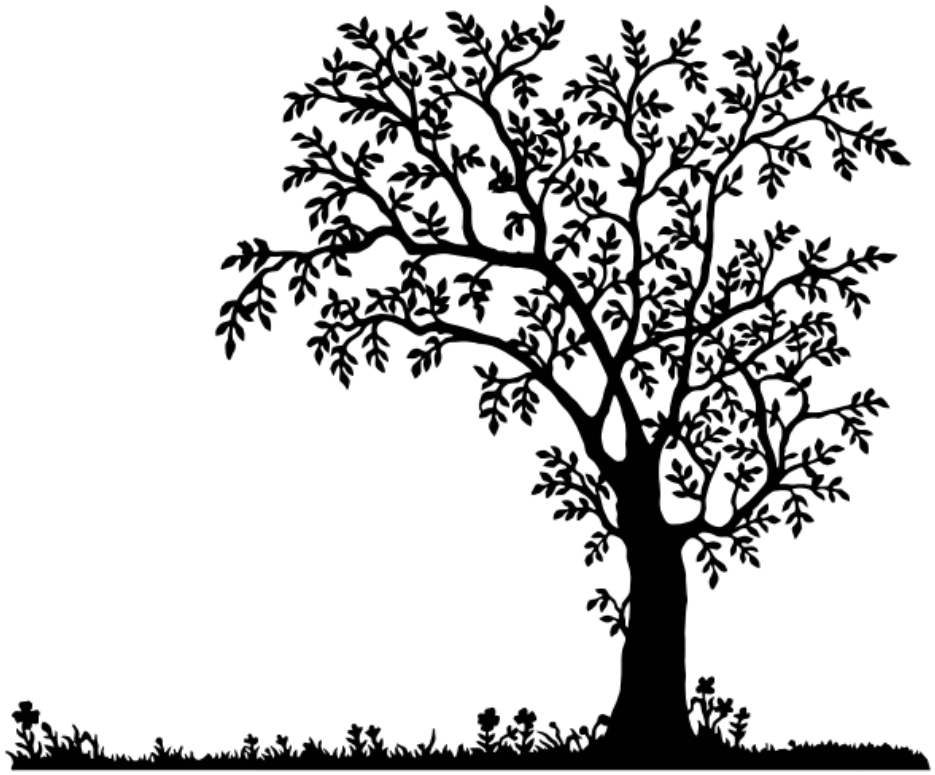
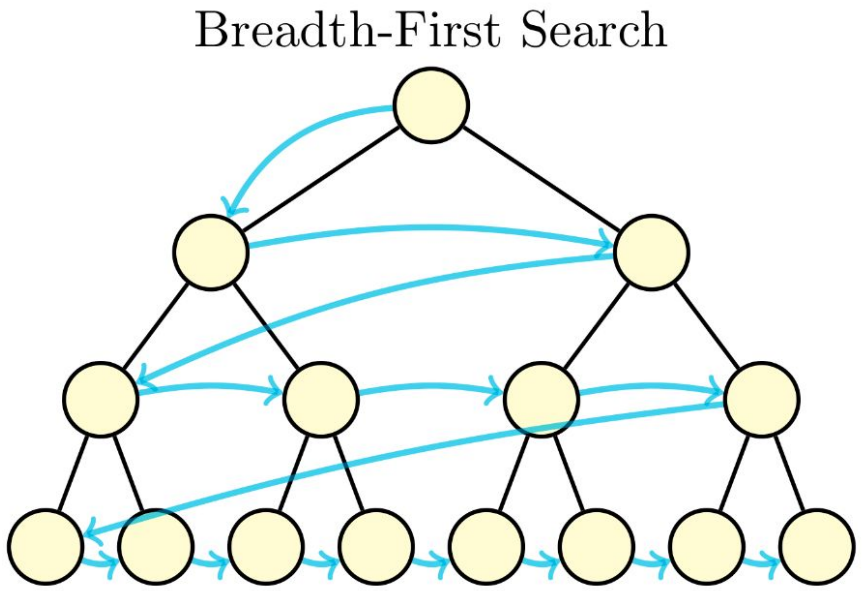
Data Structures and Algorithms (DSA) for AI

Fernanda Madeiral

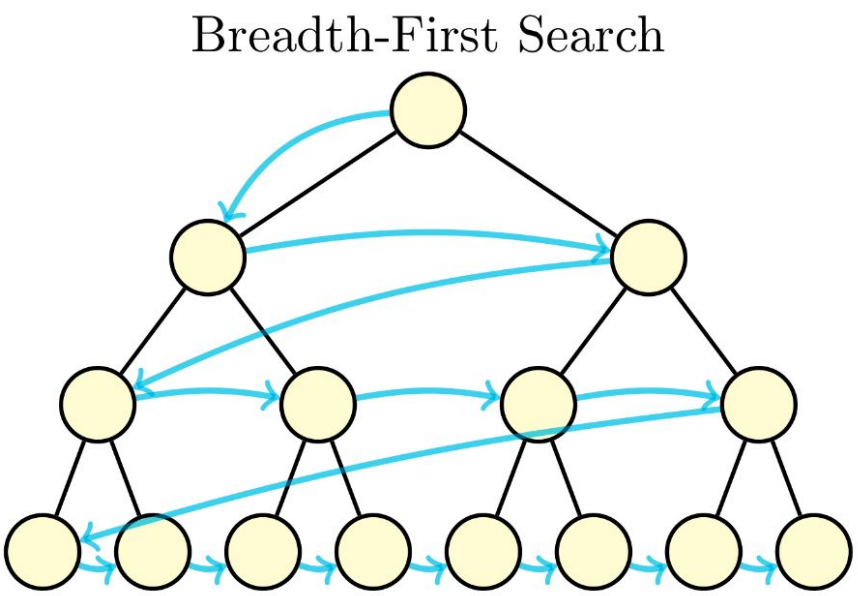


Breadth-first search (BFS)



Queue

How to implement depth-first search (DFS)?



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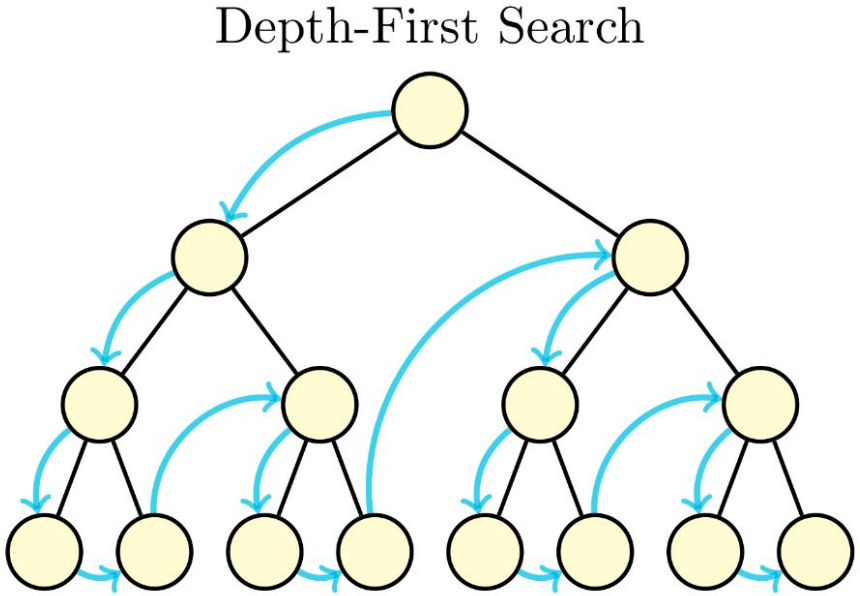

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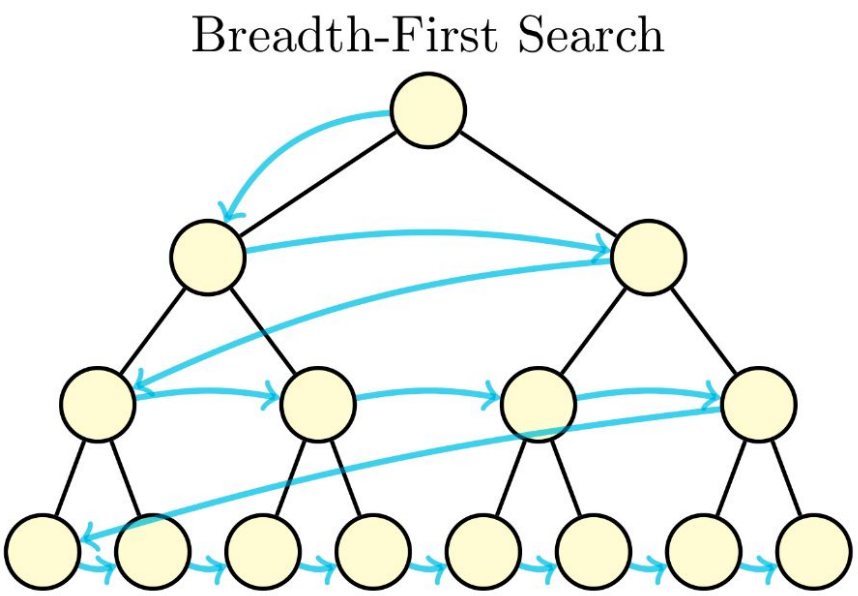
How to implement depth-first search (DFS)?



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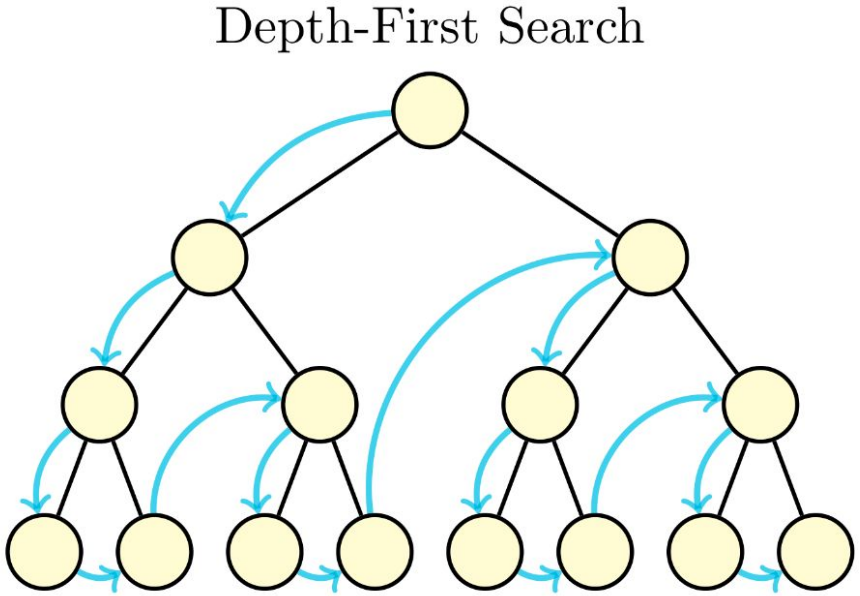

Stack

Depth-first search (DFS)

DFS (Iterative Version)

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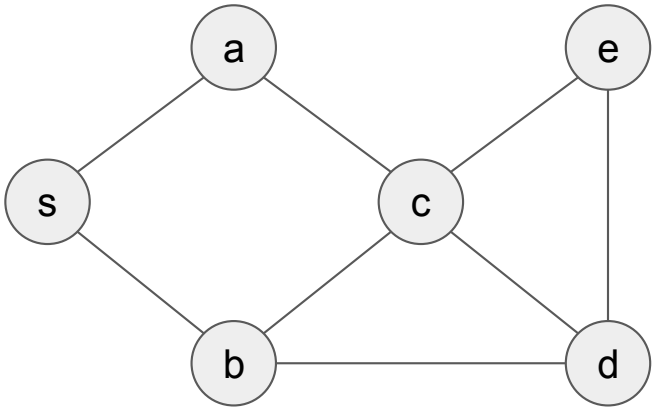
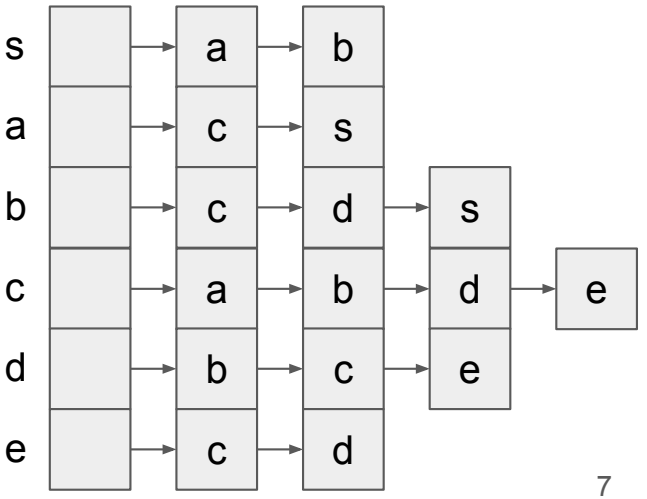


Let's assume the following adjacency-list:



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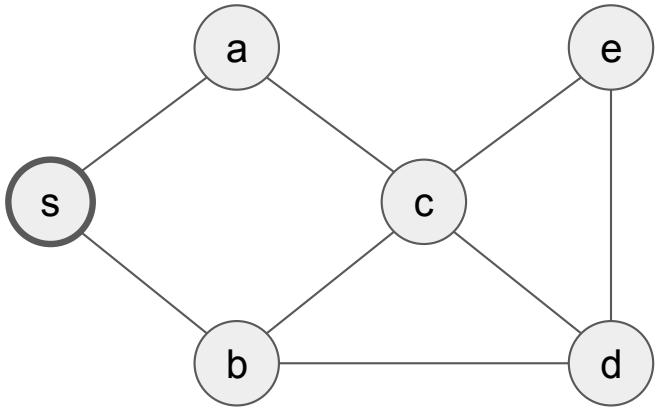

S

Explored:

∅

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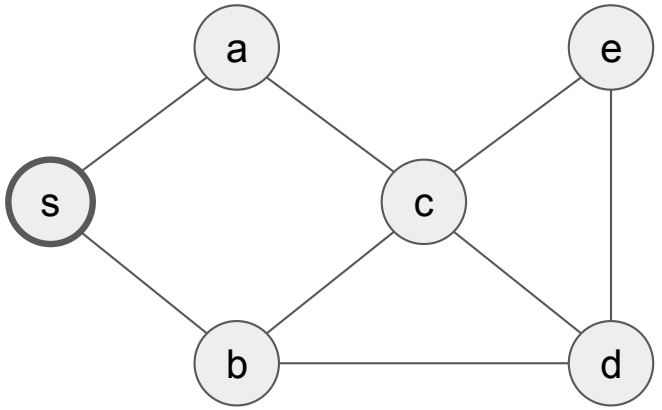

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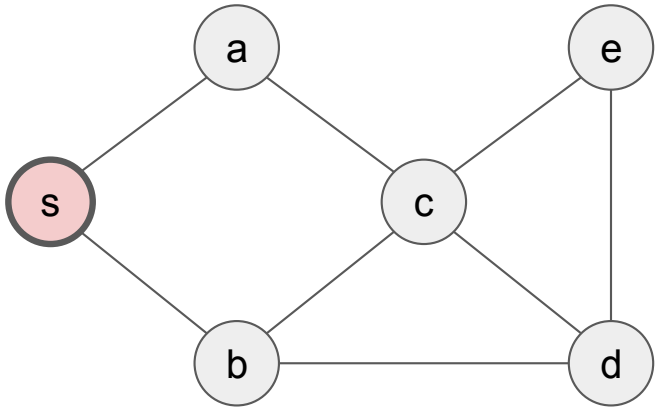

Explored:
s

S

v = s

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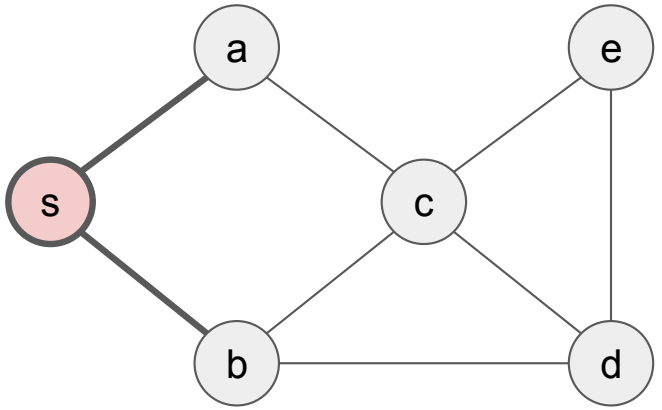

Explored:
s

S

v = s

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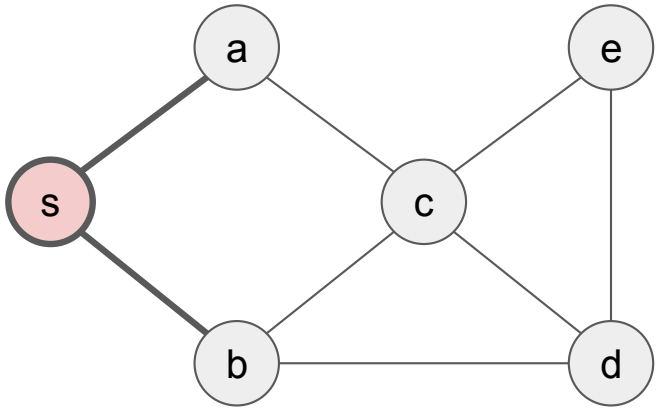

S

Explored:
s

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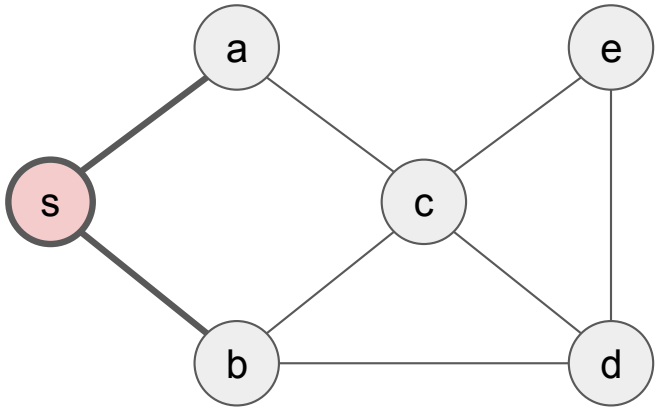

S

Explored:
s

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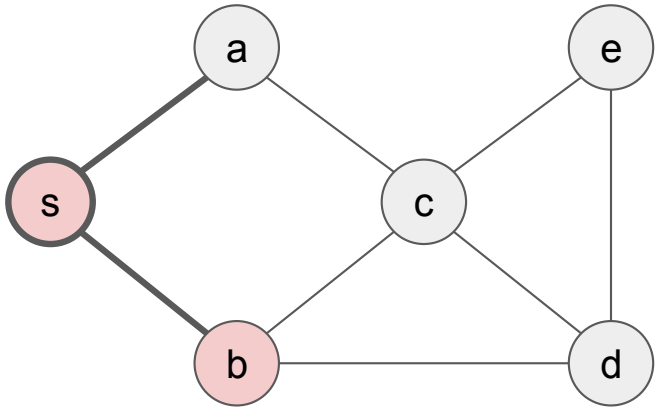

S

Explored:
s, b

v = b

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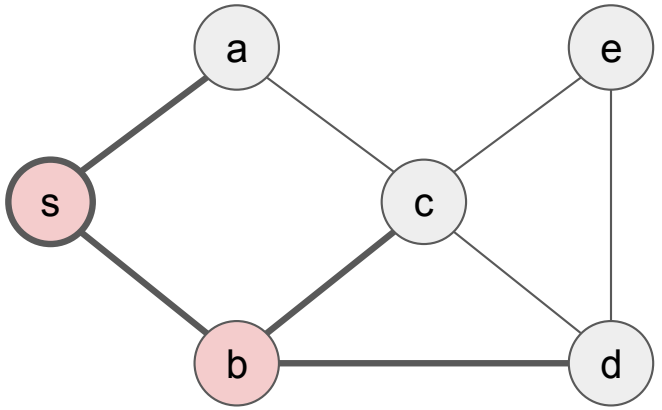

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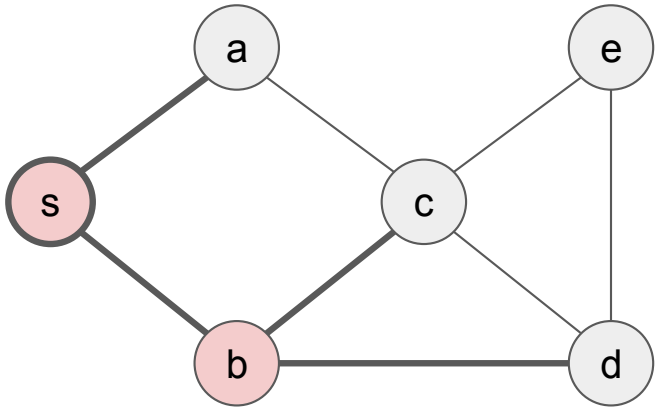

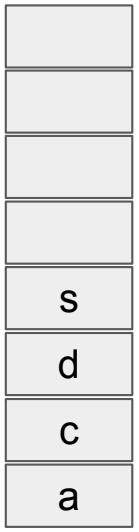

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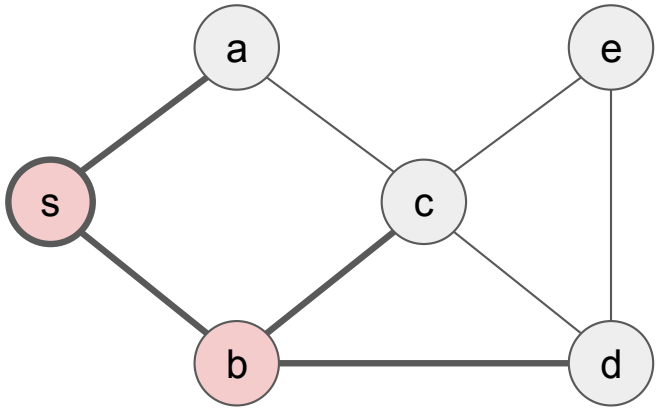

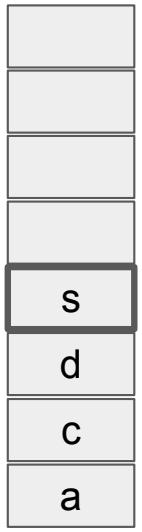

S

Explored:
s, b

v = b

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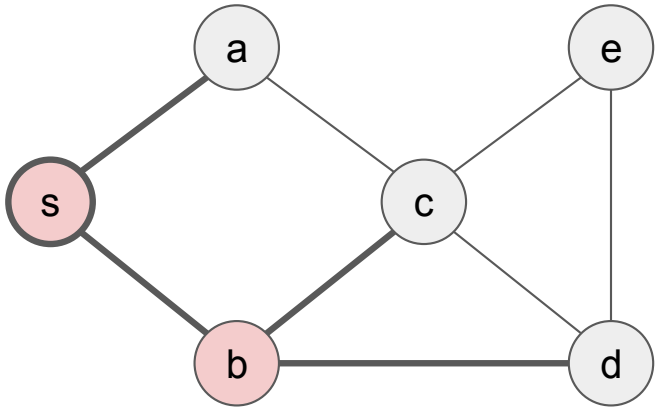
s was already explored



S

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s, b

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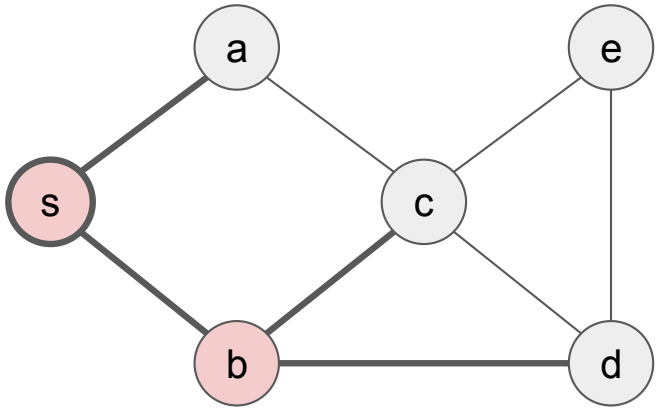

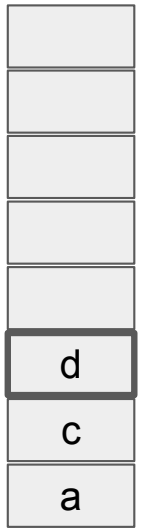

S

Explored:
s, b

v = s

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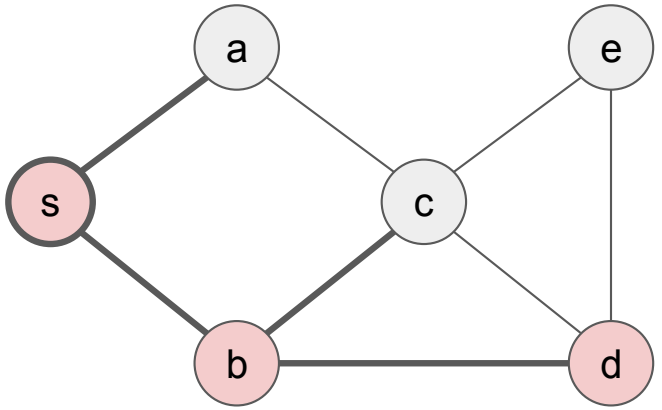

S

Explored:
s, b, d

v = d

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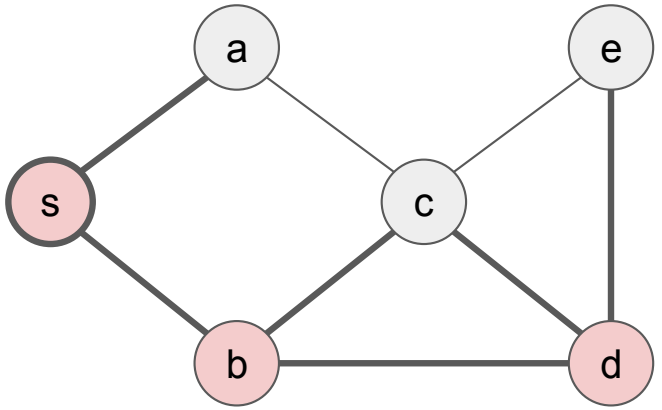

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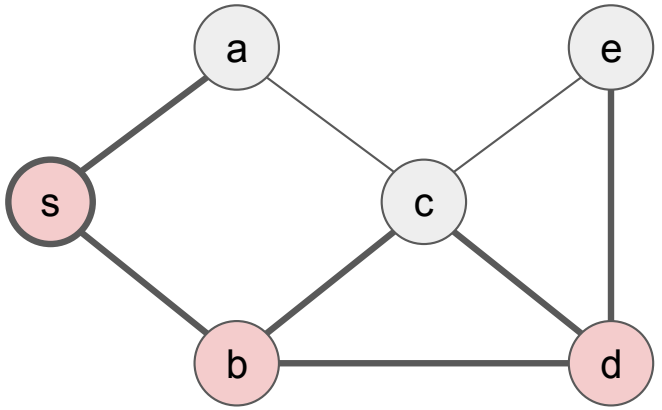

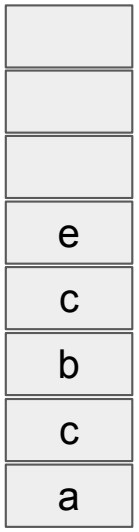

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s, b, d

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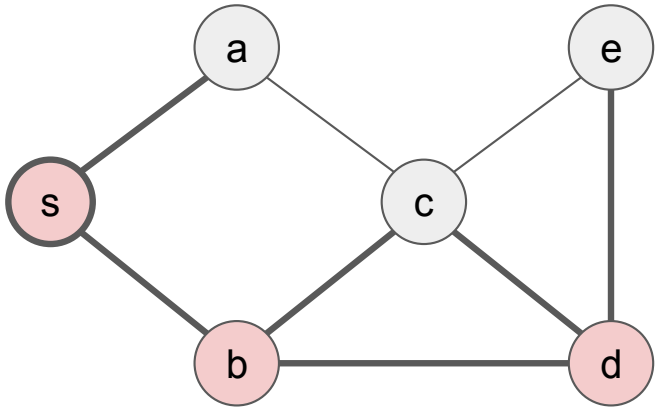

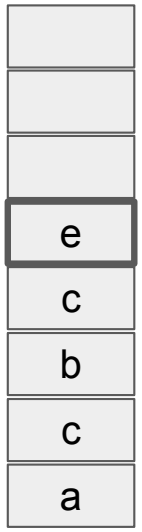

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Explored:
s, b, d

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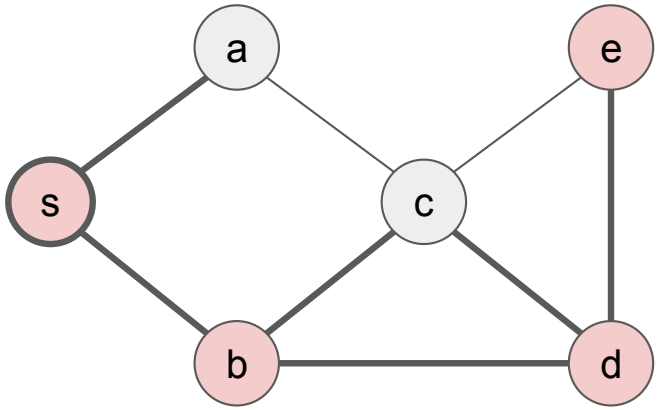

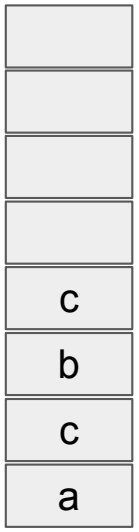

S

Explored:
s, b, d, e

v = e

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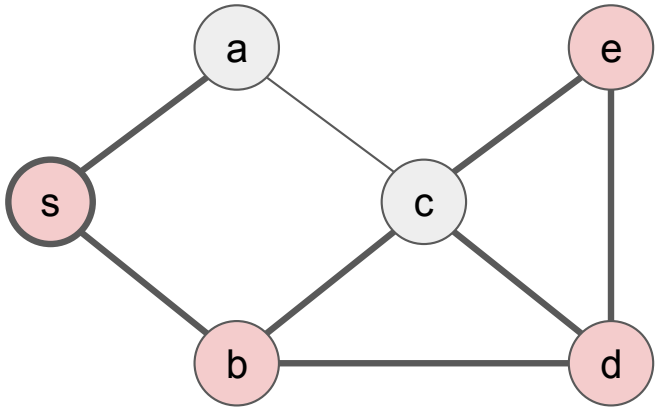

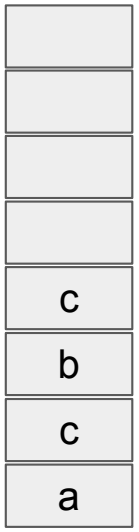

S

Explored:
s, b, d, e

v = e

Depth-first search (DFS)

DFS (Iterative Version)

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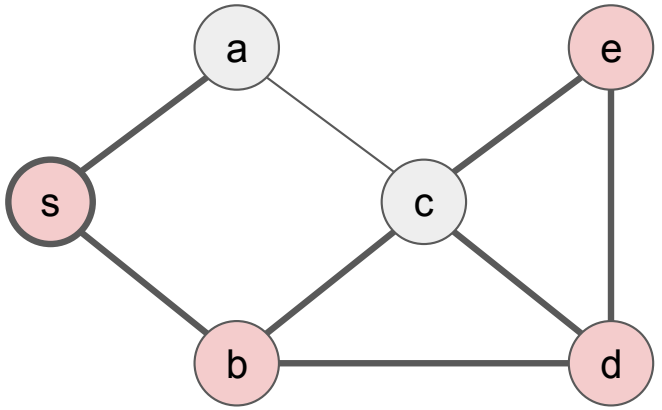

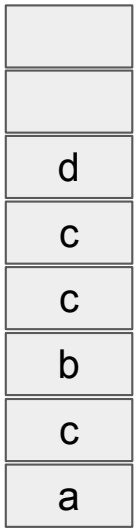

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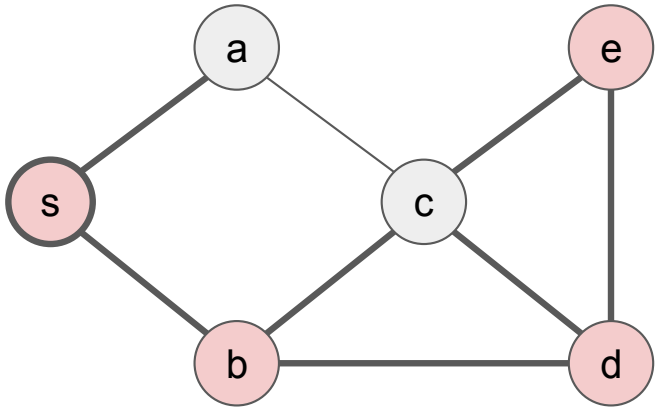

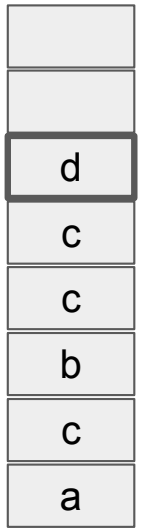

S

Explored:
s, b, d, e

v = e

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DFS (Iterative Version)

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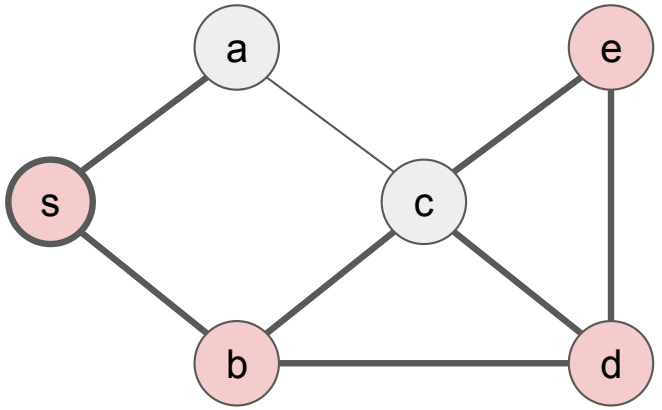
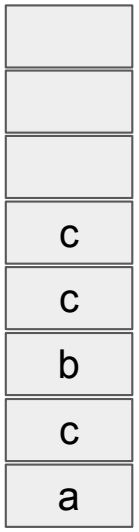
d was already explored





S

Explored:
s, b, d, e

v = d

Depth-first search (DFS)

DFS (Iterative Version)

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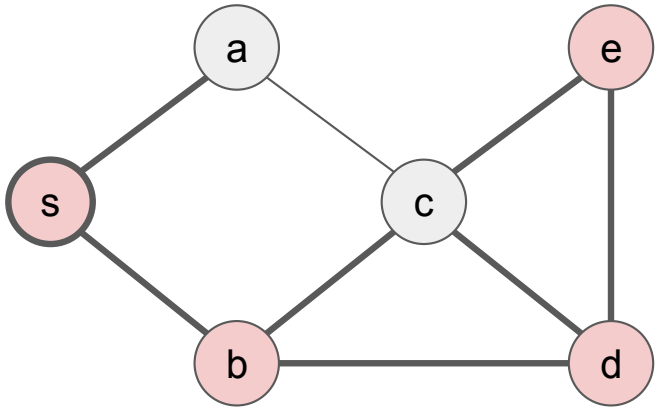

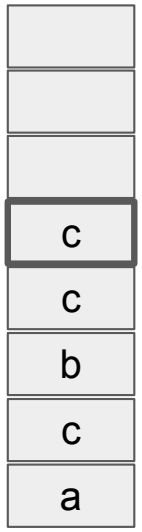

S

Explored:
s, b, d, e

v = d

Depth-first search (DFS)

DFS (Iterative Version)

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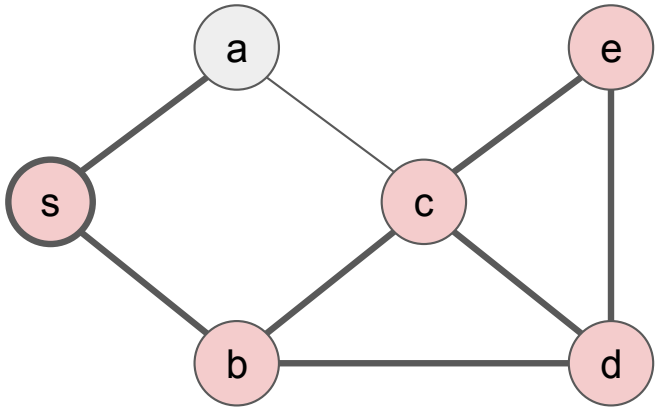

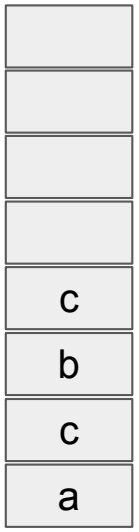

S

Explored:
s, b, d, e,
c

v = c

Depth-first search (DFS)

DFS (Iterative Version)

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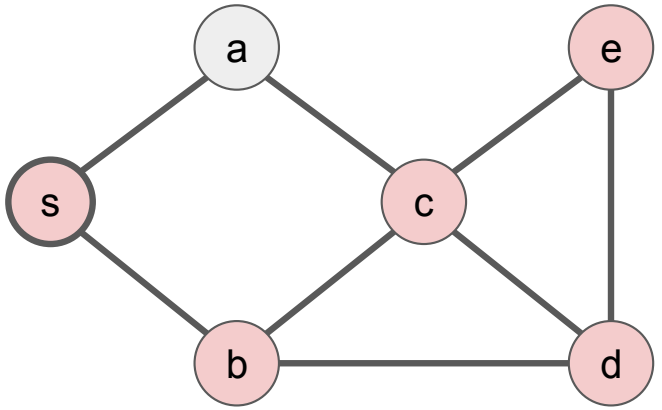

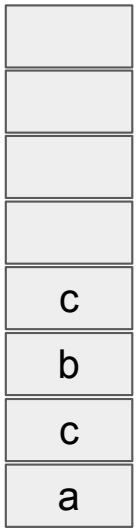

S

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s, b, d, e,
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v = c

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DFS (Iterative Version)

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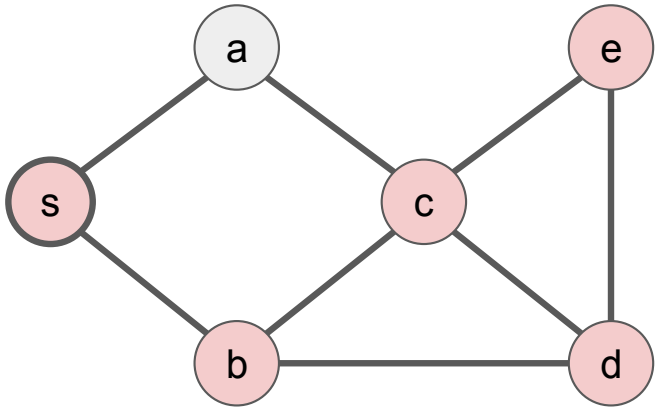

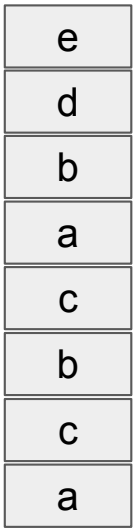

S

Explored:
s, b, d, e, c

v = c

Depth-first search (DFS)

DFS (Iterative Version)

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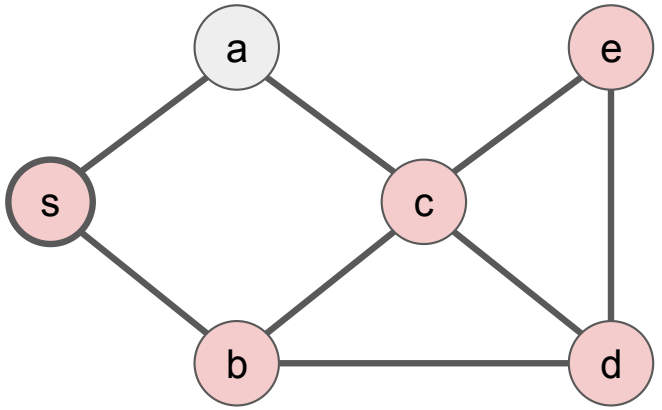

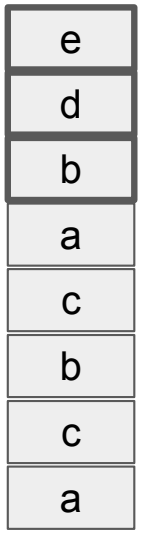

S

Explored:
s, b, d, e,
c

v = c

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DFS (Iterative Version)

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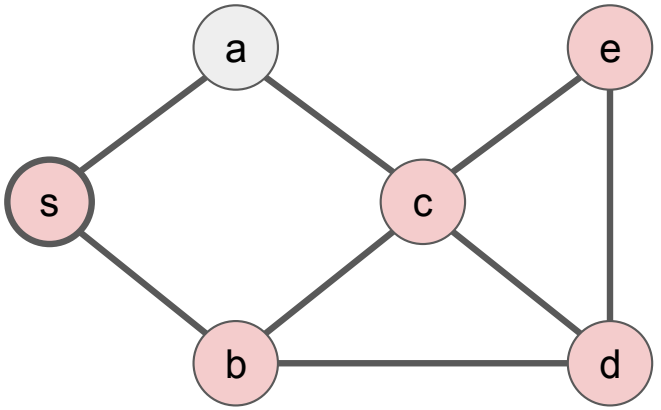
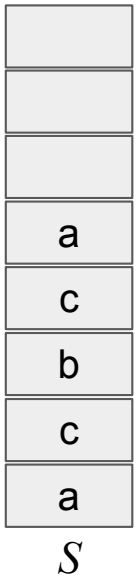
e, d, b were already explored





S

Explored:
s, b, d, e,
c

$v =$
e…d…b

Depth-first search (DFS)

DFS (Iterative Version)

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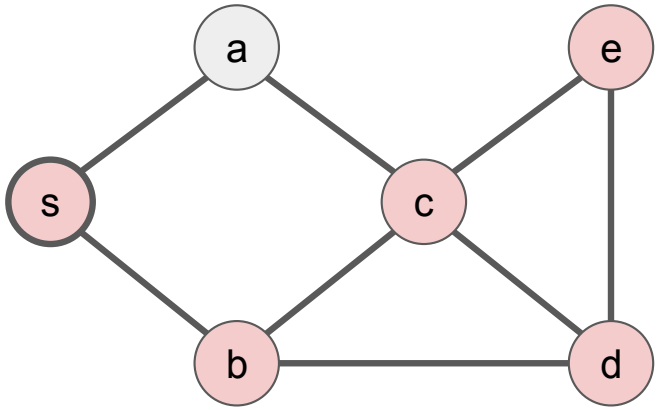

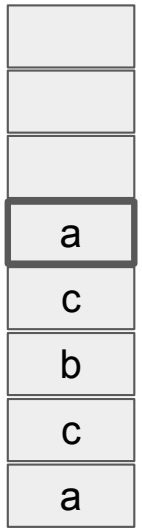

S

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s, b, d, e,
c

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e…d…b

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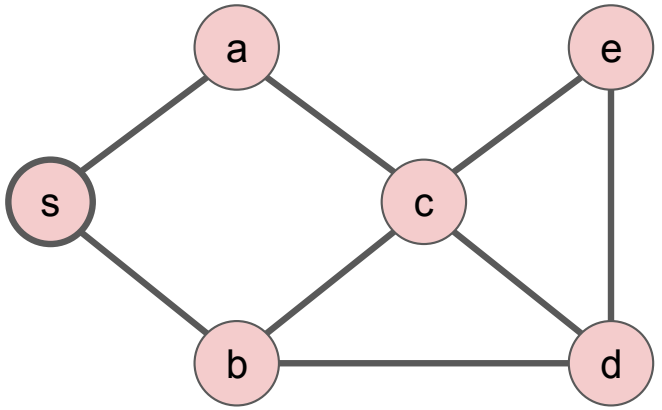

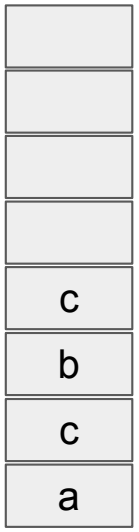

S

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s, b, d, e,
c, a

v = a

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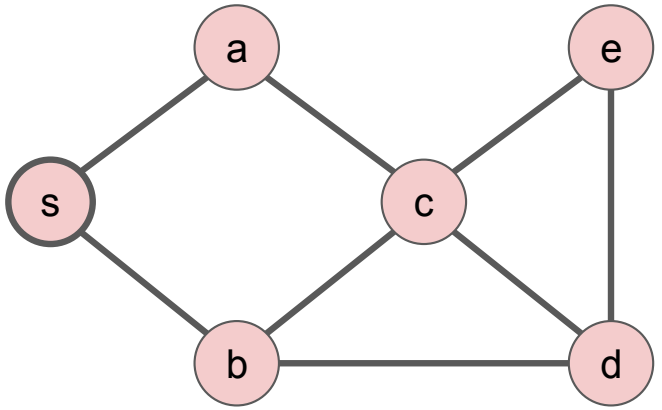

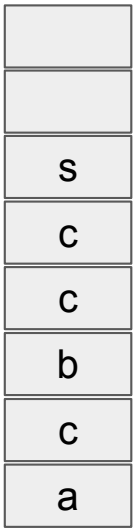

S

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v = a

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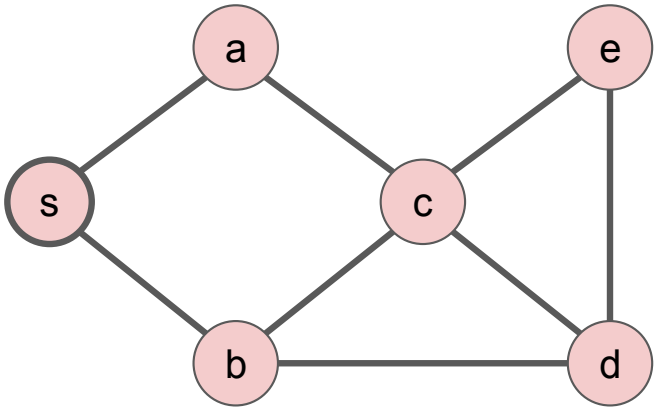

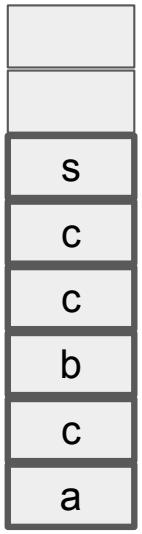

S

Explored:
s, b, d, e,
c, a

v = each
at a time

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DFS (Iterative Version)

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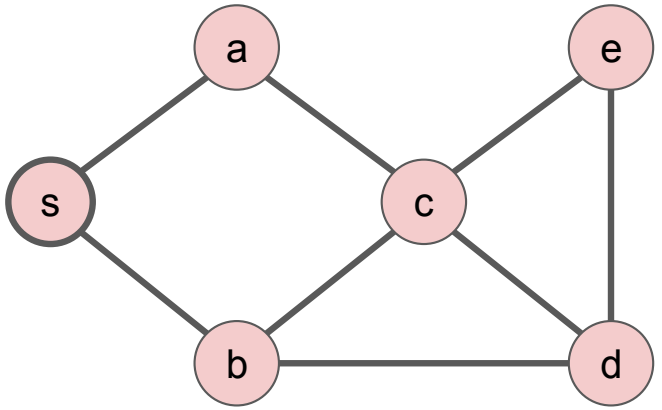

S

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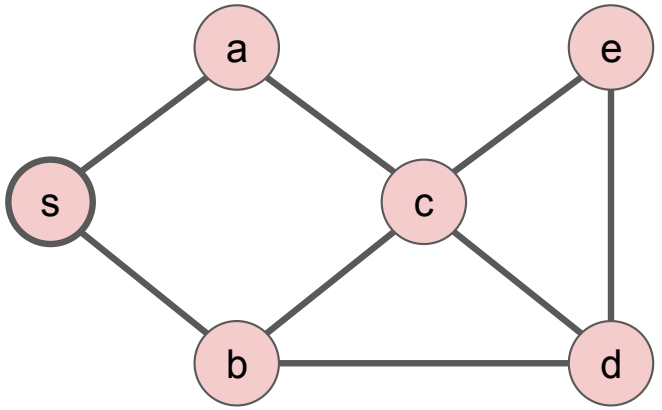

Time complexity?

Depth-first search (DFS)

DFS (Iterative Version)

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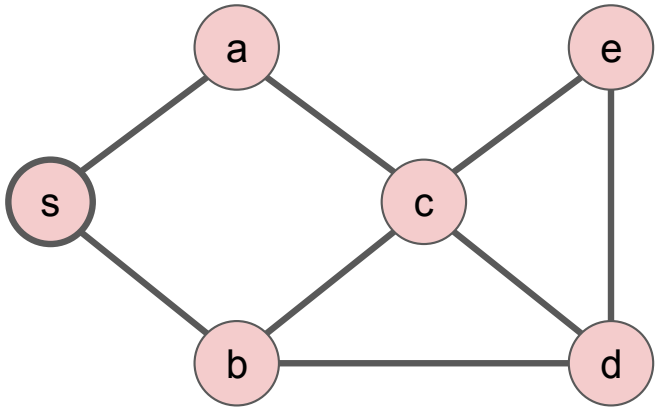

Time complexity
O(V + E)

Depth-first search (DFS) without the stack?
How do you think it is possible?

Recursive depth-first search (DFS)

DFS (Recursive Version)

Input: graph $G = (V, E)$ in adjacency-list representation, and a vertex $s \in V$.
Postcondition: a vertex is reachable from s if and only if it is marked as "explored."

```
// all vertices unexplored before outer call
```
mark s as explored
for each edge (s, v) in s's adjacency list **do**
    if v is unexplored **then**
        DFS (G, v)