Assignment 2: Multi-View Geometry

Comment: most of the written problems are designed to help with the coding part (structure-from-motion). Thus, they should be solved first. Your coding part will require figuring out how to work with numpy arrays (e.g. slicing, broadcasting). If you are new to numpy, this could be tricky. I can recommend working on small test cases. For debugging, print arrays before and after slicing (etc.) to verify that the result is correct.

Problem 1

Prove that epipoles in two images obtained at different view points correspond to left and right null vectors of the essential (or fundamental) matrix.

Solution: Note that the fundamental matrix can be stated in the alternative form x^TFx', where x and x' are 'corresponding'(x in image L and x' on the epipolar line of x in image R, or vice-versa) points. \Since every epipolar lines in L and R contains the epipoles e and e', respectively, letting $x = e$ in our last equation yields:

$$e^TFx' = (e^TF)x' = 0 \implies (e^TF) = 0 \forall x' \in imgR$$

Since for any point $0x' \in imgR$, the respective point on an epipolar line in image L will include e, as stated above. This implies that e is in the left nullspace of F, as required; a similar proof follows for e' by substituting it for x'. QED.

Problem 2

Assuming a $calibrated$ camera (that is, $K = I$) and its two views corresponding to projection matrices $P_1 = [I|0]$ and $P_2 = [R|T]$ w.r.t. some world coordinate system, show formulas for coordinates of the following 3D points (in the same world coordinate system):

Assumption: All image coordinates have their origins at the bottom left. F = focal length/projection plane z coordinate

(a) optical center for the first view: $C_1 = (0, 0, 0)$

(b) image center for the first view: $Q_1 = (0, 0, 1)$

(c) optical center for the second view: $C_2 = (T + C_1)$

(d) image center for the second view: $Q_2 = (C_2 x, C_2 y, C_2 z + 1)$

Problem 3

Using the same set up as in problem 2, show formulas for normalized coordinates of the following image points:

The epipoles can be expressed as the projections of the cameras onto the opposing image planes. We assume that the pixels are square without skew:

(a) epipole in the first camera image: $e_1=

$$\begin{bmatrix} wx \\ wy \\ wz \end{bmatrix}$$

= C_2

$$\begin{bmatrix} F & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$

(b) epipole in the second camera image: $e_2=

$$\begin{bmatrix} wx \\ wy \\ wz \end{bmatrix}$$

= R*(T+C_1

$$\begin{bmatrix} F & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

)$

Problem 4 (homogeneous and non-homogeneous line representations)

Lines in 2D images can be represented "homogeneously" as column 3-vectors $l = [l_1, l_2, l_3]^T$ that give equation $l^T x = 0$ for homogeneous points $x = [x_1, x_2, x_3]^T$ forming a line. Given l, what are the values of scalar parameters a, b in the line equation $u = av + b$ for the same 2D points based on their regular (nonhomogeneous) representation $(u, v) = (\frac{x_1}{x_3}, \frac{x_2}{x_3})$?

Expanding $l^T x$:

$$l_1 x_1 + l_2 x_2 + l_3 x_3 = 0$$

Divide by x_3 to get non-homogeneous coordinates:

$$\frac{l_1}{x_3} x_1 + \frac{l_2}{x_3} x_2 + l_3 = 0$$

$$x_2 = -\frac{l_1 x_3}{l_2 x_3} x_1 - \frac{l_3 x_3}{l_2} = -\frac{l_1}{l_2} x_1 - \frac{l_3 x_3}{l_2}$$

Hence

$a = -\frac{l_1}{l_2}$

$b = -\frac{l_3 x_3}{l_2}$

Problem 5 (epipolar lines in normalized and non-normalized images)

Given a matrix of intrinsic camera parameters K and essential matrix E between two views (A) and (B) such that $x_A^T E x_B = 0$ for any corresponding points, write expressions for the following:

(a) given homogeneous normalized point x^n_B in image B, specify 3-vector l^n_A describing the corresponding epipolar line of normalized points in image A:

$l^n_A = Ex^n_b$

(b) given homogeneous normalized point x^n_A in image A, specify 3-vector l^n_B describing the corresponding epipolar line of normalized points in image B:

$l^n_B = x^{nT}_A E$

(c) assuming line (3-vector) l^n of normalized image points, what is a 3-vector representation l for the line formed by the corresponding points on the real (unnormalized) camera image:

$l = kl^n$

Problem 6 (least squares for triangulation)

Describe your approach to triangulating two matched feature points $x_a = [u_a, v_a, 1]^T$ and $x_b = [u_b, v_b, 1]^T$ in two views with given projection matrices P_a and P_b. You should find 3D point $X = [X_1, X_2, X_3, 1]^T$ and two scalars w_a, w_b such that $P_a X \approx w_a x_a$ and $P_b X \approx w_b x_b$. Be specific as you will need this for your programming part below. Use notation like $M[i]$ to denote the i-th row vector of matrix M.

Hint: you can use approach 1 described for homograpy estimation in topic 6. In particualr, you can formulate the problem as $AX \approx 0$, define elements of $4x4$ matrix A, convert the problem to an overdetermined system of 4 linear equations $A_{1:3}[X_1, X_2, X_3]^T \approx -A_4$, and specify its solution minimizing the sum of squared errors.

Solution: Consider the two image points in images A and B, respectively:

$$w_a x_a = P_a X, w_b x_b = P_b X$$

Expanding yields 6 equations:

$$w_a u_a = P_a(1,1)X_1 + P_a(1,2)X_2 + P_a(1,3)X_3, w_a v_a = P_a(2,1)X_1 + P_a(2,2)X_2 + P_a(2,3)X_3 w_a = + P_a(3,3)X_3$$

The 3 equations for image B follow the same fashion. We convert to non-homogenous coordinates by dividing by w_a and w_b's values w.r.t the projection matrices and our real point X, leaving 4 equations remaining (the other 2 equations for each of image A and image B divided by w_a and w_b respectively), with 3 unknowns remaining (X_1, X_2 and X_3). We then apply least-squares with the 3 equations to find the minimum squared error of the linear equations, solving for our values X_1, X_2 and X_3, which can then be used to solve for w_a and w_b using the equations defined above.

Probelm 7 (the programming part)

Structure from Motion

NOTE: Steps 0-3 and 10 are given, other steps needs to be implemented.

Step 0: Loading two camera views and camera's intrinsic matrix K

In [1]:
```
%matplotlib notebook

import numpy as np
import numpy.linalg as la
import matplotlib
import matplotlib.image as image
import matplotlib.pyplot as plt
from skimage.feature import (corner_harris, corner_peaks, plot_matches, BRIEF,
 match_descriptors)
from skimage.transform import warp, ProjectiveTransform, EssentialMatrixTransf
orm, FundamentalMatrixTransform
from skimage.color import rgb2gray
from skimage.measure import ransac

# Indicate (E) inlier matches in image 1 and image 2
# loading two images (two camera views) and the corresponding matrix K (intrin
sic parameters)
imL = image.imread("images/kronan1.jpg")
imR = image.imread("images/kronan2.jpg")
imLgray = rgb2gray(imL)
imRgray = rgb2gray(imR)

K = 1.0e+03 * np.array([[2.3940, -0.0000,    0.9324],
                        [     0,  2.3981,    0.6283],
                        [     0,       0,    0.0010]])


plt.figure(0,figsize = (10, 4))
ax81 = plt.subplot(121)
plt.imshow(imL)
ax82 = plt.subplot(122)
plt.imshow(imR)
plt.show()
```

Step 1: Feature detection (e.g. corners)

In [2]:
```
# NOTE: corner_peaks and many other feature extraction functions return point
 coordinates as (y,x), that is (rows,cols)
keypointsL = corner_peaks(corner_harris(imLgray), threshold_rel=0.001, min_dis
tance=15)
keypointsR = corner_peaks(corner_harris(imRgray), threshold_rel=0.001, min_dis
tance=15)


print 'the number of features in images 1 and 2 are {:5d} and {:5d}'.format(ke
ypointsL.shape[0],keypointsR.shape[0])

fig = plt.figure(1,figsize = (10, 4))
axA = plt.subplot(111)
plt.gray()
matchesLR = np.empty((0,2))
plot_matches(axA, imL, imR, keypointsL, keypointsR, matchesLR)
axA.axis('off')

plt.show()
```

```
the number of features in images 1 and 2 are  1576 and  1661
```

Step 2: Feature matching (e.g. BRIEF descriptor, a variant of SURF, SIFT, etc)

In [3]:
```
extractor = BRIEF()

extractor.extract(imLgray, keypointsL)
keypointsL = keypointsL[extractor.mask]
descriptorsL = extractor.descriptors

extractor.extract(imRgray, keypointsR)
keypointsR = keypointsR[extractor.mask]
descriptorsR = extractor.descriptors

matchesLR = match_descriptors(descriptorsL, descriptorsR, cross_check=True)

print 'the number of matches is {:2d}'.format(matchesLR.shape[0])

fig = plt.figure(2,figsize = (10, 4))
axA = plt.subplot(111)
axA.set_title("matches")
plt.gray()
plot_matches(axA, imL, imR, keypointsL, keypointsR, matchesLR) #, matches_colo
r = 'r')
axA.axis('off')

plt.show()
```

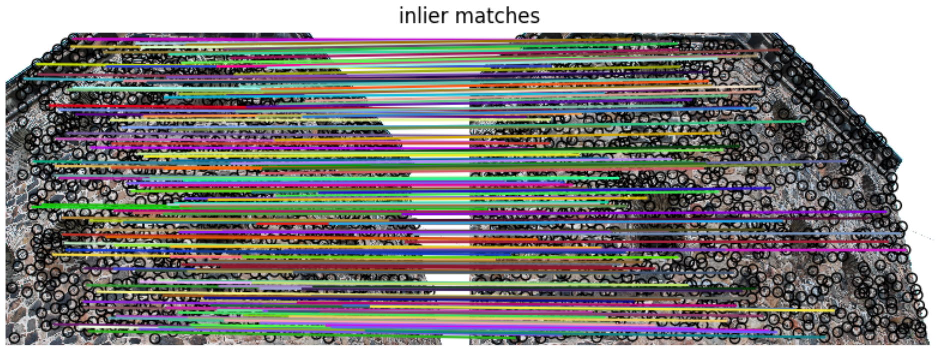
```
C:\Users\Logan\Anaconda2\lib\site-packages\skimage\feature\match.py:49: Futur
eWarning: Conversion of the second argument of issubdtype from `bool` to `np.
generic` is deprecated. In future, it will be treated as `np.bool_ == np.dtyp
e(bool).type`.
  if np.issubdtype(descriptors1.dtype, np.bool):
```

```
the number of matches is 982
```

Step 3: Fundamental Matrix estimation using RANSAC

In [8]:
```
ptsL1 = []
ptsR1 = []
for i in matchesLR:
    ptsL1.append(keypointsL[i[0]])
    ptsR1.append(keypointsR[i[1]])
ptsL1 = np.array(ptsL1)
ptsR1 = np.array(ptsR1)

# swapping columns using advanced indexing https://docs.scipy.org/doc/numpy/re
ference/arrays.indexing.html#advanced-indexing
# This changes point coordinates from (y,x) in ptsL1/ptsR1 to (x,y) in ptsL/pt
sR
ptsL = ptsL1[:,[1, 0]]
ptsR = ptsR1[:,[1, 0]]

# robustly estimate fundamental matrix using RANSAC
F_trans, F_inliers = ransac((ptsL, ptsR), FundamentalMatrixTransform, min_samp
les=8, residual_threshold=0.1, max_trials=1500)
print 'the number of inliers is {:2d}'.format(np.sum(F_inliers))
#print('ptsL: ' + str(ptsL))
ind = np.ogrid[:ptsL.shape[0]]
FmatchesRansac = np.column_stack((ind[F_inliers],ind[F_inliers]))
#print('Fmatches Ransac:' + str(FmatchesRansac))
fig = plt.figure(3,figsize = (10, 4))
axA = plt.subplot(111)
axA.set_title("inlier matches")
plt.gray()
# NOTE: function "plot matches" expects that keypoint coordinates are given as
 (y,x), that is (row, col)
plot_matches(axA, imL, imR, ptsL1, ptsR1, FmatchesRansac) #, matches_color =
 'r')
axA.axis('off')
plt.show()
```

```
the number of inliers is 182
```



singular values for F

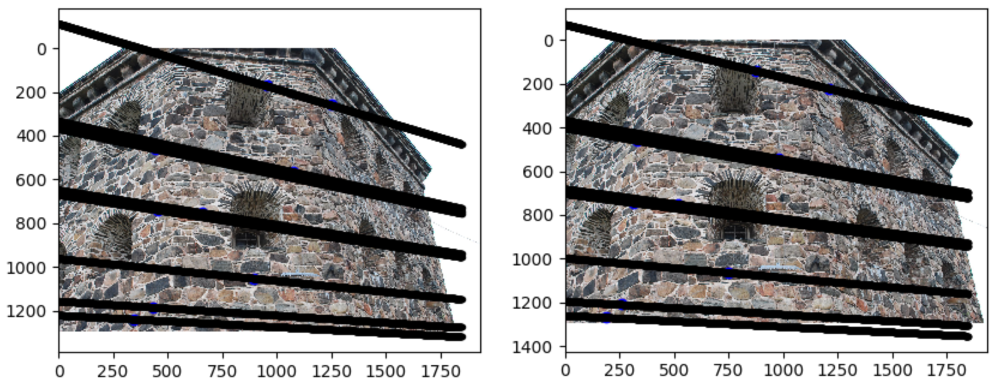
In [5]:
```
F = F_trans.params
Uf,Sf,Vf = la.svd(F, full_matrices=False)
print Sf
```

```
[8.53071480e-02 5.86098126e-05 8.22906640e-20]
```

Step 4: Epipolar lines from F

In [9]:
```
def plotLines(x_start, x_end, lines, graph):
    x = np.arange(x_start,x_end)
    # generate the "true" line points
    for line in lines:
        a = -line[0]/line[1]
        #Formula includes x_3 homogeneous term, but it is excluded in this cas
e, since it is 1.
        b = -line[2]/line[1]
        y = a * x + b
        data = np.column_stack([x, y])    # staking data points into (Nx2) arr
ay
        graph.plot(data[:,0], data[:,1], '.k')

# Randomly select 10 matches (paris of features in two images) from the set of
 inliers for F
ind_sample = np.random.choice(ind[F_inliers], 10, replace = False)

# Indicate these matching features in image 1 and image 2
plt.figure(4,figsize = (10, 4))
ax41 = plt.subplot(121)
plt.imshow(imL)
plt.plot(ptsL[ind_sample, 0], ptsL[ind_sample, 1], 'ob')
ax42 = plt.subplot(122)
plt.imshow(imR)
plt.plot(ptsR[ind_sample, 0], ptsR[ind_sample, 1], 'ob')

# generate epipolar line equations in image 2 (homogeneous 3-vectors l2 repres
enting lines l2 x  = 0)
# a. create an array of points sampled in images 1 and 2
imLSample = ptsL[ind_sample, :]
imRSample = ptsR[ind_sample, :]
transF = np.transpose(F)

#print('L sample: ' + str(imLSample) + ' imR sample: ' + str(imRSample))
# b. create an array of homogeneous points sampled in images 1 and 2
lSampleHomog = np.zeros([imLSample.shape[0],imLSample.shape[1]+1])
rSampleHomog = np.zeros([imRSample.shape[0],imRSample.shape[1] +1])
epiLinesR = np.zeros(lSampleHomog.shape)
epiLinesL = np.zeros(rSampleHomog.shape)
for i in range(imLSample.shape[0]):
    lSampleHomog[i] = np.array([imLSample[i][0], imLSample[i][1], 1])
    rSampleHomog[i] = np.array([imRSample[i][0], imRSample[i][1], 1])
    epiLinesR[i] = F.dot(lSampleHomog[i]);
    epiLinesL[i] = transF.dot(rSampleHomog[i]);
#print('R epipole lines: ' + str(epiLinesR) + ' imL epipole lines: ' + str(epi
LinesL))
# c. create an array of the corresponding epipolar lines in images 1 and 2
x_start = 0
x_end = 1850

plotLines(x_start, x_end, epiLinesL, ax41)
plotLines(x_start, x_end, epiLinesR, ax42)


# for each feature (in both images) draw a correspoindiung epipolar line in th
e other image
```

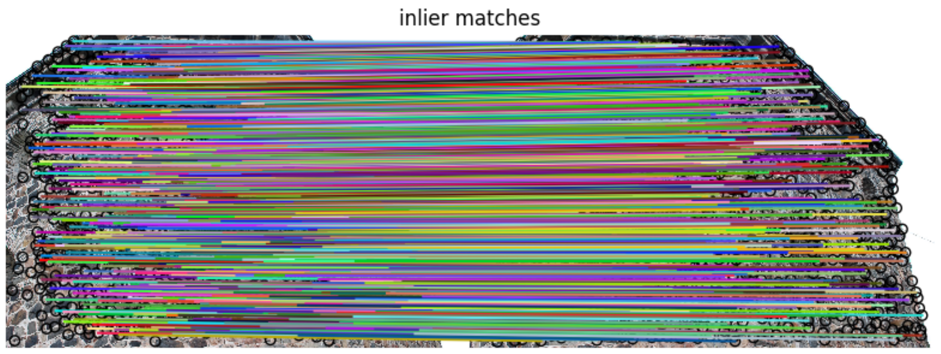
```
# see Assignment 1 (line fitting part 1) for inspiration on how to visualize l
ines
# use ax41.plot and ax42.plot


plt.show()
```



Step 5: Camera Normalization and Essential Matrix estimation using RANSAC

In [12]:
```
# normalization of points in two images using K (intrinsic parameters) e.g. in
 the following three steps
# a. convert original points to homogeneous 3-vectors (append "1" as a 3rd coo
rdinate using np.append function)
# b. transform the point by applying the inverse of K
# c. convert homogeneous 3-vectors to 2-vectors (in R2)
n_ptsL = np.zeros([ptsL.shape[0],2])
n_ptsR = np.zeros([ptsR.shape[0],2])
print('')
print('Shape : ' + str(ptsL.shape[0]))
for i in range(ptsL.shape[0]):
    temp = la.inv(K).dot(np.transpose(np.append(ptsL[i], [1])))
    n_ptsL[i] = np.array([temp[0]/temp[2], temp[1]/temp[2]])
for i in range(ptsR.shape[0]):
    temp = la.inv(K).dot(np.append(ptsR[i], [1]))
    n_ptsR[i] = np.array([temp[0]/temp[2], temp[1]/temp[2]])
#n_ptsL = n_ptsL
# robustly estimate essential matrix using normalized points and RANSAC
E_trans, E_inliers = ransac((n_ptsL, n_ptsR), EssentialMatrixTransform, min_sa
mples=8, residual_threshold=0.0005, max_trials=5000)
num_inliers = np.sum(E_inliers)
print 'the number of inliers is {:2d}'.format(num_inliers)

ind = np.ogrid[:n_ptsL.shape[0]]
EmatchesRansac = np.column_stack((ind[E_inliers],ind[E_inliers]))

fig = plt.figure(5,figsize = (10, 4))
axA = plt.subplot(111)
axA.set_title("inlier matches")
plt.gray()
# NOTE: function "plot matches" expects that keypoint coordinates are given as
 (y,x), that is (row, col)
plot_matches(axA, imL, imR, ptsL1, ptsR1, EmatchesRansac) #, matches_color =
 'r')
axA.axis('off')
plt.show()
```

```
Shape : 982
the number of inliers is 825
```



singular values for E

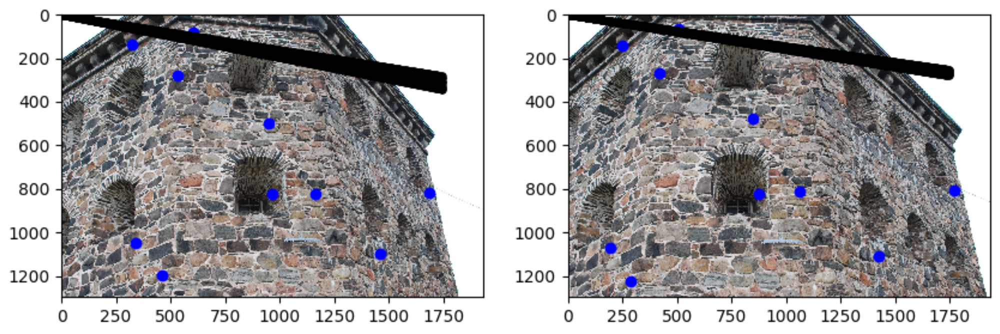
Hint: function svd from $linalg$ returns transpose V^T, not V.

In [14]:
```
E = E_trans.params
Ue,Se,Ve = la.svd(E)
print Se
```

```
[4.67105816e+00 4.56819524e+00 3.38182530e-16]
```

Step 6: Epipolar Lines from E

In [15]:
```
# Randomly select 10 matches (paris of features in two images) from the set of
 inliers for E
ind_sample = np.random.choice(ind[E_inliers], 10, replace = False)

# Indicate these matching features in image 1 and image 2
plt.figure(6,figsize = (10, 4))
ax61 = plt.subplot(121)
plt.imshow(imL)
plt.plot(ptsL[ind_sample, 0], ptsL[ind_sample, 1], 'ob')
ax62 = plt.subplot(122)
plt.imshow(imR)
plt.plot(ptsR[ind_sample, 0], ptsR[ind_sample, 1], 'ob')

# generate epipolar line equations in image 2 (homoheneous 3-vectors l2 repres
enting lines l2 x  = 0)
# a. create an array of normalized points sampled in image 1
# b. create an array of homogeneous normalized points sampled in image 1
# c. create an array of the corresponding (uncalibrated) epipolar lines in ima
ge 2
n_ptsL_sample = n_ptsL[ind_sample,:]
n_ptsR_sample = n_ptsR[ind_sample,:]

transE = np.transpose(E)

n_ptsL_sampleHomog = np.zeros([imLSample.shape[0],imLSample.shape[1]+1])
n_ptsR_sampleHomog = np.zeros([imRSample.shape[0],imRSample.shape[1] +1])
n_epiLinesR = np.zeros(n_ptsL_sampleHomog.shape)
n_epiLinesL = np.zeros(n_ptsR_sampleHomog.shape)
for i in range(imLSample.shape[0]):
    n_ptsL_sampleHomog[i] = np.array([n_ptsL_sample[i][0], n_ptsL_sample[i][1
], 1])
    n_ptsR_sampleHomog[i] = np.array([n_ptsR_sample[i][0], n_ptsR_sample[i][1
], 1])
    n_epiLinesR[i] = K.dot(E.dot(n_ptsL_sampleHomog[i]));
    n_epiLinesL[i] = K.dot(transE.dot(n_ptsR_sampleHomog[i]));
print('epiLines L: ' + str(n_epiLinesL) + ' epiLines R: ' + str(n_epiLinesR))
x_start = 0
x_end = 1750

plotLines(x_start, x_end, n_epiLinesL, ax61)
plotLines(x_start, x_end, n_epiLinesR, ax62)

# for each feature (in both images) draw a correspoindiung epipolar line in th
e other image
# use ax61.plot and ax62.plot

plt.show()
```



```
epiLines L: [[ 1.96432211e+03 -1.05213729e+04 -2.33925462e-01]
 [ 1.50500504e+03 -9.00154932e+03  6.76564969e-01]
 [ 1.70644773e+03 -1.06363409e+04  1.15839243e+00]
 [ 1.77816329e+03 -1.01285173e+04  3.38141024e-01]
 [ 1.58167601e+03 -8.77465376e+03  8.72250655e-02]
 [ 2.25505357e+03 -1.14490203e+04 -8.41491000e-01]
 [ 1.74224863e+03 -9.85811856e+03  2.71406247e-01]
 [ 2.17193250e+03 -1.13899260e+04 -4.80205371e-01]
 [ 2.27629439e+03 -1.17721959e+04 -6.53453387e-01]
 [ 1.80711220e+03 -1.09294261e+04  9.22445682e-01]] epiLines R: [[-1.45353217
e+03  9.79066068e+03  2.30249696e-01]
 [-1.27692151e+03  8.11844603e+03 -6.64763760e-01]
 [-1.58540978e+03  9.88535052e+03 -1.17463549e+00]
 [-1.43843525e+03  9.37179211e+03 -3.41977891e-01]
 [-1.21445486e+03  8.02165566e+03 -9.26212252e-02]
 [-1.57175281e+03  1.09255038e+04  8.57181781e-01]
 [-1.37779744e+03  9.00711218e+03 -2.72960635e-01]
 [-1.58993907e+03  1.08403971e+04  4.87084266e-01]
 [-1.65846601e+03  1.14009663e+04  6.75777658e-01]
 [-1.62670320e+03  1.02868214e+04 -9.46526662e-01]]
```

Step 7: Camera rotation and translation (four solutions)

Factorize essential matrix $E = [T]_x R$ where R is rotation and T is a translation. Find solutions R_1, R_2 and T_1, T_2. Use camera 1 for world coordinates. Define projection matrix for camera 1 as $P_w = [I|0]$ and compute four projection matrices for the second camera P_a, P_b, P_c, P_d.

Hint 1: for array multiplication use dot or $matmul$, never $*$.

Hint 2: function svd from $linalg$ returns V^T rather than V (the 2nd orthogonal matrix in svd decomposition $E = USV^T$).

Warning: remember that python uses 0 as a starting index for the rows or columns in arrays. For example, $A[0]$ denotes the first row of matrix A, while $P_w[2]$ stands for the 3rd row of the corresponding projection matrix and $E[:, [1]]$ is the second column of the essential matrix.

In [18]:
```
print('E test (should be 3x3 matrix): ' + str(E))
q, r = la.qr(E)
print('Q: ' + str(q) + ' R: ' + str(r))

#Convert the cross-product matrix form to the actual translation
T1 = np.array([q[2][1], q[0][2], q[1][0]])
R1 = r

iden = np.identity(3)
Pw = np.zeros([3,4])

Pw[0] = np.append(iden[0], [0])
Pw[1] = np.append(iden[1], [0])
Pw[2] = np.append(iden[2], [0])
#print('Pw '  + str(Pw))

Pa = T1.dot(R1)
```

```
E test (should be 3x3 matrix): [[ 0.00691949  1.69061489 -0.60572354]
 [-2.08987748 -0.06019168  4.03272171]
 [ 0.78176847 -4.26811697 -0.00497152]]
Q: [[-0.00310107 -0.38863907  0.92138486]
 [ 0.93660956  0.32169173  0.13884151]
 [-0.35036112  0.86340842  0.36300548]] R: [[-2.23132197e+00  1.43376344e+00
3.78070592e+00]
 [ 0.00000000e+00 -4.36153031e+00  1.52840860e+00]
 [ 0.00000000e+00  0.00000000e+00  4.44089210e-16]]
```

Step 8: Triangulation (four solutions).

Implement homogeneous least square solver (you can use svd function) and use it for each matching pair of features in left and right images to find the corresponding 3D point. Make sure to use normalized coordinates for image points. Show four solutions corresponding to cameras P_a, P_b, P_c, P_d. Specify, which solution has 3D points in front of both cameras.

Project 3D points onto each camera, convert to uncalibrated coordinates and display these projected points (use red) together with the original features (use blue). Observe if the are red and blue points are close in each image.

In [20]:
```
# Select normalized coordinates for matched features that are inliers for esse
ntial matrix E.
# Form matrix A in equation AX=0 where X represent 4 vectors (homogeneous repr
esentation of 3D point).
# Use your solution for Problem 6.
# Each camera (projection matrix P) will define its own A

# HINT: to keep it simple, first solve the problem for one match.
#Normalized features
normIn1 = n_ptsL[ind[F_inliers],:]
normIn2 = n_ptsR[ind[F_inliers],:]
#Linear equations for least-squares
#Equation should be equal to projection P, since w coordinates are 1
Aa = Pa

#Ab =

#Ac =

#Ad =
```

Solution using least squares: assume homogeneous 3D point $X = [X_1, X_2, X_3, 1]$. Then, $AX = 0$ gives 4 equations for 3 unknowns. Use approach 1 (inhomogeneous least squares) discussed for homography estimation (Topic 6).

In [ ]:
```
# least squares for solving linear system A_{0:2} X_{0:2} = - A_3
Aa_02 =        # the first 3 columns of 3x4 matrix A
Aa_3  =        # the last column on 3x4 matrix A
Ab_02 =
Ab_3  =
Ac_02 =
Ac_3  =
Ad_02 =
Ad_3  =


# Nx3 matrices: N rows with 3D point coordinates for N reconstructed points (N
=num_inliers)
Xa =
Xb =
Xc =
Xd =
```

Step 9: Compute camera positions

In [ ]:

```
# camera's optical centers (for pair of cameras)
# 2x3 matrices: two rows with 3D point coordinates for the first and second ca
mera
Ca = np.zeros((2,3))
Cb = np.zeros((2,3))
Cc = np.zeros((2,3))
Cd = np.zeros((2,3))

# image centers (for pair of cameras)
# 2x3 matrices: two rows with 3D point coordinates for the first and second ca
mera
Qa = np.zeros((2,3))
Qb = np.zeros((2,3))
Qc = np.zeros((2,3))
Qd = np.zeros((2,3))


print Cc
print Qc
```

Step 10: Visualization of reconstructed 3D points and cameras

In [ ]:

```
# visualization part
from mpl_toolkits.mplot3d import Axes3D

fig = plt.figure(10,figsize = (10, 10))

ax10_1 = plt.subplot(221, projection='3d')
plt.title('Solution a')
ax10_1.scatter(Xa[:,0],Xa[:,1],Xa[:,2], c='b', marker='p')
ax10_1.scatter(Ca[:,0],Ca[:,1],Ca[:,2], c='r', marker='p')
ax10_1.scatter(Qa[:,0],Qa[:,1],Qa[:,2], c='g', marker='p')

ax10_2 = plt.subplot(222, projection='3d')
plt.title('Solution b')
ax10_2.scatter(Xb[:,0],Xb[:,1],Xb[:,2], c='b', marker='p')
ax10_2.scatter(Cb[:,0],Cb[:,1],Cb[:,2], c='r', marker='p')
ax10_2.scatter(Qb[:,0],Qb[:,1],Qb[:,2], c='g', marker='p')

ax10_3 = plt.subplot(223, projection='3d')
plt.title('Solution c')
ax10_3.scatter(Xc[:,0],Xc[:,1],Xc[:,2], c='b', marker='p')
ax10_3.scatter(Cc[:,0],Cc[:,1],Cc[:,2], c='r', marker='p')
ax10_3.scatter(Qc[:,0],Qc[:,1],Qc[:,2], c='g', marker='p')

ax10_4 = plt.subplot(224, projection='3d')
plt.title('Solution d')
ax10_4.scatter(Xd[:,0],Xd[:,1],Xd[:,2], c='b', marker='p')
ax10_4.scatter(Cd[:,0],Cd[:,1],Cd[:,2], c='r', marker='p')
ax10_4.scatter(Qd[:,0],Qd[:,1],Qd[:,2], c='g', marker='p')

plt.show()
```

Step 11: Reprojection errors

In [ ]:
```
# Randomly select N=50 matches (pairs of features in two images) from the set
 of inliers for E
N = 50
ind_sample2 = np.random.choice(num_inliers, N, replace = False)

# Indicate (E) inlier matches in image 1 and image 2
plt.figure(11,figsize = (10, 4))
ax11_1 = plt.subplot(121)
plt.imshow(imL)
plt.plot(ptsL[ind[E_inliers][ind_sample2], 0], ptsL[ind[E_inliers][ind_sample2
], 1], 'ob')
ax11_2 = plt.subplot(122)
plt.imshow(imR)
plt.plot(ptsR[ind[E_inliers][ind_sample2], 0], ptsR[ind[E_inliers][ind_sample2
], 1], 'ob')

# project reconstructed 3D points onto both images and display them in red col
or
# a. convert correct points (Xa, Xb, Xc, or Xd) to homogeneous 4 vectors
# b. project homogeneous 3D points (onto uncalibrated cameras) using correct P
rojection matrices (KPw and, e.g. KPa)
# c. convert to regular (inhomogeneous) point
ptsL_proj = np.zeros((N,2))
ptsR_proj = np.zeros((N,2))


ax11_1.plot(ptsL_proj[:,0], ptsL_proj[:,1], '.r')
ax11_2.plot(ptsR_proj[:,0], ptsR_proj[:,1], '.r')

plt.show()
```

Question: how different are projected points for SfM solutions a, b, c, and d? Explain.

Answer: