Problem 1

(sumf (nth-power 3) 2)
(+ ((nth-power 3) 2) (sumf (nth-power 3) (- 2 1))) $[n = 2 \neq 0]$
(+ (power 2 3) (sumf (nth-power 3) 1))
(+ 8 (+ ((nth-power 3) 1) (sumf (nth-power 3) (- 1 1)))) $[n = 1 \neq 0]$
(+ 8 (+ (power 1 3) (sumf (nth-power 3) 0)))
(+ 8 (+ 1 0)) $[n = 0]$
(+ 8 1)
9

Problem 3

Show (succ two) equals three, which is (lambda (f x) (f (f (f x)))).

Proof. (succ two)
= ((lambda (n f x) (f ((n f) x))) two)
$\rightarrow_\beta$(lambda (f x) (f ((two f) x)))
= (lambda (f x) (f (((lambda (f‘ x‘) (f‘ (f‘ (x‘)))) f) x)))
$\rightarrow_\beta$(lambda (f x) (f ((lambda (x‘) (f (f (x‘)))) x)))
$\rightarrow_\beta$(lambda (f x) (f (f (f (x))))) □

Problem 5

Given m and n are the Church numeral encodings of 2 and 3 respectively, show that (mulc m n) returns the Church numeral encoding for m*n = 6. Assume (addc a b) returns the Church numeral representing a+b.

Proof. (mulc m n)
= ((m (lambda (a) (addc n a))) zero)
=((lambda (x) ((lambda (a) (addc n a)) ((lambda (a) (addc n a)) x))) zero)
=((lambda (x) ((lambda (a) (addc n a)) (addc n x))) zero)
=((lambda (x) ((addc n (addc n x)))) zero)
=(addc n (addc n zero))
=(addc n n) [by given assumption]
=(addc (lambda (f x) (f (f (f x)))) (lambda (f x) (f (f (f x)))))
=(lambda (f x) (f (f (f (f (f (f x))))))) [by given assumption]
which is the Church encoding for 6. □