ALGA _ O Determinante

①

a) $\begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} = 3\times7 - 5\times4 = 1$

b) $\begin{vmatrix} 0 & 3 \\ 1 & 2 \end{vmatrix} = 0 - 3 = -3$

c) $\begin{vmatrix} \operatorname{sen}\alpha & -\cos\alpha \\ \cos\alpha & \operatorname{sen}\alpha \end{vmatrix} = \operatorname{sen}^2\alpha + \cos^2\alpha = 1$

d) $\begin{vmatrix} 0 & 7 & 1 \\ 4 & 1 & 2 \\ 1 & 7 & 3 \end{vmatrix} = 0\times1\times3 + 4\times7\times1 + 1\times7\times2 - 1\times1\times1 - 2\times7\times0 - 3\times7\times4 = 0 + 28 + 14 - 1 - 0 - 84$

$= 42 - 85 = -43$

$\begin{matrix} 0 & 7 & 1 \\ 4 & 1 & 2 \end{matrix}$

e) $\begin{vmatrix} 1 & 7 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{vmatrix} = 1\times1\times3 + 0\times0\times1 + 0\times7\times2 - 1\times1\times0 - 2\times0\times1 - 3\times7\times0 = 3$

$\begin{matrix} 1 & 7 & 1 \\ 0 & 1 & 2 \end{matrix}$

② $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 7$

a) $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 4c_1 & 4c_2 & 4c_3 \end{vmatrix} = 4 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4\times7 = 28$

b) $\begin{vmatrix} a_1 & a_3 & a_2 \\ b_1 & b_3 & b_2 \\ c_1 & c_3 & c_2 \end{vmatrix} \overset{C_2 \leftrightarrow C_3}{=} (-1) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -7$

c) $\begin{vmatrix} a_1 - 5c_1 & a_2 - 5c_2 & a_3 - 5c_3 \\ 10b_1 & 10b_2 & 10b_3 \\ -4c_1 & -4c_2 & -4c_3 \end{vmatrix} = 10\times(-4) \begin{vmatrix} a_1 - 5c_1 & a_2 - 5c_2 & a_3 - 5c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \underset{L_1 := L_1 - 5L_3}{=} -40 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -40\times7$

$= -280$

③ $\begin{vmatrix} \lambda+2 & -1 & 3 \\ 2 & \lambda-1 & 2 \\ 0 & 0 & \lambda+4 \end{vmatrix} = 0$

$\Leftrightarrow \lambda+4\,(-1)^{3+3} \begin{vmatrix} \lambda+2 & -1 \\ 2 & \lambda-1 \end{vmatrix} = 0$

$\Leftrightarrow \lambda+4 \quad (\lambda+2)(\lambda-1) - (-1)\times2 = 0$

$\Leftrightarrow \lambda+4 = 0 \quad \vee \quad (\lambda+2)(\lambda-1) + 2 = 0$

$\Leftrightarrow \lambda = -4 \quad \vee \quad \lambda^2 - \lambda + 2\lambda - 2 + 2 = 0$

$\Leftrightarrow \lambda = -4 \quad \vee \quad \lambda^2 + \lambda = 0$

$\Leftrightarrow \lambda = -4 \quad \vee \quad \lambda = 0 \quad \vee \quad \lambda = -1$

$\lambda \in \{-4, -1, 0\}$

④ $c \in \mathbb{R}$

$A_{n \times n}$

$\det(cA) = c^n \det(A)$

⑤ A, B 5×5

$|A| = 3$

$|B| = -5$

a) $|A^T| = |A| = 3$

b) $|AB| = 3 \times (-5) = -15$

c) $|A^4| = |A||A||A||A| = (|A|)^4 = 3^4$

d) $|B^{-1}| = \frac{1}{|B|} = -\frac{1}{5}$

e) $|2A| = 2^5|A| = 2^5 \times 3$

f) $|2A^{-1}| = 2^5|A^{-1}| = 2^5 \times \frac{1}{|A|} = \frac{2^5}{3}$

g) $|(2A)^{-1}| = \frac{1}{|2A|} = \frac{1}{2^5|A|} = \frac{1}{2^5 \times 3}$

h) $|AB^{-1}A^T| = |A||B^{-1}||A^T| = |A| \times \frac{1}{|B|}|A| = 3 \times \frac{1}{-5} \times 3 = -\frac{9}{5}$

$|2A| \leftarrow 5 \times 5$

$|2A| = 2^5|A| = 2^5 \times 3$

$|2A^{-1}| = 2^5|A^{-1}| = 2^5 \frac{1}{|A|} = 2^5/3$

⑥ $A_{4 \times 4}$

$\det(A) = 3$

$\det(2(A^{-1})^T) = 2^4 |(A^{-1})^T| = 2^4 |(A^T)^{-1}| = 2^4 |A^{-1}| = 2^4 \times \frac{1}{3} = \frac{16}{3}$

⑦ A, B 5×5

B é invertível

$|AB| = 24$ $\quad |AB| = |A||B| = 24$

$|B^{-1}| = 4$

• $|B^{-1}| = \frac{1}{|B|} \Leftrightarrow 4 = \frac{1}{|B|} \Leftrightarrow |B| = \frac{1}{4}$

$|AB| = |A||B| = 24 \Leftrightarrow 24 = |A|\frac{1}{4} \Leftrightarrow |A| = 96$

⑧

a) $\begin{vmatrix} b+c & c+a & b+a \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} =$

b) $\begin{vmatrix} a_1 & b_1 & a_1+b_1+c_1 \\ a_2 & b_2 & a_2+b_2+c_2 \\ a_3 & b_3 & a_3+b_3+c_3 \end{vmatrix}^T = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1+b_1+c_1 & a_2+b_2+c_2 & a_3+b_3+c_3 \end{vmatrix} \underset{L_3 = L_3 - L_1 - L_2}{=} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^T = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ c.q.d.

c) $\begin{vmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{vmatrix} =$

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a) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 1$

$$\begin{vmatrix} a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ 3c_1+b_1 & 3c_2+b_2 & 3c_3+b_3 \\ -b_1 & -b_2 & -b_3 \end{vmatrix} = (-1) \begin{vmatrix} a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ 3c_1+b_1 & 3c_2+b_2 & 3c_3+b_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \underset{\text{troca do sinal do determinante}}{=} L_2 \leftrightarrow L_3$$

logo volta a ser positivo

$$\begin{vmatrix} a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ b_1 & b_2 & b_3 \\ 3c_1+b_1 & 3c_2+b_2 & 3c_3+b_3 \end{vmatrix}$$

$$\underset{L_3 := L_3 - L_2}{=} \begin{vmatrix} a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ b_1 & b_2 & b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix} \underset{L_1 := L_1 - 2L_2}{=} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix} = 3 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 3 \times 1 = 3$$

Não há alteração do valor do determinante

o valor do det. não altera

$|A| = |A^T|$

b) $\begin{vmatrix} 2a_1 & a_2+a_3 & -a_3 \\ 2c_1 & c_2+c_3 & -c_3 \\ 2b_1 & b_2+b_3 & -b_3 \end{vmatrix} = 10$

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \overset{T}{=} \begin{vmatrix} \\ \\ \end{vmatrix}$

a_1

c)

d)

(10) Teorema de Laplace

a) $\begin{vmatrix} 2 & 3 & 0 \\ 4 & 5 & 1 \\ 5 & 10 & 4 \end{vmatrix}$ $|A| = 0 \times (-1)^{1+3} \det(M_{13}) + 1 \times (-1)^{2+3} \det(M_{23}) + 4 \times (-1)^{3+3} \det(M_{33})$

$= 0 - \begin{vmatrix} 2 & 3 \\ 5 & 10 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -((2\times10) - (3\times5)) + 4((2\times5) - (3\times4))$

$= -5 - 8 = -13$

b) $\begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ 7 & 5 & -6 \end{vmatrix} =$ $|A| = 2 \times (-1)^{1+1} \begin{vmatrix} 1 & -2 \\ 5 & -6 \end{vmatrix} + (-1)(-1)^{1+2} \begin{vmatrix} 4 & -2 \\ 7 & -6 \end{vmatrix} + 3(-1)^{1+3} \begin{vmatrix} 4 & 1 \\ 7 & 5 \end{vmatrix}$

$= 2 \times (-6+10) + (-24+14) + 3(20-7) = 2 \times 4 - 10 + 3 \times 13$

$= 8 - 10 + 39 = 37$

c) $\begin{vmatrix} 3 & -2 & 7 & 0 \\ 1 & -2 & -3 & 8 \\ 6 & 0 & -1 & 8 \\ -1 & 2 & 5 & 2 \end{vmatrix}$

(11) $A = \begin{bmatrix} 3 & 4 & -1 \\ 0 & 5 & -4 \\ 8 & -6 & 2 \end{bmatrix}$

a) $A_{11} = (-1)^{1+1} \det \begin{vmatrix} 5 & -4 \\ -6 & 2 \end{vmatrix} = 10 - 24 = -14$

$A_{12} = (-1)^{1+2} \det \begin{vmatrix} 0 & -4 \\ 8 & 2 \end{vmatrix} = -(0+32) = -32$

$A_{13} = (-1)^{1+3} \det \begin{vmatrix} 0 & 5 \\ 8 & -6 \end{vmatrix} = -40$

$A_{21} = (-1)^{2+1} \det \begin{vmatrix} 4 & -1 \\ -6 & 2 \end{vmatrix} = -(8-6) = -2$

$A_{22} = (-1)^{2+2} \det \begin{vmatrix} 3 & -1 \\ 8 & 2 \end{vmatrix} = 6+8 = 14$

$A_{23} = (-1)^{2+3} \det \begin{vmatrix} 3 & 4 \\ 8 & -6 \end{vmatrix} = -(-18-32) = 50$

$A_{31} = (-1)^{3+1} \det \begin{vmatrix} 4 & -1 \\ 5 & -4 \end{vmatrix} = -16+5 = -11$

$A_{32} = (-1)^{3+2} \det \begin{vmatrix} 3 & -1 \\ 0 & -4 \end{vmatrix} = -(-12) = 12$

$A_{33} = (-1)^{3+3} \det \begin{vmatrix} 3 & 4 \\ 0 & 5 \end{vmatrix} = 15$

$$\operatorname{Adj} A = \begin{bmatrix} -14 & -32 & -40 \\ -2 & 14 & 50 \\ -11 & 12 & 15 \end{bmatrix}^T = \begin{bmatrix} -14 & -2 & -11 \\ -32 & 14 & 12 \\ -40 & 50 & 15 \end{bmatrix}$$

b) $\det(A) = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

$= 3\,\underbrace{(-1)^{1+1}\begin{vmatrix} 5 & -4 \\ -6 & 2 \end{vmatrix}}_{A_{11}} + 0\,\underbrace{(-1)^{2+1}\begin{vmatrix} 4 & -1 \\ -6 & 2 \end{vmatrix}}_{A_{21}} + 8\,\underbrace{(-1)^{3+1}\begin{vmatrix} 4 & -1 \\ 5 & -4 \end{vmatrix}}_{A_{31}}$

$= 3\times(-14) + 0\times(-2) + 8(-11)$

$= -42 + 0 - 88 = -130$

c) $A \operatorname{adj} A = \det(A)\, I_n$

• $A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$ multiplicamos por A

$\Leftrightarrow A A^{-1} = \frac{1}{\det(A)} A \operatorname{adj}(A)$

$\Leftrightarrow I_n = \frac{1}{\det(A)} A \operatorname{adj}(A)$

$\Leftrightarrow \det(A)\, I_n = A \operatorname{adj}(A)$ c.q.d

d) $A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) \Leftrightarrow A^{-1} = \frac{1}{-130} \operatorname{adj}(A)$

$$A^{-1} = \begin{bmatrix} \frac{14}{130} & \frac{2}{130} & \frac{11}{130} \\ \frac{32}{130} & -\frac{14}{130} & -\frac{12}{130} \\ \frac{40}{130} & -\frac{50}{130} & -\frac{15}{130} \end{bmatrix}$$

(12)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \, adj A$$

$A_{11} = (-1)^{1+1} \times 4 = 4$

$A_{12} = (-1)^{1+2} \times 3 = -3$

$A_{21} = (-1)^{2+1} \times 2 = -2$

$A_{22} = (-1)^{2+2} \times 1 = 1$

$$adj A = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

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$$A \cdot adj(A) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4-6 & -2+2 \\ 12-12 & -6+4 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

(13)

14

$$A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\qquad \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}$$

$|A| = 1\times0\times1 + 1\times1\times1 + 0\times1\times1 = 1\times0\times0 - 1\times1\times1 - 1\times1\times1$

$= 0 + 1 + 0 - 0 - 1 - 1 = 1 - 2 = -1$

$|A| \neq 0$ logo A é invertível

• o elemento (1,2) da inversa de A

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{|A|}\begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{m1} & \cdots & A_{mn} \end{bmatrix}^T$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = -1 \times 1 - 1 = 0$$

Inversa de A

o elemento na posição (1,2) da inversa de A é $\frac{1}{|A|}A_{21} = \frac{1}{-1}\times 0 = 0$

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$$A=\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

a) $|A|$ Usando o teorema de Laplace

$$\det(A) = 1\times(-1)^{1+1}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} + 0 + 1\times(-1)^{3+1}\begin{vmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2\times(-1)^{2+1}\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} + \left[1\times(-1)^{2+1}\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 1\times(-1)^{3+1}\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}\right] = -2\times(-1-1) + (-1)\times(2+1) + (2-1)$$

$$= 4 - 3 + 1 = 2$$

b) $A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -\left(1\times(-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}\right) = -\,1\times(-1-1) = 2$

↳ o elemento 2,3 da adjunta de A

• o elemento 2,3 da inversa de A

$$\frac{1}{|A|}\,\text{adj}\,A = \frac{1}{|A|}A_{32} = \frac{1}{2}\times 2 = 1$$

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$$A=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 2 & -1 & 0 & 1 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

$$\boxed{A^{-1}=\frac{1}{|A|}\,\text{adj}\,A}$$

elemento (4,1) da inversa de A ?

- $\det(A)= 1\times(-1)^{1+1}\begin{vmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} + 1\times(-1)^{1+3}\begin{vmatrix} 0 & 1 & 2 \\ 2 & -1 & 0 \\ -1 & 1 & 0 \end{vmatrix}$

$= 1\times0\times1 + 1\times0\times1 + 1\times2\times(-1) - 1\times0\times1 - (-1)\times0\times1 - 1\times2\times1$

$= -2-2=-4$

- $A_{14}=(-1)^{1+4}\begin{vmatrix} 0 & 1 & 2 \\ 2 & -1 & 0 \\ -1 & 1 & 0 \end{vmatrix} = -\left((0\times(-1)\times0)+(2\times1\times2)+(-1)\times1\times0) - (2\times(-1)\times(-1)) - 0\times1\times0 - 0\times1\times2\right)$

$= -(4-(+2)) = -2$

Vei o ERRO!

O elemento (4,1) da inversa de A

$\frac{1}{-4}\times(-2) = \frac{2}{4}=\frac{1}{2}$

17

$$\begin{bmatrix} \alpha-4 & 0 & 10 \\ 4 & \alpha+5 & 1 \\ 2 & 0 & \alpha-3 \end{bmatrix}$$

$\det(A) = (\alpha+5)(-1)^{2+2}\begin{vmatrix} \alpha-4 & 10 \\ 2 & \alpha-3 \end{vmatrix} = \alpha+5\left[(\alpha-3)(\alpha-4)-20\right]$

$(\alpha+5)\left[(\alpha-3)(\alpha-4)-20\right]=0$

$\Leftrightarrow \alpha+5=0 \vee (\alpha-3)(\alpha-4)-20=0$

$\Leftrightarrow \alpha=-5 \vee \alpha^2-4\alpha-3\alpha+12-20=0$

$\Leftrightarrow \alpha=-5 \vee \alpha^2-7\alpha-8=0$

$\Leftrightarrow \alpha=-5 \vee \alpha=\frac{7\pm\sqrt{49+4\times8}}{2}$

$\Leftrightarrow \alpha=-5 \vee \alpha=\frac{7\pm\sqrt{81}}{2}$

$\Leftrightarrow \alpha=-5 \vee \alpha=\frac{7\pm9}{2}$

$\Leftrightarrow \alpha=-5 \vee \alpha=8 \vee \alpha=-1$

$\alpha\in\{-5,-1,8\}$

18

$$A=\left[\begin{array}{ccc|c} \beta & 6 & 1 & 0 \\ 0 & \beta-1 & 1 & 0 \\ 0 & 1 & \beta+5 & 0 \end{array}\right]$$

$|A| = \beta(\beta-1)(\beta+5)-\beta$

$\beta\left[(\beta-1)(\beta+5)-\beta\right]=0$

$\Leftrightarrow \beta\left[(\beta-1)(\beta+5)-1\right]=0$

$\Leftrightarrow \beta=0 \vee (\beta-1)(\beta+5)-1=0$

$\Leftrightarrow \beta=0 \vee \beta^2+5\beta-\beta-5-1=0$

$\Leftrightarrow \beta=0 \vee \beta^2+4\beta-6=0$

$\Leftrightarrow \beta=0 \vee \beta=\frac{-4\pm\sqrt{16-4\times(-6)}}{2}$

$\Leftrightarrow \beta=0 \vee \beta=\frac{-4\pm2\sqrt{10}}{2}$

$\Rightarrow \beta=0 \vee \beta=-2+2\sqrt{10} \vee \beta=-2-2\sqrt{10}$

$\beta\in\mathbb{R}\setminus\{-2-2\sqrt{10},0,-2+2\sqrt{10}\}$

19) A Regra de Cramme pode ser usada se a matriz dos coeficientes do sistema for quadrada e se o determinante for nao nulo

20)

a) $\begin{cases} x - y - z = 0 \\ 4x + 2y - 4z = 6 \\ 3x + 2y - z = -1 \end{cases}$

$$\begin{vmatrix} 1 & -1 & -1 \\ 4 & 2 & -4 \\ 3 & 2 & -1 \end{vmatrix} = 1\times2\times(-1) + 4\times2\times(-1) + 3\times(-1)\times(-4) - (-1)\times2\times3 - (-4)\times2\times1 - (-1)\times(-1)\times4$$
$$\begin{matrix} 1 & -1 & -1 \\ 4 & 2 & -4 \end{matrix}$$
$$= -\cancel{2} - \cancel{8} + 12 + \cancel{6} + \cancel{8} - \cancel{4} = 12$$

$$x = \frac{1}{|A|}\begin{vmatrix} 0 & -1 & -1 \\ 6 & 2 & -4 \\ -1 & 2 & -1 \end{vmatrix} = \frac{1}{12} \times 0 - 12 - 4 - 2 - 0 - 6 = \frac{1}{12}\times(-24) = -\frac{24}{12} = -2$$
$$\begin{matrix} 0 & -1 & -1 \\ 6 & 2 & -4 \end{matrix}$$

$$y = \frac{1}{|A|}\begin{vmatrix} 1 & 0 & -1 \\ 4 & 6 & -4 \\ 3 & -1 & -1 \end{vmatrix} = \frac{1}{12} \times -6 + 4 + 0 + 18 - 4 - 0 = \frac{1}{12} \times 12 = 1$$
$$\begin{matrix} 1 & 0 & -1 \\ 4 & 6 & -4 \end{matrix}$$

$$z = \frac{1}{|A|}\begin{vmatrix} 1 & -1 & 0 \\ 4 & 2 & 6 \\ 3 & 2 & -1 \end{vmatrix} = \frac{1}{12} \times (-2) + 0 - 18 - 0 - 12 - 4 = \frac{1}{12}\times(-36) = -3$$
$$\begin{matrix} 1 & -1 & 0 \\ 4 & 2 & 6 \end{matrix}$$

- $x = -2,\ y = 1,\ z = -3$

b) $\begin{cases} 4x - 3z = -2 \\ 2x - y = -2 \\ x - 3y + z = 4 \end{cases}$

$$\begin{vmatrix} 4 & 0 & -3 \\ 2 & -1 & 0 \\ 1 & -3 & 1 \end{vmatrix} = -4 + 18 + 0 - 3 - 0 - 0$$
$$= 11$$
$$\begin{matrix} 4 & 0 & -3 \\ 2 & -1 & 0 \end{matrix}$$

$$x = \frac{1}{|B|}\begin{vmatrix} -2 & 0 & -3 \\ -2 & -1 & 0 \\ 4 & -3 & 1 \end{vmatrix} = \frac{1}{11} \times 2 - 18 + 0 - 12 - 0 - 0 = \frac{1}{11}\times(-28) = -\frac{28}{11}$$
$$\begin{matrix} -2 & 0 & -3 \\ -2 & -1 & 0 \end{matrix}$$

$$y = \frac{1}{|B|}\begin{vmatrix} 4 & -2 & -3 \\ 2 & -2 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \frac{1}{11} \times (-8) - 24 + 0 - 6 - 0 + 4 = \frac{1}{11}(-34) = -\frac{34}{11}$$
$$\begin{matrix} 4 & -2 & -3 \\ 2 & -2 & 0 \end{matrix}$$

$$z = \frac{1}{|B|}\begin{vmatrix} 4 & 0 & -2 \\ 2 & -1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = \frac{1}{11} \times -16 + 12 + 0 - 2 - 24 - 0 = \frac{1}{11}(-30) = -\frac{30}{11}$$
$$\begin{matrix} 4 & 0 & -2 \\ 2 & -1 & -2 \end{matrix}$$

$C.S. : \left\{ -\frac{28}{11}, -\frac{34}{11}, -\frac{30}{11} \right\}$

c) $\begin{cases} -2x - y + z = -3 \\ 2x + 4y - 2z = 8 \\ 2x + 3y - z = 1 \end{cases}$

$$\begin{vmatrix} -2 & -1 & 1 \\ 2 & 4 & -2 \\ 2 & 3 & -1 \end{vmatrix} = 8 + 6 + 4 \cancel{-8} - 12 - 2 = -4$$
$$\begin{matrix} -2 & -1 & 1 \\ 2 & 4 & -2 \end{matrix}$$

$$x = \frac{1}{|C|} \begin{vmatrix} -3 & -1 & 1 \\ 8 & 4 & -2 \\ 1 & 3 & -1 \end{vmatrix} = \frac{1}{-4} \times 12 + 24 + 2 - 4 - 18 - 8 = -\frac{1}{4} \times 8 = -2$$
$$\begin{matrix} -3 & -1 & 1 \\ 8 & 4 & -2 \end{matrix}$$

$$y = \frac{1}{|C|} \begin{vmatrix} -2 & -3 & 1 \\ 2 & 8 & -2 \\ 2 & 1 & -1 \end{vmatrix} = -\frac{1}{4} \times \cancel{16} + 2 + 12 \cancel{-16} - 4 - 6 = -\frac{1}{4} \times 4 = -1$$
$$\begin{matrix} -2 & -3 & 1 \\ 2 & 8 & -2 \end{matrix}$$

$$z = \frac{1}{|C|} \begin{vmatrix} -2 & -1 & -3 \\ 2 & 4 & 8 \\ 2 & 3 & 1 \end{vmatrix} = -\frac{1}{4} \times -8 - 18 - 16 + 24 + 48 + 2 = -\frac{1}{4} \times 32 = -8$$
$$\begin{matrix} -2 & -1 & -3 \\ 2 & 4 & 8 \end{matrix}$$

$C.S. = \{-2, -1, -8\}$

d) $\begin{cases} x + 3y + 2z + w = 0 \\ 2y + z + 3w = 4 \\ 2x + y - z + 2w = 5 \\ 3x - z + 3w = 9 \end{cases}$

$$\begin{vmatrix} 1 & 3 & 2 & 1 \\ 0 & 2 & 1 & 3 \\ 2 & 1 & -1 & 2 \\ 3 & 0 & -1 & 3 \end{vmatrix} = 1(-1)^{1+1} \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 0 & -1 & 3 \end{vmatrix} + 0 + 2(-1)^{3+1} \begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & -1 & 3 \end{vmatrix} + 3(-1)^{4+1} \begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \left(0 + (-1)(-1)^{3+2} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} + 3(-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}\right) + 2\left[0 + (-1)(-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + 3(-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}\right]$$

$$-3\left[1(-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + (-1)(-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + 2(-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}\right]$$

$$= 4 - 3 + 3(-2-1) + 2\left(6 - 2 + 3 \times (3-4)\right) - 3\left(6 - 1 + 9 - 2 + 2(3-4)\right)$$
$$= 1 - 9 + 2(4-3) - 3(5 + 7 - 2) = -8 + 2 - 30 = -28$$

$$x = \frac{1}{|D|} \begin{vmatrix} 0 & 3 & 2 & 1 \\ 4 & 2 & 1 & 3 \\ 5 & 1 & -1 & 2 \\ 9 & 0 & -1 & 3 \end{vmatrix} = \frac{1}{-28} \times \left(0 + 3 \times (-1)^{1+2} \begin{vmatrix} 4 & 1 & 3 \\ 5 & -1 & 2 \\ 9 & -1 & 3 \end{vmatrix} + 2 \times (-1)^{1+3} \begin{vmatrix} 4 & 2 & 3 \\ 5 & 1 & 2 \\ 9 & 0 & 3 \end{vmatrix} + 1(-1)^{1+4} \begin{vmatrix} 4 & 2 & 1 \\ 5 & 1 & -1 \\ 9 & 0 & -1 \end{vmatrix}\right)$$
$$\begin{matrix} 4 & 1 & 3 \\ 5 & -1 & 2 \end{matrix} \qquad \begin{matrix} 4 & 2 & 3 \\ 5 & 1 & 2 \end{matrix} \qquad \begin{matrix} 4 & 2 & 1 \\ 5 & 1 & -1 \end{matrix}$$

$$= -\frac{1}{28} \times \left(-3(-12 - 15 + 18 + 27 + 12 - 15) + 2(12 + 0 + 36 - 27 - 0 - 30) - 1(-4 + 0 - 18 - 9 - 0 + 10)\right)$$

$$= -\frac{1}{28} \times (-45 - 18 + 21) = -\frac{1}{28}$$