



Unmanned Autonomous Systems - 31390

List of Symbols – Week1

Contributors:

Kristian Weber Larsen

Ananda Nielsen

Matteo Fumagalli

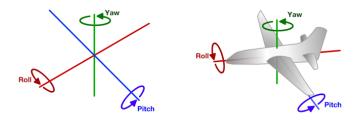
Associate Professor
Automation and Control Group
Department of Electrical Engineering
DTU Lyngby, Building 326

29 October 2019 DTU Electrical Engineering



Euler angles

Roll-Pitch-Yaw Euler angles (X-Y-Z)



- ϕ roll angle: rotation around the local x-axis
- θ pitch angle: rotation around the local y-axis
- ψ yaw angle: rotation around the local z-axis
- $\vec{\alpha} = [\phi, \theta, \psi]^{\top}$ vector of Euler angles (here called alpha, but it could take any symbol)
- $R(\vec{lpha})$ Rotation matrix describing the rotation of a frame, with respect to another, according to the chosen angular notation described by vector



Modeling of a Quadrotor: Kinematic variables

$$\vec{x} = (x, y, z)^{\top}$$
 vector describing the position of the body-fixed frame w.r.t. the inertia frame

$$\dot{\vec{x}} = (\dot{x}, \dot{y}, \dot{z})^{\top}$$
 vector describing the velocity of the body-fixed frame w.r.t. the inertia frame

$$\vec{\theta} = (\phi, \theta, \psi)^{\top}$$
 vector describing the angular position, according to our particular choice of Euler angles, of the bodyfixed frame w.r.t. the inertia frame

$$\dot{ec{ heta}} = (\dot{\phi}, \dot{ heta}, \dot{\psi})^{ op}$$
 derivative of $ec{ heta}$

 $\vec{\omega}$ vector describing the angular velocity of the body-fixed frame angular acceleration, derivative ove $\vec{\omega}$ time of

29 October 2019 DTU Electrical Engineering



Modeling of a Quadrotor

- L Distance between the propeller's axis of rotation and the z-axis of the body-fixed frame
- *k* Parameter describing the relation between the aerodynamic lift of the propellers and the propellers' angular velocity^2
- b Parameter describing the relation between the aerodynamic drag of the propellers and the propellers' angular velocity^2
- $\Omega_1 \cdots \Omega_4$ Propellers' angular velocity
 - f_i vertical force (on the axis of rotation) of the i-th propeller
 - au_i torque around the axis of rotation of the i-th propeller
 - k_d Parameter describing the viscosity of a mean (e.g. air) in which a body moves R Rotation matrix of body-fixed frame
 - gravity (9.81m/s^2) w.r.t. inertia frame



Modeling of a Quadrotor

$$\mathbf{m} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \qquad \text{matrix of mass}$$

$$I = egin{bmatrix} I_{xx} & 0 & 0 \ 0 & I_{yy} & 0 \ 0 & 0 & I_{zz} \end{bmatrix}$$
 inertia matrix of the drone (in this case diagonal, but generally full)

- $ec{F_B}$ Vector describing the net force generated by the propellers on the body-fixed frame
- $ec{ au_B}$ Vector describing the net torque generated by the propellers on the body-fixed frame
- $ec{F_D}$ Vector describing the viscous force generated by the body moving in a viscous mean (e.g. air) $F_D = egin{bmatrix} -k_d \dot{x} \\ -k_d \dot{y} \\ -k_d \dot{z} \end{bmatrix}$