Problem 2c:

Naive expansion on original equation

```
govEq = Expand[x^3 + (3 - 2 * ϵ) * x^2 + (3 + ϵ) * x + 1 - 2 ϵ]
```

$$1 + 3\,x + 3\,x^2 + x^3 - 2\,\epsilon + x\,\epsilon - 2\,x^2\,\epsilon$$

```
naive = Collect[Expand[govEq /. x -> {x_o + ϵ * x_1 + ϵ^2 * x_2}], ϵ];
```

Extract the O(1) equation

```
Collect[Normal[Series[naive /. Thread[ϵ → k * ϵ], {k, 0, 0}]] /. k → 1, ϵ]
```

$$\left\{1 + 3\,x_o + 3\,x_o^2 + x_o^3\right\}$$

```
Solve[% == 0, x_o]
```

$$\{\{x_o \to -1\}, \{x_o \to -1\}, \{x_o \to -1\}\}$$

Triple root at -1.

Proceed by using gage function

```
gage = Collect[Series[
    govEq /. x -> {x_o + ϵ^(1/3) * x_1 + ϵ^(2/3) * x_2 + ϵ^(3/3) * x_3 + ϵ^(3/3) * x_3 + ϵ^(4/3) * x_4}, {ϵ, 0, 3}], ϵ];
```

Pursue -1 triple root

```
Collect[Normal[Series[gage /. Thread[ϵ → k * ϵ], {k, 0, 1}]] /. k → 1, ϵ] /. x_o → -1
```

$$\left\{\epsilon\,\left(-5 + x_1^3\right)\right\}$$

```
Solve[% == 0, x_1]
```

$$\left\{\left\{x_1 \to -(-5)^{1/3}\right\}, \left\{x_1 \to 5^{1/3}\right\}, \left\{x_1 \to (-1)^{2/3}\,5^{1/3}\right\}\right\}$$

Compile Solutions

```
Expand[-1 + ϵ^(1/3) * -(-5)^(1/3)]
```

$$-1 - (-5)^{1/3}\,\epsilon^{1/3}$$

```
Expand[-1 + ϵ^(1/3) * 5^(1/3)]
```

$$-1 + 5^{1/3}\,\epsilon^{1/3}$$

```
Expand[-1 + ϵ^(1/3) * (-1)^(2/3) 5^(1/3)]
```

$$-1 + (-1)^{2/3}\,5^{1/3}\,\epsilon^{1/3}$$