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Exercise 1

1.1 Consider a random walk in one dimension, where the walker decides to move in either positive or negative x-direction with probability $p = 1/2$ and step size 1.

a) Create a computer visualization or animation (e.g., using bar charts) that shows how the actual distribution of the walker evolves over time. How does the hull curve look like?

b) Simulate the probabilities that the walker is more than $\sqrt{N}$ and $3\sqrt{N}$ steps away from the origin after N times steps for $N = 10$, 100, and 1000. How many runs should be done to get a reliable result (judging by experimentation experience)? Compare with the analytical results.

c) Given that there are two random walkers starting at positions with distance 5, what is the simulated probability that they meet within 10 time steps?

1.2 Consider a random walker on an $N \times N$ array with N^2 cells ($N = 2, 3, 4, \dots$). The walker is allowed to move up, down, left or right by one cell in each step, unless there is a boundary. The walker starts in one of the corners.

In scenario A the walker chooses the next cell with the same probability for all admissible options: $p = 1/4$ in the interior of the array, $p = 1/2$ in the corners of the array, and $p = 1/3$ in the remaining cells at the edges.

In scenario B the walker chooses a direction with $p = 1/4$. If the walker moves outside the array crossing a boundary, the walker stays at the current position instead.

Create a simulation that determines (for different values of N) the average number of steps that the walker needs to return to the starting corner position. Can you figure out the analytical values?

1.3* Consider the random walk of a knight on the chess board. The knight chooses among all admissible moves with the same probability. Set up a simulation that returns an estimate for the average number of steps that is needed to return to his standard starting position.

1.4 Set up a simulation for the Able Baker carhop problem as discussed in the lecture. Document and evaluate the service, idle and waiting times for a set of given arrival and service rates. Identify (reasonable) cases where the system capacity is no longer sufficient.