Phasors

Phasors will only show us the steady_state response of the circuit, not the transient response.

Eq: $v(t) = V_M \cos(\omega t + \theta) \leftrightarrow V = V_M \angle \theta_v = V_M e^{j\theta_v} = V_M (\cos\theta_v + j\sin\theta_v)$

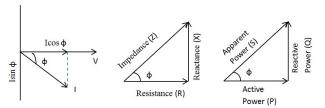
You can use phasors with ??, ??, and Thevenin and Norton Equivalencies.

 $z_1 = x_1 + y_2 = r_1 \angle \phi_1, \ z_2 = x_2 + y_2 = r_2 \angle \phi_2$

Addition	$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
Subtraction	$z_1 - z_2 = (x_1 - x_2) + \jmath (y_1 - y_2)$
Multiplication	$z_1 z_2 = r_1 r_2 \angle \left(\phi_1 + \phi_2\right)$
Division	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \left(\phi_1 - \phi_2\right)$
Reciprocal	$\frac{1}{z_1} = \frac{1}{z_1} \angle - \phi_1$
Square Root	$\sqrt{z_1} = \sqrt{r_1} \angle \frac{\phi_1}{2}$
Complex Conjugate	$z_1^* = x - y = r \angle - \phi_1 = re^{-j\phi_1}$

RMS/Complex Power/Max Power Transfer

- $C = \frac{Q_C}{\omega V_{rms}^2} = \frac{P(\tan \theta_1 \tan \theta_2)}{\omega V_{rms}^2}$
- $L = \frac{V_{rms}^2}{\omega(Q_1 Q_2)}$



Name	Symbol	$\operatorname{Equation}(\mathbf{s})$	Units
Complex Power	S	$\frac{P}{Pf} \angle \arccos(Pf) = P + jQ = \mathbf{V}_{rms} \mathbf{I}_{rms}^* = \mathbf{V}_{rms} \mathbf{I}_{rms} \angle (\theta_v - \theta_i)$	VA
Apparent Power	S	$\ \mathbf{S}\ = \mathbf{V}_{rms} \mathbf{I}_{rms} = \sqrt{P^2 + Q^2}$	VA
Real Power	P	$Re{S} = S * Pf cos [arccos (Pf)] = S cos (\theta_v - \theta_i)$	W
Reactive (Imaginary) Power	Q	$\operatorname{Im}\{\mathbf{S}\} = S * Pf \sin \left[\arccos \left(Pf\right)\right] = S \sin \left(\theta_v - \theta_i\right)$	VAR
Power Factor	Pf	$\frac{P}{S} = \cos(\theta_v - \theta_i)$	Lead/Lag

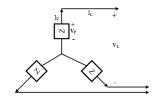
NOTE: If you are looking for 3-Phase complex power, it is in 3-Phase Circuits.

Elements

Relation	R	C	L
v-i	V = IR	$v = \frac{1}{C} \int_{t_0}^{t} i(x)dx + v(t_0)$	$v = L \frac{di}{dt}$
i-v	$I = \frac{V}{R}$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v(x) dx + i(t_0)$
P or W	$P = I^2 R = \frac{V^2}{R}$	$P = \frac{1}{2}Cv_c^2$	$W = \frac{1}{2}Li_l^2$
Series	$R_{eq} = R_1 + R_2 + \ldots + R_n$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$	$L_{eq} = L_1 + L_2 + \ldots + L_n$
Parallel	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$	$C_{eq} = C_1 + C_2 + \ldots + C_n$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$
@ Steady State	Same (Nothing Happens)	Open Circuit	Short Circuit
Phasors	$Z_R = R$	$Z_C = \frac{1}{j\omega C}$	$Z_L = j\omega L$

Thevenin and Norton Equivalencies

• ONLY independent sources - Zero all sources, find \mathbf{Z}_{eq} .

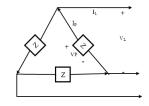


(a) 3 Phase Y-Connection

$$I_{L} = I_{P} \qquad V_{LL} = \sqrt{3}V_{P} \angle 30^{\circ} \qquad Z_{Y} = \frac{Z_{\Delta}}{3}$$

$$\mathbf{S} = \sqrt{3}\mathbf{V}_{L}\mathbf{I}_{L}^{*} \qquad \mathbf{S} = 3\mathbf{V}_{P}\mathbf{I}_{L}^{*} \qquad \phi = \theta_{V_{P}} - \theta_{I_{P}}$$

$$\mathbf{S} = S \angle \phi \qquad P = \|\mathbf{S}\|\cos(\phi) \qquad Q = \|\mathbf{S}\|\sin(\phi)$$



(b) 3 Phase Δ -Connection

$$I_{L} = \sqrt{3}I_{P} \angle -30^{\circ} \quad V_{LL} = V_{P} \qquad Z_{\Delta} = 3Z_{Y}$$

$$\mathbf{S} = \sqrt{3}\mathbf{V}_{L}\mathbf{I}_{L}^{*} \qquad \mathbf{S} = 3\mathbf{V}_{P}\mathbf{I}_{P}^{*} \qquad \phi = \theta_{V_{P}} - \theta_{I_{P}}$$

$$\mathbf{S} = S \angle \phi \qquad P = \|\mathbf{S}\|\cos(\phi) \qquad Q = \|\mathbf{S}\|\sin(\phi)$$

- 0-ing Current Sources = O.C., 0-ing Voltage Sources = S.C.
- Look at circuit from load's perspective for \mathbf{Z}_{eq}
- $\mathbf{V}_{Th} = \mathbf{V}_{OC}, \mathbf{I}_N = \mathbf{I}_{SC}$
- BOTH dependent and independent sources
- Find $\mathbf{V}_{Th} = \mathbf{V}_{OC}$, $\mathbf{I}_{N} = \mathbf{I}_{SC}$ Solve $\mathbf{Z}_{Th} = \frac{\mathbf{V}_{OC}}{\mathbf{I}_{SC}}$ ONLY dependent sources
- - $\mathbf{V}_{Th} = 0, \mathbf{I}_{N} = 0$
 - $-\mathbf{Z}_{Th} = \mathbf{Z}_N \to \text{Attach test source @ load.}$
 - * If voltage test source, find current. If current test source, find voltage
 - $-\mathbf{Z}_{Th} = \frac{\mathbf{V}_{Test}}{\mathbf{I}_{Test}}$

Maximum Power Transfer - AC

•
$$\mathbf{Z}_{Load} = \mathbf{Z}_{Th}^*, R_{Th} = \text{Re}\{\mathbf{Z}_{Th}\}, R_L = |\mathbf{Z}_{Th}| = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2}$$

$$\bullet \ P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}}$$

3-Phase Circuits

•
$$C_Y = \frac{Q_C}{3\omega \|V_{\phi,rms}\|^2}$$

• $C_{\Delta} = \frac{C_Y}{3}$

$$\bullet \ C_{\Delta} = \frac{C_Y}{3}$$

• Power lost due to line: $P_{Lost} = Z_{Wire}I_L$

You want to get everything into Y formation, because the common neutral allows you to do single-phase analysis.

Mutual Inductance

Equivalent Mutual Inductance

Series-Aiding Connection	$L = L_1 + L_2 + 2M$	• 6000 • 6000 • 6000
Series-Opposing Connection	$L = L_1 + L_2 - 2M$	• • • • • • • • • • • • • • • • • • •

Dot Convention

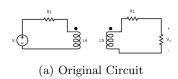
There are 2 cases:

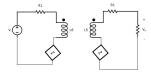
- 1. Current enters through dotted side on 1 inductor \longrightarrow POSITIVE VOLTAGE ON DOTTED SIDE OF OTHER **INDUCTOR**
 - Current flows into the dotted side of one inductor
 - Current flows out of the un-dotted side of second inductor, just like the first

- 2. Current enters through NON-dotted side of 1 inductor POSITIVE VOLTAGE ON UN-DOTTED SIDE OF OTHER INDUCTOR
 - Current flows into the un-dotted side of one inductor
 - Current flows out of the dotted side of the second inductor, just like the first

Solving Disjoint Coupled Circuits

- 1. Apply KVL
- 2. Don't forget about the Mutual Inductance Voltage Difference because of the first current
- 3. There is a second way to thing about these, shown in Figures 2a and 2b, below.





(b) Circuit "Simplified" by Adding Dependent Sources

The sign on the dependent sources depends on which side of the inductor the current is going into. Use the Dot Convention to determine which direction the source's voltage should go.

Transformers

These elements consume no power, and convert voltages and currents.

- $\frac{v_1}{v_2} = \frac{N_1}{N_2} \leftarrow$ Voltage Change $\frac{i_1}{i_2} = -\frac{N_2}{N_1} \leftarrow$ Current Change

Representations for Turns

There are 3 common ways to represent the number of turns in a transformer:

- - Both N_1 and N_2 are integers
- 2. 1:n
 - The first term might not be 1, if there isn't perfect division, i.e. 2:5 will not be reduced to $1:\frac{5}{2}$

 - $n = \frac{N_2}{N_1}$ This is the form generally used by our textbook
- 3. a:1
 - The second term might not be 1, if there isn't perfect division, i.e. 2:5 will not be reduced to $\frac{2}{5}:1$

 - $a = \frac{N_1}{N_2}$ This is the form generally used by utility companies

Reflecting Elements

There are only 3 equations:

- 1. $\frac{v_1}{v_2} = \frac{N_1}{N_2} \leftarrow \text{Voltage Change}$ 2. $\frac{i_1}{i_2} = -\frac{N_2}{N_1} \leftarrow \text{Current Change}$
- 3. $Z_1 = \frac{Z_2}{n^2}$, as seen in Figure 3
- 4. A negative can be in any one of these, depending on the dot orientation

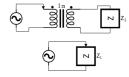


Figure 3: Transformer Reflecting Elements

Transfer Functions/Bode Plots

- Basic form of a Transfer function is $H(\omega) = \frac{X_{Out}}{X_{In}}$
 - $-H(\omega) = \frac{V_{Out}}{V_{In}}$ $-H(\omega) = \frac{I_{Out}}{I_{In}}$ $-H(\omega) = \frac{V_{Out}}{I_{In}}$

$$-H(\omega) = \frac{I_{Out}}{V_{In}}$$

 $-H\left(\omega\right)=\frac{I_{Out}}{V_{In}}$ We replace $s=\omega j\xrightarrow{}\omega=\frac{s}{j}.$

When we have the transfer function, and plug in the equivalency $s = \omega j$, we end up with something like:

$$H(s) = \frac{k(s + z_1)(s + z_2)(s + z_3)\cdots}{(s + p_1)(s + p_2)(s + p_3)\cdots}$$

Now to make the bode plot:

$$||H(s)|| = \frac{k||s + z_1|| ||s + z_2|| ||s + z_3|| \cdots}{||s + p_1|| ||s + p_2|| ||s + p_3|| \cdots}$$

$$||H(\omega)||(dB) = 20 \log (k) + 20 \log (j\omega + z_1) + 20 \log (j\omega + z_2) + 20 \log (j\omega + z_3)$$

$$- 20 \log (j\omega + p_1) - 20 \log (j\omega + p_2) - 20 \log (j\omega + p_3)$$

$$\angle \varphi = \arctan (k) + \arctan (j\omega + z_1) + \arctan (j\omega + z_2) + \arctan (j\omega + z_3)$$

$$- \arctan (j\omega + p_1) - \arctan (j\omega + p_2) - \arctan (j\omega + p_3)$$

Factor	$\mathbf{Magnitude} \; \ H\left(\omega\right)\ (dB)$	Phase $\angle \varphi$
K	$20\log_{10}K$	0°
$\left(j\omega ight) ^{N}$	20NdB/decade (Passes through 1 and continues)	90N°
$\frac{1}{(j\omega)^N}$	-20NdB/decade (Passes through 1 and continues)	-90N°
$\left(1 + \frac{j\omega}{z}\right)^N$	$\begin{cases} 0 & x \le z \\ 20N \text{dB/decade} \end{cases}$	$\begin{cases} 0 & <\frac{z}{10} \\ \frac{1}{2} (90N)^{\circ} & = z \\ 90N^{\circ} & \ge 10z \end{cases}$
$\frac{1}{(1+j\omega/p)^N}$	$\begin{cases} 0 & x \le z \\ -20N \text{dB/decade} \end{cases}$	$\begin{cases} 0 & <\frac{z}{10} \\ \frac{1}{2}(-90N)^{\circ} & = z \\ -90N^{\circ} & \ge 10z \end{cases}$
$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$	$\begin{cases} 0 & \leq \omega_n \\ 40N dB/decade & > \omega_n \end{cases}$	$\begin{cases} 0 & \leq \frac{\omega_n}{10} \\ \frac{1}{2} (180N)^{\circ} & = \omega_n \\ 180N^{\circ} & \geq 10\omega_n \end{cases}$
$\frac{1}{\left[1 + \frac{2j\omega\zeta}{\omega_k} + (j\omega/\omega_k)^2\right]^N}$	$\begin{cases} 0 & \leq \omega_k \\ -40N dB/decade & > \omega_k \end{cases}$	$\begin{cases} 0 & \leq \frac{\omega_n}{10} \\ \frac{1}{2} (-180N)^{\circ} & = \omega_k \\ -180N^{\circ} & \geq 10\omega_k \end{cases}$

Resonant Frequencies

- Remember, $\omega=2\pi f$. Also, $\mathrm{Im}\{Z_{eq}\}=0$ and $\mathrm{Im}\{Y_{eq}\}=0$. $\omega_0=\frac{1}{\sqrt{LC}}$ Imaginary portion of Transfer function vanishes
 - Half-Power Frequencies Frequencies where power dissipated is 1/2 of that dissipated at resonant frequency

$$-\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$-\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$-\omega_0 = \sqrt{\omega_1 \omega_2}$$

- $B = \omega_2 \omega_1 = \frac{R}{L}$ Bandwidth is the frequency band between half-power frequencies

 - $-\ B=\frac{R}{L}$ Series Impedance Circuit
 $-\ B=\frac{R}{RC}$ Parallel Impedance Circuit
- $Q = \frac{\omega_0}{B}$ Quality Factor: Sharpness of resonance peak

 - $-~Q=\frac{\omega_0L}{R}=\frac{1}{\omega_0RC}$ Series Impedance Circuit
 $-~Q=\omega_0RC=\frac{R}{\omega_0L}$ Parallel Impedance Circuit

Laplace Transform

Laplace Transform Properties

Property Name	Time Domain	Frequency Domain
Laplace Transform	$x(t) = \frac{1}{2j\pi} \int_{\sigma-\infty}^{\sigma+\infty} X(s)e^{st}ds$	$X(s) = \int_{0^{-}}^{\infty} x(t)e^{-st}dt$
Linearity	$x(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$	$X(s) = \alpha X_1(s) + \alpha_2 X_2(s)$
Time Scaling	x(at), where $a > 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Time Shift	x(t-a), where $a>0$	$X(s)e^{-as}$
Frequency Shift	$x(t)e^{at}$	X(s)(s-a)
Multiplication by $\sin(\omega_0 t)$	$x(t)\sin\left(\omega_0 t\right)$	$\frac{j}{2} \left[X(s+j\omega_0) - X(s-j\omega_0) \right]$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t)$	$\frac{j}{2} \left[X(s+j\omega_0) + X(s-j\omega_0) \right]$
Multiply by t in Time, Derivative in Frequency	$t^n x(t)$	$(-1)^n \frac{d^n}{ds^n} X(s)$
Mutliply by s in Frequency, Derivative in Time	$\frac{d^n}{dt^n}x(t)$	$s^{n}X(s) - \sum_{i=0}^{n-1} s^{n-1-i} \frac{d^{i}}{dt^{i}} x(t) _{t=0}$
Integration	$\int_{0}^{t} x\left(\lambda\right) d\lambda$	$\frac{1}{s}X(s)$
Multiply in Frequency, Convolution in Time	x(t) * v(t)	X(s)V(s)
Initial Value Theorem	$x(0^+) = \lim_{s \to \infty} sX(s)$	
Final Value Theorem	$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$	
Relation to Fourier Transform	If $X(\omega)$ exists, then $X(s) = X(\omega) _{\omega = \frac{s}{j}}$	

Laplace Transform Pairs

$$\mathcal{L}\{x(t)\} = X(s) \qquad \mathcal{L}\{\delta(t)\} = 1 \qquad \mathcal{L}\{\delta(t-T_0)\} = e^{-stT_0} \qquad \mathcal{L}\{\mathcal{U}(t)\} = \frac{1}{s}$$

$$\mathcal{L}\{t\mathcal{U}(t)\} = \frac{1}{s^2} \qquad \mathcal{L}\{t^n\mathcal{U}(t)\} = \frac{n!}{s^{n+1}} \qquad \mathcal{L}\{\mathcal{U}(t-T_0)\} = \frac{e^{-sT_0}}{s} \qquad \mathcal{L}\{e^{at}\mathcal{U}(t)\} = \frac{1}{(s-a)}$$

$$\mathcal{L}\{te^{at}\mathcal{U}(t)\} = \frac{1}{(s-a)^2} \qquad \mathcal{L}\{t^ne^{at}\mathcal{U}(t)\} = \frac{n!}{(s-a)^{n+1}} \qquad \mathcal{L}\{\cos(bt)\mathcal{U}(t)\} = \frac{s}{s^2 - b^2} \qquad \mathcal{L}\{\sin(bt)\mathcal{U}(t)\} = \frac{b}{s^2 + b^2}$$

$$\mathcal{L}\{e^{-at}\cos(bt)\mathcal{U}(t)\} = \frac{s+a}{(s+a)^2 + b^2} \qquad \qquad \mathcal{L}\{e^{-at}\sin(bt)\mathcal{U}(t)\} = \frac{b}{(s+a)^2 + b^2}$$

$$\mathcal{L}\{re^{-at}\cos(bt)\mathcal{U}(t)\} = \begin{cases} a: & \frac{rs\cos(\theta) + ar\cos(\theta) - br\sin(\theta)}{s^2 + 2as + (a^2 + b^2)} \\ b: & \frac{0.5re^{j\theta}}{s + a - jb} + \frac{0.5re^{-j\theta}}{s + a + jb} \\ c: & \frac{As + B}{s^2 + 2as + c} \begin{cases} r & = \sqrt{\frac{A^2c + B^2 - 2ABa}{c - a^2}} \\ \theta & = \arctan\left(\frac{Aa - B}{A\sqrt{c - a^2}}\right) \end{cases}$$

$$\mathcal{L}\{e^{-at}\left(A\cos(bt) + \frac{B - Aa}{b}\sin(bt)\right)\mathcal{U}(t)\} = \frac{As + B}{s^2 + 2as + c}b = \sqrt{c - a^2}$$

Laplace in Circuits

Steps to solve a circuit with Laplace Transforms:

- 1. Find the initial conditions in the inductor and capacitor
 - (a) v_C (init)
 - (b) i_L (init)
- 2. Convert the circuit elements to the s-domain.
 - (a) $Z_R = R$
 - (b) $Z_C = \frac{1}{Cs}$ (c) $Z_L = Ls$

- 3. Add an independent source next to the element that might have had an initial value.
 - (a) C gets Voltage source in series: $\frac{v_C(\text{init})}{}$
 - (b) L gets Current source in parallel: $\frac{i_L(\text{init})}{i_L(\text{init})}$
 - (c) You CAN perform source transformation on these to get favorable circuits

Multiport Networks

- 1. $[\mathbf{Y}] = [\mathbf{Z}]^{-1}$ 2. $[\mathbf{G}] = [\mathbf{H}]^{-1}$
- 3. $[\mathbf{t}] \neq [\mathbf{T}]^{-1}$

Multiple Multiport Networks can be arranged in 3 ways:

- 1. Multiport Networks in Parallel
- 2. Multiport Networks in Series
- 3. Multiport Networks in Cascade

Z Parameter

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

Y Parameter

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

H Parameter

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

T Parameter

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix}$$

Multiport Networks in Parallel

Admittances add.

$$\begin{bmatrix} \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix}$$

Multiport Networks in Series

Impedances add.

$$\begin{bmatrix} \mathbf{Z} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix}$$

Multiport Networks in Cascade

Transmission Parameters Multiply.

$$egin{bmatrix} \mathbf{T} \end{bmatrix} = egin{bmatrix} \mathbf{T}_{11a} & \mathbf{T}_{12a} \ \mathbf{T}_{21a} & \mathbf{T}_{22a} \end{bmatrix} egin{bmatrix} \mathbf{T}_{11b} & \mathbf{T}_{12b} \ \mathbf{T}_{21b} & \mathbf{T}_{22b} \end{bmatrix}$$

Transistors

$$V_{C} = -R_{L}I_{C}$$

$$I_{C} = \frac{h_{fe}}{1 + h_{oe}R_{L}}$$

$$I_{B} = \frac{h_{oe} (1 + h_{oe}R_{L})}{-h_{oe}R_{L}}V_{C}$$

$$V_{B} = \frac{h_{ie} + (h_{ie}h_{oe} + h_{ie}h_{fe})R_{L}}{h_{fe}R_{L}}$$

$$A_{V} = \frac{V_{C}}{V_{B}} = \frac{-h_{fe}R_{L}}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_{L}}$$

$$A_{I} = \frac{I_{C}}{I_{B}} = \frac{h_{fe}}{1 + h_{oe}R_{L}}$$

$$Z_{in} = h_{oe} + h_{fe} \left(\frac{-h_{re}}{R_{S} + h_{ie}}\right)$$

$$Z_{out} = \frac{R_{S} + h_{ie}}{h_{oe} (R_{S} + h_{ie}) - h_{fe}h_{re}}$$
MUST be Thevenin Equivalent