General Equations

- KCL: $\sum I_{in} = \sum I_{Out} \rightarrow$ Node's Input Current = Node's Output Current
- KVL: $\sum V = 0 \rightarrow$ Voltage across a loop totals to 0.
- Ohm's Law: $V = IR$

Phasors

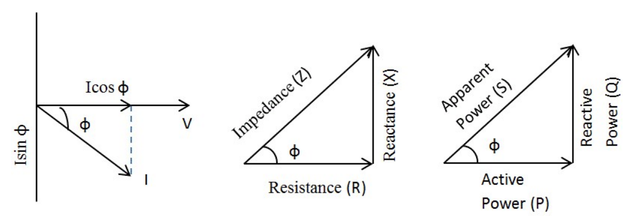
Phasors will only show us the steady state response of the circuit, not the transient response.
Eq: $v(t) = V_M \cos(\omega t + \theta) \leftrightarrow \bar{V} = V_M \angle \theta_v = V_M e^{\jmath \theta_v} = V_M (\cos\theta_v + \jmath \sin\theta_v)$
You can use phasors with Nodal Analysis, Mesh/Loop Analysis, Superposition, and Thevenin and Norton Equivalencies.

$z_1 = x_1 + \jmath y_2 = r_1 \angle \phi_1,\ z_2 = x_2 + \jmath y_2 = r_2 \angle \phi_2$

Addition	$z_1 + z_2 = (x_1 + x_2) + \jmath (y_1 + y_2)$
Subtraction	$z_1 - z_2 = (x_1 - x_2) + \jmath (y_1 - y_2)$
Multiplication	$z_1 z_2 = r_1 r_2 \angle (\phi_1 + \phi_2)$
Division	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle (\phi_1 - \phi_2)$
Reciprocal	$\frac{1}{z_1} = \frac{1}{z_1} \angle - \phi_1$
Square Root	$\sqrt{z_1} = \sqrt{r_1} \angle \frac{\phi_1}{2}$
Complex Conjugate	$z_1^* = x - \jmath y = r \angle - \phi_1 = r e^{-\jmath \phi_1}$

RMS/Complex Power/Max Power Transfer

- $X_{rms} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dt} = \frac{X_{PP}}{2\sqrt{2}} = \frac{X_{PP}}{2\sqrt{2}}$
- $P_{avg} = \frac{1}{2} \mathrm{Re}\{\mathbf{V}\mathbf{I}^*\} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$
- $\mathbf{S} = I_{rms}^2 \mathbf{Z} = \frac{V_{rms}^2}{\mathbf{Z}^*} = \mathbf{V}_{rms} \mathbf{I}_{rms}^*$
- $\sum_{k=1}^n S_k$
- $C = \frac{Q_C}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$
- $L = \frac{V_{rms}^2}{\omega (Q_1 - Q_2)}$



Name	**Symbol**	**Equation(s)**	**Units**
Complex Power	$\mathbf{S}$	$\frac{P}{Pf} \angle \arccos(Pf) = P + \jmath Q = \mathbf{V}_{rms} \mathbf{I}_{rms}^* = \lvert\mathbf{V}_{rms}\rvert \lvert\mathbf{I}_{rms}\rvert \angle (\theta_v - \theta_i)$	V A
Apparent Power	S	$\lVert\mathbf{S}\rVert = \lvert\mathbf{V}_{rms}\rvert \lvert\mathbf{I}_{rms}\rvert = \sqrt{P^2 + Q^2}$	V A
Real Power	P	$\mathrm{Re}\{\mathbf{S}\} = S * Pf \cos[\arccos(Pf)] = S \cos(\theta_v - \theta_i)$	W
Reactive (Imaginary) Power	Q	$\mathrm{Im}\{\mathbf{S}\} = S * Pf \sin[\arccos(Pf)] = S \sin(\theta_v - \theta_i)$	VAR
Power Factor	Pf	$\frac{P}{S} = \cos(\theta_v - \theta_i)$	Lead/Lag

NOTE: If you are looking for 3-Phase complex power, it is in 3-Phase Circuits.

Elements

Methods to Solve Equations

Nodal Analysis

1. # of Nodes? $\rightarrow n$
2. Make one node the reference node. Assign $n-1$ nodal voltages
3. For a **voltage** source, write a CONSTRAINT EQUATION (Con. Eq.). If there is a voltage source between 2 non-reference nodes, make that a **SUPERNODE**.
4. Write KCL at each node. $(n-1)$ equations.
5. Solve Equations.

Relation	R	C	L
v-i i-v	$V = IR$ $I = \frac{V}{R}$	$v = \frac{1}{C}\int_{t_0}^{t} i(x)dx + v(t_0)$ $i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$ $i = \frac{1}{L}\int_{t_0}^{t} v(x)dx + i(t_0)$
P or W	$P = I^2R = \frac{V^2}{R}$	$P = \frac{1}{2}Cv_c^2$	$W = \frac{1}{2}Li_l^2$
Series Parallel	$R_{eq} = R_1 + R_2 + \ldots + R_n$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$ $C_{eq} = C_1 + C_2 + \ldots + C_n$	$L_{eq} = L_1 + L_2 + \ldots + L_n$ $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$
@ Steady State	Same (Nothing Happens)	Open Circuit	Short Circuit
Phasors	$Z_R = R$	$Z_C = \frac{1}{j\omega C}$	$Z_L = j\omega L$

Mesh/Loop Analysis

1. # of Nodes? $\rightarrow n$ # of Branches? $\rightarrow b$ # of meshes/loops? $\rightarrow b - n + 1 = l$
2. Assign l loop currents.
3. For **current** sources, write a CONSTRAINT EQUATION (Con. Eq.). If there is a current source between 2 meshes, that's a **SUPERMESH**.
4. Write KVL for each mesh.
5. Solve Equations.

Superposition

- # of sources, n, determines the number of equations you will have.
- Shut off each source, one at a time, solving for the term that you want.
  - Voltage Source = S.C.
  - Current Source = O.C.
- Sum each of the individual terms together. $\sum_{i=1}^{n} x_i$
- **THIS IS THE ONLY WAY TO SOLVE FOR A CIRCUIT WITH MULTIPLE SOURCES!!**

Source Transformations

ALL source transformations obey Ohm's Law. $V = IR$. This will **ONLY** work on impedances in series with **VOLTAGE** sources, or impedances in parallel with **CURRENT** sources.

Thevenin and Norton Equivalencies

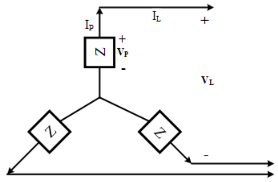
- ONLY independent sources - Zero all sources, find $\mathbf{Z}_{eq}$.
  - 0-ing Current Sources = O.C., 0-ing Voltage Sources = S.C.
  - Look at circuit from load's perspective for $\mathbf{Z}_{eq}$
  - $\mathbf{V}_{Th} = \mathbf{V}_{OC}$, $\mathbf{I}_N = \mathbf{I}_{SC}$
- BOTH dependent and independent sources
  - Find $\mathbf{V}_{Th} = \mathbf{V}_{OC}$, $\mathbf{I}_N = \mathbf{I}_{SC}$
  - Solve $\mathbf{Z}_{Th} = \frac{\mathbf{V}_{OC}}{\mathbf{I}_{SC}}$
- ONLY dependent sources
  - $\mathbf{V}_{Th} = 0$, $\mathbf{I}_N = 0$
  - $\mathbf{Z}_{Th} = \mathbf{Z}_N \rightarrow$ Attach test source @ load.
    * If voltage test source, find current. If current test source, find voltage
  - $\mathbf{Z}_{Th} = \frac{\mathbf{V}_{Test}}{\mathbf{I}_{Test}}$

Maximum Power Transfer - AC

- $\mathbf{Z}_{Load} = \mathbf{Z}_{Th}^*$, $R_{Th} = \text{Re}\{\mathbf{Z_{Th}}\}$, $R_L = |\mathbf{Z}_{Th}| = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2}$
- $P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}}$

3-Phase Circuits

- $C_Y = \frac{Q_C}{3\omega \| V_{\phi,rms} \|^2}$
- $C_\Delta = \frac{C_Y}{3}$
- Power lost due to line: $P_{Lost} = Z_{Wire} I_L$

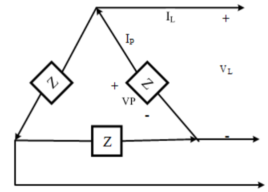


(a) **3 Phase Y-Connection**

$$I_L = I_P \qquad V_{LL} = \sqrt{3}V_P\angle 30^\circ \qquad Z_Y = \frac{Z_\Delta}{3}$$

$$\mathbf{S} = \sqrt{3}\mathbf{V}_L\mathbf{I}_L^* \qquad \mathbf{S} = 3\mathbf{V}_P\mathbf{I}_L^* \qquad \phi = \theta_{V_P} - \theta_{I_P}$$

$$\mathbf{S} = S\angle\phi \qquad P = \|\mathbf{S}\|\cos(\phi) \qquad Q = \|\mathbf{S}\|\sin(\phi)$$



(b) **3 Phase Δ-Connection**

$$I_L = \sqrt{3}I_P\angle -30^\circ \qquad V_{LL} = V_P \qquad Z_\Delta = 3Z_Y$$

$$\mathbf{S} = \sqrt{3}\mathbf{V}_L\mathbf{I}_L^* \qquad \mathbf{S} = 3\mathbf{V}_P\mathbf{I}_P^* \qquad \phi = \theta_{V_P} - \theta_{I_P}$$

$$\mathbf{S} = S\angle\phi \qquad P = \|\mathbf{S}\|\cos(\phi) \qquad Q = \|\mathbf{S}\|\sin(\phi)$$

You want to get everything into Y formation, because the common neutral allows you to do single-phase analysis.

Mutual Inductance

Equivalent Mutual Inductance

Series-**Aiding** Connection	$L = L_1 + L_2 + 2M$	
Series-**Opposing** Connection	$L = L_1 + L_2 - 2M$	

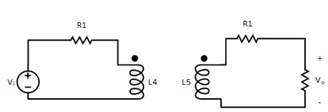
Dot Convention

There are 2 cases:

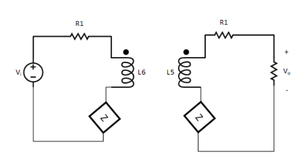
1. Current enters through dotted side on 1 inductor $\longrightarrow$ **POSITIVE VOLTAGE ON DOTTED SIDE OF OTHER INDUCTOR**
   - Current flows into the dotted side of one inductor
   - Current flows out of the un-dotted side of second inductor, just like the first
2. Current enters through **NON**-dotted side of 1 inductor $\longrightarrow$ **POSITIVE VOLTAGE ON UN-DOTTED SIDE OF OTHER INDUCTOR**
   - Current flows into the un-dotted side of one inductor
   - Current flows out of the dotted side of the second inductor, just like the first

Solving Disjoint Coupled Circuits

1. Apply KVL
2. Don't forget about the Mutual Inductance Voltage Difference because of the first current
3. There is a second way to thing about these, shown in Figures 2a and 2b, below.



(a) Original Circuit



(b) Circuit "Simplified" by Adding Dependent Sources

The sign on the dependent sources depends on which side of the inductor the current is going into. Use the Dot Convention to determine which direction the source's voltage should go.

Transformers

These elements consume no power, and convert voltages and currents.

- $\frac{v_1}{v_2} = \frac{N_1}{N_2} \longleftarrow$ Voltage Change
- $\frac{i_1}{i_2} = -\frac{N_2}{N_1} \longleftarrow$ Current Change

Representations for Turns

There are 3 common ways to represent the number of turns in a transformer:

1. $N_1 : N_2$
   - Both N_1 and N_2 are integers
2. $1 : n$
   - The first term might not be 1, if there isn't perfect division, i.e. $2 : 5$ will not be reduced to $1 : \frac{5}{2}$
   - $n = \frac{N_2}{N_1}$
   - This is the form generally used by our textbook
3. $a : 1$
   - The second term might not be 1, if there isn't perfect division, i.e. $2 : 5$ will not be reduced to $\frac{2}{5} : 1$
   - $a = \frac{N_1}{N_2}$
   - This is the form generally used by utility companies

Reflecting Elements

There are only 3 equations:

1. $\frac{v_1}{v_2} = \frac{N_1}{N_2} \longleftarrow$ Voltage Change
2. $\frac{i_1}{i_2} = -\frac{N_2}{N_1} \longleftarrow$ Current Change
3. $Z_1 = \frac{Z_2}{n^2}$, as seen in Figure 3
4. A negative can be in any one of these, depending on the dot orientation

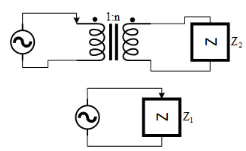


Figure 3: Transformer Reflecting Elements

Transfer Functions/Bode Plots

- Basic form of a Transfer function is $H(\omega) = \frac{X_{Out}}{X_{In}}$
  - $H(\omega) = \frac{V_{Out}}{V_{In}}$
  - $H(\omega) = \frac{I_{Out}}{I_{In}}$
  - $H(\omega) = \frac{V_{Out}}{I_{In}}$
  - $H(\omega) = \frac{I_{Out}}{V_{In}}$

We replace $s = \omega j \rightarrow \omega = \frac{s}{j}$.

When we have the transfer function, and plug in the equivalency $s = \omega j$, we end up with something like:

$$H(s) = \frac{k(s+z_1)(s+z_2)(s+z_3)\cdots}{(s+p_1)(s+p_2)(s+p_3)\cdots}$$

Now to make the bode plot:

$$\begin{aligned}
\|H(s)\| &= \frac{k\|s+z_1\|\|s+z_2\|\|s+z_3\|\cdots}{\|s+p_1\|\|s+p_2\|\|s+p_3\|\cdots} \\
\|H(\omega)\|(dB) &= 20\log(k) + 20\log(j\omega+z_1) + 20\log(j\omega+z_2) + 20\log(j\omega+z_3) \\
&\quad - 20\log(j\omega+p_1) - 20\log(j\omega+p_2) - 20\log(j\omega+p_3) \\
\angle\varphi &= \arctan(k) + \arctan(j\omega+z_1) + \arctan(j\omega+z_2) + \arctan(j\omega+z_3) \\
&\quad - \arctan(j\omega+p_1) - \arctan(j\omega+p_2) - \arctan(j\omega+p_3)
\end{aligned}$$

Factor	**Magnitude** $\|H(\omega)\|(dB)$	**Phase** $\angle\varphi$
K	$20\log_{10} K$	0°
$(j\omega)^N$	$20N$dB/decade (Passes through 1 and continues)	$90N^\circ$
$\frac{1}{(j\omega)^N}$	$-20N$dB/decade (Passes through 1 and continues)	$-90N^\circ$
$\left(1+\frac{j\omega}{z}\right)^N$	$\begin{cases} 0 & x \leq z \\ 20N\text{dB/decade} & \end{cases}$	$\begin{cases} 0 & < \frac{z}{10} \\ \frac{1}{2}(90N)^\circ & = z \\ 90N^\circ & \geq 10z \end{cases}$
$\frac{1}{(1+j\omega/p)^N}$	$\begin{cases} 0 & x \leq z \\ -20N\text{dB/decade} & \end{cases}$	$\begin{cases} 0 & < \frac{z}{10} \\ \frac{1}{2}(-90N)^\circ & = z \\ -90N^\circ & \geq 10z \end{cases}$
$\left[1+\frac{2j\omega\zeta}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]^N$	$\begin{cases} 0 & \leq \omega_n \\ 40N\text{dB/decade} & > \omega_n \end{cases}$	$\begin{cases} 0 & \leq \frac{\omega_n}{10} \\ \frac{1}{2}(180N)^\circ & = \omega_n \\ 180N^\circ & \geq 10\omega_n \end{cases}$
$\frac{1}{\left[1+\frac{2j\omega\zeta}{\omega_k}+(j\omega/\omega_k)^2\right]^N}$	$\begin{cases} 0 & \leq \omega_k \\ -40N\text{dB/decade} & > \omega_k \end{cases}$	$\begin{cases} 0 & \leq \frac{\omega_n}{10} \\ \frac{1}{2}(-180N)^\circ & = \omega_k \\ -180N^\circ & \geq 10\omega_k \end{cases}$

Resonant Frequencies

Remember, $\omega = 2\pi f$. Also, $\text{Im}\{Z_{eq}\} = 0$ and $\text{Im}\{Y_{eq}\} = 0$.

- $\omega_0 = \frac{1}{\sqrt{LC}}$ - Imaginary portion of Transfer function vanishes
- Half-Power Frequencies - Frequencies where power dissipated is 1/2 of that dissipated at resonant frequency
  - $\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$
  - $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$
  - $\omega_0 = \sqrt{\omega_1 \omega_2}$
- $B = \omega_2 - \omega_1 = \frac{R}{L}$ - Bandwidth is the frequency band between half-power frequencies
  - $B = \frac{R}{L}$ - Series Impedance Circuit
  - $B = \frac{1}{RC}$ - Parallel Impedance Circuit
- $Q = \frac{\omega_0}{B}$ - Quality Factor: Sharpness of resonance peak
  - $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$ - Series Impedance Circuit
  - $Q = \omega_0 RC = \frac{R}{\omega_0 L}$ - Parallel Impedance Circuit

Laplace Transform

Laplace Transform Properties

Property Name	Time Domain	Frequency Domain
Laplace Transform	$x(t) = \frac{1}{2j\pi}\int_{\sigma-\infty}^{\sigma+\infty} X(s)e^{st}ds$	$X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$
Linearity	$x(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$	$X(s) = \alpha X_1(s) + \alpha_2 X_2(s)$
Time Scaling	$x(at)$, where $a > 0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Time Shift	$x(t-a)$, where $a > 0$	$X(s)e^{-as}$
Frequency Shift	$x(t)e^{at}$	$X(s)(s-a)$
Multiplication by $\sin(\omega_0 t)$	$x(t)\sin(\omega_0 t)$	$\frac{j}{2}\left[X(s+j\omega_0) - X(s-j\omega_0)\right]$
Multiplication by $\cos(\omega_0 t)$	$x(t)\cos(\omega_0 t)$	$\frac{j}{2}\left[X(s+j\omega_0) + X(s-j\omega_0)\right]$
Multiply by t in Time, Derivative in Frequency	$t^n x(t)$	$(-1)^n \frac{d^n}{ds^n}X(s)$
Mutliply by s in Frequency, Derivative in Time	$\frac{d^n}{dt^n}x(t)$	$s^n X(s) - \sum_0^{n-1} s^{n-1-i}\frac{d^i}{dt^i}x(t)\rvert_{t=0^-}$
Integration	$\int_0^t x(\lambda)\,d\lambda$	$\frac{1}{s}X(s)$
Multiply in Frequency, Convolution in Time	$x(t) * v(t)$	$X(s)V(s)$
Initial Value Theorem	$x(0^+) = \lim_{s\to\infty} sX(s)$	
Final Value Theorem	$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$	
Relation to Fourier Transform	If $X(\omega)$ exists, then $X(s) = X(\omega)\rvert_{\omega=\frac{s}{j}}$	

Laplace Transform Pairs

$$\mathcal{L}\{x(t)\} = X(s) \qquad \mathcal{L}\{\delta(t)\} = 1 \qquad \mathcal{L}\{\delta(t-T_0)\} = e^{-stT_0} \qquad \mathcal{L}\{\mathcal{U}(t)\} = \frac{1}{s}$$

$$\mathcal{L}\{t\mathcal{U}(t)\} = \frac{1}{s^2} \qquad \mathcal{L}\{t^n\mathcal{U}(t)\} = \frac{n!}{s^{n+1}} \qquad \mathcal{L}\{\mathcal{U}(t-T_0)\} = \frac{e^{-sT_0}}{s} \qquad \mathcal{L}\{e^{at}\mathcal{U}(t)\} = \frac{1}{(s-a)}$$

$$\mathcal{L}\{te^{at}\mathcal{U}(t) = \frac{1}{(s-a)^2} \qquad \mathcal{L}\{t^n e^{at}\mathcal{U}(t) = \frac{n!}{(s-a)^{n+1}} \qquad \mathcal{L}\{\cos(bt)\mathcal{U}(t) = \frac{s}{s^2-b^2} \qquad \mathcal{L}\sin(bt)\mathcal{U}(t) = \frac{b}{s^2-b^2}$$

$$\mathcal{L}\{e^{-at}\cos(bt)\mathcal{U}(t) = \frac{s+a}{(s+a)^2+b^2} \qquad \mathcal{L}\{e^{-at}\sin(bt)\mathcal{U}(t) = \frac{b}{(s+a)^2+b^2}$$

$$\mathcal{L}\{re^{-at}\cos(bt+\theta)\mathcal{U}(t) = \begin{cases} a: & \frac{rs\cos(\theta)+ar\cos(\theta)-br\sin(\theta)}{s^2+2as+(a^2+b^2)} \\ b: & \frac{0.5re^{j\theta}}{s+a-jb} + \frac{0.5re^{-j\theta}}{s+a+jb} \\ c: & \frac{As+B}{s^2+2as+c} \begin{cases} r & = \sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}} \\ \theta & = \arctan\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right) \end{cases} \end{cases}$$

$$\mathcal{L}\{e^{-at}\left(A\cos(bt) + \frac{B-Aa}{b}\sin(bt)\right)\mathcal{U}(t) = \frac{As+B}{s^2+2as+c} b = \sqrt{c-a^2}$$