

# EDAN20: Language Technology - Reference Sheet

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# 1 Linguistics

**Defn 1** (Linguistics). *Linguistics* is the study and the description of human languages. Linguistics have been developed since ancient times and the Middle Ages. *Modern Linguistics* developed between the end of the 19<sup>th</sup> century and beginning of the 20<sup>th</sup> century. Its founder and prominent figure was Ferdinand de Saussure.

There is something else that we will be working with, called Computational Linguistics.

**Defn 2** (Computational Linguistics). *Computational Linguistics* is a subset of both Linguistics and computer science. Its goal is to design mathematical models of language structures enabling the automation of language processing by a computer. We can consider computational linguistics as the formalization of linguistic theories and models, or their implementation in a machine. New linguistic theories can be developed with the aid of a computer too.

Historically, there are 3 disciplines of Linguistics.

1. Phonetics
2. Words
3. Syntax

## 1.1 Phonetics

**Defn 3** (Phonetics). *Phonetics* concerns the production and perception of acoustic sounds that form the speech signal. In every language sounds can be classified into a finite set of Phonemes

**Defn 4** (Phonemes). *Phonemes* are the building blocks of Phonetics. Traditionally, Phonemes include Vowels and Consonants. Phonemes are assembled into Syllables to build words.

Examples include: *pa, pi, po*.

**Defn 5** (Vowels). *Vowels* are a speech sound that is produced by a comparatively open configuration of the vocal tract, with vibration of the vocal cords, but without audible friction. They are a unit of the sound system of a language that forms the nucleus of Syllables.

Examples include: *a, e, i, o*.

**Defn 6** (Consonants). *Consonants* are a speech sound in which the breath is at least partly obstructed and which can be combined with Vowels to form Syllables.

Examples include: *p, f, r, m*.

**Defn 7** (Syllables). *Syllables* are a unit of pronunciation having one vowel sound, with or without surrounding consonants, forming the whole or part of a word.

## 1.2 Words

**Defn 8** (Words). *Words*

## 1.3 Syntax

**Defn 9** (Syntax). *Syntax*

## A Trigonometry

### A.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{A.2})$$

### A.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm j\alpha} = \cos(\alpha) \pm j \sin(\alpha) \quad (\text{A.3})$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{A.4})$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{A.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{A.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{A.7})$$

### A.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{A.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{A.9})$$

### A.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{A.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{A.11})$$

### A.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{A.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{A.13})$$

### A.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2} \quad (\text{A.14})$$

$$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2} \quad (\text{A.15})$$

### A.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{A.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{A.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{A.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{A.19})$$

## A.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{A.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.22})$$

## A.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{A.23})$$

## A.10 Rectangular to Polar

$$a + jb = \sqrt{a^2 + b^2} e^{j\theta} = r e^{j\theta} \quad (\text{A.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{A.25})$$

## A.11 Polar to Rectangular

$$r e^{j\theta} = r \cos(\theta) + j r \sin(\theta) \quad (\text{A.26})$$

## B Calculus

### B.1 Fundamental Theorems of Calculus

**Defn B.1.1** (First Fundamental Theorem of Calculus). The *first fundamental theorem of calculus* states that, if  $f$  is continuous on the closed interval  $[a, b]$  and  $F$  is the indefinite integral of  $f$  on  $[a, b]$ , then

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{B.1})$$

**Defn B.1.2** (Second Fundamental Theorem of Calculus). The *second fundamental theorem of calculus* holds for  $f$  a continuous function on an open interval  $I$  and  $a$  any point in  $I$ , and states that if  $F$  is defined by

$$F(x) = \int_a^x f(t) dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t) dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \quad (\text{B.2})$$

**Defn B.1.3** (argmax). The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname{argmax}_x$$

### B.2 Rules of Calculus

#### B.2.1 Chain Rule

**Defn B.2.1** (Chain Rule). The *chain rule* is a way to differentiate a function that has 2 functions multiplied together. If

$$f(x) = g(x) \cdot h(x)$$

then,

$$\begin{aligned} f'(x) &= g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ \frac{df(x)}{dx} &= \frac{dg(x)}{dx} \cdot h(x) + g(x) \cdot \frac{dh(x)}{dx} \end{aligned} \quad (\text{B.3})$$