General Equations

- KCL: $\sum I_{in} = \sum I_{Out}$
  Node's Input Current = Node's Output Current
- KVL: $\sum V = 0$
  Voltage across a loop totals to 0.
- Conservation of Power: $\sum P = 0$
- Power: $P = VI$
- Ohm's Law: $V = IR$

Resistors

- $V = IR$
- Equivalent Series Resistor: $R_{eq} = \sum_{n=1}^{m} R_n$
- Equivalent Parallel Resistor: $\frac{1}{R_{eq}} = \sum_{n=1}^{m} \frac{1}{R_n}$
- Series Voltage Division: $V_1 = \frac{R_1}{R_1+R_2} V_{Source}$
- Parallel Current Division: $I_1 = \frac{R_2}{R_1+R_2} I_{Source}$

Methods to Solve Equations

Nodal Analysis

1. # of Nodes? $\rightarrow n$
2. Make one node the reference node.
3. Assign $n-1$ nodal voltages
4. For a **voltage** source, write a CONSTRAINT EQUATION (Con. Eq.). If there is a voltage source between 2 non-reference nodes, make that a **SUPERNODE**.
5. Write KCL at each node. $(n-1)$ equations.
6. Solve Equations.

Mesh/Loop Analysis

1. # of Nodes? $\rightarrow n$ # of Branches? $\rightarrow b$
2. # of meshes/loops? $\rightarrow b - n + 1 = l$
3. Assign l loop currents.
4. For **current** sources, write a CONSTRAINT EQUATION (Con. Eq.). If there is a current source between 2 meshes, that's a **SUPERMESH**.
5. Write KVL for each mesh.
6. Solve Equations.

Source Transformations

ALL source transformations obey Ohm's Law. $V = IR$. This will **ONLY** work on resistors in series with **VOLTAGE** sources, or resistors in parallel with **CURRENT** sources.

Figure 1: Left: Voltage Source in Series with a Resistor, Right: Current Source in Parallel with a Resistor

Capacitors and Inductors

Relation	R	C	L
v-i i-v	$V = IR$ $I = \frac{V}{R}$	$v = \frac{1}{C}\int_{t_0}^{t} i(x)dx + v(t_0)$ $i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$ $i = \frac{1}{L}\int_{t_0}^{t} v(x)dx + i(t_0)$
P or W	$P = I^2R = \frac{V^2}{R}$	$P = \frac{1}{2}Cv_c^2$	$W = \frac{1}{2}Li_l^2$
Series Parallel	$R_{eq} = R_1 + R_2 + \ldots + R_n$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$ $C_{eq} = C_1 + C_2 + \ldots + C_n$	$L_{eq} = L_1 + L_2 + \ldots + L_n$ $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$
@ Steady State	Same (Nothing Happens)	Open Circuit	Short Circuit

Thevenin and Norton Equivalents

- ONLY independent sources - Zero all sources, find R_{eq}.
  - 0-ing Current Sources = OC
  - 0-ing Voltage Sources = SC
  - Look at circuit from load's perspective for R_{eq}
  - $V_{Th} = V_{OC}$
  - $I_N = I_{SC}$
- BOTH dependent and independent sources
  - Find $V_{Th} = V_{OC}$
  - Find $I_N = I_{SC}$
  - Solve $R_{Th} = \frac{V_{OC}}{I_{SC}}$
- ONLY dependent sources
  - $V_{Th} = 0$
  - $I_N = 0$
  - $R_{Th} = R_N \rightarrow$ Attach test source @ load.
    - If voltage test source, find current
    - If current test source, find voltage
  - $R_{Th} = \frac{V}{I}$
- **Maximum Power Transfer**
  $R_{Load} = R_{Th}$

  $P_{Max} = \frac{V_{Th}^2}{4R_{Th}}$

Superposition

- # of sources, n, determines the number of equations you will have.
- Shut off each source, one at a time, solving for the term that you want.

- Voltage Source = SC
- Current Source = OC

- Sum each of the individual terms together. $\sum_{i=1}^{n} x_i$

Solving for RL, RC Circuits

Shortcut Method

$x(t) = x(\infty) + (x(0) - x(\infty))e^{\frac{-t}{\tau}}$, where x could be voltage or current.
$\tau = RC$ or $\tau = \frac{L}{R}$

Differential Method

1. Start by finding the initial value of the voltage across the capacitor or current through the inductor. $t = 0^-$, if switch closes at $t = 0$.
2. Use KCL or KVL on the portion of the circuit with the capacitor or inductor.
3. Solve the first-order homogenous linear differential equation, with its characteristic equation.
4. You should end with a solution that is similar to this: $x(t) = K_1 e^m + x_s$, where m is the solution to the characteristic equation.

Solving for RLC Circuits

Shortcut Method

$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = \omega_0^2 A_S$

$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
There are 3 cases.
Where A_S is the source, and K_1 and K_2 are constants that are found:

1. $\alpha > \omega_0 \longrightarrow$ Overdamped
   $x(t) = A_S + K_1 e^{S_1 t} + K_2 e^{S_2 t}$
2. $\alpha = \omega_0 \longrightarrow$ Critically Damped
   $x(t) = A_S + (K_1 + K_2 t)\, e^{-\alpha t}$
3. $\alpha < \omega_0 \longrightarrow$ Underdamped
   $x(t) = A_S + (K_1 \cos(\omega_d t) + K_2 \sin(\omega_d t))\, e^{-\alpha t}$
   $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Differential Method

1. Start by finding the voltage across the capacitor or current through the inductor. $(t = 0^-)$, this is your first initial condition.
2. Finding a basic equation for the circuit. $v_R + v_C + v_L = V_S$, or $i_R + i_C + i_L = I_S$.
3. Using as an example, $v_R + v_C + v_L = 0$.
   (a) Substitute in what you want to find, using the equations present in the relation table.
   $L\frac{di}{dt} + \frac{1}{C}\int_0^t i_C(x)dx + v_C(0) + iR = V_S$
   (b) Find the second initial condition by putting $t = 0$ into the above equation, and solving for $\frac{di}{dt}$.
   $L\frac{di}{dt} + \frac{1}{C}\int_0^0 i_C(x)dx + v_C(0) + iR = 0 \longrightarrow L\frac{di}{dt} + \frac{1}{C}(0) + v_C(0) + iR = V_S$
   (c) Remove the integral by differentiating the equation.
   $\{L\frac{di}{dt} + \frac{1}{C}\int_0^t i_C(x)dx + v_C(0) + iR = V_S\}\frac{d}{dt} \longrightarrow L\frac{d^2i}{dt^2} + \frac{1}{C}i_C(t) + R\frac{di}{dt} = 0$
   (d) Solve the 2nd Order, Linear, Homogenous Differential Equation. (Characteristic Equation $m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$)
   $\frac{d^2i}{dt} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$

(e) Solving this equation will generate an equation for $i(t)$.
$i(t) = K_1 e^{m_1} + K_2 e^{m_2} + I_S$

(f) Solve for K_1 and K_2 by using the 2 initial conditions you found earlier.

4. If you want to find the other half of the equation, voltage across the inductor, or current through a capacitor, use the appropriate relation equation.

Op Amps

ONLY 2 rules:
The two terminals of the op amp are denoted by the $+$ and $-$.

1. $v^+ = v^-$
2. $i^+ = i^- = 0$

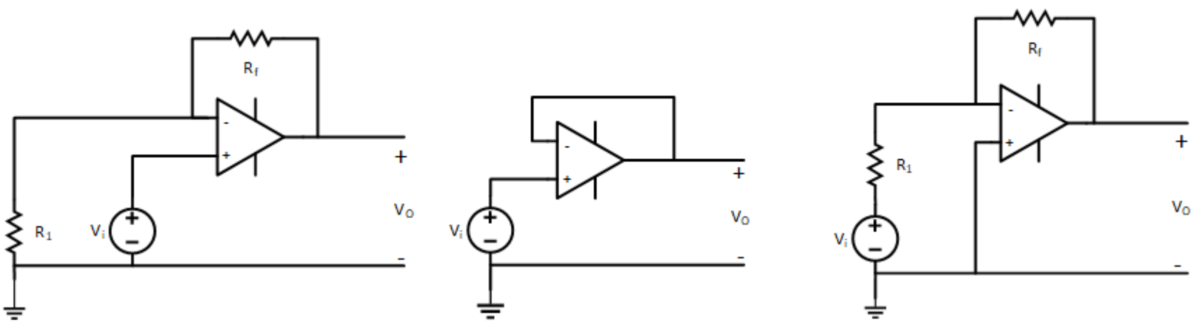


Figure 2: Left: Non-Inverting Amplifier, Middle: Buffer, Right: Inverting Amplifier

$V_O = \left(1 + \frac{R_f}{R_1}\right) V_i$ $\quad$ $V_O = V_i$ $\quad$ $V_O = \frac{-R_f}{R_1} V_i, R_f \neq R_1$

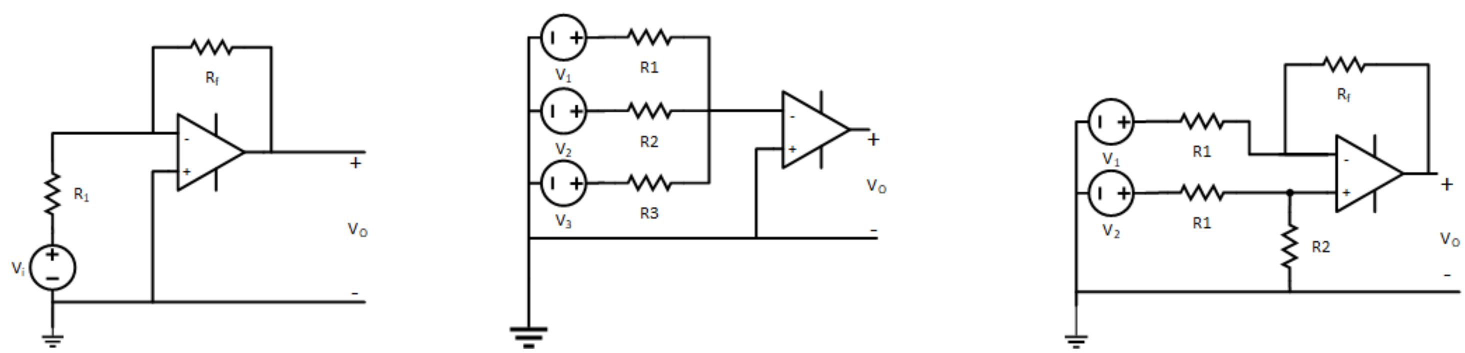


Figure 3: Left: Inverter, Middle: Summer, Right: Difference Amplifier

$V_O = \frac{-R_f}{R_1} V_i, R_f = R_1$ $\quad$ $V_O = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$ $\quad$ $V_O = \frac{R_2}{R_1}\left(V_2 - V_1\right)$

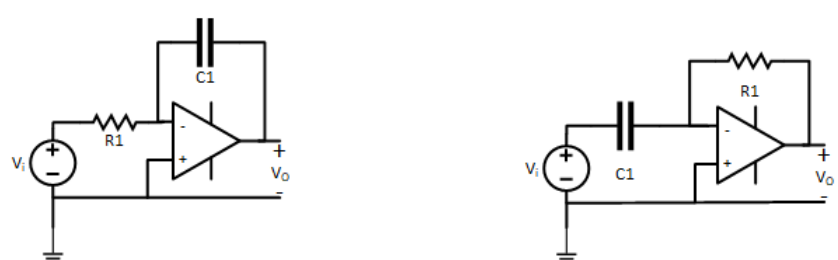
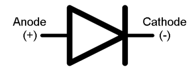


Figure 4: Left: Integrator, Right: Differentiator

$V_O(t) - V_O(0) = \frac{-1}{RC} \int_0^t v_i dt$ $\quad$ $V_O = -R_f C \frac{dv_i}{dt}$

Diodes

In an ideal diode, current is only allowed to flow in one direction, the direction of the arrow.



In an ideal diode, there are 2 cases:

1. $V_{Anode} - V_{Cathode} > 0$, Anode Voltage > Cathode Voltage
   The diode is **ON**. Current will flow.

2. $V_{Anode} - V_{Cathode} < 0$, Anode Voltage < Cathode Voltage
   The diode is **OFF**. Current will not flow.

In reality, a silicon diode needs a $0.7V$ drop across the diode. A germanium diode needs a $0.3V$ drop.