1 General Stuff

- Density - $\rho = \frac{\Delta m}{\Delta V}$
  - Uniform Density - $\rho = \frac{m}{V}$
- Pressure - $p = \frac{\Delta F}{\Delta A}$
  - Uniform Force on Flat Area - $\rho = \frac{F}{A}$
  - Conversions - $1atm = 1.01 \times 10^5 Pa = 760torr = 14.7lb/in^2$
- Trig Relations are in Trigonometric Formulas, Section A.2.1.

2 Fluids

We must satisfy several parameters to make life easier, and to use most of these formulae.

1. Incompressible - Density of the fluid is constant
2. Non-turbulent Flow - Think of fluids swirling around an object
3. Isostatic Pressure - Pressure inside the fluid is the same in all directions

- Pressure at Some Depth - $p_2 = p_1 + \rho g \left(y_1 - y_2\right)$
  - Pressure at Depth $h \rightarrow p = p_0 + \rho g h$
- Pascal's Principle - 2 Parts
  1. $\vec{F}_o = \vec{F}_i \, \frac{A_o}{A_i}$
  2. $d_o = d_i \frac{A_i}{A_o}$
  - When 2 pressures should be equal, their forces are inversely proportional
  - Set each pressure equal to each other, then solve for the missing variable.
- Archimedes' Principle - $\vec{F}_{Up} = \vec{F}_{Down}$
  - Usually breaks down to $\vec{F}_{Bouyant} = \vec{F}_{g,Object}$
  - $\vec{F}_{Bouyant} = m_{Object} g$
  - $\vec{F}_{Bouyant} = \rho_{Object} V_{Object} g$
- Continuity of Fluids
  - $A_1 v_1 = A_2 v_2$
- Bernoulli's Equation - $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$
  - Fluids at Rest - $p_2 = p_1 + \rho g \left(y_1 - y_2\right)$
  - Fluids not Changing Height - $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

3 Waves

Usually of form $y = y_m \sin\left(kx \pm \omega t\right)$ There are two types of waves:

1. Transverse Waves - Waves where displacement from equilibrium is orthogonal to direction of propagation
   - String Waves
   - Electromagnetic Waves
2. Longitudinal Waves - Waves where displacement from equilibrium is parallel to direction of propagation
   - Pressure Waves
   - Sound Waves (Which are a type of pressure wave)

- y_m - Amplitude, m
- k - Angular Wave Number, rad/m
  - $k = \frac{2\pi}{\lambda}$
  - λ is wavelength, m
- ω - Angular Frequency, rad/s
  - $\omega = 2\pi f$
  - f is frequency, Hz
  - Sign of this goes the opposite the direction the wave is going
    1. Wave going in positive direction $(+)$, then the sign should be negative $(-)$
    2. Wave going in negative direction $(-)$, then the sign should be positive $(+)$

- $v = \lambda f$, Wave Velocity, m/s
  - $v = \frac{\omega}{2\pi} * \frac{2\pi}{k} = \frac{\omega}{k}$
  - This can be proven with the angular portion of any wave (inside the parentheses of trig function)

$$
\begin{aligned}
kx - \omega t &= \text{Constant} \\
\frac{d}{dt}\left[kx - \omega t\right] &= \frac{d}{dt}\left[\text{Constant}\right] \\
k\frac{dx}{dt} - \omega\frac{dt}{dt} &= 0 \\
kv - \omega &= 0 \\
kv &= \omega \\
v &= \frac{\omega}{k}
\end{aligned}
$$

Since $\omega = 2\pi f$, then $k = \frac{\lambda}{2\pi}$

Wave Interference

Waves are nice, and they just sum when they interfere. Let:

$$y_1\left(x,t\right) = y\sin\left(kx - \omega t\right) \tag{3.1}$$
$$y_2\left(x,t\right) = y\sin\left(kx + \omega t + \varphi\right) \tag{3.2}$$
$$Y\left(x,t\right) = y\left[\sin\left(kx - \omega t\right) + \sin\left(kx + \omega t + \varphi\right)\right] \tag{3.3}$$

You can usually use Equation A.1 to simplify Equation 3.3.

Constructive/Destructive Interference

$$
\begin{aligned}
\phi &= 2\pi\frac{PathLengthDiff}{\lambda} = 2\pi\frac{\Delta\text{PLD}}{\lambda} \\
n &= \frac{\phi}{2\pi} = 4\frac{PathLengthDiff}{\lambda}
\end{aligned}
$$

- $PathLengthDiff$ - Is the Difference in path lengths that the waves must travel
- phi - Angular Location of points of Complete Constructive/Destructive Interference
- n - Number of locations where there is Complete Constructive/Destructive Interference

Standing Waves

This is actually the superposition of 2 waves, traveling in opposite directions, on a medium that is fixed at both ends, i.e. a taut string held by a wall.

Location of Nodes and Antinodes

- Nodes - $x = n\frac{\lambda}{2}$ for $n = 0, 1, 2, \ldots$
  - Always at closed ends of tubes
- Antinodes - $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$ for $n = 0, 1, 2, \ldots$
  - Always at open ends of tubes

Resonant Frequencies/Harmonics

These can also be called harmonics. There is a resonant frequency for every number of nodes/antinodes on tthe standing wave.
$f = \frac{v}{\lambda} = n\frac{v}{2L}$

- L is the length of the medium (The String).
- λ is the wavelength of the wave formed.

This can be extended to find the base resonant frequency, if you know how many node levels are between the two resonant frequencies given, i.e. they say that the **NEXT** frequency, means $n + 1$.
$f_{n+m} - f_n = \left(n + m\right)\frac{v}{2L} - n\frac{v}{2L} = m\frac{v}{2L}$

Reflecting Sound

- $D = (n+1)\, d = vt$
  - n is the number of reflections that occurred
  - $n+1$ is used when we want the distance the wave covers

Sound in Different Mediums

Frequency is a property of a wave, and **CANNOT BE ALTERED.** This means that:

$$\begin{aligned} v &= \lambda f \\ v_{Sound,Material1} &= \lambda_{Material1} f_{Unique,Material1} \\ v_{Sound,Material2} &= \lambda_{Material2} f_{Unique,Material2} \\ f_{Unique,Material1} &= f_{Unique,Material2} \\ \frac{v_{Sound,Material1}}{\lambda_{Material1}} &= \frac{v_{Sound,Material2}}{\lambda_{Material2}} \end{aligned}$$

Doppler Effect

$$f' = f\frac{v \pm v_D}{v \pm v_S}$$

- Moving ***TOWARDS*** each other: Frequency Increase
- Moving ***AWAY*** from each other: Frequency Decrease
- f - Initial Frequency, Hz
- v - Sound Speed, m/s
- v_D - Detector Speed, m/s
- v_S - Source Speed, m/s
- **For Numerator**:
  - If detector is moving towards the source, $+$
  - If detector is moving away from the source, $-$
- **For Denominator**:
  - If source is moving away from detector, $+$
  - If source is moving towards the detector, $-$

4 Thermodynamics

Defn 1 (Thermodynamics)**.** *Thermodynamics* is the study of energy transfer between two macroscopic bodies driven by temperature differences.

Defn 2 (Temperature)**.** *Temperature* is a direct measurement of internal energy of a system

Laws of Thermodynamics

Defn 3 (0th Law of Thermodynamics)**.** If 2 bodies, A and B are in thermal equilibrium with a third body "T", then they are in thermal equilibrium with each other.

Defn 4 (1st Law of Thermodynamics)**.**

$$\begin{aligned} dE_{Internal} &= dQ - dT, \; dQ \text{ and } dT \text{ are inexact (path-dependent) differentials.} \\ \Delta E_{Internal} &= Q - T \end{aligned} \tag{4.1}$$

Note 4.1. **Special Cases for the 1st Law of Thermodynamics**:

1. Adiabatic Processes - $dE_{Int} = -dW$
   - No heat exchange
   - Insulating
   - Something happens too quickly for system to keep up
2. Isothermal Processes - $dT = 0 \rightarrow dE_{Int} = 0 \rightarrow dQ = dW$
3. Isobaric Processes - $dW = pdV$, p(pressure) is constant
4. Constant Volume - $W = 0$

5. Cyclical Processes - $dE_{Int} = 0 \rightarrow dQ = dW$
   - You end a cycle with the same internal energy when the cycle started

Defn 5 (2nd Law of Thermodynamics)**.** If a *cyclical process occurs in a CLOSED system*, the entropy of the system increases for irreversible processes and remains constant for reversible processes. **IT NEVER DECREASES!!**

$$\Delta S \geq 0 \tag{4.2}$$

$$\Delta S = \int_a^b \frac{dQ\,(T)}{T} \tag{4.3}$$

Note 5.1. **ΔS is a state-function, meaning it is path-independent.**

Heat and Work

- $dW = \vec{F} \cdot \vec{ds}$
- $\vec{F} = p\,(V, T)\,dV$
- $W = \int p\,(V, T)\,dV$
- $W = \frac{dQ}{dt}$
- $W = \frac{\Delta Q}{\Delta t}$

Work done by thermal energy is path independent.

Thermal Expansion

Occurs because the "springs" between each of the atoms in a lattice have energy applied by Heat (Temperature Change).

- $\frac{\Delta L}{L_0} = \alpha \Delta T$ (One-Dimensional Expansion)
- $\frac{dL}{L_0} = \alpha dT$ (One-Dimensional Expansion)
  - α is a material-specific constant

Specific Heat/Heat Capacity

- $C = \frac{dQ}{dT} \leftarrow$ Specific Heat
- $c = \frac{dQ}{mdT} \leftarrow$ Mass Specific Heat

Heat of Phase Transitions

This is a constant unique to the material and the phase transition it is going through.

- $Q = Lm$
- $Q = \int_{m_i}^{m_f} L_f dm$
  - l_{Fusion} = Heat required to turn things from **SOLID TO LIQUID**
  - l_{Vapor} = Heat required to turn things from **LIQUID TO GAS**

Conduction Heat Transfer

$$P_{Conduction} = \frac{(T_H - T_C)}{L} Ak \tag{4.4}$$

- L = Length
- A = Cross-Sectional Area
- k = Material's Thermal Conductivity

For multiple materials between 2 thermal reservoirs:

- $P_1 = P_2 = \ldots = P_n$
- Heat will only flow as fast as the slowest thermal conductor

5 Kinetic Theory of Ideal Gases

Defn 6 (Ideal Gas)**.** An *ideal gas* is a gas that obeys the ideal gas law.

$$pV = nRT,\ R \approxeq 8.31\mathrm{J/mol\,K} \tag{5.1}$$

$$E_{Internal} = K_{Translate} + K_{Rotate} \tag{5.2}$$
$$\Delta E_{Internal} = \Delta K_{Translate} + \Delta K_{Rotate} \tag{5.3}$$

Work Done by Ideal Gases

Isothermally

$$\begin{aligned}
W &= \int \vec{F}\vec{ds} \\
W &= \int_{V_1}^{V_2} p\left(V,T\right)dV \\
W &= \int_{V_1}^{V_2} \frac{nRT}{V}dV \\
W &= nRT\int_{V_1}^{V_2} \frac{1}{V}dV \\
W &= nRT\ln\left(\frac{V_2}{V_1}\right)
\end{aligned} \tag{5.4}$$

Constant Pressure

$$W = p\left(V_{final} - V_{init}\right) \tag{5.5}$$

Translational Kinetic Energy

Defn 7 (Degrees of Freedom)**.** *Degrees of freedom* represent the number of variables that are needed to describe a system. Represented with d, occasionally.

Note 7.1. In an ideal gas, these are a means to store energy.

$$\begin{aligned}
K_{Translate} &= \frac{3}{2}nRT \\
\Delta K_{Translate} &= \frac{3}{2}nR\Delta T
\end{aligned} \tag{5.6}$$

(Blank)	Translational	Rotational	Total
Monatomic	3	0	3
Diatomic	3	2	5
Polyatomic	3	3	6

Table 1: Degrees of Freedom Table for Gases

- An ideal gas has ***ONLY*** kinetic energy
- Completely elastic collisions

$$E_{int} = \frac{DoF}{2}nRT, \text{ where } DoF = \text{Degrees of Freedom} \tag{5.7}$$

Molar Specific Heats of Ideal Gases

Molar Specific Heat @ Constant Volume

$$\begin{aligned} C_V &= \frac{\Delta E}{n \Delta T} \\ C_V &= \frac{dE}{dT} \\ C_V &= \left(\frac{DoF}{2}\right) R \end{aligned} \tag{5.8}$$

$$Q = n C_V \Delta T \tag{5.9}$$

Molar Specific Heat @ Constant Pressure

$$C_P = C_V + R \tag{5.10}$$

$$Q = n C_P \Delta T \tag{5.11}$$

Adiabatic Processes in Ideal Gases

Defn 8 (Adiabatic Process)**.** An *adiabatic process* is one in which no heat exchange occurs, namely:

$$\begin{aligned} dE_{Internal} &= dQ - dW \\ dQ &= 0 \\ dE_{Internal} &= -dW \end{aligned} \tag{5.12}$$

This leads to:

$$pV^{\gamma} = \text{Constant}, \gamma = \frac{C_P}{C_V} \tag{5.13}$$

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \tag{5.14}$$

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1} \tag{5.15}$$

You get Equation (5.15) by plugging $p = \frac{nRT}{V}$ into Equation (5.14).

6 Entropy

There is a heavy relationship between this and the 2nd Law of Thermodynamics.

Defn 9 (Entropy)**.** *Entropy* is a measure of the number of available states.

$$S = k_B \ln(\Omega) \tag{6.1}$$

Note 9.1. **Entropy is NOT disorder.**

7 Light

Defn 10 (Photon)**.** A *photon* is a wave propagating through an electric field.

The various θ used in Equations (7.1), (7.2) are measured from the surface normal.

Chromatic Dispersion is the breaking up of polychromatic light by spectra. Think of Pink Floyd's *Dark Side of the Moon* album cover. This happens because shorter wavelength, higher frequency light has a slightly higher index of refraction.

Reflection

$$\theta_{reflected} = \theta_{incident} \tag{7.1}$$

Refraction

Defn 11 (Snell's Law).

$$n_{refract} \sin(\theta_{refract}) = n_{incident} \sin(\theta_{incident}) \tag{7.2}$$

Defn 12 (Index of Refraction).

$$\begin{aligned} n_i &= \frac{c}{v_i} \\ \lambda &= \frac{\lambda_0}{n} \end{aligned} \tag{7.3}$$

Total Internal Reflection

Total Internal Reflection occurs when the *refracted light's angle is* $\frac{\pi}{2}$.

$$n_{refract} \sin(\theta_{refract}) = n_{incident} \sin(\theta_{incident}), \text{ where } \theta_{refract} = \frac{\pi}{2} \tag{7.4}$$

Interference

Defn 13 (Huygen's Principle). Any point on a plane wavefront can be treated as a source of outgoing spherical waves.

Note 13.1. This is a mathematical construct/model.

8 Quantum Mechanics

A Reference Material

A.1 Physical Constants

Constant Name	Variable Letter	Value
Boltzmann Constant	R	8.314J/mol K

A.2 Trigonometry

A.2.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \tag{A.1}$$