

EDAN40/EDAN95: Functional Programming — Reference Sheet

Karl Hallsby

Last Edited: March 20, 2020

Contents

List of Theorems	ii
A Complex Numbers	1
A.1 Complex Conjugates	1
A.1.1 Complex Conjugates of Exponentials	1
A.1.2 Complex Conjugates of Sinusoids	1
B Trigonometry	2
B.1 Trigonometric Formulas	2
B.2 Euler Equivalents of Trigonometric Functions	2
B.3 Angle Sum and Difference Identities	2
B.4 Double-Angle Formulae	2
B.5 Half-Angle Formulae	2
B.6 Exponent Reduction Formulae	2
B.7 Product-to-Sum Identities	2
B.8 Sum-to-Product Identities	3
B.9 Pythagorean Theorem for Trig	3
B.10 Rectangular to Polar	3
B.11 Polar to Rectangular	3
C Calculus	4
C.1 Fundamental Theorems of Calculus	4
C.2 Rules of Calculus	4
C.2.1 Chain Rule	4
D Laplace Transform	5

List of Theorems

A.1.1Defn (Complex Conjugate) 1

C.1.1Defn (First Fundamental Theorem of Calculus) 4

C.1.2Defn (Second Fundamental Theorem of Calculus) 4

C.1.3Defn (argmax) 4

C.2.1Defn (Chain Rule) 4

D.0.1Defn (Laplace Transform) 5

A Complex Numbers

Complex numbers are numbers that have both a real part and an imaginary part.

$$z = a \pm bi \quad (\text{A.1})$$

where

$$i = \sqrt{-1} \quad (\text{A.2})$$

Remark (i vs. j for Imaginary Numbers). Complex numbers are generally denoted with either i or j . Since this is an appendix section, I will denote complex numbers with i , to make it more general. However, electrical engineering regularly makes use of j as the imaginary value. This is because alternating current i is already taken, so j is used as the imaginary value instead.

$$Ae^{-ix} = A [\cos(x) + i \sin(x)] \quad (\text{A.3})$$

A.1 Complex Conjugates

If we have a complex number as shown below,

$$z = a \pm bi$$

then, the conjugate is denoted and calculated as shown below.

$$\bar{z} = a \mp bi \quad (\text{A.4})$$

Defn A.1.1 (Complex Conjugate). The conjugate of a complex number is called its *complex conjugate*. The complex conjugate of a complex number is the number with an equal real part and an imaginary part equal in magnitude but opposite in sign.

The complex conjugate can also be denoted with an asterisk (*). This is generally done for complex functions, rather than single variables.

$$z^* = \bar{z} \quad (\text{A.5})$$

A.1.1 Complex Conjugates of Exponentials

$$\overline{e^z} = e^{\bar{z}} \quad (\text{A.6})$$

$$\overline{\log(z)} = \log(\bar{z}) \quad (\text{A.7})$$

A.1.2 Complex Conjugates of Sinusoids

Since sinusoids can be represented by complex exponentials, as shown in Appendix B.2, we could calculate their complex conjugate.

$$\begin{aligned} \overline{\cos(x)} &= \cos(x) \\ &= \frac{1}{2} (e^{ix} + e^{-ix}) \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \overline{\sin(x)} &= \sin(x) \\ &= \frac{1}{2i} (e^{ix} - e^{-ix}) \end{aligned} \quad (\text{A.9})$$

B Trigonometry

B.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{B.2})$$

B.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm j\alpha} = \cos(\alpha) \pm j \sin(\alpha) \quad (\text{B.3})$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{B.4})$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{B.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{B.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{B.7})$$

B.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{B.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{B.9})$$

B.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{B.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{B.11})$$

B.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{B.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{B.13})$$

B.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2} \quad (\text{B.14})$$

$$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2} \quad (\text{B.15})$$

B.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{B.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{B.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{B.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{B.19})$$

B.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{B.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.22})$$

B.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{B.23})$$

B.10 Rectangular to Polar

$$a + jb = \sqrt{a^2 + b^2} e^{j\theta} = r e^{j\theta} \quad (\text{B.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{B.25})$$

B.11 Polar to Rectangular

$$r e^{j\theta} = r \cos(\theta) + j r \sin(\theta) \quad (\text{B.26})$$

C Calculus

C.1 Fundamental Theorems of Calculus

Defn C.1.1 (First Fundamental Theorem of Calculus). The *first fundamental theorem of calculus* states that, if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{C.1})$$

Defn C.1.2 (Second Fundamental Theorem of Calculus). The *second fundamental theorem of calculus* holds for f a continuous function on an open interval I and a any point in I , and states that if F is defined by

$$F(x) = \int_a^x f(t) dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t) dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \quad (\text{C.2})$$

Defn C.1.3 (argmax). The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname{argmax}_x$$

C.2 Rules of Calculus

C.2.1 Chain Rule

Defn C.2.1 (Chain Rule). The *chain rule* is a way to differentiate a function that has 2 functions multiplied together.

If

$$f(x) = g(x) \cdot h(x)$$

then,

$$\begin{aligned} f'(x) &= g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ \frac{df(x)}{dx} &= \frac{dg(x)}{dx} \cdot h(x) + g(x) \cdot \frac{dh(x)}{dx} \end{aligned} \quad (\text{C.3})$$

D Laplace Transform

Defn D.0.1 (Laplace Transform). The *Laplace transformation* operation is denoted as $\mathcal{L}\{x(t)\}$ and is defined as

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \tag{D.1}$$

References

- [Hut07] Graham Hutton. *Programming in Haskell*. English. 1st ed. Cambridge University Press, 2007. 183 pp. ISBN: 9781316626221.
- [Lip11] Miran Lipovača. *Learn You a Haskell for Great Good! A Beginner's Guide*. English. Apr. 2011. 290 pp. ISBN: 9781593272838. URL: <http://learnyouahaskell.com/>.
- [OGS08] Bryan O'Sullivan, John Goerzen, and Donal Bruce Stewart. *Real World Haskell*. Code You Can Believe In. English. O'Reilly Media, Inc., Nov. 2008. 712 pp. ISBN: 9780596554309.