

## General Equations

- KCL:  $\sum I_{in} = \sum I_{out} \rightarrow$  Node's Input Current = Node's Output Current
- KVL:  $\sum V = 0 \rightarrow$  Voltage across a loop totals to 0.
- Ohm's Law:  $V = IR$

## Phasors

Phasors will only show us the steady state response of the circuit, not the transient response.

**Eq:**  $v(t) = V_M \cos(\omega t + \theta) \leftrightarrow \bar{V} = V_M \angle \theta_v = V_M e^{j\theta_v} = V_M (\cos \theta_v + j \sin \theta_v)$

Impedances add together, like resistors. You can use phasors with Nodal Analysis, Mesh/Loop Analysis, Superposition, and Thevenin and Norton Equivalencies.

Operation	Formula
Addition	$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
Subtraction	$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
Multiplication	$z_1 z_2 = r_1 r_2 \angle (\phi_1 + \phi_2)$
Division	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle (\phi_1 - \phi_2)$
Reciprocal	$\frac{1}{z_1} = \frac{1}{r_1} \angle -\phi_1$
Square Root	$\sqrt{z_1} = \sqrt{r_1} \angle \frac{\phi_1}{2}$
Complex Conjugate	$z_1^* = x - jy = r \angle -\phi_1 = r e^{-j\phi_1}$

$$z_1 = x_1 + jy_2 = r_1 \angle \phi_1, z_2 = x_2 + jy_2 = r_2 \angle \phi_2$$

## RMS/Complex Power/Max Power Transfer

- $X_{rms} = \sqrt{\frac{1}{T} \int_0^T x(t)^2 dt}$
- $\sum_{k=1}^n S_k$

Name	Symbol	Equation(s)
Complex Power	<b>S</b>	$= P + jQ = \mathbf{V}_{rms} \mathbf{I}_{rms}^* =  \mathbf{V}_{rms}   \mathbf{I}_{rms}  \angle (\theta_v - \theta_i)$
Apparent Power	<i>S</i>	$\ \mathbf{S}\  =  \mathbf{V}_{rms}   \mathbf{I}_{rms}  = \sqrt{P^2 + Q^2}$
Real Power	<i>P</i>	$\text{Re}\{\mathbf{S}\} = S \cos(\theta_v - \theta_i)$
Reactive (Imaginary) Power	<i>Q</i>	$\text{Im}\{\mathbf{S}\} = S \sin(\theta_v - \theta_i)$
Power Factor	<i>Pf</i>	$\frac{P}{S} = \cos(\theta_v - \theta_i)$

## Elements

## Methods to Solve Equations

### Nodal Analysis

1. # of Nodes?  $\rightarrow n$
2. Make one node the reference node. Assign  $n - 1$  nodal voltages
3. For a **voltage** source, write a CONSTRAINT EQUATION (Con. Eq.). If there is a voltage source between 2 non-reference nodes, make that a **SUPERNODE**.
4. Write KCL at each node.  $(n - 1)$  equations.
5. Solve Equations.

Relation	R	C	L
v-i	$V = IR$	$v = \frac{1}{C} \int_{t_0}^t i(x) dx + v(t_0)$	$v = L \frac{di}{dt}$
i-v	$I = \frac{V}{R}$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v(x) dx + i(t_0)$
P or W	$P = I^2 R = \frac{V^2}{R}$	$P = \frac{1}{2} C v_c^2$	$W = \frac{1}{2} L i_l^2$
Series	$R_{eq} = R_1 + R_2 + \dots + R_n$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$	$L_{eq} = L_1 + L_2 + \dots + L_n$
Parallel	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$	$C_{eq} = C_1 + C_2 + \dots + C_n$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$
@ Steady State	Same (Nothing Happens)	Open Circuit	Short Circuit
Phasors	$Z_R = R$	$Z_C = \frac{1}{j\omega C}$	$Z_L = j\omega L$

## Mesh/Loop Analysis

1. # of Nodes?  $\rightarrow n$  # of Branches?  $\rightarrow b$  # of meshes/loops?  $\rightarrow b - n + 1 = l$
2. Assign  $l$  loop currents.
3. For **current** sources, write a CONSTRAINT EQUATION (Con. Eq.). If there is a current source between 2 meshes, that's a **SUPERMESH**.
4. Write KVL for each mesh.
5. Solve Equations.

## Superposition

- # of sources,  $n$ , determines the number of equations you will have.
- Shut off each source, one at a time, solving for the term that you want.
  - Voltage Source = S.C.
  - Current Source = O.C.
- Sum each of the individual terms together.  $\sum_{i=1}^n x_i$

## Source Transformations

**ALL** source transformations obey Ohm's Law.  $V = IR$ . This will **ONLY** work on resistors in series with **VOLTAGE** sources, or resistors in parallel with **CURRENT** sources.



Figure 1: Left: Voltage Source in Series with a Resistor, Right: Current Source in Parallel with a Resistor

## Thevenin and Norton Equivalencies

- **ONLY** independent sources - Zero all sources, find  $\mathbf{Z}_{eq}$ .
  - 0-ing Current Sources = O.C., 0-ing Voltage Sources = S.C.
  - Look at circuit from load's perspective for  $\mathbf{Z}_{eq}$
  - $\mathbf{V}_{Th} = \mathbf{V}_{OC}$ ,  $\mathbf{I}_N = \mathbf{I}_{SC}$
- **BOTH** dependent and independent sources
  - Find  $\mathbf{V}_{Th} = \mathbf{V}_{OC}$ ,  $\mathbf{I}_N = \mathbf{I}_{SC}$
  - Solve  $\mathbf{Z}_{Th} = \frac{\mathbf{V}_{OC}}{\mathbf{I}_{SC}}$
- **ONLY** dependent sources
  - $\mathbf{V}_{Th} = 0$ ,  $\mathbf{I}_N = 0$
  - $\mathbf{Z}_{Th} = \mathbf{Z}_N \rightarrow$  Attach test source @ load.
    - \* If voltage test source, find current. If current test source, find voltage
  - $\mathbf{Z}_{Th} = \frac{\mathbf{V}_{Test}}{\mathbf{I}_{Test}}$
- **Maximum Power Transfer - AC**
  - $\mathbf{Z}_{Load} = \mathbf{Z}_{Th}^*$ ,  $R_{Th} = \text{Re}\{\mathbf{Z}_{Th}\}$ ,  $R_L = |\mathbf{Z}_{Th}| = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2}$
  - $P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}}$