

Chem 122: Introduction to Chemistry — Reference Sheet

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1 Introduction

1.1 Basic Chemistry Things

Defn 1 (Chemistry). *Chemistry* is the study of Matter and its changes.

Remark 1.1. We tend to use the macroscopic world to visualize the microscopic world.

Defn 2 (Matter). *Matter* is “stuff” that has both mass and volume.

Defn 3 (Scientific Method). The *scientific method* is a systematic approach to research that utilizes qualitative or quantitative measurements.

Defn 4 (Hypothesis). A *hypothesis* is a tentative explanation that will be tested using the Scientific Method.

Defn 5 (Law). A *law* is a statement of a relationship between phenomena that is always the same, under the same conditions.

Remark 5.1. These tend to be drawn from large amounts of data.

Example 1.1: Law 1.
Chlorine (Cl) is a highly reactive gas.

Example 1.2: Law 2.
Matter is neither created nor destroyed.

Defn 6 (Theory). A *theory* is a unifying principle that explains a body of facts based on facts and laws. These are constantly tested for validity.

Remark 6.1. A Hypothesis can turn into a Theory with enough experimentation and acceptance.

Example 1.3: Theory 1.
Reactivity of elements depends on the element’s electron (e^-) configuration.

Example 1.4: Theory 2.
All matter is made up of tiny, indestructible particles, called atoms.

1.2 Matter

As Definition 2 said, matter must have both volume and mass. Matter can have several Matter States.

Defn 7 (Matter State). A *matter state* or *state of matter* is just the configuration of atoms in a particular material. There are 3 common states:

1. Solid
2. Liquid
3. Gas

But, Matter can be categorized in different ways as well. The Matter Tree is one way to categorize them.

Figure 1.1: Matter Tree

Others include:

- Atomic Weight
- Chemical Properties
- Physical Properties
- The Periodic Table
- And many others

1.3 Significant Figures

Defn 8 (Significant Figures). *Significant Figures* or *Sig Figs* are ways to handle uncertainty in our measurements. In general, we treat the data that we receive as inexact numbers, thus we must confirm our suspicions several times. Additionally, Precision and Accuracy are used interchangeably when they shouldn't.

Defn 9 (Precision). *Precision* is defined as the closeness of data points to each other. If you think about a dartboard, this would be all the darts landing right next to each other.

Defn 10 (Accuracy). *Accuracy* is defined as how close your data is to the predicted true real value.

Remark 10.1. Generally, this must be done with a minimum of 3 trials, but more will yield more accurate data.

1.3.1 Rules for Significant Figures

1. 0s between any non-zero digit is significant (100, both 0s are significant)
2. 0s at the beginning of an integer are not significant (010 = 10)
3. 0s at the end are significant if the number is a decimal (0.003050 has 4 sig figs)
4. 0s at the end, if there is no decimal/fractional portion, are not significant (16000 has 2 sig figs)

Example 1.5: Addition and Subtraction of Significant Figures.

Add 20.3056, 1.34, and 54.2 and keeping in mind significant figures.

You want to find the least precise number first, in this case it is 54.2 because it only has one decimal place. This also determines how many decimal places to go past on the solution. Adding these 3 together gives 75.8456, but because of 54.2, it becomes 75.8.

Example 1.6: Multiplication and Division of Significant Figures.

Multiply 3.4456 and 2.15 keeping in mind significant figures.

You find the number with the least number of significant figures and use that. So, 2.15 has 3 sig figs, that's the same amount your answer must have.

$$3.4456 \times 2.15 = 7.40804$$

But because we can only have 3 sig figs in our answer, 7.41 is our solution.

2 History of Chemistry

2.1 Dalton

Dalton created the first meaningful definition of an atom. He made several claims:

1. Atoms are very small
2. The same element's atoms are identical, but different elements have different atoms.
3. Atoms are neither created, nor destroyed (Law of Conservation of Matter)
4. Compounds are 2 or more elements together.

Defn 11 (Law of Conservation of Matter). Matter is neither created, nor destroyed. It can *only* change forms.

2.2 Thomson

Thomson made several discoveries about atoms and their constituent particles. For his experimentation, he used a cathode ray (a beam of positively charged ions) and magnets. He discovered the Electron.

Defn 12 (Electron). The *electron* is one of 3 particles that make up an atom. Electrons are negatively charged particles that are contained *outside* of the nucleus. An electron's position and velocity can not be known simultaneously. This is known as the Heisenberg's Uncertainty Principle

The cathode ray deflected from the "negative" magnetic plate to the "positive". From this he calculated the Magnetic Deflection.

Defn 13 (Magnetic Deflection). When Thomson deflected his cathode ray with magnets, he measured how far it deviated from the starting line.

$$1.76 \times 10^8 \text{C/g} \tag{2.1}$$

2.3 Millikan

Millikan made 2 significant contributions to the model of the atom. He discovered the charge of a single Electron and the mass of a single Electron.

$$1.602 \times 10^{-19} \text{C} \quad (2.2)$$

$$9.10938 \times 10^{-28} \text{g} \quad (2.3)$$

Both the Millikan and Millikan were drawn from Thomson's work with Magnetic Deflection.

2.4 Becquerel

Becquerel did his work with high energy radiation caused by radioactivity. He found that there were 3 types of particles released by radioactive decay.

1. Alpha Particles (α) - Positively charged particles that are charged helium atoms.
2. Beta Particles (β) - Negatively charged particles that are essentially high speed electrons.
3. Gamma particles (γ) - Uncharged particles that have next to no mass and are quite energetic.

2.5 Johnson

Johnson developed one of the first models for single atoms. This was called the Plum Pudding Model.

Defn 14 (Plum Pudding Model). The *Plum Pudding Model* is a visualization of an atom. It is based off the plum pudding desert, which was one of Johnson's favorites. The positive charges were held together in a "soft" shell. The electrons were evenly distributed on the outer surface of the positively charged "plum." One of the hallmarks of this model was that the entire atom was *not* empty space.

2.6 Rutherford

Rutherford performed experiments with α -particles. He "shot" these particles at a piece of gold foil and observed what happened with after the particles passed through.

Defn 15 (Rutherford Model). This is, more or less, the next model of the atom. Rutherford challenged Johnson's Plum Pudding Model of the atom. When Rutherford sent the beam of alpha particles through the gold foil, he found most of them didn't deflect, i.e. hit any thing. However some did, and were scattered in all directions. Rutherford proved that atoms are mostly empty space, with the positive charges being held in a small dense area he called the *nucleus*.

Remark 15.1. One thing to note about the *nucleus* in the Rutherford Model is that there was no concept of the neutron yet. Since neutrons are uncharged particles, they were not discovered until much later.

2.7 Atomic Mass Units, u

Eventually the neutron was discovered and the current understanding of the fundamental particles in atoms was completed. These include the:

- Proton (p^+)
- Neutron (N^0)
- Electron (e^-)

The mass of each of these particles was found and the Atomic Mass Unit was developed to make calculations easier.

$$1\text{u} = 1.66054 \times 10^{-24} \text{g} \quad (2.4)$$

Thus, the atomic mass for each particles is as follows:

- Proton (p^+) - $1.672623 \times 10^{-24} \text{g} = 1.0074\text{u}$
- Neutron (N^0) - $1.674927 \times 10^{-24} \text{g} = 1.0087\text{u}$
- Electron (e^-) - $9.109383 \times 10^{-28} \text{g} = 5.486 \times 10^{-4}\text{u}$

3 The Periodic Table

The Periodic Table was developed as a way to categorize the many chemical elements in the world. Most of the elements on the table are naturally occurring, but some of the heaviest elements have been synthesized in laboratories.

Defn 16 (The Periodic Table). The *Periodic Table* is an arrangement of elements by their atomic number, Z , or the number of protons in the nucleus.

Remark 16.1. The number of protons is the *ONLY* thing that determines what an element is and where it is located on The Periodic Table. If an element has a different number of neutrons, that is an Isotope. If an element has a different number of electrons, that is an Ion.

Remark 16.2. On The Periodic Table, the elements are always in their electroneutral form. This means they have the same number of protons and electrons.

A single element drawn out of The Periodic Table will look like Figure.

Figure 3.1: Example Element from Periodic Table

There are 4 parts of each element in The Periodic Table.

1. The element's name
2. The element's atomic number, Z , which is also the number of protons in the element
3. The element's symbol
4. The element's *AVERAGE* atomic mass (g/mol), A

An element is usually written as such: ${}^A_Z\text{X}$.

Example 3.1: Element Notation.
What is Copper's (Cu) notation in text?
${}^{63.546}_{29}\text{Cu}$

3.1 Variations of Atoms

Defn 17 (Isotope). An *isotope* is the same element, but with a different number of neutrons. While this does *NOT* change the element, it may change some of the properties of this element.

Example 3.2: Number of Neutrons.
What is the number of neutrons in the average isotope of these elements: ${}^{28}_{14}\text{Si}$, ${}^{29}_{14}\text{Si}$, ${}^{30}_{14}\text{Si}$?
Since Silicon, Si has 14 protons, so subtract 14 from each of the total nuclei weight.
<ol style="list-style-type: none"> 1. ${}^{28}_{14}\text{Si}$ has 14 neutrons 2. ${}^{29}_{14}\text{Si}$ has 15 neutrons 3. ${}^{30}_{14}\text{Si}$ has 16 neutrons

Defn 18 (Ion). An *ion* is the same element as the non-ionic form, however, the number of electrons and protons is different. This means that an ion can be positively or negatively charged.

Remark 18.1. The number of electrons in relation to protons determines the type of ion it is.

- If there are more electrons than protons, it is a negatively charged ion.
- If there are more protons than electrons, it is a positively charged ion.

Remark 18.2. In general:

- Metals are usually cations
- Non-Metals are usually anions

Example 3.3: What is that Element?.
Given that an element has 38 protons, 50 neutrons, and 36 electrons, what is the element?
Since there are 38 protons, that makes it Strontium (Sr). The atomic weight will be $38 + 50 = 88$. The ion is $38 - 36 = +2$. Therefore, the written atom would be ${}^{88}_{38}\text{Sr}^{+2}$.

Example 3.4: An Ion's Protons Neutrons and Electrons.

Given the element ${}^{75}_{33}\text{As}^{-3}$, how many protons, neutrons and electrons are there?

Astatine has 33 protons. The number of neutrons is $75 - 33 = 42$. Since this is Astatine, the number of electrons is $|-33 - -3| = 36$.

3.2 Types of Materials

Defn 19 (Metal). A *metal* is a type of material that:

- Conducts heat well
- Conducts electricity well by freely losing electrons
- Their nuclei are congregated and their electrons form an outer electron cloud

Remark 19.1. For Metals, there are variations of some metals. For example, Iron (II) and Iron (III). These are *NOT* different ions. Rather, they are Iron in different Oxidation States.

Defn 20 (Non-Metal). A *non-metal* is a type of material that:

- Do not conduct heat well
- Do not conduct electricity well
- Gain electrons easily due to high electronegativity

Defn 21 (Metalloid). A *metalloid* is a type of material that has properties of both Metal and Non-Metal. These elements on right on the edge between Metals and Non-Metals on the periodic table. These include:

- Boron, B
- Silicon, Si
- Germanium, Ge
- Astatine, As
- Antimony, Sb
- Tellurium, Te

Example 3.5: Metal Nonmetal or Metalloid?.

Are these elements Metals, Non-Metals, or Metalloids?

1. Phosphorus, P
2. Osmium, Os
3. Selenium, Se
4. Thallium, Tl
5. Nickel, Ni
6. Argon, Ar
7. Tellurium, Te

1. Phosphorus, P is a Non-Metal
2. Osmium, Os is a Metal
3. Selenium, Se is a Non-Metal
4. Thallium, Tl is a Metal
5. Nickel, Ni is a Metal
6. Argon, Ar is a Non-Metal
7. Tellurium, Te is a Metalloid

4 Naming Compounds

There is a specific way to name compounds, however, it depends on the type of elements in the compound.

4.1 Naming Inorganic Compounds

Inorganic compounds are non-carbon-based compounds. There are a 2 rules for naming these compounds:

1. If there are only 2 elements in the compound and the first element only has 1 atom, then *DO NOT USE A PREFIX*

2. Otherwise, use the numerical prefix for the number of atoms present
 - (a) 1 atom \rightarrow Mono-
 - (b) 2 atoms \rightarrow Di-
 - (c) 3 atoms \rightarrow Tri-
 - (d) 4 atoms \rightarrow Tetra-
 - (e) so on and so forth

4.2 Naming Metallic Compounds

Metallic compounds are ones that have Metals in them. The rules for naming metallic compounds are similar to the inorganic ones. The only difference is naming the metal itself. You use the roman numerals when writing the compound and say the number out loud. For example, rust is Fe_2O_3 , is written Iron (III) Oxide.

5 Chemical Equations

Chemical Equations form one of the key pillars of chemistry.

Defn 22 (Chemical Equations). Just like in mathematical equations, *chemical equations* define the way certain chemical reactions take place. Just like in mathematics, there are some rules that are followed.

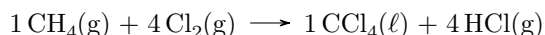
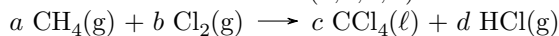
1. You need to have the same elements on each side of the reaction
2. There must be reactants (input) and products (output)
3. There must be a *Species* on each element/compound
4. There must be an arrow, \rightarrow or \leftrightarrow showing the direction of reaction
 - \leftrightarrow is for reactions that can occur in both directions. Nearly all do not follow this.
5. Elements **MUST** follow the Law of Conservation of Matter. Refer to Section 6, Stoichiometry for further information.

Defn 23 (Species). An element's or compound's *species* is the current state of matter. It can only be one of the following:

- Solid - (s)
- Liquid - (ℓ)
- Gas - (g)
- Aqueous - (aq)

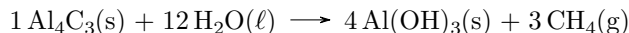
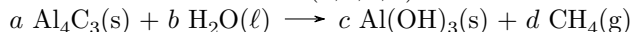
Example 5.1: Chemical Equation 1.

What are the coefficients (a, b, c, d) for this chemical equation?



Example 5.2: Chemical Equation 2.

What are the coefficients (a, b, c, d) for this chemical equation?



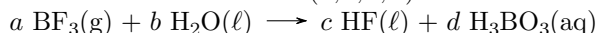
Example 5.3: Chemical Equation 3.

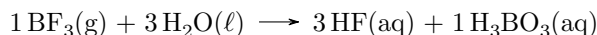
What are the coefficients (a, b, c, d) for this chemical equation?



Example 5.4: Chemical Equation 4.

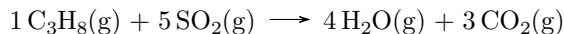
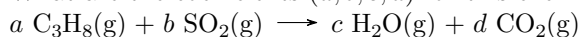
What are the coefficients (a, b, c, d) for this chemical equation?





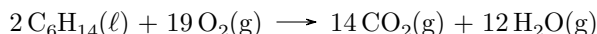
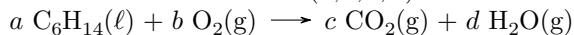
Example 5.5: Chemical Equation 5.

What are the coefficients (a, b, c, d) for this chemical equation?



Example 5.6: Chemical Equation 6.

What are the coefficients (a, b, c, d) for this chemical equation?



5.1 Limiting Reactants

Defn 24 (Limiting Reactants). The *limiting reactant(s)* is the reactant(s) that limits a reaction's total output. This means that there is some amount of excess reagent from the other, no limited reactant.

Example 5.7: Limiting Reactants 1.

Given the chemical equation $2 \text{Al}(\text{s}) + 3 \text{Cl}_2(\text{g}) \longrightarrow 2 \text{AlCl}_3(\text{s})$ and 1.50 mol of Al and 3.0 mol of Cl_2 . Which reactant limits the reaction?

$$1.50 \text{molAl} \left(\frac{3 \text{molCl}_2}{2 \text{molAl}} \right) = 2.25 \text{molCl}_2$$

$$3.00 \text{molCl}_2 \left(\frac{2 \text{molAl}}{3 \text{molCl}_2} \right) = 2.00 \text{molAl}$$

Since 2.25 mol of Cl_2 would be consumed when we have 1.50 mol of Al, and 2.00 mol Al would be consumed when we have 3.00 mol Cl_2 , the Al is the limiting reactant.

This means that 1.50 mol of AlCl_3 will be created from this reaction.

$$1.50 \text{molAl} \left(\frac{2 \text{molAlCl}_3}{2 \text{molAl}} \right) = 1.50 \text{molAlCl}_3$$

We will have .75 mol Cl_2 left after the reaction takes place.

$$3.00 \text{molCl}_2 - 2.25 \text{molCl}_2 = .75 \text{molCl}_2$$

Example 5.8: Limiting Reactants 2.

Given the chemical equation $\text{Zn}(\text{s}) + 2 \text{AgNO}_3(\text{aq}) \longrightarrow 2 \text{Ag}(\text{s}) + \text{Zn}(\text{NO}_3)_2(\text{aq})$ and 2.00 g of Zn and 2.50 g of AgNO_3 . Which reactant limits the reaction?

6 Stoichiometry

Defn 25 (Stoichiometry). *Stoichiometry* is the calculation of reactants and products in chemical reactions. *Stoichiometry* is founded on the Law of Conservation of Matter, where the total mass of the reactants equals the total mass of the products.

Stoichiometry fits together with Chemical Equations very closely.

Some terms that are thrown around a lot, seemingly interchangeably are Formula Weight and Molecular Weight.

Defn 26 (Formula Weight). *Formula weight* is the sum of atomic weights in a *chemical formula*. Since chemical formulae might be in a reduced form, it's formula weight might be different.

Defn 27 (Molecular Weight). *Molecular Weight* is the sum of atomic weights in the *molecule*. This is the molecule as would be found in nature, not the reduced formula, like in the chemical formula. Therefore, the Molecular Weight might be different than the Formula Weight.

One question that can be asked is what is the percent composition of a molecule based on weight. This is generally referred to as Percent Composition.

Defn 28 (Percent Composition). *Percent Composition* is the percentage of weight that certain atoms are a part of in a molecule. The equation for Percent Composition is shown below.

$$\% \text{ Element} = \frac{(\# \text{ of atoms}) (\text{Atomic Weight of Element})}{\text{Molecular Weight}} \times 100 \quad (6.1)$$

Example 6.1: Percent Composition of Carbon in Ethane.

What is the percentage of carbon in ethane, by weight?

Ethane has the chemical formula C_2H_6 . Therefore, the Percent Composition of carbon in ethane is as shown:

$$\% \text{ C} = \frac{2 * 12.0107}{30.0690} \times 100 = 79.8876\%$$

One other thing that could be asked is to calculate the Empirical Formula.

Defn 29 (Empirical Formula). The *empirical formula* of a molecule is the smallest possible whole number ratio on each of the molecule's constituent elements. For example the Molecular Weight of B_2H_6 is B_2H_6 . However, it's Empirical Formula is BH_3 .

Calculating the Empirical Formula is done with the steps and equation below.

1. You can assume that you have a 100 g sample, to make things easier.
- 2.

$$\frac{\text{Sample Mass}}{\text{Sample Molar Mass}} = \text{Empirical Formula Factor} \quad (6.2)$$

3. Divide each Empirical Formula factor by the smallest factor, and found to relatively nice numbers.
4. Multiply your resultant, roughly nice, numbers to whole integers.

Example 6.2: Calculate Empirical Formula 1.

Calculate the empirical formula if the molecule's percent composition is as follows:

- 75.69% Carbon
- 8.80% Hydrogen
- 15.51% Oxygen

First, assume that you have a 100g sample, to make calculating the percentages into mass more easily.

$$\begin{aligned} \frac{75.69\text{g}}{12.0\text{g/mol}} &= 6.31\text{mol} \\ \frac{8.80\text{g}}{1.0\text{g/mol}} &= 8.80\text{mol} \\ \frac{15.51\text{g}}{16.0\text{g/mol}} &= .969\text{mol} \end{aligned}$$

Then, you have to divide each mole by the lowest number, in this case .969mol.

$$\begin{aligned}\frac{6.31\text{mol}}{.969\text{mol}} &= 6.5 \\ \frac{8.80\text{mol}}{.969\text{mol}} &= 9 \\ \frac{.969\text{mol}}{.969\text{mol}} &= 1\end{aligned}$$

Then, multiply each number until you reach a whole integer, in this case, 2.

$$\begin{aligned}6.5 \times 2 &= 13 \\ 9 \times 2 &= 18 \\ 1 \times 2 &= 2\end{aligned}$$

So, we end up with $\text{C}_{13}\text{H}_{18}\text{O}_2$ as the empirical formula.

Example 6.3: Calculate Empirical Formula 2.

Calculate the empirical formula if the molecule's percent composition is as follows:

- 38.7% Carbon
- 9.7% Hydrogen
- 51.6% Oxygen

6.1 Theoretical Yield

Defn 30 (Theoretical Yield). *Theoretical Yield* is the theoretical maximum amount of a product that can be made from the input reagents.

Defn 31 (Percent Yield). *Percent Yield* is a percentage of the amount of product that you actually received from a reaction in comparison to the Theoretical Yield.

$$\% \text{ Yield} = \frac{\text{Actual Yield}}{\text{Theoretical Yield}} \times 100 \quad (6.3)$$

Remark 31.1. The use of Percent Yield can be combined with limiting reactants to find out some other information.

Example 6.4: Percent Yield.

Given the chemical equation $\text{Fe}_2\text{O}_3 + 3 \text{CO} \longrightarrow 2\text{Fe} + 3 \text{CO}_2$ and a limiting reactant of 150 g of Fe_2O_3 , what is the theoretical yield of Fe? Given that the reaction actually produced 87.9 g of Fe, what was the percent yield?

$$150\text{gFe}_2\text{O}_3 \left(\frac{1\text{molFe}_2\text{O}_3}{159.688\text{gFe}_2\text{O}_3} \right) \left(\frac{2\text{molFe}}{1\text{molFe}_2\text{O}_3} \right) \left(\frac{55.845\text{gFe}}{1\text{molFe}} \right) = 105\text{gFe}$$

Therefore, the theoretical yield of Fe is 105 g.

It's percent yield is:

$$\frac{87.9\text{gFe}}{105\text{gFe}} \times 100 = 83.7\%$$

7 Eletrolytes

Defn 32 (Eletrolyte). An *electrolyte* is an ion that dissociates into its constituent ions when dissolved in water. For example, salt.

Defn 33 (Strong Eletrolyte). A *strong electrolyte* is an Eletrolyte that *completely* dissociates into its constituent ions when dissolved in water.

Defn 34 (Weak Electrolyte). A *weak electrolyte* is an Electrolyte that *somewhat* dissociates into its constituent ions when dissolved in water.

Defn 35 (Non-Electrolyte). A *non-electrolyte* is a molecule that **DOES NOT** dissolve into its constituent ions when dissolved in water. For example, sugar.

There are some easy and general rules for determining if something is an electrolyte.

1. *Ionic* compound means it is a Strong Electrolyte
2. *Molecular* compounds have 3 potential electrolytic states
 - (a) Strong Acids/Bases are Strong Electrolytes
 - (b) Weak Acids/Bases are Weak Electrolytes
 - (c) All others are Non-Electrolytes

Make sure you refer to your solubility chart when referring to the strong and weak electrolytes above. Your solubility chart/table is in your textbook or online. These will give you which molecules are soluble in water.

7.1 Acids

Defn 36 (Acid). An *acid* is an Electrolyte that dissociates in water. It is a compound that when dissociated makes the solution end up with additional H^+ ions. These ions will bond with any free hydroxide molecule (OH^-).

Remark 36.1. You may hear these solutions being called “corrosive”.

Remark 36.2. Acids can be dangerous chemicals! Depending on the concentration of the acid, they can eat through skin, concrete, and even glass!! **Handle these with extreme caution!!**

There are 2 types of acids:

1. Brønstad-Lowry Acid
 - Arrhenius Acids are a special form of Brønstad-Lowry Acids
2. Lewis Acid

Defn 37 (Brønstad-Lowry Acid). A *Brønstad-Lowry acid* is a general name for compounds that are proton (H^+) donors.

Remark 37.1. In general, Acids refer to Brønstad-Lowry Acids.

Defn 38 (Arrhenius Acid). An *Arrhenius acid* are aqueous Brønstad-Lowry Acids that are proton donors. Usually when they donate their proton, they form a Hydronium ion H_3O^+ .

Defn 39 (Lewis Acid). A *Lewis acid* is a molecule that forms a covalent bond with an electron pair. This generalizes such that an acid is a chemical species that accepts electron pairs directly or by releasing protons.

Remark 39.1. Lewis Acids will usually be referred to as Lewis acids.

Defn 40 (Hydronium). *Hydronium* is a water molecule with an extra proton (H_3O^+). In Reduction-Oxidation Reactions that utilize Arrhenius Acids the proton donor gives the water solution the extra proton. The water becomes an Hydronium ion.

7.2 Bases

Defn 41 (Base). A *base* is an Electrolyte that dissociates in water. It is a compound that when dissociated makes the solution end up with additional OH^- ions. These ions will bond with any free hydrogen ions (H^+).

Remark 41.1. You may hear these solutions being called “caustic”.

Remark 41.2. Bases can be dangerous chemicals! Depending on the concentration of the base, they can eat through plastic, skin, and many other items!! **Handle these with extreme caution!!**

7.3 Dilution

There is a simple formula to follow when diluting acids and bases.

$$M_1 V_1 = M_2 V_2 \quad (7.1)$$

- M is the molarity of the solution
- V is the volume of the solution

8 Reduction-Oxidation Reactions

These reactions are somewhat unique because the electrons *ONLY* move around but do not change the output of the reaction.

Defn 42 (Reduction-Oxidation Reactions). *Reduction-Oxidation Reactions* or *Redox Reactions* are a type of chemical reaction. These reactions take place due to differences in electronegativity and each element's Oxidation State. The compounds that undergo a Redox Reaction are either Reduced/Oxidizing Agent or Oxidized/Reducing Agent.

Defn 43 (Oxidation State). An *oxidation state* describes the degree of loss of electrons in a chemical compound.

There are some rules for how elements in a compound are assigned their Oxidation State.

1. Elemental form of element $\rightarrow 0$
2. Monoatomic ion \rightarrow Oxidation State = Charge
 - Peroxides O_2^{2-} are always -2
 - Hydrides H^- are always -1
3. Fluorine is always -1
4. Groups 1 and 2 are always +1 and +2, respectively
5. Hydrogen, H, will usually be +1
6. Oxygen, O, will usually be -2
7. The sum of all Oxidation States **MUST** = Charge

Defn 44 (Reduced). A compound that is *reduced* is one that **GAINS** electrons. A Reduced compound is the one that is Oxidizing Agent.

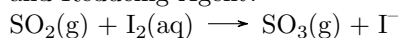
Defn 45 (Oxidized). A compound that is *oxidized* is one that **LOSES** electrons. A Oxidized compound is the Reducing Agent.

Defn 46 (Oxidizing Agent). An *oxidizing agent* or *oxidant* is one that **GAINS** electrons. A Oxidizing Agent is one that is Reduced.

Defn 47 (Reducing Agent). A *reducing agent* or *reductant* is one that **LOSES** electrons. A Reducing Agent is one that is Oxidized.

Example 8.1: Redox Reaction 1.

Given the chemical equation below, which compounds are Reduced and Oxidized, and which are the Oxidizing Agent and Reducing Agent?



Solution from Redox and More.

9 Quantum Chemistry

This section is a brief introduction to how we have discovered certain properties of atoms due to quantum mechanics and physics. One of the biggest ideas in quantum mechanics is Heisenberg's Uncertainty Principle.

Defn 48 (Heisenberg's Uncertainty Principle). *Heisenberg's Uncertainty Principle* states that it is impossible to accurately know both the position and velocity of a particle in a system.

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \quad (9.1)$$

Remark 48.1. The parameters for Equation (9.1) are below.

- Δx is the change in position of the "thing"
- Δp is the change in the momentum of the "thing"
- h is Planck's Constant.

A Physical Constants

Constant Name	Variable Letter	Value
Boltzmann Constant	R	8.314J/mol K
Universal Gravitational	G	$6.67408 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
Planck's Constant	h	$6.62607004 \times 10^{-34} \text{mkg/s} = 4.163 \times 10^{-15} \text{eV s}$
Speed of Light	c	$299792458 \text{m/s} = 2.998 \times 10^8 \text{ m/s}$
Charge of Electron	e	$1.602 \times 10^{-19} \text{C}$
Mass of Electron	m_{e-}	$9.11 \times 10^{-31} \text{kg}$
Mass of Neutron	m_{n^0}	$1.67 \times 10^{-31} \text{kg}$
Mass of Earth	m_{Earth}	$5.972 \times 10^{24} \text{kg}$
Diameter of Earth	d_{Earth}	12742km

B Trigonometry

B.1 Trigonometric Formulas

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{B.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{B.2})$$

B.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm j\alpha} = \cos(\alpha) \pm j \sin(\alpha) \quad (\text{B.3})$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{B.4})$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{B.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{B.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{B.7})$$

B.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{B.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{B.9})$$

B.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{B.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{B.11})$$

B.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{B.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{B.13})$$

B.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = (\sin(\alpha))^2 = \frac{1 - \cos(2\alpha)}{2} \quad (\text{B.14})$$

$$\cos^2(\alpha) = (\cos(\alpha))^2 = \frac{1 + \cos(2\alpha)}{2} \quad (\text{B.15})$$

B.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{B.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{B.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{B.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{B.19})$$

B.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{B.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.22})$$

B.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{B.23})$$

B.10 Rectangular to Polar

$$a + jb = \sqrt{a^2 + b^2} e^{j\theta} = r e^{j\theta} \quad (\text{B.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{B.25})$$

B.11 Polar to Rectangular

$$r e^{j\theta} = r \cos(\theta) + j r \sin(\theta) \quad (\text{B.26})$$

C Calculus

C.1 L'Hopital's Rule

L'Hopital's Rule can be used to simplify and solve expressions regarding limits that yield irreconcilable results.

Lemma C.0.1 (L'Hopital's Rule). *If the equation*

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \begin{cases} \frac{0}{0} \\ \frac{\infty}{\infty} \end{cases}$$

then Equation (C.1) holds.

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad (\text{C.1})$$

C.2 Fundamental Theorems of Calculus

Defn C.2.1 (First Fundamental Theorem of Calculus). The *first fundamental theorem of calculus* states that, if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{C.2})$$

Defn C.2.2 (Second Fundamental Theorem of Calculus). The *second fundamental theorem of calculus* holds for f a continuous function on an open interval I and a any point in I , and states that if F is defined by

$$F(x) = \int_a^x f(t) dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t) dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \quad (\text{C.3})$$

Defn C.2.3 (argmax). The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname{argmax}_x$$

C.3 Rules of Calculus

C.3.1 Chain Rule

Defn C.3.1 (Chain Rule). The *chain rule* is a way to differentiate a function that has 2 functions multiplied together.

If

$$f(x) = g(x) \cdot h(x)$$

then,

$$\begin{aligned} f'(x) &= g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ \frac{df(x)}{dx} &= \frac{dg(x)}{dx} \cdot h(x) + g(x) \cdot \frac{dh(x)}{dx} \end{aligned} \quad (\text{C.4})$$

C.4 Useful Integrals

$$\int \cos(x) dx = \sin(x) \quad (\text{C.5})$$

$$\int \sin(x) dx = -\cos(x) \quad (\text{C.6})$$

$$\int x \cos(x) dx = \cos(x) + x \sin(x) \quad (\text{C.7})$$

Equation (C.7) simplified with Integration by Parts.

$$\int x \sin(x) dx = \sin(x) - x \cos(x) \quad (\text{C.8})$$

Equation (C.8) simplified with Integration by Parts.

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x) \quad (\text{C.9})$$

Equation (C.9) simplified by using Integration by Parts twice.

$$\int x^2 \sin(x) dx = 2x \sin(x) - (x^2 - 2) \cos(x) \quad (\text{C.10})$$

Equation (C.10) simplified by using Integration by Parts twice.

$$\int e^{\alpha x} \cos(\beta x) dx = \frac{e^{\alpha x} (\alpha \cos(\beta x) + \beta \sin(\beta x))}{\alpha^2 + \beta^2} + C \quad (\text{C.11})$$

$$\int e^{\alpha x} \sin(\beta x) dx = \frac{e^{\alpha x} (\alpha \sin(\beta x) - \beta \cos(\beta x))}{\alpha^2 + \beta^2} + C \quad (\text{C.12})$$

$$\int e^{\alpha x} dx = \frac{e^{\alpha x}}{\alpha} \quad (\text{C.13})$$

$$\int x e^{\alpha x} dx = e^{\alpha x} \left(\frac{x}{\alpha} - \frac{1}{\alpha^2} \right) \quad (\text{C.14})$$

Equation (C.14) simplified with Integration by Parts.

$$\int \frac{dx}{\alpha + \beta x} = \int \frac{1}{\alpha + \beta x} dx = \frac{1}{\beta} \ln(\alpha + \beta x) \quad (\text{C.15})$$

$$\int \frac{dx}{\alpha^2 + \beta^2 x^2} = \int \frac{1}{\alpha^2 + \beta^2 x^2} dx = \frac{1}{\alpha \beta} \arctan \left(\frac{\beta x}{\alpha} \right) \quad (\text{C.16})$$

$$\int \alpha^x dx = \frac{\alpha^x}{\ln(\alpha)} \quad (\text{C.17})$$

$$\frac{d}{dx} \alpha^x = \frac{d\alpha^x}{dx} = \alpha^x \ln(\alpha) \quad (\text{C.18})$$

C.5 Leibnitz's Rule

Lemma C.0.2 (Leibnitz's Rule). *Given*

$$g(t) = \int_{a(t)}^{b(t)} f(x, t) dx$$

with $a(t)$ and $b(t)$ differentiable in t and $\frac{\partial f(x, t)}{\partial t}$ continuous in both t and x , then

$$\frac{d}{dt} g(t) = \frac{dg(t)}{dt} = \int_{a(t)}^{b(t)} \frac{\partial f(x, t)}{\partial t} dx + f[b(t), t] \frac{db(t)}{dt} - f[a(t), t] \frac{da(t)}{dt} \quad (\text{C.19})$$

D Complex Numbers

Complex numbers are numbers that have both a real part and an imaginary part.

$$z = a \pm bi \quad (\text{D.1})$$

where

$$i = \sqrt{-1} \quad (\text{D.2})$$

Remark (i vs. j for Imaginary Numbers). Complex numbers are generally denoted with either i or j . Since this is an appendix section, I will denote complex numbers with i , to make it more general. However, electrical engineering regularly makes use of j as the imaginary value. This is because alternating current i is already taken, so j is used as the imaginary value instead.

$$Ae^{-ix} = A [\cos(x) + i \sin(x)] \quad (\text{D.3})$$

D.1 Complex Conjugates

If we have a complex number as shown below,

$$z = a \pm bi$$

then, the conjugate is denoted and calculated as shown below.

$$\bar{z} = a \mp bi \quad (\text{D.4})$$

Defn D.1.1 (Complex Conjugate). The conjugate of a complex number is called its *complex conjugate*. The complex conjugate of a complex number is the number with an equal real part and an imaginary part equal in magnitude but opposite in sign.

The complex conjugate can also be denoted with an asterisk (*). This is generally done for complex functions, rather than single variables.

$$z^* = \bar{z} \quad (\text{D.5})$$

D.1.1 Complex Conjugates of Exponentials

$$\overline{e^z} = e^{\bar{z}} \quad (\text{D.6})$$

$$\overline{\log(z)} = \log(\bar{z}) \quad (\text{D.7})$$

D.1.2 Complex Conjugates of Sinusoids

Since sinusoids can be represented by complex exponentials, as shown in Appendix B.2, we could calculate their complex conjugate.

$$\begin{aligned} \overline{\cos(x)} &= \cos(x) \\ &= \frac{1}{2} (e^{ix} + e^{-ix}) \end{aligned} \quad (\text{D.8})$$

$$\begin{aligned} \overline{\sin(x)} &= \sin(x) \\ &= \frac{1}{2i} (e^{ix} - e^{-ix}) \end{aligned} \quad (\text{D.9})$$