

# Phys 123: Classical Mechanics - Reference Sheet

Karl Hallsby

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# 1 Vectors

**Defn 1** (Vector). A *vector* is a way to show both magnitude of displacement and direction of displacement. Vectors are drawn as rays.

*Remark 1.1.* Vectors and Scalars may seem similar, but are different.

**Defn 2** (Scalar). A *scalar* is a way to show **ONLY** the magnitude of a displacement, without any direction information.

## 1.1 Vector Properties

- (i)  $\vec{A} + \vec{B} = \vec{C}$
- (ii)  $\vec{0} = \langle 0, 0, 0, \dots, 0 \rangle$
- (iii)  $\vec{A} + \vec{0} = \vec{A}$
- (iv)  $\vec{A} + -\vec{A} = \vec{0}$
- (v)  $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$
- (vi)  $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- (vii) Magnitude of vector:  $\|\vec{A}\| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

### 1.1.1 Getting Components

Getting the components of a vector involves solving the imaginary pythagorean triangle around the vector.

For a 2-dimensional vector,  $\vec{V}$ , you have the components  $\langle V_x, V_y \rangle$ . You find their values with this equation:

$$\begin{aligned} V_x &= V \cos \theta \\ V_y &= V \sin \theta \end{aligned} \tag{1.1}$$

### 1.1.2 3D Unit Vectors

3-dimensional vectors shouldn't be any too crazy by this point. They are just another variable that can be thrown around in the vector. However, the three 3D Unit Vectors are special. You can also use these to describe any lower-dimensional vector as well.

$$\begin{aligned} \hat{i} &= \langle 1, 0, 0 \rangle \\ \hat{j} &= \langle 0, 1, 0 \rangle \\ \hat{k} &= \langle 0, 0, 1 \rangle \end{aligned} \tag{1.2}$$

### 1.1.3 Addition

Vectors are additive, and are done from head-to-tail. This means that

$$\vec{A} + \vec{B} = \vec{C} \tag{1.3}$$

This means that in 3-dimensional vectors, they are added like this:

$$\begin{aligned} \vec{A} &= \langle A_x, A_y, A_z \rangle \\ \vec{B} &= \langle B_x, B_y, B_z \rangle \\ \vec{A} + \vec{B} &= \langle A_x + B_x, A_y + B_y, A_z + B_z \rangle \end{aligned} \tag{1.4}$$

### 1.1.4 Scalar Multiplication

When applying multiplication between a scalar and a vector, you perform Scalar Multiplication.

$$2 \times \vec{V} = 2\langle V_x, V_y \rangle = \langle 2V_x, 2V_y, 2V_z \rangle \tag{1.5}$$

This means that you do **NOT** modify the direction of the vector, you only change its magnitude.

### 1.1.5 Scalar (Dot) Product

The Scalar (Dot) Product is the first of two ways to multiply 2 vectors. The other is the Vector (Cross) Product. There are 2 ways to calculate the Scalar (Dot) Product.

The first involves using the magnitudes of each vector and multiplying those by the cosine of the angle between them.

$$\vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos(\theta) \quad (1.6)$$

The second is done by adding the product of each component of each vector.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad (1.7)$$

*Remark.* This means that when you apply the Scalar (Dot) Product to 2 vectors, you return a Scalar.

#### Properties of Scalar (Dot) Product

- (i)  $(\vec{A})^2 = \vec{A} \cdot \vec{A}$
- (ii)  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

### 1.1.6 Vector (Cross) Product

The Vector (Cross) Product is the second of two ways to multiply 2 vectors. The other is the Scalar (Dot) Product. There are 2 ways to calculate the Vector (Cross) Product.

The first involves using the magnitudes of each vector and multiplying those by the sine of the angle between them.

$$\vec{A} \times \vec{B} = \|\vec{A}\| \|\vec{B}\| \sin \theta \quad (1.8)$$

The second is done by taking the determinant of a  $2 \times 2$  or  $3 \times 3$  matrix.

$$\begin{aligned} \vec{A} \times \vec{B} &= \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \\ &= \langle A_y B_z - A_z B_y, -(A_x B_z - A_z B_x), A_x B_y - A_y B_x \rangle \end{aligned} \quad (1.9)$$

*Remark.* This means that when you apply the Vector (Cross) Product to 2 vectors, you return a Vector.

#### Properties of Vector (Cross) Product

- (i)  $\vec{A} \times \vec{A} = \vec{0}$
- (ii)  $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$
- (iii)  $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$
- (iv)  $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{C} \times \vec{A})$

## 2 Kinematics

**Defn 3** (Kinematics). *Kinematics* is a way to describe macroscopic motion with equations. This includes anything moving, falling, thrown, shot, launched, etc. This forms the fundamental basis for all of classical mechanics.

### 2.1 1-D Kinematics

**Defn 4** (1-D Displacement). *One dimensional displacement* is calculated based on the change in position of the ‘thing.’

$$s = x_2 - x_1 \quad (2.1)$$

*Remark 4.1.* *Displacement is different than path!* Displacement is the change in position of an object. Path is the length of the path takes between its starting and end point.

**Defn 5** (1-D Velocity). *One dimensional velocity* is calculated as the displacement per unit time. There is instantaneous velocity and average velocity. Average velocity is calculated with Equation (2.2).

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad (2.2)$$

Instantaneous velocity is calculated by reducing the time interval  $\Delta t$  to 0. This can be summarized in Equation (2.3).

$$\begin{aligned} v &= \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \\ &= \frac{dx}{dt} \end{aligned} \quad (2.3)$$

**Defn 6** (Acceleration). *One dimensional acceleration* is the change in velocity over time. Again, there is average acceleration and instantaneous acceleration. Average acceleration is calculated with Equation (2.4)

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad (2.4)$$

Instantaneous acceleration is calculated by reducing the time interval  $\Delta t$  to 0. This can be summarized by Equation (2.5).

$$\begin{aligned} a &= \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} \\ &= \frac{dv}{dt} = \frac{d^2x}{dt^2} \end{aligned} \quad (2.5)$$

## 2.2 Multi-Dimensional Kinematics

Because we can represent a two-dimensional and three-dimensional space in sets, and movement through this space as their respectively dimensioned vectors, we can construct multi-dimensional problems with multi-dimensional vectors! This is a massive simplification, because instead of solving for one equation with three variables, we can solve three equations for one variable each!!

**For the following definitions, I have assumed that we are in a 3-dimensional space  $(x, y, z)$ .**

**Defn 7** (Multi-Dimensional Position). *Position* in multiple dimensions is done by referring to each of the constituent dimensions.

$$\vec{s} = (x(t), y(t), z(t)) \quad (2.6)$$

**Defn 8** (Multi-Dimensional Displacement). *Displacement* in multiple dimensions can be broken down into several 1-D Displacements. Since 1-D Displacement is calculated as the difference between the start and end position, the same is true for the multi-dimensional case.

$$\begin{aligned} \vec{r} &= \Delta \vec{s} = \vec{s}_2 - \vec{s}_1 \\ &= \langle x_2(t) - x_1(t), y_2(t) - y_1(t), z_2(t) - z_1(t) \rangle \\ &= \langle r_x(t), r_y(t), r_z(t) \rangle \end{aligned} \quad (2.7)$$

**Defn 9** (Multi-Dimensional Velocity). *Velocity* in multiple dimensions is described in much the same way as 1-D Velocity.

$$\begin{aligned} \vec{v} &= \frac{d\vec{r}}{dt} \\ &= \left\langle \frac{dr_x(t)}{dt}, \frac{dr_y(t)}{dt}, \frac{dr_z(t)}{dt} \right\rangle \\ &= \langle r'_x(t), r'_y(t), r'_z(t) \rangle \end{aligned} \quad (2.8)$$

**Defn 10** (Multi-Dimensional Acceleration). *Acceleration* in multiple dimensions is described in much the same way as Acceleration.

$$\begin{aligned} \vec{a} &= \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \\ &= \left\langle \frac{dv_x(t)}{dt}, \frac{dv_y(t)}{dt}, \frac{dv_z(t)}{dt} \right\rangle = \left\langle \frac{d^2r_x(t)}{dt^2}, \frac{d^2r_y(t)}{dt^2}, \frac{d^2r_z(t)}{dt^2} \right\rangle \\ &= \langle v'_x(t), v'_y(t), v'_z(t) \rangle = \langle r''_x(t), r''_y(t), r''_z(t) \rangle \end{aligned} \quad (2.9)$$

## 2.3 Projectile Motion

**Defn 11** (Projectile). A *projectile* is any body given an initial velocity that then follows a path determined by gravity and air resistance.

*Remark 11.1.* However, for most of our calculations, we will neglect air resistance. Air resistance can be a difficult thing to calculate for, especially in the variable cases that we will have.

There are a few things to keep in mind with projectiles in motion.

1. Origin is where the projectile starts from
2. The x-axis is the *distance* that the projectile travels. This is its displacement.
3. The y-axis is the *height* that the projectile travels.
4. The end point (landing point) is the only thing that may change on the x-axis.
5. The acceleration vector is as follows:  $\langle 0, -g \rangle$ .
6. *Trajectory* depends on  $\vec{v}_0$  and  $\vec{a}$  *ONLY*.
7. The two components of the projectile's initial velocity are *independent*  $(v_{0,x}, v_{0,y})$ .

### 2.3.1 Projectile Motion Equations

The following equations are used to solve for various questions that could be asked about projectile motion.

#### Initial Velocity Components

$$v_{0,x} = v_0 \cos(\theta) \quad v_{0,y} = v_0 \sin(\theta) \quad (2.10)$$

#### Velocity Components

$$v_x = v_{0,x} \cos(\theta) \quad v_y = v_{0,y} \sin(\theta) - gt \quad (2.11)$$

#### Projectile Position

$$x = v_0 t \cos(\theta) \quad y = v_0 t \sin(\theta) - \frac{1}{2}gt^2 \quad (2.12)$$

#### Projectile Time

$$t = \frac{x}{v_0 \cos(\theta)} \quad t = \frac{v_0 \sin(\theta)}{g} \quad (2.13)$$

#### Projectile Range

$$R = \frac{v_0}{g} \cos(\theta) \sin(\theta) \quad (2.14)$$

#### Projectile Maximum Range

$$R_{\text{Max}} = \frac{v_0^2}{g} \quad (2.15)$$

This means that the  $\theta$  in Equation (2.14) is  $45^\circ$ .

#### Projectile Height

$$h = \frac{v_0^2}{2g} \sin^2(\theta) \quad (2.16)$$

#### Projectile Maximum Height

$$h = \frac{v_0^2}{2g} \quad (2.17)$$

The lack of  $\sin^2(\theta)$  from Equation (2.16) means that there is **NO** y-component to the velocity, meaning the projectile is at its instant of maximum height.

### 3 Uniform Circular Motion

**Defn 12** (Uniform Circular Motion). *Uniform circular motion* is when an object is moving in a perpetual circular motion. There is no outside source of acceleration changing the state of the system.

*Remark 12.1.* This does *not* happen in real life. However, it is useful for modelling things under ideal conditions that do happen in real life.

#### 3.1 Angular Part of Circular Motion

During Uniform Circular Motion, your terminology changes a little bit.

**Defn 13** (Angular Position). *Angular position* is determined with radians around a circle. It is denoted with

$$\vec{\theta}$$

**Defn 14** (Angular Velocity). *Angular velocity* is orthogonal to the flat 2-D plane that the object is traveling in. It is the derivative of the Angular Position.

$$\begin{aligned}\vec{\omega} &= \frac{d\theta}{dt} \\ &= \langle 0, 0, \omega \rangle\end{aligned}\tag{3.1}$$

**Defn 15** (Angular Acceleration). *Angular acceleration* is the derivative of the Angular Velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}\tag{3.2}$$

*Remark 15.1.* Note that under Uniform Circular Motion, by its very definition, there cannot be any acceleration on the object. Therefore, when an object is in uniform circular motion,  $\vec{\alpha} = 0$ .

However, when the object is **NOT** in Uniform Circular Motion the object is undergoing Linear Acceleration.

#### 3.2 Linear Part of Circular Motion

**Defn 16** (Linear Position). *Linear position* relates the position of an object from the cartesian coordinate plane to the polar. This means that:

$$x = r \cos(\theta) \quad y = r \sin(\theta)\tag{3.3}$$

**Defn 17** (Linear Velocity). *Linear velocity* relates the velocity of an object in a line to its Angular Velocity.

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} \\ \frac{d\vec{r}}{dt} &= \left\langle \frac{dx}{dt}, \frac{dy}{dt}, 0 \right\rangle = \left\langle \frac{d}{dt}r \cos(\theta), \frac{d}{dt}r \sin(\theta), 0 \right\rangle \\ &= \left\langle -r \sin(\theta)\omega, r \cos(\theta)\omega, 0 \right\rangle \\ &= \vec{\omega} \times \vec{r}\end{aligned}\tag{3.4}$$

**Defn 18** (Linear Acceleration). *Linear acceleration* is the derivative of Linear Velocity. It relates the acceleration of an object in a line is relative to its Angular Acceleration

$$\begin{aligned}\frac{d\vec{a}}{dt} &= \frac{d\vec{v}}{dt} \\ &= \langle -r\omega^2 \cos(\theta), -r\omega^2 \sin(\theta), 0 \rangle = \omega^2 \langle -r \cos(\theta), -r \sin(\theta), 0 \rangle \\ &= -\omega^2 \vec{r}\end{aligned}\tag{3.5}$$

#### 3.3 Relation Between the Angular Part of Circular Motion and the Linear Part of Circular Motion

There are a few equations that relate both the Angular Part of Circular Motion and the Linear Part of Circular Motion.

**Velocity and Angular Velocity**

$$v = \omega r\tag{3.6}$$

## Acceleration and Angular Acceleration

$$\begin{aligned} a &= \omega^2 r \\ &= \frac{v^2}{r} \end{aligned} \tag{3.7}$$

## 4 Reference Frames

**Defn 19** (Reference Frames). An *inertial reference frame* is a frame for the world. It is easiest to think of of an inertial reference frame with an example. For instance, when you're in a car going 40 mph and you see someone going 45, you can only tell they're going 5 mph than you.

**Defn 20** (Galileo Transformation/Relativity Principle). This is a transformation that happens when you're calculating in one Reference Frames and the event is happening in a different Reference Frames.

$$\begin{aligned} \frac{d\vec{R}}{dt} &= \frac{d\vec{r}}{dt} + \frac{d\vec{r}'}{dt} \\ \vec{V} &= \vec{v} + \vec{v}' \end{aligned} \tag{4.1}$$

## 5 Newton's Laws

There are 3 fundamental laws of classical mechanics.

1. An object in motion/at rest stays as such, unless acted upon by an outside force(s).
2. Force is equal to the change in momentum ( $p = mv$ ) per change in time. For a constant mass, force equals mass times acceleration ( $\vec{F} = m\vec{a}$ ).
3. For every force there is an equal and opposite force.

There are a few forces that are fundamental in our universe.

- Gravitational Force
- Magnetic Force
- Electric Force
- Frictional Force
- Normal Force
- Tension Force

*Remark.* **REMEMBER** that forces are vectors!!

To solve any force problem, you should construct a free-body diagram.

### 5.1 Newton's Laws in 1-D

#### Example 5.1: Atwood Machine.

Find the acceleration that is the result of dropping one side of the machine? Neglect any friction that may occur because of the pulley.

$$\begin{aligned} m_2 \vec{a}_2 &= m_2 \vec{g} + \vec{T}_2 \\ m_1 \vec{a}_1 &= m_1 \vec{g} + \vec{T}_1 \end{aligned}$$

One thing to remember is that both masses will have the same acceleration, because the pulley has no friction. So because  $\vec{a}_2 = \vec{a}_1$ , we can solve for a single  $\vec{a}$  variable.

$$\begin{aligned} m_2 \vec{a} &= m_2 \vec{g} + \vec{T}_2 \\ m_1 \vec{a} &= m_1 \vec{g} + \vec{T}_1 \\ \vec{a} (m_2 + m_1) &= \vec{g} (m_2 - m_1) \\ \vec{a} &= \frac{g (m_2 - m_1)}{m_2 + m_1} \end{aligned}$$



## 5.2 Newton's Laws in Multi-D

For a multi-dimensional situation, you break the free-body problem's vectors down into their components.

### Example 5.2: Sliding Block.

If a block of mass  $m$  slides down a ramp of angle  $\theta$ , what is the coefficient of static and kinetic friction?

## 5.3 Common Forces

This section will discuss various forces in greater detail.

### 5.3.1 Gravitational Force

**Defn 21** (Gravitational Force). *Gravitational force* is a force that arises from Gravity. Gravity will be covered more in Section 10, Gravitation. This force arises when one object is being pulled towards another due to Gravity. Gravitational Force is generally the acceleration vector due to Earth's gravity.

$$\vec{g} = 9.81\text{m/s}^2 \quad (5.1)$$

### 5.3.2 Magnetic Force

This force is discussed in much greater detail in the Physics 221: Electromagnetics and Optics course.

### 5.3.3 Electric Force

This force is discussed in much greater detail in the Physics 221: Electromagnetics and Optics course.

### 5.3.4 Frictional Force

Frictional force is the common culprit for the "Equal and Opposite Force" in classical mechanics.

**Defn 22** (Frictional Force). Frictional force is defined as

$$\vec{F}_f = \mu \vec{N} \quad (5.2)$$

- $\mu$  - The coefficient of the type of friction you are dealing with (Either the Coefficient of Static Friction or Coefficient of Kinetic Friction)
- $\vec{N}$  - The Normal Force

There are 2 types of frictional force:

1. Static Friction
2. Kinetic Friction

**Defn 23** (Static Friction). *Static friction* is friction that arises when an object is starting from a static position, and is being moved. This force tends to be stronger than Kinetic Friction because of electrostatic bonds between the object and its supporting surface, along with other reasons. However, static friction is drawn from the Coefficient of Static Friction. This is a Scalar number that represents how much the object *does not* want to move.

$$\vec{F}_{f,s} = \mu_s \vec{N} \quad (5.3)$$

- $\mu_s$  - The Coefficient of Static Friction
- $\vec{N}$  - The Normal Force

**Defn 24** (Coefficient of Static Friction). *Coefficient of static friction* is a Scalar number that represents how much the object does not want to **START** moving. This value tends to be greater than the Coefficient of Kinetic Friction. This is because there are additional bonds and forces at play that resist the start of an objects motion. The Coefficient of Kinetic Friction is denoted as such:

$$\mu_s \quad (5.4)$$

**Defn 25** (Kinetic Friction). *Kinetic friction* is a friction that arises when an object is in motion. This force is weaker than Static Friction. The value we may use for kinetic friction is drawn from the Coefficient of Kinetic Friction.

$$\vec{F}_{f,k} = -\mu_k \vec{N} \quad (5.5)$$

- $\mu_k$  - The Coefficient of Kinetic Friction
- $\vec{N}$  - The Normal Force that is going **IN** the direction of motion. You might need to break the normal force's vector down into its components.

**Defn 26** (Coefficient of Kinetic Friction). *Coefficient of kinetic friction* is Scalar number that represents how much the object does not want to **CONTINUE** moving. This value tends to be smaller than its Coefficient of Static Friction counterpart. The Coefficient of Kinetic Friction is denoted as such:

$$\mu_k \quad (5.6)$$

### 5.3.5 Normal Force

**Defn 27** (Normal Force). The *normal force* is, as the name implies, a normalizing force. This does not necessarily make it special, as both Kinetic Friction and Static Friction can be considered normalizing forces as well. However, the normal force is generally considered whenever something is being held against something. For example, a book on a table. The table is exerting a Normal Force on the book to prevent it from falling through the table. Likewise, the ground is exerting a Normal Force on the table to prevent it from falling through the ground.

This normal force is defined as

$$\vec{N} = -m\vec{g} \quad (5.7)$$

## 6 Dynamics of Circular Motion

There are three cases when the Dynamics of Circular Motion are easily visible. Each of these is illustrated with an example.

1. Turning on a flat curve, neglecting friction (Example 6.1)
2. Turning on an angled curve, neglecting friction (Example 6.2)
3. Turning on an angled curve, with friction (Example 6.3)

### Example 6.1: Car Turning on a Flat Curve.

A car of mass  $m$  is turning on a flat curve. What is its maximum velocity,  $v_{\text{Max}}$ ?

Solution from Dynamics of Circular Motion Notes Page.

### Example 6.2: Car Turning on Angled Curve with No Friction.

A car of mass  $m$  is turning on a curve with an angle  $\theta$  from the horizontal. What is its maximum velocity,  $v_{\text{Max}}$ ?

Solution from Dynamics of Circular Motion Notes Page.

### Example 6.3: Car Turning on Angled Curve with Friction.

A car of mass  $m$  is turning on a curve with an angle  $\theta$  from the horizontal. There is friction, with the coefficient of static friction being  $\mu_s$  and coefficient of kinetic friction being  $\mu_k$ . What is its maximum velocity,  $v_{\text{Max}}$ ?

Solution from Dynamics of Circular Motion Notes Page.

## 7 Springs

### 7.1 Hooke's Law

## 8 Energy

### 8.1 Kinetic Energy

### 8.2 Potential Energy

### 8.3 Conservation of Energy

## 9 Systems of Particles

### 9.1 Center of Mass

## 10 Gravitation

**Defn 28** (Gravity). *Gravity* is one of the 4 fundamental forces in the universe. It is also one of the least understood. However, it is the thing that is keeping us “glued” to this planet.

## A Physical Constants

Constant Name	Variable Letter	Value
Boltzmann Constant	$R$	8.314J/mol K
Universal Gravitational	$G$	$6.67408 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
Planck's Constant	$h$	$6.62607004 \times 10^{-34} \text{mkg/s} = 4.163 \times 10^{-15} \text{eV s}$
Speed of Light	$c$	$299792458 \text{m/s} = 2.998 \times 10^8 \text{ m/s}$
Charge of Electron	$e$	$1.602 \times 10^{-19} \text{C}$
Mass of Electron	$m_{e-}$	$9.11 \times 10^{-31} \text{kg}$
Mass of Neutron	$m_{n^0}$	$1.67 \times 10^{-31} \text{kg}$
Mass of Earth	$m_{Earth}$	$5.972 \times 10^{24} \text{kg}$
Diameter of Earth	$d_{Earth}$	12742km

## B Trigonometry

### B.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{B.2})$$

### B.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm i\alpha} = \cos(\alpha) \pm i \sin(\alpha) \quad (\text{B.3})$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \quad (\text{B.4})$$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad (\text{B.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{B.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{B.7})$$

### B.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{B.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{B.9})$$

### B.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{B.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{B.11})$$

### B.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{B.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{B.13})$$

### B.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2} \quad (\text{B.14})$$

$$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2} \quad (\text{B.15})$$

### B.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{B.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{B.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{B.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{B.19})$$

## B.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{B.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.22})$$

## B.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{B.23})$$

## B.10 Rectangular to Polar

$$a + ib = \sqrt{a^2 + b^2} e^{i\theta} = r e^{i\theta} \quad (\text{B.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{B.25})$$

## B.11 Polar to Rectangular

$$r e^{i\theta} = r \cos(\theta) + ir \sin(\theta) \quad (\text{B.26})$$

## C Calculus

### C.1 Fundamental Theorems of Calculus

**Defn C.1.1** (First Fundamental Theorem of Calculus). The *first fundamental theorem of calculus* states that, if  $f$  is continuous on the closed interval  $[a, b]$  and  $F$  is the indefinite integral of  $f$  on  $[a, b]$ , then

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{C.1})$$

**Defn C.1.2** (Second Fundamental Theorem of Calculus). The *second fundamental theorem of calculus* holds for  $f$  a continuous function on an open interval  $I$  and  $a$  any point in  $I$ , and states that if  $F$  is defined by

$$F(x) = \int_a^x f(t) dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t) dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \quad (\text{C.2})$$

**Defn C.1.3** (argmax). The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname{argmax}_x$$

## D Complex Numbers

$$Ae^{-ix} = A [\cos (x) + i \sin (x)] \tag{D.1}$$