EDAN20: Language Technology — Reference Sheet
Lund University

Karl Hallsby

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Contents

1 Linguistics

Defn 1 (Linguistics)**.** *Linguistics* is the study and the description of human languages. Linguistics have been developed since ancient times and the Middle Ages. *Modern Linguistics* developed between the end of the 19^{th} century and beginning of the 20^{th} century. It founder and prominent figure was Ferdinand de Saussure.

There is something else that we will be working with, called Computational Linguistics.

Defn 2 (Computational Linguistics)**.** *Computational Linguistics* is a subset of both Linguistics and computer science. Its goal is to design mathematical models of language structures enabling the automation of language processing by a computer. We can consider computational linguistics as the formalization of linguistic theories and models, or their implementation in a machine. New linguistic theories can be developed with the aid of a computer too.

Historically, there are 3 disciplines of Linguistics.

1. Phonetics
2. Words
3. Syntax

1.1 Phonetics

Defn 3 (Phonetics)**.** *Phonetics* concerns the production and perception of acoustic sounds that form the speech signal. In every language sounds can be classified into a finite set of Phonemes

Defn 4 (Phonemes)**.** *Phonemes* are the building blocks of Phonetics. Traditionally, Phonemes include Vowels and Consonants. Phonemes are assembled into Syllables to build words.

Examples include: *pa, pi, po.*

Defn 5 (Vowels)**.** *Vowels* are a speech sound that is produced by a comparatively open configuration of the vocal tract, with vibration of the vocal cords, but without audible friction. They are a unit of the sound system of a language that forms the nucleus of Syllables.

Examples include: *a, e, i, o.*

Defn 6 (Consonants)**.** *Consonants* are a speech sound in which the breath is at least partly obstructed and which can be combined with Vowels to form Syllables.

Examples include: *p, f, r, m.*

Defn 7 (Syllables)**.** *Syllables* are a unit of pronunciation having one vowel sound, with or without surrounding consonants, forming the whole or part of a word.

1.2 Words

Defn 8 (Words)**.** *Words*

1.3 Syntax

Defn 9 (Syntax)**.** *Syntax*

A Trigonometry

A.1 Trigonometric Formulas

$$\sin(\alpha) \pm \sin(\beta) = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cos\left(\frac{\alpha \mp \beta}{2}\right) \tag{A.1}$$

$$\cos(\theta)\sin(\theta) = \frac{1}{2}\sin(2\theta) \tag{A.2}$$

A.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm j\alpha} = \cos(\alpha) \pm j\sin(\alpha) \tag{A.3}$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \tag{A.4}$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \tag{A.5}$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2} \tag{A.6}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \tag{A.7}$$

A.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \tag{A.8}$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \tag{A.9}$$

A.4 Double-Angle Formulae

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \tag{A.10}$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \tag{A.11}$$

A.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \tag{A.12}$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \tag{A.13}$$

A.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \big(\sin(\alpha)\big) = \frac{1 - \cos(2\alpha)}{2} \tag{A.14}$$

$$\cos^2(\alpha) = \big(\cos(\alpha)\big) = \frac{1 + \cos(2\alpha)}{2} \tag{A.15}$$

A.7 Product-to-Sum Identities

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \tag{A.16}$$

$$2\sin(\alpha)\sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \tag{A.17}$$

$$2\sin(\alpha)\cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \tag{A.18}$$

$$2\cos(\alpha)\sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \tag{A.19}$$

A.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cos\left(\frac{\alpha \mp \beta}{2}\right) \tag{A.20}$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right) \tag{A.21}$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right) \tag{A.22}$$

A.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \tag{A.23}$$

A.10 Rectangular to Polar

$$a + jb = \sqrt{a^2 + b^2}e^{j\theta} = re^{j\theta} \tag{A.24}$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \tag{A.25}$$

A.11 Polar to Rectangular

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta) \tag{A.26}$$

B Calculus

B.1 L'Hopital's Rule

L'Hopital's Rule can be used to simplify and solve expressions regarding limits that yield irreconcialable results.

Lemma B.0.1 (L'Hopital's Rule)**.** *If the equation*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \begin{cases} \frac{0}{0} \\ \frac{\infty}{\infty} \end{cases}$$

then Equation (B.1) *holds.*

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \tag{B.1}$$

B.2 Fundamental Theorems of Calculus

Defn B.2.1 (First Fundamental Theorem of Calculus)**.** The *first fundamental theorem of calculus* states that, if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a) \tag{B.2}$$

Defn B.2.2 (Second Fundamental Theorem of Calculus)**.** The *second fundamental theorem of calculus* holds for f a continuous function on an open interval I and a any point in I, and states that if F is defined by

$$F(x) = \int_a^x f(t)\, dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t)\, dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \tag{B.3}$$

Defn B.2.3 (argmax)**.** The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname*{argmax}_x$$

B.3 Rules of Calculus

B.3.1 Chain Rule

Defn B.3.1 (Chain Rule)**.** The *chain rule* is a way to differentiate a function that has 2 functions multiplied together.

If

$$f(x) = g(x) \cdot h(x)$$

then,

$$\begin{aligned} f'(x) &= g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ \frac{df(x)}{dx} &= \frac{dg(x)}{dx} \cdot g(x) + g(x) \cdot \frac{dh(x)}{dx} \end{aligned} \tag{B.4}$$

B.4 Useful Integrals

$$\int \cos(x)\, dx = \sin(x) \tag{B.5}$$

$$\int \sin(x)\, dx = -\cos(x) \tag{B.6}$$

$$\int x \cos(x)\, dx = \cos(x) + x \sin(x) \tag{B.7}$$

Equation (B.7) simplified with Integration by Parts.

$$\int x \sin(x)\; dx = \sin(x) - x\cos(x) \tag{B.8}$$

Equation (B.8) simplified with Integration by Parts.

$$\int x^2 \cos(x)\; dx = 2x\cos(x) + (x^2 - 2)\sin(x) \tag{B.9}$$

Equation (B.9) simplified by using Integration by Parts twice.

$$\int x^2 \sin(x)\; dx = 2x\sin(x) - (x^2 - 2)\cos(x) \tag{B.10}$$

Equation (B.10) simplified by using Integration by Parts twice.

$$\int e^{\alpha x} \cos(\beta x)\; dx = \frac{e^{\alpha x}\big(\alpha\cos(\beta x) + \beta\sin(\beta x)\big)}{\alpha^2 + \beta^2} + C \tag{B.11}$$

$$\int e^{\alpha x} \sin(\beta x)\; dx = \frac{e^{\alpha x}\big(\alpha\sin(\beta x) - \beta\cos(\beta x)\big)}{\alpha^2 + \beta^2} + C \tag{B.12}$$

$$\int e^{\alpha x}\; dx = \frac{e^{\alpha x}}{\alpha} \tag{B.13}$$

$$\int x e^{\alpha x}\; dx = e^{\alpha x}\left(\frac{x}{\alpha} - \frac{1}{\alpha^2}\right) \tag{B.14}$$

Equation (B.14) simplified with Integration by Parts.

$$\int \frac{dx}{\alpha + \beta x} = \int \frac{1}{\alpha + \beta x}\; dx = \frac{1}{\beta}\ln(\alpha + \beta x) \tag{B.15}$$

$$\int \frac{dx}{\alpha^2 + \beta^2 x^2} = \int \frac{1}{\alpha^2 + \beta^2 x^2}\; dx = \frac{1}{\alpha\beta}\arctan\left(\frac{\beta x}{\alpha}\right) \tag{B.16}$$

$$\int \alpha^x\; dx = \frac{\alpha^x}{\ln(\alpha)} \tag{B.17}$$

$$\frac{d}{dx}\alpha^x = \frac{d\alpha^x}{dx} = \alpha^x \ln(x) \tag{B.18}$$

B.5 Leibnitz's Rule

Lemma B.0.2 (Leibnitz's Rule)**.** *Given*

$$g(t) = \int_{a(t)}^{b(t)} f(x,t)\, dx$$

with $a(t)$ and $b(t)$ differentiable in t and $\frac{\partial f(x,t)}{\partial t}$ continuous in both t and x, then

$$\frac{d}{dt}g(t) = \frac{dg(t)}{dt} = \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t}\, dx + f\big[b(t), t\big]\,\frac{db(t)}{dt} - f\big[a(t), t\big]\,\frac{da(t)}{dt} \tag{B.19}$$