

# 1 Exam 1 Equations

## 1.1 Ch. 2 - Interatomic Forces

$$E_N = E_A + E_R = -\frac{A}{r} + \frac{B}{r^n} \quad (1.1)$$

$$A = \frac{1}{4\pi\epsilon_0} (Z_1 e) (Z_2 e) \quad (1.2)$$

- This equation works for both  $A$  and  $B$
- $\epsilon_0 = 8.85 \times 10^{-12} \text{F/m}$
- $e = 1.602 \times 10^{-19} \text{C}$
- $r$  - Radius in m

$$F_A = \frac{(1.602 \times 10^{-19})^2}{4\pi(8.85 \times 10^{-12})r^2} (\|Z_1\|)(\|Z_2\|) \quad (1.3)$$

- $F_A$  - Force of Attraction
- $r$  - Distance in m
- $Z$  - Number of Valence Electrons
- $F_A$  - Interatomic Force in N
- $-F_A = F_R$  - Attractive and Repulsive Force Equal and Opposite

$$\text{Force} = \frac{dE}{dr} \quad (1.4)$$

$$\text{Elastic Modulus} = \frac{dF}{dr} \quad (1.5)$$

$$\%IC = \left(1 - e^{\frac{(x_A - x_B)^2}{4}}\right) \times 100\% \quad (1.6)$$

- %IC - % Ionic Character
- $x$  - Electronegativities

## 1.2 Ch. 3 - Structures of Metals/Ceramics

### 1.2.1 Lattice Parameters

$$a_{\text{BCC}} = \frac{4r}{\sqrt{3}} \quad (1.7)$$

$$a_{\text{FCC}} = \frac{4r}{\sqrt{2}} \quad (1.8)$$

$$a_{\text{HCP}} = \frac{c}{1.633} \quad (1.9)$$

- $a$  - Lattice Parameter
- $r$  - Radius of atom

### 1.2.2 Volume of Hexagonal Prism

$$V_H = \frac{3\sqrt{3}}{2} a^2 h \quad (1.10)$$

### 1.2.3 Densities

$$\rho = \frac{nA}{V_C N_A} \quad (1.11)$$

- $n$  - Number of atoms/unit cell
- $A$  - Molar Mass of Material
- $V_C$  - Volume of Unit Cell in cm

- $N_A$  - Avogadro's Number ( $6.022 \times 10^{23}$ )

$$\text{Planar Density} = \frac{\frac{\text{Atoms}}{2\text{D Unit Area}}}{\frac{\text{Area}}{2\text{D Repeat Unit}}} \quad (1.12)$$

$$\text{Linear Density} = \frac{\# \text{ of Atoms in a Direction}}{\text{Magnitude of Linear Vector}} \quad (1.13)$$

- The repeat units/vector magnitude are in terms of atomic radii

$$\text{APF} = \frac{\frac{\text{Atoms}}{\text{Unit Cell}} \left(\frac{4}{3}\pi (\text{atom radius})^3\right)}{\text{Unit Cell Volume}} \quad (1.14)$$

### 1.2.4 Thermal Expansion

$$\frac{\Delta L}{L_0} = \alpha (T_2 - T_1) \quad (1.15)$$

- $E \uparrow, T_m \uparrow$
- $E \uparrow, \alpha \downarrow$

### 1.2.5 Convert between Coordinates

$$a_1 = \frac{1}{3} (2X - Y)$$

$$a_2 = \frac{1}{3} (2Y - X)$$

$$a_3 = -(a_1 + a_2)$$

$$c = Z$$

$$a_1 + a_2 + a_3 = 0$$

$$[XYZ] = [a_1 a_2 a_3 c] \quad (1.16)$$

### 1.2.6 Planes

1. Given  $x, y, z$  as intercepts
2. Convert to  $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$
3. Reduce to smallest common denominator
4. Leave as  $\left(\frac{1}{x} \frac{1}{y} \frac{1}{z}\right)$

### 1.2.7 Light Refraction

$$D = \frac{n\lambda}{2 \sin \theta} \quad (1.17)$$

- $n = 1$
- $\lambda$  - Wavelength in nm
- $\theta$  - Angle of Incidence
- $\theta$  is usually given as  $2\theta$ . Be careful

### 1.2.8 Randoms

$$D_{HKL} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (1.18)$$

- ONLY for cubic structures

## 2 Exam 2 Equations

### 2.1 Ch. 4 Imperfections

#### 2.1.1 Vacancies

Number of Vacancies:

$$N_V = \text{Vacancy Frac.} \times \frac{N_A \rho_{\text{Ele}}}{A_{\text{Ele}}} \quad (2.1)$$

- $A_{\text{Ele}}$  - Atomic Weight (g/mol)
- $\rho_{\text{Ele}}$  - Element Density (g/cm<sup>3</sup>)
- $N_A$  - Avogadro's Number ( $6.022 \times 10^{23}$ )

Number of Potential Vacancy Sites:

$$N = \frac{\rho_{\text{Ele}} N_A}{A_{\text{Ele}}} \quad (2.2)$$

- $A_{\text{Ele}}$  - Atomic Weight (g/mol)
- $\rho_{\text{Ele}}$  - Element Density (g/cm<sup>3</sup>)
- $N_A$  - Avogadro's Number ( $6.022 \times 10^{23}$ )

Grains per Area:

$$n_M = \left( \frac{100}{M} \right)^2 = 2^{G-1} \quad (2.3)$$

- $n_M$  - Grains/in<sup>2</sup>
- $G$  - Grain Size Number
  - $G = -6.6457 \log(\bar{\ell}) - 3.298$  for mm
  - $G = -6.6353 \log(\bar{\ell}) - 12.6$  for in.

#### 2.1.2 Mean Intercept Length

$$\bar{\ell} = \frac{L_T}{PM} \quad (2.4)$$

- $L_T$  - Total Length of all Lines
- $P$  - Number of Grain Intersections
- $M$  - Magnification

$$M = \frac{\text{Scale Length}}{\# \text{ on Scale Bar}} \quad (2.5)$$

#### 2.1.3 Complete Substitution

To have a complete substitution, it must be:

- $\Delta r < 15\%$
- Electronegativity  $\leq .4$
- SAME Crystal Structure
- SAME Valence

#### 2.1.4 Edges

Burger's Vector:

- Burger's Vector
  - $\perp$  for Edge Dislocations
  - $\parallel$  for Screw Dislocations
- Twin Boundary - Symmetric Around Fault
- Stacking Fault - NOT Symmetric Around Fault
  - High Angle -  $E \uparrow$
  - Low Angle -  $E \downarrow$

### 2.2 Ch. 5 Diffusion

#### 2.2.1 Diffusion Coefficient

$$D = D_0 \times e^{\frac{-Q_d}{RT}} \quad (2.6)$$

- $D$  - Diffusion Coefficient (m<sup>2</sup>/s)
- $Q_d$  - Activation Energy (J/mol, eV/atom)
- $R$  - 8.314 (J/molK)
- $T$  - Temperature (K)

$$D_1 t_1 = D_2 t_2 \quad (2.7)$$

- $D$  - Diffusion Coefficients
- $t$  - Time

#### 2.2.2 Flux

$$J = -DA \frac{dC}{dx} \quad (2.8)$$

- For Steady State Diffusion
- $D$  - Diffusion Coefficient
- $dC$  -  $\Delta$  Concentration (Low-High)
- $dx$  - Distance to Cross
- $A$  - Area

#### 2.2.3 Diffusion Concentration

$$\frac{C(x, t) - C_0}{C_s - C_0} = 1 - \text{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \quad (2.9)$$

$$Z = \frac{x}{2\sqrt{Dt}} = \frac{z - \text{point below}}{\text{point above} - \text{point below}} \quad (2.10)$$

- $C(x, t)$  - Concentration at a point AND time
- $C_0$  - Initial Concentration
- $C_s$  - Surface Concentration of DIFFUSING species
- $x$  - Position
- $D$  - Diffusion Coefficient
- $t$  - Time

## 3 Exam 3 Equations

### 3.1 Ch. 6 - Mechanical Properties

#### 3.1.1 Stress

##### Tensile Stress

$$\sigma = \frac{F_t}{A_0} \quad (3.1)$$

- Think of amount of force required to pull ends of paper apart
- $\sigma$  has units  $lb_f/in^2$  or  $N/m^2$
- $F_t$  - Normal Force
- $A_0$  - Original Cross-Sectional Area
- $\sigma < 0$  - Compressive Force
- $\sigma > 0$  - Tensile Force

##### Shear Stress

$$\tau = \frac{F_S}{A_0} \quad (3.2)$$

- Think of amount of force required to rip a piece of paper
- $\tau$  has units  $lb_f/in^2$  or  $N/m^2$
- $F_S$  - Shear Force
- $A_0$  - Original Cross-Sectional Area

#### 3.1.2 Strain

##### Tensile Strain

$$\varepsilon = \frac{\delta}{L_0} = \frac{L - L_0}{L_0} \quad (3.3)$$

- Think of pulling ends of paper apart and seeing how much it stretches
- $\varepsilon$  - Tensile Strain
- $\delta$  - Change in Length of Material
- $L_0$  - Original Length of Material

##### Shear Strain

$$\gamma = \frac{\Delta x}{y} = \tan \theta \quad (3.4)$$

- Change of length of material compared to height when ripped apart
- $\gamma$  - Shear Strain
- $\Delta x$  - Change in length
- $y$  - Height of Material Tested

#### 3.1.3 Moduli

##### Young's Modulus

$$E = \frac{\sigma}{\varepsilon} = \frac{dF}{dr} \quad (3.5)$$

- How Stiff a Material is from being pulled apart
- $E$  - Young's Modulus
- $\sigma$  - Tensile Stress
- $\varepsilon$  - Tensile Strain

##### Elastic Shear Modulus

$$G = \frac{\tau}{\gamma} \quad (3.6)$$

- How Stiff a material is from Ripping
- $\tau$  - Shear Stress
- $\gamma$  - Shear Strain

##### Elastic Bulk Modulus

$$K = -P \frac{V_0}{\Delta V} \quad (3.7)$$

- $K$  - Elastic Bulk Modulus
- $P$  -
- $V_0$  - Original Volume
- $\Delta V$  - Change in Volume

##### Poisson's Ratio

$$v = -\frac{\varepsilon_L}{\varepsilon} \quad (3.8)$$

- $v$  - Poisson's Ratio
- $\varepsilon_L$  - Tensile Strain at the length  $L$
- $\varepsilon$  - Tensile Strain

#### 3.1.4 Isotropic Materials

If a material is isotropic, these equations apply to Elastic Shear Modulus and Elastic Bulk Modulus.

$$G = \frac{E}{2(1+v)} \quad (3.9)$$

$$K = \frac{E}{3(1-2v)} \quad (3.10)$$

- $G$  - Elastic Shear Modulus of isotropic material
- $K$  - Elastic Bulk Modulus of isotropic material
- $E$  - Young's Modulus of material
- $v$  - Poisson's Ratio of Material

#### 3.1.5 Deflection

$$\delta = \frac{FL_0}{EA_0} \quad (3.11)$$

- $F$  - Force Applied
- $L_0$  - Original Length of Material
- $E$  - Young's Modulus
- $A_0$  - Original Cross-Sectional Area of Material

##### Simple Tension

$$\delta_L = -v \frac{Fw_0}{EA_0} \quad (3.12)$$

- $\delta_L$  - Deflection
- $v$  - Poisson's Ratio
- $F$  - Force Applied
- $w_0$  - Width of Thing applying the force
- $E$  - Young's Modulus
- $A_0$  - Original Cross-Sectional Area of Material

#### 3.1.6 Simple Torsion

$$\alpha = \frac{2ML_0}{\pi(r_0)^4 G} \quad (3.13)$$

- $\alpha$  - Simple Torsion
- $M$  -
- $L_0$  - Original Length of Material
- $r_0$  -
- $G$  - Elastic Shear Modulus

### 3.1.7 Working with Stress Curve

- $\sigma_y$  = Yield Strength
- Tensile Strength = Max Height on Curve (Plastic Deformation)
- Toughness = Area Beneath the Stress Curve (Energy Absorbed)
- Percent Elongation (%EL)
- Percent Reduction in Area
- $U_r \cong \frac{1}{2}\sigma_y\epsilon_y$  = Resilience (Energy Absorbed in Elastic Deformation)
- $\sigma_T = \frac{F}{A_0} = K\epsilon_T^n$  = True Stress
- $\epsilon_T = \ln\left(\frac{L}{L_0}\right)$  = True Strain
- $\sigma_{\text{Working}} = \frac{\sigma_y}{N}$  = Safety Measure Measure

### Percent Elongation (%EL)

$$\%EL = \frac{L_f - L_0}{L_0} \times 100\% \quad (3.14)$$

- %EL - Percent Elongation
- $L_f$  - Final Length of Material
- $L_0$  - Starting Length of Material

### Percent Reduction in Area

$$\%RA = \frac{A_0 - A_f}{A_0} \times 100\% \quad (3.15)$$

- %RA - Percent Reduction in Area
- $A_f$  - Final Cross-Sectional Area of Material
- $A_0$  - Starting Cross-Sectional Area of Material

### 3.1.8 Random Equations

$$HB = \frac{2P}{\pi D (D - \sqrt{D^2 - d^2})} \quad (3.16)$$

$$HV = 1.854 \times \frac{P}{d^2} \quad (3.17)$$

$$HK = 14.2 \times \frac{P}{l^2} \quad (3.18)$$

## 3.2 Ch. 7 - Deformation & Strengthening Mechanisms

### 3.2.1 Burger's Vector Explained

$$\begin{aligned} \|\vec{b}\| &= \frac{a}{2} \sqrt{u^2 + v^2 + w^2} \\ \vec{b}_{BCC} &= \frac{a}{2} [111] \\ \vec{b}_{FCC} &= \frac{a}{2} [110] \\ \vec{b}_{HCP} &= \frac{a}{2} [11\bar{2}0] \end{aligned} \quad (3.19)$$

### 3.2.2 Angled Stresses

$$\phi/\lambda = \arccos\left(\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{(u_1^2 + v_1^2 + w_1^2)(u_2^2 + v_2^2 + w_2^2)}}\right) \quad (3.20)$$

- $\phi$  - Angle of stress to normal
- $\lambda$  - Angle of stress to slip direction

### 3.2.3 Strengthening Mechanisms

1. Grain-Size Reduction - Increase grain boundaries, increase misalignment
2. Solid-Solution - Add interstitial atoms
3. Cold Work/Annealing - Increase the number of dislocations
4. Precipitation Strengthening - Shear precipitate or bend the slip line

### Grain-Size Reduction

$$\sigma_y = \sigma_0 + kdy^{-\frac{1}{2}} \quad (3.21)$$

- $\rho_{\text{disloc}} \uparrow = \sigma_y \uparrow$
- $\rho_{\text{disloc}} = \frac{\text{Total disloc. Length}}{\text{Unit Volume}}$

### Solid-Solution

$$\sigma_y = c^{\frac{1}{2}} \quad (3.22)$$

- $c$  - Impurity Concentration

### Cold Work/Annealing

$$\%CW = \frac{A_0 - A}{A_0} \times 100 \quad (3.23)$$

$$d^n - d_0^n = kt \quad (3.24)$$

- Equation (3.24) only works if heat treatment occurs
- $n = 2$ , usually

### Precipitation Strengthening

$$\sigma_y = \frac{1}{s} \quad (3.25)$$

- $s$  - Space in pinning sites

## 4 Exam 4 Equations

### 4.1 Ch. 9 - Phase Diagrams

$$W_L = \frac{s}{R + S} \quad (4.1)$$

$$W_\alpha = \frac{R}{R + S} \quad (4.2)$$

- Hypo [Eutectic Composition] - Before
- Hyper [Eutectic Composition] - After
- Eutectic  $\rightarrow S \rightleftharpoons \alpha + \beta$ , Solid  $\rightleftharpoons$  2 solids
- Eutectoid  $\rightarrow S \rightleftharpoons \alpha + \beta$ , Solid  $\rightleftharpoons$  2 solids
- Peritectic  $\rightarrow S \rightleftharpoons \alpha + L$ , Solid  $\rightleftharpoons$  1 Solid, 1 Liquid
- Proeutectoid Ferrite - Temp  $i$  Forming temp of eutectoid composition
- Eutectoid Ferrite - Temp  $j$  Forming temp of eutectoid composition

### 4.2 Ch. 10 - Phase Transformations

$$r^* = \frac{-2\gamma T_m}{\Delta H_f \Delta T} \quad (4.3)$$

$$\Delta T = (T_m - T) k$$

$$\Delta G_T = 4\pi r^2 \gamma + \frac{4}{3}\pi r^3 \left( \frac{\Delta H_f (T_m - T)}{T_m} \right) \quad (4.4)$$

$$\Delta G^* = \left( \frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2} \right) \cdot \frac{1}{(T_m - T)^2} \quad (4.5)$$

$$y = 1 - e^{-kt^n} \quad (4.6)$$

$$\ln \left( \ln \left( \frac{1}{1-y} \right) \right) = \ln(k) + n \ln(t) \text{ Rate} = \frac{1}{t_{.5}} \quad (4.7)$$

- Coarse Pearlite - Heat  $\rightarrow$  Furnace Cool
- Fine Pearlite - Heat  $\rightarrow$  Air Cool
- Spheroidite - Heat for a long time @ eutectoid Temp, then furnace cooled
- Martensite - Heat  $\rightarrow$  Quench
- Tempered Martensite - Heat  $\rightarrow$  Quench  $\rightarrow$  Heat  $\rightarrow$  Air cool

Ductility INCREASES as you go *up* this list.

Tensile Strength INCREASES as you go *down* this list.

## 5 Exam 5 Equations

### 5.1 Ch. 12 - Mechanical Properties of Ceramics

Overall, ceramics are categorized by:

1. Ionic Bonds
2. Few Slip Systems
3. Dislocations **cannot** move

#### 5.1.1 Strength of Ceramics

$$\sigma_{\text{Max}} = 2\sigma_0 \sqrt{\frac{a}{\rho}} \quad (5.1)$$

- $\sigma_{\text{Max}}$  - Maximum Strength
- $a$  - Half major axis length
- $\rho$  - Radius of Crack tip

$$\sigma_{\text{FS}} = A \sqrt{\frac{E\gamma}{a}} \quad (5.2)$$

- $\sigma_{\text{FS}}$  - Fracture Strength
- $\gamma$  - Surface Energy
- $A$  - Constant

$$\sigma_{\text{FXS}} = \frac{3F_f L}{2bd^2} \quad (5.3)$$

$$\sigma_{\text{FXS}} = \frac{F_f L}{\pi R^3} \quad (5.4)$$

- $\sigma_{\text{FXS}}$  - Flexural Strength

#### 5.1.2 Elasticity of Ceramics

$$E = \frac{F}{\delta} \cdot \frac{L^3}{4bd^2} \quad (5.5)$$

$$E = \frac{F}{\delta} \cdot \frac{L^3}{12\pi R^4} \quad (5.6)$$

- $\delta$  - Midpoint Deflection

### 5.2 Ch. 15 - Mechanical Properties of Polymers

$$\sigma_{\text{TS}} = \sigma_{\text{TS}\infty} - \frac{A}{M_n} \quad (5.7)$$

#### 5.2.1 Molecular Weight

- $E$  - No relationship
- $\sigma_{\text{TS}}$  - Increases
- More carbon chain entanglement

#### 5.2.2 Degree of Crystallinity

- $E$  - Increases
- $\sigma_{\text{TS}}$  - Increases
- Stronger secondary bonds between carbon chains

#### 5.2.3 Deformation by Drawing

- $E$  - Increases
- $\sigma_{\text{TS}}$  - Increases
- Carbon chains are aligning, effectively testing the Carbon-Carbon bonds

#### Annealing/Pre-Drawing

- $E$  - Increases
- $\sigma_{\text{TS}}$  - Increases

#### Post-Drawing

- $E$  - Decreases
- $\sigma_{\text{TS}}$  - Decreases

### 5.3 Ch. 18 - Electrical Properties

#### 5.3.1 Resistivity/Conductance

$$\rho = \frac{RA}{\ell} \quad (5.8)$$

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA} \quad (5.9)$$

- $\rho$  - Resistivity
- $\sigma$  - Conductance
- $R$  - Resistance
- $A$  - Cross-Sectional Area
- $\ell$  - Length

$$\rho = \rho_{\text{Thermal}} + \rho_{\text{Impurity}} + \rho_{\text{Deform}} \quad (5.10)$$

#### 5.3.2 Conductance with Doping

$$\sigma = n|e|\mu_e + p|e|\mu_h \quad (5.11)$$

- $p = 0$  In Metals
- If  $p = n$  it is intrinsic
- $e = 1.602 \times 10^{-19} \text{C}$

#### 5.3.3 Electron Drift Velocity

$$v_d = \frac{\mu_e}{\mu_h} \xi \quad (5.12)$$

$$t = \frac{\ell}{v_d} \quad (5.13)$$

## A Trigonometry

### A.1 Trigonometric Formulas

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{A.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{A.2})$$

### A.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm j\alpha} = \cos(\alpha) \pm j \sin(\alpha) \quad (\text{A.3})$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{A.4})$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{A.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{A.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{A.7})$$

### A.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{A.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{A.9})$$

### A.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{A.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{A.11})$$

### A.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{A.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{A.13})$$

### A.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = (\sin(\alpha))^2 = \frac{1 - \cos(2\alpha)}{2} \quad (\text{A.14})$$

$$\cos^2(\alpha) = (\cos(\alpha))^2 = \frac{1 + \cos(2\alpha)}{2} \quad (\text{A.15})$$

### A.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{A.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{A.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{A.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{A.19})$$

### A.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{A.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.22})$$

### A.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{A.23})$$

### A.10 Rectangular to Polar

$$a + jb = \sqrt{a^2 + b^2} e^{j\theta} = r e^{j\theta} \quad (\text{A.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{A.25})$$

### A.11 Polar to Rectangular

$$r e^{j\theta} = r \cos(\theta) + jr \sin(\theta) \quad (\text{A.26})$$