Contents

1 General Information

1.1 Vectors

2 Kinematics

Defn 1 (Kinematics)**.** *Kinematics* is a way to describe macroscopic motion with equations. This includes anything moving, falling, thrown, shot, launched, etc. This forms the fundamental basis for all of classical mechanics.

2.1 1-D Kinematics

Defn 2 (1-D Displacement)**.** *One dimensional displacement* is calculated based on the change in position of the 'thing.'

$$s = x_2 - x_1 \tag{2.1}$$

Remark 2.1. *Displacement is different than path!* Displacement is the change in position of an object. Path is the length of the path takes between its starting and end point.

Defn 3 (1-D Velocity)**.** *One dimensional velocity* is calculated as the displacement per unit time. There is instantaneous velocity and average velocity. Average velocity is calculated with Equation (2.2).

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{2.2}$$

Instantaneous velocity is calculated by reducing the time interval Δt to 0. This can be summarized in Equation (2.3).

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.3}$$

Defn 4 (Acceleration)**.** *One dimesional acceleration* is the change in velocity over time. Again, there is average acceleration and instantaneous acceleration. Average acceleration is calculated with Equation (2.4)

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \tag{2.4}$$

Instantaneous acceleration is calculated by reducing the time interval Δt to 0. This can be summarized by Equation (2.5).

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2} \tag{2.5}$$

A Physical Constants

Constant Name	Variable Letter	Value
Boltzmann Constant	R	8.314J/mol K
Universal Gravitational	G	$6.67408 \times 10^{-11}\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$
Planck's Constant	h	$6.62607004 \times 10^{-34}\text{mkg/s} = 4.163 \times 10^{-15}\text{eV s}$
Speed of Light	c	$299792458\text{m/s} = 2.998 \times 10^8\ \text{m/s}$
Charge of Electron	e	$1.602 \times 10^{-19}\text{C}$
Mass of Electron	m_{e^-}	$9.11 \times 10^{-31}\text{kg}$
Mass of Neutron	m_{n^0}	$1.67 \times 10^{-31}\text{kg}$
Mass of Earth	m_{Earth}	$5.972 \times 10^{24}\text{kg}$
Diameter of Earth	d_{Earth}	12742km

B Trigonometry

B.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \tag{B.1}$$

$$\cos(\theta)\sin(\theta) = \frac{1}{2}\sin(2\theta) \tag{B.2}$$

B.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm\imath\alpha} = \cos(\alpha) \pm \imath\sin(\alpha) \tag{B.3}$$

$$\sin(x) = \frac{e^{\imath x} + e^{-\imath x}}{2} \tag{B.4}$$

$$\cos(x) = \frac{e^{\imath x} - e^{-\imath x}}{2\imath} \tag{B.5}$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2} \tag{B.6}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \tag{B.7}$$

B.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \tag{B.8}$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \tag{B.9}$$

B.4 Double-Angle Formulae

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \tag{B.10}$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \tag{B.11}$$

B.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1-\cos(\alpha)}{2}} \tag{B.12}$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1+\cos(\alpha)}{2}} \tag{B.13}$$

B.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2} \tag{B.14}$$

$$\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2} \tag{B.15}$$

B.7 Product-to-Sum Identities

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta) \tag{B.16}$$

$$2\sin(\alpha)\sin(\beta) = \cos(\alpha-\beta) - \cos(\alpha+\beta) \tag{B.17}$$

$$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta) \tag{B.18}$$

$$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta) \tag{B.19}$$

B.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cos\left(\frac{\alpha \mp \beta}{2}\right) \tag{B.20}$$

$$\cos(\alpha) + \cos(\alpha) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right) \tag{B.21}$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right) \tag{B.22}$$

B.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \tag{B.23}$$

B.10 Rectangular to Polar

$$a + \imath b = \sqrt{a^2 + b^2}e^{\imath\theta} = re^{\imath\theta} \tag{B.24}$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \tag{B.25}$$

B.11 Polar to Rectangular

$$re^{\imath\theta} = r\cos(\theta) + \imath r\sin(\theta) \tag{B.26}$$

C Calculus

C.1 Fundamental Theorems of Calculus

Defn C.1.1 (First Fundamental Theorem of Calculus)**.** The *first fundamental theorem of calculus* states that, if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a) \tag{C.1}$$

Defn C.1.2 (Second Fundamental Theorem of Calculus)**.** The *second fundamental theorem of calculus* holds for f a continuous function on an open interval I and a any point in I, and states that if F is defined by

$$F(x) = \int_a^x f(t)\,dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t)\,dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \tag{C.2}$$

Defn C.1.3 (argmax)**.** The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\underset{x}{\operatorname{argmax}}$$

D Complex Numbers

$$Ae^{-ix} = A\left[\cos\left(x\right) + i\sin\left(x\right)\right] \tag{D.1}$$