1 Exam 1 Equations

1.1 Ch. 2 - Interatomic Forces

$$E_N = E_A + E_R = -\frac{A}{r} + \frac{B}{r^n} \tag{1.1}$$

$$A = \frac{1}{4\pi\epsilon_0}\left(Z_1 e\right)\left(Z_2 e\right) \tag{1.2}$$

- This equation works for both A and B
- $\epsilon_0 = 8.85 \times 10^{-12}\text{F/m}$
- $e = 1.602 \times 10^{-19}\text{C}$
- r - Radius in m

$$F_A = \frac{\left(1.602 \times 10^{-19}\right)^2}{4\pi\left(8.85 \times 10^{-12}\right) r^2}\left(\|Z_1\|\right)\left(\|Z_2\|\right) \tag{1.3}$$

- F_A - Force of Attraction
- r - Distance in m
- Z - Number of Valence Electrons
- F_A - Interatomic Force in N
- $-F_A = F_R$ - Attractive and Repulsive Force Equal and Opposite

$$\text{Force} = \frac{dE}{dr} \tag{1.4}$$

$$\text{Elastic Modulus} = \frac{dF}{dr} \tag{1.5}$$

$$\%\text{IC} = \left(1 - e^{\frac{(x_A - x_B)^2}{4}}\right) \times 100\% \tag{1.6}$$

- %IC - % Ionic Character
- x - Electronegativities

1.2 Ch. 3 - Structures of Metals/Ceramics

1.2.1 Lattice Parameters

$$a_{\text{BCC}} = \frac{4r}{\sqrt{3}} \tag{1.7}$$

$$a_{\text{FCC}} = \frac{4r}{\sqrt{2}} \tag{1.8}$$

$$a_{\text{HCP}} = \frac{c}{1.633} \tag{1.9}$$

- a - Lattice Parameter
- r - Radius of atom

1.2.2 Volume of Hexagonal Prism

$$V_H = \frac{3\sqrt{3}}{2}a^2 h \tag{1.10}$$

1.2.3 Densities

$$\rho = \frac{nA}{V_C N_A} \tag{1.11}$$

- n - Number of atoms/unit cell
- A - Molar Mass of Material
- V_C - Volume of Unit Cell in cm
- N_A - Avogadro's Number (6.022×10^{23})

$$\text{Planar Density} = \frac{\frac{\text{Atoms}}{\text{2D Unit Area}}}{\frac{\text{Area}}{\text{2D Repeat Unit}}} \tag{1.12}$$

$$\text{Linear Density} = \frac{\#\text{ of Atoms in a Direction}}{\text{Magnitude of Linear Vector}} \tag{1.13}$$

- The repeat units/vector magnitude are in terms of atomic radii

$$\text{APF} = \frac{\frac{\text{Atoms}}{\text{Unit Cell}}\left(\frac{4}{3}\pi\left(\text{atom radius}\right)^3\right)}{\text{Unit Cell Volume}} \tag{1.14}$$

1.2.4 Thermal Expansion

$$\frac{\Delta L}{L_0} = \alpha\left(T_2 - T_1\right) \tag{1.15}$$

- $E \uparrow$, $T_m \uparrow$
- $E \uparrow$, $\alpha \downarrow$

1.2.5 Convert between Coordinates

$$[XYZ] = [a_1 a_2 a_3 c] \qquad \begin{aligned} a_1 &= \frac{1}{3}\left(2X - Y\right) \\ a_2 &= \frac{1}{3}\left(2Y - X\right) \\ a_3 &= -\left(a_1 + a_2\right) \\ c &= Z \end{aligned} \tag{1.16}$$

$$a_1 + a_2 + a_3 = 0$$

1.2.6 Planes

1. Given x, y, z as intersects
2. Convert to $\frac{1}{x}$, $\frac{1}{y}$, $\frac{1}{z}$
3. Reduce to smallest common denominator
4. Leave as $\left(\frac{1}{x}\frac{1}{y}\frac{1}{z}\right)$

1.2.7 Light Refraction

$$D = \frac{n\lambda}{2\sin\theta} \tag{1.17}$$

- $n = 1$
- λ - Wavelength in nm
- θ - Angle of Incidence
- θ is usually given as 2θ. Be careful

1.2.8 Randoms

$$D_{HKL} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \tag{1.18}$$

- ONLY for cubic structures

2 Exam 2 Equations

2.1 Ch. 4 Imperfections

2.1.1 Vacancies

Number of Vacancies:

$$N_V = \text{Vacancy Frac.} \times \frac{N_A \rho_{\text{Ele}}}{A_{\text{Ele}}} \tag{2.1}$$

- A_{Ele} - Atomic Weight (g/mol)
- ρ_{Ele} - Element Density (g/c^{3}m)
- N_A - Avogadro's Number (6.022×10^{23})

Number of Potential Vacancy Sites:

$$N = \frac{\rho_{\text{Ele}} N_A}{A_{\text{Ele}}} \tag{2.2}$$

- A_{Ele} - Atomic Weight (g/mol)
- ρ_{Ele} - Element Density (g/c^{3}m)
- N_A - Avogadro's Number (6.022×10^{23})

Grains per Area:

$$n_M = \left(\frac{100}{M}\right)^2 = 2^{G-1} \tag{2.3}$$

- n_M - Grains/in^2
- G - Grain Size Number
  - $G = -6.6457 \log\left(\bar{\ell}\right) - 3.298$ for mm
  - $G = -6.6353 \log\left(\bar{\ell}\right) - 12.6$ for in.

2.1.2 Mean Intercept Length

$$\bar{\ell} = \frac{L_T}{PM} \tag{2.4}$$

- L_T - Total Length of all Lines
- P - Number of Grain Intersections
- M - Magnification

$$M = \frac{\text{Scale Length}}{\text{\# on Scale Bar}} \tag{2.5}$$

2.1.3 Complete Substitution

To have a complete substitution, it must be:

- $\Delta r < 15\%$
- Electronegativity $\leq .4$
- SAME Crystal Structure
- SAME Valence

2.1.4 Edges

Burger's Vector:

- Burger's Vector
  - $\perp$ for Edge Dislocations
  - $\|$ for Screw Dislocations
- Twin Boundary - Symmetric Around Fault
- Stacking Fault - NOT Symmetric Around Fault
  - High Angle - $E \uparrow$
  - Low Angle - $E \downarrow$

2.2 Ch. 5 Diffusion

2.2.1 Diffusion Coefficient

$$D = D_0 \times e^{\frac{-Q_d}{RT}} \tag{2.6}$$

- D - Diffusion Coefficient (m^2/s)
- Q_d - Activation Energy (J/mol, eV/atom)
- R - 8.314 (J/molK)
- T - Temperature (K)

$$D_1 t_1 = D_2 t_2 \tag{2.7}$$

- D - Diffusion Coefficients
- t - Time

2.2.2 Flux

$$J = -DA\frac{dC}{dx} \tag{2.8}$$

- For Steady State Diffusion
- D - Diffusion Coefficient
- dC - Δ Concentration (Low-High)
- dx - Distance to Cross
- A - Area

2.2.3 Diffusion Concentration

$$\frac{C\left(x,t\right) - C_0}{C_s - C_0} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \tag{2.9}$$

$$Z = \frac{x}{2\sqrt{Dt}} = \frac{z - \text{point below}}{\text{point above} - \text{point below}} \tag{2.10}$$

- $C\left(x,t\right)$ - Concentration at a point AND time
- C_0 - Initial Concentration
- C_2 - Surface Concentration of DIFFUSING species
- x - Position
- D - Diffusion Coefficient
- t - Time

3 Exam 3 Equations

3.1 Ch. 6 - Mechanical Properties

3.1.1 Stress

Tensile Stress

$$\sigma = \frac{F_t}{A_0} \tag{3.1}$$

- Think of amount of force required to pull ends of paper apart
- σ has units lb_f/in^2 or N/m^2
- F_t - Normal Force
- A_0 - Original Cross-Sectional Area
- $\sigma < 0$ - Compressive Force
- $\sigma > 0$ - Tensile Force

Shear Stress

$$\tau = \frac{F_S}{A_0} \tag{3.2}$$

- Think of amount of force required to rip a piece of paper
- τ has units lb_f/in^2 or N/m^2
- F_S - Shear Force
- A_0 - Original Cross-Sectional Area

3.1.2 Strain

Tensile Strain

$$\varepsilon = \frac{\delta}{L_0} = \frac{L - L_0}{L_0} \tag{3.3}$$

- Think of pulling ends of paper apart and seeing how much it stretches
- ε - Tensile Strain
- δ - Change in Length of Material
- L_0 - Original Length of Material

Shear Strain

$$\gamma = \frac{\Delta x}{y} = \tan\theta \tag{3.4}$$

- Change of length of material compared to height when ripped apart
- γ - Shear Strain
- Δx - Change in length
- y - Height of Material Tested

3.1.3 Moduli

Young's Modulus

$$E = \frac{\sigma}{\varepsilon} = \frac{dF}{dr} \tag{3.5}$$

- How Stiff a Material is from being pulled apart
- E - Young's Modulus
- σ - Tensile Stress
- ε - Tensile Strain

Elastic Shear Modulus

$$G = \frac{\tau}{\gamma} \tag{3.6}$$

- How Stiff a material is from Ripping
- τ - Shear Stress
- γ - Shear Strain

Elastic Bulk Modulus

$$K = -P\frac{V_0}{\Delta V} \tag{3.7}$$

- K - Elastic Bulk Modulus
- P -
- V_0 - Original Volume
- ΔV - Change in Volume

Poisson's Ratio

$$v = -\frac{\varepsilon_L}{\varepsilon} \tag{3.8}$$

- v - Poisson's Ratio
- ε_L - Tensile Strain at the length L
- ε - Tensile Strain

3.1.4 Isotropic Materials

If a material is isotropic, these equations apply to Elastic Shear Modulus and Elastic Bulk Modulus.

$$G = \frac{E}{2(1+v)} \tag{3.9}$$

$$K = \frac{E}{3(1-2v)} \tag{3.10}$$

- G - Elastic Shear Modulus of isotropic material
- K - Elastic Bulk Modulus of isotropic material
- E - Young's Modulus of material
- v - Poisson's Ratio of Material

3.1.5 Deflection

$$\delta = \frac{FL_0}{EA_0} \tag{3.11}$$

- F - Force Applied
- L_0 - Original Length of Material
- E - Young's Modulus
- A_0 - Original Cross-Sectional Area of Material

Simple Tension

$$\delta_L = -v\frac{Fw_0}{EA_0} \tag{3.12}$$

- $delta_L$ - Deflection
- v - Poisson's Ratio
- F - Force Applied
- w_0 - Width of Thing applying the force
- E - Young's Modulus
- A_0 - Original Cross-Sectional Area of Material

3.1.6 Simple Torsion

$$\alpha = \frac{2ML_0}{\pi (r_0)^4 G} \tag{3.13}$$

- α - Simple Torsion
- M -
- L_0 - Original Length of Material
- r_0 -
- G - Elastic Shear Modulus

3.1.7 Working with Stress Curve

- σ_y = Yield Strength
- Tensile Strength = Max Height on Curve (Plastic Deformation)
- Toughness = Area Beneath the Stress Curve (Energy Absorbed)
- Percent Elongation (%EL)
- Percent Reduction in Area
- $U_r \cong \frac{1}{2}\sigma_y \varepsilon_y$ = Resilience (Energy Absorbed in Elastic Deformation)
- $\sigma_T = \frac{F}{A_0} = K\varepsilon_T^n$ = True Stress
- $\epsilon_T = \ln\left(\frac{L}{L_0}\right)$ = True Strain
- $\sigma_{\text{Working}} = \frac{\sigma_y}{N}$ = Safety Measure Measure

Percent Elongation (%EL)

$$\%EL = \frac{L_f - L_0}{L_0} \times 100\% \tag{3.14}$$

- $\%EL$ - Percent Elongation
- L_f - Final Length of Material
- L_0 - Starting Length of Material

Percent Reduction in Area

$$\%RA = \frac{A_0 - A_f}{A_0} \times 100\% \tag{3.15}$$

- $\%RA$ - Percent Reduction in Area
- A_f - Final Cross-Sectional Area of Material
- A_0 - Starting Cross-Sectional Area of Material

3.1.8 Random Equations

$$\text{HB} = \frac{2P}{\pi D\left(D - \sqrt{D^2 - d^2}\right)} \tag{3.16}$$

$$\text{HV} = 1.854 \times \frac{P}{d^2} \tag{3.17}$$

$$\text{HK} = 14.2 \times \frac{P}{l^2} \tag{3.18}$$

3.2 Ch. 7 - Deformation & Strengthening Mechanisms

3.2.1 Burger's Vector Explained

$$\begin{aligned} \|\vec{b}\| &= \frac{a}{2}\sqrt{u^2 + v^2 + w^2} \\ \vec{b}_{BCC} &= \frac{a}{2}[111] \\ \vec{b}_{FCC} &= \frac{a}{2}[110] \\ \vec{b}_{HCP} &= \frac{a}{2}[11\bar{2}0] \end{aligned} \tag{3.19}$$

3.2.2 Angled Stresses

$$\phi/\lambda = \arccos\left(\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right) \tag{3.20}$$

- ϕ - Angle of stress to normal
- λ - Angle of stress to slip direction

3.2.3 Strengthening Mechanisms

1. Grain-Size Reduction - Increase grain boundaries, increase misalignment
2. Solid-Solution - Add interstitial atoms
3. Cold Work/Annealing - Increase the number of dislocations
4. Precipitation Strengthening - Shear precipitate or bend the slip line

Grain-Size Reduction

$$\sigma_y = \sigma_0 + kdy^{\frac{-1}{2}} \tag{3.21}$$

- $\rho_{\text{disloc}} \uparrow = \sigma_y \uparrow$
- $\rho_{\text{disloc}} = \frac{\text{Total disloc. Length}}{\text{Unit Volume}}$

Solid-Solution

$$\sigma_y = c^{\frac{1}{2}} \tag{3.22}$$

- c - Impurity Concentration

Cold Work/Annealing

$$\%\text{CW} = \frac{A_0 - A}{A_0} \times 100 \tag{3.23}$$

$$d^n - d_0^n = kt \tag{3.24}$$

- Equation (3.24) only works if heat treatment occurs
- $n = 2$, usually

Precipitation Strengthening

$$\sigma_y = \frac{1}{s} \tag{3.25}$$

- s - Space in pinning sites

4 Exam 4 Equations

4.1 Ch. 9 - Phase Diagrams

$$W_L = \frac{s}{R+S} \tag{4.1}$$

$$W_\alpha = \frac{R}{R+S} \tag{4.2}$$

- Hypo [Eutectic Composition] - Before
- Hyper [Eutectic Composition] - After
- Eutectic $\rightarrow S \rightleftharpoons \alpha + \beta$, Solid $\rightleftharpoons$ 2 solids
- Eutectoid $\rightarrow S \rightleftharpoons \alpha + \beta$, Solid $\rightleftharpoons$ 2 solids
- Peritectic $\rightarrow S \rightleftharpoons \alpha + L$, Solid $\rightleftharpoons$ 1 Solid, 1 Liquid
- Proeutectoid Ferrite - Temp ¿ Forming temp of eutectoid composition
- Eutectoid Ferrite - Temp ¡ Forming temp of eutectoid composition

4.2 Ch. 10 - Phase Transformations

$$\begin{gathered} r^* = \frac{-2\gamma T_m}{\Delta H_F \Delta T} \\ \Delta T = \left(T_m - T\right) k \end{gathered} \tag{4.3}$$

$$\Delta G_T = 4\pi r^2 \gamma + \frac{4}{3}\pi r^3 \left(\frac{\Delta H_f \left(T_m - T\right)}{T_m}\right) \tag{4.4}$$

$$\Delta G^* = \left(\frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2}\right) \cdot \frac{1}{\left(T_m - T\right)^2} \tag{4.5}$$

$$y = 1 - e^{-kt^n} \tag{4.6}$$

$$\ln\left(\ln\left(\frac{1}{1-y}\right)\right) = \ln\left(k\right) + n\ln\left(t\right) \text{Rate} = \frac{1}{t_{.5}} \tag{4.7}$$

- Coarse Pearlite - Heat $\rightarrow$ Furnace Cool
- Fine Pearlite - Heat $\rightarrow$ Air Cool
- Spheroidite - Heat for a long time @ eutectoid Temp, then furnace cooled
- Martensite - Heat $\rightarrow$ Quench
- Tempered Martensite - Heat $\rightarrow$ Quench $\rightarrow$ Heat $\rightarrow$ Air cool

Ductility INCREASES as you go *up* this list.
Tensile Strength INCREASES as you go *down* this list.

5 Exam 5 Equations

5.1 Ch. 12 - Mechanical Properties of Ceramics

Overall, ceramics are categorized by:

1. Ionic Bonds
2. Few Slip Systems
3. Dislocations **cannot** move

5.1.1 Strength of Ceramics

$$\sigma_{\text{Max}} = 2\sigma_0\sqrt{\frac{a}{\rho}} \tag{5.1}$$

- σ_{Max} - Maximum Strength
- a - Half major axis length
- ρ - Radius of Crack tip

$$\sigma_{\text{FS}} = A\sqrt{\frac{E\gamma}{a}} \tag{5.2}$$

- σ_{FS} - Fracture Strength
- γ - Surface Energy
- A - Constant

$$\sigma_{\text{FXS}} = \frac{3F_f L}{2bd^2} \tag{5.3}$$

$$\sigma_{\text{FXS}} = \frac{F_f L}{\pi R^3} \tag{5.4}$$

- σ_{FXS} - Flexural Strength

5.1.2 Elasticity of Ceramics

$$E = \frac{F}{\delta}\cdot\frac{L^3}{4bd^2} \tag{5.5}$$

$$E = \frac{F}{\delta}\cdot\frac{L^3}{12\pi R^4} \tag{5.6}$$

- δ - Midpoint Deflection

5.2 Ch. 15 - Mechanical Properties of Polymers

$$\sigma_{\text{TS}} = \sigma_{\text{TS}\infty} - \frac{A}{\bar{M}_n} \tag{5.7}$$

5.2.1 Molecular Weight

- E - No relationship
- σ_{TS} - Increases
- More carbon chain entanglement

5.2.2 Degree of Crystallinity

- E - Increases
- σ_{TS} - Increases
- Stronger secondary bonds between carbon chains

5.2.3 Deformation by Drawing

- E - Increases
- σ_{TS} - Increases
- Carbon chains are aligning, effectively testing the Carbon-Carbon bonds

Annealing/Pre-Drawing

- E - Increases
- σ_{TS} - Increases

Post-Drawing

- E - Decreases
- σ_{TS} - Decreases

5.3 Ch. 18 - Electrical Properties

5.3.1 Resistivity/Conductance

$$\rho = \frac{RA}{\ell} \tag{5.8}$$

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA} \tag{5.9}$$

- ρ - Resistivity
- σ - Conductance
- R - Resistance
- A - Cross-Sectional Area
- ℓ - Length

$$\rho = \rho_{\text{Thermal}} + \rho_{\text{Impurity}} + \rho_{\text{Deform}} \tag{5.10}$$

5.3.2 Conductance with Doping

$$\sigma = n|e|\mu_e + p|e|\mu_h \tag{5.11}$$

- $p = 0$ In Metals
- If $p = n$ it is intrinsic
- $e = 1.602 \times 10^{-19}\text{C}$

5.3.3 Electron Drift Velocity

$$v_d = \frac{\mu_e}{\mu_h}\xi \tag{5.12}$$

$$t = \frac{\ell}{v_d} \tag{5.13}$$

A Trigonometry

A.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \tag{A.1}$$

$$\cos(\theta)\sin(\theta) = \frac{1}{2}\sin(2\theta) \tag{A.2}$$

A.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm\imath\alpha} = \cos(\alpha) \pm \imath\sin(\alpha) \tag{A.3}$$

$$\sin(x) = \frac{e^{\imath x} + e^{-\imath x}}{2} \tag{A.4}$$

$$\cos(x) = \frac{e^{\imath x} - e^{-\imath x}}{2\imath} \tag{A.5}$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2} \tag{A.6}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \tag{A.7}$$

A.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta) \tag{A.8}$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \tag{A.9}$$

A.4 Double-Angle Formulae

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \tag{A.10}$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \tag{A.11}$$

A.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1-\cos(\alpha)}{2}} \tag{A.12}$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1+\cos(\alpha)}{2}} \tag{A.13}$$

A.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2} \tag{A.14}$$

$$\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2} \tag{A.15}$$

A.7 Product-to-Sum Identities

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha-\beta) + \cos(\alpha+\beta) \tag{A.16}$$

$$2\sin(\alpha)\sin(\beta) = \cos(\alpha-\beta) - \cos(\alpha+\beta) \tag{A.17}$$

$$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta) \tag{A.18}$$

$$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta) \tag{A.19}$$

A.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2\sin\left(\frac{\alpha\pm\beta}{2}\right)\cos\left(\frac{\alpha\mp\beta}{2}\right) \tag{A.20}$$

$$\cos(\alpha) + \cos(\alpha) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \tag{A.21}$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \tag{A.22}$$

A.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \tag{A.23}$$

A.10 Rectangular to Polar

$$a + \imath b = \sqrt{a^2+b^2}e^{\imath\theta} = re^{\imath\theta} \tag{A.24}$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \tag{A.25}$$

A.11 Polar to Rectangular

$$re^{\imath\theta} = r\cos(\theta) + \imath r\sin(\theta) \tag{A.26}$$