

EDAN20: Language Technology — Reference Sheet

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1 Linguistics

Defn 1 (Linguistics). *Linguistics* is the study and the description of human languages. Linguistics have been developed since ancient times and the Middle Ages. *Modern Linguistics* developed between the end of the 19th century and beginning of the 20th century. Its founder and prominent figure was Ferdinand de Saussure.

There is something else that we will be working with, called Computational Linguistics.

Defn 2 (Computational Linguistics). *Computational Linguistics* is a subset of both Linguistics and computer science. Its goal is to design mathematical models of language structures enabling the automation of language processing by a computer. We can consider computational linguistics as the formalization of linguistic theories and models, or their implementation in a machine. New linguistic theories can be developed with the aid of a computer too.

Historically, there are 3 disciplines of Linguistics.

1. Phonetics
2. Words
3. Syntax

1.1 Phonetics

Defn 3 (Phonetics). *Phonetics* concerns the production and perception of acoustic sounds that form the speech signal. In every language sounds can be classified into a finite set of Phonemes

Defn 4 (Phonemes). *Phonemes* are the building blocks of Phonetics. Traditionally, Phonemes include Vowels and Consonants. Phonemes are assembled into Syllables to build words.

Examples include: *pa, pi, po*.

Defn 5 (Vowels). *Vowels* are a speech sound that is produced by a comparatively open configuration of the vocal tract, with vibration of the vocal cords, but without audible friction. They are a unit of the sound system of a language that forms the nucleus of Syllables.

Examples include: *a, e, i, o*.

Defn 6 (Consonants). *Consonants* are a speech sound in which the breath is at least partly obstructed and which can be combined with Vowels to form Syllables.

Examples include: *p, f, r, m*.

Defn 7 (Syllables). *Syllables* are a unit of pronunciation having one vowel sound, with or without surrounding consonants, forming the whole or part of a word.

1.2 Words

Defn 8 (Words). *Words*

1.3 Syntax

Defn 9 (Syntax). *Syntax*

A Trigonometry

A.1 Trigonometric Formulas

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{A.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{A.2})$$

A.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm j\alpha} = \cos(\alpha) \pm j \sin(\alpha) \quad (\text{A.3})$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{A.4})$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{A.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{A.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{A.7})$$

A.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{A.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{A.9})$$

A.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{A.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{A.11})$$

A.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{A.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{A.13})$$

A.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = (\sin(\alpha))^2 = \frac{1 - \cos(2\alpha)}{2} \quad (\text{A.14})$$

$$\cos^2(\alpha) = (\cos(\alpha))^2 = \frac{1 + \cos(2\alpha)}{2} \quad (\text{A.15})$$

A.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{A.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{A.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{A.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{A.19})$$

A.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{A.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{A.22})$$

A.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{A.23})$$

A.10 Rectangular to Polar

$$a + jb = \sqrt{a^2 + b^2} e^{j\theta} = r e^{j\theta} \quad (\text{A.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{A.25})$$

A.11 Polar to Rectangular

$$r e^{j\theta} = r \cos(\theta) + jr \sin(\theta) \quad (\text{A.26})$$

B Calculus

B.1 L'Hopital's Rule

L'Hopital's Rule can be used to simplify and solve expressions regarding limits that yield irreconcilable results.

Lemma B.0.1 (L'Hopital's Rule). *If the equation*

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \begin{cases} \frac{0}{0} \\ \frac{\infty}{\infty} \end{cases}$$

then Equation (B.1) holds.

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad (\text{B.1})$$

B.2 Fundamental Theorems of Calculus

Defn B.2.1 (First Fundamental Theorem of Calculus). The *first fundamental theorem of calculus* states that, if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{B.2})$$

Defn B.2.2 (Second Fundamental Theorem of Calculus). The *second fundamental theorem of calculus* holds for f a continuous function on an open interval I and a any point in I , and states that if F is defined by

$$F(x) = \int_a^x f(t) dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t) dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \quad (\text{B.3})$$

Defn B.2.3 (argmax). The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname{argmax}_x$$

B.3 Rules of Calculus

B.3.1 Chain Rule

Defn B.3.1 (Chain Rule). The *chain rule* is a way to differentiate a function that has 2 functions multiplied together.

If

$$f(x) = g(x) \cdot h(x)$$

then,

$$\begin{aligned} f'(x) &= g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ \frac{df(x)}{dx} &= \frac{dg(x)}{dx} \cdot h(x) + g(x) \cdot \frac{dh(x)}{dx} \end{aligned} \quad (\text{B.4})$$

B.4 Useful Integrals

$$\int \cos(x) dx = \sin(x) \quad (\text{B.5})$$

$$\int \sin(x) dx = -\cos(x) \quad (\text{B.6})$$

$$\int x \cos(x) dx = \cos(x) + x \sin(x) \quad (\text{B.7})$$

Equation (B.7) simplified with Integration by Parts.

$$\int x \sin(x) dx = \sin(x) - x \cos(x) \quad (\text{B.8})$$

Equation (B.8) simplified with Integration by Parts.

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x) \quad (\text{B.9})$$

Equation (B.9) simplified by using Integration by Parts twice.

$$\int x^2 \sin(x) dx = 2x \sin(x) - (x^2 - 2) \cos(x) \quad (\text{B.10})$$

Equation (B.10) simplified by using Integration by Parts twice.

$$\int e^{\alpha x} \cos(\beta x) dx = \frac{e^{\alpha x} (\alpha \cos(\beta x) + \beta \sin(\beta x))}{\alpha^2 + \beta^2} + C \quad (\text{B.11})$$

$$\int e^{\alpha x} \sin(\beta x) dx = \frac{e^{\alpha x} (\alpha \sin(\beta x) - \beta \cos(\beta x))}{\alpha^2 + \beta^2} + C \quad (\text{B.12})$$

$$\int e^{\alpha x} dx = \frac{e^{\alpha x}}{\alpha} \quad (\text{B.13})$$

$$\int x e^{\alpha x} dx = e^{\alpha x} \left(\frac{x}{\alpha} - \frac{1}{\alpha^2} \right) \quad (\text{B.14})$$

Equation (B.14) simplified with Integration by Parts.

$$\int \frac{dx}{\alpha + \beta x} = \int \frac{1}{\alpha + \beta x} dx = \frac{1}{\beta} \ln(\alpha + \beta x) \quad (\text{B.15})$$

$$\int \frac{dx}{\alpha^2 + \beta^2 x^2} = \int \frac{1}{\alpha^2 + \beta^2 x^2} dx = \frac{1}{\alpha \beta} \arctan \left(\frac{\beta x}{\alpha} \right) \quad (\text{B.16})$$

$$\int \alpha^x dx = \frac{\alpha^x}{\ln(\alpha)} \quad (\text{B.17})$$

$$\frac{d}{dx} \alpha^x = \frac{d\alpha^x}{dx} = \alpha^x \ln(\alpha) \quad (\text{B.18})$$

B.5 Leibnitz's Rule

Lemma B.0.2 (Leibnitz's Rule). *Given*

$$g(t) = \int_{a(t)}^{b(t)} f(x, t) dx$$

with $a(t)$ and $b(t)$ differentiable in t and $\frac{\partial f(x, t)}{\partial t}$ continuous in both t and x , then

$$\frac{d}{dt} g(t) = \frac{dg(t)}{dt} = \int_{a(t)}^{b(t)} \frac{\partial f(x, t)}{\partial t} dx + f[b(t), t] \frac{db(t)}{dt} - f[a(t), t] \frac{da(t)}{dt} \quad (\text{B.19})$$