

# Phys 123: Classical Mechanics - Reference Sheet

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# 1 General Information

## 1.1 Vectors

## 2 Kinematics

**Defn 1** (Kinematics). *Kinematics* is a way to describe macroscopic motion with equations. This includes anything moving, falling, thrown, shot, launched, etc. This forms the fundamental basis for all of classical mechanics.

### 2.1 1-D Kinematics

**Defn 2** (1-D Displacement). *One dimensional displacement* is calculated based on the change in position of the ‘thing.’

$$s = x_2 - x_1 \quad (2.1)$$

*Remark 2.1. Displacement is different than path!* Displacement is the change in position of an object. Path is the length of the path takes between its starting and end point.

**Defn 3** (1-D Velocity). *One dimensional velocity* is calculated as the displacement per unit time. There is instantaneous velocity and average velocity. Average velocity is calculated with Equation (2.2).

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad (2.2)$$

Instantaneous velocity is calculated by reducing the time interval  $\Delta t$  to 0. This can be summarized in Equation (2.3).

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (2.3)$$

**Defn 4** (Acceleration). *One dimensional acceleration* is the change in velocity over time. Again, there is average acceleration and instantaneous acceleration. Average acceleration is calculated with Equation (2.4)

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad (2.4)$$

Instantaneous acceleration is calculated by reducing the time interval  $\Delta t$  to 0. This can be summarized by Equation (2.5).

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad (2.5)$$

## A Physical Constants

Constant Name	Variable Letter	Value
Boltzmann Constant	$R$	8.314J/mol K
Universal Gravitational	$G$	$6.67408 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
Planck's Constant	$h$	$6.62607004 \times 10^{-34} \text{mkg/s} = 4.163 \times 10^{-15} \text{eV s}$
Speed of Light	$c$	$299792458 \text{m/s} = 2.998 \times 10^8 \text{ m/s}$
Charge of Electron	$e$	$1.602 \times 10^{-19} \text{C}$
Mass of Electron	$m_{e-}$	$9.11 \times 10^{-31} \text{kg}$
Mass of Neutron	$m_{n^0}$	$1.67 \times 10^{-31} \text{kg}$
Mass of Earth	$m_{Earth}$	$5.972 \times 10^{24} \text{kg}$
Diameter of Earth	$d_{Earth}$	12742km

## B Trigonometry

### B.1 Trigonometric Formulas

$$\sin(\alpha) + \sin(\beta) = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.1})$$

$$\cos(\theta) \sin(\theta) = \frac{1}{2} \sin(2\theta) \quad (\text{B.2})$$

### B.2 Euler Equivalents of Trigonometric Functions

$$e^{\pm i\alpha} = \cos(\alpha) \pm i \sin(\alpha) \quad (\text{B.3})$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \quad (\text{B.4})$$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad (\text{B.5})$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (\text{B.6})$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (\text{B.7})$$

### B.3 Angle Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta) \quad (\text{B.8})$$

$$\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta) \quad (\text{B.9})$$

### B.4 Double-Angle Formulae

$$\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha) \quad (\text{B.10})$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) \quad (\text{B.11})$$

### B.5 Half-Angle Formulae

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}} \quad (\text{B.12})$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos(\alpha)}{2}} \quad (\text{B.13})$$

### B.6 Exponent Reduction Formulae

$$\sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2} \quad (\text{B.14})$$

$$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2} \quad (\text{B.15})$$

### B.7 Product-to-Sum Identities

$$2 \cos(\alpha) \cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta) \quad (\text{B.16})$$

$$2 \sin(\alpha) \sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta) \quad (\text{B.17})$$

$$2 \sin(\alpha) \cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta) \quad (\text{B.18})$$

$$2 \cos(\alpha) \sin(\beta) = \sin(\alpha + \beta) - \sin(\alpha - \beta) \quad (\text{B.19})$$

## B.8 Sum-to-Product Identities

$$\sin(\alpha) \pm \sin(\beta) = 2 \sin\left(\frac{\alpha \pm \beta}{2}\right) \cos\left(\frac{\alpha \mp \beta}{2}\right) \quad (\text{B.20})$$

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.21})$$

$$\cos(\alpha) - \cos(\beta) = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \quad (\text{B.22})$$

## B.9 Pythagorean Theorem for Trig

$$\cos^2(\alpha) + \sin^2(\alpha) = 1^2 \quad (\text{B.23})$$

## B.10 Rectangular to Polar

$$a + ib = \sqrt{a^2 + b^2} e^{i\theta} = r e^{i\theta} \quad (\text{B.24})$$

$$\theta = \begin{cases} \arctan\left(\frac{b}{a}\right) & a > 0 \\ \pi - \arctan\left(\frac{b}{a}\right) & a < 0 \end{cases} \quad (\text{B.25})$$

## B.11 Polar to Rectangular

$$r e^{i\theta} = r \cos(\theta) + ir \sin(\theta) \quad (\text{B.26})$$

## C Calculus

### C.1 Fundamental Theorems of Calculus

**Defn C.1.1** (First Fundamental Theorem of Calculus). The *first fundamental theorem of calculus* states that, if  $f$  is continuous on the closed interval  $[a, b]$  and  $F$  is the indefinite integral of  $f$  on  $[a, b]$ , then

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{C.1})$$

**Defn C.1.2** (Second Fundamental Theorem of Calculus). The *second fundamental theorem of calculus* holds for  $f$  a continuous function on an open interval  $I$  and  $a$  any point in  $I$ , and states that if  $F$  is defined by

$$F(x) = \int_a^x f(t) dt,$$

then

$$\begin{aligned} \frac{d}{dx} \int_a^x f(t) dt &= f(x) \\ F'(x) &= f(x) \end{aligned} \quad (\text{C.2})$$

**Defn C.1.3** (argmax). The arguments to the *argmax* function are to be maximized by using their derivatives. You must take the derivative of the function, find critical points, then determine if that critical point is a global maxima. This is denoted as

$$\operatorname{argmax}_x$$

## D Complex Numbers

$$Ae^{-ix} = A [\cos (x) + i \sin (x)] \tag{D.1}$$