

**Question 1:**

- a) Explain the cost trade-off that is solved by lot-sizing models. (3 points)
- b) Consider the general newsvendor problem. How does a change in variance affect the optimal order quantity? (4 points)

The Weighty trash Bag Company has fairly constant demand level of 600 units per year. The accounting department estimates that the fixed cost of placing an order is €8. Holding costs are based on a 20 percent annual interest rate. The supplier charges €0.30 per bag for orders of less than 500 bags and €0.29 per bag for orders of 500 bags or more.

- c) Determine the optimal order quantity for the company under this price schedule. (5 points)

Now, consider the problem without quantity discount (€0.30 per bag for all size of orders) and suppose that the orders are delivered in boxes. Each of these boxes holds 250 units. Thus, the order quantity can only be a multiple of 250.

- d) Using the convexity of the cost function in  $Q$ , compute the optimal order quantity. (4 points)
- e) Assume that we use the order quantity calculated in part d) and there is a lead time of 2 months. What should be the reorder point to ensure that there are no stock outs? (4 points)

Consider two newsvendors A and B. Their demands are assumed to be normally distributed. The two newsvendors calculated the respective means as  $\mu_A = 80$ ,  $\mu_B = 60$  and standard deviations  $\sigma_A = \sigma_B = 20$ . The demand of newsvendors A and B is negatively correlated with coefficient -0.5. The sales price is 100 €, the procurement price is 30 € and leftover inventory is discarded at no cost.

- f) Determine the individual optimal order quantities and the related profits if newsvendors procure independently. (6 points)
- g) If newsvendors A and B cooperate and give joint orders, what should be the joint order quantity? Without calculating associated (total) profit, explain how it would be affected and why? Name this effect and give a short explanation. (4 points)

**Question 2:**

- a) Explain (without using formulas) the differences between  $(R, S)$ ,  $(s, Q)$  and  $(s, S)$  inventory control rules. Discuss the parameters  $s$ ,  $Q$  and  $S$  and how these parameters should be set? (6 points)

Canadian Wheel Ltd. continuously reviews its inventory and establishes safety stocks to satisfy a certain service level. For a basic item whose demand is normally distributed, an order quantity of 200 units is used. Annual demand for the item is 1000 units and its weekly standard deviation is 50. The supplier of the item ensures a constant lead time of 4 weeks to Canadian Wheel. The current purchase price of the item from the supplier is €0.80/unit. Receiving and handling costs add €0.20/unit. Annual carrying charge is 20 percent and penalty cost of  $p = €0.50$  per unit short applies.

The supplier offers to reduce the lead time to a new constant level of one week, but in so doing she will increase the selling price to Canadian by €0.05/unit. Canadian management is faced with the decision of whether or not to accept the supplier's offer.

- b) Discuss the economic tradeoff in the decision. (Note: Canadian would not change the order quantity) (6 points)
- c) Determine the expected number of stock-outs per replenishment cycle and calculate the safety stocks required in the present and offered contract if the company aims a cycle fill-rate of 0.94. (5 points)
- d) Determine the expected number of stock-outs per replenishment cycle and calculate the safety stocks required in the present and offered contract, if the company aims an  $\alpha$ -service level of 0.90. (4 points)
- e) What should be the decision of whether or not to accept the supplier's offer considering the annual cost of each contract for the case that the company aims a cycle fill-rate of 0.94? (9 points)

**Question 3:**

- a) Explain (without using formulas) the approach of dedicated capacity, average utilization, and common cycle. (8 points)

Consider two products with constant demand rate of  $d_1 = 300$  and  $d_2 = 350$  units per period which are stored in a warehouse with a total capacity of 500 units. The products require  $a_1 = 3$  and  $a_2 = 1$  units of warehouse space. The ordering costs are  $A_1 = 150\text{€}$  and  $A_2 = 111\text{€}$ . Additionally, the products cause holding costs of  $h_1 = 1.5\text{€}$  and  $h_2 = 2\text{€}$  per period.

- b) Determine, using the strategy of dedicated space, the optimal order quantities. (**Hint:** Consider only integer values for the Lagrange multiplier) (7 points)
- c) How much would you be willing to pay to increase warehouse space to 1000 units? (5 points)
- d) Determine, using the Common-cycle method, the optimal replenishment cycle for all products. (5 points)
- e) Use the results in d) to determine how many units of product 1 are in stock when you replenish product 2? (5 points)

**Question 4:**

A company has collected the following sales data over the last ten months for product A and B.

t	1	2	3	4	5	6	7	8	9	10
Product A	4	8	0	3	0	0	0	0	4	0
Product B	99	155	75	81	88	150	101	74	65	48

- a) Classify the products according to an XYZ analysis. (4 points)
- b) Explain the meaning of the  $R^2$  in regression analysis. (4 points)
- c) Consider the standard newsvendor model. How does the quality of your forecast influence your order decision? (4 points)

Consider a two-stage serial inventory system where the retailer orders from the warehouse. Both parties in the supply chain use an (R, S) inventory control rule. Review period of the retailer is 1 week, and it is immediately supplied by the warehouse with the ordered amount. On the other hand, review period of the warehouse is 0 and it is supplied by an outside supplier after a lead time of 1 week. Demand is normally distributed with unknown level and known standard deviation of 15. The initial guess for the demand level is 50. Both parties are supposed to satisfy an  $\alpha$ -service level of 95%. In the following table weekly demand observations of the retailer are given.

Week	1	2	3	4	5
Demand	56	40	61	42	51

- d) Perform an ex-post forecast for the retailer using exponential smoothing with  $\alpha=0.1$  and determine the order-up-to level that would have been optimal given the forecast. Calculate the retailer's order quantities. (4 points)
- e) Perform an ex-post forecast for the warehouse using exponential smoothing with  $\alpha=0.1$  and determine the order-up-to level that would have been optimal given the forecast. Calculate the warehouse's order quantities. (4 points)
- f) Calculate the variance of demand and orders of retailer and the warehouse. (3 points)
- g) Which effect can be observed from the results obtained part f)? Name and describe the effect briefly. What are possible strategies to mitigate the effect? (7 points)