

Inventory Management

Summer 2025

- Assignment 3 -

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Exercise 1:

BrightEyes is a manufacturer of eyewear. The company's best-selling item is a black frame made of acetate. The manager responsible for inventory decisions at BrightEyes has installed an **(s,Q) policy** with a lead time of **3 months**. If the product is out-of-stock, there is a **shortage penalty of 500 €** per occasion. Inventory holding costs are **2 € per unit per year**. Experience from previous years has shown that annual demand is normally distributed with **mean 1000 and standard deviation 200**. The supplier charges a fixed cost of **300 € per order**.

- a) Using the successive approach, determine the order quantity Q and order point s as well as the resulting costs.
- b) How are costs affected if the simultaneous approach is used instead (up to 5 iterations)?

Exercise 1.a) Using the **successive approach**, determine the order quantity Q and order point s as well as the resulting costs.

μ	1000
σ	200
L	0,25/year
Fixed ordering cost	300
h	2
shortage penalty p	500/occasion

$$EOQ = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2 * 1000 * 300}{2}} = 547,72$$

$$z = \sqrt{2 \ln \left(\frac{\mu p}{\sqrt{2\pi} Q h \sigma \sqrt{L}} \right)} = \sqrt{2 \ln \left(\frac{1000 * 500}{\sqrt{2\pi} * 547,72 * 2 * 200 \sqrt{0,25}} \right)} = 1,095$$

$$\text{if } \frac{\mu p}{\sqrt{2\pi} Q h \sigma \sqrt{L}} = \frac{1000 * 500}{\sqrt{2\pi} * 547,72 * 2 * 200 \sqrt{0,25}} = 1,821 \geq 1$$

$$\therefore \text{Safety Stock} = z\sigma\sqrt{L} = 1,095 * 200 * \sqrt{0,25} = 109,5$$

$$\& \text{Reorder point } (s) = ss + L * \mu = 109,5 + 0,25 * 1000 = 359,5$$

Exercise 1.a) Using the successive approach, determine the order quantity Q and order point s as well as the resulting costs.

μ	1000
σ	200
L	0,25/year
Fixed ordering cost	300
h	2
shortage penalty p	500/occasion

k	$\phi(k)$	$\Phi(k)$	$G(k)$
1.0000	0.24197	0.84134	0.08332
1.0100	0.23955	0.84375	0.08174
1.0200	0.23713	0.84614	0.08019
1.0300	0.23471	0.84849	0.07866
1.0400	0.23230	0.85083	0.07716
1.0500	0.22988	0.85314	0.07568
1.0600	0.22747	0.85543	0.07422
1.0700	0.22506	0.85769	0.07279
1.0800	0.22265	0.85993	0.07138
1.0900	0.22025	0.86214	0.06999
1.1000	0.21785	0.86433	0.06862

$$F_{0,1}(1,095) = 0,8632$$

$$C(z) = \frac{\mu}{Q}A + h\left(\frac{Q}{2} + k\sigma\sqrt{L}\right) + \frac{\mu}{Q}p(1 - F_{0,1}(z))$$

$$\begin{aligned}
C(1,095) &= \frac{1000}{547,72} 300 + 2\left(\frac{547,72}{2} + 1,095 * 200\sqrt{0,25}\right) + \frac{1000}{547,72} 500(1 - 0,8632) \\
&= 1439,29
\end{aligned}$$

Exercise 1.b) How are costs affected if **the simultaneous approach** is used instead (up to 5 iterations)?

μ	1000
σ	200
L	0,25/year
Fixed ordering cost	300
h	2
shortage penalty p	500/occasion

$$Q = \sqrt{\frac{2\mu(A + p(1 - F_{0,1}(z)))}{h}}$$

$$z = \sqrt{2 \ln \left(\frac{\mu p}{\sqrt{2\pi} Q h \sigma \sqrt{L}} \right)}$$

- 1st Iteration**

$$EOQ = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2 \cdot 1000 \cdot 300}{2}} = 547,72$$

$$z(547,72) = \sqrt{2 \ln \left(\frac{\mu p}{\sqrt{2\pi} Q h \sigma \sqrt{L}} \right)} = \sqrt{2 \ln \left(\frac{1000 \cdot 500}{\sqrt{2\pi} \cdot 547,72 \cdot 2 \cdot 200 \sqrt{0,25}} \right)} = 1,095 \quad \text{where, } F_{0,1}(1,095) = 0,8632$$

$$C(s, Q) = \frac{\mu}{Q} A + h \left(\frac{Q}{2} + k \sigma \sqrt{L} \right) + \frac{\mu}{Q} p (1 - F_{0,1}(z))$$

$$= \frac{1000}{547,72} 300 + 2 \left(\frac{547,72}{2} + 1,095 \cdot 200 \sqrt{0,25} \right) + \frac{1000}{547,72} 500 (1 - 0,8632) = 1439,29$$

$$Q(1,095) = \sqrt{\frac{2\mu(A + p(1 - F_{0,1}(z)))}{h}} = \sqrt{\frac{2 \cdot 1000 (300 + 500 (1 - 0,8632))}{2}} = 606,96$$

Exercise 1.b) How are costs affected if **the simultaneous approach** is used instead (up to 5 iterations)?

- **2nd Iteration**

$$EOQ = Q_{Iteration\ 1.} = 606,96$$

$$z(606,96) = \sqrt{2 \ln \left(\frac{1000 * 500}{\sqrt{2\pi} * 606,96 * 2 * 200 \sqrt{0,25}} \right)} = 1,00 \quad \text{where, } F_{0,1}(1,00) = 0,84134$$

$$C(s, Q) = \frac{1000}{606,96} 300 + 2 \left(\frac{606,96}{2} + 1,00 * 200 \sqrt{0,25} \right) + \frac{1000}{606,96} 500 (1 - 0,84134) = 1431,92$$

$$Q(1,00) = \sqrt{\frac{2 * 1000 (300 + 500 (1 - 0,84134))}{2}} = 616,23$$

Exercise 1.b) How are costs affected if **the simultaneous approach** is used instead (up to 5 iterations)?

- **3rd Iteration**

$$EOQ = Q_{Iteration\ 2.} = 616,23$$

$$z(616,23) = \sqrt{2 \ln \left(\frac{1000 * 500}{\sqrt{2\pi} * 616,23 * 2 * 200 \sqrt{0,25}} \right)} = 0,981 \quad \text{where, } F_{0,1}(0,981) = 0,8367$$

$$C(s, Q) = \frac{1000}{616,23} 300 + 2 \left(\frac{616,23}{2} + 0,981 * 200 \sqrt{0,25} \right) + \frac{1000}{616,23} 500 (1 - 0,8367) = 1431,76$$

$$Q(0,981) = \sqrt{\frac{2 * 1000 (300 + 500 (1 - 0,8367))}{2}} = 617,74$$

Exercise 1.b) How are costs affected if **the simultaneous approach** is used instead (up to 5 iterations)?

- **4th Iteration**

$$EOQ = Q_{Iteration\ 3.} = 617,74$$

$$z(617,74) = \sqrt{2 \ln \left(\frac{1000 * 500}{\sqrt{2\pi} * 617,74 * 2 * 200 \sqrt{0,25}} \right)} = 0,979 \quad \text{where, } F_{0,1}(0,979) = 0,8362$$

$$C(s, Q) = \frac{1000}{617,74} 300 + 2 \left(\frac{617,74}{2} + 0,979 * 200 \sqrt{0,25} \right) + \frac{1000}{617,74} 500 (1 - 0,8362) = 1431,75$$

$$Q(0,979) = \sqrt{\frac{2 * 1000 (300 + 500 (1 - 0,8362))}{2}} = 618$$

Exercise 1.b) How are costs affected if **the simultaneous approach** is used instead (up to 5 iterations)?

- **5th Iteration**

$$EOQ = Q_{Iteration\ 4.} = 618$$

$$z(618) = \sqrt{2 \ln \left(\frac{1000 \cdot 500}{\sqrt{2\pi} \cdot 618 \cdot 2 \cdot 200 \sqrt{0,25}} \right)} = 0,978 \quad \text{where, } F_{0,1}(0,979) = 0,8359$$

$$C(s, Q) = \frac{1000}{618} 300 + 2 \left(\frac{618}{2} + 0,978 \cdot 200 \sqrt{0,25} \right) + \frac{1000}{618} 500 (1 - 0,8359) = \mathbf{1431,753}$$

$$Q(0,978) = \sqrt{\frac{2 \cdot 1000 (300 + 500 (1 - 0,8359))}{2}} = 618,04$$

$$\textbf{\textit{Total Cost (Successive Approach) = 1439,29}}$$

$$\textbf{\textit{Total Cost (Simultaneous Approach) = 1431,75}}$$

Exercise 2:

A café is open every day and observes **independent, normally distributed demand** for Arabica coffee beans per day. **Weekly demand is 700 packs** of Arabica coffee beans. The café places **orders of 300 packs**, holding costs are **2.5 €/week**. Replenishment costs are **20 € per order**. The following demands (in packs) have been recorded during the past 15 days:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand	83	101	105	145	179	65	79	121	81	103	75	89	61	123	85

- Estimate mean and standard deviation per day. Assuming that each unit short incurs a cost of 3 €, determine the reorder point for coffee with one-day lead time.
- Determine the total costs per week that will be incurred using the reorder point calculated in a).
- The manager of the café is very satisfied with the current service level. He has measured the α -service level over several weeks and it turns out to be 95%. Determine the corresponding fill-rate which will be achieved in this case.

Exercise 2.a) Estimate **mean and standard deviation per day**. Assuming that each unit short incurs a cost of **3 €**, determine **the reorder point for coffee with one-day lead time**.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand	83	101	105	145	179	65	79	121	81	103	75	89	61	123	85

$$\bar{x} = \frac{83 + 101 + 105 + 145 + 179 + 65 + 79 + 121 + 81 + 103 + 75 + 89 + 61 + 123 + 85}{15} = 99,67$$

$$\sigma_s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2} = \sqrt{\frac{(83 - 99,67)^2 + (101 - 99,67)^2 + (105 - 99,67)^2 + \dots + (85 - 99,67)^2}{15 - 1}} = 31,71$$

Exercise 2.a) Estimate mean and standard deviation per day. Assuming that each **unit short** incurs a cost of **3 €**, determine **the reorder point for coffee with one-day lead time**.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand	83	101	105	145	179	65	79	121	81	103	75	89	61	123	85

$$\frac{dC}{dz} = h\sigma\sqrt{L} - \frac{\mu p \sigma \sqrt{L}}{Q} (F_{0,1}(z) - 1) = 0$$

$$F_{0,1}(z) = 1 - \frac{hQ}{p\mu} \text{ if } \frac{hQ}{p\mu} < 1$$

$$\bar{x} = 99,67$$

$$\sigma_s = 31,71$$

shortage penalty p	3/unit short
replenishment quantity Q	300
weekly demand D	700
fixed order cost	20
holding cost	2,5/week

$$F_{0,1}(z) = 1 - \frac{2,5 \cdot 300}{3 \cdot 700} = 1 - 0,3571 = 0,6429 \text{ and } \frac{hQ}{p\mu} = 0,3571 < 1$$

$$z = 0,3661$$

$$\therefore \text{Reorder point } (s) = d_{L/T} + z \cdot \sigma_{L/T} = 99,67 + 0,3661 \cdot 31,71 = 111,28$$

k	$\phi(k)$	$\Phi(k)$	G(k)
0.3400	0.37654	0.63307	0.25178
0.3500	0.37524	0.63683	0.24813
0.3600	0.37391	0.64058	0.24452
0.3700	0.37255	0.64431	0.24094
0.3800	0.37115	0.64803	0.23740
0.3900	0.36973	0.65173	0.23390

Exercise 2.b) Determine the total costs per week that will be incurred using the reorder point calculated in a).

shortage penalty p	3/unit short
replenishment quantity Q	300
weekly demand D	700
fixed order cost	20
holding cost	2,5/week

$$z = 0,3661 \quad G(z) = G(0,3661) \approx 0,241$$

$$\text{Reorder point } (s) = 111,28$$

$$\bar{x} = 99,67$$

$$\sigma_s = 31,71$$

k	$\phi(k)$	$\Phi(k)$	G(k)
0.3400	0.37654	0.63307	0.25178
0.3500	0.37524	0.63683	0.24813
0.3600	0.37391	0.64058	0.24452
0.3700	0.37255	0.64431	0.24094
0.3800	0.37115	0.64803	0.23740
0.3900	0.36973	0.65173	0.23390

$$C(z) = \frac{\mu}{Q}A + h\left(\frac{Q}{2} + z\sigma\sqrt{L}\right) + \frac{\mu}{Q}p\sigma\sqrt{L}G(z)$$

$$C(0,3661) = \frac{700}{300} \cdot 20 + 2,5 \cdot \left(\frac{300}{2} + 0,3661 \cdot 31,71 \cdot \sqrt{1}\right) + \frac{700}{300} \cdot 3 \cdot 31,71 \cdot \sqrt{1} \cdot 0,241$$

$$= 46,67 + 404,022 + 53,50 = \mathbf{504,20}$$

Exercise 2.c) The manager of the café is very satisfied with the current service level. He has measured the **α -service level** over several weeks and it turns out to be **95%**. Determine the corresponding **fill-rate** which will be achieved in this case.

$$\alpha - \text{service level} = 0,95$$

$$\bar{x} = 99,67$$

$$F(z) = 0,95$$

$$\sigma_s = 31,71$$

$$z = 1,65 \rightarrow G(z) = 0,021$$

$$\text{Safety Stock (SS)} = \sigma_{L/T} \cdot k = 31,71 \cdot 1,65 = 52,32$$

$$\text{Reorder Point (s)} = d_{L/T} + z \cdot \sigma_{L/T} = 99,67 + 52,32 = 151,99 (\approx 152)$$

$$G(z) = \frac{Q(1 - \beta)}{\sigma\sqrt{L}}$$

$$\beta = 1 - \frac{\sigma\sqrt{L}G(z)}{Q} = 1 - \frac{31,71 \cdot 0,021}{300} = 0,99778 (\approx 99,8\%)$$

k	$\phi(k)$	$\Phi(k)$	G(k)
1.6100	0.10915	0.94630	0.02270
1.6200	0.10741	0.94738	0.02217
1.6300	0.10567	0.94845	0.02165
1.6400	0.10396	0.94950	0.02114
1.6500	0.10226	0.95053	0.02064

Exercise 3:

A manufacturer of kitchenware expects **annual demand to be 5000** units for a product that costs **30 € per unit**. The holding cost is estimated to be **9 € per unit per year** and the order cost is **75 € per order**. Stockouts are assumed to cost **2 € per unit**. The lead time demand is as follows:

Demand d	100	150	200	250	300
$P(D=d)$	0,11	0,25	0,27	0,25	0,11

Consider the following six reorder points $s=\{100, 150, 200, 250, 300, 350\}$.

- Determine the expected overage cost and the expected underage cost within the lead time for each of the six reorder points s .
- Calculate the optimal order quantity Q for each reorder point.
- Which reorder point of the six mentioned above is the best solution?

Exercise 3.a) Determine the expected overage cost and the expected underage cost within the lead time for each of the six reorder points s .

Demand / year	5000
Price / unit	30
h / unit / year	9
A / order	75
Penalty / unit	2

$$\text{Total Yearly Cost } C(z) = \frac{\mu}{Q}A + h\left(\frac{Q}{2} + \underbrace{z\sigma\sqrt{L}}_{\substack{\text{Safety stock} \\ \text{(expected overage)}}}\right) + \frac{\mu}{Q}p\underbrace{\sigma\sqrt{L}G(z)}_{\substack{\text{Number of units shortage} \\ \text{(expected underage)}}$$

$$\text{Exp. overage} = \sum_{d \leq s} (s - d)P(D = d)$$

$$\text{Exp. underage} = \sum_{d > s} (d - s)P(D = d)$$

Leadtime Demand	P(D=d)	Reorder Points (s)					
		100	150	200	250	300	350
100	0,11	$(s-d)*p=(100-100)*0,11=0$	5,5	11,0	16,5	22,0	27,5
150	0,25	$(100-150)*0,25=-12,5$	0,0	12,5	25,0	37,5	50,0
200	0,27	$(100-200)*0,27=-27$	-13,5	0,0	13,5	27,0	40,5
250	0,25	$(100-250)*0,25=-37,5$	-25,0	-12,5	0,0	12,5	25,0
300	0,11	$(100-300)*0,11=-22,0$	-16,5	-11,0	-5,5	0,0	5,5
Exp. underage		$12,5+27+37,5+22=99$	55	23,5	5,5	0	0
Exp. overage		0,0	5,5	23,5	55,0	99,0	148,5
Exp. underage cost/cycle		Qty*Penalty cost = $99*2 = 198$	110	47	11	0	0
Exp. overage cost/year		Qty*Holding cost = $0*9 = 0$	49,5	211,5	495	891	1336,5

Exercise 3.b) Calculate the optimal order quantity Q for each reorder point.

Demand / year	5000
Price / unit	30
h / unit / year	9
A / order	75
Penalty / unit	2

$$C(z) = \frac{\mu}{Q}A + h\left(\frac{Q}{2} + z\sigma\sqrt{L}\right) + \frac{\mu}{Q}p\sigma\sqrt{L}G(z)$$

$$Q^* = \sqrt{\frac{2\mu(A + p(1 - F_{0,1}(z)))}{h}} = \sqrt{\frac{2\mu(A + \text{Underage cost})}{h}}$$

	Reorder Points (s)					
	100	150	200	250	300	350
Exp. underage cost/cycle	198	110	47	11	0	0
Exp. overage cost/year	0	49,5	211,5	495	891	1336,5
EOQ	$\sqrt{\frac{2 \cdot 5000(75 + 198)}{9}}$ <p>= 550,76</p>	453,4	368,2	309,12	288,7	288,7

Exercise 3.c) Which reorder point of the six mentioned above is **the best solution**?

	Reorder Points (s)					
	100	150	200	250	300	350
Exp. underage	99	55	23,5	5,5	0	0
Exp. overage	0	5,5	23,5	55,0	99,0	148,5
EOQ	550,76	453,4	368,2	309,12	288,68	288,67

Demand / year	5000
Price / unit	30
h / unit / year	9
A / order	75
Penalty / unit	2

$$\text{Total Yearly Cost } C(z) = \frac{\mu}{Q} A + h \left(\frac{Q}{2} + \underbrace{z\sigma\sqrt{L}}_{\text{Expected overage}} \right) + \frac{\mu}{Q} p \underbrace{\sigma\sqrt{L}G(z)}_{\text{Expected underage}}$$

$$C_{s=100} = \frac{5000}{550,76} \cdot 75 + 9 \left(\frac{550,76}{2} + 0 \right) + \frac{5000}{550,76} \cdot 2 \cdot 99 = 4956,813$$

$$C_{s=150} = \frac{5000}{453,4} \cdot 75 + 9 \left(\frac{453,4}{2} + 5,5 \right) + \frac{5000}{453,4} \cdot 2 \cdot 55 = 4130$$

$$C_{s=200} = \frac{5000}{368,2} \cdot 75 + 9 \left(\frac{368,2}{2} + 23,5 \right) + \frac{5000}{368,2} \cdot 2 \cdot 23,5 = 3525$$

$$C_{s=250} = 3277,1$$

$$C_{s=300} = 3489,1$$

$$C_{s=350} = 3934,58$$

∴ The best solution is s = 250

Exercise 4:

A supermarket has installed an **(R,S) inventory control rule**. The manager places orders for chocolate-chip cookies every Monday morning before the store opens. It then takes **three weeks** until the order arrives. The supermarket faces **normally distributed demand with a mean of 250 and a standard deviation of 30 units per week**.

- a) The supermarket aims to achieve an (adjusted) fill-rate of 98%. Determine the required base-stock level to fulfil this objective.
- b) Which α -service level will the supermarket achieve using the base-stock level obtained in a)?
- c) An employee observes that the delivery time of the cookies varies, e.g., due to delays in transportation or in the production process of the supplier. Determine the safety stock required if lead time is also normally distributed with mean 3 weeks and variance 1.44 weeks for an α -service level of 90%.

Exercise 4.a) The supermarket aims to achieve an (adjusted) fill-rate of 98%. Determine **the required base-stock level** to fulfil this objective.

mean	250
std dev	30
L	3
R	1
Service level (γ)	0,98

k	$\phi(k)$	$\Phi(k)$	G(k)
0.9900	0.24439	0.83891	0.08491
1.0000	0.24197	0.84134	0.08332

$$\gamma = 1 - \frac{\sigma\sqrt{L+R}G(z)}{\mu}$$

$$G(z) = \frac{\mu(1-\gamma)}{\sigma\sqrt{L+R}} = \frac{250(1-0,98)}{30\sqrt{3+1}} = 0,08333 \rightarrow z = 1,0036$$

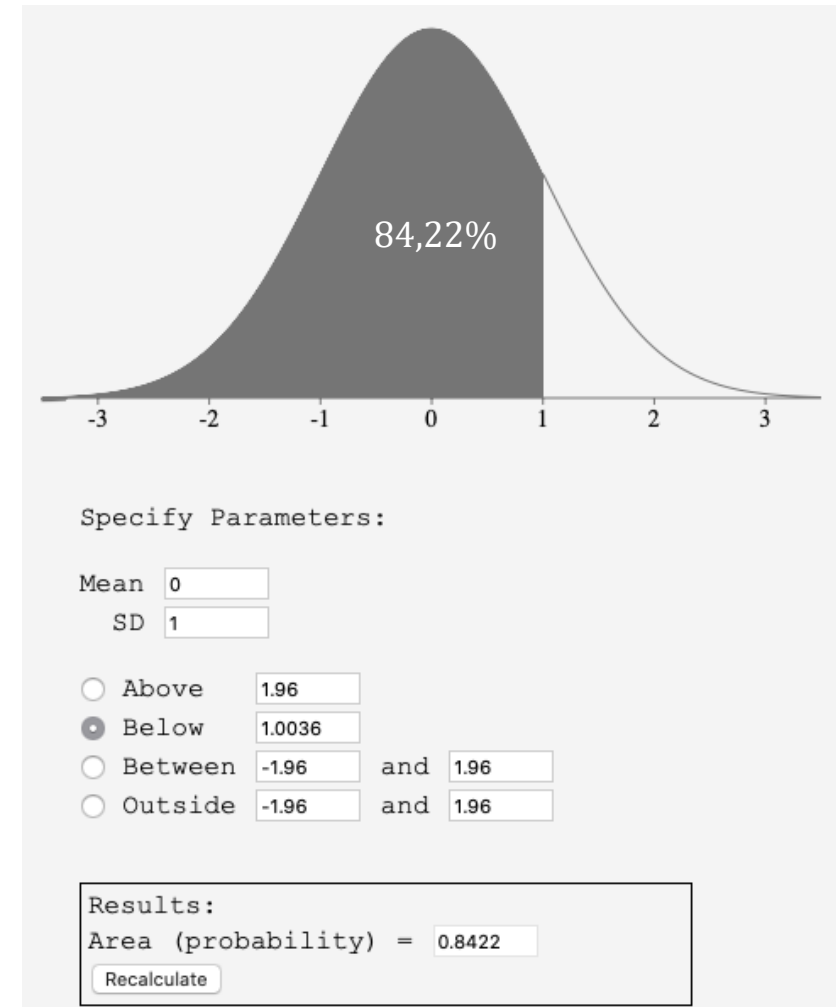
$$\therefore S = \mu(L+R) + z\sigma\sqrt{L+R} = 250(3+1) + 1,0036 * 30 * \sqrt{3+1} = 1060,216 (\approx 1060)$$

Exercise 4.b) Which α -service level will the supermarket achieve using the base-stock level obtained in a)?

$$z = 1,0036$$

$$\alpha\text{-service level} = F(z) = F(1,0036) = 0,8422 (\approx 84,2\%)$$

k	$\phi(k)$	$\Phi(k)$	G(k)
1.0000	0.24197	0.84134	0.08332
1.0100	0.23955	0.84375	0.08174
1.0200	0.23713	0.84614	0.08019
1.0300	0.23471	0.84849	0.07866
1.0400	0.23230	0.85083	0.07716



Exercise 4.c) An employee observes that the delivery time of the cookies varies, e.g., due to delays in transportation or in the production process of the supplier. Determine **the safety stock** required if **lead time is also normally distributed with mean 3 weeks and variance 1.44 weeks** for an α -service level of **90%**.

$$F(z) = 0,9 (= 90\%)$$

$$z = 1,28155$$

$$\mu_L = 3 \text{ weeks}$$

$$\sigma_L^2 = 1,44 \text{ weeks}$$

mean	250
std dev	30
L	3
R	1

k	$\phi(k)$	$\Phi(k)$	G(k)
1.2700	0.17810	0.89796	0.04851
1.2800	0.17585	0.89973	0.04750
1.2900	0.17360	0.90147	0.04650
1.3000	0.17137	0.90320	0.04553

\therefore Safety Stock under demand and lead time uncertainty

$$SS = z \cdot \sqrt{(L + R) \cdot \sigma_D^2 + \mu_D^2 \cdot \sigma_L^2}$$

$$= 1,28155 \cdot \sqrt{(3 + 1) \cdot 30^2 + 250^2 \cdot 1,44} = 392,08$$

$$\text{Reorder Point } (S') = \mu(L + R) + SS = 250(3 + 1) + 392,08 = \mathbf{1392,08}$$

Exercise 5:

Krusty aims to improve his inventory system and revisits his assumptions made in exercise sheet 1. He decides to install a periodic-review, order-up-to level control system for one of his specialties, the Royal Spicy Burger. He now places weekly orders which take another week to arrive at the burger restaurant ($R=1$, $L=1$). The demand is **gamma distributed with mean 1500 and standard deviation 750**.

What is Krusty's order-up-to level given an in-stock probability of 95%?

Exercise 5.a) What is Krusty's **order-up-to level** given an in-stock probability of **95%**?

L	1
R	1
alpha(SL)	0,95

$$\mu = 1500$$

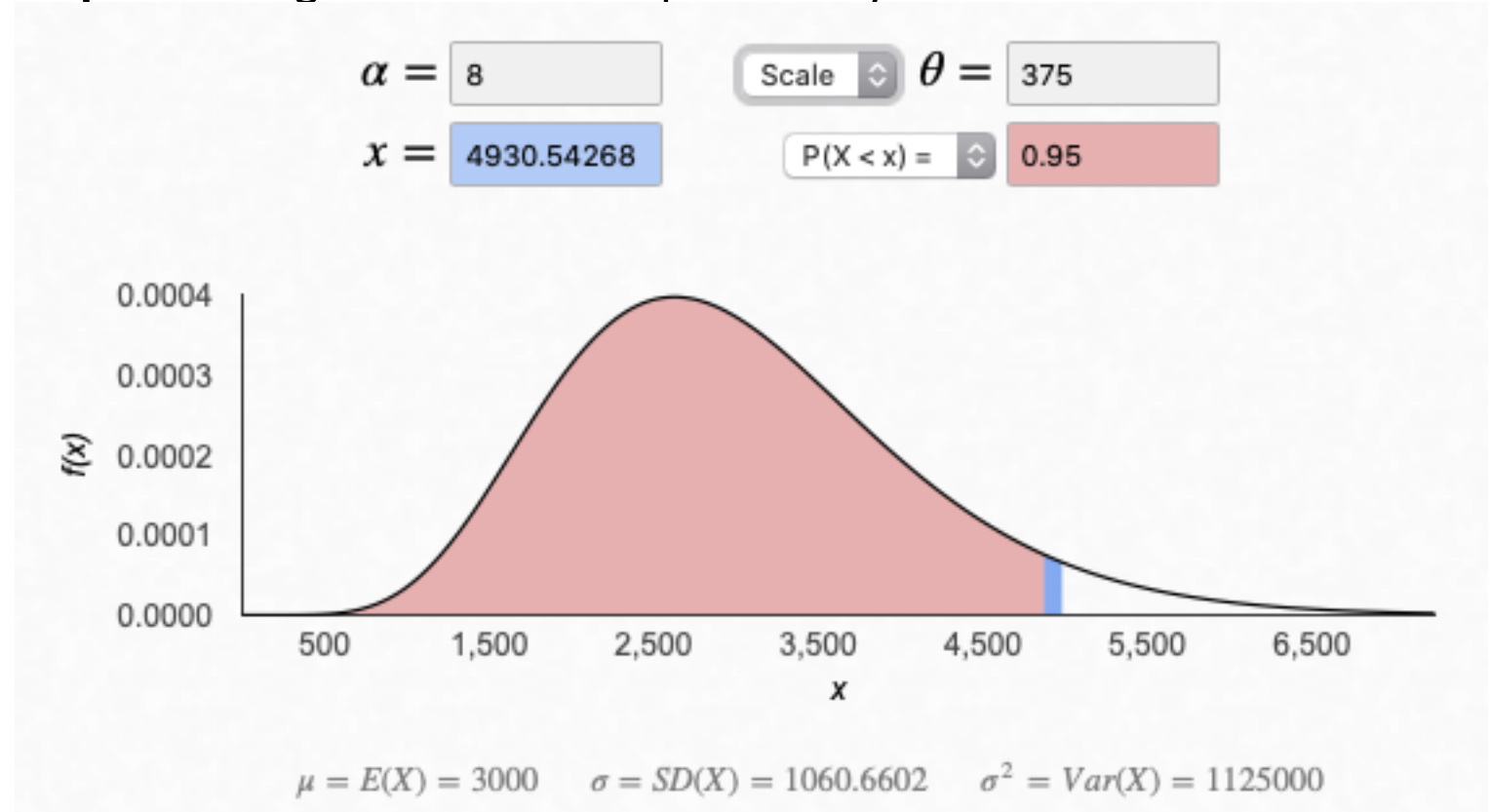
$$\sigma = 750$$

$$\mu_{L+R} = 1500(1 + 1) = 3000$$

$$\sigma_{L+R} = 750\sqrt{1 + 1} = 1060,66$$

$$\alpha = \frac{\mu_{(L+R)}^2}{\sigma_{(L+R)}^2} = \frac{3000^2}{1060,66^2} = 8,0099 (\approx 8), \quad \beta = \frac{\sigma_{(L+R)}^2}{\mu_{L+R}} = \frac{1060,66^2}{3000} = 374,99 (375)$$

$$\therefore S = \text{GAMMAINV}(0,95; 8; 375) = \mathbf{4930,543}$$



Thank you!