Inventory Management

Summer 2025

- Assignment 6 -

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Exercise 1:

A manufacturer produces ten products on one machine. No other products are assigned to this equipment. The data for the items are given in the following table.

Item	1	2	3	4	5	6	7	8	9	10
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001
Setup cost (€)	75	120	110	60	200	150	80	300	115	95
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3

a) Determine the independent optimal production lot-sizes.

b) Determine the production policy for the common cycle approach.

c) Use the basic period approach to find a solution.

Exercise 1.a) Determine **the independent** optimal production lot-sizes.

Item	1	2	3	4	5	6	7	8	9	10
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001
Setup cost (€)	75	120	110	60	200	150	80	300	115	95
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3

Machine		1	2	3	4	5	6	7	8	9	10
Q (EPQ)	$\sqrt{\frac{2dA}{h(1-\frac{d}{p})}}$	62.50 $(=\sqrt{\frac{2*200*75}{8(1-\frac{200}{5000})}})$	29.65 $(=\sqrt{\frac{2*50*120}{14(1-\frac{50}{2000})}})$	209.76	180.40	18.52	247.02	52.98	189.74	36.69	174.32
T (Cycle Length)	$\frac{EPQ}{D}$	0.31 (=62.50/200)	0.59 (=29.65/50)	0.26	0.36	0.93	0.41	0.53	0.19	0.46	0.39
Total Time (Manufacturing + Setup)	$\frac{Q}{p} + Setup$	0.013 (=62.5/5000 + 0.0005)	0.015 (=29.65/2000 + 0.0006)	0.053	0.029	0.062	0.007	0.027	0.032	0.041	0.024
Cost	$\frac{d}{Q}\cdot A+\frac{h}{2}(p-d)\cdot\frac{Q}{p}$	480.00 $(=\frac{200*75}{62.5}+\frac{8}{2}*\frac{(5000-200)62.5}{5000})$	404.72 $(=\frac{50*120}{29.65}+\frac{14}{2}*\frac{(2000-50)29.65}{2000})$	839.05	332.60	432.05	728.70	301.99	3162.28	501.46	490.49

Exercise 1.a) Determine **the independent** optimal production lot-sizes.

Machine	1	2	3	4	5	6	7	8	9	10	**Total**
T (Cycle Length)	0.31	0.59	0.26	0.36	0.93	0.41	0.53	0.19	0.46	0.39	
Q (EPQ)	62.50	29.65	209.76	180.40	18.52	247.02	52.98	189.74	36.69	174.32	
Total Time (Manufacturing + Setup)	0.013	0.015	0.053	0.029	0.062	0.007	0.027	0.032	0.041	0.024	0.30
Cost	480.00	404.72	839.05	332.60	432.05	728.70	301.99	3162.28	501.46	490.49	7673.34

Total cost (lower bound)	**7673.34**
Required manufacturing time	0.30
Minimum cycle time	0.19

Exercise 1.b) Determine the production policy for **the common cycle approach**.

Item	1	2	3	4	5	6	7	8	9	10	Sum
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450	3800
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250	69850
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001	0.0036
Setup cost (€)	75	120	110	60	200	150	80	300	115	95	1305
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3	101
$h_j d_j (1-\frac{d_j}{p_j})$	1536.00 (=8*200(1-200/5000))	682.50	3200.00	921.88	466.67	1770.00	570.00	16666.67	1093.33	1266.21	28173.25
$\frac{d_j}{p_j}$	0.04(=200/5000)	0.025	0.2	0.0781	0.0667	0.0167	0.05	0.1667	0.0889	0.0621	0.79

Unconstrained optimum	$T = \sqrt{\frac{2\sum_{j=1}^{n} A_j}{\sum_{j=1}^{n} h_j d_j (1-\frac{d_j}{p_j})}}$	$0.304 = \sqrt{\frac{2 * 1305}{28173.25}}$
Minimum cycle length (= Constraint)	$T^{\min} = \frac{\sum_{j=1}^{n} r_j}{1-\sum_{j=1}^{n} \frac{d_j}{p_j}}$	$0.017 = \frac{0.0036}{1-0.79}$
Optimal cycle	$Max\{Uncons\ Opt, Min\ Cycle\}$	**0.304** $= Max\{0.304, 0.017\}$

Exercise 1.b) Determine the production policy for **the common cycle approach**.

Item	1	2	3	4	5	6	7	8	9	10	Sum
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450	3800
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250	69850
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001	0.0036
Setup cost (€)	75	120	110	60	200	150	80	300	115	95	1305
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3	101
$h_j d_j (\frac{p_j - d_j}{p_j})$	1536.00	682.50	3200.00	921.88	466.67	1770.00	570.00	16666.67	1093.33	1266.21	28173.25

***Optimal Common Cycle: 0.304**

Item		1	2	3	4	5	6	7	8	9	10
Q	$= \mathrm{D} * \mathrm{Opt\ Cycle}$	60.87 (=0.304*200)	15.22	243.50	152.18	6.09	182.62	30.44	304.37	24.35	136.97
Cost	$= \frac{A_j}{T} + \frac{h_j}{2}(p_j - d_j)\frac{d_j}{p_j}T$	480.18 $(= \frac{75}{0.304} + \frac{0.304}{2} * 1536)$	498.12	848.39	337.42	728.11	762.19	349.58	3522.06	544.22	504.82
Total cost (upper bound)	$= \sum_{j=1}^{n} \left(\frac{A_j}{T} + \frac{h_j}{2}(p_j - d_j)\frac{d_j}{p_j}T \right)$	**8575.09**									

Exercise 1.c) **Use the basic period approach** to find a solution.

Algorithm Recap

1. Determine W: Minimum cycle length from independent optimization
2. Determine n_i, given W (n_i=2^m)
3. Determine new W
4. Back to Step 2 unless the procedure has converged (multipliers do not change anymore).
5. If converged, check feasibility. If not feasible, adjust ni and determine W from 3.

Exercise 1.c) **Use the basic period approach** to find a solution.

Item	1	2	3	4	5	6	7	8	9	10
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001
Setup cost (€)	75	120	110	60	200	150	80	300	115	95
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3
$h_j d_j (\frac{p_j - d_j}{p_j})$	1536.00	682.50	3200.00	921.88	466.67	1770.00	570.00	16666.67	1093.33	1266.21

1. Determine W: Minimum cycle length from independent optimization & 2. Determine n_i, given W

$$C(W, n_i) = \sum_{j=1}^{n} \left(\frac{A_j}{n_j W} + \frac{h_j}{2} (p_j - d_j) \frac{d_j}{p_j} n_j W \right)$$

Basic period (W)_initial	0.19									
Item	1	2	3	4	5	6	7	8	9	10
$n_i = 2^m$	2	4	1	2	4	2	2	1	2	2
Cost	489.208 $= \left(\frac{75}{2 * 0.19} + \frac{2 * 0.19}{2} 1536 \right)$	417.245 $= \left(\frac{120}{4 * 0.19} + \frac{4 * 0.19}{2} 682.5 \right)$	882.947	333.051	440.491	731.037	318.826	3162.281	510.365	490.579
Total cost	**7776.60**									

Exercise 1.c) **Use the basic period approach** to find a solution.

$$Basic\ period(W)_{new}?\ \ W = \sqrt{\frac{2\sum_{i=1}^{N}\frac{A_i}{n_i}}{\sum_{i=1}^{N}h_i(p_i-d_i)\frac{d_i}{p_i}n_i}}$$

3. Determine new W & 4. Back to Step 2 unless the multipliers do not change anymore

Item	1	2	3	4	5	6	7	8	9	10
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001
Setup cost (€)	75	120	110	60	200	150	80	300	115	95
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3
$n_i=2^m$	2	4	1	2	4	2	2	1	2	2
$h_j d_j(\frac{p_j-d_j}{p_j})$	1536.00	682.50	3200.00	921.88	466.67	1770.00	570.00	16666.67	1093.33	1266.21

Item	1	2	3	4	5	6	7	8	9	10	**Sum**
$\frac{A_i}{n_i}$	37.5 (=75/2)	30 (=120/4)	110	30.0	50.0	75.0	40.0	300.0	57.5	47.5	777.5
$h_i(p_i-d_i)\frac{d_i}{p_i}n_i$	3072 $(=1536*2)$	2730 $(=682.5*4)$	3200	1843.8	1866.7	3540.0	1140.0	16666.7	2186.7	2532.4	38778.2

$$\therefore\ W = \sqrt{\frac{2*777.5}{38778.2}} = 0.2002$$

Exercise 1.c) **Use the basic period approach** to find a solution.

Item	1	2	3	4	5	6	7	8	9	10
Demand rate (units/year)	200	50	800	500	20	600	100	1000	80	450
Production rate (units/year)	5000	2000	4000	6400	300	36000	2000	6000	900	7250
Setup time × 10^{-4} (years)	0.0005	0.0006	0.0008	0.0004	0.0001	0.0002	0.0001	0.0006	0.0002	0.0001
Setup cost (€)	75	120	110	60	200	150	80	300	115	95
Holding cost (€/year)	8	14	5	2	25	3	6	20	15	3
$n_i=2^m$	2	4	1	2	4	2	2	1	2	2
$h_j d_j (\frac{p_j - d_j}{p_j})$	1536.00	682.50	3200.00	921.88	466.67	1770.00	570.00	16666.67	1093.33	1266.21

$* \; W \approx 0.2002$

$$C(W, n_i) = \sum_{j=1}^{n} \left(\frac{A_j}{n_j W} + \frac{h_j}{2}(p_j - d_j)\frac{d_j}{p_j} n_j W \right)$$

5. If converged, check feasibility. If not feasible, adjust ni and determine W from 3.

	1	2	3	4	5	6	7	8	9	10
Q (=$d_j * n_j * W$)	80.08 (=200*2*0.2002)	40.04	160.16	200.20	16.02	240.24	40.04	200.20	32.03	180.18
Time (=$\frac{r_j}{n_j} + \frac{Q_j}{n_j p_j}$)	0.00826 (=$\frac{0.0005}{2} + \frac{80.08}{2*5000}$)	0.00516	0.04085	0.01584	0.01337	0.00344	0.01006	0.03397	0.01790	0.01248
Cost	494.82 (=$\frac{75}{2*0.2002} + \frac{1536*2*0.2002}{2}$)	423.12 (=$\frac{120}{4*0.2002} + \frac{682.5*4*0.2002}{2}$)	869.77	334.41	436.60	728.98	313.91	3166.83	506.10	490.76

$$\sum_{j=1}^{n} \left(\frac{r_j}{n_j} + \frac{Q_j}{n_j p_j} \right) \le W \,? \rightarrow 0.16134 \le 0.2002$$

$$\therefore \text{Total cost} = 7765.31$$

Independent Approach: 7673.34
Common Cycle: 8575.09

Exercise 2:

Consider two products with constant demand rate of **d1 = 200 and d2 = 250** units per period which are stored in a warehouse with a **total capacity of 300** units. The products require **a1 = 3 and a2 = 1** units of warehouse space. The ordering costs are **A1=150€ and A2=111€**. Additionally, the products cause holding costs **of h1 = 1€** per unit per period and **h2 = 2€** per unit per period.

a) Determine the optimal order quantities using the strategy of dedicated space.

b) How much would you be willing to pay to obtain additional warehouse space of 700 units?

c) Use the common-cycle method to determine the optimal replenishment cycle for all products.

d) Use the results in c) to determine how many units of product 1 are in stock when you replenish product 2.

Exercise 2.a) Determine the **optimal order quantities** using the strategy of **dedicated space.**

Product	1	2
d	200	250
A	150	111
h	1	2
a	3	1

*Capacity(W) = 300

$$L = \sum_{i=1}^{N}\left[\frac{d_i}{Q_i}A_i + \frac{h_i}{2}Q_i\right] + \lambda\left[\sum_{i=1}^{N} a_i Q_i - W\right]$$

$$\frac{\partial L}{\partial Q_i} = \frac{-d_i}{Q_i^2}A_i + \frac{h_i}{2} + \lambda a_i = 0$$

$$\Rightarrow Q_i^*(\lambda) = \sqrt{\frac{2d_iA_i}{h_i + 2\lambda a_i}} \qquad i = 1,2,\ldots,N$$

Solution

Case 1: Unconstrained solution λ = 0

Case 2: Constrained solution, choose λ such that $\sum_{i=1}^{N} a_i Q_i^*(\lambda) = W$

Case 1: Unconstrained solution

$$Q_1^* = \sqrt{\frac{2*200*150}{1}} = 244.95, \qquad Q_2^* = \sqrt{\frac{2*250*111}{2}} = 166.58$$

Total used space? = 244.95*3 + 166.58*1 = 901.43 ≈ 902 (> 300!!!)

Total Cost = 244.95 + 333.17 = **578.12**

Cost (Q_1^*) = $\frac{d_i}{Q_i}A_i + \frac{h_i}{2}Q_i = \frac{200}{244.95} * 150 + \frac{244.95}{2} * 1 \approx 244.95$

Cost $(Q_2^*) = \frac{250}{166.58} * 111 + \frac{166.58}{2} * 2 \approx 333.17$

Exercise 2.a) Determine the **optimal order quantities** using the strategy of **dedicated space.**

Product	1	2
d	200	250
A	150	111
h	1	2
a	3	1

*Capacity(W) = 300

Case 2: Constrained solution

$$Q_1^*= \sqrt{\frac{2*200*150}{1+2*2*3}} = 67.94, \qquad Q_2^* = \sqrt{\frac{2*250*111}{2+2*2*1}} = 96.18$$

How to derive λ ?

$$\sum_{i=1}^{N} a_i Q_i^*(\lambda) = W$$

$$a_1\sqrt{\frac{2d_1A_1}{h_1+2\lambda a_1}} + a_2\sqrt{\frac{2d_2A_2}{h_2+2\lambda a_2}} = 300$$

$$3\sqrt{\frac{2*200*150}{1+2\lambda*3}} + 1\sqrt{\frac{2*250*111}{2+2\lambda*1}} = 300$$

$\therefore\ \lambda = 2$

Total used space? = 67.94*3 + 96.18*1 = **300**

$$L = \sum_{i=1}^{N}\left[\frac{d_i}{Q_i}A_i + \frac{h_i}{2}Q_i\right] + \lambda\left[\sum_{i=1}^{N} a_i Q_i - W\right]$$

Cost $(Q_1^*) = \frac{d_i}{Q_i}A_i + \frac{h_i}{2}Q_i = = \frac{200*150}{67.94} + \frac{1*67.94}{2} \approx 475.54$

Cost $(Q_2^*) = \frac{250*111}{96.18} + \frac{96.18*2}{2} \approx 384.70$

Total Cost = $475.54 + 384.70 \approx$ **860.2**

Exercise 2.b) How much would you be willing **to pay to obtain additional warehouse space of 700 units**?

Product	1	2
d	200	250
A	150	111
h	1	2
a	3	1

*Capacity(W) **= 1000**

From the unconstrained solution...

∴ Solution: Q_1^* = 244.95, Q_2^* = 166.58

Total used space? = 244.95*3 + 166.58*1 = 901.43 (< 1000)

Total Cost = **578.12**

By contrained with W= 300...

∴ Solution: Q_1^* = 67.94, Q_2^* = 96.18

Total used space? = 67.94*3 + 96.18*1 = **300**

Total Cost = **860.24**

W=300	860.24
W=1000	578.12
Delta	**282.12**

Exercise 2.c) Use the **common-cycle method** to determine **the optimal replenishment cycle** for all products.

Product	1	2
d	200	250
A	150	111
h	1	2
a	3	1

*Capacity(W) = 300

$$T^* = \min\left\{\sqrt{\frac{2\sum_{i=1}^{N} A_i}{\sum_{i=1}^{N} h_i d_i}}; W\frac{\sum_{i=1}^{N} a_i d_i}{\sum_{i=1}^{N}\sum_{j=1}^{i} a_i a_j d_i d_j}\right\}$$

$$T_{uncons} = \sqrt{\frac{2\sum_{i=1}^{N} A_i}{\sum_{i=1}^{N} h_i d_i}} = \sqrt{\frac{2(150+111)}{(200*1+250*2)}} = 0.86355$$

$$T_{cons} = W\frac{\sum_{i=1}^{N} a_i d_i}{\sum_{i=1}^{N}\sum_{j=1}^{i} a_i a_j d_i d_j}$$

$$= 300\frac{(3\cdot 200 + 1\cdot 250)}{(3\cdot 3\cdot 200\cdot 200)+(1\cdot 1\cdot 250\cdot 250)+(1\cdot 3\cdot 250\cdot 200)}$$

$$= 0.4454$$

$\therefore\ T^* = \min\{0.86355; 0.445\}$ = 0.445

Exercise 2.c) Use the **common-cycle method** to determine **the optimal replenishment cycle** for all products.

Product	1	2
d	200	250
A	150	111
h	1	2
a	3	1

*Capacity(W) = 300

$$C = \sum_{i=1}^{N} \left(\frac{A_i}{T} + \frac{h_i d_i}{2} T \right) \qquad where, T^* = 0.4454$$

Cost $(Q_1^*) = \frac{A_1}{T} + \frac{h_1 d_1}{2} T = \frac{150}{0.445} + \frac{1*200}{2} * 0.445 \approx 381.58$

Cost $(Q_2^*) = \frac{A_2}{T} + \frac{h_2 d_2}{2} T = \frac{111}{0.445} + \frac{2*250}{2} * 0.445 \approx 360.69$

Total Cost = $381.58 + 360.69$ = **742.27**

Exercise 2.d) Use the results in c) to determine **how many units of product 1 are in stock when you replenish product 2.**

$$t_i = \frac{\sum_{j=1}^{i} a_j d_j}{\sum_{j=1}^{N} a_j d_j} T$$

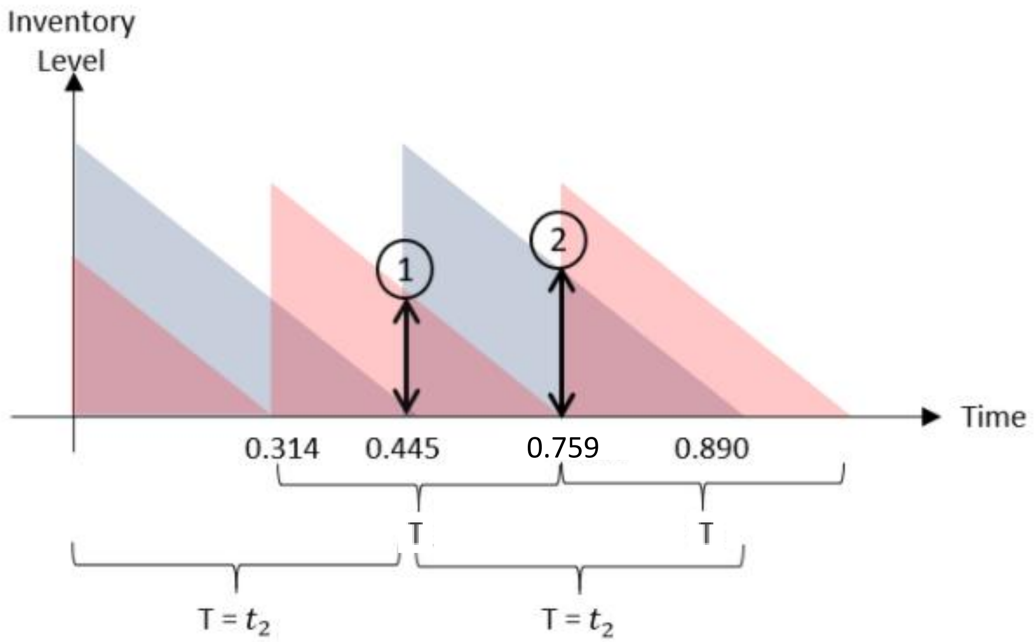
$$t_1 = \frac{200 \cdot 3}{(200 \cdot 3\ + 250 \cdot 1)} 0.445 \approx 0.314$$

$$t_2 = \frac{(200 \cdot 3\ + 250 \cdot 1)}{(200 \cdot 3\ + 250 \cdot 1)} 0.445 \approx 0.445$$

$$t_{gap} = t_2 - t_1 = 0.131$$

$$Q_i = d_i \cdot T \ \rightarrow Q_1 = 200\ \cdot 0.4454 \approx 89$$

$$\therefore Q_{1L/O} = Q_1 - d_1 \cdot t_{gap} = 89 - 200 \cdot 0.131 \approx 62.8$$



Exercise 3:

Ten products are ordered by a distributor from a single supplier. The specific product data are given in the following table and the following general information has been gathered. The major ordering cost is **A_0 = 30€** and the minor ordering cost for each product is **Ai = 15€.** The delivery lead time is **one week.**

Product	Monthly demand (units)	Holding cost (€)
1	8	1
2	25	2
3	4	0.6
4	63	5.2
5	67	1.6
6	46	0.4
7	54	0.098
8	2	12
9	83	2
10	82	1

a) Find the optimal order frequencies for each product.

b) What are the corresponding overall costs?

Exercise 3.a) Find **the optimal order frequencies** for each product.

Iterative solution approach recap

1. Let T_1 the smallest cycle time from independent planning
2. Determine initial multipliers by rounding non-integer values $n_i = \sqrt{\frac{A_i h_1 d_1}{h_i d_i (A_0 + A_1)}}$
3. Determine T_1 $T_1^* = \sqrt{\frac{2(A_0 + \sum_{i=1}^{N} \frac{A_i}{n_i})}{\sum_{i=1}^{N} h_i d_i n_i}}$
4. Determine new integers from cost function
   – Minimum integer that satisfies $n_i(n_i + 1) \geq \frac{2A_i}{h_i d_i T_1^2}$
5. Back to step 2 if any integer changed

Exercise 3.a) Find **the optimal order frequencies** for each product.

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1

Major Setup=30€
Minor Setup=15€

1) Determine initial multipliers by rounding non-integer values

Product	1	2	3	**4**	5	6	7	8	9	10
Independent cycle times $T_i = \sqrt{\frac{2A_i}{h_i d_i}}$	1.9365 $=\sqrt{\frac{2\cdot 15}{1\cdot 8}}$	0.7746 $=\sqrt{\frac{2\cdot 15}{2\cdot 25}}$	3.5355 $=\sqrt{\frac{2\cdot 15}{0.6\cdot 4}}$	0.3026 $=\sqrt{\frac{2\cdot 15}{5.2\cdot 63}}$	0.5290 $=\sqrt{\frac{2\cdot 15}{1.6\cdot 67}}$	1.2769	2.3810	1.1180	0.4251	0.6049
Optimal multipliers $n_i = \sqrt{\frac{A_i h_4 d_4}{h_i d_i (A_0 + A_4)}}$	3.695 $=\sqrt{\frac{15\cdot 5.2\cdot 63}{1\cdot 8\cdot(30+15)}}$	1.478 $=\sqrt{\frac{15\cdot 5.2\cdot 63}{2\cdot 25\cdot(30+15)}}$	6.745 $=\sqrt{\frac{15\cdot 5.2\cdot 63}{0.6\cdot 4\cdot(30+15)}}$	0.577 $=\sqrt{\frac{15\cdot 5.2\cdot 63}{5.2\cdot 63\cdot(30+15)}}$	1.009	2.436	4.543	2.133	0.811	1.154
Rounded*	4	1	7	1	1	2	5	2	1	1

Exercise 3.a) Find **the optimal order frequencies** for each product.

1) Determine initial multipliers by rounding non-integer values

Major Setup=30€
Minor Setup=15€

$$\underline{C} = \sqrt{2(A_0 + A_4)h_4 d_4} + \sum_{i=2}^{N} \sqrt{2A_i h_i d_i}$$

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1
$\sqrt{2A_i h_i d_i}$	15.49	38.73	8.49	99.14	56.71	23.49	12.60	26.83	70.57	49.60

$$\therefore \underline{C} = \sqrt{2(A_0 + A_4)h_4 d_4} + \sum_{i=2}^{N} \sqrt{2A_i h_i d_i} = \sqrt{2(30+15)5.2 * 63} + (15.49 + 38.73 + 8.49 + 56.71 + 23.49 + 12.6 + 26.83 + 70.57 + 49.6) = \mathbf{474.22}$$

Exercise 3.a) Find **the optimal order frequencies** for each product.

2) Determine T_1

Major Setup=30€
Minor Setup=15€

$$T^* = \sqrt{\frac{2(A_0 + \sum_{i=1}^{N} \frac{A_i}{n_i})}{\sum_{i=1}^{N} h_i d_i n_i}}$$

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1
n_i	4	1	7	1	1	2	5	2	1	1

											Sum
$\frac{A_i}{n_i}$	3.75 $(=\frac{15}{4})$	15 $(=\frac{15}{1})$	2.14286 $(=\frac{15}{7})$	15 $(=\frac{15}{1})$	15 $(=\frac{15}{1})$	7.5 $(=\frac{15}{2})$	3 $(=\frac{15}{5})$	7.5 $(=\frac{15}{2})$	15 $(=\frac{15}{1})$	15 $(=\frac{15}{1})$	98.89
$h_i d_i n_i$	32 (=1*8*4)	50 (=2*25*1)	16.8 (=0.6*4*7)	327.6	107.2	36.8	26.46	48	166	82	892.86

$$\therefore T^* = \sqrt{\frac{2(A_0 + \sum_{i=1}^{N} \frac{A_i}{n_i})}{\sum_{i=1}^{N} h_i d_i n_i}} = \sqrt{\frac{2(30 + 98.89)}{892.86}} = \mathbf{0.53732}$$

Exercise 3.a) Find **the optimal order frequencies** for each product.

3) Determine new integers from cost function

$T^* =$ **0.53732**

Major Setup=30€
Minor Setup=15€

*Multiplier of the 2nd and 7th item changed!

$$n_i(n_i + 1) >= \frac{2A_i}{h_i d_i T^2}$$

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1

RH ($\frac{2A_i}{h_i d_i T^2}$)	12.988 (=$\frac{2*15}{1*8*0.53732^2}$)	2.078 (=$\frac{2*15}{2*25*0.53732^2}$)	43.295	0.317	0.969	5.647	19.635	4.329	0.626	1.267
New multipliers	4	2	7	1	1	2	4	2	1	1
LF ($n_i(n_i + 1)$)	20	6	56	2	2	6	20	6	2	2

Initial n_i	4	1	7	1	1	2	5	2	1	1

Exercise 3.a) Find **the optimal order frequencies** for each product.

3) Determine new integers from cost function

Product	1	2	3	4	5	6	7	8	9	10	Total sum
New multipliers	4	2	7	1	1	2	4	2	1	1	
$\frac{A_i}{n_i}$	3.75 $(=\frac{15}{4})$	7.5 $(=\frac{15}{2})$	2.14286 $(=\frac{15}{7})$	15 $(=\frac{15}{1})$	15 $(=\frac{15}{1})$	7.5 $(=\frac{15}{2})$	3.75 $(=\frac{15}{4})$	7.5 $(=\frac{15}{4})$	15 $(=\frac{15}{1})$	15 $(=\frac{15}{1})$	92.14
$h_i d_i n_i$	32 (=1*8*4)	100 (=2*25*2)	16.8 (=0.6*4*7)	327.6	107.2	36.8	26.46 (=0.098*54*4)	48	166	82	937.57

$$\therefore T^* = \sqrt{\frac{2(A_0 + \sum_{i=1}^{N} \frac{A_i}{n_i})}{\sum_{i=1}^{N} h_i d_i n_i}} = \sqrt{\frac{2(30 + 92.14)}{937.57}} = \mathbf{0.51044}$$

Exercise 3.a) Find **the optimal order frequencies** for each product.

3) Determine new integers from cost function

$T^* = \mathbf{0.51044}$

*Multiplier of the 6th and 7th item changed!

$$n_i(n_i + 1) >= \frac{2A_i}{h_i d_i T^2}$$

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1

RH ($\frac{2A_i}{h_i d_i T^2}$)	14.392 (=$\frac{2*15}{1*8*0.51044^2}$)	2.303 (=$\frac{2*15}{2*25*0.51044^2}$)	47.975	0.351	1.074	6.258	21.757	4.797	0.694	1.404
New multipliers	4	2	7	1	1	3	5	2	1	1
LF ($n_i(n_i + 1)$)	20	6	56	2	2	12	30	6	2	2

Previous n_i	4	2	7	1	1	2	4	2	1	1

Exercise 3.a) Find **the optimal order frequencies** for each product.

3) Determine new integers from cost function

Product	1	2	3	4	5	6	7	8	9	10	Total sum
New multipliers	4	2	7	1	1	3	5	2	1	1	
$\frac{A_i}{n_i}$	3.75 $(=\frac{15}{4})$	7.5 $(=\frac{15}{2})$	2.14286 $(=\frac{15}{7})$	15 $(=\frac{15}{1})$	15 $(=\frac{15}{1})$	5 $(=\frac{15}{3})$	3 $(=\frac{15}{5})$	7.5 $(=\frac{15}{4})$	15 $(=\frac{15}{1})$	15 $(=\frac{15}{1})$	88.89
$h_i d_i n_i$	32 (=1*8*4)	100 (=2*25*2)	16.8 (=0.6*4*7)	327.6	107.2	36.8 (=0.4*46*3)	26.46 (=0.098*54*5)	48	166	82	961.26

$$\therefore T^* = \sqrt{\frac{2(A_0 + \sum_{i=1}^{N} \frac{A_i}{n_i})}{\sum_{i=1}^{N} h_i d_i n_i}} = \sqrt{\frac{2(30 + 88.89)}{961.26}} = \mathbf{0.4974}$$

Exercise 3.a) Find **the optimal order frequencies** for each product.

4) Back to step 2 if any integer changed

$T^* = \mathbf{0.4974}$

*Multipliers did not change!

$$n_i(n_i+1) >= \frac{2A_i}{h_i d_i T^2}$$

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1

RH ($\frac{2A_i}{h_i d_i T^2}$)	15.16 (=$\frac{2*15}{1*8*0.4974^2}$)	2.426 (=$\frac{2*15}{2*25*0.4974^2}$)	50.532	0.370	1.131	6.591	22.917	5.053	0.731	1.479
New multipliers	4	2	7	1	1	3	5	2	1	1
LF ($n_i(n_i+1)$)	20	6	56	2	2	12	30	6	2	2

Previous n_i	4	2	7	1	1	3	5	2	1	1

Exercise 3.b) What are the corresponding overall **costs**?

Product	1	2	3	4	5	6	7	8	9	10
demand	8	25	4	63	67	46	54	2	83	82
holding cost	1	2	0.6	5.2	1.6	0.4	0.098	12	2	1

Excel File
will be provided

$T^* =$ **0.4974**

$$C = \frac{A_0 + A_1 + \sum_{i=2}^{N} \frac{A_i}{n_i}}{T_1} + \frac{T_1(h_1 d_1 + \sum_{i=2}^{N} h_i d_i n_i)}{2}$$

Product	1	2	3	4	5	6	7	8	9	10	Total sum
Final multipliers	**4**	**2**	**7**	**1**	**1**	**3**	**5**	**2**	**1**	**1**	
$\frac{A_i}{n_i}$	3.75	7.5	2.14286	15	15	5	3	7.5	15	15	88.89
$h_i d_i$	8	50	2.4	327.6	107.2	18.4	5.292	24	166	82	790.89
$h_i d_i n_i$	32	100	16.8	327.6	107.2	55.2	26.46	48	166	82	961.26

$$\therefore C = \frac{A_0+A_1+\sum_{i=2}^{N}\frac{A_i}{n_i}}{T_1} + \frac{T_1(h_1 d_1+\sum_{i=2}^{N} h_i d_i n_i)}{2} = \frac{30+15+(88.89-15)}{0.49736} + \frac{0.49736(327.6+(961.26-327.6))}{2} \approx \mathbf{478.09}$$

$$\underline{C} = \mathbf{474.22}$$

Thank you!