Poster-Experiment

#1

#2

#3

Poster-Experiment I

Phase 1: Consumer choice

– Choose exactly one of the three posters you like most !

My choice:

Poster-Experiment II

Phase 2: Procurement decision

How many posters y_i of each type would you buy to sell them to your classmates?

Revenues and cost data

- **Procurement** price for each unit: **1.50 €**
- **Salvage** value for each unsold poster: **0.50 €**
- **Sales** price for a poster: **5.50 €**

My decision: Order quantities

#1:

#2:

#3:

Poster-Experiment III

Phase 3: Determine profit

- Demand realization d
- $d \geq y$: $\Pi = 4*y$
- $d < y$: $\Pi = 5d - y$

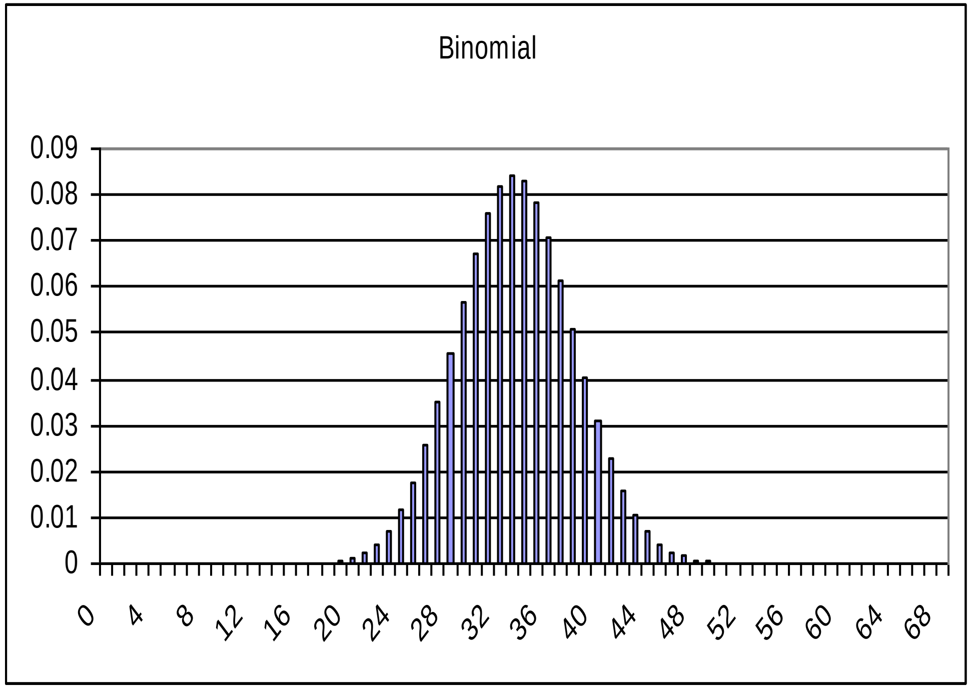
My profit:

The Newsvendor Model

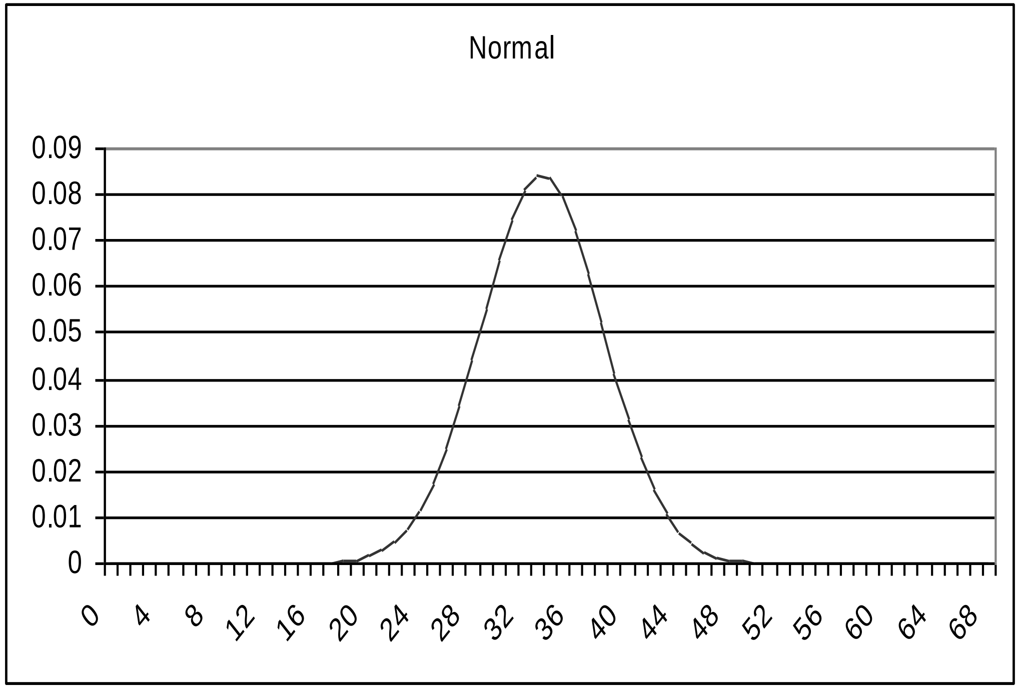
- Arrow, K. J., Harris, T., & Marschak, J. (1951). Optimal inventory policy. *Econometrica: Journal of the Econometric Society*, 250-272.
- Original idea: newspaper vendor
- One product, one period
- Question: how many units of the product to buy?
- Any units left over are either
    - Sold at a discount, or
    - Scrapped at possibly an additional cost
- Demand is stochastic, but the distribution is known

Example distributions (demand)

n=100, p=1/3



μ=33.33, σ=4.71



Revenues and costs

Procurement price: c

Sales price: p

Salvage value: g

Ranges: $g \leq c \leq p$.

- $c > p$: product is not profitable
- $g > c$: arbitrage
- Note: only unique, interior solutions for $g<c<p$. Boundary cases are ``trivial“.

Cost definitions (too many – too few)

- **Overage**: $c_o = c - g$

  extra cost of having one unit too many

- **Underage**: $c_u = p - c$

  extra cost of having one unit too few (lost margin of not having procured this item)

Decision rule

Costs weighting with the chance:

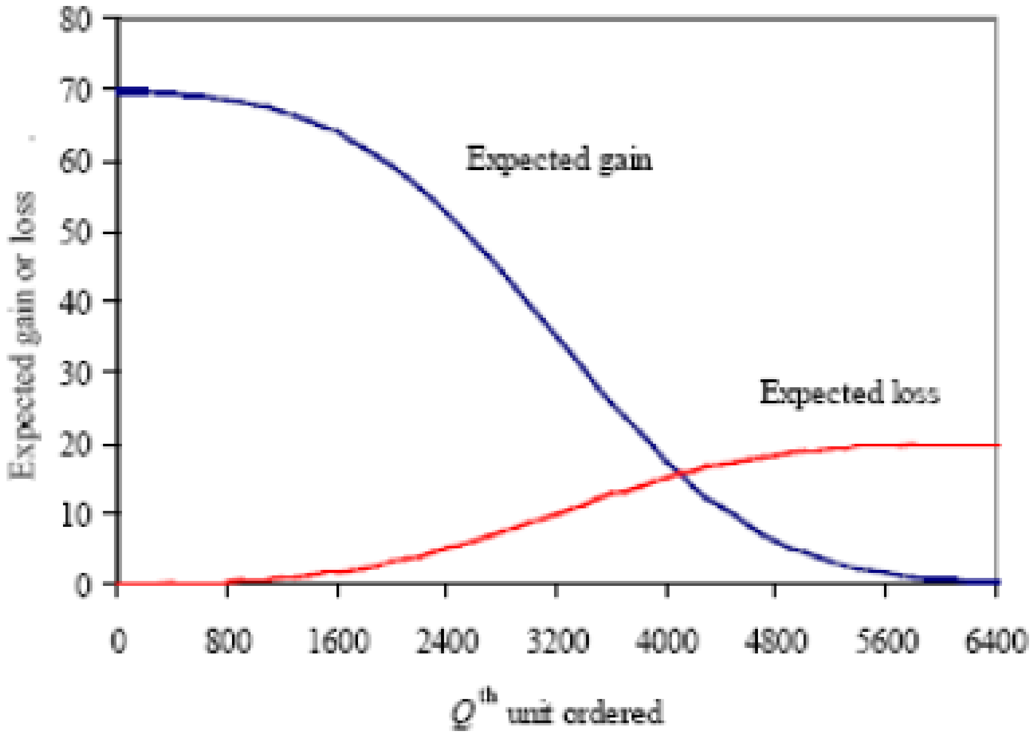
- of having an overage (demand smaller or equal to y): F(y) (CDF of demand)
- of having an underage (demand exceeds y): 1-F(y)

Increasing the order quantity by one unit

- Increases the chance of an overage
- Decreases the chance of an underage (gain)

Decision rule: increase the order quantity until the reduction of underage costs does not offset the increase in overage costs anymore

$$c_o F(y) = c_u(1 - F(y)) \Leftrightarrow F(y) = \frac{c_u}{c_u + c_o}$$



Decision model

Order quantity *y*

Profit function

- discrete $$\Pi(y) = -c \cdot y + \sum_{d=0}^{y-1} \big(p \cdot d + g \cdot (y-d)\big) \cdot \mathrm{P}(D=d) + \sum_{d=y}^{\infty} p \cdot y \cdot \mathrm{P}(D=d)$$
- continuous $$\Pi(y) = -c \cdot y + \int_{0}^{y} \big(p \cdot d + g \cdot (y-d)\big) \cdot f(d)\mathrm{d}d + \int_{y}^{\infty} p \cdot y \cdot f(d)\mathrm{d}d$$

Solution

- discrete $$F(y^* - 1) \leq \frac{p-c}{p-g} \leq F(y^*)$$
- continuous $$F(y^*) = \frac{p-c}{p-g}$$

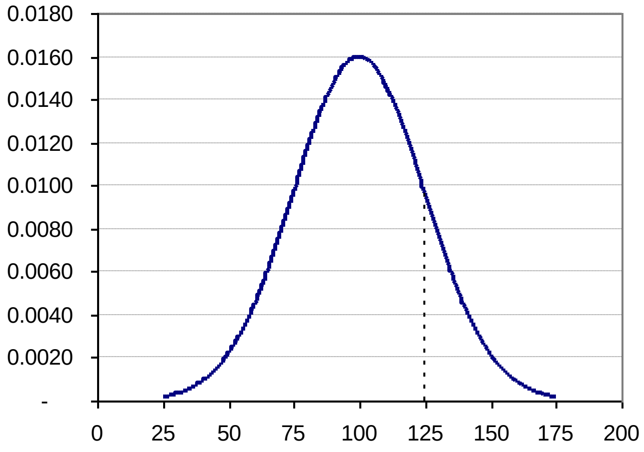
Normal distribution tutorial

- All normal distributions are characterized by two parameters, mean = μ and standard deviation = σ
- All normal distributions are related to the standard normal that has mean = 0 and standard deviation = 1.
- For example:
  - Let *Q* be the order quantity, and (μ, σ) the parameters of the normal demand distribution.
  - *Prob*{demand is *Q* or lower} = *Prob*{the outcome of a standard normal is *z* or lower}, where

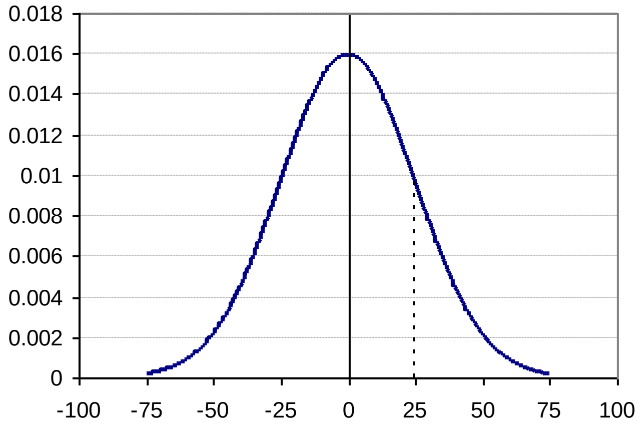
$$z = \frac{Q - \mu}{\sigma} \quad \text{or} \quad Q = \mu + z \times \sigma$$

  - Look up *Prob*{the outcome of a standard normal is *z* or lower} in the Standard Normal Distribution Function Table. Either find the *z* for which the probability comes closest, or interpolate for higher precision. You will always make a slight error.
  - Use that $F_{0,1}^{-1}(y) = -F_{0,1}^{-1}(1 - y)$

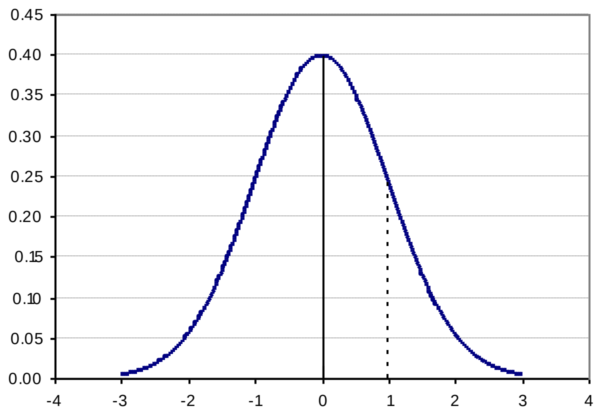
Start with
$\mu = 100$,
$\sigma = 25$,
$Q = 125$



Center the distribution over 0 by subtracting the mean



$$z = \frac{Q-\mu}{\sigma} = \frac{125-100}{25} = 1$$



Rescale the x and y axes by dividing by the standard deviation

Numerical example (continuous)

Data:

- Normally distributed demand with mean 100 and standard deviation 30
- p=10, c=4 or c=8, g=0

Procedure:

- Critical ratio = (p-c)/(p-g)
- Find z such that $F_{0,1}(z) = \frac{p-c}{p-g}$, i.e. $z = F_{0,1}^{-1}\left(\frac{p-c}{p-g}\right)$.
- Optimal order quantity $y^* = \mu + \sigma z$

Note: only if you assume normally distributed demand!

Results (note the slight errors because z-values come from the table):

- c=4: $y* = 100 + 30 \cdot F_{0,1}^{-1}(0.6) = 100 + 30 \cdot 0.25 = 107.5 \approx 108$
- c=8: $y* = 100 + 30 \cdot F_{0,1}^{-1}(0.2) = 100 + 30 \cdot (-0.84) = 74.8 \approx 75$

Performance measures (continuous)

Expected lost sales $$ELS(y) = \int_{y}^{\infty} (d-y) f(d) \mathrm{d}d$$

– Normal distribution: $$z = \frac{y-\mu}{\sigma}, ELS(y) = \sigma G(z), G(z) = f_{0,1}(z) - z(1 - F_{0,1}(z))$$

Expected sales $$ES(y) = \int_{0}^{y} d f(d) \mathrm{d}d + y \int_{y}^{\infty} f(d) \mathrm{d}d = \mu - ELS(y)$$

Expected left over inventory $$ELO(y) = \int_{0}^{y} (y-d) f(d) \mathrm{d}d = y - ES(y)$$

Expected profit $$EP(y) = -cy + pES(y) + gELO(y)$$

Non-stockout probability $$\alpha = F(y) \text{ (normal distribution: } F_{0,1}(z) \text{)}$$

Fill-rate $$\beta = \frac{ES(y)}{\mu} = 1 - \frac{ELS(y)}{\mu}$$

Distribution functions

- Excel
- Any statistical software (R, Matlab)
- Normal distribution table (e.g. exam, see Moodle)

Performance measures (example)

Normally distributed demand: μ = 100, σ = 30, p=10, c=4, g=0 → y* = 107.6004

(Here we use exact values from the distributions, not from the table)

- Expected lost sales
  - z=0.2533471, G(z)= 0.2850037, ELS(108) = 30* 0.2850037 = 8.550111
- Expected sales
  - ES(107.6004)=100-ELS(107.6004)= 91.44989
- Expected left over
  - ELO(107.6004)=y-ES(107.6004)=107.6004-91.44989 =16.15051
- Expected profit
  - EP=-c*107.6004+p*91.44989+g*16.15051= 484.0973
- Non-stockout probability
  - α=60%
- Fill rate
  - β=91.44989%

	c = 4	c = 2
y*	107.6004	125.2486
ELS	8.55	3.3588
ES	91.45	96.6412
ELO	16.15	28.6074
EP	484.10	715.9148
α	60%	80%
β	91.45%	96.64%

Note that here, if you don‘t round off y*, α is exactly your critical ratio

Numerical example (discrete)

Data:

- Poisson distributed demand with λ = 3
- p=10, c=4 or c=8, g=0

$$F(y^* - 1) \leq \frac{p-c}{p-g} \leq F(y^*)$$

demand d_i	0	1	2	3	4	5	...
$P(D = d_i)$	0.05	0.15	0.22	0.22	0.17	0.10	0.08
$P(D \leq d_i)$	0.05	0.20	0.42	0.65	0.82	0.92	1

Results

- c=4: $F(y^*) \geq 0.6 \Rightarrow y^* = 3$
- c=8: $F(y^*) \geq 0.2 \Rightarrow y^* = 1$

Performance measures (discrete)

Expected lost sales
$$ELS(y) = \sum_{d=y+1}^{\infty} (d-y)\mathrm{P}(d)$$

Expected sales
$$ES(y) = \sum_{d=0}^{y-1} d\ \mathrm{P}(d) + y\sum_{d=y}^{\infty} \mathrm{P}(d)$$

Expected left over inventory
$$ELO(y) = \sum_{d=0}^{y-1} (y-d)\mathrm{P}(d) = y - ES(y)$$

Expected profit
$$EP(y) = -cy + pES(y) + gELO(y)$$

Non-stockout probability
$$\alpha = \sum_{d=0}^{y} \mathrm{P}(d)$$

Fill-rate
$$\beta = \frac{ES(y)}{\mu} = 1 - \frac{ELS(y)}{\mu}$$

Poisson distribution

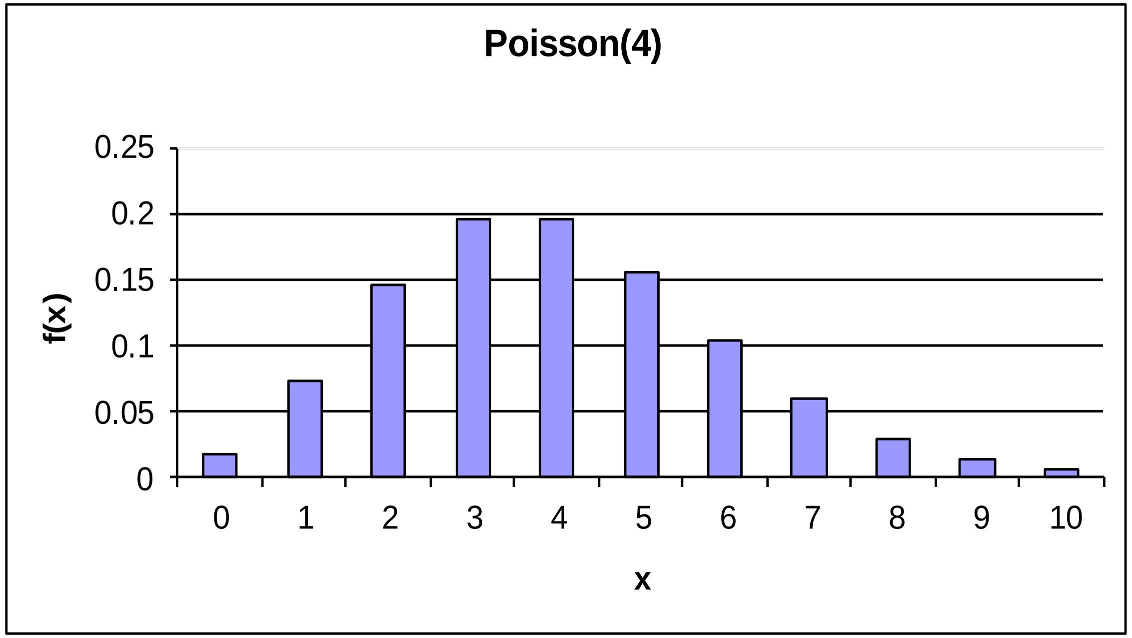
- Probability Mass Function

$$f(x) = \frac{\lambda^x}{x!} \cdot e^{-\lambda} \quad x = 0,1,2,...$$

- Distribution

$$F(X) = e^{-\lambda} \cdot \sum_{i=0}^{x} \frac{\lambda^i}{i!} \quad x = 0,1,2,...$$

$$\mathrm{E}(X) = \mathrm{Var}(X) = \lambda$$



Performance measures (example)

Poisson distributed demand: $\lambda = 3$, p=10, c=4, g=0 → y* = 3

- Expected lost sales
  - ELS(3) = 1*0.1680+2*0.1008+... = 0.672125
- Expected sales
  - ES(3) = 3-ELS(3) = 2.327875
- Expected left over
  - ELO(3) = y-ES(3) = 3-2.327875=0.672125
- Expected profit
  - EP = -c*3+p* 2.327875 +g*0.672125 = 11.27875
- Non-stockout probability
  - α = P(0)+P(1)+P(2)+P(3) = 64.7231889%
- Fill rate
  - β = ES(3) / 3 = 77.59583%

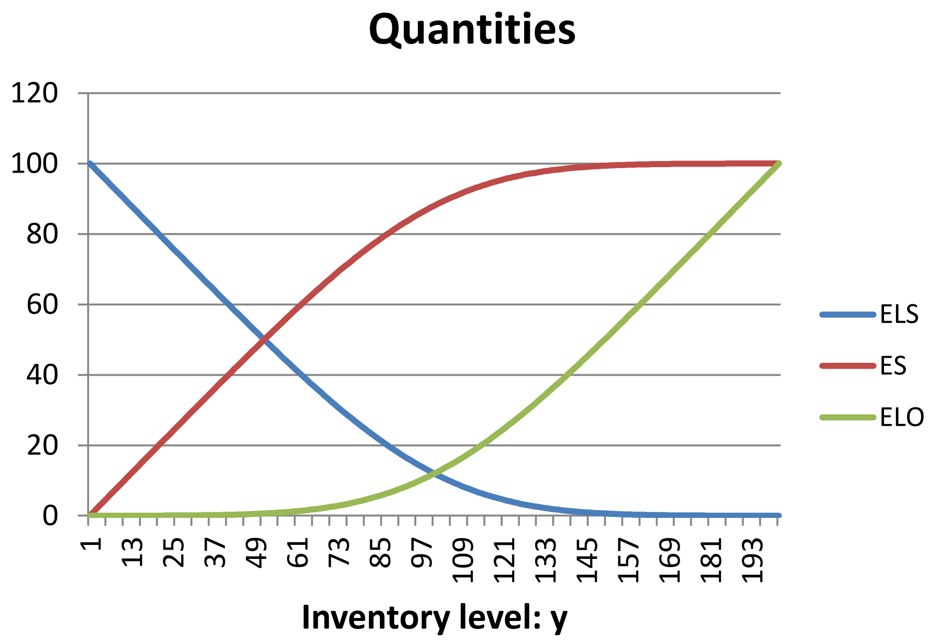
Try in Excel!

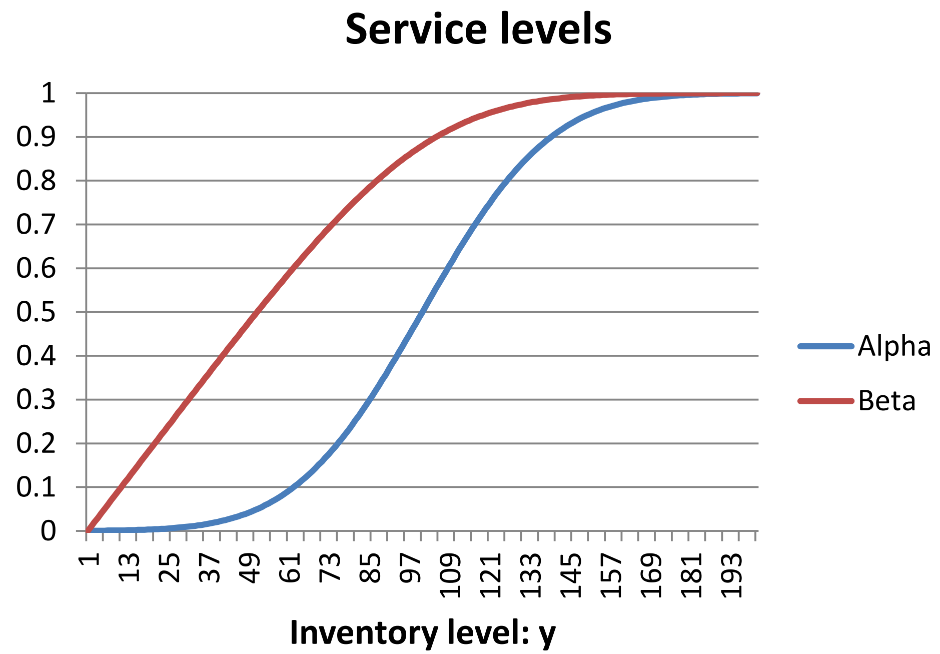
	c = 4	c = 2
y*	3	4
ELS	0.67	0.32
ES	2.33	2.68
ELO	0.67	1.32
EP	11.3	18.81
α	64.7%	81.5%
β	77.6%	89.4%

Note that here, α typically overshoots your critical ratio

Performance measures (example)

Normally distributed demand: μ = 100, σ = 30





Decision criteria

- **Decision making under uncertainty**
  - Decision maker‘s attitude towards risk
  - Operations literature: predominantly assumes risk neutrality
  - Optimization of expected profit, expected cost etc.

- **Newsvendor simulation**: See Excel file

- **Recent research trend**
  - How do results change if the decision maker is risk averse?

The risk averse newsvendor

- **Assumptions**
  - As in the standard newsvendor
  - Utility function $u(x), u'(x) > 0, u''(x) < 0$
  - Maximize expected utility
- **Objective function**

$$\max \mathrm{E}\,(u[y]) = \int_0^y u[pd - cy]f(d)\mathrm{d}d + (1 - F(y))u[(p - c)y]$$

- **Result: Order quantity decreases**

Other criteria

Probability distribution of profit

$$\prod(y) = -cy + p\min(D, y), \qquad \prod(y) \in \{-cy, (p-c)y\}$$

$$P(\prod = x) = \begin{cases} P\left(D = \dfrac{x+cy}{p}\right) & -cy \le x < (p-c)y \\ P(D \ge y) & x = (p-c)y \end{cases}$$

Normally distributed demand: μ = 100, σ = 30, p=10, c=4 or c=8, g=0

Probability of selling all goods:

x = (p – c) y*

	c = 4 y* = 107.6004 x = 645.6024	c = 8 y* = 74.75136 x = 149.5027
x = (p – c) y*	40%	80%

Loss probability

Expression

$$P(\prod \leq 0) = P\left(D \leq \frac{c}{p} y\right)$$

Normally distributed demand: μ = 100, σ = 30, p=10, c=4 or c=8, g=0

c = 4 → y* = 107.6004:

$\boldsymbol{y}^*\colon\ P(\Pi \leq 0) = P\left(D \leq \frac{4}{10} * 107.6004\right) \approx \mathbf{2.88}\%$

$\mathbf{2}\boldsymbol{y}^*\colon\ P(\Pi \leq 0) = P\left(D \leq \frac{4}{10} * 215.2008\right) \approx \mathbf{32.13}\%$

c = 8 → y* = 74.75136:

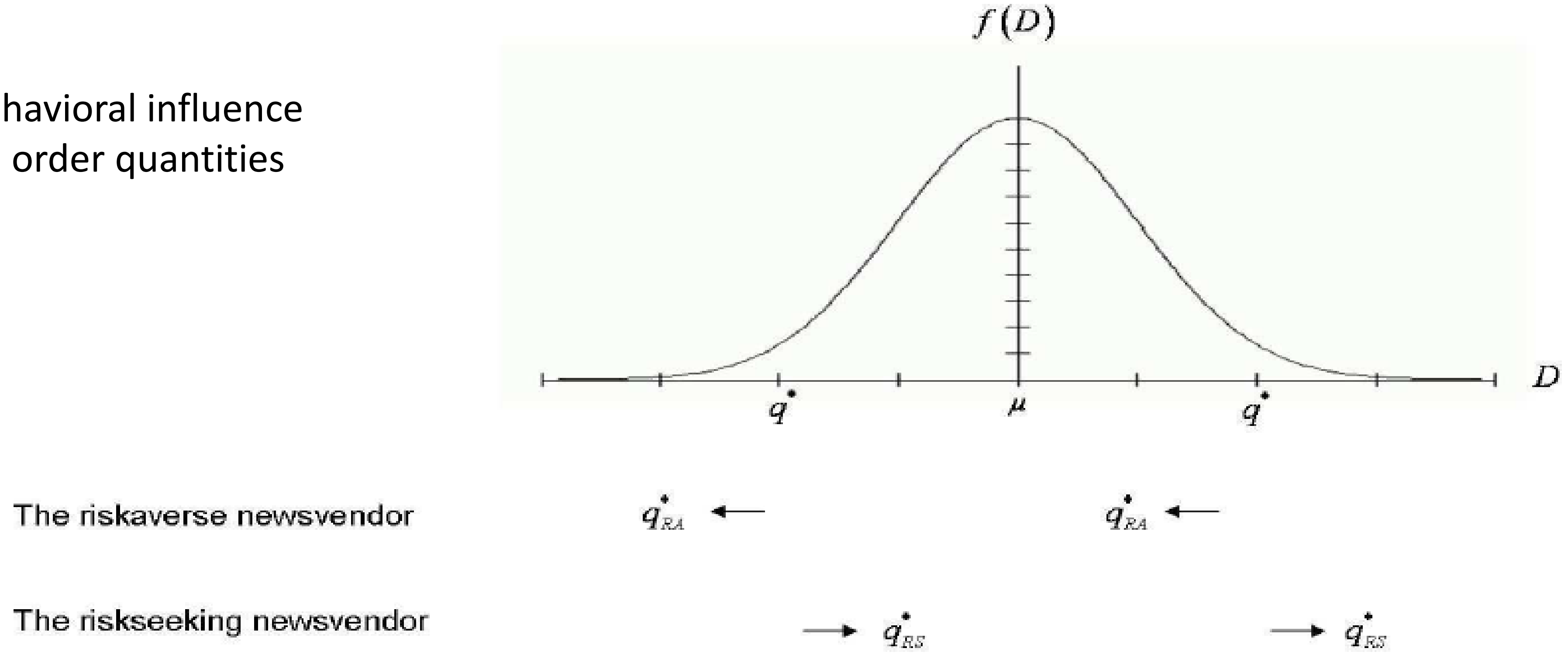
$\boldsymbol{y}^*\colon\ P(\Pi \leq 0) = P\left(D \leq \frac{8}{10} * 74.75136\right) \approx \mathbf{9.01}\%$

$\mathbf{2}\boldsymbol{y}^*\colon\ P(\Pi \leq 0) = P\left(D \leq \frac{8}{10} * 149.5027\right) \approx \mathbf{74.33}\%$

	c = 4	c = 8
$y_0 = y^*$	2.88%	9.01%
$y_0 = 2y^*$	32.13%	74.33%

Experimental results

Behavioral influence
on order quantities



Found effect in experiments: „Anchoring“ on mean demand

Schweiter, M.E, Cachon, G.P. (2000), Decision Bias in the Newsvendor Problem with a Known Demand Distribution: Experimental Evidence, Management Science 46(3):404-420

Other extensions

- **Price-setting newsvendor**
- **Selling to the newsvendor**
- **Dual sourcing newsvendor**
- **Multi-product newsvendor**
- **Newsvendor games**
- **Newsvendor inventory pooling**
- **The remanufacturing newsvendor**

See e.g. Choi, T. M. (Ed.). (2012). *Handbook of Newsvendor problems: Models, extensions and applications* (Vol. 176). Springer Science & Business Media.