Inventory Management

Summer 2025

- Assignment 1 -

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Exercise 1:

A supermarket faces **weekly demand for apples of 600 kg**. The setup cost for placing an **order to replenish inventory is 26 €.** The order is delivered by a supplier who charges the supermarket 0.20 € per kg for the cost of transportation. This transportation cost increases **the cost of apples to 1.35 € per kg**. The apples lose their freshness while stored at the supermarket. To account for this, the supermarket charges **an annual holding cost of 2.2 € per kg.**

a) Determine how often an order should be placed by the supermarket and the optimal order quantity.

b) How are costs affected if an order quantity of 500 kg is placed instead?

c) Determine the optimal policy using the power-of-two-approach.

Exercise 1.a) Determine **how often** an order should be placed by the supermarket and **the optimal order quantity**.

Demand (D)	600	kg per week* / 31200 (=600*52) kg per year
Setup cost (A)	26	Euro
Procurement cost (c)	1,35	Euro
Holding cost (h)	2,2	Euro per year* / 0,042307692 (=2,2/52) Euro per week

Economic order quantity Q

$$Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2*31200*26}{2,2}} = \sqrt{\frac{2*600*26}{0,042307}} = 858,75$$

Order frequency

$$T^* = \frac{Q^*}{d} = \frac{858,75}{600} = 1,43 \text{ (Every 1,4 weeks)}$$

Exercise 1.b) How are **costs** affected if **an order quantity of 500 kg** is placed instead?

Demand (D)	600	kg per week* / 31200 (=600*52) kg per year
Setup cost (A)	26	Euro
Procurement cost (c)	1,35	Euro
Holding cost (h)	2,2	Euro per year* / 0,042307692 (=2,2/52) Euro per week

→**Sensitivity Analysis of EOQ!**

Weekly Cost

$$TRC(Q^*) = \sqrt{2dAh} = \sqrt{2 * 600 * 26 * 0{,}0423} = 36{,}33$$

$$TRC(Q') = \frac{d}{Q}A + \frac{h}{2}Q \rightarrow TRC(500) = \frac{600}{500} * 26 + \frac{0{,}0423}{2} * 500 = 41{,}78$$

$$\therefore \textit{Percentage cost penalty} = \frac{\mathrm{TRC}(Q') - TRC(Q^*)}{TRC(Q^*)} = \frac{41{,}78 - 36{,}33}{36{,}33} \approx 0{,}15\ (15\%)$$

Exercise 1.b) How are **costs** affected if **an order quantity of 500 kg** is placed instead?

Demand (D)	600	kg per week* / 31200 (=600*52) kg per year
Setup cost (A)	26	Euro
Procurement cost (c)	1,35	Euro
Holding cost (h)	2,2	Euro per year* / 0,042307692 (=2,2/52) Euro per week

→**Sensitivity Analysis of EOQ!**

Yearly Cost

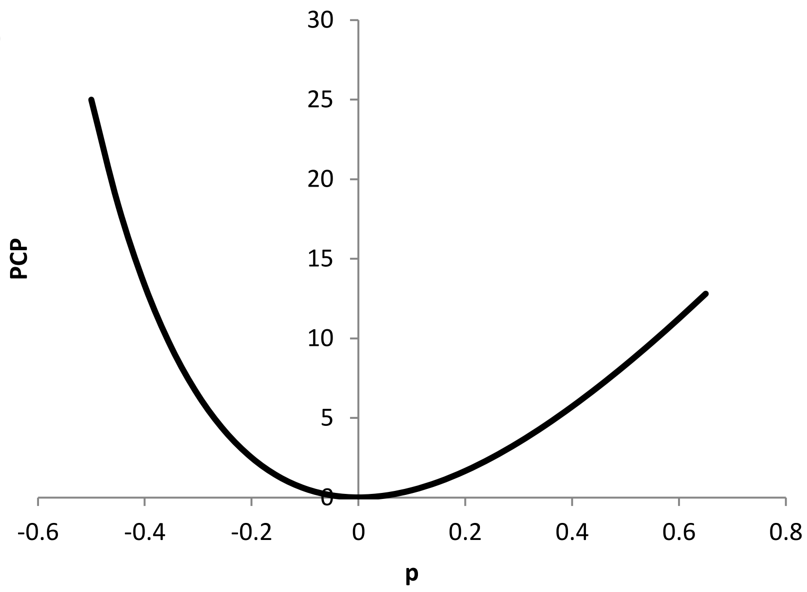
$$TRC(Q^*) = \sqrt{2dAh} = \sqrt{2 * 31200 * 26 * 2{,}2} = 1889{,}25$$

$$TRC(Q') = \frac{d}{Q}A + \frac{h}{2}Q \rightarrow TRC(500) = \frac{31200}{500} * 26 + \frac{2{,}2}{2} * 500 = 2172{,}40$$

$$\therefore \, Percentage\ cost\ penalty = \frac{\mathrm{TRC}(Q') - TRC(Q^*)}{TRC(Q^*)} = \frac{2172{,}40 \ - 1889{,}25}{1889{,}25} \approx 0{,}15\ (15\%)$$

Exercise 1.b) How are **costs** affected if **an order quantity of 500 kg** is placed instead?

- Order quantity Q‘ that deviates from EOQ: $Q' = (1+\mathrm{p})Q^*$
- Percentage cost penalty: $\mathrm{PCP} = \frac{TRC(Q')-TRC(Q^*)}{TRC(Q^*)} * 100 = 50(\frac{p^2}{1+p})$



$$\mathrm{p} = \frac{Q'}{Q^*} - 1 = \frac{500}{858{,}75} - 1 = -0{,}41776$$

$$\therefore \mathrm{PCP} = 50\left(\frac{p^2}{1+p}\right) = 50\left(\frac{-0{,}41776^2}{1+(-0{,}41776)}\right) = 14{,}99\% \approx 15\%$$

$$C(Q^*) = \sqrt{2dAh} + cd = \sqrt{2 * 31200 * 26 * 2{,}2} + 1{,}35 * 31200 = 44009{,}25$$

$$C(Q') = \frac{d}{Q}A + \frac{h}{2}Q + cd = \frac{31200}{500} * 26 + \frac{2{,}2}{2} * 500 + 1{,}35 * 31200 = 44292{,}40$$

$$\therefore \text{Total Cost Comparison: } \frac{44292{,}40 - 44009{,}25}{44009{,}25} \approx 0{,}0064\ (0{,}64\%)$$

Exercise 1.c) Determine the optimal policy using **the power-of-two-approach.**

Demand (D)	600	kg per week
Setup cost (A)	26	Euro
Procurement cost (c)	1,35	Euro
Holding cost (h)	2,2	Euro per year* / 0,0423 per week

$$\min_{T\geq 0} \quad C(T) = \frac{A}{T} + \frac{1}{2}hdT,$$

$$s.t. \quad T = 2^l T_L, \quad l = \{0,1,2,3,\ldots\}$$

$$C(2^{l^*}T_L) \leq C(2^{l^*+1}T_L)$$

$$C(2^{l^*}T_L) \leq \max\left\{C(\sqrt{2}T^*), C\left(\frac{1}{\sqrt{2}}T^*\right)\right\}$$

l	T	Cost (weekly)	Cost (yearly)
0	1	$\frac{26}{1} + \frac{1*600*0{,}0423}{2} = 38{,}69$	38,69*52 ≈ 2012
1	**2**	$\frac{26}{2} + \frac{2*600*0{,}0423}{2} = \mathbf{38{,}38}$	**38,38*52 ≈ 1996**
2	4	57,27	2978
3	8	104,79	5449

$$C(\sqrt{2}T^*) = \frac{26}{1{,}43*\sqrt{2}} + \frac{1{,}43*\sqrt{2}*600*0{,}0423}{2} = 38{,}52$$

$$\rightarrow 38{,}52 * 52 \approx 2003$$

$$C\left(\frac{1}{\sqrt{2}}T^*\right) = \frac{26}{1{,}43*\frac{1}{\sqrt{2}}} + \frac{1{,}43*\frac{1}{\sqrt{2}}*600*0{,}0423}{2} = 38{,}54$$

$$\rightarrow 38{,}52 * 52 \approx 2004$$

$$C(2^{l^*}T_l) = 1996 \leq max\left\{C(\sqrt{2}T^*), C\left(\frac{1}{\sqrt{2}}T^*\right)\right\} = 2004$$

Exercise 2:

A mining company routinely replaces a specific part on a certain type of equipment. **The usage rate is forty per week**, and there is no significant seasonality. The supplier of the part offers the following all-units discount structure. The fixed cost of **a replenishment is estimated to be 25 €**, and **a carrying charge of 0.26 €/€/yr** is used by the company.

Range of Q	Purchasing Cost
0<Q<300	10Q
300≤Q	9,7Q

a) What replenishment size should be used?

b) Now, determine replenishment size by assuming that the supplier of the part offers the following incremental discount structure.

Range of Q	Purchasing Cost
0<Q<300	10Q
300≤Q	10*300 + 9.7(Q-300)

Exercise 2.a) What **replenishment size** should be used?

Demand (D)	40	units per week* / 2080 (=40*52) per year
Setup cost (A)	25	Euro
Procurement cost (C1)	10	Euro (0<Q<300)
Procurement cost (C2)	9,7	Euro (300≤Q)
Carrying Charge/ Interest rate (I)	0,26	Euro per year* / 0,005 (=0,26/52) per week

Step 1) Q^* with C2: $Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2*40*25}{9{,}7*0{,}005}} = 203{,}07$

Q = 203 is NOT under the range of 300≤Q

Step 2) Q^* with C1: $Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2*40*25}{10*0{,}005}} = 200$

Q = 200 is under the range of 0<Q<300

Step 3) Total Cost Comparison with step2 and Q=300

$$C(Q) = \frac{d}{Q}A + \frac{h}{2}Q + cd$$

$$C(200) = \frac{d}{Q}A + \frac{h}{2}Q + cd = \frac{40}{200} * 25 + \frac{10*0{,}005}{2} * 200 + 10 * 40$$

$$= \; 410\ per\ week\ (410 * 52 = 21320\ per\ year)$$

$$C(\mathbf{300}) = \frac{d}{Q}A + \frac{h}{2}Q + cd = \frac{40}{300} * 25 + \frac{9{,}7*0{,}005}{2} * 300 + 9{,}7 * 40$$

$$= 398{,}6\ per\ week\ (398{,}6 * 52 = \mathbf{20728}\ per\ year)$$

Exercise 2.b) Now, determine replenishment size by assuming that the supplier of the part offers the following **incremental discount structure.**

Procurement cost (C1)	10	Euro (0<Q<300)
Procurement cost (C2)	10*300 + 9.7(Q-300)	Euro (300≤Q)
q1	0	
q2	300	
R1	0	
R2	3000	$= C_1(q_2 - q_1)$

Step 1) Q^* with C1: $Q_1{}^* = \sqrt{\frac{2(R_1 - C_1 q_1 + A)d}{IC_1}} = \sqrt{\frac{2(0-10*0+25)40}{0,005*10}} = 200$

Q = 200 is under the range of 0<Q<300

Step 2) Q^* with C2: $Q_2{}^* = \sqrt{\frac{2(R_2 - C_2 q_2 + A)d}{IC_2}} = \sqrt{\frac{2(3000-9,7*300+25)40}{0,005*9,7}} = 435,53$

Q = 435,53 is under the range of 300≤Q

Step 3) Total Cost Comparison $C(Q) = \frac{d}{Q}A + \frac{h}{2}Q + \boldsymbol{cd}$

$C(200) = 410\ per\ week\ (410 * 52 = 21320\ per\ year)$

$C(\mathbf{435,53}) = \frac{d}{Q}A + \frac{h}{2}Q + cd = \frac{40}{435,53} * 25 + \frac{9,907*0,005}{2} * 435,53 + 9,907 * 40$

$= 409,36\ per\ week\ (409,36 * 52 = \mathbf{21286,87}\ per\ year)$

$$c' = \frac{10 * 300 + 9,7 * (435,53 - 300)}{435,53} = 9,907$$

Exercise 3:

A company manufactures sunshades in fashionable colors. Since sunshades are usually sold in summer, retailers have to submit their order decision by mid May in order to have the goods delivered in June. Sales from previous years have been recorded by the Point-of-sale scanner system so that you know that **demand is normally distributed with mean 2000 and standard deviation of 250**. The sunshades are **produced for 3 € and sold at a price of 6 €** to the customer. Any sunshades that cannot be sold until August 31st will be shipped to Australia for the Australian summer season. The sunshades are **sold at 3 € in Australia** in order to make sure that all leftover inventories can be sold. However, shipping costs incur for the goods being transported to the Australian market which are **0.50 € per piece**. It can be assumed that all sunshades shipped to Australia will also be sold.

a) Which order quantity maximizes the company's expected profit?

b) Determine the expected number of sunshades that will be shipped to Australia given your optimal order quantity in a) What is the expected profit?

Exercise 3.a) Which **order quantity** maximizes the company's expected profit?

Selling price (p)	6
Procurement price (c)	3
Salvage value (g)	2,5
Mean Demand	2000
st.dev Demand	250

Critical ratio	$= \frac{C_u}{C_u + C_0} = \frac{p-c}{p-g} = \frac{6-3}{6-2{,}5}$	0,857142857
z	$F(z) = 0{,}857142$	1,067570524
Optimal order quantity (y*)	$= \mu + z * \sigma$ $= 2000 + 1{,}06757 * 250$	2266,89

Functions of the standard normal distribution

k	ϕ(k)	Φ(k)	G(k)	k	ϕ(k)	Φ(k)	G(k)
1.0000	0.24197	0.84134	0.08332	1.5000	0.12952	0.93319	0.02931
1.0100	0.23955	0.84375	0.08174	1.5100	0.12758	0.93448	0.02865
1.0200	0.23713	0.84614	0.08019	1.5200	0.12566	0.93574	0.02800
1.0300	0.23471	0.84849	0.07866	1.5300	0.12376	0.93699	0.02736
1.0400	0.23230	0.85083	0.07716	1.5400	0.12188	0.93822	0.02674
1.0500	0.22988	0.85314	0.07568	1.5500	0.12001	0.93943	0.02612
1.0600	0.22747	0.85543	0.07422	1.5600	0.11816	0.94062	0.02552
1.0700	0.22506	0.85769	0.07279	1.5700	0.11632	0.94179	0.02494
1.0800	0.22265	0.85993	0.07138	1.5800	0.11450	0.94295	0.02436
1.0900	0.22025	0.86214	0.06999	1.5900	0.11270	0.94408	0.02380
1.1000	0.21785	0.86433	0.06862	1.6000	0.11092	0.94520	0.02324
1.1100	0.21546	0.86650	0.06727	1.6100	0.10915	0.94630	0.02270
1.1200	0.21307	0.86864	0.06595	1.6200	0.10741	0.94738	0.02217
1.1300	0.21069	0.87076	0.06465	1.6300	0.10567	0.94845	0.02165
1.1400	0.20831	0.87286	0.06336	1.6400	0.10396	0.94950	0.02114
1.1500	0.20594	0.87493	0.06210	1.6500	0.10226	0.95053	0.02064
1.1600	0.20357	0.87698	0.06086	1.6600	0.10059	0.95154	0.02015
1.1700	0.20121	0.87900	0.05964	1.6700	0.09893	0.95254	0.01967

Exercise 3.b) Determine **the expected number of sunshades that will be shipped to Australia** given your optimal order quantity in a). What is **the expected profit**?

$$\text{Expected Profit} = -\mathrm{c} * \mathrm{y} + \mathrm{p} * \mathrm{ES(y)} + gELO(\mathrm{y})$$

$$\text{Expected Sale} = \mu - ELS(y)$$
$$\text{Expected Lost Sales} = \sigma * G(z)$$
$$\text{Expected Leftover} = \mathrm{y} - \mathrm{ES(y)}$$

G(z)	=NORM.DIST(z;0;1;0) - z*(1-NORM.DIST(z;0;1;1))	0,0728
ELS	$= \sigma * G(z) = 250 * 0{,}0728$	18,2
ES	$= \mu - ELS(y) = 2000 - 18{,}2$	1981,8
ELO	$= \mathrm{y} - \mathrm{ES(y)} = 2266.89 - 1981{,}8$	285,09
EP	$= -\mathrm{c} * \mathrm{y} + \mathrm{p} * \mathrm{ES(y)} + gELO(\mathrm{y})$ $= -3 * 2266{,}89 + 6 * 1981{,}8 + 2{,}5 * 285{,}09$	5802,86

Functions of the standard normal distribution

k	ϕ(k)	Φ(k)	G(k)	k	ϕ(k)	Φ(k)	G(k)
1.0000	0.24197	0.84134	0.08332	1.5000	0.12952	0.93319	0.02931
1.0100	0.23955	0.84375	0.08174	1.5100	0.12758	0.93448	0.02865
1.0200	0.23713	0.84614	0.08019	1.5200	0.12566	0.93574	0.02800
1.0300	0.23471	0.84849	0.07866	1.5300	0.12376	0.93699	0.02736
1.0400	0.23230	0.85083	0.07716	1.5400	0.12188	0.93822	0.02674
1.0500	0.22988	0.85314	0.07568	1.5500	0.12001	0.93943	0.02612
1.0600	0.22747	0.85543	0.07422	1.5600	0.11816	0.94062	0.02552
1.0700	0.22506	0.85769	0.07279	1.5700	0.11632	0.94179	0.02494
1.0800	0.22265	0.85993	0.07138	1.5800	0.11450	0.94295	0.02436
1.0900	0.22025	0.86214	0.06999	1.5900	0.11270	0.94408	0.02380
1.1000	0.21785	0.86433	0.06862	1.6000	0.11092	0.94520	0.02324
1.1100	0.21546	0.86650	0.06727	1.6100	0.10915	0.94630	0.02270
1.1200	0.21307	0.86864	0.06595	1.6200	0.10741	0.94738	0.02217
1.1300	0.21069	0.87076	0.06465	1.6300	0.10567	0.94845	0.02165
1.1400	0.20831	0.87286	0.06336	1.6400	0.10396	0.94950	0.02114
1.1500	0.20594	0.87493	0.06210	1.6500	0.10226	0.95053	0.02064
1.1600	0.20357	0.87698	0.06086	1.6600	0.10059	0.95154	0.02015
1.1700	0.20121	0.87900	0.05964	1.6700	0.09893	0.95254	0.01967

Exercise 4:

Krusty owns a burger restaurant. His burgers are made of a secret ingredient. Krusty is very busy with his store, so he decided to place only a single order for May. **The demand is uniformly distributed on the interval [4000; 8000].**

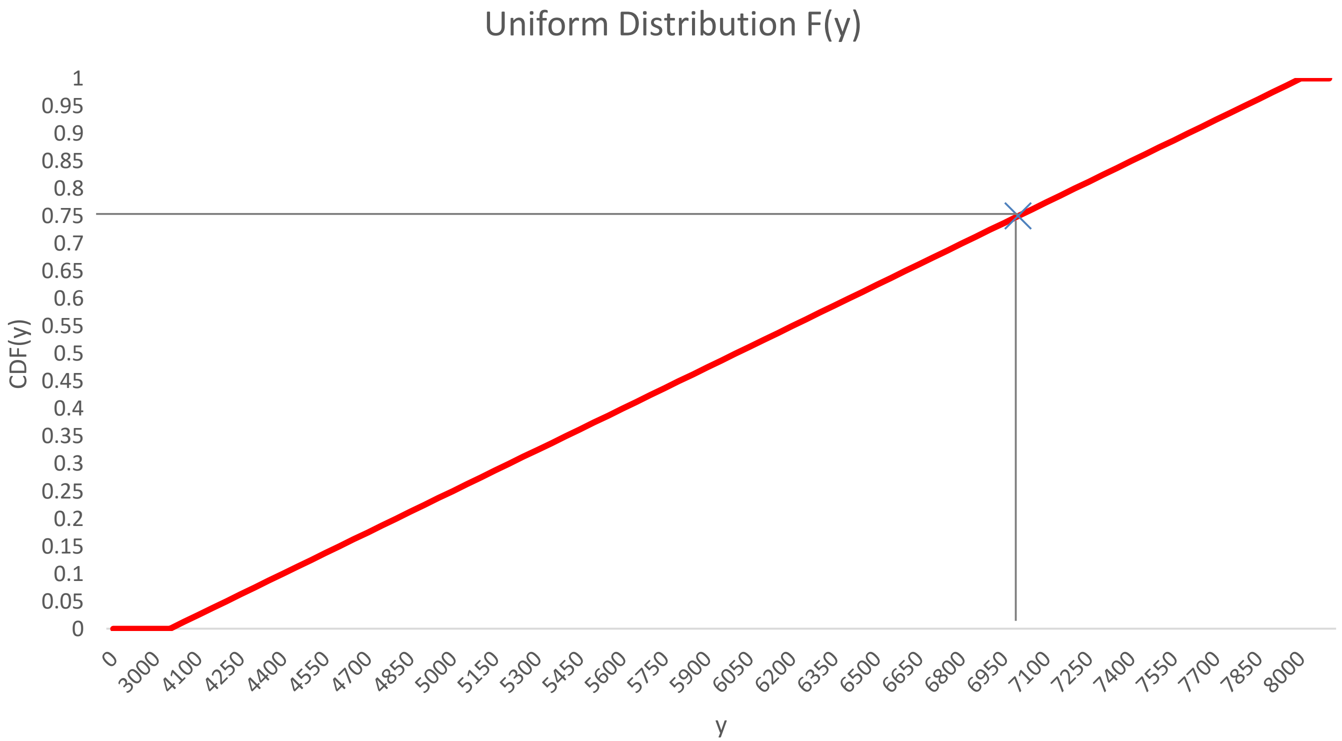
a) What is Krusty's optimal order quantity given a critical fractile of 0.75?

b) How does the result in a) change if demand is Poisson with λ=6000?

c) Now assume demand is Gamma-distributed with mean 6000 and a standard deviation being equal to the one of the uniform distribution used in a). What is the optimal order quantity?

Exercise 4.a) What is Krusty's **optimal order quantity** given a critical fractile of 0.75?

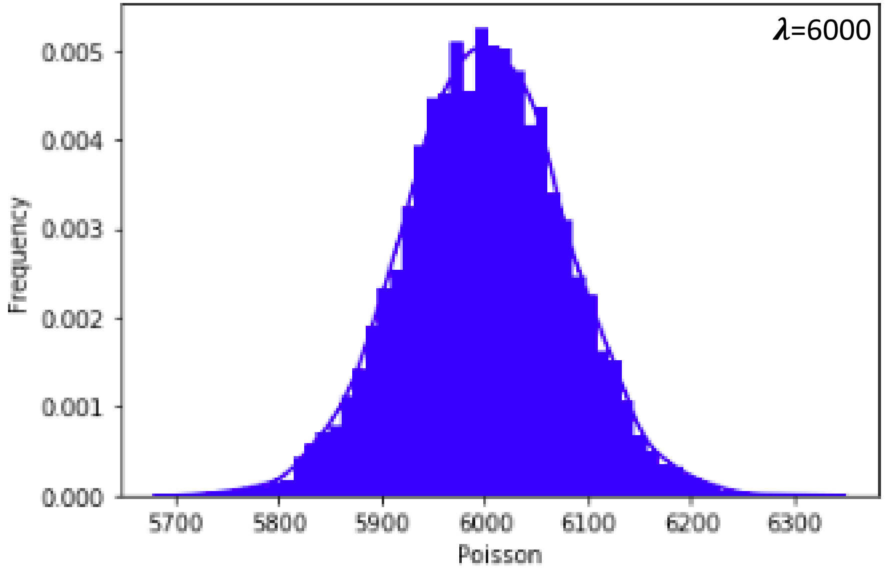
Critical ratio	-	0,75
Optimal quantity at least	$= 4000 + 0{,}75 * 4000\ (\text{Uniform Dist.})$	7000



Exercise 4.b) How does the result in a) change if demand is **Poisson with λ=6000**?

Critical ratio (obtained), P{D<=y*}	0,75
Optimal quantity at least	6052

X	P(D<=y)
6000	0,503
6010	0,555
6020	0,605
6040	0,700
6050	0,743
6051	**0,747**
6052	**0,751**

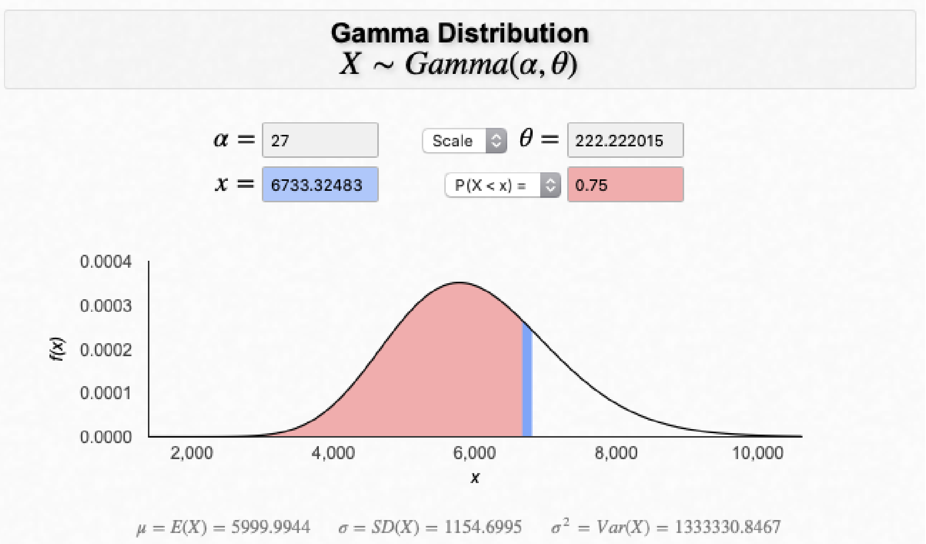


Exercise 4.c) Now assume demand is **Gamma-distributed** with mean 6000 and a standard deviation being equal to the one of the uniform distribution used in a). What is **the optimal order quantity**?

- Mean= 6000
- Standard Deviation = $\sqrt{\frac{1}{12}(8000-4000)^2} = 1154{,}7$

$$\therefore \alpha = \frac{\mu^2}{\sigma^2} = \frac{6000^2}{1154{,}7^2} = 27$$

$$\beta = \frac{\sigma^2}{\mu} = \frac{1154{,}7^2}{6000} = 222{,}222$$



Critical ratio P{D<=y*}	0,75
Optimal quantity at least	=GAMMA.INV(CF; α; β) **= 6733,33**

Exercise 5:

Anton is the cook of a Viennese restaurant that is famous for its delicious cakes. Every Monday morning, Anton prepares the chocolate icing and the cakes according to a special recipe. The cakes need to cool down before the chocolate icing can be applied. Anton estimates that **the ingredients cost 20 € for 1 kg of chocolate** cake. A cake of 1 kg can be split up into 12 pieces. **Each piece of cake is sold for 3 €.** Any cakes leftover at the end of the week have to be **discarded at no cost.** Since Anton is too busy during the week to prepare new cakes, he wants to make a careful decision on Monday to ensure that his guests are satisfied. From previous weeks, he knows that demand for cakes is **normally distributed with mean 20 and standard deviation 8.**

a) If Anton is told by his boss to satisfy at least 95 % of all incoming demand, how many cakes should he prepare?

b) How many cakes should he prepare if he has to ensure that cakes are available at least on 95 % of all days?

c) Since Anton is also responsible for the recipe, he receives a bonus for every week when more than 25 cakes are sold. His boss considers this an indicator that customers like his cakes more than those of the competitors'. What is the probability that he receives a bonus with his recipe given the above demand estimates? If the bonus is 100 € and the restaurant is open 48 weeks per year, how much money will Anton probably make on his bonus in a year?

Exercise 5.a) If Anton is told by his boss to satisfy **at least 95 % of all incoming demand**, how many cakes should he prepare?

mean	20
st.dev	8

$$\beta = \frac{ES(y)}{\mu} = 1 - \frac{ELS(y)}{\mu} = 1 - \frac{\sigma G(z)}{\mu}$$

G(z)	$\beta = 1 - \sigma * \frac{G(z)}{M}$ $0{,}95 = 1 - 8 * \frac{G(z)}{20}$	0,125
z	$G(z) = 0{,}125$	0,780
Optimal Quantity	$= \mu + z * \sigma$ $= 20 + 0{,}78 * 8$	26,24

k	ϕ(k)	Φ(k)	G(k)
0.7300	0.30563	0.76730	0.13576
0.7400	0.30339	0.77035	0.13345
0.7500	0.30114	0.77337	0.13117
0.7600	0.29887	0.77637	0.12892
0.7700	0.29659	0.77935	0.12669
0.7800	0.29431	0.78230	0.12450
0.7900	0.29200	0.78524	0.12234
0.8000	0.28969	0.78814	0.12021

Exercise 5.b) How many cakes should he prepare if he has to ensure that cakes are available **at least on 95 % of all days?**

Critical ratio (alpha)		0,95
z	F(z) = 0,95	1,645
Optimal quantity	$= \mu + z * \sigma$ $= 20 + 1{,}645 * 8$	33,16

Observation	Inventory Level	Demand	Service Level (α)	Fill Rate (β)
1	24	17	1	0
2	28	25	1	0
3	18	23	0	5
4	24	25	0	1
5	19	23	0	4
6	23	17	1	0
7	23	18	1	0
8	24	21	1	0
9	20	25	0	5
10	25	22	1	0
		216	60,00%	93,06%

k	ϕ(k)	Φ(k)	G(k)
1.5000	0.12952	0.93319	0.02931
1.5100	0.12758	0.93448	0.02865
1.5200	0.12566	0.93574	0.02800
1.5300	0.12376	0.93699	0.02736
1.5400	0.12188	0.93822	0.02674
1.5500	0.12001	0.93943	0.02612
1.5600	0.11816	0.94062	0.02552
1.5700	0.11632	0.94179	0.02494
1.5800	0.11450	0.94295	0.02436
1.5900	0.11270	0.94408	0.02380
1.6000	0.11092	0.94520	0.02324
1.6100	0.10915	0.94630	0.02270
1.6200	0.10741	0.94738	0.02217
1.6300	0.10567	0.94845	0.02165
1.6400	0.10396	0.94950	0.02114
1.6500	0.10226	0.95053	0.02064
1.6600	0.10059	0.95154	0.02015
1.6700	0.09893	0.95254	0.01967

Exercise 5.c) Since Anton is also responsible for the recipe, he receives a bonus for every week when **more than 25 cakes are sold**. His boss considers this an indicator that customers like his cakes more than those of the competitors'. What is the probability that he receives a bonus with his recipe given the above demand estimates? If the bonus is **100 € and the restaurant is open 48 weeks per year**, how much money will Anton probably make on his bonus in a year?

Target Sales Number		25
z	$= \frac{x - \mu}{\sigma} = \frac{25 - 20}{8}$	0,625
P{D<25}	$F(0{,}625)$	0,734014
P{D>25}	$1 - F(0{,}625)$	0,265986
Bonus	$= P(D > 25) * bonus * working\ weeks$ $= (1 - F(0{,}625)) * 100 * 48$	1276,8

k	$\phi(k)$	$\Phi(k)$	$G(k)$
0.5000	0.35207	0.69146	0.19780
0.5100	0.35029	0.69497	0.19473
0.5200	0.34849	0.69847	0.19170
0.5300	0.34667	0.70194	0.18870
0.5400	0.34482	0.70540	0.18573
0.5500	0.34294	0.70884	0.18281
0.5600	0.34105	0.71226	0.17991
0.5700	0.33912	0.71566	0.17705
0.5800	0.33718	0.71904	0.17422
0.5900	0.33521	0.72240	0.17143
0.6000	0.33322	0.72575	0.16867
0.6100	0.33121	0.72907	0.16595
0.6200	0.32918	0.73237	0.16325
0.6300	0.32713	0.73565	0.16059

Exercise 6:

The famous furniture producer IDEA introduces a new couch named “SITZMAL”. The new couch is produced in a large factory together with hundreds of other products. Every time IDEA has to set up production of the new couch, many machines have to be **cleaned and maintained causing costs of €10.000**. As the couches are stored in efficiently packed packages that do not take up much of the available inventory space, IDEA **estimates holding costs of 20%** of a couch’s value per unit and period. The value of one couch is listed as **€120**. The demand forecast for SITSGUD for the next 10 months is given in the table below.

Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
550	200	400	110	430	980	400	300	200	650

a) Solve the problem using the least unit cost heuristic.

b) Solve the problem using the Silver-Meal heuristic.

c) Solve the problem using the Wagner-Whitin model.

Exercise 6.a) Solve the problem using **the least unit cost** heuristic.

A	10000	
h	24	per unit and period (=0,20*120)

Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
1	2	3	4	5	6	7	8	9	10
550	200	400	110	430	980	400	300	200	650

Unit Cost Calculation: $C_{tz} = \frac{A + h * \sum_{\mathcal{T}=t}^{z} (\mathcal{T} - t) * d_{\mathcal{T}}}{\sum_{\mathcal{T}=t}^{z} d_{\mathcal{T}}}$

T [1;1] = (10000 + 24*0) / 550 = 18,18
T [1;2] = (10000 + 24*200) / (550+200) = 19,73 / Stop

T [2;2] = (10000 + 24*0) / 200 = 50,00
T [2;3] = (10000 + 24*400) / (200+400) = 32,66
T [2;4] = (10000 + 24*400 + 2*24*110) / (200 + 400 + 110) = 35,04 / Stop
…

Exercise 6.a) Solve the problem using **the least unit cost** heuristic.

		Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
		1	2	3	4	5	6	7	8	9	10
	Demand	550	200	400	110	430	980	400	300	200	650
Jun	1	**18,18**	19,73								
Jul	2		50,00	**32,67**	35,04						
Aug	3										
Sep	4				90,91	**37,63**	44,32				
Oct	5										
Nov	6						**10,20**	14,20			
Dec	7							25,00	**24,57**	29,78	
Jan	8										
Feb	9									50,00	**30,12**
Mar	10										
Setup???		1	1	0	1	0	1	1	0	1	0

Total unit cost

= 6*10000+(400+430+300+650)*24

= (550*18,18) + (600*32,67) + (540*37,63) + (980*10,20) + (700*24,57) + (850*30,12) = **102720**

Exercise 6.b) Solve the problem using **the Silver-Meal** heuristic.

A	10000	
h	24	per unit and period (=0,20*120)

Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
1	2	3	4	5	6	7	8	9	10
550	200	400	110	430	980	400	300	200	650

Period Cost Calculation: $C_{tz} = \frac{A+h*\sum_{\mathcal{T}=t}^{z}(\mathcal{T}-t)*d_{\mathcal{T}}}{z-t+1}$

T [1;1] = (10000 + 24*0) / 1 = 10000
T [1;2] = (10000 + 24*200) / 2 = 7400
T [1;3] = (10000 + 24*200 + 2*24*400) / 3 = 11333,33 /Stop

T [3;3] = (10000 + 24*0) / 1 = 10000
T [3;4] = (10000 + 24*110) / 2 = 6320
T [3;5] = (10000 + 24*110+ 2*24*430) / 3 = 11093,33 / Stop
…

Exercise 6.b) Solve the problem using **the Silver-Meal** heuristic.

		Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
		1	2	3	4	5	6	7	8	9	10
	Demand	550	200	400	110	430	980	400	300	200	650
Jun	1	10000,00	**7400,00**	11333,33							
Jul	2										
Aug	3			10000,00	**6320,00**	11093,33					
Sep	4										
Oct	5					**10000,00**	16760,00				
Nov	6						10000,00	**9800,00**	11333,33		
Dec	7										
Jan	8								10000,00	**7400,00**	15333,33
Feb	9										
Mar	10										**10000,00**
Setup???		1	0	1	0	1	1	0	1	0	1

Total period cost

= 6*10000+(200+110+400+200)*24

= (7400*2) + (6320*2) + 10000 + (9800*2) + (7400*2) + 10000 = **81840**

Exercise 6.c) Solve the problem using **the Wagner-Whitin model for the first 6 periods (Jun-Nov).**

A	10000	
h	24	per unit and period (=0,20*120)

Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar
1	2	3	4	5	6	7	8	9	10
550	200	400	110	430	980	400	300	200	650

F(t): total costs of the best replenishment strategy that satisfies the demand requirements in periods 1, 2, . . . , t.

F(1) = (10000 + 24*0) = 10000

F(2) = min{Opt1, Opt2} = 14800
- Option 1 (Produce in this period) = F(1)+A = 10000 + 10000 = 20000
- Option 2 (Produce in period 1) = A +h*200 = 10000 + 24*200 = 14800

F(3) = min {Opt1, Opt2, Opt3}

Exercise 6.c) Solve the problem using **the Wagner-Whitin model for the first 6 periods (Jun-Nov).**

F(3) = min {Opt1, Opt2, Opt3} = 24800
- Option 1 (Produce in this period) = F(2)+A = 14800 + 10000 = 24800
- Option 2 (Produce in period 2) = F(1) + A +h*400 = 10000 + 10000 + 24*400 = 29600
- Option 3 (Produce in period 1) = A +h*200 + h*2*400 = 10000 + 24*200 + 24*2*400 = 34000

F(4) = min {Opt1, Opt2, Opt3, Opt4} = 27440
- Option 1 (Produce in this period) = F(3)+A = 248000 + 10000 = 34800
- Option 2 (Produce in period 3) = F(2) + A +h*110 = 14800 + 10000 + 24*110 = 27440
- Option 3 (Produce in period 2) = F(1) + A +h*400 + h*2*110 = 10000 + 10000 + 24*400 + 24*2*110 = 34880
- Option 4 (Produce in period 1) = A +h*200 + h*2*400 + h*3*110 = 10000 + 24*200 + 24*2*400 + 24*3*110 = 41920

F(5) = min {Opt1, Opt2, Opt3, Opt4, Opt5} = 37440
- Option 1 (Produce in this period) = F(4)+A = 27440 + 10000 = 37440
- Option 2 (Produce in period 4) = F(3) + A +h*430 = 45120
- Option 3 (Produce in period 3) = F(2) + A +h*110 + h*2*430 = = 48080
- Option 4 (Produce in period 2) = F(1) + A +h*400 + h*2*110 + h*3*430 = 65840
- Option 5 (Produce in period 1) = A +h*200 + h*2*400 + h*3*110 + h*4*430 = 83200

Exercise 6.c) Solve the problem using **the Wagner-Whitin model for the first 6 periods (Jun-Nov).**

F(6) = min {Opt1, Opt2, Opt3, Opt4, Opt5, Opt6} = **47440**

- Option 1 (Produce in this period) = F(5)+A = 37440 + 10000 = 47440
- Option 2 (Produce in period 5) = F(4) + A +h*980 = 60960
- Option 3 (Produce in period 4) = F(3) + A +h*430+ h*2*980 = 92160
- Option 4 (Produce in period 3) = F(2) + A +h*110 + h*2*430 + h*3*980 = 118640
- Option 5 (Produce in period 2) = F(1) +A +h*400 + h*2*110 + h*3*430 + h*4*980 = 159920
- Option 6 (Produce in period 1) = A +h*200 + h*2*400 + h*3*110 + h*4*430 + h*5*980 = 200800

Demand	550	200	400	110	430	980
Period	1	2	3	4	5	6
1	10000	**14800**	34000	41920	83200	200800
2		20000	29600	34880	65840	159920
3			24800	**27440**	48080	118640
4				34800	45120	92160
5					**37440**	60960
6						**47440**
Setup???	1	0	1	0	1	**1**

Total period cost

= 4*10000+(200+110)*24 = **47440**

Exercise 6.c) Solve the problem using **the Wagner-Whitin model for the first 6 periods (Jun-Nov).**

Total planning Horizon for Comparison

Demand	550	200	400	110	430	980	400	300	200	650
Period	1	2	3	4	5	6	7	8	9	10
1	10000	**14800**	34000	41920	83200	200800	258400	308800	347200	487600
2		20000	29600	34880	65840	159920	207920	251120	284720	409520
3			24800	**27440**	48080	118640	157040	193040	221840	331040
4				34800	45120	92160	120960	149760	173760	267360
5					**37440**	60960	80160	101760	120960	198960
6						47440	**57040**	71440	85840	148240
7							57440	64640	74240	121040
8								67040	**71840**	103040
9									74640	90240
10										**81840**
Setup	1	0	1	0	1	1	0	1	0	1

6.a	LUC	102720	
6.b	SM	81840	20%
6.c	WW	81840	20%

Total period cost

= 6*10000+(200+110+400+200)*24 = **81840**

Exercise 7:

A producer of customer gifts sells large amounts of USB sticks. It has to deal with irregular and large orders. **Setting up** the production of USB sticks causes costs of **€180**. Storing the USB sticks per 1000 units and one period causes **10% costs of the value of 1000 USB sticks, which is €35**. The available machines can produce **at most 25.000 USB sticks per week**. The demand forecast for the next 7 weeks is given in the table below. Quantities are given in thousands of USB sticks.

Week	1	2	3	4	5	6	7
Demand	12	12	1	8	15	2	7

a) Model the problem to minimize the total costs. (Solving the problem is not needed)

Exercise 7.a) Model the problem to minimize the total costs.

- **Set**

t (Week) = 1, …, 7

- **Parameters**

A (Setup Cost) = €180
h (Holding Cost) = €3.5 (€ 35*0.1)
C (Capacity Limit) = 25000
M (Sufficiently Large Nr) = 100000
d_t(Demand in Period t) Ref. Table

- **Decision Variables**

y_t: Inventory level at the end of period t
γ_t: Setup decision at the begining of period t
q_t: Replenishment Quantity (Lot-size) at the period t

- **Constraints**

1. Inventory Balance at the end of each period
2. Order quantity occurs if the setup decision is made in that period
3. Zero beginning and end inventory
4. Limited Capacity
5. Domain Restriction

- **Objective Function**

Minimize the total costs

Exercise 7.a) Model the problem to minimize the total costs.

Excel file will be provided!

Obj.

$$\min \sum_{t=1}^{T} \left(A \cdot \gamma_t + h \cdot y_t \right)$$

S.t.

$$y_t = y_{t-1} + q_t - d_t \quad t = 1,2,\ldots,T$$

$$q_t \leq M\gamma_t \qquad t = 1,2,\ldots,T$$

$$y_0 = y_T = 0$$

$$q_t \leq C \quad t = 1,2,\ldots,T$$

$$q_t, y_t \geq 0, \ \gamma_t \in \{0,1\} \quad t = 1,2,\ldots,T$$

Period	Demand	Decision Variables		
		Setup	Quantity	Inventory
1	12	1	25	13
2	12	0	0	1
3	1	0	0	0
4	8	1	8	0
5	15	1	24	9
6	2	0	0	7
7	7	0	0	0
Total Cost		645		

Thank you!