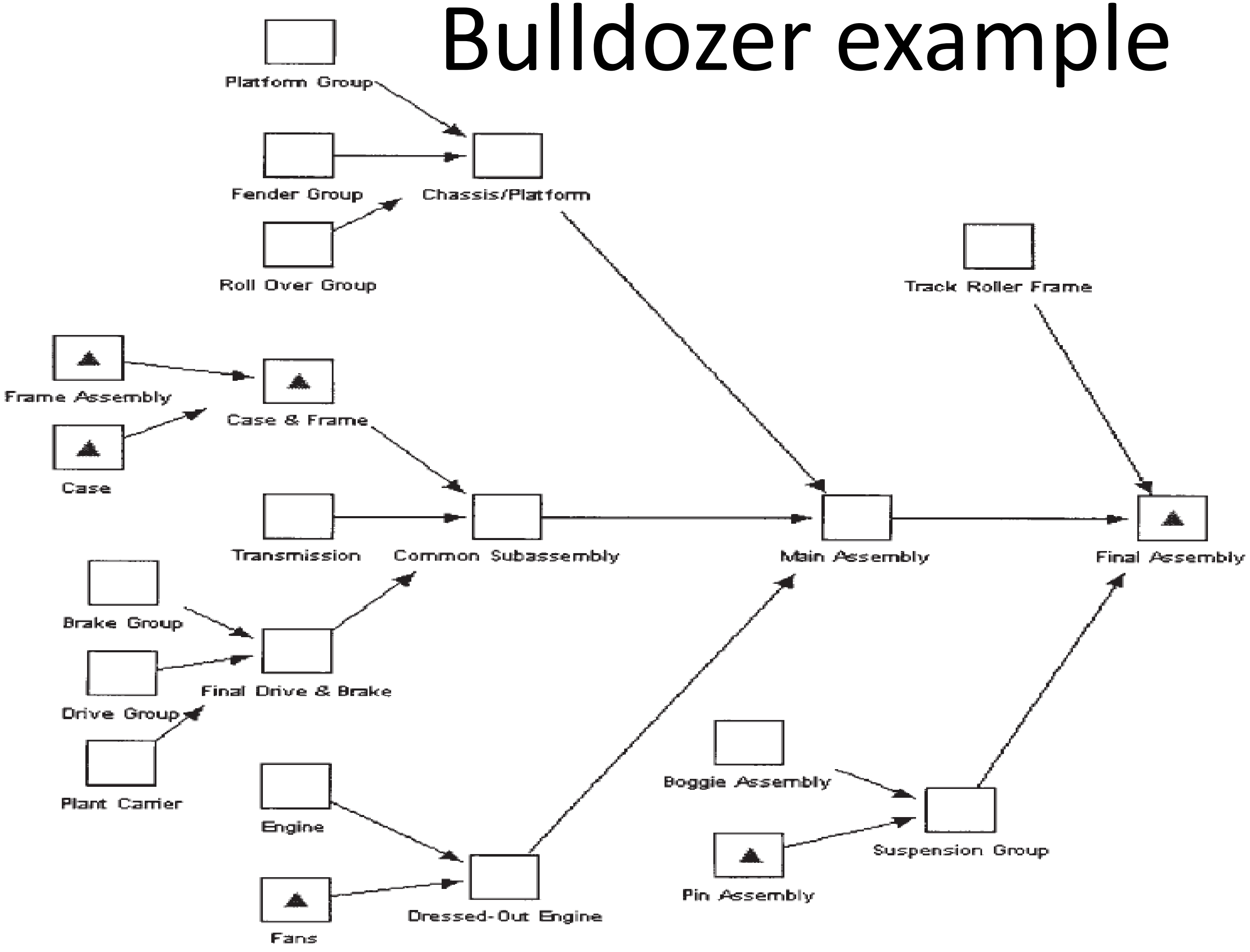
5. Supply chain inventory control

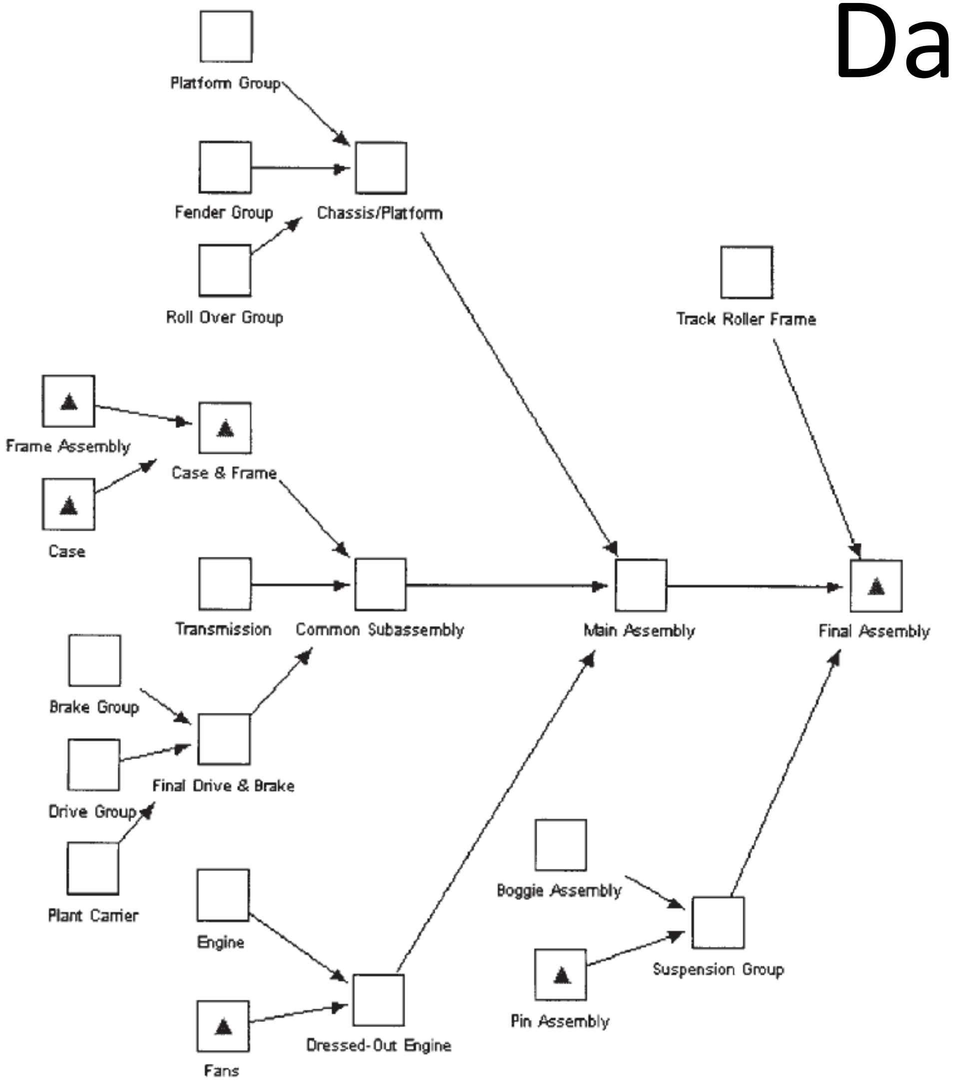
Prof. Dr. Stefan Minner
Logistics & Supply Chain Management
TUM School of Management

Bulldozer example



Source: Graves and Willems (2003)

Data and results



Parameters for Bulldozer Supply Chain

Stage name	Nominal time	Stage cost ($)
Boggie assembly	11	575
Brake group	8	3850
Case	15	2200
Case & frame	16	1500
Chassis/platform	7	4320
Common subassembly	5	8000
Dressed-out engine	10	4100
Drive group	9	1550
Engine	7	4500
Fans	12	650
Fender group	9	900
Final assembly	4	8000
Final drive & brake	6	3680
Frame assembly	19	605
Main assembly	8	12,000
Pin assembly	35	90
Plant carrier	9	155
Platform group	6	725
Roll over group	8	1150
Suspension group	7	3600
Track roller frame	10	3000
Transmission	15	7450

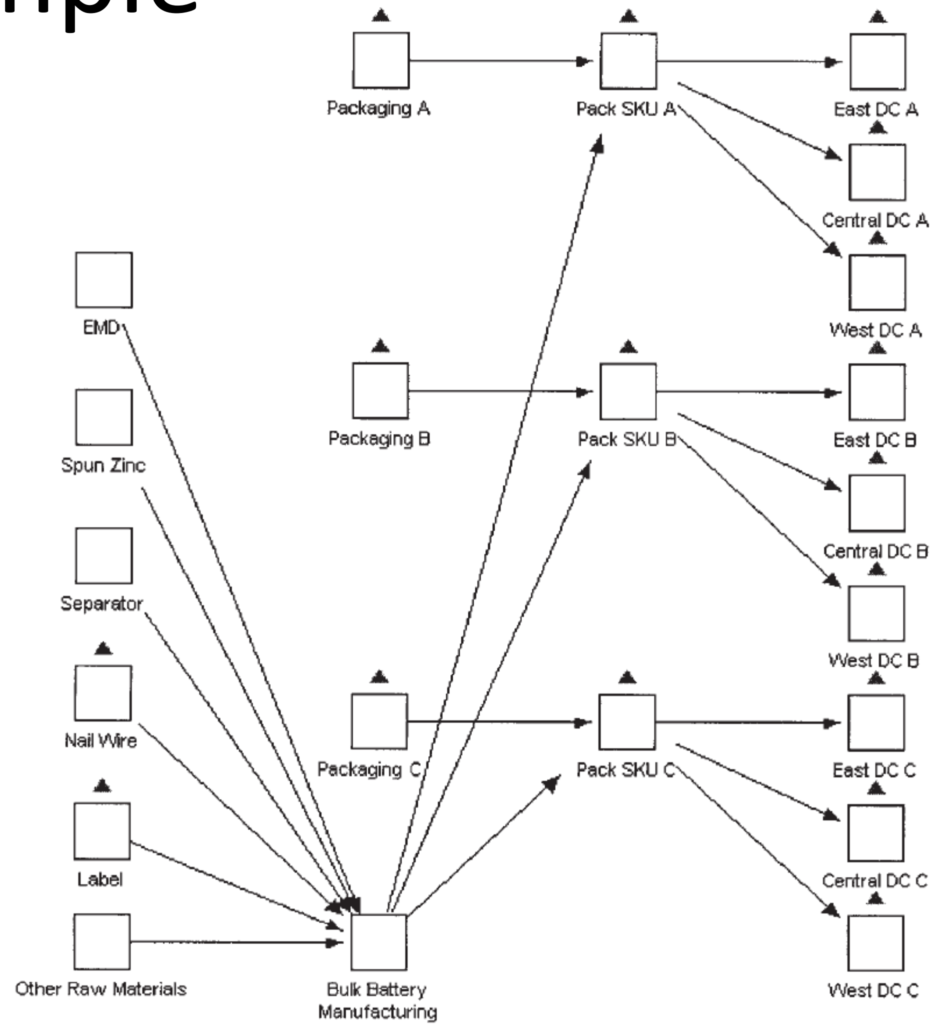
Optimal Service Times and Safety Stock Costs under Guaranteed-Service Model

Stage name	Service time	Stage safety stock cost ($)
Boggie assembly	11	0
Brake group	8	0
Case	0	12,614
Case & frame	15	6373
Chassis/platform	16	0
Common subassembly	20	0
Dressed-out engine	20	0
Drive group	9	0
Engine	7	0
Fans	10	1361
Fender group	9	0
Final assembly	0	607,969
Final drive & brake	15	0
Frame assembly	0	3904
Main assembly	28	0
Pin assembly	21	499
Plant carrier	9	0
Platform group	6	0
Roll over group	8	0
Suspension group	28	0
Track roller frame	10	0
Transmission	15	0

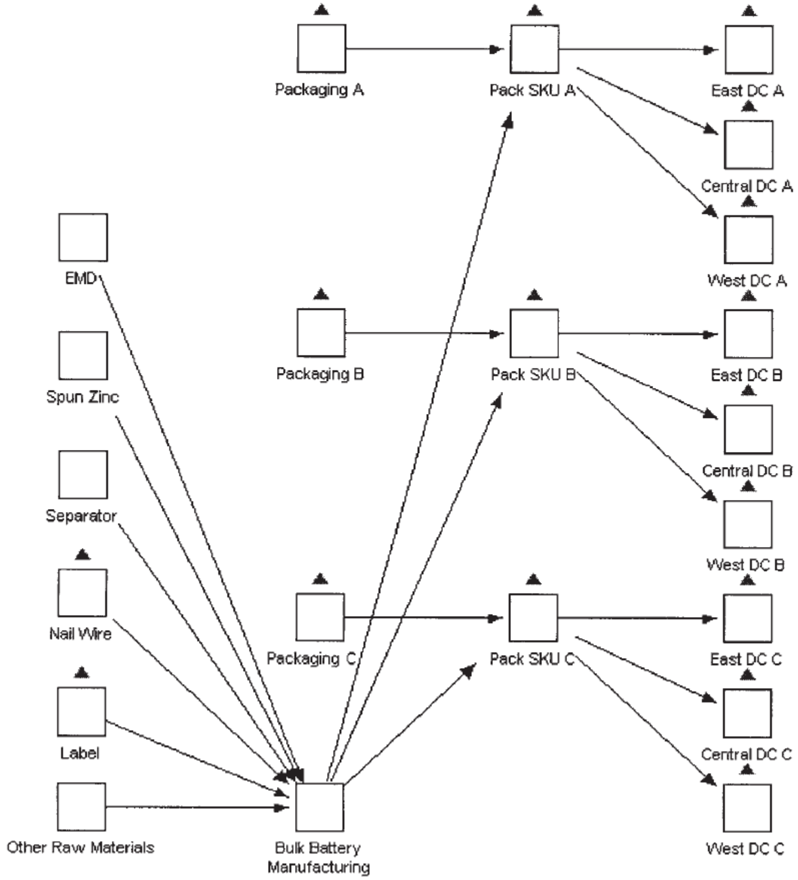
Battery example

Demand Information for Battery Supply Chain

Stage name	Mean demand	Standard deviation of demand
Central DC A	43,422	67,236
Central DC B	16,350	39,552
Central DC C	5536	11,213
East DC A	67,226	109,308
East DC B	15,765	34,079
East DC C	6416	14,125
West DC A	65,638	119,901
West DC B	10,597	23,277
West DC C	3519	6576



Data and results



Parameters for Battery Supply Chain

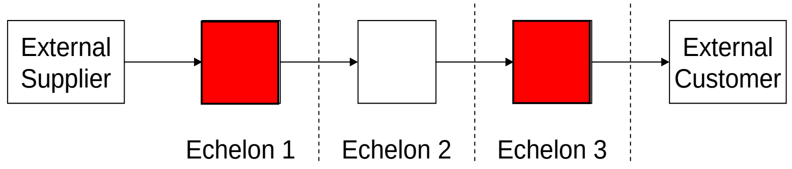
Stage name	Nominal time	Stage cost ($)
Bulk battery manufacturing	5	0.07
Central DC A	6	0.02
Central DC B	6	0.01
Central DC C	4	0.01
East DC A	4	0.00
East DC B	4	0.01
East DC C	4	0.01
EMD	2	0.13
Label	28	0.06
Nail wire	24	0.02
Other raw materials	1	0.24
Pack SKU A	11	0.07
Pack SKU B	11	0.12
Pack SKU C	9	0.24
Packaging A	28	0.16
Packaging B	28	0.24
Packaging C	28	0.36
Separator	2	0.02
Spun zinc	2	0.05
West DC A	5	0.01
West DC B	8	0.03
West DC C	6	0.06

Optimal Service Times using Guaranteed-Service Model

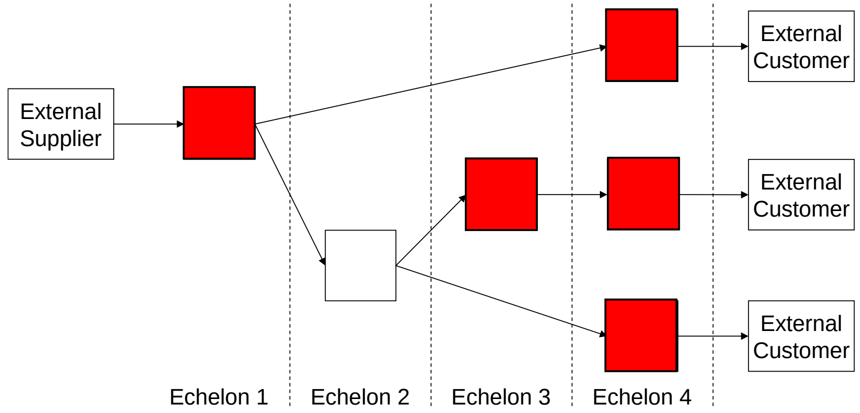
Stage name	Service time	Stage safety stock cost ($)
Bulk battery manufacturing	7	0
Central DC A	0	56,889
Central DC B	0	38,245
Central DC C	0	11,066
East DC A	0	73,716
East DC B	0	26,907
East DC C	0	13,940
EMD	2	0
Label	2	23,361
Nail wire	2	7163
Other raw materials	1	0
Pack SKU A	0	251,253
Pack SKU B	0	94,741
Pack SKU C	0	37,573
Packaging A	7	52,953
Packaging B	7	25,852
Packaging C	7	13,022
Separator	2	0
Spun zinc	2	0
West DC A	0	91,507
West DC B	0	26,531
West DC C	0	8279

5.1 Ordering policies

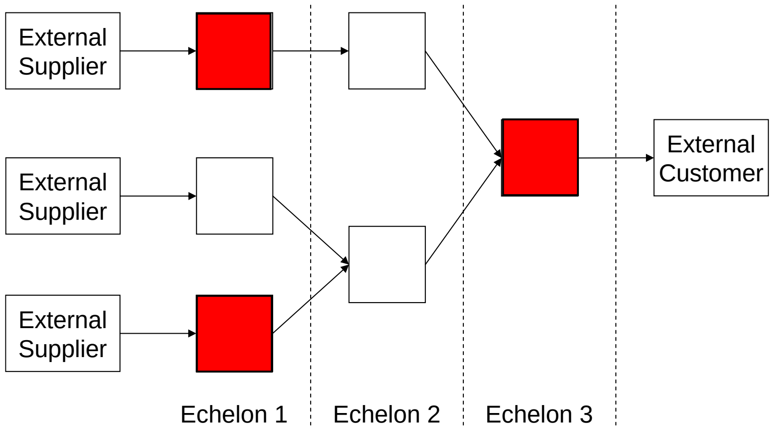
Serial Chain



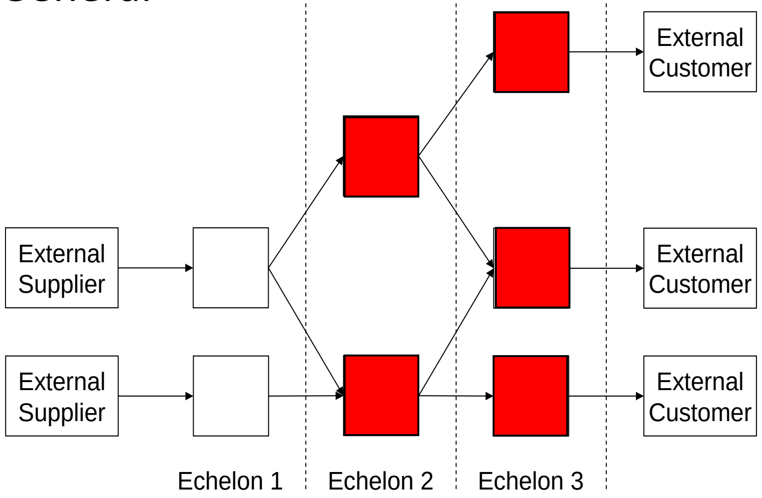
Distribution



Assembly



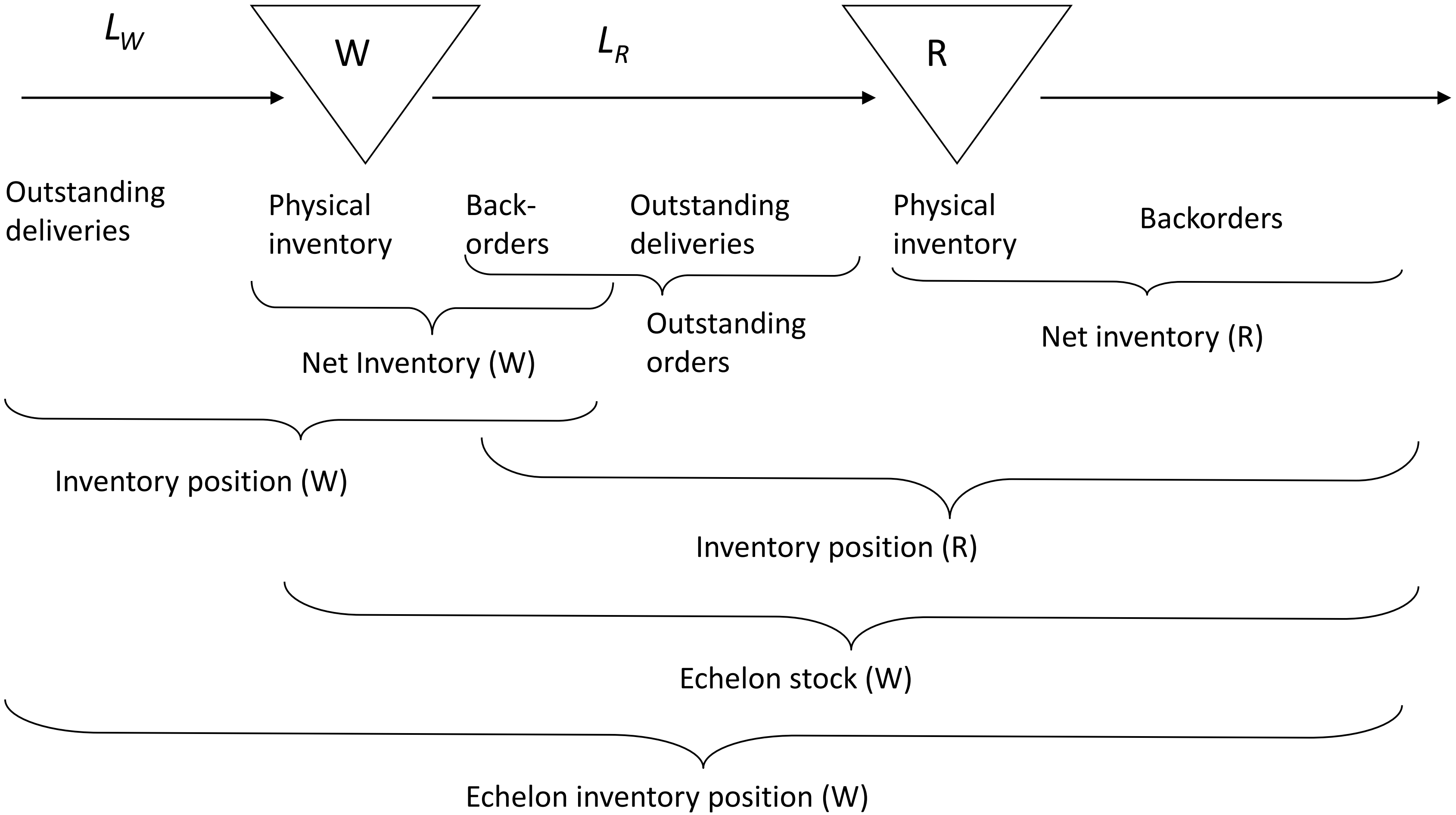
General



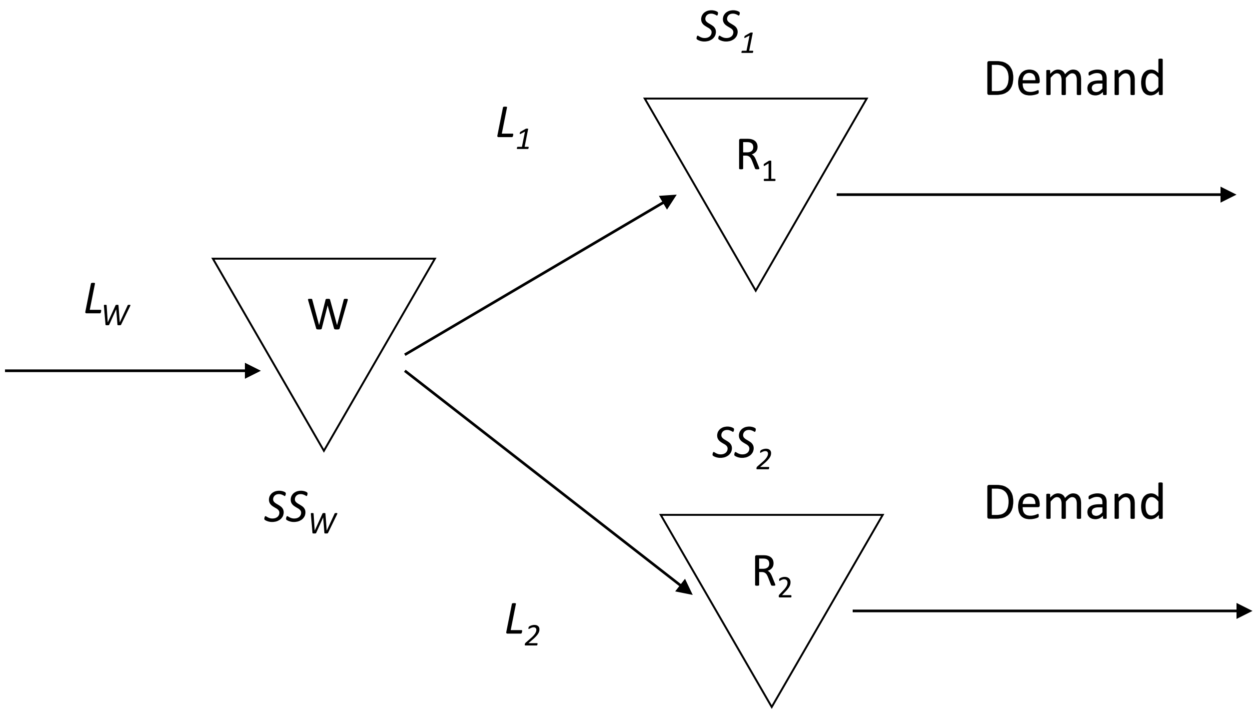
Ordering systems

- Centralized
  - Material/Distribution Requirements Planning
  - Echelon stock policies (basing on inventories of that and all downstream installations)
- Decentralized
  - Installation stock policies (basing only on inventories of the respective installation)
  - Coordination of local reorder-point systems
- Review
  - Periodic vs. continuous

Stock definitions



Example

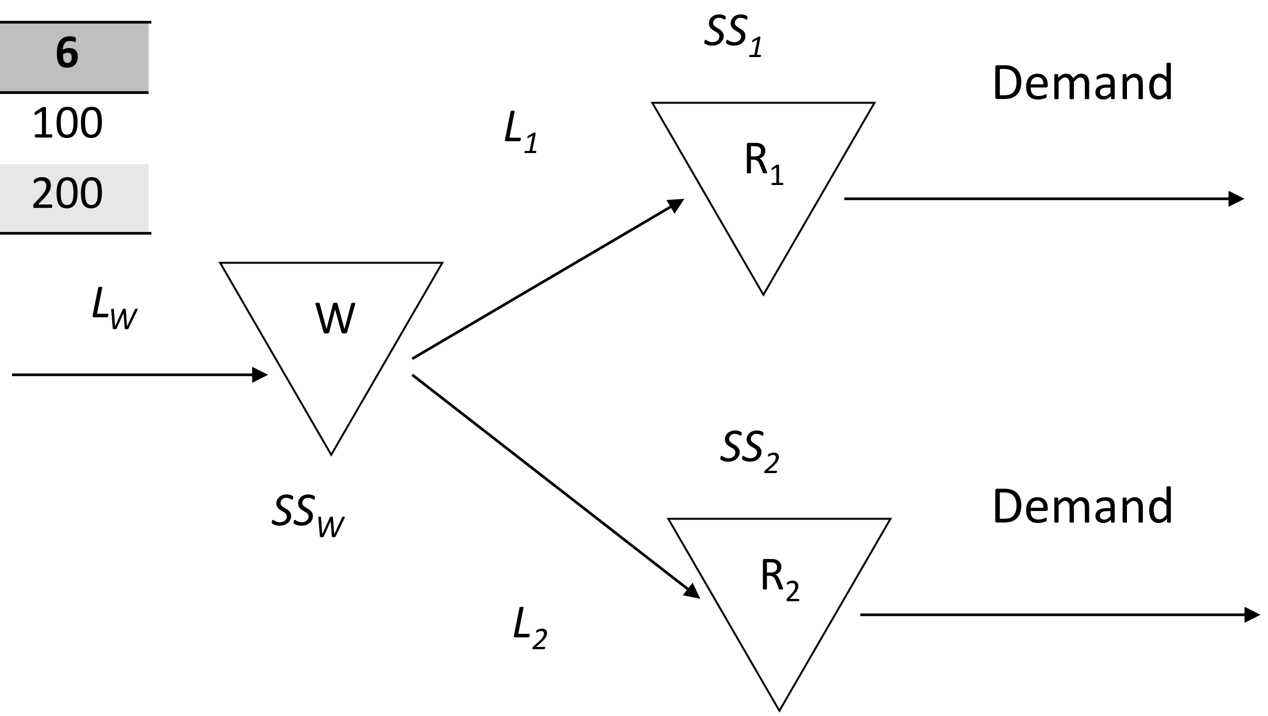


Data

- Demands

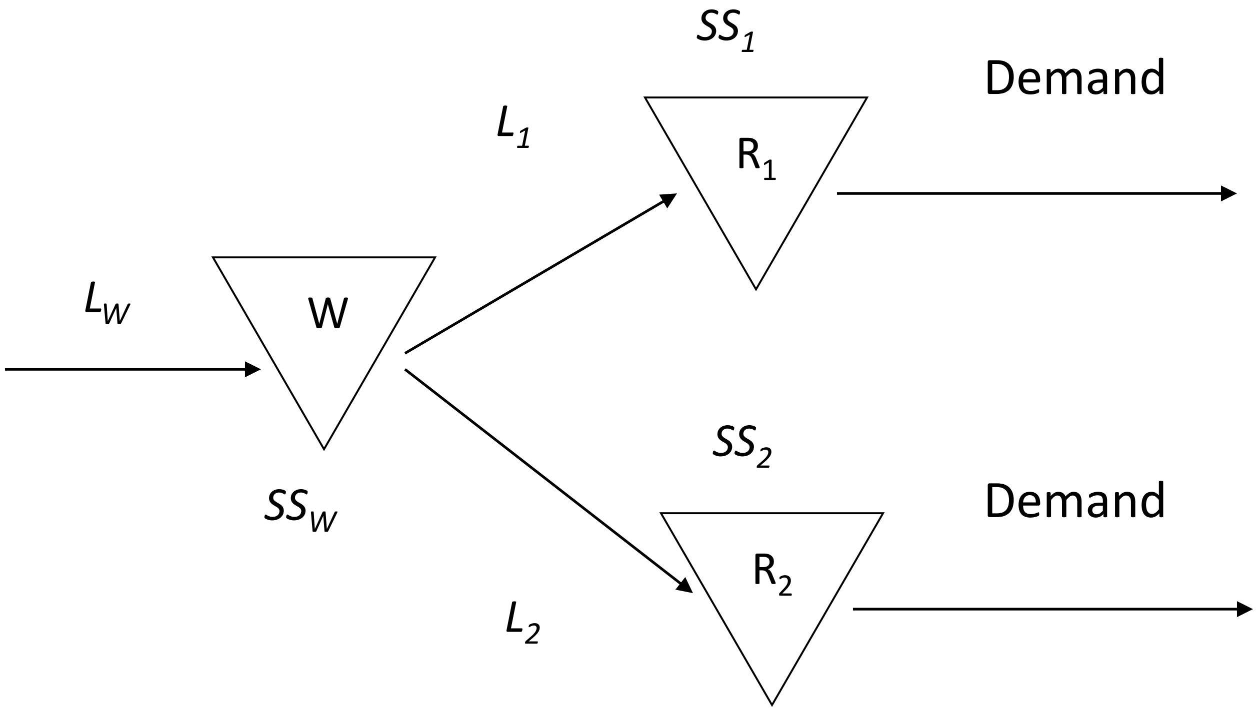
t	1	2	3	4	5	6
D1	100	150	120	80	50	100
D2	200	150	300	250	100	200

- Lead times: L_W=1, L_1=2, L_2=1
- Initial inventories
  - y_W=300, y_1=150, y_2=250
  - Outstanding order retailer 1: 200



Replenishment policies

- Distribution Requirements Planning
  - Demand forecasts
  - Safety stocks: SS_W=0, SS_1=20, SS_2=40
- Local (R,S)-policies (installation stock policy)
  - R=1, S_W=300, S_1=320, S_2=440
- Echelon-stock policy
  - R=1, S_W=1000, S_1=300, S_2=400



DRP results

	Period					
Demand	1	2	3	4	5	6
1	100	150	120	80	50	100
2	200	150	300	250	100	200

	1	**2**	**3**	**4**	**5**	**6**
Retailer 2	L_2=1					
Gross	200	150	300	250	100	200
Inventory	250	50	40	40	40	40
Replenishment	0	140	300	250	100	200
Order	140	300	250	100	200	0
Retailer 1	L_1=2					
Gross	100	150	120	80	50	100
Inventory	150	250	100	20	20	20
Replenishment	0	0	40	80	50	100
Order	40	80	50	100	0	0
Wholesaler	L_W=1					
Gross	180	380	300	200	200	0
Inventory	300	120	0	0	0	0
Replenishment	0	260	300	200	200	0
Order	260	300	200	200	0	0

SS_W=0, SS_1=20, SS_2=40

y_W=300, y_1=150, y_2=250
Outstanding order retailer 1: 200

Inventory rationing

- Simple Policies
  - Priority rationing
  - Proportional rationing

- Optimization
  - Fair share

- Other policies
  - Balanced stock rationing

Installation stock policy

S_W=300,
S_1=320,
S_2=440

Lead times:
L_W=1, L_1=2, L_2=1
Initial inventories
y_W=300, y_1=150,
y_2=250
Outstanding order retailer 1: 200

		Inventory	Inv. Position	Order	Demand	Final Inventory	Fulfilled demand
P1	R1	150	350	0	100	50	100
	R2	250	250	190	200	50	200
	W	300	110	190	190	110	0,190
P2	R1	250	250	70	150	100	150
	R2	240	240	200	150	90	150
	W	300	30	270	270	30	70,200
P3	R1	100	170	150	120	-20	100
	R2	290	290	150	300	-10	290
	W	300	0	300	300	0	150,150
P4	R1	50	200	120	80	-30	70 (20+50)
	R2	140	140	300	250	-110	150 (10+140)
	W	300	-120	420	420	-120	86,214
P5	R1	120	240	80	50	70	80
	R2	104	190	250	100	4	210
	W	300	-30	330	330	-30	107,313
P6	R1	156	270	50	100	56	100
	R2	317	340	100	200	117	200
	W	300	150	150	150	150	57,123

Echelon stock policy

S_W=1000,
S_1=300,
S_2=400

Lead times:
L_W=1, L_1=2, L_2=1
Initial inventories
y_W=300, y_1=150,
y_2=250
Outstanding order retailer 1: 200

		Inventory	Inventory Position	Order	Demand	Final inventory	Fulfilled Demand	
P1	R1	150	350	0	100	50	100	
	R2	250	250	150	200	50	200	
	W	300	900	100	150	150	0,150	
P2	R1	250	250	50	150	100	150	
	R2	200	200	200	150	50	150	
	W	250	700	300	250	0	50,200	
P3	R1	100	150	150	120	-20	100	
	R2	250	250	150	300	-50	250	
	W	300	700	300	300	0	150,150	
P4	R1	30	180	120	80	-50	50	
	R2	100	100	300	250	-150	150	
	W	300	580	420	420	-120	86,214	
P5	R1	100	220	80	50	50	100	
	R2	64	150	250	100	-36	214	
	W	300	670	330	330	-30	107,313 (tot)	73,227 (this)
P6	R1	136	250	50	100	36	100	
	R2	277	300	100	200	77	236	
	W	300	850	150	150	150	57,123	

Multi-echelon inventory control decisions

- Deterministic models
  - Coordination of lot-sizes

- Stochastic models
  - Safety stock placement and sizes

Depot-Effect

- Model: Economic order quantity
  - Demand rates d_i, fixed cost per replenishment A, inventory holding cost h
- Demand consolidation $D = \sum_{i=1}^{n} d_i$
- Transaction effect
  - Individual replenishments $Q_i = \sqrt{\frac{2 d_i A}{h}}; \quad C_i = \sqrt{2 d_i A h}$
  - Central replenishments $Q = \sqrt{\frac{2DA}{h}}; \quad C = \sqrt{2DAh}$
  - Cost comparison $\Delta = \sqrt{2Ah} \cdot \left(\sum_{i=1}^{n} \sqrt{d_i} - \sqrt{\sum_{i=1}^{n} d_i} \right)$
  - Special case: identical demand rates (square root law)

$$C(D) = \sqrt{n} \cdot C(d)$$

Portfolio-Effect

- Statistical context
  - Expected demand μ_i, standard deviation σ_i
  - Correlation: covariance, coefficient of correlation
    - ρ = +1: perfect positive correlation
    - ρ = -1: perfect negative correlation
- Joint demand

$$\sigma(2)=\sqrt{\sigma_1^2+2\rho_{12}\sigma_1\sigma_2+\sigma_2^2}$$

$$\rho_{12}=1: \quad \sigma(2)=\sigma_1+\sigma_2$$

$$\rho_{12}=-1: \sigma(2)=\left|\sigma_1-\sigma_2\right|$$

$$\mu(n)=\sum_{i=1}^{n}\mu_i$$

$$\sigma(n)=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}}=\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\cdot\sigma_i\cdot\sigma_j}; \quad \rho_{ii}=1$$

  - Special case: identical demands, no correlation $\sigma(n)=\sqrt{n}\cdot\sigma$

Example

	1	2	3	4	mu	sigma	
D1	80	130	120	70	100	29,44	
D2	150	250	220	180	200	43,97	
Sum	230	380	340	250	300	71,65	0,90
	1	2	3	4	mu	sigma	
D1	80	130	120	70	100	29,44	
D2	220	180	150	250	200	43,97	
Sum	300	310	270	320	300	21,60	-0,90
	1	2	3	4	mu	sigma	
D1	80	130	120	70	100	29,44	
D2	180	250	150	220	200	43,97	
Sum	260	380	270	290	300	54,77	0,08

Parameter estimation

- Covariance

$$COV(D_1, D_2) = \sum_{d_1=0}^{\infty} \sum_{d_2=0}^{\infty} (d_1 - \mu_1) \cdot (d_2 - \mu_2) \cdot f(D_1 = d_1, D_2 = d_2)$$

- Coefficient of correlation

$$\rho_{ij} = \frac{COV(i, j)}{\sigma_i \cdot \sigma_j} \qquad -1 \le \rho_{ij} \le +1$$

- *n* observations (d_{1i}, d_{2i}), $i=1,2,\ldots,n$

$$C\hat{O}V(D_1, D_2) = \frac{\sum_{i=1}^{n} (d_{1i} - \hat{\mu}_1) \cdot (d_{2i} - \hat{\mu}_2)}{n-1}$$