Inventory Management

Summer 2025

- Assignment 5 -

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Exercise 1:

Consider a **three-stage** serial system with **normally distributed demand** (mean 50, standard deviation 20). All locations review inventory periodically **(R=1)** and face a **non-stockout probability constraint of 90%.** Inventory holding costs are **1, 2 and 5** for the respective locations from upstream to downstream. The lead times are **L1=2** for the most upstream location and **L_i=1** for the other two locations.

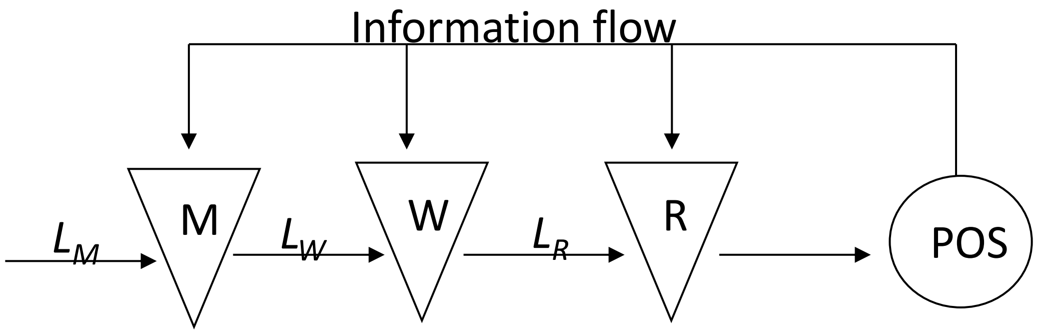
a) What are candidate values for the safety stock coverage times?

b) Determine the optimal allocation.

Exercise 1.a) What are **candidate values** for the safety stock coverage times?

	Up	→	Down
Holding costs	1	2	5
Lead Times	2	1	1
Review period	1		

Service level	0.9
Z	1.282
Mean	50
st.Dev	20



Possible Options (Candidates)?

	M	**W**	**R**
1	x	x	o
2	x	o	o
3	o	x	o
4	o	o	o

		M	W	R	COST
Opt. 1	Cover	0	0	5 (2+1+1+1)	
Opt. 2	Cover	0	3 (2+1)	2 (1+1)	
Opt. 3	Cover	2	0	3 (1+1+1)	
Opt. 4	Cover	2	1	2 (1+1)	

Exercise 1.a) What are **candidate values** for the safety stock coverage times?

$$SS = Z * \delta * \sqrt{Coverage\ Time}$$

	Up	→	Down
Holding costs	1	2	5
Lead Times	2	1	1
Review period	1		

Service level	0.9
Z	1.282
Mean	50
st.Dev	20

Possible Options (Candidates)?

	M	W	R
1	x	x	o
2	x	o	o
3	o	x	o
4	o	o	o

		M	W	R	COST
Opt. 1	Cover	0	0	5 (2+1+1+1)	
	SS	0	0	≈ 57.3 (=20*1.282*√5)	286.5 (0*0+0*0+5*57.3)
Opt. 2	Cover	0	3 (2+1)	2 (1+1)	
	SS	0	≈ 44.4 (=20*1.282*√3)	≈ 36.26 (=20*1.282*√2)	270.1 (2*44.4+5*36.26)
Opt. 3	Cover	2	0	3 (1+1+1)	
	SS	≈ 36.26 (=20*1.282*√2)	0	≈ 44.4 (=20*1.282*√3)	258.26 (1*36.26+5*44.4)
Opt. 4	Cover	2	1	2 (1+1)	
	SS	≈ 36.26 (=20*1.282*√2)	≈ 25.64 (=20*1.282*√1)	≈ 36.26 (=20*1.282*√2)	268.84 (1*36.26+2*25.64+5*36.26)

Exercise 1.b) Determine **the optimal allocation.**

Coverage Time	1	2	3	COST
Candidate 1	0	0	5	286.5
Candidate 2	0	3	2	270.1
Candidate 3	2	0	3	**258.26**
Candidate 4	2	1	2	268.84

∴ Candidate 3 has the minimum holding cost with the allocation policy of Loc1 and Loc3

Tips!

1. Design possible options (Safety stock holding location)
2. Evaluate coverage time for each echelon member (*if it holds any) and for each option
3. Calculate total holding costs of SC with corresponding holding cost in each location
4. Conclude with the best option

Exercise 2:

Assume **a two-stage serial** supply chain **with periodic control and normally distributed** demand with mean 100 and standard deviation 40. The lead times for each stage are **L=1**. The backorder penalty is **b=20**, inventory holding costs are **h_W=1, h_R=3**.

a) Determine the optimal parameters of an echelon-order-up-to-policy.

b) How does the analysis change if a non-stockout probability of 95% has to be ensured?

Exercise 2:

Clark-Scarf-model

Average cost analysis

- Stage 1

$$C_1(S_1,S_2)=h_1E(S_1-D(L_1)-S_2)^+$$

$$=h_1\int_0^{S_1-S_2}(S_1-S_2-d)f_{L_1}(d)\mathrm{d}d$$

- Stage 2

$$C_2(S_1,S_2)=h_2E((S-D(L_2+1))^+)+bE((S-D(L_2+1))^-)$$

$$=h_2\int_0^S(S-d)f_{L_2+1}(d)\mathrm{d}d+\mathrm{b}\int_S^\infty(d-S)f_{L_2+1}(d)\mathrm{d}d$$

Optimal solution

- Stage 2

$$F_{L_2+1}(S_2)=\frac{h_1+b}{h_2+b}$$

- Stage 1

$$\int_0^{S_2}F_{L_1}(S_1-x)f_{L_2+1}(x)\mathrm{d}x=\frac{b}{h_2+b}$$

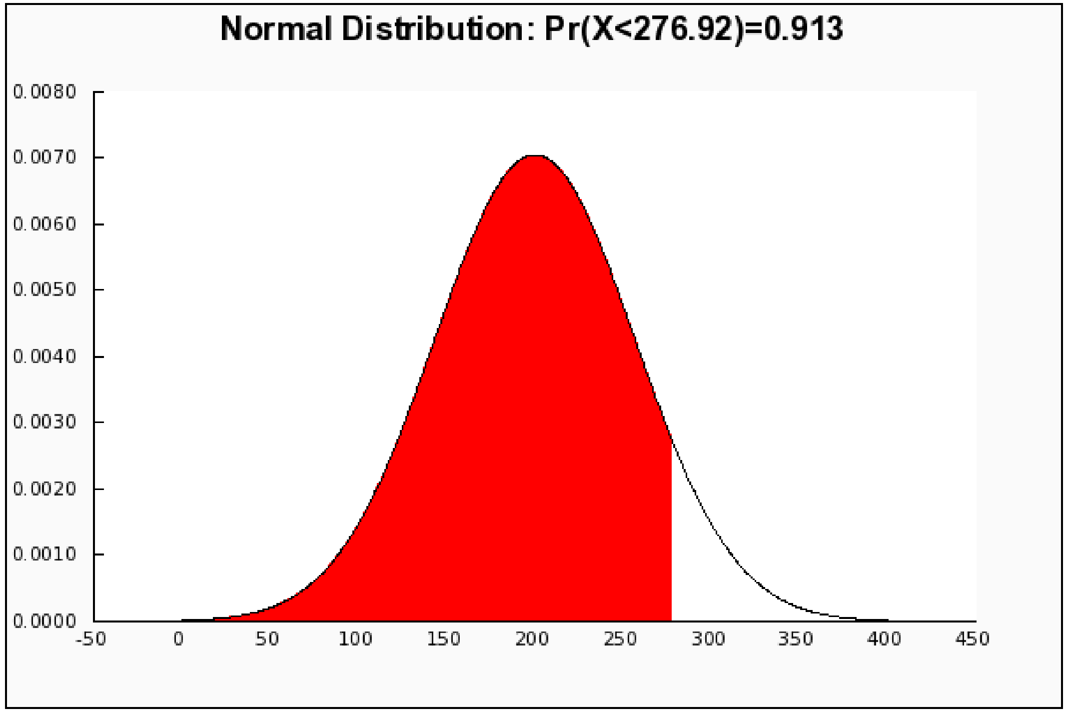
Exercise 2.a) Determine the optimal parameters of an **echelon-order-up-to-policy.**

mean	100
sigma	40
h1	1
h2	3
b	20
L1	1
L2	1
R	1

$$F_{L_2+1}(S_2) = \frac{h_1 + b}{h_2 + b} \rightarrow \frac{1 + 20}{3 + 20} = 0.913$$

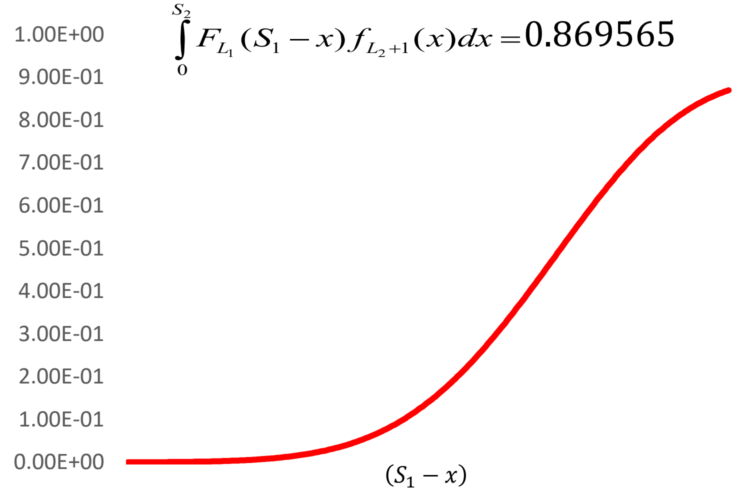
$$S_2 = \mu * (L_2 + 1) + z * \sigma * \sqrt{L_2 + 1}$$
$$= 100 * (1 + 1) + 1.3597 * 40 * \sqrt{1 + 1}$$
$$= \mathbf{276.92}$$

F(k)	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922



Exercise 2.a) Determine the optimal parameters of an **echelon-order-up-to-policy.**

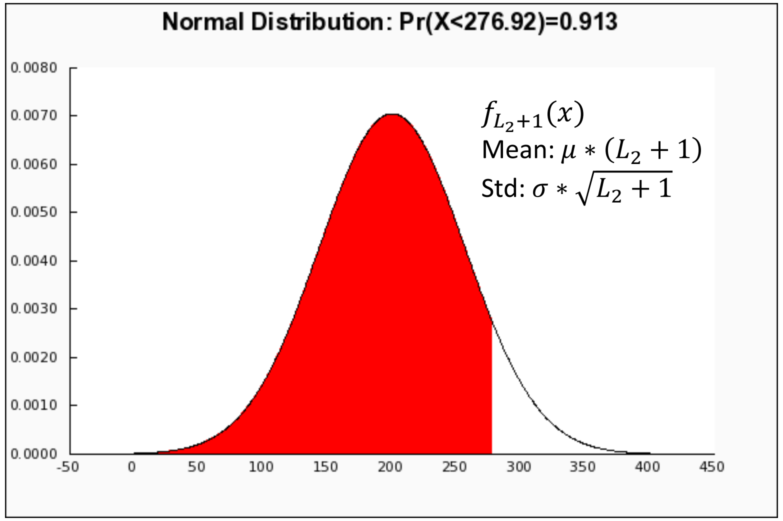
mean	100
sigma	40
h1	1
h2	3
b	20
L1	1
L2	1
R	1

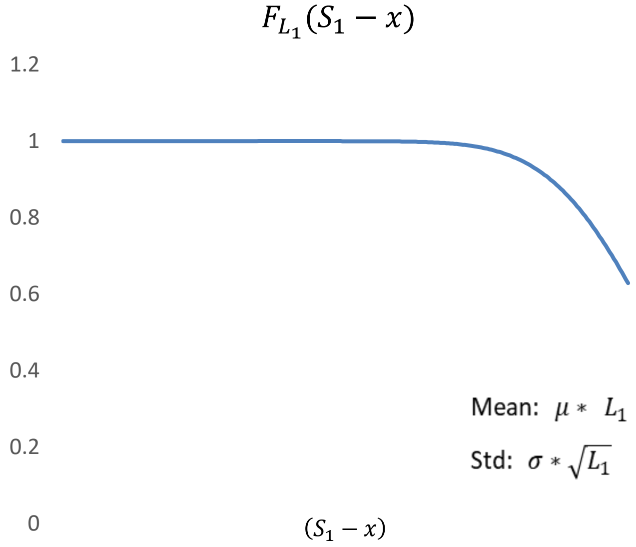


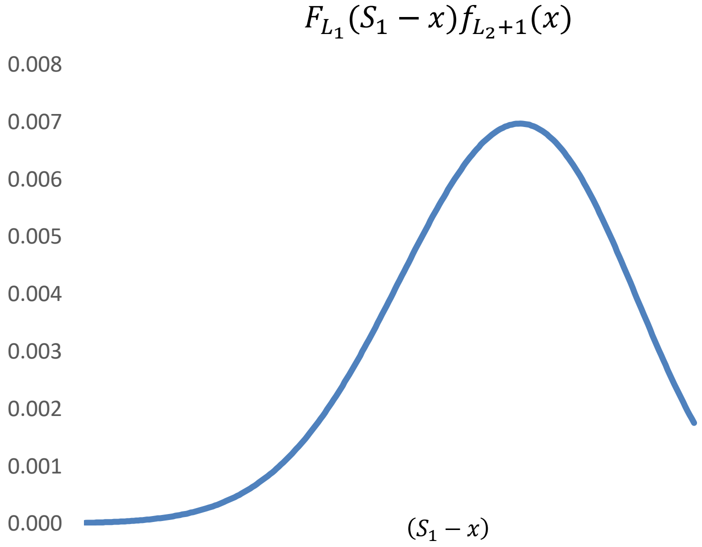
$$\int_0^{S_2} F_{L_1}(S_1 - x) f_{L_2+1}(x)dx = \frac{b}{h_2 + b} \rightarrow \frac{20}{3 + 20} = 0.869565$$

$$\therefore \boldsymbol{S_1 = 390.15}$$

By. Python/Excel Solver
Excel Sheet/Python will be provided.







Exercise 2.b) How does the analysis change if **a non-stockout probability of 95%** has to be ensured?

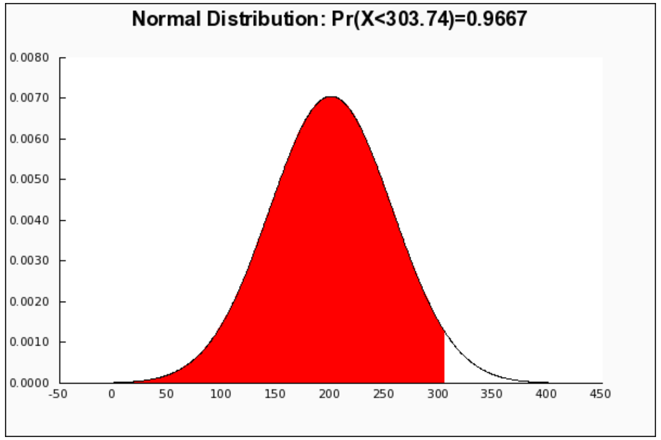
$$\int_0^{S_2} F_{L_1}(S_1 - x) f_{L_2+1}(x)dx = \frac{b}{h_2 + b} \geq \color{red}{\mathbf{0.95}} \quad \rightarrow b \geq \frac{0.95*3}{0.05} \qquad \therefore \boldsymbol{b = 57}$$

mean	100
sigma	40
h1	1
h2	3
b	20
L1	1
L2	1
R	1

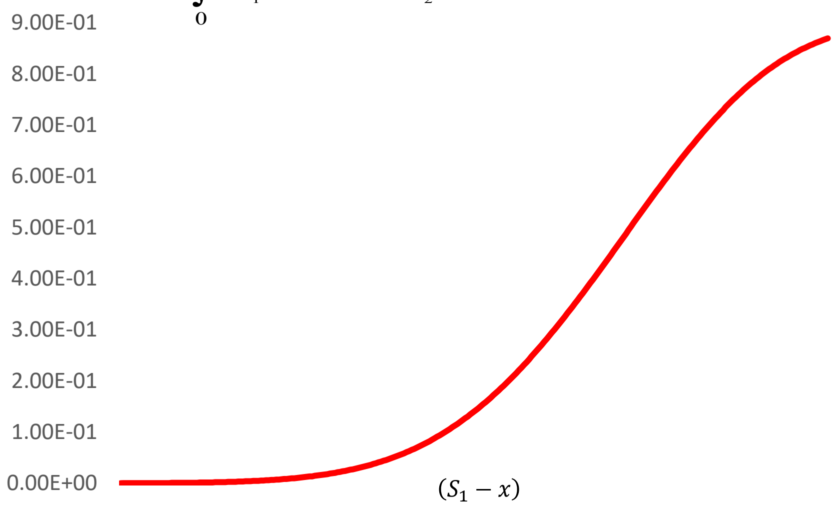
$$F_{L_2+1}(S_2) = \frac{h_1 + b}{h_2 + b} \rightarrow \frac{1+57}{3+57} = 0.9667$$

$$S_2 = \mu * (L_2 + 1) + z * \sigma * \sqrt{L_2 + 1}$$
$$= 100 * (1 + 1) + 1.8339 * 40 * \sqrt{1 + 1}$$
$$= \mathbf{303.74}$$

F(k)	0	0.01	0.02	0.03	0.04
1.2	0.88493	0.88686	0.88877	0.89065	0.89251
1.3	0.90320	0.90490	0.90658	0.90824	0.90988
1.4	0.91924	0.92073	0.92220	0.92364	0.92507
1.5	0.93319	0.93448	0.93574	0.93699	0.93822
1.6	0.94520	0.94630	0.94738	0.94845	0.94950
1.7	0.95543	0.95637	0.95728	0.95818	0.95907
1.8	0.96407	0.96485	0.96562	0.96638	0.96712
1.9	0.97128	0.97193	0.97257	0.97320	0.97381



$$\int_0^{S_2} F_{L_1}(S_1 - x) f_{L_2+1}(x)dx = 0.95$$



$$\therefore \boldsymbol{S_1 = 428.43}$$

By. Python/Excel Solver
Excel Sheet/Python will be provided.

Exercise 3:

A **two-echelon serial** systems reviews inventories **continuously** and places replenishment orders following an **(S-1, S)** policy at both locations. Customer demand is assumed to follow a **Poisson-process with a mean of 5** customers per period. Unsatisfied demand is **backordered**, the penalty cost per unit and unit of time is **b=20.** Inventories at both locations are subject to inventory holding costs**, 1 at the upper location and 3 at the downstream** location. The lead times are equal to **one for the upstream location (warehouse) and two periods for downstream** location (retailer). Determine the optimal parameters S for each location following the METRIC-approach.

Exercise 3:

- Optimization $$C = C_0(S_0) + \sum_{i=1}^{N} (h_i \mathrm{E}(IL_i^+) + b_i \mathrm{E}(IL_i^-))$$

Properties for the Approximation Method

- Convexity of objective function for given S_0
- Bounds on inventory level
  - Lower: find S_i for lowest possible lead time L_i
  - Upper: find S_i for largest possible lead time L_0+L_i
  - Independent optimization for each retailer by successively increasing S_i
- Warehouse parameter S_0
  - Not necessarily convex
  - Bounds
    - Lower: Optimize w.r.t. to upper bounds on S_i
    - Upper: Optimize w.r.t. to lower bounds on S_i
  - Enumeration of values

Exercise 3.a) Determine the optimal parameters **S** for each location following the **METRIC-approach.**

Lambda	5
b	20
hW	1
hR	3
LW	1
LR	2

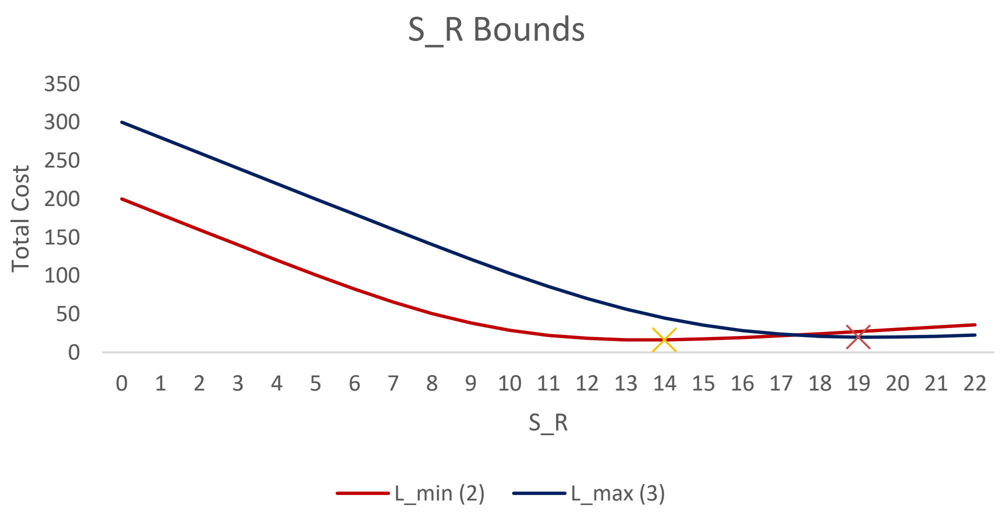
1) Bound of retailer

Bounds of lead time

L_lower = L_i	2
L_upper = L_0+L_i	3

Bounds of stock level S

S(i)_min	14
S(i)_max	19



$$E(IL_i^+) = \sum^{S_i} jP(IL_i = j)$$

$$E(IL_i^-) = E(IL_i^+) - (S_i - \lambda_i \bar{L}_i)$$

$$C = C_0(S_0) + \underbrace{\sum_{i=1}^{N} \left(h_i E(IL_i^+) + b_i E(IL_i^-)\right)}_{\text{Retailer}}$$

Exercise 3.a) Determine the optimal parameters S for each location following the METRIC-approach.

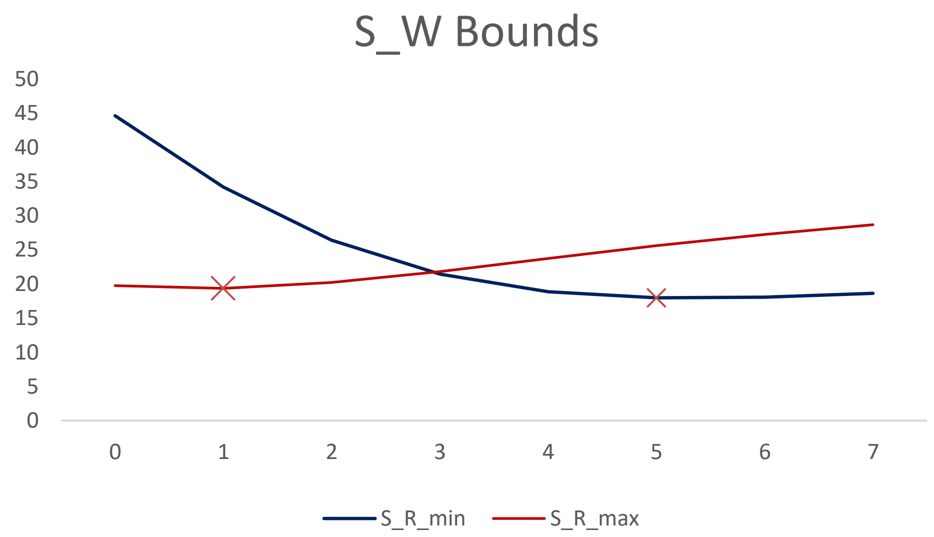
Lambda	5
b	20
hW	1
hR	3
LW	1
LR	2

2) Approximate stochastic lead time $\overline{L}_i = L_i + \frac{E(IL_0^-)}{\lambda_0}$

Bounds of warehouse

S(R) min	14
S(R) max	19

S(W) max	5
S(W) min	1



3) Enumarate

∴

S_R	18	17	16	**15**	14
S_W	2	3	3	**4**	5
Overall Cost	18.87	18.41	17.96	**17.74**	17.98

Jupyter notebook will be provided.

Thank you!