2 Lot-sizing and Safety Stocks

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The Trade-Off

Large Lots
reduce the setup costs by requiring less frequent setup

Small Lots
reduce inventory by bringing in product closer to the time it is used

Economic Order Quantity (EOQ) Model

Assumptions

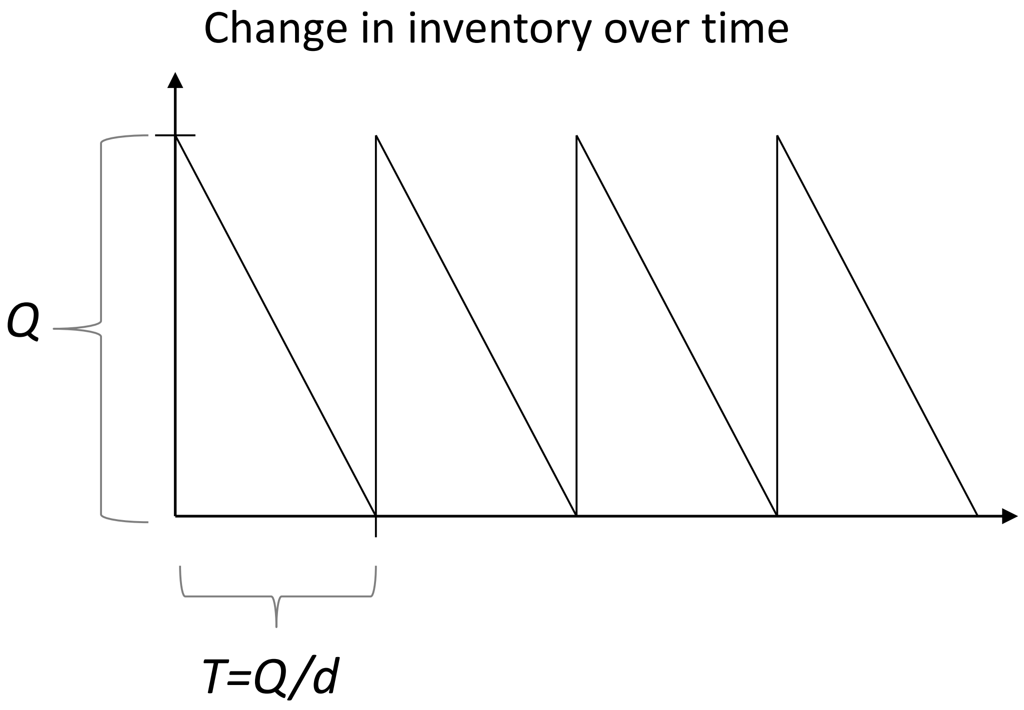
1. Continuous time, infinite planning horizon
2. Constant demand rate d (units/time)
3. No backorders, no lead time, infinite supply rate
4. Inventory holding cost per unit and unit of time: h
5. Fixed ordering (setup) cost per order/batch: A
6. Procurement cost per unit: c
7. Orders in constant batches of size Q

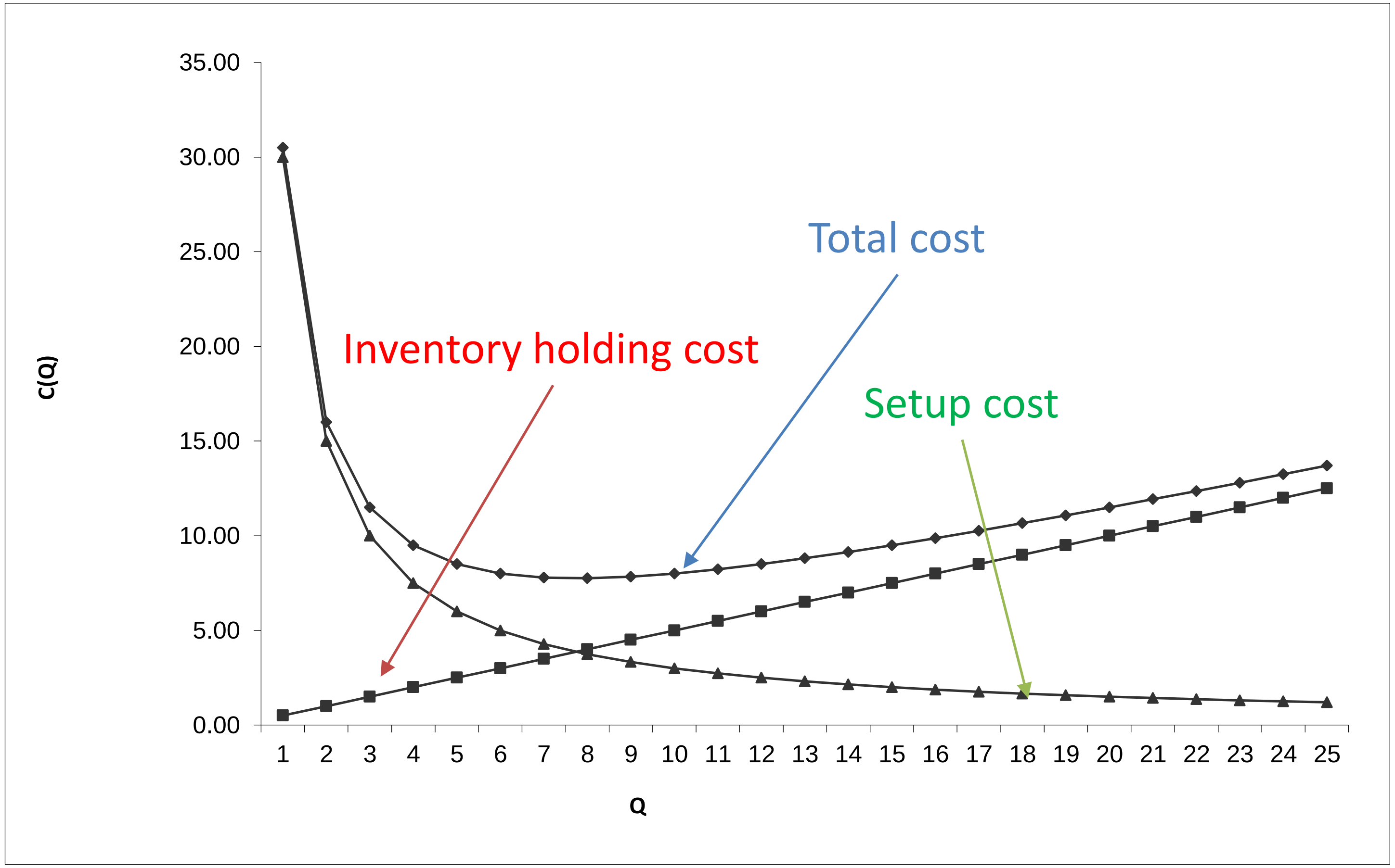
Ford Whitman Harris
1877-1962

Economic Order Quantity (EOQ) Model

- Inventory management of a single item
- Restrictive modeling assumptions
- But, in many practical situations model performs well
- Leads to solution in closed form:
  - Easy to compute
  - Provides managerial insights

Economic Order Quantity (EOQ)





Economic Order Quantity (EOQ)

- Decision variable: Economic order quantity Q
- Average cost formulation

$$C(Q) = \boxed{\frac{d}{Q}}A + \boxed{\frac{h}{2}Q} + \boxed{cd}$$

#of orders in a year

Holding cost in a year

Direct cost Independent of decision Q

- Total relevant cost: $TRC(Q) = \frac{d}{Q}A + \frac{h}{2}Q$
- Cost trade-off : Fixed costs per order vs Inventory holding costs

Economic Order Quantity (EOQ)

- Decision variable: Economic order quantity Q
- Average cost formulation

$$C(Q) = \frac{d}{Q}A + \frac{h}{2}Q + cd$$

$$\frac{\mathrm{d}C(Q)}{\mathrm{d}Q} = -\frac{dA}{Q^2} + \frac{h}{2} = 0 \Rightarrow Q^* = \sqrt{\frac{2dA}{h}}$$

$$C(Q^*) = \sqrt{2dAh} + cd$$

- Total relevant cost

$$TRC(Q^*) = \sqrt{2dAh}$$

Economic order quantity

- Cost function per time unit

$$C(Q) = \frac{d}{Q} \cdot A + \frac{h}{2} \cdot Q + c \cdot d$$

- Optimality condition

$$\frac{dC}{dQ} = -\frac{d}{Q^2} \cdot A + \frac{h}{2} = 0$$

- Solution
  - Optimal lot size

$$Q^* = \sqrt{\frac{2dA}{h}}$$

  - Optimal order interval

$$T^* = \sqrt{\frac{2A}{hd}}$$

  - Minimal costs per time unit

$$C^* = \sqrt{2dhA} + cd$$

Example

- Demand of 1000 units/year
- Unit variable cost c = $250/unit
- Metalworking shop charges a fixed cost of $ 500 per order
- Interest rate I = 0.1 $/$/yr
- Assume that h = Ic

$$Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2 \cdot 500 \cdot 1000}{250 \cdot 0.1}} = 200$$

- Total relevant cost:

$$TRC(Q*) = \sqrt{2dAh} = \sqrt{2 \cdot 500 \cdot 1{,}000 \cdot 250 \cdot 0.1} = 5{,}000$$

Sensitivity Analysis of EOQ

- Assume we set an order quantity Q‘ that deviates from EOQ

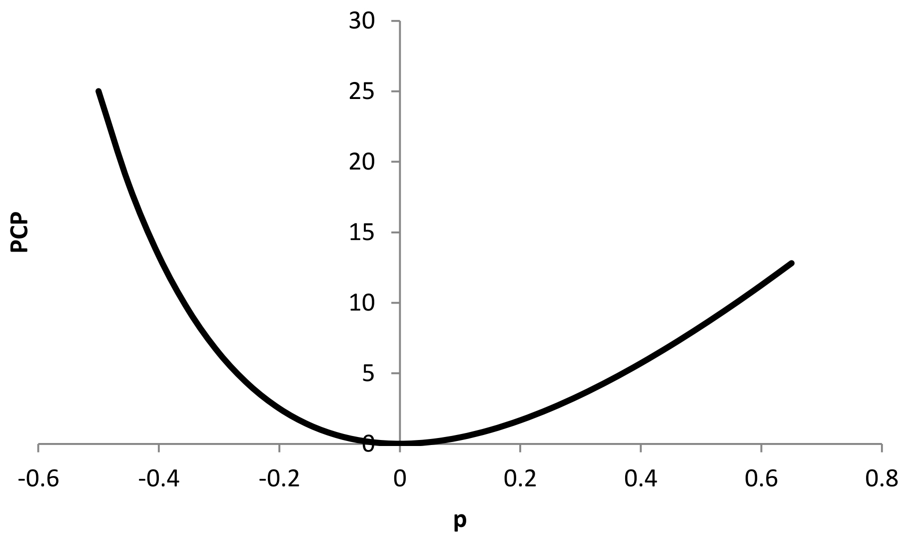
$$Q' = (1 + p)Q^*$$

- Percentage cost penalty:

$$PCP = \frac{TRC(Q') - TRC(Q^*)}{TRC(Q^*)} \times 100$$
$$= 50\left(\frac{p^2}{1+p}\right)$$

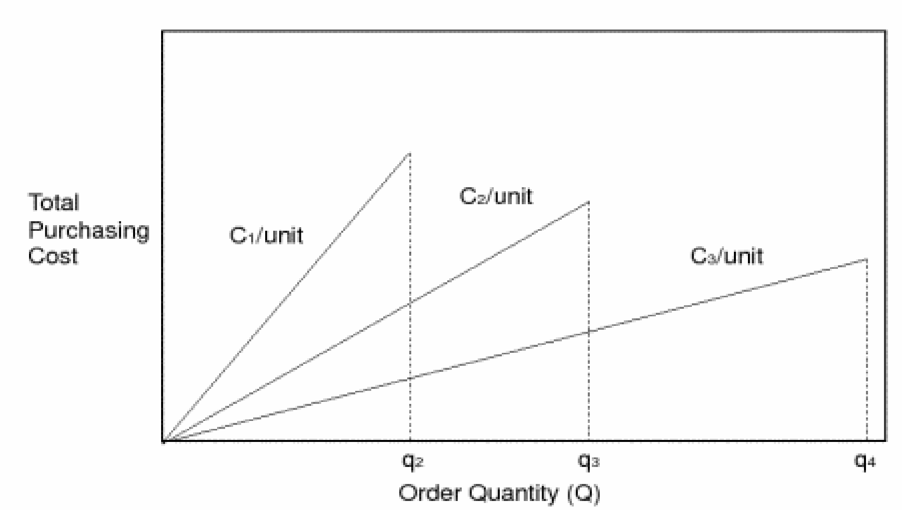
- Example continued: Q‘=250

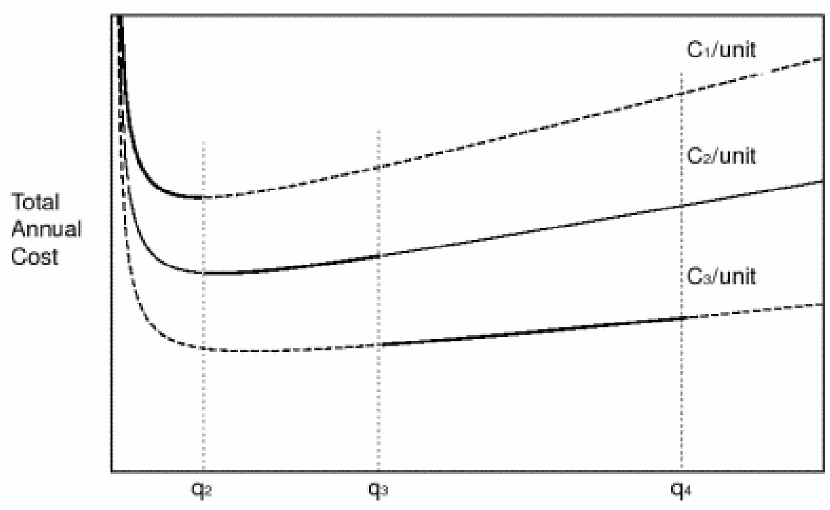
→ p=0.25 → PCP=2.5%



Quantity Discount Models

- All-unit Quantity Discounts
- In this model as the order quantity increases, the unit purchasing cost decreases for every unit purchased.
- Purcasing cost = $\begin{cases} C_1 Q \ for \ \ q_1 \leq Q < q_2 \\ C_2 Q \ for \ \ q_2 \leq Q < q_3 \\ C_3 Q \ for \ \ q_3 \leq Q \end{cases}$





Algorithm: All-unit discount

- Step 1: Calculate EOQ for the discounted price:

$$Q^*_2 = \sqrt{\frac{2dA}{I \cdot C_2}}$$

Note: assumption is that $h = I \cdot C$
I: Interest rate
C: Procurement price

- Step 2: Check if $Q_2^* \geq q_2$. Yes? Order Q_2^*. No? Continue to step 3
- Step 3: Calculate EOQ without discount:

$$Q^*_1 = \sqrt{\frac{2dA}{I \cdot C_1}}$$

- Step 4: Compare $C(Q^*_1)$ with $C(q_2)$. Order Q^*_1 if $C(Q^*_1) \leq C(q_2)$, else order q_2
- Note: $C(Q) = \frac{d}{Q}A + \frac{IC}{2}Q + Cd$. Select the right *C* for each order quantity!

Example (see Silver Pyke Thomas 4.5 p. 158):

- Consider three components in the below table.
- The supplier offers a **2% discount** on any replenishment of **100 units or higher** of a single item.
- What are the optimal order sizes for item A, B and C ?

Item	*D (Units/Year)*	v_0 *($/Unit)*	*A ($)*	*r ($/$/Year)*
A	416	14.20	1.50	0.24
B	104	3.10	1.50	0.24
C	4,160	2.40	1.50	0.24

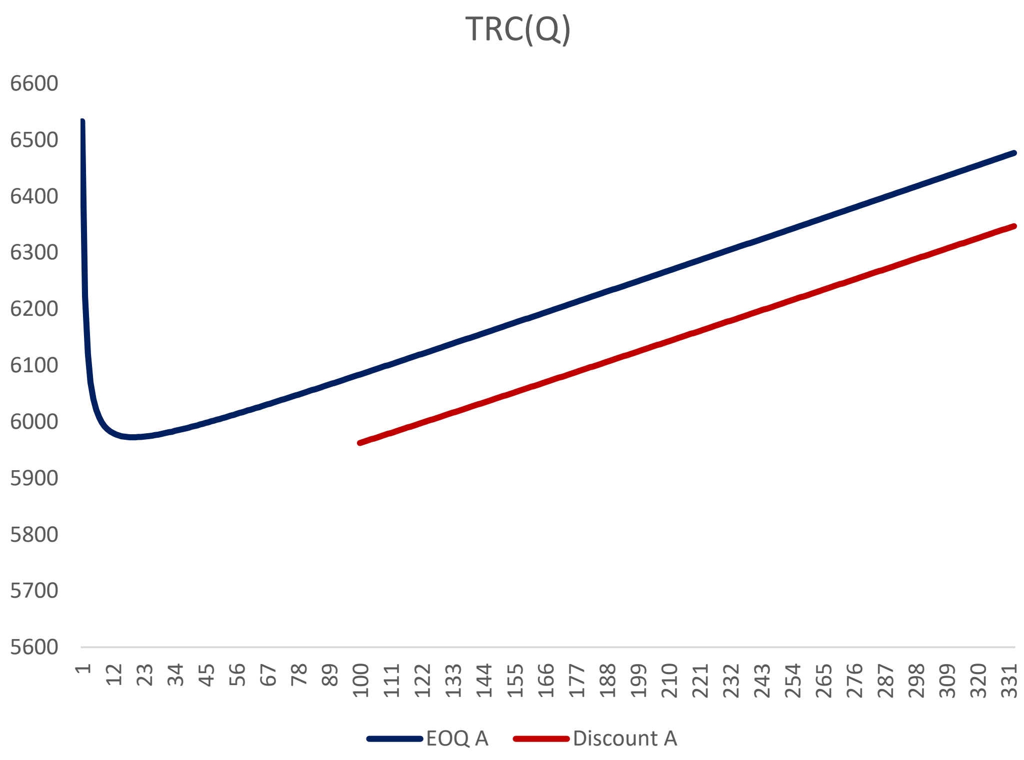
D : Annual Demand

v_0 : Unit Cost

A: Ordering Cost/Setup Cost

r: Carrying Charge

• Item A



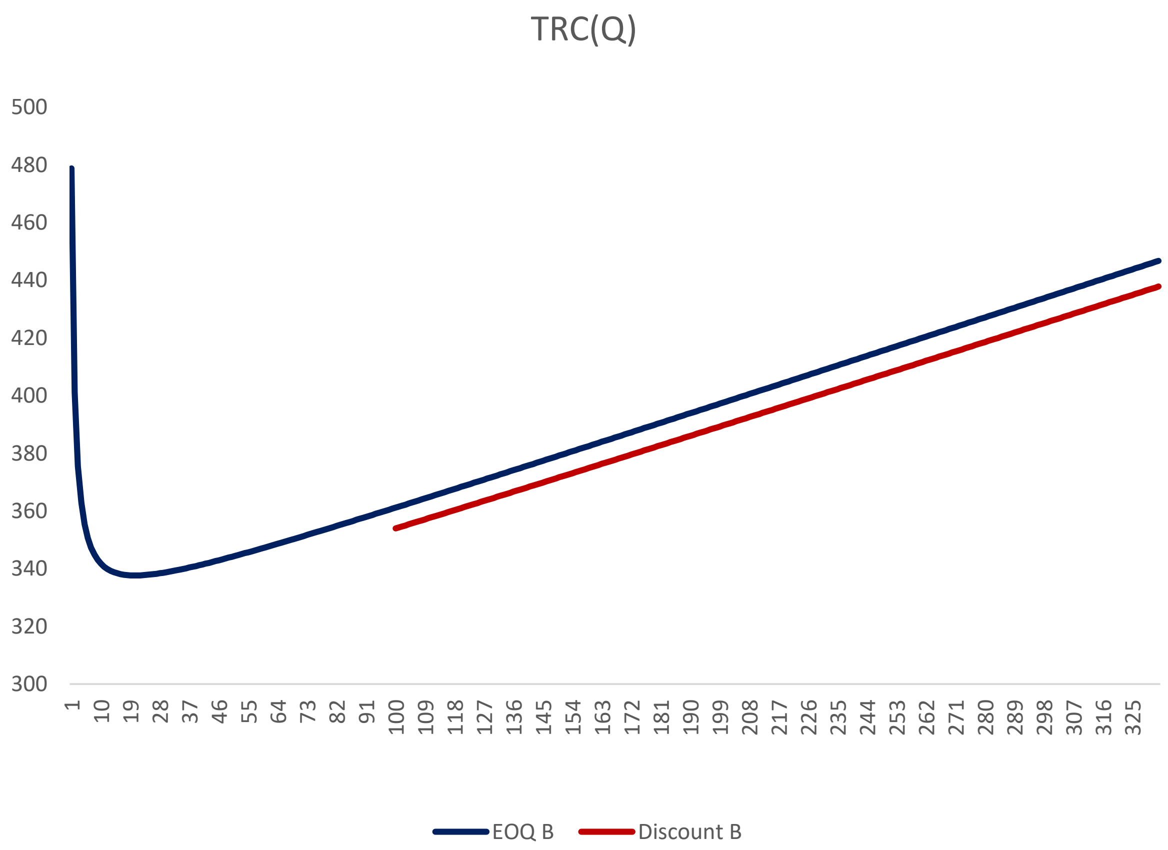
Step 1 EOQ (discount) = 19 units < 100 units.
Step 2 EOQ (discount) < Q_b; therefore, go to Step 3.
Step 3

$$\text{TRC(EOQ)} = \sqrt{2 \times 1.50 \times 416 \times 14.20 \times 0.24} + 416 \times 14.20$$
$$= \$5{,}972.42/\text{year}$$

$$\text{TRC}(Q_b) = \text{TRC}(100) = \frac{100 \times 14.20 \times 0.98 \times 0.24}{2} + \frac{1.50 \times 416}{100}$$
$$+ 416 \times 14.20 \times 0.98$$
$$= \$5{,}962.29/\text{year}$$

TRC(EOQ) > TRC (Q_b). Therefore, the best order quantity to use is Q_b, that is, 100 units.

- # Item B



Step 1 EOQ (discount) = 21 units < 100 units.
Step 2 EOQ (discount) < Q_b; therefore, go to Step 3.
Step 3

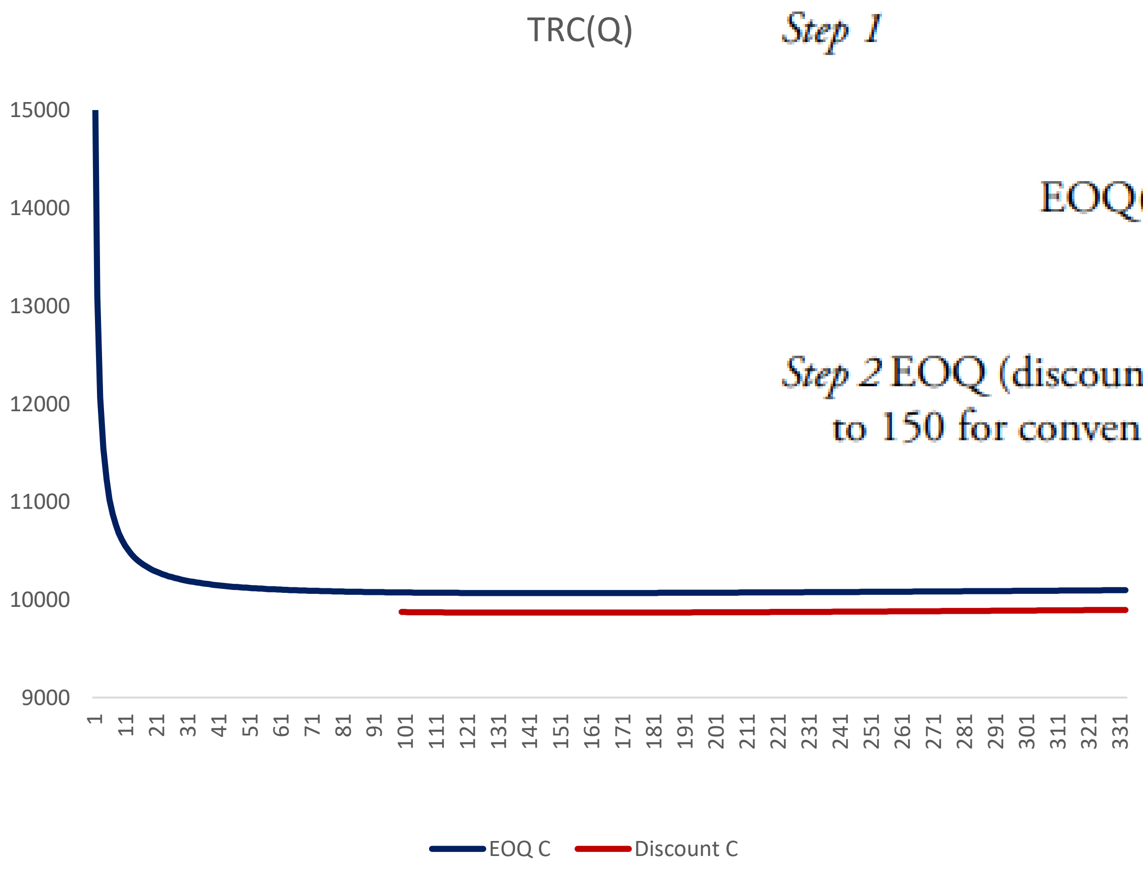
$$\begin{aligned}\text{TRC(EOQ)} &= \sqrt{2 \times 1.50 \times 104 \times 3.10 \times 0.24} + 104 \times 3.10 \\ &= \$337.64/\text{year}\end{aligned}$$

$$\begin{aligned}\text{TRC}(Q_b) = \text{TRC}(100) &= \frac{100 \times 3.10 \times 0.98 \times 0.24}{2} + \frac{1.50 \times 104}{100} \\ &\quad + 104 \times 3.10 \times 0.98 \\ &= \$353.97/\text{year}\end{aligned}$$

TRC(EOQ) < TRC (Q_b). Therefore, use the EOQ without a discount; that is,

$$\text{EOQ} = \sqrt{\frac{2 \times 1.50 \times 104}{3.10 \times 0.24}} \approx 20 \text{ units}$$

- # Item C



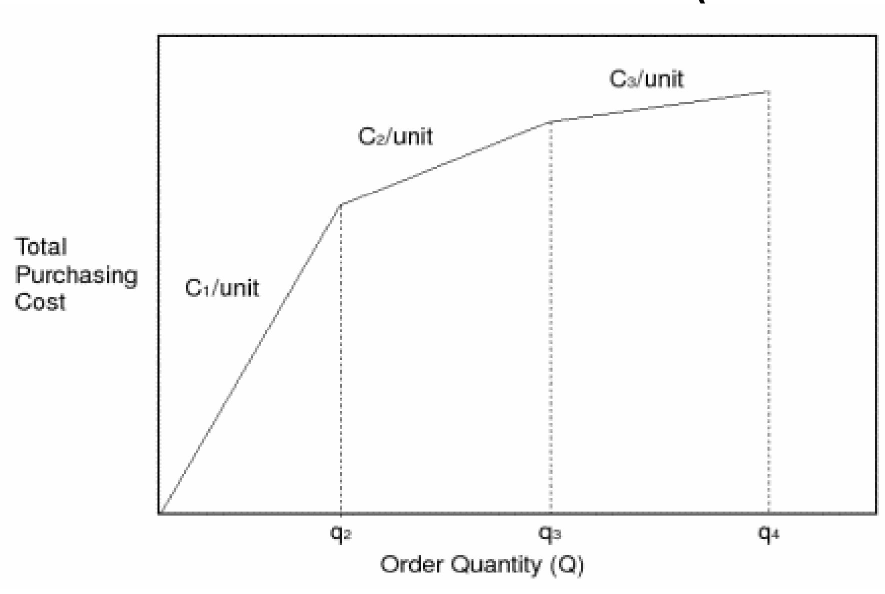
Step 1

$$\text{EOQ(discount)} = \sqrt{\frac{2 \times 1.50 \times 4{,}160}{2.40 \times 0.98 \times 0.24}} = 149 \text{ units} > 100 \text{ units}$$

Step 2 EOQ (discount) is greater than Q_b. Therefore, the Q to use is 149 units (perhaps rounded to 150 for convenience).

Incremental quantity discounts

- In this model, unit purchasing cost decreases only for units beyond a certain threshold and not for every unit.
- Purchasing cost c(Q)= $\begin{cases} C_1(Q - q_1) & for\ q_1 \leq Q < q_2 \\ C_1(q_2 - q_1) + C_2(Q - q_2) & for\ q_2 \leq Q < q_3 \\ C_1(q_2 - q_1) + C_2(q_3 - q_2) + C_3(Q - q_3) & for\ q_3 \leq Q \end{cases}$



Holding cost is now:

$h = I * c(Q)/Q$

See e.g. Muckstadt & Sapra 2.3.3

Algorithm: Incremental

R_j denotes the sum of the terms that are independent of Q in purchasing cost, if $q_j \leq Q < q_{j+1}$

R_j = C_1 (q_2 - q_1) + C_2 (q_3 - q_2) + ⋯ +$C_{j-1}(q_j - q_{j-1})$, j ≥ 2

1) Compute $Q_j^* = \sqrt{\dfrac{2(R_j - C_j q_j + A)d}{IC_j}}$ for all j

2) Check if $q_{j+1} > Q_j^* \geq q_j$ and disregard the ones that do not satisfy this inequality

3) For each remaining Q_j^* compute the corresponding costs $C(Q_j^*)$

Q_j^* that produces the least cost is the optimal order quantity

Example (see Muckstadt & Sapra 2.3.3 p. 39):

- Consider incremental discount table offered by a supplier
- The retailer sells one product. What are the optimal order sizes for the retailer ?

Quantity (Q)	Price (C)	Demand (d)	Ordering Cost (A)	Carrying Charge (I%)
$0 \le Q < 110$	\$ 5	520	10	20%
$110 \le Q < 150$	\$ 4.75			
$150 \le Q$	\$ 4.5			

R1 = 0

R2 = C1(q2 − q1) = (5)(110 − 0) = 550

R3 = C1(q2 − q1) + C2(q3 −q2) = R2 +(4.75)(150−110) = 740

1) Compute Q_j^* for all j

$$Q_1^* = \sqrt{\frac{2(0-0+10)(520)}{(0.2)(5)}} = 101.98,$$

$$Q_2^* = \sqrt{\frac{2(550-(4.75)(110)+10)(520)}{(0.2)(4.75)}} = 202.61,$$

$$Q_3^* = \sqrt{\frac{2(740-(4.5)(150)+10)(520)}{(0.2)(4.5)}} = 294.39.$$

Example (Cont.):

Quantity (Q)	Price (C)	Demand (d)	Ordering Cost (A)	Carrying Charge (I%)
$0 \leq Q < 110$	\$ 5	520	10	20%
$110 \leq Q < 150$	\$ 4.75			
$150 \leq Q$	\$ 4.5			

2) Check if $q_{j+1} > Q^*_j \geq q_j$ and disregard the ones that do not satisfy this inequality

- $Q_1^* \in [0, 110)$
- $\boldsymbol{Q_2^* \notin [110, 150)}$ $\longrightarrow$ Only Q_1^* and Q_3^* are feasible
- $Q_3^* \in [150, \infty)$

3) For Q_1^* and Q_3^* compute the corresponding costs $C(Q_j^*)$

$$C(Q_1^*) = (5)(520) + (0 - 0 + 10)\frac{520}{101.98} + \frac{(0.2)(5)(101.98)}{2} + \frac{(0.2)(0-0)}{2}$$

$$= 2701.98,$$

$$C(Q_3^*) = (4.5)(520) + (740 - (4.5)(150) + 10)\frac{520}{294.39} + \frac{(0.2)(4.5)(294.39)}{2} + \frac{(0.2)(740 - (4.5)(150))}{2}$$

$$= 2611.45.$$

∴ The retailer should order Q_3^* = 294.39

Power-of-Two Policies

Assume we are interested in the optimal reorder interval rather than the optimal order quantity.

$$T^* = \sqrt{\frac{2A}{hd}}$$

For practical reasons we may need the reorder interval to be an integer multiple of a base planning period

$$T = n.T_L$$

In a power-of-two policy, further, n can only be a power of two.

$$T = \{T_L, 2T_L, 4T_L, 8T_L, \dots\}$$

For example, joint ordering of multiple different items.

Power-of-Two Policies

- Inventory Management problem under a power of two policy is:

$$\min_{T\geq 0} C(T) = \frac{A}{T} + \frac{1}{2} hdT\,,$$

$$s.t.\;\; T = 2^{l} T_L, l = \{0,1,2,3,\ldots\}$$

T=Q/d

- l^* is the smallest non-negative integer (including zero) that satisfies

$$C(2^{l^*} T_L) \leq C(2^{l^*+1} T_L)$$

Sensitivity of EOQ with respect to T

- One can show that $\frac{C(T)}{C(T^*)} = \frac{1}{2}\left(\frac{T^*}{T} + \frac{T}{T^*}\right)$

- From this, and the condition $C(2^{l^*}T_L) \le C(2^{l^*+1}T_L)$

  it can be shown that $\frac{T^*}{\sqrt{2}} \le 2^{l^*}T_L \le T^*\sqrt{2}$.

  Hint: the solution should satisfy $\frac{C(T)}{C(T^*)} \le \frac{C(2T)}{C(T^*)}$.

  Thus: $\frac{C(2^{l^*}T_L)}{C(T^*)} \le \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) \approx 1.06$

- So, a power-of-two policy leads to at most a 6% cost disadvantage compared to the optimal order interval

Marketing – Operations Interface

- Sequential marketing-operation decision versus simultaneous planning
- Sequential planning
  - Stage 1: Price optimization

$$\Pi = (p-c)(a-bp), \quad p^* = \frac{a+bc}{2b}$$

  - Stage 2: Lot-size optimization

$$C(d) = \sqrt{2Ah(a-bp^*)}$$

- Simultaneous planning

$$\Pi = (p-c)(a-bp) - \sqrt{2Ah(a-bp)}$$

Numerical example

Price response				
a	50			
b	2			
Cost				
c	10			
A	450			
h	1			
Sequential			**Simultaneous**	
p	17.50		p	19.83
d (a-bp)	15.00		d	10.33
Profit (π-C(d))	-3.69		Profit	5.17
Revenue (p*d)	262.50			204.95
Variable cost (c*d)	150.00			103.34
Overhead (C(d))	116.19			96.44

Dynamic single product lot-sizing

- Finite-horizon
- Discrete-time t=1,2,...,T
- Deterministic, non-stationary demand: d_t
- Single product at a single stage
- Other assumptions as in EOQ model
- Planning problem (Wagner/Whitin)
  - Decision variables
    - q_t Lot-size (Production quantity) in t
    - y_t Inventory level at the end of period t
    - γ_t Setup indicator, γ_t=1 if a lot is placed in period t, γ_t=0 otherwise
  - Cost minimization (fixed order cost A, holding cost h for inventory at the end of a period)
  - Constraints

Mixed-integer Linear Program

- Model

$$\min \sum_{t=1}^{T} (A \cdot \gamma_t + h \cdot y_t)$$

$$\begin{aligned} & y_t = y_{t-1} + q_t - d_t && t = 1,2,\ldots,T \\ & q_t \leq M\gamma_t && t = 1,2,\ldots,T \\ & y_0 = y_T = 0 && \\ & q_t, y_t \geq 0,\ \gamma_t \in \{0,1\} && t = 1,2,\ldots,T \end{aligned}$$

- No fixed order cost: optimal to place an order in every period.
- Positive fixed cost: combine multiple periods' demands into a single order.

Mixed-integer Linear Program

- Solution Properties

If out of inventory after t-1, then order in t for exactly some number of periods ahead

$$q_t^* = \begin{cases} \sum_{\tau=t}^{z} d_\tau & if \quad y_{t-1}^* = 0 \\ 0 & else \end{cases} \qquad t = 1,2,\ldots,T$$

Positive order in period t only if no inventory left at end of t-1

$$q_t^* \cdot y_{t-1}^* = 0 \quad t = 1,2,\ldots,T$$

Wagner-Whitin Algorithm

- Algorithm guarantees an optimal solution.
- An application of dynamic programming.
- *F(t): t*otal costs of the best replenishment strategy that satisfies the demand in periods 1, 2, . . . , *t*.
- For period t, there are t possible options to evaluate.

Period	1	2	3	4	5	6
Demand	750	100	50	100	400	1000

A=400, *h*=2

- F(1)=A=400
- F(2)=min{Option 1, Option 2}=600
  - Option 1 (Produce in this period)→ F(1)+A=800
  - Option 2 (Produce in period 1)→ A+h*100=600

Wagner-Whitin Algorithm

Period	1	2	3	4	5	6
Demand	750	100	50	100	400	1000

A=400, *h*=2

- F(3)=min{Option 1, Option 2, Option 3}=800
  - Option 1(Produce in this period)→ F(2)+A=1000
  - Option 2 (Produce in period 2)→ F(1)+A+ h*50=900
  - Option 3 (Produce in period 1)→ A+ h*100+h*2*50=800

- F(4)=min{Option 1, Option 2, Option 3, Option 4}=1200
  - Option 1(Produce in this period)→ F(3)+A=1200
  - Option 2 (Produce in period 3)→ F(2)+A+ h*100=1200
  - Option 3 (Produce in period 2)→ F(1)+A+h*50+h*2*100=1300
  - Option 4 (Produce in period 1)→ A+h*100+h*2*50+h*3*100=1400

- Option 4 is actually redundant (no need to compute), since

$$h * 3 * 100 > A$$

Wagner-Whitin Algorithm

- If $d_j . h > A$ the optimal solution will have a replenishment at the beginning of period j.
- Since $d_5 . h > A$ and $d_6 . h > A$, for F(5) and F(6), the only meaningful option is the first one (Produce in the period).
- F(5)=F(4)+A=1600
- F(6)=F(5)+A=2000

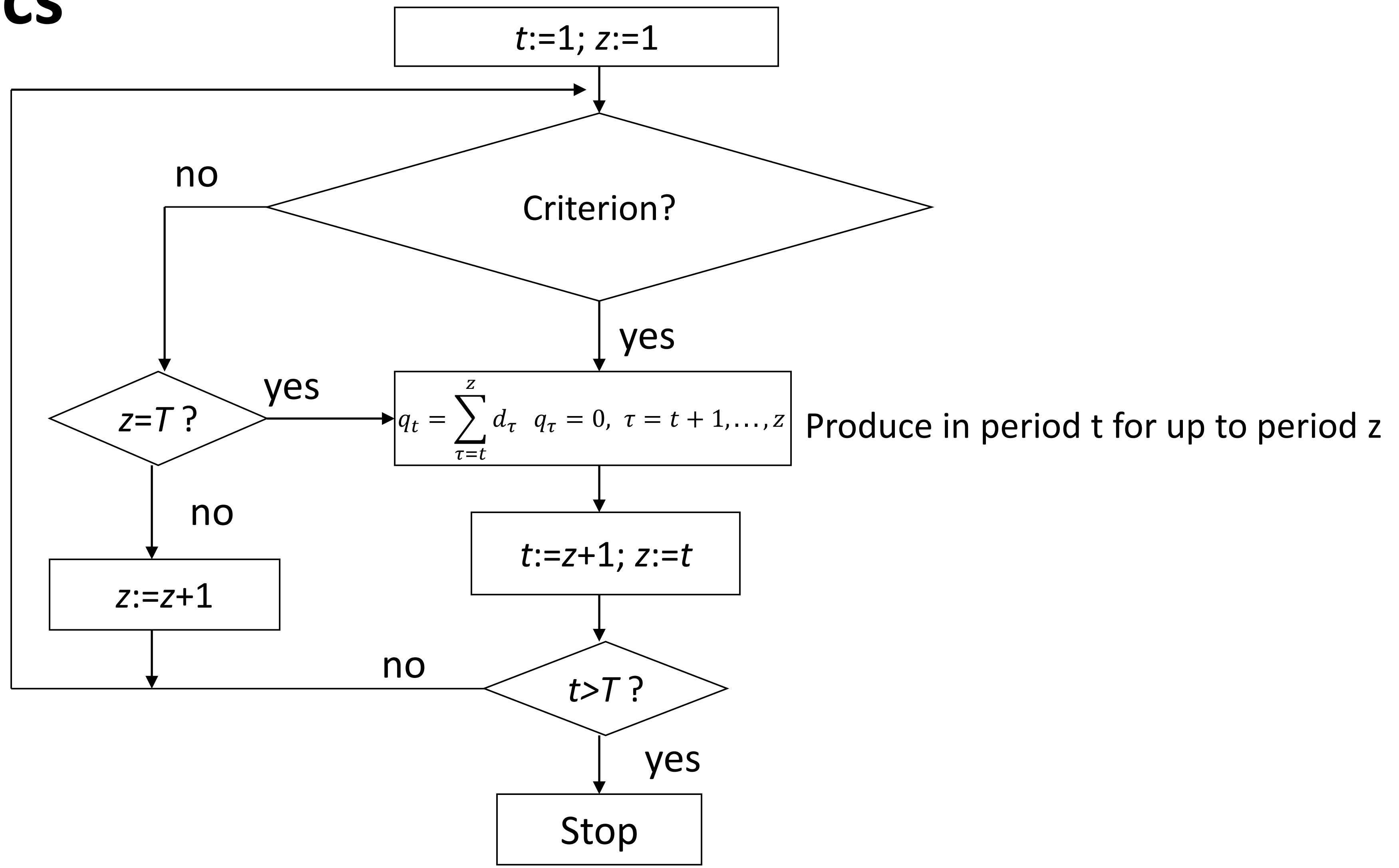
Wagner-Whitin Algorithm

Demand	750	100	50	100	400	1000
Make Period	1	2	3	4	5	6
1	**400**	**600**	800	1400	4600	14600
2		800	900	1300	3700	11700
3			1000	**1200**	2800	8800
4				1200	2000	6000
5					**1600**	3600
6						**2000**
Order	850	0	150	0	400	1000

Lot-sizing heuristics

<u>Algorithm:</u>

Successive extension of a lot by a future demand until termination criterion fulfilled



Lot-sizing heuristics

- Average demand $\bar{d} = \frac{1}{T}\sum_{t=1}^{T} d_t$

- Economic order interval (EOI) heuristic
  - Combine demands of EOI periods

$$EOI = \sqrt{\frac{2A}{\bar{d}h}} \qquad r = \max\{1; \text{round}(EOI)\} \qquad q_t = \begin{cases} \sum_{\tau=t}^{t+r-1} d_\tau & t = k \cdot r + 1, k = 0,1,2,\ldots \\ 0 & else \end{cases}$$

- Economic order quantity heuristic
  - Combine demands until lot-size comes closest to EOQ

$$EOQ = \sqrt{\frac{2\bar{d}A}{h}} \qquad z(t) := \text{argmin}\left\{ i = t, t+1, \ldots, T \,\middle|\, \left| \sum_{\tau=t}^{i} d_\tau - EOQ \right| \right\} \qquad q_t = \sum_{\tau=t}^{z(t)} d_\tau$$

Lot-sizing heuristics

- Least unit cost (LUC)
  - Extend z, until average cost per unit increases ($k_{t,z+1}$>k_{tz})

$$k_{tz} = \frac{A + h \cdot \sum_{\tau=t}^{z} (\tau - t) \cdot d_\tau}{\sum_{\tau=t}^{z} d_\tau}$$

- Silver-Meal (SM)
  - Extend z, until average cost per period increases ($k_{t,z+1}$>k_{tz})

$$k_{tz} = \frac{A + h \cdot \sum_{\tau=t}^{z} (\tau - t) \cdot d_\tau}{z - t + 1}$$

- Part period balancing (PP)
  - Extend z, until fixed cost and cumulative holding costs are (almost) equal

$$A \geq h \cdot \sum_{\tau=t}^{z} (\tau - t) \cdot d_\tau$$

$$A < h \cdot \sum_{\tau=t}^{z+1} (\tau - t) \cdot d_\tau$$

Example Silver-Meal (SM)

Period	1	2	3	4	5	6
Demand	750	100	50	100	400	1000

A=400, *h*=2

Include d_2 when producing in period 1?

- $k_{11} = A = 400,\ k_{12} = \frac{A+h*100}{2}$ =300
- $k_{12} < \mathrm{k}_{11}$

Include d_3 when producing in period 1?

- $k_{13} = \frac{A+h*100+2*h*50}{3}$ =266.67
- $k_{13} < \mathrm{k}_{12}$

Include d_4 when producing in period 1?

- $k_{14} = \frac{A+h*100+2*h*50+3*h*100}{4}$ =350
- $k_{14} > \mathrm{k}_{13}$

$$k_{tz} = \frac{A + h \cdot \sum_{\tau=t}^{z}(\tau - t) \cdot d_\tau}{z - t + 1}$$

Example Silver-Meal (SM) continued

- So, produce in period 1 for periods 1,2, and 3.
- Start again in period 4:
  - Include d_5 when producing in period 4?
  - $k_{44} = A = 400,\ k_{45} = \frac{A+h*400}{2} = 600$ ✗
- Start again in period 5:
  - Include d_6 when producing in period 5?
  - $k_{55} = A = 400,\ k_{56} = \frac{A+h*1000}{2} = 1200$ ✗
- So, produce in period 1 for periods 1,2, and 3; produce in period 4 for period 4; produce in period 5 for period 5; produce in period 6 for period 6

All heuristic solutions to this example

- Demands over the next 6 months

  750, 100, 50, 100, 400, 1000
- Setup cost: A=400
- Inventory holding cost per unit and month: h=2
- Results of the heuristics

See the Excel file!

	1	2	3	4	5	6	Cost
EOI	750	100	50	100	400	1000	2400
EOQ	750	250	0	0	400	1000	2100
LUC	750	150	0	500	0	1000	2500
SM	900	0	0	100	400	1000	2000
PP	900	0	0	100	400	1000	2000
Optimal	850	0	150	0	400	1000	2000

- A rolling-horizon & Demand Uncertainty: Why to use heuristics ?

Rank	Rule	Mean	Std. Dev.
1	WW	0	0
2	GMR	2·24	2·47
3	SM	3·06	3·83
4	WMR1	3·34	2·85
5	PPB w. LA-LB	4·09	3·70
6	WMR3	4·89	5·06
7	OM	4·93	5·10
8	PPB	5·74	5·18
9	WMR2	5·78	4·87
10	POQ	10·72	9·35
11	EOQ-D	13·06	12·69
12	LUC	17·16	18·02
13	EOQ	33·87	29·53
14	LFL	108·27	97·57

Mean and Std of relative cost increase (%) when forecast errors are zero

Rank	Rule	Mean	Std. Dev.
1	PPB	−0·67	4·91
2	WMR3	−0·57	4·94
3	WMR2	−0·26	4·95
4	OM	−0·25	4·25
5	WW	0	0
6	EOQ	0·19	9·24
7	WMR1	1·24	4·17
8	EOQ-D	1·27	8·41
9	GMR	1·45	4·34
10	PPB w. LA-LB	1·73	4·39
11	POQ	2·58	5·29
12	SM	2·71	5·89
13	LUC	6·02	14·59
14	LFL	63·71	69·70

Mean and Std of relative cost increase (%) when forecast errors are present

- WW: Wagner-Whitin Algorithm
- SM: Silver-Meal Procedure
- PPB w. LA-LB: Part Period Balancing with Look-Ahead & Look-Back
- POQ: Period Order Quantity
- EOQ: Economic Order Quantity
- EOQ-D: Discrete Economic Order Quantity
- OM: Order Moment Procedure
- PPB: Part Period Balancing
- LUC: Least Unit Cost
- LFL: Lot-for-Lot
- GMR: Groff Method
- WMR1: Wemmerloev Method 1
- WMR2: Wemmerloev Method 2
- WMR3: Wemmerloev Method 3

If uncertainty and a rolling schedule are present, it is no longer obvious that WW should be used as a reference rule

Wemmerlöv, U., & Whybark, D. C. (1984). Lot-sizing under uncertainty in a rolling schedule environment. *The International Journal Of Production Research*, *22*(3), 467-484.

• A rolling-horizon & Demand Uncertainty: Why to use heuristics ?

Percentage Deviation from Optimality
Uniform Distribution, $P_t = .2$, Set-Up Cost = 800

	R = 0				R = 35				R = 75				R = 150			
Window	WW	MSM	SM	PP	WW	MSM	SM	PP	WW	MSM	SM	PP	WW	MSM	SM	PP
2	25.04	25.04	25.04	25.04	25.61	25.61	25.61	25.61	26.55	26.55	26.55	26.55	29.74	29.74	29.74	29.74
3	5.29	5.29	5.29	5.29	5.70	5.70	5.70	5.70	6.50	6.50	6.50	6.50	9.26	9.32	9.32	9.26
4	3.34	3.99	3.99	3.34	3.70	4.05	4.05	3.70	4.49	3.72	3.72	4.49	7.30	4.40	4.40	7.26
5	6.81	2.46	3.99	4.44	6.74	2.18	4.05	4.65	6.27	2.02	3.61	5.27	6.93	2.48	3.49	7.42
6	3.72	1.40	3.99	4.67	3.90	1.37	4.05	5.09	4.27	1.59	3.61	5.74	3.89	1.90	3.35	7.58
7	2.00	1.40	3.99	4.93	1.56	1.35	4.05	5.26	1.59	1.46	3.61	5.96	1.85	1.81	3.27	7.76
8	1.23	1.31	3.99	4.93	1.31	1.28	4.05	5.26	1.34	1.48	3.61	5.96	1.22	1.77	3.27	7.76
9	.56	1.31	3.99	4.93	1.03	1.34	4.05	5.26	.85	1.46	3.61	5.96	.67	1.71	3.27	7.76
10	.66	1.32	3.99	4.93	.57	1.35	4.05	5.26	.64	1.51	3.61	5.96	.68	1.76	3.27	7.76

- WW: Wagner-Whitin Algorithm / MSM: Modified Silver-Meal Procedure / SM: Silver-Meal Procedure / PPB w. LA-LB: Part Period Balancing

- With short forecast horizons, SM heuristics outperform WW

- While heuristics may provide less effective than WW in a static framework, their myopia reduces the amount of schedule instability in a rolling-horizon.

Blackburn, J. D., & Millen, R. A. (1980). Heuristic lot-sizing performance in a rolling-schedule environment. *Decision Sciences*, *11*(4), 691-701.