A simple interactions-based economic model

Maxim Gusev and Dimitri Kroujiline

Overview

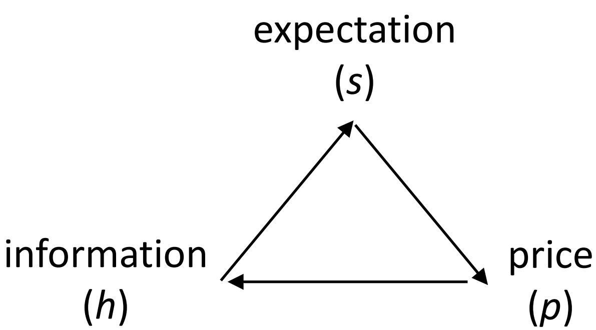
I. News-driven stock market model

II. Solow model with interactions

III. Research agenda

Interactions-based stock market model (*Gusev et al., 2015*)

The core of the model: feedback mechanism



Evidence of price feedback:
Shiller (2003); Greenwood & Shleifer (2014)

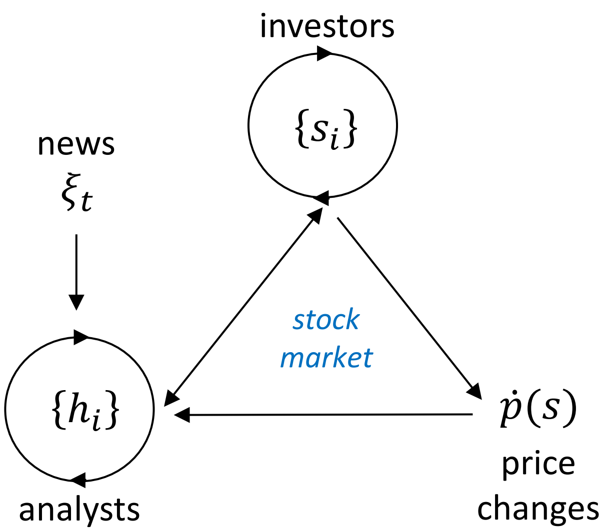
Micro-level formulation: generalized Ising model

$\{s_i\}$ - investors ($s_i = \pm 1$)

$\{h_i\}$ - "analysts" ($h_i = \pm 1$)

ξ_t - exogenous news (noise)

$\dot{p}$ - price changes



Agents (investors and analysts) interact by co-aligning expectations

Investors invest or divest based on their expectations

Analysts' expectations are influenced by news about market price changes and by exogenous news

Expectations are subject to random fluctuations due to idiosyncratic influences

Macro-level equations: noise-driven dynamical system

Analytic solution to the generalized Ising model in mean-field approximation:

$$\tau_s \dot{s} = -s + \tanh(\beta_1 s + \beta_2 h) \qquad (-1 \le s \le 1)$$

$$\tau_h \dot{h} = -h + \tanh(\cancel{\beta_3 s + \beta_4 h} + \gamma_1 \dot{p} + \zeta_t) \qquad (-1 \le h \le 1)$$

$\zeta_t = \varepsilon + \xi_t; \quad \varepsilon > 0; \quad \xi_t$ - noise

Phenomenological relation for log price p:

$\dot{p} \sim \dot{s}$ for $t \ll \tau_s$

$\dot{p} \sim s$ for $t \gg \tau_s$

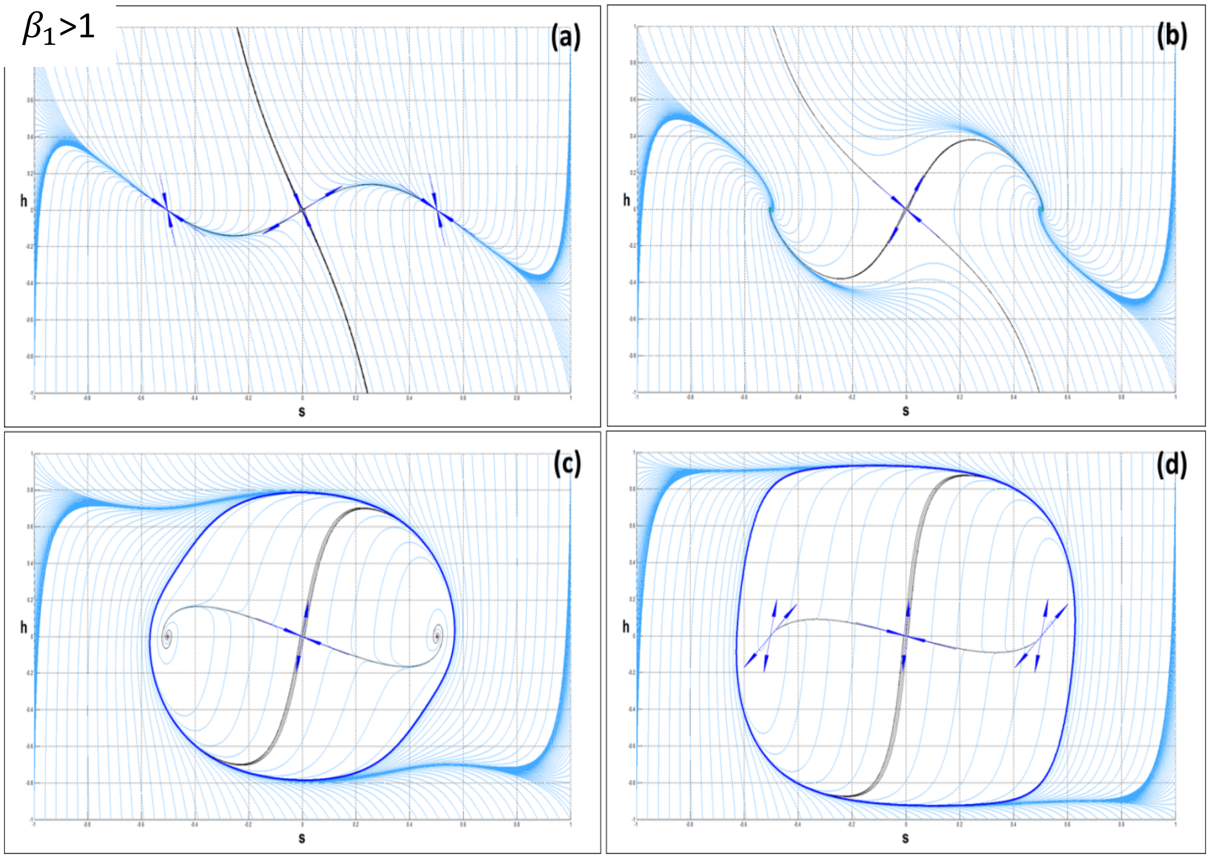
$$\rightarrow \boxed{\dot{p} = c_1 \dot{s} + c_2 (s - s_*)}$$

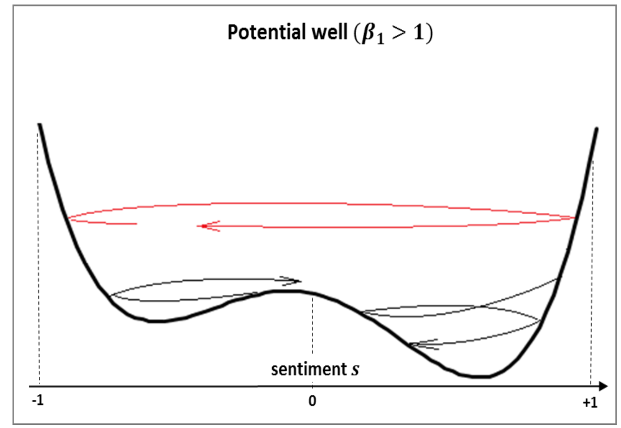
Detailed derivation is provided in Gusev et al. (2015) and Kroujiline et al. (2018)

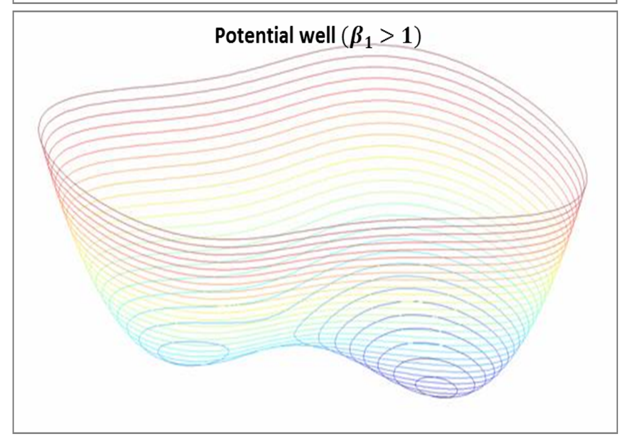
Phase portraits for different parameters

$$\tau_s \ddot{s} + G(s,\dot{s})\dot{s} + \frac{\mathrm{d}U(s)}{\mathrm{d}s} = F(\xi(t))$$

β_1>1

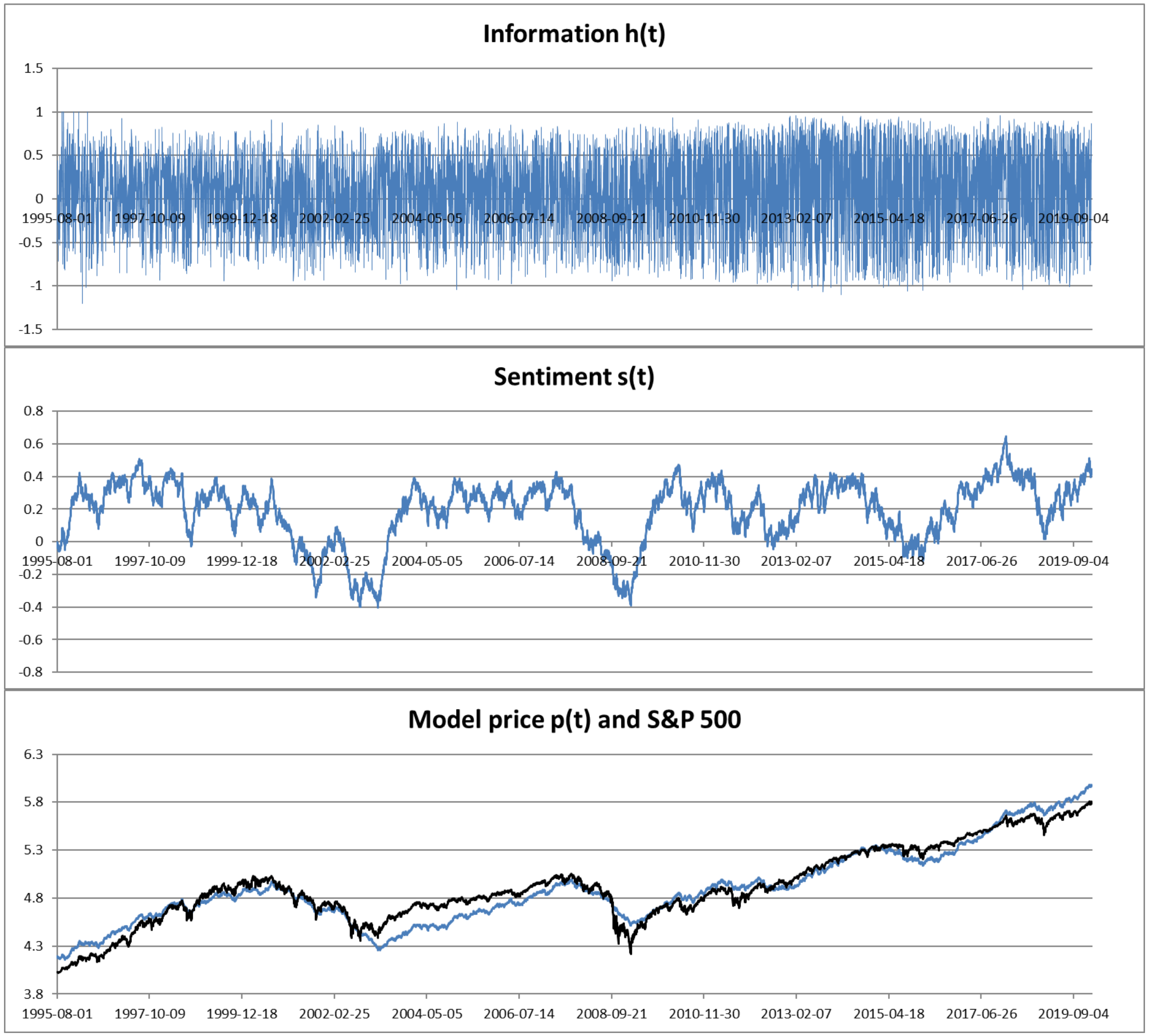






Empirical data: calibration of parameters

US stock market example



empirical $h(t)$ measured in daily news flow as the ratio of the number of news items with positive return expectations minus the number of news items with negative return expectations over the total number of relevant news items

$$\tau_s \dot{s} = -s + \tanh(\beta_1 s + \beta_2 h(t))$$

(β_1=1.1, β_2=1.0, τ_s=1 month)

$$\dot{p} = c_1 \dot{s} + c_2 (s - s_*)$$

Quasiperiodic market cycles as coherence resonance

Theoretical model

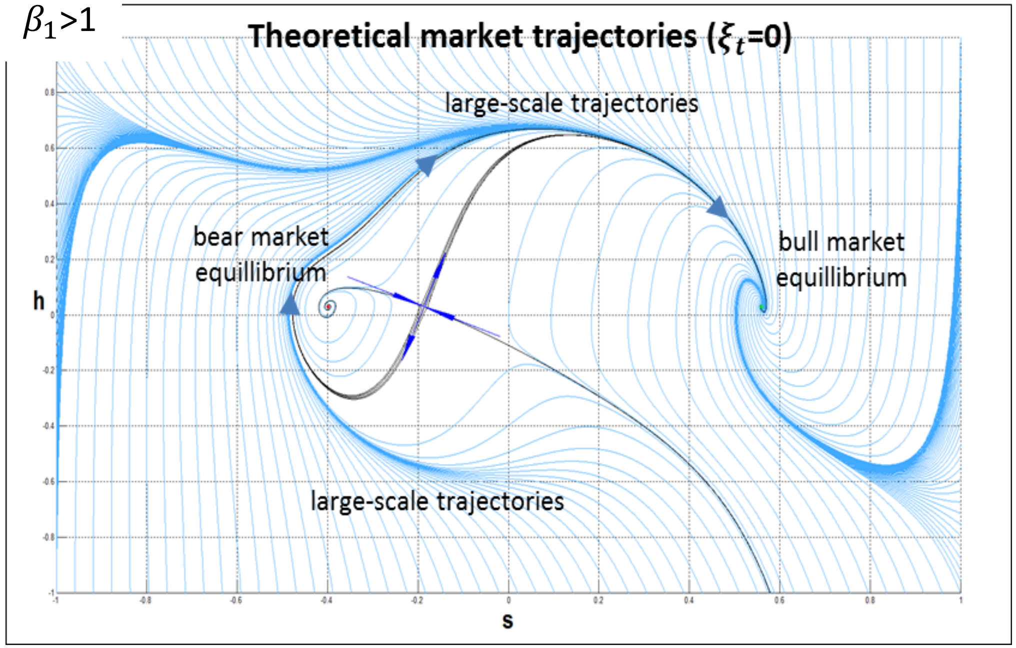
$$\tau_s \dot{s} = -s + \tanh(\beta_1 s + \beta_2 h)$$

$$\tau_h \dot{h} = -h + \tanh\big(\gamma_1 \dot{p} + \zeta(t)\big)$$

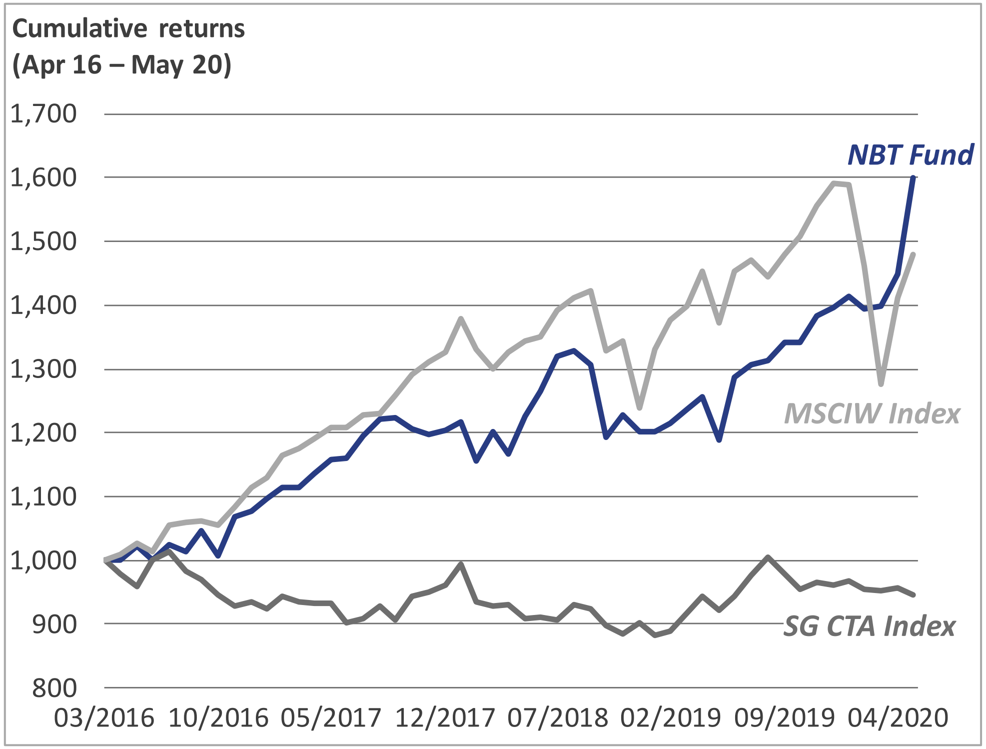
$$\dot{p} = c_1 \dot{s} + c_2 (s - s_*)$$

$\zeta_t = \varepsilon + \xi_t;\quad \varepsilon > 0;\quad \xi_t$ - noise

β_1>1



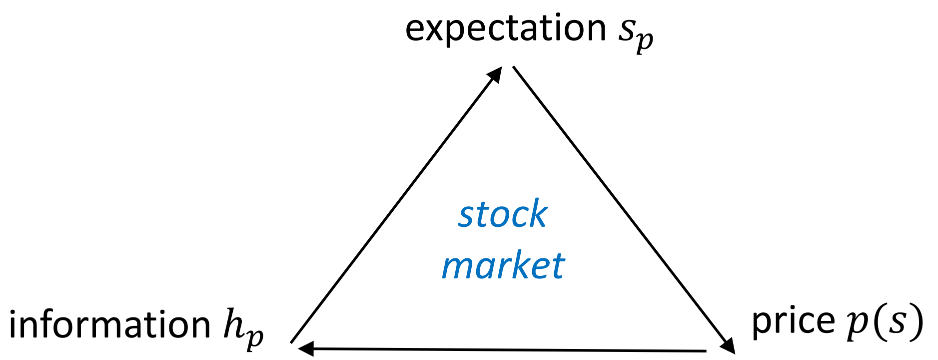
Return forecasting (live track)

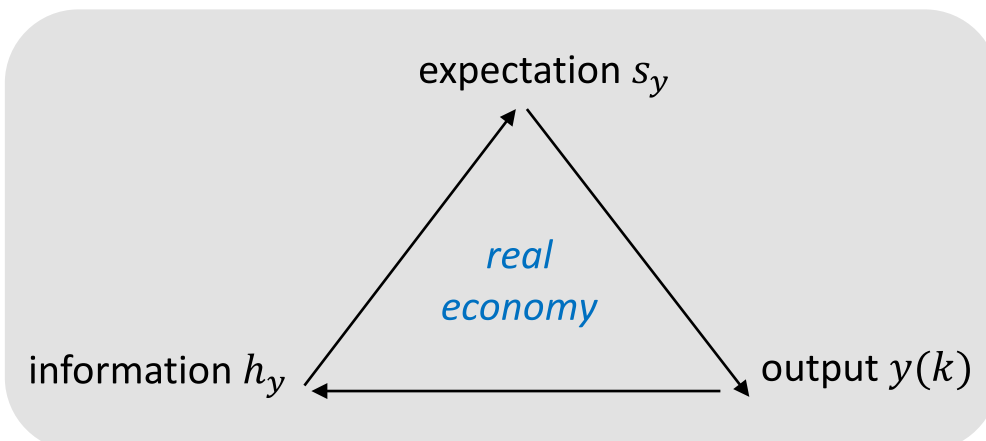


Source: LGT Capital Partners, Datastream.
LGT AI News-Based Trading Sub-Fund ("**NBT Fund**") data from 31 Mar 2016 to 31 May 2020 is the actual performance track record in USD net of management and performance fees.

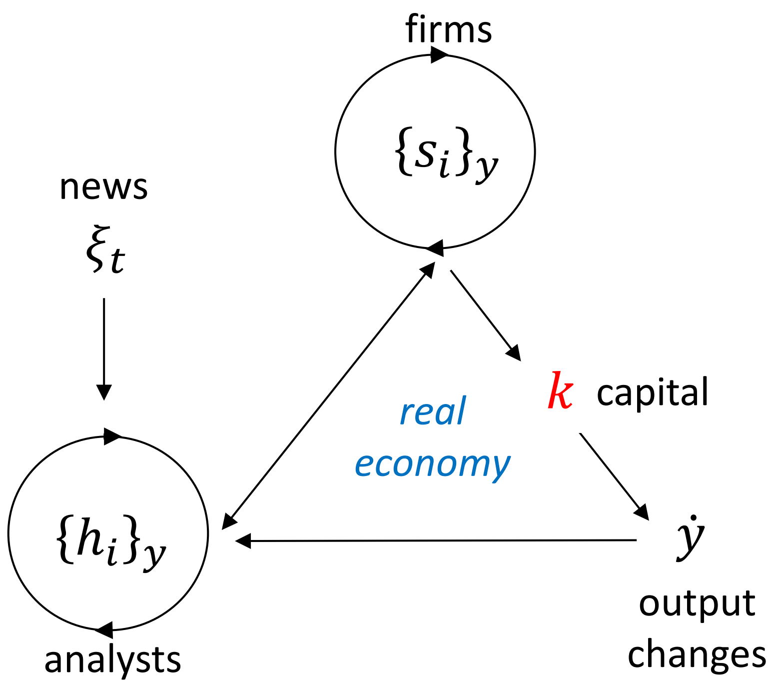
Overview

Similar feedbacks at work





Dynamics of expectations via interactions



Agents interact by co-aligning expectations

Firms make investment decisions based on their expectations and deploy capital k *subject to borrowing constraints*

Analysts' expectations are influenced by news about changes in economic output and by exogenous news

Expectations are subject to random fluctuations due to idiosyncratic influences

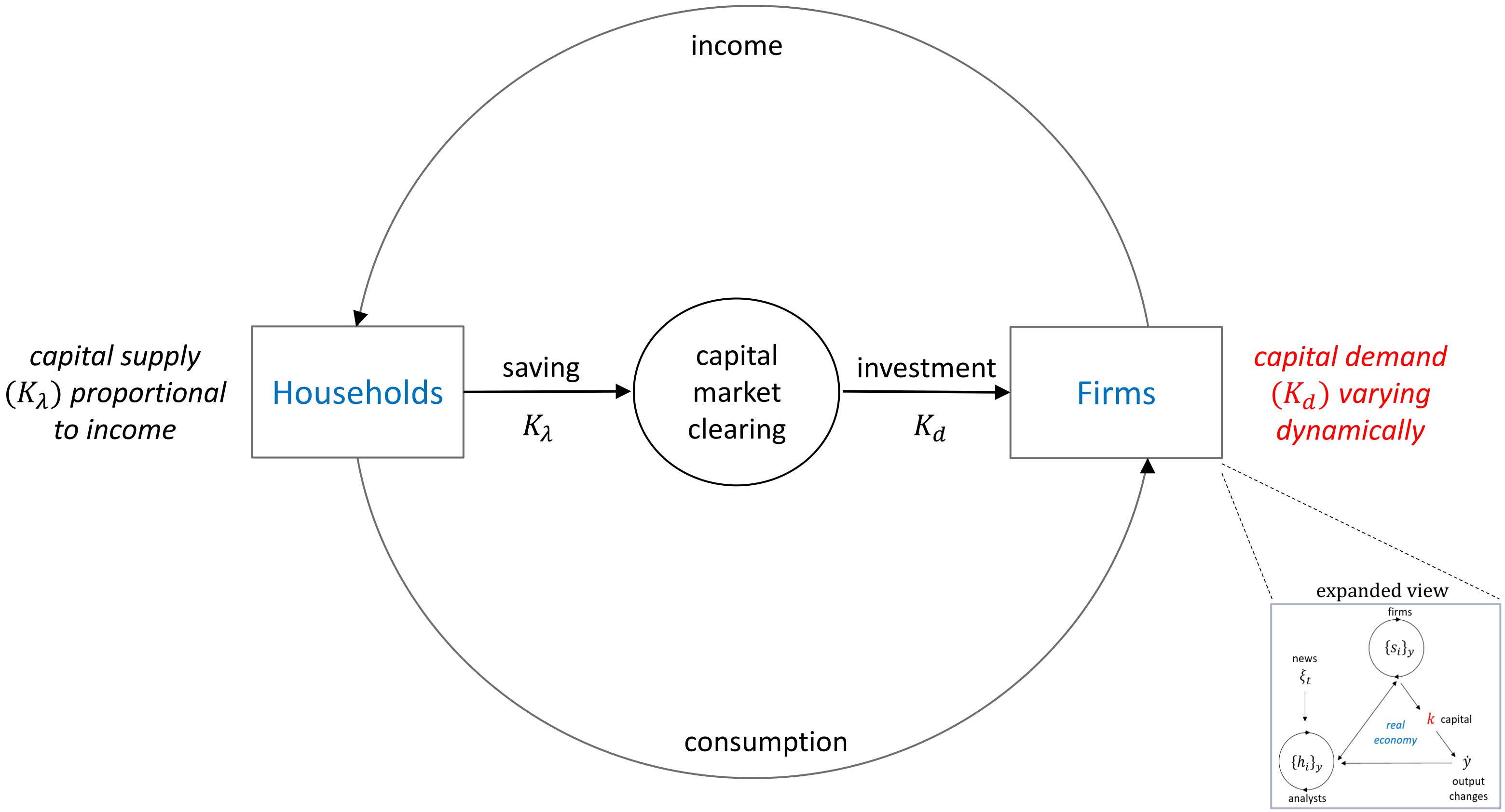
$\{s_i\}_y$ - firms with expectations: $s_i = \pm 1$

$\{h_i\}_y$ - analysts with expectations: $h_i = \pm 1$

ξ_t- industry-relevant news flow

$\dot{y}$ - economic growth (as a function of k)

Modified Solow model: dynamic capital demand



Model equations adapted to work outside equilibrium

Output production $(\boldsymbol{Y})$

$$Y = F(K) = cK^{\alpha}e^{\varepsilon t}$$

Equilibrium relation with Cobb-Douglas production

$$\tau_y\dot{Y} = -Y + cK^{\alpha}e^{\varepsilon t}$$

Dynamic extension: Y adjusts to changes in K over time τ_y

Capital supply $(\boldsymbol{K_\lambda})$

$$\dot{K}_\lambda = \lambda Y - \delta K_\lambda$$

Constant saving and capital depreciation

Market clearing $(\boldsymbol{K})$

$$K = \begin{cases} K_\lambda\,, & K_d > K_\lambda \\ K_d\,, & K_d < K_\lambda \end{cases}$$

Inelastic supply K_λ and demand K_d: binary clearing

Limit 1: classic growth regime $(K_d > K_\lambda)$

Market clearing: invested capital driven by **supply dynamics**

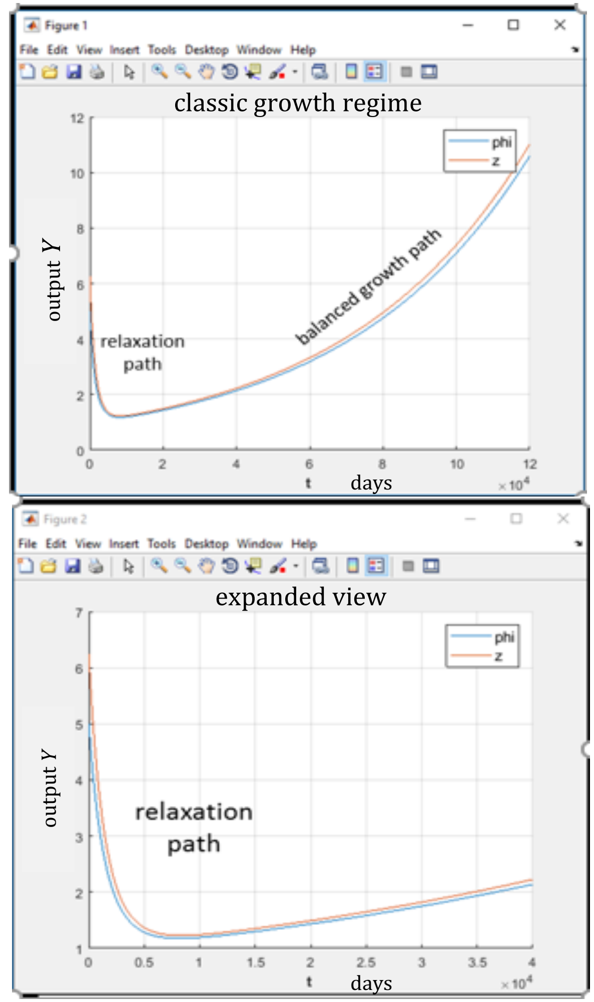
$$K = K_\lambda$$

Model equations: (i) capital supply & (ii) output production

$$\tau_y \ddot{K} + (1 + \tau_y \delta)\dot{K} + \delta K = \lambda K^\alpha e^{\varepsilon t} \qquad (1 \ll \tau_y \ll 1/\varepsilon)$$

Approximate solution:

$$Y(t) = \left(\frac{\lambda}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\left(A e^{-\left(\frac{1-\alpha}{\tau_y}\right)t} + 1 \right)^{\frac{1}{1-\alpha}} + e^{\left(\frac{\varepsilon}{1-\alpha}\right)t} - 1 \right)$$



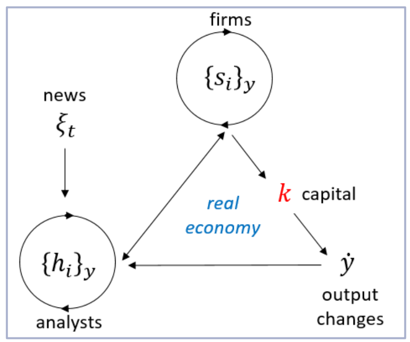
$\alpha = 0.5, \tau_y = 1'000$ days, $1/\varepsilon = 100'000$ days

Limit 2: business cycle regime $(K_d < K_\lambda)$

Market clearing:
invested capital driven
by **demand dynamics**

$$K = K_d$$

Demand dynamics
stemming from firms'
expectations dynamics
caused by interactions



$y = \ln Y, k = \ln K$

$$\dot{k} = c_1 \dot{s} + c_2 (s - s_*)$$

$$\tau_s \dot{s} = -s + \tanh(\beta_1 s + \beta_2 h))$$

$$\tau_h \dot{h} = -h + \tanh(\gamma \dot{y} + \xi_t)$$

Output production

$$\tau_y \dot{y} = c e^{\alpha k + \varepsilon t - y} - 1$$

Noise-driven dynamical system

Introduce $z = k + \varepsilon t - y$ to obtain a *bounded* 3D system:

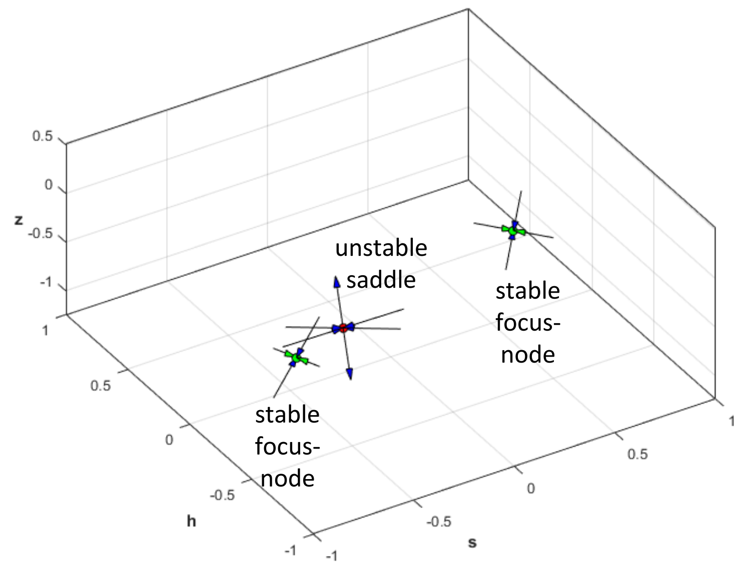
$$\dot{z} = c_1 \dot{s} + c_2(s - s_*) - \omega_y(ce^z - 1) + \varepsilon$$

$$\tau_s \dot{s} = -s + \tanh(\beta_1 s + \beta_2 h)$$

$$\tau_h \dot{h} = -h + \tanh\big(\gamma \omega_y(ce^z - 1) + \xi_t\big)$$

(with $\omega_y = 1/\tau_y$; $\tau_h \ll \tau_s \ll \tau_y \ll 1/\varepsilon$)

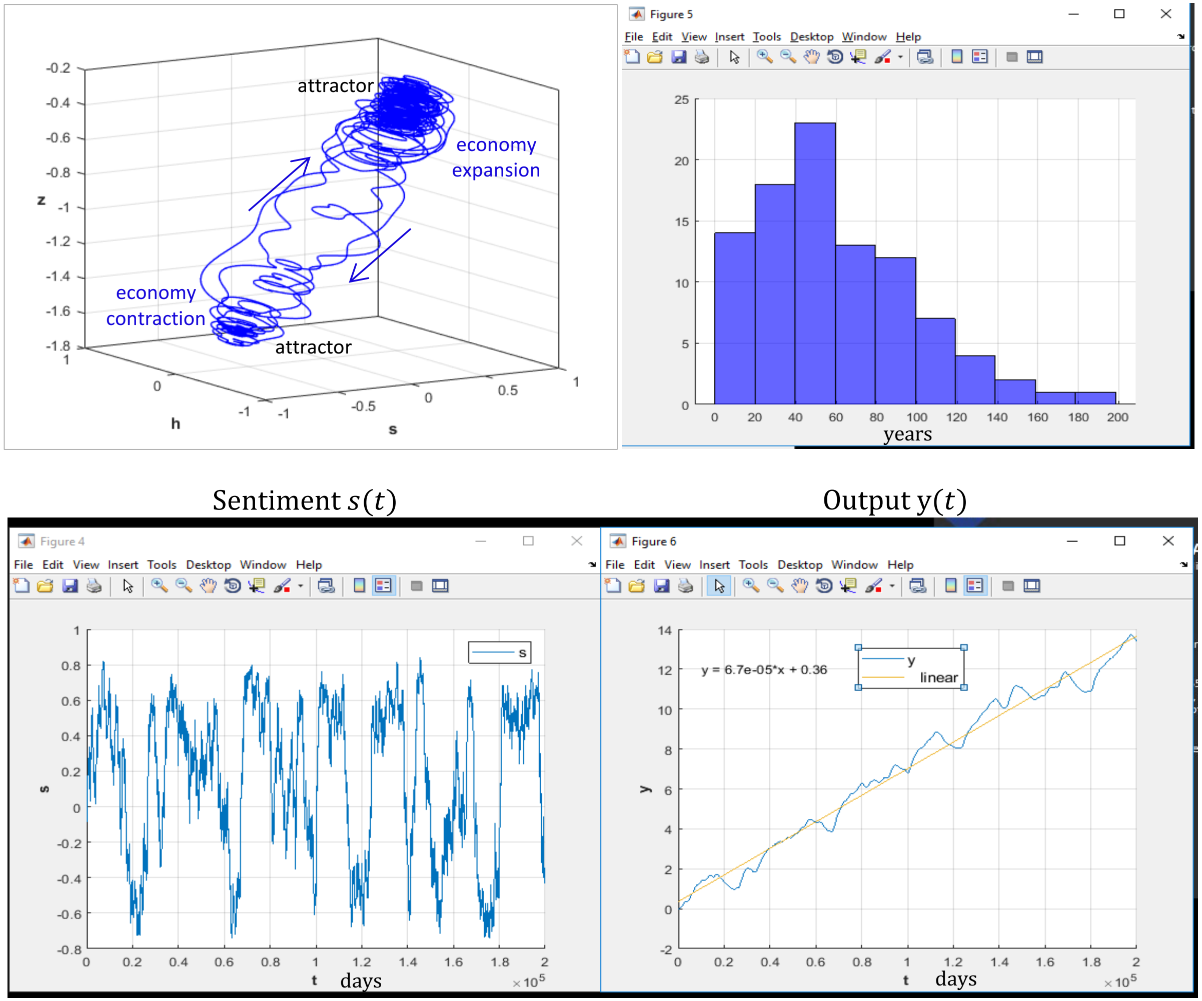
Equilibrium points in the 3D phase space



Business cycles as quasiperiodic fluctuations

Simulated trajectory in 3D phase space

Histogram of business cycle lengths

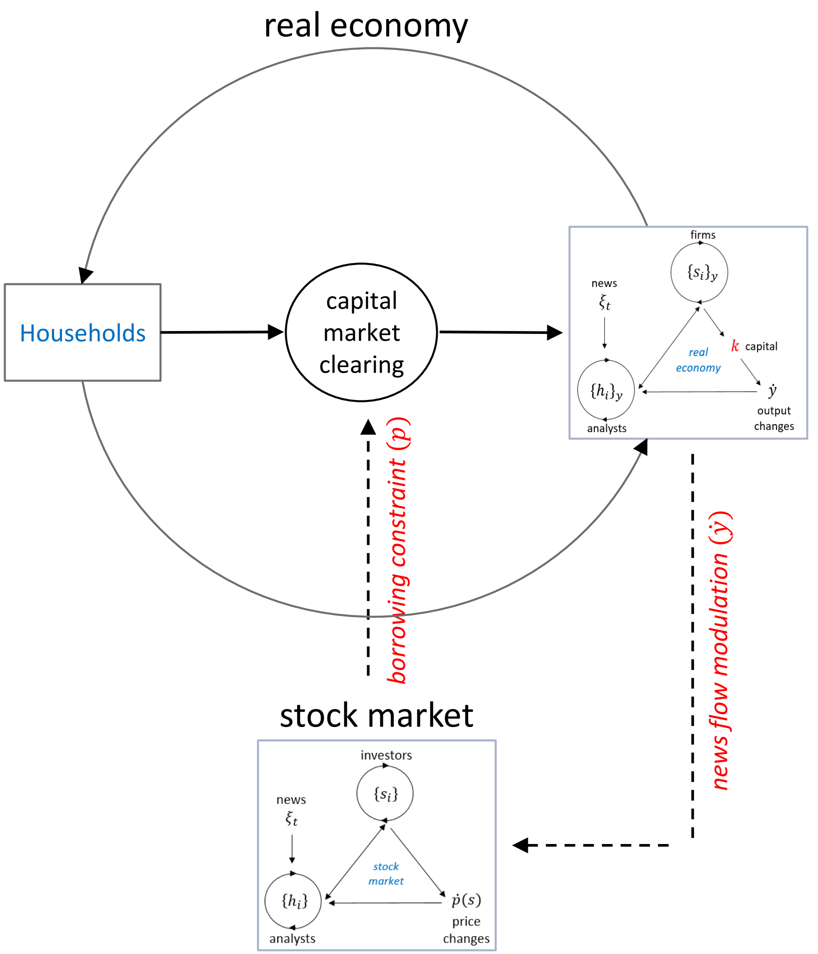


Overview

Project 1: Study of the modified Solow model

- *Two limiting cases examined: (i) supply-driven balanced growth and (ii) demand-driven business cycles*
- Investigate a general case where supply and demand dynamics can replace each other
- Study the impact of a variable policy-driven saving rate
- Compare with empirical data (e.g. inventory investment dynamics)
- Introduce further modifications/additions (e.g. labor)

Project 2: Coupled real economy and stock market



Real economy and stock market interlinked via

- credit frictions in the form of borrowing constraints dependent on firms' market valuation
- modulation of news flow relevant to the stock market by economic growth

More realistic, diverse behaviors*

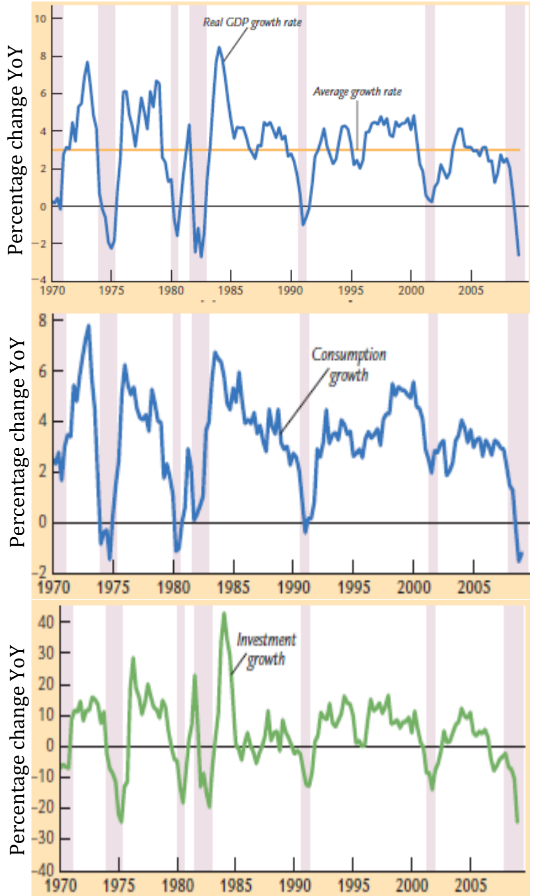
** Kroujiline et al. (2018) considered a simplified coupled real economy – stock market system, which is a limiting case of this model, obtaining some interesting dynamics.*

Project 3: Closed-form *realistic* macro model with interactions

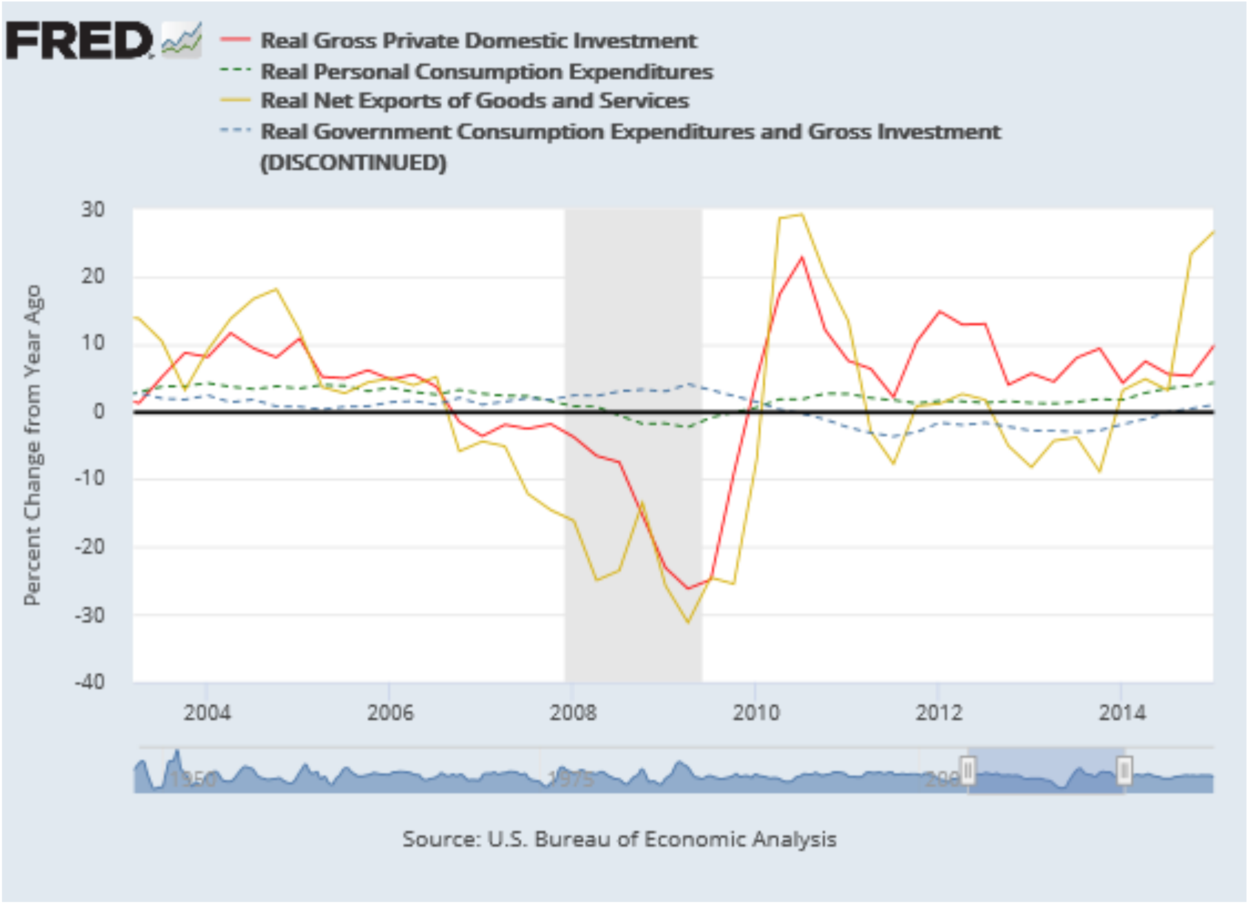
Can the modified Solow model serve as an elementary building block for developing a realistic (but tractable) macro model with interactions to help better understand disequilibrium behaviors and economic instability?

Appendix

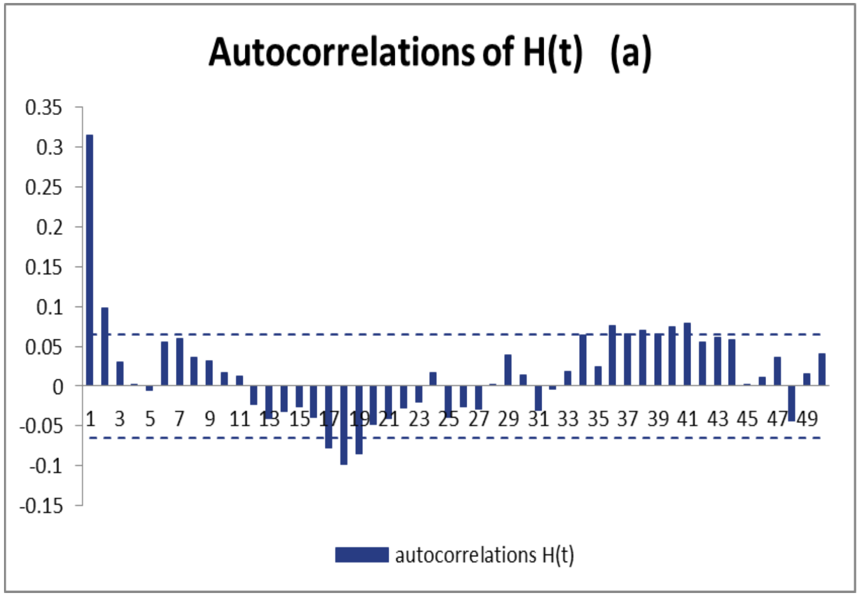
GDP components

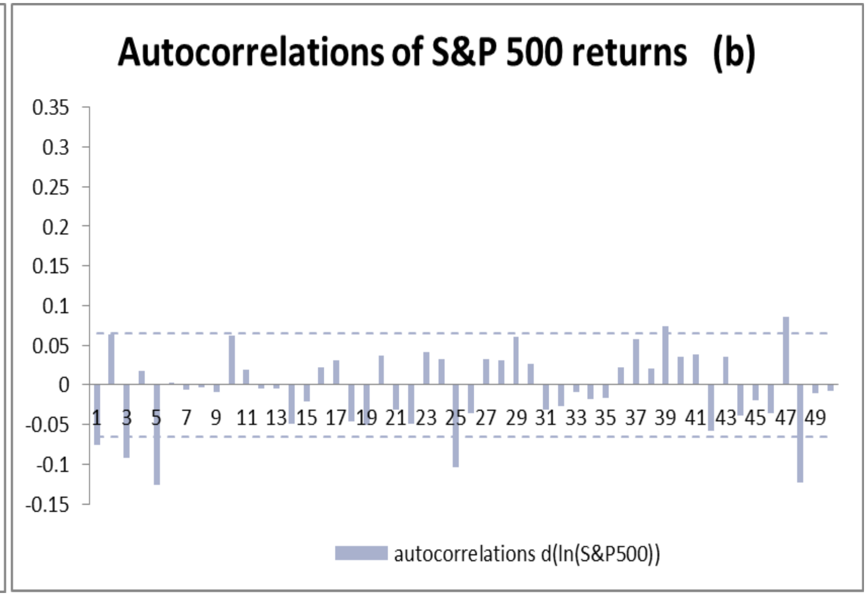


Source: Mankiw "Macroeconomics"

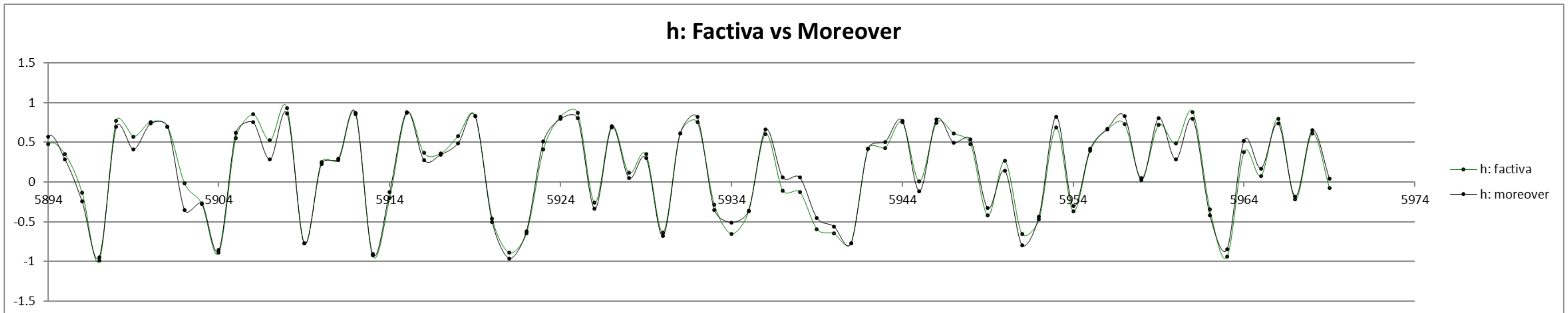


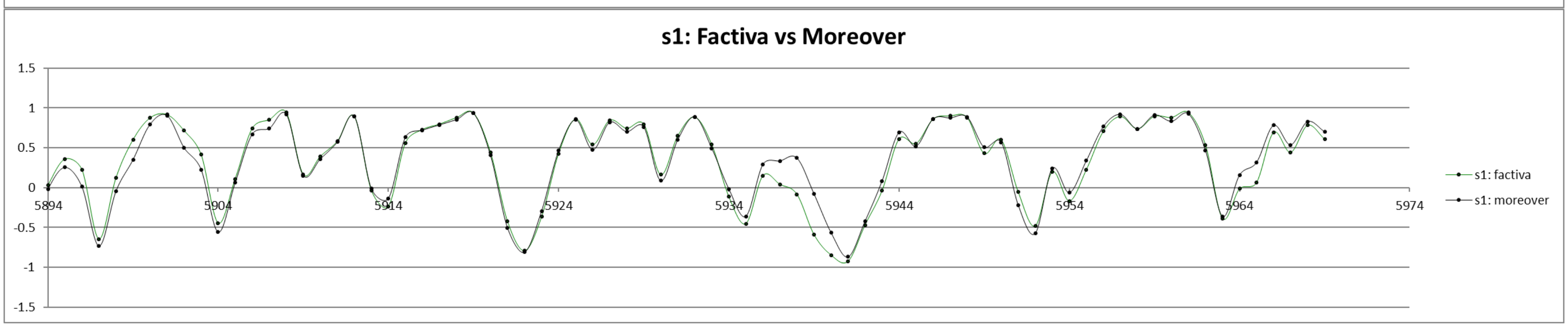
Serial correlation of empirical $h(t)$ across different time lags





H and short-term sentiment (S1)





Equation for sentiment dynamics

N_+ is the number of investors who expect that the market will rise ("+") -> optimists

N_- is the number of investors who expect that the market will fall ("-") -> pessimists

$N = (N_+ + N_-) \gg 1$

$n_+ = N_+/N \qquad n_- = N_-/N \qquad \rightarrow\ n_+(t) + n_-(t) = 1$

p^{-+} and p^{+-} are transition probabilities

$$n_+(t+\Delta t) = n_+(t) + \Delta t\big(n_-(t)p^{-+}(t) - n_+(t)p^{+-}(t)\big)$$

$$n_-(t+\Delta t) = n_-(t) + \Delta t\big(n_+(t)p^{+-}(t) - n_-(t)p^{-+}(t)\big)$$

$s(t) = n_+(t) - n_-(t)$ - average sentiment per investor at time t

$\rightarrow\ \dot{s} = (1-s)p^{-+} - (1+s)p^{+-}$

Equation for sentiment dynamics (continued)

First condition on p^{-+} and p^{+-}: $\frac{\mathrm{d}(p^{-+}/p^{+-})}{p^{-+}/p^{+-}} = \alpha \mathrm{d}F$ $\Rightarrow$ $\frac{p^{-+}}{p^{+-}} = e^{\alpha F}$ (*Weidlich and Haag, 1983*)

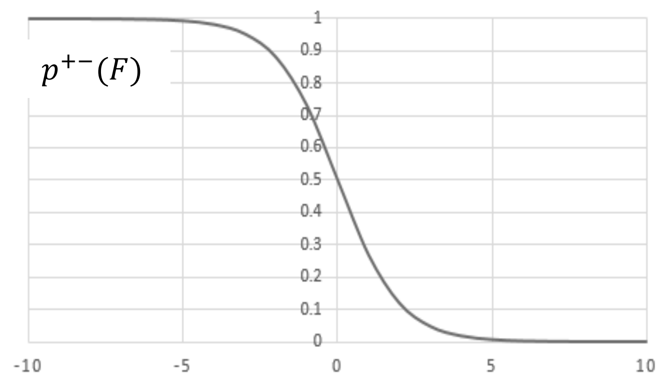
Second condition to determine p^{-+} and p^{+-} uniquely: $(p^{-+} + p^{+-})\tau_s = 1$

(τ_s is average time over which individual sentiments flip)

Final transition probabilities:

$$p^{+-} = \frac{1}{\tau_s(1 + e^{\alpha F})}$$

$$p^{-+} = \frac{1}{\tau_s(1 + e^{-\alpha F})}$$



Define $F = \beta_1 s(t) + \beta_2 h(t)$ $\Rightarrow$ $\tau_s \dot{s} = -s + \tanh(\beta_1 s + \beta_2 h)$ (*Suzuki and Kubo, 1968*)

mean field mean field

Credit frictions

Assume credit frictions where firms' access to credit depends on their market value; on aggregate level, stock market price imposes a borrowing constraint (e.g. *Winkler, 2016*). In a simple form:

$$\bar{K} = \begin{cases} K, & K < aP \\ aP, & K \geq aP \end{cases} \qquad (0 < a \leq 1)$$

Or written in log variables:

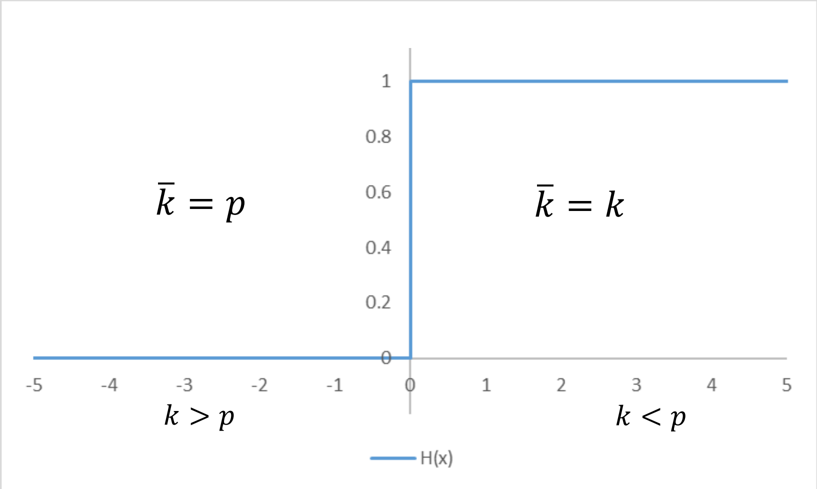
$$\bar{k} = \begin{cases} k, & k < p \\ p, & k \geq p \end{cases} \qquad (a = 1 \text{ for simplicity})$$

Borrowing constraint

Rewrite the borrowing constraint as

$$\bar{k} = p - (p - k)\mathrm{H}(p - k)$$

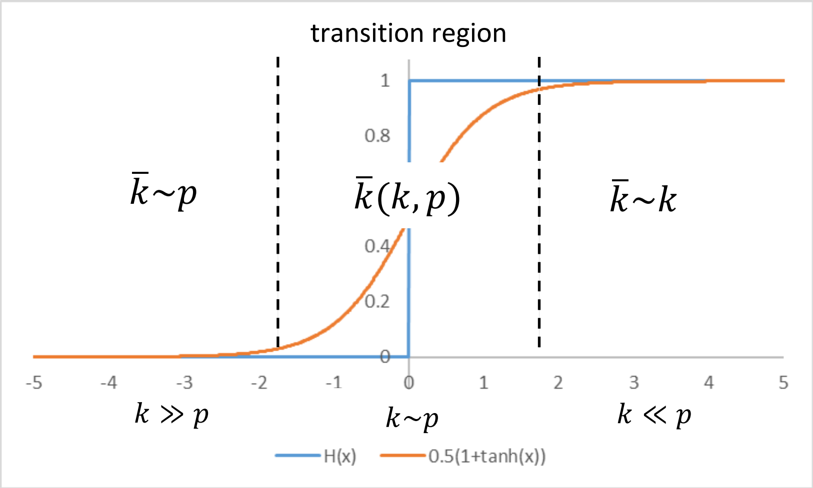
where H(x) is the Heaviside step function.



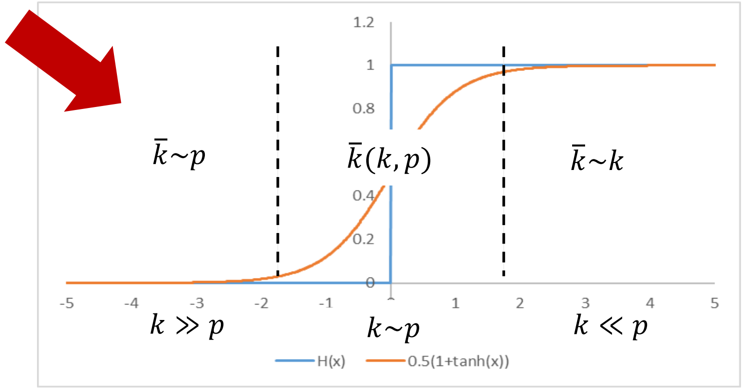
More realistic to replace H(x) by $\frac{1}{2}(1 + \tanh(\mu x))$:

$$\boxed{\bar{k} = p - (p - k)\frac{1}{2}(1 + \tanh(p - k))}$$

where $\mu = 1$ for simplicity.



Case 1: borrowing constraint severely restricts investment



$\rightarrow \tau_y' \dot{y} = e^{\alpha\bar{k}-y} - b = e^{\alpha p - y} - b$ (*Blanchard, 1981*)

Macroeconomic model

Stock market

$$\dot{p} = c_1 \dot{s}_p + c_2\left(s_p - s_p^*\right)$$

$$\tau_s \dot{s}_p = -s_p + \tanh\left(\beta_1 s_p + \beta_2 h_p\right)$$

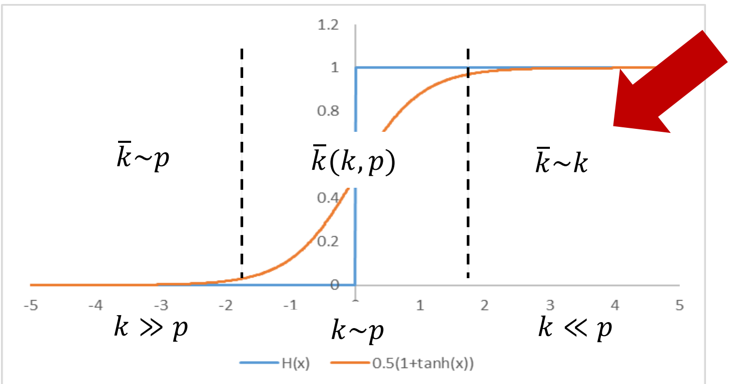
$$\tau_h \dot{h}_p = -h_p + \tanh\left(\gamma_1 \dot{p} + \gamma_2 \dot{y} + \xi_p\right)$$

Real economy

$$\tau_y' \dot{y} = e^{\alpha p - y} - b$$

The model generates quasiperiodic endogenous fluctuations at business cycle frequencies (*Kroujiline et al., 2018*).

Case 2: borrowing constraint is immaterial



$$\rightarrow \tau_y'\dot{y} = e^{\alpha\bar{k}-y} - b = e^{\alpha k - y} - b$$

Macroeconomic model

Stock market

$$\dot{p} = c_1\dot{s}_p + c_2\left(s_p - s_p^*\right)$$

$$\tau_s\dot{s}_p = -s_p + \tanh\left(\beta_1 s_p + \beta_2 h_p\right)$$

$$\tau_h\dot{h}_p = -h_p + \tanh\left(\gamma_1\dot{p} + \gamma_2\dot{y} + \xi_p\right)$$

Real economy

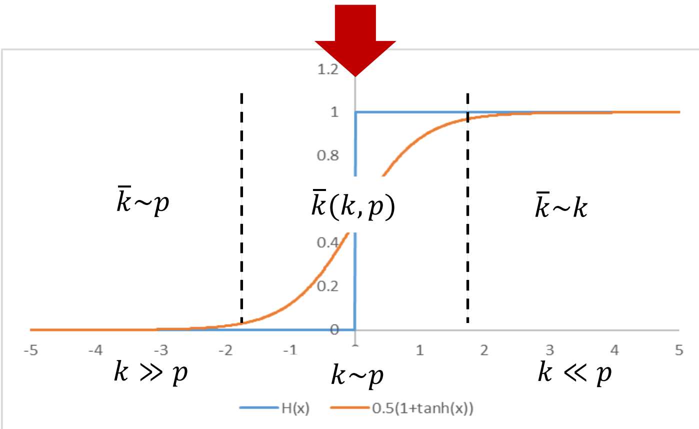
$$\tau_y'\dot{y} = e^{\alpha k - y} - b$$

$$\dot{k} = c_1'\dot{s}_y + c_2'\left(s_y - s_y^*\right)$$

$$\tau_s'\,\dot{s}_y = -s_y + \tanh\left(\beta_1' s_y + \beta_2' h_y\right)$$

$$\tau_h'\,\dot{h}_y = -h_y + \tanh\left(\gamma_1'\dot{y} + \xi_y\right)$$

Case 3: general situation



$$\rightarrow \tau_y' \dot{y} = e^{\alpha \bar{k} - y} - b$$

$$\rightarrow \bar{k} = p - (p-k)\frac{1}{2}(1 + \tanh(p-k))$$

Macroeconomic model

Stock market

$$\dot{p} = c_1 \dot{s}_p + c_2 (s_p - s_p^*)$$

$$\tau_s \dot{s}_p = -s_p + \tanh(\beta_1 s_p + \beta_2 h_p)$$

$$\tau_h \dot{h}_p = -h_p + \tanh(\gamma_1 \dot{p} + \gamma_2 \dot{y} + \xi_p)$$

Real economy

$$\tau_y' \dot{y} = e^{\alpha \bar{k} - y} - b$$

$$\dot{k} = c_1' \dot{s}_y + c_2' (s_y - s_y^*)$$

$$\tau_s' \dot{s}_y = -s_y + \tanh(\beta_1' s_y + \beta_2' h_y)$$

$$\tau_h' \dot{h}_y = -h_y + \tanh(\gamma_1' \dot{y} + \xi_y)$$

References

Gusev, M., Kroujiline, D., Ushanov D. 2019. *Tractable interactions-based macroeconomic model with micro-foundations*. Presentation at the Instability Workshop of Rebuilding Macroeconomics Initiative, Warwick University (https://www.rebuildingmacroeconomics.ac.uk/wp-content/uploads/2019/06/Gusev-Kroujiline-Slides-Warwick-June-2019_compressed.pdf).

Gusev, M., Kroujiline, D., Govorkov, B., Sharov, S. V., Ushanov D., Zhilyaev, M. 2015. *Predictable markets? A news-driven model of the stock market*. Algorithmic Finance, 4, 5-51.

Kroujiline, D., Gusev, M., Ushanov, D., Sharov, S. V., Govorkov, B. 2018. *An endogenous mechanism of business cycles*. Forthcoming in Algorithmic Finance.

Kroujiline, D., Gusev, M., Ushanov, D., Sharov, S. V., Govorkov, B. 2016. *Forecasting stock market returns over multiple time horizons*. Quantitative Finance, 16, 1695-1712.