Taking Banks to Solow*

Hans Gersbach
CER-ETH – Center of Economic Research at ETH Zurich and CEPR
Zürichbergstrasse 18
8032 Zurich, Switzerland
hgersbach@ethz.ch

Jean-Charles Rochet
Institute of Banking and Finance at University of Zurich
Plattenstrasse 14
8032 Zurich, Switzerland
jean-charles.rochet@bf.uzh.ch

Martin Scheffel
CMR – Center for Macroeconomic Research at University of Cologne
Albertus-Magnus-Platz
50923 Cologne, Germany
scheffel@wiso.uni-koeln.de

This Version: October 2014

Abstract

We develop a simple integration of banks in the Solow model. The objective is to provide a tractable benchmark. A fraction of firms have to rely on banks for financing their investments while banks face themselves an endogenous leverage constraint. Informed lending by banks and uninformed lending through capital markets spur capital accumulation. The ensuing coupled accumulation rules for household wealth and bank equity yield a uniquely determined steady state. We highlight three properties when shocks to wealth, productivity or trust affect the economy. First, typically bond and loan financing react in opposite directions to such shocks. Second, negative temporary shocks to household wealth(financial crisis) or negative sectoral production shocks can cause persistent booms of banking and even of the entire economy – after an initial bust. Third shocks to bank equity (banking crisis), however, lead to large and persistent downturns associated with high output losses.

JEL: E21, E32, F44, G21, G28

Keywords: Solow, financial intermediation, impact of banking and financial crises

*We would like to thank Tobias Adrian, Salomon Faure, Kamali Wickramage and seminar participants at the New York Fed, Oxford University, the European university Institute, the Bank of Korea and at the IEA World Congress for valuable comments.

1 Introduction

The integration of financial intermediaries with own balance sheets into macroeconomic models is an important scientific endeavor. It promises not only new insights into the role of banks for economic activity, but it might also help to identify suitable regulations of the banking system and policies to deal with fluctuations or default in this system that could spill over to the other parts of the economy. Since the first attempts,[1] the literature has advanced significantly and is briefly discussed below. In this paper we pursue a complementary route. We examine whether and how banks make a difference in standard macroeconomic models. In particular, the Solow and Ramsey (or Ramsey-Cass-Koopmans) models are starting points for the study of accumulation and growth processes in economics and inspired a large branch of dynamic macroeconomic models. In the sequel, we study how these models can be combined with banks and their role in the intermediation of funds.

We pursue this route for three reasons. First, the integration delivers simple and tractable coupled accumulation processes of bank capital and household wealth. Second, it allows to clarify whether such processes fundamentally change when a fraction of firms has no access to capital markets and thus rely on banks to finance their investments, and when all prices (commodity prices, wages and interest rates) flexibly adjust in the presence of shocks. Third, the simplicity of the ensuing process allows to study the impact of shocks on economic activity without the need to linearize the system around a steady state. The focus on smooth intertemporal savings/investment decisions and full recognition of non-linear effects can help to clarify whether persistence and amplification of temporary shocks is a robust phenomenon and it helps to identify new phenomena such as bust-boom or boom-bust cycles in such models.

For our purpose, we start with the simplest version of the Solow model and combine it with the simplest but quite general micro-founded form of banking. Specifically, we assume that a fraction of firms operating a neoclassical production technology have higher total factor productivity and depend on bank lending as banks alleviate the moral hazard problems of the entrepreneurs running the technology. Banks are run by bankers and their lending in each period is limited as bankers can only pledge part of their revenues from entrepreneurs to households. As a consequence, bankers face an endogenous leverage constraint and the amount of funds they can attract from households is proportional

[1] For a long time, banks have been neglected in macroeconomic theory. For instance, Bernanke, Gertler, and Gilchrist (1999), page 1376, noted "Nor do we consider research focusing on the role of banks in business cycles, primarily because there has been little work on the "bank lending channel" and related effects in an explicitly dynamic context. Interesting recent exceptions are Gersbach (1997) and Krishnamurthy (1997)."

to the amount of bank equity capital, which is the wealth of bankers. The particular form of the financial friction does not matter. Well-known formulations such as moral hazard of bankers, non-alienability of human capital or asset diversion can be mapped into the formal structure.

While the standard Solow model is fully described by the evolution of the aggregate accumulation rule, Solow cum banks is characterized by the evolution of two coupled accumulation rules: capital owned by households and bank equity capital owned by bankers. The analysis of these coupled accumulation rules yields the following insights: First, there exists a unique intertemporal equilibrium and a unique steady state with intuitive dependencies of parameters. Second, shocks to wealth, productivity or trust often lead to strongly asymmetric reactions of bankers' equity/consumption or household wealth/consumption on the one side and bond and loan financing on the other side. Third, negative temporary shocks to household wealth or negative sectoral productivity shocks can cause persistent booms – after an initial bust.

Of course, the simple Solow model and the ensuing two coupled accumulation rules can only be a start and benchmark for the investigation of micro-founded versions of capital accumulation and growth processes. We thus outline the research program in this area and we report the first results in such subsequent analysis performed in Gersbach, Rochet, and Scheffel (2014).

The present paper is motivated by the role of banks in dynamic macroeconomics and in particular how accumulation evolves and is affected by shocks to the economy in the presence of such banks. It is related to the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1996) who have shown that small, temporary shocks can have persistent effects on macroeconomic variables through their impact on net worth of levered agents in economies with financial frictions. Moreover, shocks can be amplified as leverage or prices are affected.[2] A more recent literature discussed in Gersbach, Rochet, and Scheffel (2014) has significantly broadened these insights.[3] Our work is complementary. We focus on the behavior of coupled accumulation processes that arise from the simple Solow framework, when agents take smooth consumption/investment decisions, output and factor prices, including the

[2]The quantitative magnitudes of such effects have been assessed in a variety of papers (see Carlstrom and Fuerst (1997)) including the interplay of monetary policy and financial frictions in a series of recent contributions (see Christiano, Eichenbaum, and Evans (2005), Christiano, Eichenbaum, and Evans (2007), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)).

[3]In the context of DSGE models, Meh and Moran (2010), Angeloni and Faia (2013) and Gertler and Kiyotaki (2010) have set benchmarks. Brunnermeier and Sannikov (2014) and Rampini and Viswanathan (2014) are well-known contributions for the coupled dynamics of net worth when agents are risk-neutral and for the potentially large effects of shocks. Martinez-Miera and Suarez (2013) focus on the build-up of systematic risk and the role of capital requirements to prevent it.

real interest rate, adjust to shocks and the non-linearities in the system can easily be tracked.

2 Solow Cum Banks

We integrate banks into the Solow capital accumulation model. The minimal desiderata for such an integration are: two production modes in which one type is associated with financial frictions and requires financial intermediation, constant returns to scale production using capital and labor, homogeneous labor and a constant savings rate[4]. The details of the model are set out in this section.

2.1 Macroeconomic Environment

Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. There is one physical good that can be produced by capital and labor. The good can be either consumed in the same period or invested as capital for production in future periods. The consumption/investment good is the numéraire in the economy and its price is normalized to 1. There are two types of agents: households and bankers. There is a continuum of households with mass L ($L > 0$). Labor is supplied inelastically. Each household is endowed with one unit of labor and ω_0 units of the capital good. Total endowment of households is Ω_0 ($\Omega_0 > 0$). There is a continuum of banks with unit mass. Each banker is endowed with e_0 of the good. Total endowment of bankers is E_0 ($E_0 > 0$).

2.2 Production

Production takes place in two different sectors: We label them as sector M and sector I. Both sectors consist of a continuum of identical firms. The production technologies exhibit constant returns to scale in the production factors capital and labor, have positive and diminishing returns and satisfy the Inada conditions. The assumption of constant returns to scale allows to represent each sector by an aggregate production technology. Specifically, we assume Cobb-Douglas technologies

$$Y_t^j = z^j \left(K_t^j\right)^\alpha \left(L_t^j\right)^{1-\alpha}, \quad j \in \{M, I\}$$

where z^j denotes total factor productivity and K_t^j and L_t^j denote capital and labor input in sector $j \in \{M, I\}$, respectively.

[4]Following the standard program in macroeconomics, in a Ramsey version of the model, the savings rate will be endogenized.

The sectors differ with respect to their access to financing. Firms in sector M have direct access to financial markets and obtain capital directly from the households. In contrast, firms in sector I have no access to the financial markets and obtain capital exclusively from financial intermediaries, the banking sector.[5] The following assumption ensures that bank lending is essential in the economy:

Assumption 1 (Productivity Difference)**.**
Total factor productivities satisfy $z^I > z^M$.

If $z^I \leq z^M$, banks are inessential in the sense that the allocation, and thus the evolution, of capital are identical to the standard version of the Solow model without banks. This is obvious for $z^I < z^M$ and can also be established for $z^M = z^I$ once we have characterized intra-temporal equilibria.

The representative firms choose capital and labor to maximize their profits, taking interest and wage rates as given. As usual in the Solow model this is equivalent to maximize the profit in a particular period. Specifically, the optimization problems in period t read

$$\max_{\{K_t^j, L_t^j\}} \left\{ z^j \left(K_t^j\right)^\alpha \left(L_t^j\right)^{1-\alpha} - r_t^j K_t^j - w_t^j L_t^j \right\}, \quad j \in \{M, I\} \tag{1}$$

where w_t^j is the prevailing wage in sector j in period t and r_t^j is the (gross) rental rate of capital.

The total amount of capital invested in both sectors is denoted by K_t:

$$K_t = K_t^M + K_t^I.$$

We assume that capital depreciates at rate δ ($0 < \delta < 1$). Since labor is homogeneous, wages will be the same in both sectors and denoted by w_t.[6]

2.3 Labor Markets, Capital Markets and Frictions

An individual household supplies his labor endowment inelastically, from which he supplies l_t^M and $l_t^I = 1 - l_t^M$ units of labor to sector M and I, respectively. Hence, his total labor income is $w_t \, l_t^M + w_t \, l_t^I = w_t$.

There are no financial frictions in sector M and an individual household directly lends k_t^M to firms in a competitive market. In contrast, firms in sector I cannot raise funds from

[5] Typically, sector I consists of newer or smaller firms that cannot pledge repayment to investors in the capital market because of moral hazard but may have high productivity.

[6] Strictly speaking, there is a unique wage at which all labor is employed. There could be circumstances, however, when labor is only employed in one sector.

households as they cannot pledge their output. In other words, frictions are so severe that they prevent direct financing. Each banker, however, manages a financial intermediary that can alleviate the moral hazard problem of the entrepreneurs. We assume that a bank evaluates and monitors entrepreneurs and enforces contractual obligations. For simplicity, the costs of these activities are neglected.[7]

Bankers themselves raise funds from households at the deposit interest rate r_t^D. A banker, however, cannot pledge the entire amount of repayments from entrepreneurs to households. More specifically, the non-pledgeable part is θk_t^I if the banker has granted a loan of size k_t^I to entrepreneurs in period t. The lending rate is denoted by r_t^I. Hence, a banker can only pledge $(1 + r_t^I - \theta)\ k_t^I$ to households.

The need for informed lending coupled with the lack of full pledgeability is the only financial friction in our model. The foundation of this friction can be traced back to moral hazard à la Holmstrom and Tirole (1997), asset diversion (like Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or non-alienability of human capital (like Hart and Moore (1994) and Diamond and Rajan (2000)).[8]

2.4 Households

We next describe households. As in the Solow model, households save a constant fraction of their income. The savings rate is denoted by s $(0 < s < 1)$.

As all households behave in the same way, we can focus on a representative household and obtain for their aggregate consumption and capital accumulation:

$$\begin{aligned} C_t^H &= (1-s)(r_t^M K_t^M + r_t^D D_t + w_t L) \\ \Omega_{t+1} &= s(r_t^M K_t^M + r_t^D D_t + w_t L) + (1-\delta)\Omega_t \end{aligned}$$

where D_t denotes the aggregate amount of deposits and Ω_{t+1} the aggregate amount of capital of households at the beginning of period $t+1$. As markets for deposits are competitive and households are at their participation constraint when they fund banks,

[7] In Gersbach, Rochet, and Scheffel (2014), we show how costs of intermediation can be integrated in Solow or Ramsey cum banks models.

[8] While all formulations of frictions lead to similar versions of the above formula in a single period, in Ramsey versions of the model with explicit inter-temporal optimization, a specific formulation of the friction has to be chosen. In addition, we assume that bankers that shirk in this period cannot be excluded from seeking new funds from households in the next period. This rules out that bankers can pledge revenues from future periods in order to attract more funds today.

r_t^D will be equal to r_t^M. Hence, the previous equations can be written as

$$C_t^H = (1-s)(r_t^M \Omega_t + w_t L) \tag{2}$$

$$\Omega_{t+1} = s(r_t^M \Omega_t + w_t L) + (1-\delta)\Omega_t \tag{3}$$

2.5 Bankers

Like households, bankers also save a constant fraction s of their income. At the beginning of period t, a typical banker owns e_t which he uses as equity funding for his bank. He attracts additional funds from households and lends k_t^I to entrepreneurs in sector I: hence, the bank contributes e_t and households provide $k_t^I - e_t$.[9]

As households can invest into sector M, a banker needs to be able to pledge (at least) $(1+r_t^M)(k_t^I - e_t)$ to households in order to attract $(k_t^I - e_t)$. However due to the financial friction, θk_t^I cannot be pledged. Hence, the banker faces the market imposed leverage constraint

$$(1 + r_t^M)(k_t^I - e_t) \leq k_t^I (1 + r_t^I - \theta). \tag{4}$$

When (4) is binding it can be rewritten as

$$k_t^I = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} e_t.$$

Bankers are price-takers. Hence, since a binding leverage constraint holds for all banks and it is linear in equity, the behavior of the banking system in such circumstances can be described by the behavior of a representative price-taking bank facing the aggregate leverage constraint

$$K_t^I = \frac{1 + r_t^M}{r_t^M - r_t^I + \theta} E_t = \lambda_t E_t \tag{5}$$

where λ_t is the leverage realized in period t. Intuitively, (5) is binding in a particular period if bank equity is sufficiently scarce relative to the wealth of households, also implying $r_t^I > r_t^M$. The formal condition on scarcity of bank equity will be established in Section 3. We note that with $r_t^I > r_t^M$, a banker is always better off attracting loanable funds and investing in sector I thereby earning θk_t^I than investing only e_t in sector I and earning $(1 + r_t^I)e_t$ or investing in sector M.[10]

[9] In principle, bankers could also invest their resources in sector M. However, this will never occur in equilibrium since returns on bank equity are above the return to capital in sector M.

[10] The leverage constraint implies $\theta k_t^I = (1 + r_t^I)e_t + (k_t^I - e_t)(r_t^I - r_t^M)$.

With a binding leverage constraint, aggregate consumption of bankers is given by

$$C_t^B = (1-s)(\theta\lambda_t - 1)E_t$$

and aggregate bank equity evolves according to[11]

$$E_{t+1} = s(\theta\lambda_t - 1)E_t + (1-\delta)E_t. \tag{6}$$

We note that $\theta K_t^I = (1+r_t^I)E_t + (K_t^I - E_t)(r_t^I - r_t^M)$ and thus adding up (6) and (3) implies

$$K_{t+1} = s(Lw_t + r_t^M K_t^M + r_t^I K_t^I) + (1-\delta)K_t$$

which conforms with the standard accumulation rule in the Solow model.

3 Intra-Temporal Competitive Equilibrium

In this section, we focus on the intra-temporal (period-wise) competitive equilibrium for any given initial endowments (Ω_t, E_t). It will turn out to be convenient to use (K_t, E_t) with $K_t = \Omega_t + E_t$ as the state variables for characterizing the competitive equilibrium in period t. As we focus on a particular period in this section, we omit the time index.

3.1 The Firm's Problem and Labor Market Equilibrium

We start with the problem of firms in both sectors.

The first-order conditions yield

$$r^j = \alpha z^j \left(\frac{K^j}{L^j}\right)^{\alpha-1}, \quad j \in \{M, I\} \tag{7}$$

$$w = (1-\alpha) z^j \left(\frac{K^j}{L^j}\right)^{\alpha}, \quad j \in \{M, I\} \tag{8}$$

where we have used the fact that competitive labor markets require that wages are equalized across sectors. It is useful to introduce $x := \frac{K^M}{L^M}$ as the capital-to-labor ratio

[11]The current formulation is in the spirit of the Solow model. We assume that households and bankers bear the depreciation of capital according to their initial capital share. Handling of depreciation, however, is subtle as it may depend on the underlying source of the financial friction. An alternative formulation assumes that capital is already depreciated when loans of bankers are repaid. Then, the leverage constraint amounts to $(1 + r_t^M - \delta)(K_t^I - E_t) = K_t^I(1 + r_t^I - \delta - \theta)$ and thus leverage amounts to $\lambda_t = \frac{1+r_t^M-\delta}{r_t^M - r_t^I + \theta}$.

in sector M. As a consequence of (8), the marginal product conditions in both sectors yield

$$x := \frac{K^M}{L^M} = z\frac{K^I}{L^I} \quad \text{and} \tag{9}$$

$$w = (1-\alpha)z^M x^\alpha \tag{10}$$

with $z \doteq \left(\frac{z^I}{z^M}\right)^{1/\alpha} > 1$. Hence, we obtain:

$$x = \frac{K^M + zK^I}{L} = \frac{K + (z-1)K^I}{L} \tag{11}$$

$$L^I = \frac{zK^I}{K^M + zK^I}L; \quad L^M = \frac{K^M}{K^M + zK^I}L \tag{12}$$

The interest rates can be expressed as:

$$r^M = \alpha z^M \left(\frac{K^M}{L^M}\right)^{\alpha-1} = \alpha z^M x^{\alpha-1} \tag{13}$$

$$r^I = \alpha z^M z^\alpha \left(\frac{K^I}{L^I}\right)^{\alpha-1} = \alpha z^M z x^{\alpha-1} = \frac{\alpha z^I x^{\alpha-1}}{z^{\alpha-1}} \tag{14}$$

We note that $r^I > r^M$. The output is given by

$$Y = z^M (K^M)^\alpha (L^M)^{1-\alpha} + z^M (zK^I)^\alpha (L^I)^{1-\alpha} \tag{15}$$

which can be simply written as

$$Y = Lz^M x^\alpha. \tag{16}$$

Moreover,

$$K^I = \frac{xL - K}{z-1} \in [0, K], \quad K^M = K - K^I. \tag{17}$$

This implies

$$\frac{K}{L} \leq x \leq z\frac{K}{L}. \tag{18}$$

This condition is easy to understand. Since $z > 1$ and $x = \frac{K^M}{L^M} = \frac{zK^I}{L^I}$, the capital-to-labor ratio is always larger in sector M than in sector I, and thus than in the whole economy: $x \geq \frac{K}{L}$. Conversely, $\frac{K^I}{L^I} = \frac{x}{z}$ is smaller than $\frac{K^M}{L^M} = \alpha$ and thus than $\frac{K}{L}$. Hence, the equilibrium capital-to-labor ratio lies between the two polar cases when all capital and labor are either used only in sector M or only in sector I. As $E_0 > 0$, it will turn out that $x = \frac{K}{L}$ cannot occur in equilibrium as a positive amount of capital will be employed in sector I.

Proposition 1.
In any period, the competitive allocation of labor, capital and consumption is completely determined by x, the capital-to-labor ratio in sector M and aggregate capital:

$$K^I = \frac{xL - K}{z - 1}; \qquad K^M = K - K^I \tag{19}$$

$$L^I = \frac{zK^I}{x}; \qquad L^M = \frac{K^M}{x}. \tag{20}$$

We note that both interest rates r^M, r^I and the spread $r^I - r^M$ decrease in x.

3.2 Capital Market Equilibrium

To derive the capital market equilibrium we distinguish two cases. We start with the simple case when frictions do not matter for aggregate capital investments.[12]

3.2.1 When Financial Frictions are Irrelevant

When frictions do not matter for aggregate capital investments, all capital and labor are employed in sector I as total factor productivity, expressed by factor z^I, is higher in this sector than in sector M. Hence, in such circumstances we have $K^M = L^M = 0$, which implies

$$x = z\frac{K}{L}, \quad Y = z^I(K^\alpha)L^{1-\alpha} \; = z^M(zK)^\alpha L^{1-\alpha}. \tag{21}$$

Although $K^M = L^M = 0$, we can determine the alternative investment opportunity of households by (13) which would be the interest rate if an arbitrarily small amount of capital K^M were invested in sector M and labor markets cleared. Hence, the wealth of bankers amount to

$$(1 + r^M)E + (r^I - r^M)K \tag{22}$$

[12] We continue to assume that banks are needed to monitor entrepreneurs in sector I. Accordingly, bankers will earn higher return on capital. An alternative scenario would be the complete absence of differences between bankers and households when they invest their capital.

and earning amounts to

$$r^M E + (r^I - r^M)K. \tag{23}$$

We note that the split between Ω and E of total capital K does not matter for aggregate capital investments in a particular period.[13] Still, banks are needed to channel funds to sector I and thus are able to obtain higher returns per unit of bank equity than households earn. Incentive compatibility requires that the wealth is at least as large as the non-pledgeable part θK which implies

$$E \geq \frac{\theta - (r^I - r^M)}{1 + r^M} K \equiv \bar{E}(K). \tag{24}$$

Using the equilibrium expressions for the interest rate factors (13) and (14) yields

$$\bar{E}(K) = \frac{K\left[\theta - (z-1)\alpha z^M \left(z\frac{K}{L}\right)^{\alpha-1}\right]}{1 + \alpha z^M \left(z\frac{K}{L}\right)^{\alpha-1}}. \tag{25}$$

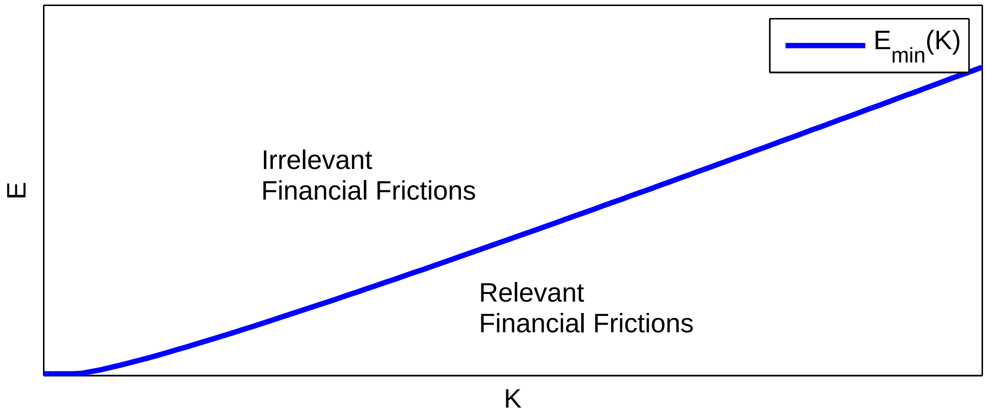
We note that $\bar{E}(K)$ is the lower bound of bank equity to make financial frictions irrelevant, given some overall capital K in the economy. From (25), one can deduce equivalently a maximal amount of capital owned by households such that financial frictions do not matter for a given level of bank equity E.

In Figure 1, we illustrate the function $\bar{E}(K)$. We note that $\bar{E}(K)$ increases more than θK for low values of K and that it approaches θK for higher values of K.Moreover, for given θ the function $\bar{E}(K)$ starts at some positive value of K. Below this critical value financial frictions are irrelevant for any value of equity. The reason is that for low values of K, interest rates and the spread $r^I - r^M$ are so large such that the leverage constraint is never binding. From (25) we observe that financial frictions can only matter if $\theta - (z-1)\alpha z^M x^{\alpha-1} > 0$, which yields

$$x > \tilde{x} = \left(\frac{(z-1)\alpha z^M}{\theta}\right)^{1/(1-\alpha)}. \tag{26}$$

[13]This also holds across time if bankers and households have the same savings rate.

Figure 1: $\bar{E}(K)$-Function



3.2.2 When Financial Frictions Matter

We next turn to the case when financial frictions matter and thus the leverage constraint is binding.

$$K^I = \frac{xL - K}{z-1} = \lambda(x)E \tag{27}$$

where

$$\lambda(x) := \frac{1 + \alpha z^M x^{\alpha-1}}{\theta - (z-1)\alpha z^M x^{\alpha-1}} \tag{28}$$

is the leverage of banks, expressed as a function of $x = \frac{K^M}{L^M}$. Hence, an equilibrium value of x is the solution of

$$F(x; K, E) := xL - K - (z-1)E\lambda(x) = 0. \tag{29}$$

We only consider circumstances for which $\tilde{x} < \frac{K}{L}$, i.e. aggregate capital is not too small. Otherwise, equilibria with a positive and binding leverage constraint are excluded.

We next determine the equilibrium value of x. Clearly, leverage $\lambda(x)$ is monotonically decreasing for $x \in (\tilde{x}, \infty)$. We observe that $F(x; K, E)$ as defined in (29) approaches minus infinity when x approaches $\tilde{x}$ from above, as $\lambda(x)$ approaches infinity. As $\lim_{x\to\infty} \lambda(x) = \frac{1}{\theta}$, $F(x; K, E)$ becomes very large when x approaches infinity. As $F(x; K, E)$ is continuous in x, the application of the intermediate value theorem in combination with strict monotonicity of the leverage delivers the existence and uniqueness of x^* that solves (29).

With x^* determined by (29), the remaining equilibrium quantities (K^I, K^M, L^M, L^I, Y) are given by equations (17), (12) and (16) and the equilibrium prices are given by (10), (13) and (14).

Finally, we determine the range of values for E such that financial frictions matter. We have

$$F\Big(z\frac{K}{L};K,E\Big) = (z-1)\Big[K - E\lambda\Big(z\frac{K}{L}\Big)\Big]. \tag{30}$$

If financial frictions matter, K^M and L^M are positive. Hence, $x < z\frac{K}{L}$. Since $F(x;K,E)$ is monotonically increasing in x, we have

$$F\Big(z\frac{K}{L};K,E\Big) > 0 \;\Leftrightarrow\; E < \frac{K}{\lambda\Big(z\frac{K}{L}\Big)} = \bar{E}(K). \tag{31}$$

3.2.3 Existence and Uniqueness

The preceding considerations are summarized in the following proposition.

Proposition 2 (Intra-Temporal Equilibrium)**.**
For all pairs (E, K) with $0 < E \leq K$, there exists a unique equilibrium, parameterized by $x \in \Big(\frac{K}{L}, z\frac{K}{L}\Big]$.

(i) If $E \geq \bar{E}(K)$, we obtain the corner solution $x = z\frac{K}{L}$ and financial frictions do not matter.

(ii) If $E < \bar{E}(K)$, x is the unique solution, denoted by $x(K,E)$ of

$$F(x;K,E) \equiv xL - K - (z-1)E\lambda(x) = 0. \tag{32}$$

Several remarks are in order. In any equilibrium with financial frictions, Ω strictly exceeds the amount of funds channeled from households to the banking sector as some capital is invested in sector M. We also note that the spread $r^I - r^M$, given by

$$r^I - r^M = \alpha z^M x^{\alpha-1}(z-1), \tag{33}$$

is positive and decreasing in x.

3.2.4 Comparative Statics and Equity Shocks

The equilibrium in the case when financial frictions matter allows straight-forward comparative statics exercises. From

$$F(x;K,E) = xL - K - E\frac{1+\alpha z^M x^{\alpha-1}}{\frac{\theta}{z-1} - \alpha z^M x^{\alpha-1}} \tag{34}$$

and the derivatives $F_x > 0$, $F_K < 0$, $F_E < 0$, $F_\theta > 0$, $F_{z^I} < 0$, $F_{z^M} < 0$, we obtain

Corollary 1.

(i) x increases in E, K, z^I, z^M and decreases in θ.

(ii) λ decrease in x.

The properties are intuitive. It is also useful to consider consequences when E increases and the overall amount of capital K in the economy increases by the same amount. Hence, we next investigate the equilibrium response to a shock in bank equity when Ω remains unaffected.

We first note that a positive equity shock (and thus a simultaneous increase of E and K) has two opposing effects on K^I. On the one hand, a higher amount of equity increases *ceteris paribus* the capital K^I that can be employed in the banking sector. On the other hand, as x increases (Corollary 1), leverage declines which lowers, *ceteris paribus*, capital used in sector I. However, for the calibrated version in Section 5, we obtain that an increase in bank equity (and total capital) raises K^I.

4 Dynamics and Steady States

In this section, we explore the dynamics of the model. It is useful to start with the case when bankers are not subject to non-pledgeability constraints.

4.1 When Financial Frictions do not Matter

We assume in this subsection that financial frictions do not bind. As we will see below this imposes a lower bound on the value θ. However, as in subsection 3.2.1, bankers are needed to channel funds from households to firms in sector I and thus bankers earn higher return on equity than households earn on capital. As there are no financial frictions, all capital will be employed in sector I. Thus in any period

$$x_t = z\frac{K_t}{L}. \tag{35}$$

The laws of motion for bank equity and total capital are given by

$$\begin{aligned} E_{t+1} &= s[r_t^M E_t + (r_t^I - r_t^M)K_t] + (1-\delta)E_t \\ &= s[\alpha z^M x_t^{\alpha-1} E_t + \alpha(z-1)z^M x_t^{\alpha-1} K_t] + (1-\delta)E_t \end{aligned} \tag{36}$$

$$\begin{aligned} K_{t+1} &= sLz^M x_t^\alpha + (1-\delta)K_t \\ &= sLz^I\Big(\frac{K_t}{L}\Big)^\alpha + (1-\delta)K_t. \end{aligned} \tag{37}$$

A steady state of the economy is any pair (E^*, K^*) such that $E_t = E^*$, $K_t = K^*$, $\forall t \geq 1$ if the economy starts with $(E_0 = E^*, K_0 = K^*)$. Equivalently, a steady state is a fixed point of the system (36) and (37).

We obtain:

Proposition 3.
Suppose $\theta = 0$. Then, there exists a unique, globally stable steady state (E^*, K^*), with $K^* = L\Big(\frac{sz^I}{\delta}\Big)^{\frac{1}{1-\alpha}}$.

Proof of Proposition 3.
We note that the second equation (37) of the laws of motion is independent of E_t. A steady state value K^* satisfies

$$K^* = sLz^I\Big(\frac{K^*}{L}\Big)^\alpha + (1-\delta)K^* \tag{38}$$

which implies uniquely

$$K^* = L\Big(\frac{sz^I}{\delta}\Big)^{\frac{1}{1-\alpha}}. \tag{39}$$

Using (39) in the first equation of the laws of motion yields a uniquely determined value of E^*.[14]

$$E^* = \frac{s\alpha(z-1)z^M(x^*)^{\alpha-1}K^*}{\delta - s\alpha z^M(x^*)^{\alpha-1}} \tag{40}$$

where $x^* = z\frac{K^*}{L} = z\big(\frac{sz^I}{\delta}\big)^{1/(1-\alpha)}$ and $z^M(x^*)^{\alpha-1} = z^M\frac{\delta}{sz^I}z^{\alpha-1} = \frac{\delta}{sz}$.

For the stability, it is sufficient to observe that K_t converges to K^* for any initial value

[14] The share of bank equity in total capital can become small or can even vanish in extreme cases.

of K_0, as the convergence of K_t (and thus x_t) implies the convergence of E_t. From the second law of motion we obtain directly that $K_{t+1} > K_t$ if $K_t < K^*$ and $K_{t+1} < K_t$ if $K_t > K^*$ which implies global stability.

□

We stress that the behavior and stability of the system regarding aggregate capital accumulation is the same as in the Solow model with only sector I. The dynamics of bank equity, however, are more complicated as earnings of bankers are determined by $r_t^M E_t$ and by the spread $r_t^I - r_t^M$ they earn on the entire invested capital. This can introduce more complex dynamic accumulation of bank equity. This will become crucial for aggregate capital accumulation in cases where financial frictions matter and will be detailed in the following subsection.

In order that (E^*, K^*) constitute a steady state without frictions, condition (25) has to hold, i.e. $E^* > \bar{E}(K^*)$ which, after using $z^M(x^*)^{\alpha-1} = \frac{\delta}{sz}$ and algebraic manipulations, translates into

$$\theta < \frac{(z-1)\alpha}{z-\alpha}\frac{s+\delta}{s}. \tag{41}$$

4.2 When Financial Frictions Matter

When θ is positive and the condition from subsection 3.2.2 for positive leverage is fulfilled, we can encounter two cases. When $E_t \geq \bar{E}(K_t)$, the evolution from period t to period $t+1$ is the same as in the preceding section since financial frictions do not matter. We thus focus in the following on the case where $E_t < \bar{E}(K_t)$.

The laws of motion for bank equity and total capital in this case are given by

$$E_{t+1} = s(\theta\lambda_t - 1)E_t + (1-\delta)E_t \tag{42}$$

$$K_{t+1} = sLz^M x_t^\alpha + (1-\delta)K_t. \tag{43}$$

Equivalently, one could use the laws of motion described in (3) and (42) as (42) is the sum of (3) and (42). We obtain:

Proposition 4.
The system described by (42) *and* (43) *has a unique steady state* $(\hat{E}, \hat{K})$ *characterized*

by

$$\lambda(\hat{x}) = \frac{s+\delta}{s\theta}; \quad \hat{x} = \left(\frac{\alpha z^M \Big(s\theta + (s+\delta)(z-1)\Big)}{\theta\delta}\right)^{\frac{1}{1-\alpha}} \tag{44}$$

$$\hat{K} = \frac{s z^M \hat{x}^\alpha L}{\delta}; \quad \hat{E} = \frac{(\hat{x}L - \hat{K})}{(z-1)} \frac{s\theta}{(s+\delta)}. \tag{45}$$

Proposition 4 is derived as follows: For $E_{t+1} = E_t$, the first equation in the laws of motion implies $\lambda(\hat{x}) = \frac{s+\delta}{s\theta}$. From (28) we obtain $\hat{x}$. $\hat{K}$ follows from the second equation (43) of the laws of motion. From Proposition 4 we can derive $\hat{K}^I$ using (17) and E from $\hat{E} = \frac{\hat{K}^I}{\lambda(\hat{x})}$. For a steady state in which financial frictions matter, $\hat{x} < z\frac{K}{L}$ and $\hat{E} < \bar{E}(\hat{K})$ have to hold.[15]

There are a variety of simple and more sophisticated relationships how changes of parameter values θ, δ, s, z and α affect the steady state values. For instance, an increase of θ lowers the steady state value of total capital, but not necessarily bank equity.

We note that leverage and $\hat{x}$ do not depend on the amount of labor in the economy while both $\hat{K}$ and $\hat{E}$ are linear in L. Hence, the share of capital intermediated by banks in the economy is independent of the size of the economy. Similarly, technological progress, that varies both z^M and z^I by the same factor, does not affect the split between bond and loan financing. Both, $\hat{K}$ and $\hat{E}$ are proportional to $(z^M)^{1/(1-\alpha)}$ and $z-1$ is not affected by an economy wide increase of total productivity.

The dynamics of the system and the implications for the role of bank equity in the economy in this section are non-trivial and will be at the center of the discussion in the next sections.[16] Before, we show that different savings rates of bankers and households can be dealt with easily.

4.3 Role of Savings Rates

It is also useful to stress that the existence and uniqueness of the steady state derived from the system (42) and (43) do not depend on the assumption of equal savings rates of bankers and households. Indeed suppose that there are two different savings rates, denoted by s_B and s_H, for bankers and households, respectively. The laws of motion in

[15]This can be verified.

[16]Uniqueness and stability of steady states in Ramsey versions of the model are dealt with in Gersbach, Rochet, and Scheffel (2014).

the case when frictions matter are modified accordingly:

$$E_{t+1} = s_B(\theta\lambda_t - 1)E_t + (1-\delta)E_t \tag{46}$$

$$K_{t+1} = s_B(\theta\lambda_t - 1)E_t + s_H\Big((K_t - E_t)r_t^M + Lw_t\Big) + (1-\delta)K_t \tag{47}$$

which yields

Proposition 5.
When the savings rates of bankers and households are s_B and s_H, respectively, the unique steady state of the system (46) and (47) $(\hat{E}, \hat{K})$ is characterized by[17]

$$\lambda(\hat{x}) = \frac{s_B+\delta}{s_B\theta}; \quad \hat{x} = \left(\frac{\alpha z^M\Big(s_B\theta + (s_B+\delta)(z-1)\Big)}{\theta\delta}\right)^{\frac{1}{1-\alpha}} \tag{48}$$

$$\hat{K} = \frac{\hat{x}L\Big[\delta s_B\theta - s_H s_B\theta\alpha z^M\hat{x}^{\alpha-1} + s_H\hat{x}^{\alpha-1}(1-\alpha)(z-1)z^M(s_B+\delta)\Big]}{\delta(z-1)(s_B+\delta) + \delta s_B\theta - s_H\alpha z^M\hat{x}^{\alpha-1}\big[(z-1)(s_B+\delta)+s_B\theta\big]} \tag{49}$$

$$\hat{E} = \frac{(\hat{x}L - \hat{K})}{(z-1)}\frac{s_B\theta}{(s_B+\delta)}. \tag{50}$$

Proposition 5 follows from an iterative solution procedure. The first law of motion yields $\lambda(\hat{x})$ and $\hat{x}$ follows from (28). The factor prices r_t^M and w_t follow from (13) and (10). Using $\hat{E} = \frac{\hat{K}^I}{\lambda(\hat{x})}$ and $\hat{K}^I = \frac{\hat{x}L-\hat{K}}{z-1}$ allows to transform (47) as follows:

$$\hat{K} = \delta\frac{\hat{x}L-\hat{K}}{(z-1)\lambda(\hat{x})} + s_H\left[\left(\hat{K} - \frac{\hat{x}L-\hat{K}}{(z-1)\lambda(\hat{x})}\right)\hat{r^M} + L\hat{w}\right] + (1-\delta)\hat{K}, \tag{51}$$

which yields $\hat{K}$. Then, $\hat{K}^I$ and $\hat{E}$ follow at once.

5 An Example

In this section we illustrate the behavior of the system outside the steady state in order to illustrate the typical pattern when the economy is hit by wealth, productivity or trust shocks. For this purpose, we use a calibrated version of the model. The calibration is based on annual US data from 1998 to 2014 and proceeds as follows. Without loss of generality, we normalize the labor force to unity, $L = 1$. We further normalize $z^M = 1$. The output elasticity of capital is set to $\alpha = 0.36$, which is in the range of values

[17] The steady state values have to fulfill the conditions that financial frictions matter which puts constraints on the admissible parameters of the model.

suggested in the literature.

We choose a saving rate of $s = 0.1761$ to align total saving with the average gross-saving-to-GNP ratio taken from the FRED NIPA accounts. Using the steady state condition for total capital, the capital to-output-ratio simplifies to $\overline{K/Y} = s/\delta$. Given the capital and output series from the Penn World Table, we obtain an average capital-to-output ratio of 3 such that $\delta = 0.0587$. The calibration target for bank leverage $\overline{\lambda} = 10.1091$ is taken from the aggregated Call Report Data provided by the FDIC. Rewriting the steady state condition for leverage yields $\theta = (s+\delta)/(s\overline{\lambda}) = 0.1319$. Finally, we calibrate z^I to match the bond-to-loan finance ratio $\overline{K^M/K^I} = 1.5000$ taken from De Fiore and Uhlig (2011). Specifically, given the target definition the aggregate resource constraint yields $K^I = K/(1 + \overline{K^M/K^I})$. Together with the intra-temporal equilibrium condition for investment in sector I, $K^I = (xL - K)/(z - 1)$, we get

$$x = \left(1 + \frac{z-1}{1 + \overline{K^M/K^I}}\right)\frac{K}{L} \quad \Leftrightarrow \quad z^M x^{\alpha-1} = \frac{1}{\left(1 + \frac{z-1}{1+\overline{K^M/K^I}}\right)\overline{K/Y}}$$

where the second equation uses the calibration target for the capital-to-output ratio. Furthermore, the leverage constraint can be rewritten as

$$z^M x^{\alpha-1} = \frac{\overline{\lambda}\theta - 1}{\alpha(1 + \overline{\lambda}(z-1))}.$$

Equalizing and solving for z finally leads to

$$z = \frac{(1 + \overline{K^M/K^I})\alpha(\lambda - 1) + \overline{K^M/K^I}(\lambda\theta - 1)\overline{K/Y}}{\alpha(1 + \overline{K^M/K^I})\lambda - (\lambda\theta - 1)\overline{K/Y}}.$$

Plugging in the values finally delivers $z = 1.1976$ and thus $z^I = 1.0671$. The calibrated parameter values and the calibration targets are summarized in Table 5.

We consider two further non-targeted equilibrium statistics to assess the calibration strategy: the bank-size-to-GDP ratio and the implied return on equity. $\hat{K}^I$ is equivalent the asset side of the bank balance sheet, such that the relative size of the banking sector to GDP amounts to $\hat{K}^I/\hat{Y} = (1 + \overline{K^M/K^I})^{-1}\overline{K/Y} = 1.2000$. Because we calibrate to the steady state we have to compare the model outcome with the pre-crisis average. Specifically, choosing 1998-2008 as reference period, we get a relative bank sector size of 0.7839. The overestimation can stem from various sources. For instance, our model abstracts from retained earnings as means of finance investments. Return on equity is $R^B = \theta\overline{\lambda} - \delta = 1.2747$, i.e. 27.47 percent, which is rather high. We stress that equity is inside equity in our model and that we neglect costs of intermediation. Both aspects,

Table 1: Parameters and Calibration Targets

Parameters						
α	z^M	z^I	s	δ	θ	L
0.3600	1.0000	1.0671	0.1761	0.0587	0.1319	1.0000
Calibration Targets						
λ	$\overline{K/Y}$	$\overline{K^M/K^I}$				
10.1081	3.0000	1.5000				

among other possible reasons, could explain why returns are so high. The steady state allocation and further non-targeted equilibrium statistics are provided in Table 5.

Table 2: Steady State Allocation and Non-Targeted Statistics

Steady State Allocation						
$\hat{E}$	$\hat{K}$	$\hat{K}^M$	$\hat{K}^I$	$\hat{L}^M$	$\hat{L}^I$	
0.2299	5.8088	3.4853	2.3235	0.5561	0.4439	
Non-Targeted Statistic						
$\hat{C}^H$	$\hat{C}^B$	$\hat{r}^M$	$\hat{r}^I$	$\hat{R}^B$	$\hat{w}$	$\hat{K}^I/\hat{Y}$
1.5322	0.0631	0.1112	0.1332	1.2746	1.2392	1.2000

5.1 Wealth Shocks

Wealth shocks are parameterized as follows. We either reduce equity or household wealth by 1 percent of total capital to make both shocks comparable in size. Specifically, the equity and household wealth shocks amount to 25.27 and 1.04 percent of equity and household wealth, and could be associated with a banking crisis and financial crisis, respectively. The equity shock size is large but not unreasonable for periods like the Great Recession in which inside and outside equity measured in book or market values have dropped substantially.

There are three main results from this exercise and illustrated in Figure 2: First, the economy exhibits substantially different dynamics, depending on whether the shock affects bank equity or household wealth. Negative bank equity shocks raise leverage and reduce loan finance K_t^I. In contrast, bond finance increases.[18] In the subsequent periods, bank equity recovers monotonically but slowly. Household wealth starts to decrease as the drop in wages dominates the increase in capital income, thereby pushing household saving below its steady state value. As bank equity recovers, interest rates and wages adjust and household wealth reverts towards the steady state value. Output dynamics are dominated by the initial drop in equity and the monotonic recovery.

Similarly to the bank equity shocks, household wealth shocks raise leverage. In contrast, however, loan finance increases and bond finance $K_t^M = K_t - K_t^I$ decreases. Household wealth is put on a monotonic recovery. Since $s(\theta\lambda^* - 1) - \delta = 0$, the increase in bank leverage induces an increase in bank equity $(E_{t+1} - E_t)/E_t = s(\theta\lambda_t - 1) - \delta > 0$ above its steady state value. Interestingly, the accumulation of bank equity above its steady state value can only be reversed in the long-run when at some point in time, leverage falls below its steady state. As a consequence, more capital will be employed in the more productive sector. Flexible labor markets induce a corresponding reallocation of labor. After some time, these reallocations overcompensate the drop in output caused by the initial decline of capital in the economy. In other words, the initial bust turns into a moderate and long-lasting boom.

Second, wealth shocks impact the economy for many periods. As observed before, however, the adjustments are very different for shocks to bank equity and household wealth. One of the reasons why the adjustment processes are slow is the assumption of constant savings rates.

Third, even if household wealth shocks are similar in size, equity shocks lead to much larger output losses. When the economy is hit by a negative household wealth shock, the ensuing early bust-boom cycle exhibits early output losses accompanied by late output gains, which can be of similar size. In contrast, a negative bank equity shock causes substantially lower investment and labor employed in the more productive sector which leads to large output losses. Accumulated relative output losses in relation to relative capital losses in the economy can easily be a three-digit multiple.[19]

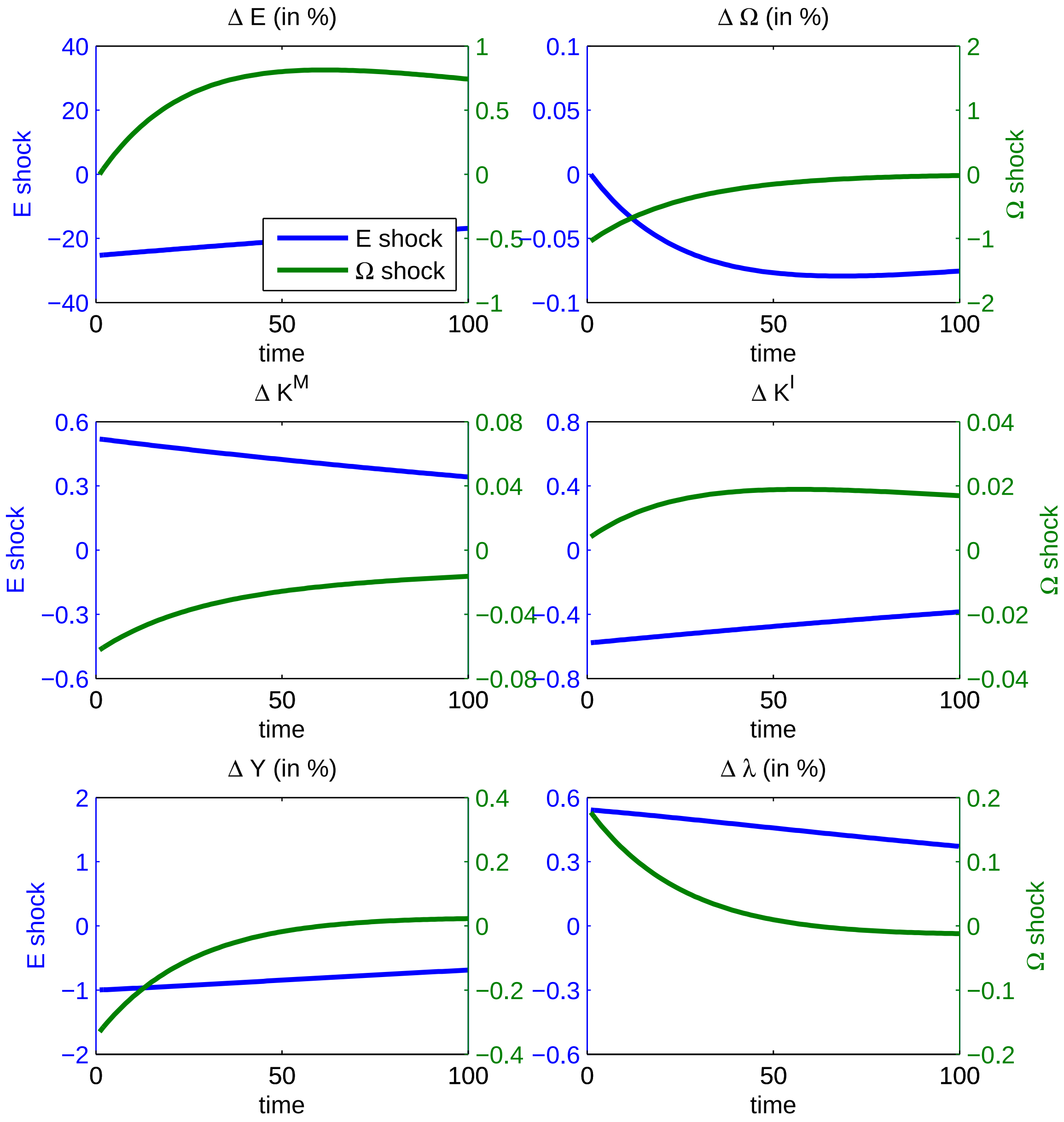
Equalizing the household wealth to the bank equity shock in absolute terms allowed us

[18] Formally, when leverage is rather insensitive to wealth shocks

$$\frac{\partial K^M}{\partial E} = \frac{\partial K}{\partial E} - \frac{\partial K^I}{\partial E} = 1 - \frac{\partial \lambda}{\partial x}\frac{\partial x}{\partial E}E - \lambda \approx 1 - \lambda < 0.$$

[19] Of course appropriate discounting in a Ramsey version will reduce the multiple, but the results in Gersbach, Rochet, and Scheffel (2014) suggest that the multiple remains large.

Figure 2: Response to Wealth Shock



to highlight the differences in the typical recovery patterns, in particular with respect to shock amplification and persistence. As household wealth in the real world is the value of all assets including real estate and investment in the stock market, substantially larger shocks to household wealth are plausible. While the pattern of the recovery remains unchanged, larger shocks lead to more substantial reversals. For instance, a 15 percent drop in household wealth ensues an overshooting of total output of up to 0.4 percent as the bust-boom cycle proceeds. Positive shocks to household wealth exhibit similar patterns, although the reversal is slightly dampened.

5.2 Productivity Shocks

Productivity shocks are parameterized as a two percent decrease in productivity for one period. Provided that x is sufficiently insensitive to changes in z^I, a negative productivity shock in sector I reduces the bank leverage.[20] In such circumstances, loan finance $K_t^I = \lambda_t E_t$ decreases and bond finance $K_t^M = K_t - K_t^I$ increases. Starting from the steady state, the drop in leverage leads to a reduction of bank equity $(E_{t+1} - E_t)/E_t = s(\theta\lambda_t - 1) - \delta < 0$. In addition, the productivity decrease also depresses the household's return on investment and wage rate such that household wealth decreases, as well. Moreover, the productivity loss directly translates into an immediate drop in output. With equity and household wealth below steady state, the further dynamics after the productivity returned to its pre-shock level, somewhat resembles the dynamics discussed for equity shocks.[21]

In contrast, a negative shock to the productivity in sector M increases bank leverage and induces a shift from bond to loan finance. With leverage increasing, bank equity increases whereas household wealth still decreases. As before, output decreases. When productivity returns to its pre-shock value, the economy exhibits a behaviour somewhat similar to the household wealth shock analyzed in the previous subsection.

Finally, if a productivity shock hits both sectors and leaves the relative productivity z unaffected, bank leverage increases. Compared to a similar shock that hits only sector M, the increase in bank leverage is only modest, whereas the reduction in output is more pronounced. The positive effect on next period bank equity is attenuated and, as productivities return to their pre-shock values, the recovery pattern is similar to the one observed when only sector M is hit by the shock. In contrast, however, deviations from trend are much lower, which is due to the milder reaction of bank leverage.

[20] The leverage response to a change in z^I is

$$\frac{\mathrm{d}\lambda}{\mathrm{d}z^I} = \frac{\partial\lambda}{\partial z}\frac{\partial z}{\partial z^I} + \frac{\partial\lambda}{\partial x}\frac{\partial x}{\partial z^I}.$$

The positive first term dominates the negative second term as long as x is sufficiently insensitive to changes in productivity.

[21] When productivity shocks are set at levels that produce the same initial output decline as for wealth shocks, the speed of adjustment to the steady state tends to be faster for productivity shocks.

5.3 Trust Shocks

"Trust shocks" are parameterized as a 30 percent increase in the financial friction θ over 3 periods. Provided that x is sufficiently insensitive to changes in θ, a positive trust shock reduces the bank leverage.[22] This in turn reduces loan finance $K_t^I = \lambda_t E_t$ and raises bond finance $K_t^M = K_t - K_t^I$. In such circumstances, equity declines. Furthermore, according to Corollary 1, the capital-to-labor ratio in sector M declines, driving down the wage rate that dominates the effect on household income. Hence, household wealth declines as well. Once trust is restored, the economic starts with equity and household wealth below the steady state levels and exhibits similar dynamics to the bank equity shock analyzed previously.

6 Discussion and Conclusion

In this paper, we performed a simple integration of banks into the standard Solow model that could serve as a tractable benchmark. Numerous issues await further examination. Even within the simple framework, a variety of properties shown in the example could be established formally and the stability analysis along the lines of Gersbach, Rochet, and Scheffel (2014) could be undertaken for the simple version of this paper. Moreover, a much richer set of numerical examples could be developed.

However, many issues related to welfare, regulation and policies to prevent and possibly stabilize negative shocks to household wealth, bank equity, productivity or trust require an explicit treatment of households and bankers as intertemporal utility maximizers. This is performed in Gersbach, Rochet, and Scheffel (2014). A variety of further interesting extensions could be pursued in Ramsey-type models. Allowing for outside equity issuance and possible bank defaults are obvious desiderata. Moreover, combining the model with explicit innovation activities of firms and the role of banks and capital markets in financing such investments would allow to combine banking simultaneously to crisis and growth and to policies to prevent the former and to foster the latter.

There are several ways to integrate banks in macroeconomic models. The approach pursued in this paper focuses on the interplay between accumulation of bank equity and household wealth and how this interplay is affected by shocks. We have mentioned

[22] The leverage response to a change in θ is

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\theta} = \frac{\partial\lambda}{\partial\theta} + \frac{\partial\lambda}{\partial x}\frac{\partial x}{\partial\theta}.$$

The negative first term dominates the positive second term as long as x is sufficiently insensitive to changes in the financial friction.

several fruitful directions how such models could be used to understand macroeconomic developments and to design policies. Ultimately, this route and the older and recent literature will help to close a serious gap in macroeconomics – a gap which should have been closed a long time ago.

References

Angeloni, I. and E. Faia (2013). Capital regulation and monetary policy with fragile banks. *Journal of Monetary Economics 60*(3), 3111–382.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *American Economic Review 79*(1), 14–31.

Bernanke, B., M. Gertler, and S. Gilchrist (1996). The financial accelerator and the flight to quality. *The Review of Economics and Statistics 78*(1), 1–15.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1, Chapter 21, pp. 1341–1393. Elsevier.

Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *American Economic Review*. (forthcoming).

Carlstrom, C. T. and T. S. Fuerst (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review 87*(5), 893–910.

Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Christiano, L., M. Eichenbaum, and C. Evans (2007). Shocks, structures or monetary policies? the euro area and us after 2001. *Journal of Economic Dyanmics and Control 32*(8), 2476–2506.

De Fiore, F. and H. Uhlig (2011, October). Bank Finance versus Bond Finance. *Journal of Money, Credit and Banking, Blackwell Publishing 43*(7), 1399–1421.

Diamond, D. W. and R. G. Rajan (2000). A theory of bank capital. *Journal of Finance 55*(6), 2431–2465.

Gersbach, H. (1997). Financial intermediation, capital spillovers and business fluctuations. Discussion Paper 273, University of Heidelberg.

Gersbach, H., J.-C. Rochet, and M. Scheffel (2014). Financial intermediation and capital accumulation. *mimeo*.

Gertler, M. and P. Karadi (2011). A model of unconventional monetary policy. *Journal of Monetary Economics 58*(1), 17–34.

Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3 of *Handbook of Monetary Economics*, Chapter 11, pp. 547–599. Elsevier.

Hart, O. and J. Moore (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics 109*(4), 841–79.

Holmstrom, B. and J. Tirole (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics 112*(3), 663–91.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–48.

Krishnamurthy, A. (1997). Collateral constraints and the credit channel. *unpublished*.

Martinez-Miera, D. and J. Suarez (2013). A macroeconomic model of endogenous systemic risk taking. *Working Paper*.

Meh, C. A. and K. Moran (2010). The role of bank capital in the propagation of shocks. *Journal of Economic Dynamics and Control 34*(3), 555–576.

Rampini, A. A. and S. Viswanathan (2014). Financial intermediary capital. *Working Paper*.