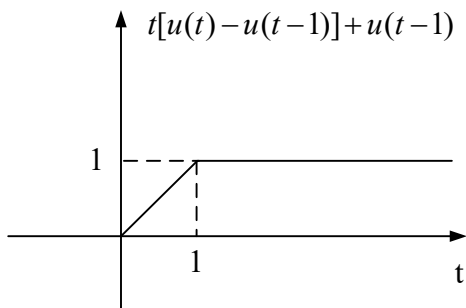


一、

$$1 \frac{\pi}{6} + \sin \frac{\pi}{6} = \frac{\pi}{6} + \frac{1}{2}$$

2



3

$$\begin{aligned} f_1(t) &= u(t) - u(t-1) & f_2(t) &= u(t) - u(t-2) \\ [u(t) - u(t-1)] * [u(t) - u(t-2)] \\ &= [tu(t) - (t-1)u(t-1)] * [\delta(t) - \delta(t-2)] \\ &= tu(t) - (t-1)u(t-1) - (t-2)u(t-2) + (t-3)u(t-3) \end{aligned}$$

4

$$\begin{aligned} \Rightarrow f(-t) &\leftrightarrow F(-\omega) \\ \Rightarrow -jtf(-t) &\leftrightarrow \frac{dF(-\omega)}{d\omega} \\ \Rightarrow -tf(-t) &\leftrightarrow -j \frac{dF(-\omega)}{d\omega} \\ \Rightarrow (1-t)f(1-t) &\leftrightarrow -e^{-j\omega} j \frac{dF(-\omega)}{d\omega} \end{aligned}$$

5 $F(0) = \int_{-\infty}^{+\infty} f(t)e^{-j0t} dt = \int_{-\infty}^{+\infty} f(t)dt$ ，即是 $f(t)$ 围成的面积，由图可得面积为

$$\frac{1}{2} * 4 * 2 = 4, \text{ 所以 } F(0) = 4$$

$$\int_{-\infty}^{+\infty} F(\omega)d\omega = \int_{-\infty}^{+\infty} F(\omega)e^{j\omega 0}d\omega = 2\pi f(0) = 2\pi$$

6

$$\begin{aligned} \Rightarrow f(t/a) &\leftrightarrow aF(sa) \\ \Rightarrow e^{-t/a} f(t/a) &\leftrightarrow aF(sa+1) \end{aligned}$$

7

$$\frac{1}{s^2 - 3s + 2} = \frac{A_1}{s-1} + \frac{A_2}{s-2}$$

$$A_1 = \frac{1}{s-2} \Big|_{s=1} = -1$$

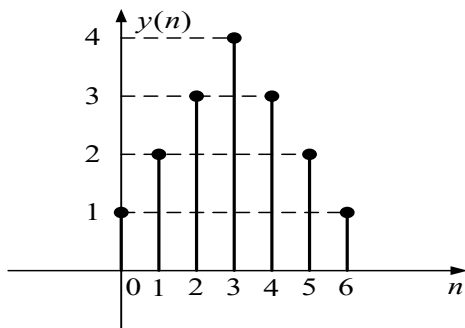
$$A_2 = \frac{1}{s-1} \Big|_{s=2} = 1$$

$$\Rightarrow F^{-1}\left(\frac{1}{s^2 - 3s + 2}\right) = (e^{2t} - e^t)u(t)$$

$$\Rightarrow F^{-1}\left(\frac{e^{-s}}{s^2 - 3s + 2}\right) = (e^{2(t-1)} - e^{t-1})u(t-1)$$

8

$$\begin{aligned} h(n) * x(n) &= [u(n) - u(n-4)] * [u(n) - u(n-4)] \\ &= [\delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)] * [\delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)] \\ &= \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) \\ &\quad + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4) \\ &\quad + \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5) \\ &\quad + \delta(n-3) + \delta(n-4) + \delta(n-5) + \delta(n-6) \\ &= \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3) + 3\delta(n-4) + 2\delta(n-5) + \delta(n-6) \end{aligned}$$



二

1

$$f(t)x(t) = (\cos 100t \cos 2000t) * \cos 2000t = \frac{1}{2} \cos 100t (\cos 4000t + 1)$$

$$= \frac{1}{4} \cos 3900t + \frac{1}{4} \cos 4100t + \frac{1}{2} \cos 100t$$

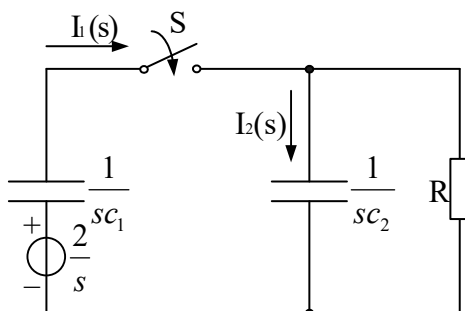
$$F\{f(t)x(t)\} = \frac{1}{4} [\pi\delta(\omega + 3900) + \pi\delta(\omega - 3900)] + \frac{1}{4} [\pi\delta(\omega + 4100) + \pi\delta(\omega - 4100)]$$

$$+ \frac{1}{2} [\pi\delta(\omega + 100) + \pi\delta(\omega - 100)]$$

$$Y(\omega) = F\{f(t)x(t)\}H(\omega) = \frac{1}{2}[\pi\delta(\omega + 100) + \pi\delta(\omega - 100)]$$

$$\therefore y(t) = F^{-1}(Y(\omega)) = \frac{1}{2}\cos 100t$$

2



设流经电容 c_2 上的电流为 $I_2(s)$ （方向如图所示），可列出 s 域的方程如下：

$$\frac{2}{s} = \frac{1}{sc_1} I_1(s) + \frac{1}{sc_2} I_2(s)$$

$$\frac{1}{sc_2} I_2(s) = R[I_1(s) - I_2(s)]$$

$$\Rightarrow I_2(s) = \frac{sc_2 R}{sc_2 R + 1} I_1(s)$$

$$\Rightarrow I_1(s) = \frac{1+4s}{1+6s} \cdot 2 = \frac{4}{3} + \frac{\frac{1}{9}}{s + \frac{1}{6}}$$

$$\Rightarrow i_1(t) = \frac{4}{3}\delta(t) + \frac{1}{9}e^{-\frac{t}{6}}u(t)$$

3

设系统的零输入响应为 $r_{zi}(t)$ ，激励为 $e(t)$ 时引起的零状态响应为 $r_{zs}(t)$ ，则利用系统的线性性质有：

$$r_{zi}(t) + r_{zs}(t) = r_1(t) = (e^{-t} + 2\cos \pi t)u(t)$$

$$r_{zi}(t) + 2r_{zs}(t) = r_2(t) = (3\cos \pi t)u(t)$$

$$\Rightarrow \begin{cases} r_{zs}(t) = (\cos \pi t - e^{-t})u(t) \\ r_{zi}(t) = (\cos \pi t + 2e^{-t})u(t) \end{cases}$$

$$\Rightarrow r_3(t) = r_{zi}(t) + 3_{zs}r(t-3) = (\cos \pi t + 2e^{-t})u(t) + 3\{\cos \pi(t-3) - e^{-(t-3)}\}u(t-3)$$

4

由 $\frac{d^2 r(t)}{dt^2} + 3\frac{dr(t)}{dt} + 2r(t) = \frac{de(t)}{dt} + 3e(t)$ ，知：

系统的特征方程 $\lambda^2 + 3\lambda + 2 = 0$ ， $\therefore \lambda_1 = -1, \lambda_2 = -2$

可设 $r_{zi}(t) = A_1 e^{-t} + A_2 e^{-2t}$ ，由 $r(0_-) = 1, r'(0_-) = 2$ ，得：

$$\begin{aligned} A_1 + A_2 &= 1 \\ -A_1 - 2A_2 &= 2 \end{aligned} \Rightarrow A_1 = 4, A_2 = -3$$

$$\therefore r_{zi}(t) = [4e^{-t} - 3e^{-2t}]u(t)$$

对 $\frac{d^2 r(t)}{dt^2} + 3\frac{dr(t)}{dt} + 2r(t) = \frac{de(t)}{dt} + 3e(t)$ 取零状态下的拉氏变换，得：

$$s^2 R_{zs}(s) + 3sR_{zs}(s) + 2R_{zs}(s) = (s+3)E(s)$$

$$= \frac{s+3}{s^2 + 3s + 2} E(s)$$

$$e(t) = u(t) \Rightarrow E(s) = \frac{1}{s}$$

$$\Rightarrow R_{zs}(s) = \frac{s+3}{s(s^2 + 3s + 2)} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2}$$

$$A_1 = \left. \frac{s+3}{(s+1)(s+2)} \right|_{s=0} = \frac{3}{2}$$

$$A_2 = \left. \frac{s+3}{s(s+2)} \right|_{s=-1} = -2$$

$$A_3 = \left. \frac{s+3}{s(s+1)} \right|_{s=-2} = \frac{1}{2}$$

$$\therefore R_{zs}(s) = \frac{3}{2s} + \frac{-2}{s+1} + \frac{\frac{1}{2}}{s+2}$$

$$\therefore r_{zs}(t) = \left[\frac{3}{2} - 2e^{-t} + \frac{1}{2}e^{-2t} \right] u(t)$$

$$\therefore r(t) = r_{zi}(t) + r_{zs}(t) = \left(\frac{3}{2} + 2e^{-t} - \frac{5}{2}e^{-2t} \right) u(t)$$

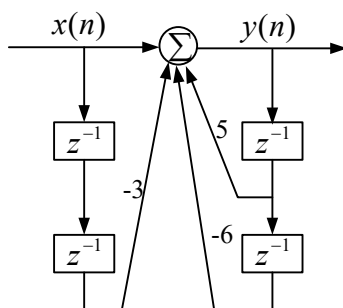
由 $H(s) = \frac{s+3}{s^2 + 3s + 2}$ 知：其极点为 $z_1 = -1, z_2 = -2$ ，均位于 s 平面的左半平面，因此系统是稳定的。

其中，自由响应由系统的极点决定，为 $(2e^{-t} - \frac{5}{2}e^{-2t})u(t)$ ，强迫响应由激励决定，

对应特解，为 $\frac{3}{2}u(t)$ 。

5

1 框图如下：



2 对 $y(n) - 5y(n-1) + 6y(n-2) = x(n) - 3x(n-2)$ 取零状态下的 Z 变换，得：

$$Y_{zs}(z) - 5z^{-1}Y_{zs}(z) + 6z^{-2}Y_{zs}(z) = X(z) - 3z^{-2}X(z)$$

$$\Rightarrow H(z) = \frac{z^2 - 3}{z^2 - 5z + 6}$$

$$\Rightarrow \frac{H(z)}{z} = \frac{z^2 - 3}{z(z^2 - 5z + 6)} = \frac{A_1}{z} + \frac{A_2}{z-2} + \frac{A_3}{z-3}$$

$$A_1 = \left. \frac{z^2 - 3}{(z-2)(z-3)} \right|_{z=0} = -\frac{1}{2}$$

$$A_2 = \left. \frac{z^2 - 3}{z(z-3)} \right|_{z=2} = -\frac{1}{2}$$

$$A_3 = \left. \frac{z^2 - 3}{z(z-2)} \right|_{z=3} = 2$$

$$\therefore H(z) = -\frac{1}{2} + \frac{-\frac{1}{2}z}{z-2} + \frac{2z}{z-3}$$

$$\therefore h(n) = -\frac{1}{2}\delta(n) + \left[-\frac{1}{2}(2)^n + 2 \cdot 3^n \right] u(n)$$