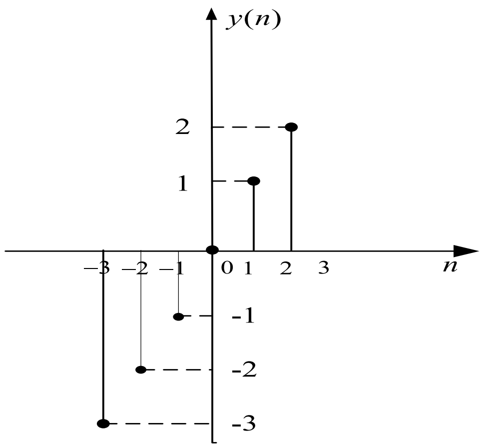
2007-2008 学年第二学期《信号与线性系统》试卷 A 答案

一.

1.

$$\int_{-\infty}^{\infty}\frac{\sin 2t}{t}\delta(t)dt$$
$$=2\int_{-\infty}^{\infty}\frac{\sin 2t}{2t}\delta(t)dt$$
$$=2\frac{\sin 2t}{2t}\Big|_{t=0}$$
$$=2$$

2.



3.

$f_1(t)=\varepsilon(t)-\varepsilon(t-2)$　　　　　　$f_2(t)=\varepsilon(t)-\varepsilon(t-3)$

$$[\varepsilon(t)-\varepsilon(t-2)]*[\varepsilon(t)-\varepsilon(t-3)]$$
$$=[t\varepsilon(t)-(t-2)\varepsilon(t-2)]*[\delta(t)-\delta(t-3)]$$
$$=t\varepsilon(t)-(t-2)\varepsilon(t-2)-(t-3)\varepsilon(t-3)+(t-5)\varepsilon(t-5)$$

4.

$$F(j\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$
$$=\int_{-\infty}^{\infty} e^{-\alpha t}\varepsilon(t)e^{-j\omega t}dt$$
$$=\int_{0}^{\infty} e^{-\alpha t}e^{-j\omega t}dt$$
$$=\frac{1}{\alpha+j\omega}$$

5.

$F(0)=\int_{-\infty}^{+\infty}f(t)e^{-j0t}dt=\int_{-\infty}^{+\infty}f(t)dt$，即是$f(t)$围成的面积，由图可得面积为

$\frac{1}{2}*4*2=4$，所以$F(0)=4$

$$\int_{-\infty}^{+\infty}F(\omega)d\omega=\int_{-\infty}^{+\infty}F(\omega)e^{j\omega 0}d\omega=2\pi f(0)=2\pi$$

6.

$\because f(t)\leftrightarrow F(s)$

根据拉氏变换的性质可知，

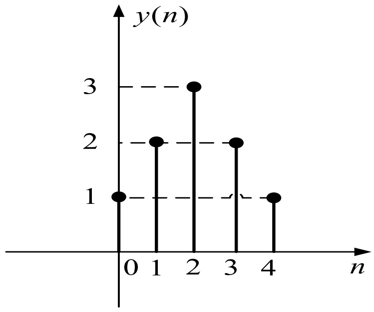
$\therefore e^{at}f(t)\leftrightarrow F(s-a)$

7.

$$F(s)=\frac{s}{s^2+5s+6}=\frac{A_1}{s+2}+\frac{A_2}{s+3}$$
$$A_1=\left.\frac{s}{s+3}\right|_{s=-2}=-2$$
$$A_2=\left.\frac{s}{s+2}\right|_{s=-3}=3$$
$$\Rightarrow F(s)=\frac{-2}{s+2}+\frac{3}{s+3}$$
$$\Rightarrow f(t)=(3e^{-3t}-2e^{-2t})u(t)$$

8.

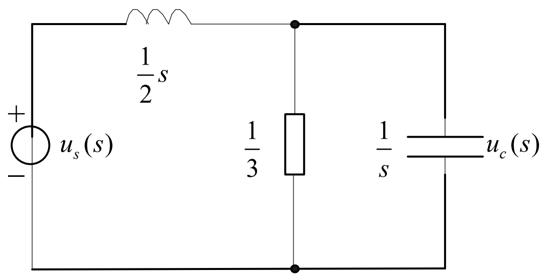
$$
\begin{aligned}
y(n) &= h(n)*x(n) = [u(n)-u(n-3)]*[u(n)-u(n-3)] \\
&= [\delta(n)+\delta(n-1)+\delta(n-2)]*[\delta(n)+\delta(n-1)+\delta(n-2)] \\
&= \delta(n)+\delta(n-1)+\delta(n-2) \\
&+\delta(n-1)+\delta(n-2)+\delta(n-3) \\
&+\delta(n-2)+\delta(n-3)+\delta(n-4) \\
&= \delta(n)+2\delta(n-1)+3\delta(n-2)+2\delta(n-3)+\delta(n-4)
\end{aligned}
$$



二.

1.

复频域等效电路如图：



可列出 s 域的方程如下：

$$\frac{u_s(s)}{\frac{1}{2}s+\frac{1}{3+s}}=\frac{u_c(s)}{\frac{1}{3+s}}$$

$$\Rightarrow H(s)=\frac{u_c(s)}{u_s(s)}=\frac{2}{s^2+3s+2}$$

所以，系统的冲激响应为 $h(t)=2(e^{-t}-e^{-2t})u(t)$

2.

设系统的初始状态 $x_1(0)$ 和 $x_2(0)$ 引起的零输入响应分别为 $r_{zi1}(t)$ 和 $r_{zi2}(t)$，激励为 $e(t)$ 时引起的零状态响应为 $r_{zs}(t)$，则利用系统的线性性质有：

$$5r_{zi1}(t)+2r_{zi2}(t)=e^{-t}(7t+5)u(t)$$
$$r_{zi1}(t)+4r_{zi2}(t)=e^{-t}(5t+1)u(t)$$
$$\Rightarrow\begin{cases}r_{zi1}(t)=(te^{-t}+e^{-t})u(t)\\ r_{zi2}(t)=te^{-t}u(t)\end{cases}$$
$$\Rightarrow r_{zs}(t)=-te^{-t}u(t)$$

所以，当 $e(t)=\begin{cases}2(t>0)\\0(t<0)\end{cases}$ 时系统的零状态响应为 $r(t)=-2te^{-t}u(t)$

3.

由 $\frac{d^2r(t)}{dt^2}+5\frac{dr(t)}{dt}+6r(t)=2\frac{de(t)}{dt}+e(t)$，知：

系统的特征方程 $\lambda^2+5\lambda+6=0$， $\therefore \lambda_1=-3,\lambda_2=-2$

可设 $r_{zi}(t)=A_1e^{-3t}+A_2e^{-2t}$，由 $r(0_-)=0, r'(0_-)=1$，得：

$$\begin{matrix}A_1+A_2=0\\-3A_1-2A_2=1\end{matrix}\Rightarrow A_1=-1,A_2=1$$

$$\therefore r_{zi}(t)=[-e^{-3t}+e^{-2t}]u(t)$$

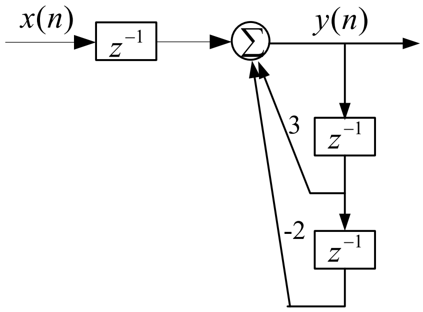
对 $\frac{d^2r(t)}{dt^2}+5\frac{dr(t)}{dt}+6r(t)=2\frac{de(t)}{dt}+e(t)$ 取零状态下的拉氏变换，得：

$$s^2R_{zs}(s)+5sR_{zs}(s)+6R_{zs}(s)=(2s+1)E(s)$$
$$H(s)=\frac{2s+1}{s^2+5s+6}$$
$$e(t)=u(t)\Rightarrow E(s)=\frac{1}{s}$$
$$\Rightarrow R_{zs}(s)=\frac{2s+1}{s(s^2+5s+6)}=\frac{A_1}{s}+\frac{A_2}{s+2}+\frac{A_3}{s+3}$$
$$A_1=\frac{1}{6}$$
$$A_2=\frac{3}{2}$$
$$A_3=-\frac{5}{3}$$
$$\therefore R_{zs}(s)=\frac{\frac{1}{6}}{s}+\frac{\frac{3}{2}}{s+2}+\frac{-\frac{5}{3}}{s+3}$$
$$\therefore r_{zs}(t)=[\frac{1}{6}-\frac{5}{3}e^{-3t}+\frac{3}{2}e^{-2t}]u(t)$$

$$\therefore r(t)=r_{zi}(t)+r_{zs}(t)=(\frac{1}{6}-\frac{8}{3}e^{-3t}+\frac{5}{2}e^{-2t})u(t)$$

4.

（1）框图如下：



（2）对 $y(n)-3y(n-1)+2y(n-2)=x(n-1)$ 取零状态下的 Z 变换，得：

$$Y_{zs}(z)-3z^{-1}Y_{zs}(z)+2z^{-2}Y_{zs}(z)=z^{-1}X(z)$$

$$\Rightarrow H(z)=\frac{Y_{zs}(z)}{X(z)}=\frac{z}{z^2-3z+2}$$

$$\Rightarrow \frac{H(z)}{z}=\frac{1}{z^2-3z+2}=\frac{A_1}{z-2}+\frac{A_2}{z-1}$$

$$A_1=1$$

$$A_2=-1$$

$$\therefore H(z)=\frac{z}{z-2}-\frac{z}{z-1}$$

$$\therefore h(n)=(2^n-1)u(n)$$